A TWO DIMENSIONAL PHOTOELASTIC STUDY
OF THE LOAD DISTRIBUTION AMONG
THE ROLLER BEARINGS IN A
ROCK BIT

BY

Arthur A. E. Bleimeyer, Jr.

A THESIS
SUBMITTED TO THE FACULTY
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

Houston, Texas
April, 1955
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgement</td>
<td>1</td>
</tr>
<tr>
<td>Abstract</td>
<td>2</td>
</tr>
<tr>
<td>Introduction</td>
<td>3</td>
</tr>
<tr>
<td>Experimental Procedure</td>
<td>12</td>
</tr>
<tr>
<td>Mathematical Analysis</td>
<td>13</td>
</tr>
<tr>
<td>Theoretical load distribution for roller bearings assuming negligible radial clearance</td>
<td>13</td>
</tr>
<tr>
<td>Effect of large radial clearance on theoretical load distribution</td>
<td>13</td>
</tr>
<tr>
<td>Quantitative Analysis of Photoelastic Experiments</td>
<td>24</td>
</tr>
<tr>
<td>Qualitative Analysis of Photoelastic Experiments</td>
<td>27</td>
</tr>
<tr>
<td>Analysis of Case I</td>
<td>27</td>
</tr>
<tr>
<td>Analysis of Case II</td>
<td>27</td>
</tr>
<tr>
<td>Analysis of Case III</td>
<td>27</td>
</tr>
<tr>
<td>Comparison of the three cases</td>
<td>33</td>
</tr>
<tr>
<td>Summary and Conclusions</td>
<td>33</td>
</tr>
<tr>
<td>Appendix</td>
<td>36</td>
</tr>
<tr>
<td>Selection of photoelastic material</td>
<td>36</td>
</tr>
<tr>
<td>Description of MR - 31C</td>
<td>36</td>
</tr>
<tr>
<td>Casting the sheet of plastic</td>
<td>36</td>
</tr>
<tr>
<td>Description of the model</td>
<td>36</td>
</tr>
<tr>
<td>Machining the model</td>
<td>36</td>
</tr>
<tr>
<td>Calibration of the loading frame</td>
<td>36</td>
</tr>
<tr>
<td>Calibration of the photoelastic material</td>
<td>36</td>
</tr>
<tr>
<td>Photographing the isochromatics</td>
<td>36</td>
</tr>
<tr>
<td>Sketching the isoclinics</td>
<td>36</td>
</tr>
<tr>
<td>Bibliography</td>
<td>48</td>
</tr>
</tbody>
</table>
List of Illustrations

Figure 1. Cutaway view of cross cutter rock bit showing bearing arrangement

Figure 2. Parts of the photoelastic model

Figure 3. Calibration curve for loading frame

Figure 4. Calibration curve for MR - 31C

Figure 5. Loading frame of the polariscope with tension specimen

Figure 6. General view of polariscope

Figure 7. Model mounted in loading frame

Figure 8. Isochromatics using plastic cutter and standard journal

Figure 9. Isochromatics using steel disk and standard journal

Figure 10. Isochromatics using steel disk and oversize journal

Figure 11. Cross-section of roller bearing with negligible radial clearance

Figure 12. Plot of $\frac{c \pi}{4(1 - \nu^2)} D$ vs. $\frac{W}{3 L E D}$

Figure 13. Plot of $\log_{10} B$ vs. $\log_{10} A$

Figure 14. Cross-section of roller bearing with relatively large radial clearance

Figure 15. Enlarged view of shaded triangle shown in Figure 14

Figure 16. Comparison of load distribution for roller bearing with no radial clearance and roller bearing with relatively large radial clearance
ACKNOWLEDGEMENT

The author wishes to express his sincere gratitude to the following: Dr. James C. Wilhoit of The Rice Institute for his guidance and assistance during the course of this investigation; Mr. Henry C. Tooley of the Reed Roller Bit Company for having made this work possible; Mr. Wiley B. Noble of the Reed Roller Bit Company for his cooperation and interest; Mr. J. C. Staples for his assistance with the illustrations; Messrs. W. M. Koch, H. W. Murdock, and G. L. Schliefer for their helpful suggestions.
ABSTRACT

The photoelastic technique was employed to investigate the adverse effect on the load distribution among the roller bearings in a rock bit cutter as a result of the relatively large radial clearance in the bearings. A mathematical solution was obtained for the load distribution among the bearings assuming negligible radial clearance, and parts of this solution were then applied to the case of relatively large radial clearance. The possibility of achieving a more equitable load distribution through the use of hollow roller bearings was also studied.
INTRODUCTION

The oil industry is constantly searching for improved methods of drilling holes in the earth's crust for the purpose of producing oil, gas, sulphur, or other minerals which may be mined in this manner. This search encompasses an extremely diversified field of study, inasmuch as each individual phase of the drilling operation is subject to revision and improvement. For example, a study of the behavior of various geological formations subjected to the hydrostatic pressures which are commonly encountered in drilling a well is necessary for a better understanding of the over-all problems which may be expected when drilling through such formations. On the other hand, attention may be directed primarily to the development of some drilling technique which may be more efficient than the presently used rotary drilling method. Another aspect of this search is concerned with improvements in the performance of the rock bit, with the consequent reduction of costs and increase in drilling efficiency. Such improvements are made in an attempt to achieve the following principal objectives: (1) increased footage drilled per bit, and (2) increased rates of penetration.

The life of a rock bit is terminated by failure either of teeth, or bearings, or both. It is the hope of the author that this investigation will result in a broader knowledge of the problem of load distribution in rock bit bearings; hence, we will be concerned only with such improvements in rock bit performance which pertain to the bearings in the bit. In order to have a better understanding of the difficulties encountered in attempting to improve bearing per-
formance, it would be well to discuss some of the major problems which face the rock bit designer in this regard.

The first commercial rock bit was manufactured in 1900. From that time until 1929, friction bearings were used rather than ball or roller bearings. This design was very unsatisfactory because of the short life of the bearing surfaces which, in turn, resulted in short life and poor performance of the entire bit. The use of roller bearings in rock bits began in 1929, followed in 1934 by the use of ball bearings in conjunction with the roller bearings. The design most commonly used at the present time employs the use of a row of roller bearings with a row of ball bearings adjacent to it. In many cases, a row of ball bearings with a row of roller bearings on either side of it is used. Such an arrangement is shown in Figure 1. The ball bearings primarily serve the purpose of carrying the thrust load while the roller bearings carry the radial load.

Perhaps the greatest single problem confronting the rock bit designer is that which concerns the environment in which the bit must operate. The problem of the size limitations which are imposed on the physical dimensions of the bit probably ranks next in importance and difficulty. The combination of these two major factors makes it quite likely that the bearings in a rock bit experience operating conditions which are more destructive than the operating conditions encountered by any other ball or roller bearings in common use today. Improvement in the performance of the bearings in a rock bit is made even more difficult by a consideration of the extremely large loads
CUTAWAY VIEW OF CROSS CUTTER ROCK BIT
SHOWING BEARING ARRANGEMENT

FIG. 1
which must be transmitted by them.

First, let us briefly consider the general problem of size limitations. It must be recognized that the over-all size of the rock bit is dictated by the size of the hole being drilled. Consequently, an increase in the size of the bearings may be achieved only by decreasing the size of the adjacent parts. Hence the designer must strive for a satisfactory balance between the size of the journal, bearings, and thickness of the cutter shell. Carefully controlled heat treatment of these parts, together with exacting metallurgical specifications, plays a very important role in making this problem less acute.

Next, a study of the extremely large loads applied to the bit indicates that no appreciable decrease in these loads is likely to be forthcoming. This statement is based on the fact that the load applied to the rock bit (and consequently to the bearings) is determined by a consideration of the compressive force required to fracture the particular geological formation being drilled. It is known that the compressive strength of a majority of geological formations is greatly increased by the application of a hydrostatic pressure to the surface of the formation. In the drilling operation, the column of drilling mud subjects the formation to such hydrostatic pressures, thereby making the rock more difficult to fracture than it would be at atmospheric pressure. The use of compressed air as a substitute for the commonly used water-base drilling muds greatly reduces this pressure on the formation face, and makes it possible in some instances to apply lighter loads to the bit; however, there is a general tendency
to maintain large loads on the bit in order to obtain increased rates of penetration. It should also be mentioned that in some cases, such as high pressure gas zones, air cannot be used, and then the more conventional types of drilling mud must be employed.

Turning our attention to the environment in which the rock bit operates, we find that the bearings and bearing races are subjected to the pitting action of small particles of sand and rock which are held in suspension in the drilling mud. It is fully recognized that it would be very desirable to prevent these abrasive particles from coming into contact with the bearings in the first place, but, up until the present time, all attempts to develop an effective seal for this purpose have resulted in failure. As a result, the rock bit designer must search for other methods of bringing about an improvement of environment.

One approach to the solution of this problem may be concerned with the drilling mud which is used. A noticeable improvement in bearing life has been observed when compressed air has been substituted for the drilling mud. This is partially explained by the fact that the cuttings are removed more rapidly, resulting in less recutting of the loosened particles. Also, due to the absence of moisture, these fine abrasive particles are not held in suspension to act as a lapping compound on bearings and bearing races. Improved bearing performance has also been noticed when an oil-emulsion type of drilling mud is used. A primary reason for this is that the oil in the mud actually serves as a lubricant for the bearings, thus removing the
possibility of the bearings operating with a complete lack of lubrication. Another fairly recent innovation which has aided greatly in overcoming the problem of lack of lubrication is the development of a grease which possesses good wetting and rust proof action, but does not wash out readily while the bit is being run. This grease is applied to the bearings and races during the process of assembling the rock bit.

Another important consideration is the fact that no satisfactory cage or retainer has yet been devised to maintain the desired spacing and alignment of the rock bit bearings. This is primarily due to the severe washing action of the drilling mud as it passes through the rock bit cutters at extremely high velocities. The presence of particles of rock in the cutter is also a factor here. The absence of a cage or retainer increases the possibility of skewing in the roller bearings, and also introduces additional bearing wear due to the rubbing of adjacent bearings in opposite directions at their point of contact.

As a result of some of the problems already mentioned, it has been found necessary to employ a relatively large radial clearance in rock bit bearings as compared with the radial clearance normally encountered in conventional ball or roller bearings. This means that the inner bearing race is eccentric with respect to the outer bearing race even when no load is applied to the bit. By comparison, this eccentricity may be considered to be negligible in the case of an unloaded conventional bearing with a relatively small radial clearance. It is believed that this initial eccentricity results in a load distribution among the bearings in a rock bit which is much less favorable
than the load distribution in a conventional bearing. Henry W. Murdock, in his Master's Thesis, submitted to the Mechanical Engineering Department of The Rice Institute in 1954 (Ref. #8)* made the observance that in a rock bit it is probable that only one or two bearings are loaded at any one time as compared with conventional ball or roller bearings in which approximately one-half of the bearings are loaded.

It is the purpose of this investigation to obtain a mathematical expression for the load distribution among roller bearings assuming negligible radial clearance, and then to determine analytically the effect of the previously-mentioned initial eccentricity on this load distribution. Henceforth all discussion of rock bit bearings will refer to the roller bearings only, with no consideration being given to the ball bearings present in the cutter. An attempt will be made to confirm the results of the analytical solution by use of the photoelastic method of experimental stress analysis. In addition to this, an experimental attempt will be made to gain a more favorable load distribution among the bearings by drilling a hole through the longitudinal axis of each bearing with the idea of inducing greater bearing deflections for a given load. The effect of such holes of various sizes will be compared photoelastically with the case where solid bearings are used. Also, a photoelastic comparison will be made for the case of small radial clearance versus the case of large radial clearance.

There is an abundance of empirical data in the literature dealing with all types of conventional ball or roller bearings (Ref. #5, 9, 11, 12). This data includes information concerning such prob-

*All references are numbered in the Bibliography.
lems as static capacity, rated life, coefficients of friction, recommended lubrication practices, load distribution, and deflections in bearings. In addition, there is also considerable mathematical analysis of problems dealing with surface compressive stresses, elastic deformations, and areas of contact for different combinations of elastic bodies in contact with one another (Ref. 3, 12, 13). It is felt that the empirical data does not apply to the roller bearings in a rock bit because of the vast differences in their design, manufacture, and use as compared with the conventional bearings for which the data is compiled. Certain additions and modifications must be made in the mathematical analysis in order for it to apply to the problem under consideration.

The works of Heinrich Hertz (Ref. 3) and Richard Stribeck (Ref. 12) will form the basis for the mathematical analysis of this particular problem. Hertz was probably the first person to succeed in obtaining a mathematical solution for the contact stresses and deformations produced when two elastic, isotropic, curved bodies are pressed against one another. Previous to Hertz, Winkler and Grashof had both attempted mathematical solutions of the same problem, but their results were either approximate or involved some unknown empirical factor. The work done by Hertz on this problem was published in Germany in 1881 in a paper entitled "On the Contact of Elastic Solids" (Ref. 3). In this paper, he made mention of the fact that a practical application for this particular investigation was the calculation of the stresses encountered in ball and roller bearings and in bridge rollers, and at the same time he pointed out that such calculations were impossible by the common formulas of mechanics. He also conducted a number of experiments in which
he confirmed the validity of his analysis. Later, Auerbach conducted a more extensive experimental investigation which also confirmed the theory formulated by Hertz.

In 1900, Richard Striibeck was called upon by the German Small Arms and Ammunition Factories of Berlin to make a thorough investigation of ball bearings in order to eliminate much of the guess-work from their design, manufacture, and use. In this investigation (Ref. #12) Striibeck attempted to correlate his experimental results with the Hertzian Theory. Although Hertz's equations are based on the assumption that the material is not stressed beyond the proportional limit, Striibeck found substantial agreement even when the proportional limit of the material was exceeded. In short, Striibeck broadened the scope of Hertz's work and applied it directly to problems involving ball bearings. His work was so widely accepted that for many years the ball bearing industry used it as their only basis for load rating. A more detailed discussion of his work will be found in the mathematical analysis of this particular investigation.

In more recent years, however, the equations of Arvid Palmgren (Ref. #9) have, to a large extent, replaced those of Striibeck as the standard for load rating in the ball and roller bearing industry. Striibeck's equations give only the static load rating which may be applied to any given bearing, while Palmgren's equations give the "dynamic specific capacity" of the bearing. This "dynamic specific capacity" was found to depend upon the following factors:

1. The diameter of the rolling elements
2. The number of rolling elements
3. Properties of the bearing materials

4. The curvatures of the bearing elements

Much of Palmgren's work employed the use of statistical analysis and resulted in empirical load rating equations which agree quite well with the actual results which are obtained. But the reader is reminded that these equations do not apply strictly to the bearings in a rock bit because of the vast differences which have already been discussed in some detail.
EXPERIMENTAL PROCEDURE

In the experimental analysis, a two-dimensional approximation is made for a truly three-dimensional problem, thus enabling us to consider the problem as being one of plane stress. This approximation is commonly made in work of this nature.

The plastic model used in these experiments represents a section through a rock bit cutter perpendicular to its center line. It was made from a sheet of MR-31C which was cast between pieces of plate glass (Figure 2). The loading frame of the polariscope was calibrated (Figure 3), and a tensile specimen of the photoelastic material was tested in order to determine the photoelastic constant (Figures 4 and 5). The model was then assembled and mounted in the loading frame (Figure 7), and the isochromatics (lines of constant difference in principal stresses) were photographed (Figures 8, 9, and 10). A load of 85 pounds was applied to the model for all of these photographs. A number of attempts were made to obtain an accurate sketch of the isoclinics (lines of constant inclination of the principal stresses), but all such attempts resulted in failure. Time did not permit additional work to be done in this regard. Consequently, no quantitative results were obtained from the photoelastic experiments.

A more detailed description of the experimental procedure is given in the Appendix.
LOADING FRAME OF THE POLARISCOPE WITH TENSION SPECIMEN
GENERAL VIEW
OF POLARISCOPE

FIG. 6
A. USING PLASTIC CUTTER SHELL AND PLASTIC JOURNAL

B. USING STEEL DISK AND ALUMINUM JOURNAL

MODEL MOUNTED IN LOADING FRAME

FIG. 7
FIGURE 8

ISOCHROMATICS USING PLASTIC CUTTER AND STANDARD JOURNAL

A. SOLID WHEEL

B. WHEEL WITH 3/8" HOLES

C. WHEEL WITH 5/16" HOLES
ISOCHROMATICS USING STEEL DISK AND STANDARD JOURNAL

FIGURE 9

4. Grooves with 5/64 inch
5. Grooves with 3/32 inch
6. Grooves with 1/8 inch
7. No grooves
ISOCHROMATICS USING STEEL DISK
AND OVERSIZE JOURNAL

FIGURE 10
MATHEMATICAL ANALYSIS

Theoretical Load Distribution for Roller Bearings Assuming Negligible Radial Clearance

The works of Hertz and Striebeck form the basis for the mathematical analysis of this problem. In this regard, it should be mentioned that Striebeck made a complete mathematical analysis for the case of a ball bearing in which the internal radial clearance was assumed to be negligible, and the inner and outer bearing rings were assumed to remain true circles when loaded. Using these basic assumptions, and referring to the Hertzian Theory for elastic spheres in contact, Striebeck derived the following expression for the external radial load on the bearing \( W_R \) as a function of the load on the most heavily loaded ball \( W_A \) and the position of the balls in the race, which is determined by the angle \( \beta \):

Eqn. (1) \[ W_R = W_A (1 + 2 \cos(\frac{\beta}{2}) + 2 \cos(2\frac{\beta}{2}) + \ldots) \]

Striebeck began the derivation of this equation by considering the equation of static equilibrium for the given problem:

Eqn. (2) \[ W_R = W_A + 2W_B \cos \beta + 2W_C \cos 2\beta + \ldots \]

where the angle \( \beta \) is defined by the following equation:

Eqn. (3) \[ \beta = \frac{360^\circ}{Z} \]

where \( Z = \) total number of balls in the bearing.

Striebeck reasoned that the application of the external load \( W_R \) to the bearing will cause the inner bearing race to move toward the outer race, in the direction of the applied load, a distance equal to \( \delta_A \) with a resulting radial deformation of balls B and C which may be
FIGURE 11. CROSS-SECTION OF ROLLER BEARING WITH NEGligible RADIAL CLEARANCE
expressed in the following form:

\[ \text{Eqn. (4)} \quad \sigma_B = \sigma_A \cos \beta \quad \text{Or:} \quad \frac{\sigma_B}{\sigma_A} = \cos \beta \]

\[ \text{Eqn. (5)} \quad \sigma_C = \sigma_A \cos 2 \beta \quad \text{Or:} \quad \frac{\sigma_C}{\sigma_A} = \cos 2 \beta \]

Referring to the work of Hertz, Stribeck noted that the normal deflection between any two spherical surfaces varies as \( W^{2/3} \). It is of interest to note that this relationship is valid for the following arrangements of elastic spheres in contact under a compressive load:

1. Sphere on flat
2. Convex spheres
3. Concave and convex spheres

Taking advantage of this fact, it is possible to write the following expression for deflections as a function of loads:

\[ \text{Eqn. (6)} \quad \frac{W_B}{W_A}^{2/3} = \frac{\sigma_B}{\sigma_A} \]

\[ \text{Eqn. (6a)} \quad \frac{W_B}{W_A} = \frac{\sigma_B^{3/2}}{\sigma_A^{3/2}} \]

By substitution of Equation (4) in Equation (6):

\[ \text{Eqn. (7)} \quad \frac{W_B}{W_A} = \cos \beta^{3/2} \quad \text{Or:} \quad W_B = W_A \cos \beta^{3/2} \]

Similarly:

\[ \text{Eqn. (8)} \quad \frac{W_C}{W_A} = \cos 2 \beta^{3/2} \quad \text{Or:} \quad W_C = W_A \cos 2 \beta^{3/2} \]

Equation (1) may now be obtained by substituting Equations (7) and (8) into Equation (2). With this information, we are able to calculate the magnitude of the load being carried by each loaded ball merely by knowing the magnitude of the external radial load on the bearing (\( W_R \))
and the total number of balls in the bearing \((Z)\), provided the bearing satisfies the restrictions imposed by Stribeck.

However this analysis applies only to ball bearings, whereas we are especially concerned with roller bearings in the present problem. Although an extensive search of the literature was conducted, no similar analysis for the case of roller bearings was found. However, the same approach to the problem may be employed, provided the same basic assumptions are made. The one notable difference in the two problems is found in the expressions for the normal deflections of the balls (or rollers) as a function of the load acting on them.

For example, the normal deflection for the case of a sphere of diameter \(D\) being compressed against a flat surface by a load \(W\) was determined by Hertz to be:

\[
S = 1.550 \sqrt[3]{\frac{W}{DE^2}}
\]

As we have already seen, Stribeck was able to employ this relationship directly in his analysis for ball bearings. But in the case of a cylinder of diameter \(D\) and length \(L\) being compressed against a flat surface by a load \(W\), the corresponding expression for the normal deflection is:

\[
S = \frac{h (1 - \nu^2)}{\frac{W}{E}} \frac{W}{L} \left( \frac{1}{3} + \log_e \frac{D}{b} \right)
\]

where \(b\) is the semi-width of the rectangle of contact:

\[
b = \sqrt[3]{\frac{WDn}{L}}
\]  

(Ref. #6, pg. 16)

where: \(n = \frac{h (1 - \nu^2)}{E} = 3.64h \frac{W}{E}\)  

When: \(\nu = \) Poisson's Ratio = 0.3  
\(E = \) Modulus of Elasticity

Therefore:

\[
b = \sqrt[3]{\frac{3.64WDE}{WLE}} = 1.075 \sqrt[3]{\frac{WD}{L}}
\]
Substituting Equation (11 a) into Equation (10):

\[ \text{Eqn. (12)} \quad \sigma = \frac{4}{\pi} \left( 1 - \frac{V^2}{E} \right) \frac{W}{L} \left( \frac{1}{3} + \log_e \frac{1}{1.075} \right) \sqrt{\frac{LED}{W}} \]

Although we recognize that a roller bearing does not represent the case of a cylinder compressed against a flat surface, we feel justified in using this equation inasmuch as the same relationship between load and deflection is valid for convex cylinders and for concave and convex cylinders in contact. Also, the algebraic operations are greatly simplified in this way without affecting the end result. This is completely analogous to the Stribeck analysis for ball bearings. However, in this case we do not have the relatively simple relationship between normal deflection and applied load which was encountered in the case of spheres in contact, but some such relationship must be determined if we are to be successful in deriving an equation for roller bearings which will correspond to Equation (1) for ball bearings.

In order to obtain this desired relationship, Equation (12) may be written in the following dimensionless form:

\[ \text{Eqn. (13)} \quad \frac{\sigma}{4 / (1 - V^2)} D = \frac{W}{3 \cdot LED} \left( 1 + 3 \log_e \frac{1}{1.075} \right) \sqrt{\frac{LED}{W}} \]

Now, \( \frac{\sigma}{4 / (1 - V^2)} D \) may be plotted as a function of \( \frac{W}{3 \cdot LED} \) for the range of values of \( \frac{W}{3 \cdot LED} \) from 0 to 0.0006. This range was chosen because the following values apply for this particular experimental work:

\[ W = 85 \text{ lbs.} \quad E = 30 \times 10^4 \text{ lbs./in.}^2 \]

\[ L = 0.235 \text{ inch} \quad D = 1 \text{ inch} \]

resulting in a value for \( \frac{W}{3 \cdot LED} \) of 0.0004. This plot is shown in Figure 12.
Letting \[ \frac{\sigma \pi}{4 (1 - \nu^2)} = A \] and \[ \frac{W}{3 LE D} = B, \] we may write:

Eqn. (14) \[ A = k B^n \]

Also:

Eqn. (15) \[ \log_e A = \log_e (k B^n) = \log_e k + n \log_e B \]

For values of A and B obtained in the preceding calculation, we may now plot \( \log_e A \) versus \( \log_e B \), thereby obtaining values for \( n \) and \( k \). This plot is shown in Figure 13. The following equation was thus obtained:

Eqn. (16) \[ A = 3.29 B^{0.85} \]

Or:

Eqn. (16 a) \[ \frac{\sigma \pi}{4 (1 - \nu^2)} D = 3.29 \left( \frac{W}{3 LE D} \right)^{0.85} \]

As a check, a plot of this equation was superimposed on the plot of \( \frac{\sigma \pi}{4 (1 - \nu^2)} D \) versus \( \frac{W}{3 LE D} \) in Figure 12. The two curves are in very close agreement.

We may now write Equation (16 a) in the following form:

Eqn. (17) \[ \sigma = K D \left( \frac{W}{3 LE D} \right)^{0.85} \]

where: \[ K = \frac{(3.29)(4)(1 - \nu^2)}{\pi} = 3.82 \]

Therefore:

Eqn. (17 a) \[ \sigma = 3.82 D \left( \frac{W}{3 LE D} \right)^{0.85} \]

This gives us a relationship for compressed cylinders which corresponds to the relationship for the normal deflection of compressed spheres. In the case of the spheres, the normal deflection was seen to vary as \( W^{2.5} \), whereas in the case of the cylinders, this deflection is seen to vary as \( W^{0.85} \). We may now employ this relationship in an analysis similar to Stribeck's.
Analogous to Equations (7) and (8), we may now write:

Eqn. (19) \( W_B = w_A \cos \beta \cdot 1.18 \)

And:

Eqn. (20) \( W_C = w_A \cos 2\beta \cdot 1.18 \)

Substituting Equations (19) and (20) into Equation (2), we obtain:

Eqn. (21) \( W_R = w_A (1 + 2 \cos \beta \cdot 1.18 + 2 \cos 2\beta \cdot 1.18 + \ldots) \)

Solving for \( w_A \):

Eqn. (22) \( w_A = \frac{W_R}{1 + 2 \cos \beta \cdot 1.18 + 2 \cos 2\beta \cdot 1.18 + \ldots} \)

For this particular experiment:

\( W_R = 85 \text{ lbs.} \)

\( \beta = \frac{360^\circ}{Z} = \frac{360^\circ}{13} = 27.7^\circ \)

By substitution in Equation (22), we obtain: \( w_A = 27.1 \text{ lbs.} \)

From Equation (19): \( W_B = 23.5 \text{ lbs.} \)

And from Equation (20): \( W_C = 13.9 \text{ lbs.} \)

Similarly: \( W_D = 2.22 \text{ lbs.} \)

This is the theoretical load distribution for the case shown in Figure 11. In this case the journal was larger than the journal used in the other two cases so that the requirement of no radial clearance would be met. Also, the steel disk was substituted for the plastic cutter shell in order to be assured that the outer bearing ring would
remain approximately a true circle when loaded. It was felt that the inner bearing ring (or journal) would remain a true circle when loaded because of its relatively great thickness and diameter as compared with the bearings.

**Effect of Large Radial Clearance on Theoretical Load Distribution**

The analytical solution for determining the effect of a relatively large radial clearance on the load distribution among the bearings is basically a problem in geometry. In Figure 14, it is apparent that only one roller will be in contact with the inner race of an unloaded bearing with a relatively large radial clearance. Upon application of an external load on the bearing, this roller, designated by the letter "A" in the drawing, will be the only roller supporting the load until it deflects a sufficient amount to allow rollers B to contact the inner race. Then three rollers will support the load, with roller A continuing to be the most heavily loaded of the three. In the same manner it would be possible for rollers C to begin to carry some of the load, provided the external load becomes great enough to cause rollers B and A to deflect a sufficient amount. If the radial clearance is sufficiently large, it is reasonable to suppose that the yield point of roller A will be exceeded before roller C ever comes into contact with the inner race. In this instance, three rollers would be the maximum number that could support any load without plastic deformation occurring in the most heavily loaded roller.

First, we must derive an expression for the normal deflection which roller A must experience before rollers B will come into contact with the inner race. We designate this deflection as \( \delta_A \), where:
FIGURE 14. CROSS-SECTION OF ROLLER BEARING WITH
RELATIVELY LARGE RADIAL CLEARANCE
FIGURE 15. ENLARGED VIEW OF SHARED TRIANGLE SHOWN IN FIGURE 14
Eqn. (23) \[ \sigma_A = S - R_B \] (See Figures 14 and 15)

But in Equation (23), \( \sigma_A \) and \( S \) are both unknowns, so we consider triangle XYZ in Figure 15. In order to obtain an expression for \( S \) in terms of known quantities. Applying the Law of Cosines:

Eqn. (24) \[ S^2 = (R_A - D)^2 + \Delta^2 - 2(R_A - D)\Delta \cos \beta \]

Or:

Eqn. (25) \[ S = \sqrt{(R_A - D)^2 + \Delta^2 - 2(R_A - D)\Delta \cos \beta} \]

Because the term \((R_A - D)\) is quite large relative to \( \Delta \), we note:

Eqn. (26) \[ \frac{\Delta^2}{(R_A - D)^2} \ll \frac{2\Delta}{R_A - D} \cos \beta \]

Consequently, we may expand the radical by the Binomial Theorem. Retaining only the first two terms of this series, we obtain:

Eqn. (27) \[ S = (R_A - D) (1 + \frac{1}{2} \frac{\Delta^2}{(R_A - D)^2} - \frac{\Delta}{R_A - D} \cos \beta) \]

Now that \( S \) is expressed in terms of known quantities, we may substitute it into Equation (23) and obtain:

Eqn. (28) \[ \sigma_A = (R_A - D)(1) + (R_A - D) \left[ \frac{1}{2} \frac{\Delta^2}{(R_A - D)^2} - \frac{\Delta}{R_A - D} \cos \beta \right] - R_B \]

But:

Eqn. (29) \[ \Delta = R_A - R_B - D \]

Substituting Equation (29) into Equation (28) and simplifying:

Eqn. (30) \[ \sigma_A = \Delta + \frac{1}{2} \frac{\Delta^2}{R_A - D} - \Delta \cos \beta \]

Or:

Eqn. (31) \[ \sigma_A = \Delta(1 - \cos \beta) + \frac{1}{2} \frac{\Delta^2}{R_A - D} \]
Once more taking advantage of the fact that $R - D$ is quite large relative to $\Delta$, we may write:

Eqn. (32) $\Delta_A \approx \Delta (1 - \cos \beta)$

This same analysis may be made for rollers C with the following result:

Eqn. (33) $\Delta_A'^{*} \approx \Delta (1 - \cos 2 \beta)$

In which $\Delta_A'^{*}$ represents the total normal deflection which roller A must experience in order for rollers C to come into contact with the inner race. But in order for roller A to deflect this amount, rollers B must experience a radial deflection which may be expressed:

Eqn. (34) $\sigma_B = \Delta_A'^{*} \cos \beta$

Where $\Delta_A'^{*}$ represents only that deflection which roller A experiences after rollers B have come into contact with the inner race. Hence:

Eqn. (35) $\Delta_A'^{*} = \Delta (1 - \cos 2 \beta) - \Delta (1 - \cos \beta)$

$= \Delta (\cos \beta - \cos 2 \beta)$

By substituting Equation (35) into Equation (34):

Eqn. (36) $\sigma_B = \Delta \cos \beta (\cos \beta - \cos 2 \beta)$

We are now able to determine $\sigma_A$ by substituting the known values for $\Delta$ and $\beta$ into Equation (32). For this particular problem, the following values apply:

$\Delta = 1/32$ inch \hspace{1cm} $\beta = 27.7^\circ$

Therefore:

Eqn. (37) $\sigma_A = 1/32 (1 - 0.885) = 0.0036$ inch

By applying Equation (17 a) from our previous analysis, we may determine the magnitude of the load required to cause roller A to deflect this
amount. Solving Equation (17 a) for \( W \):

\[
W = \frac{3 L E D \sigma_A^{1.18}}{(3.82 D)^{1.18}}
\]

Where:

- \( L \) = length of roller = 0.235 inch
- \( E \) = Modulus of Elasticity of MR = 310
- \( = 30 \times 10^4 \) lbs./in.\(^2\)
- \( D \) = diameter of roller = 1 inch
- \( \sigma_A = 0.0036 \) inch (from Eqn. 35)

Therefore:

\[
W = \frac{(3)(0.235)(30 \times 10^4)(1)(0.0036)^{1.18}}{(3.82)^{1.18}} = 56.5 \text{ lbs.}
\]

This represents the external radial load which must act on the bearing in order to bring three rollers into contact with the inner race. This result is in good agreement with experimental observations as the load on the model was being increased by increments. Figure 8A indicates that rollers B have come into contact with the inner race under an external radial load of 85 lbs. with roller A carrying a major portion of the load. From Equation (35) we see that the normal deflection which roller A must experience before rollers C come into contact with the inner race is:

\[
\sigma_A^{**} = \frac{1}{32} (0.885 - 0.568) = 0.0102\text{ inch}
\]

which would require an external load of:

\[
W = \frac{(3)(0.235)(30 \times 10^4)(1)(0.0102)^{1.18}}{(3.82)^{1.18}} = 196 \text{ lbs.}
\]

Although the model was loaded to 200 pounds, rollers C did not come into contact with the inner race as would be expected from the above result. The load was not increased beyond this point for fear of breaking the model.
With this information it is possible to compare the load distribution for the case of a negligible radial clearance with the case of a relatively large radial clearance. This comparison is shown in Figure 16 for an external radial load of 85 pounds and for a radial clearance of 1/32 inch in the latter case. This illustrates quite well the adverse affect on the load distribution in a roller bearing as a result of a relatively large radial clearance.
FIGURE 16. COMPARISON OF LOAD DISTRIBUTION FOR ROLLER BEARING WITH NO RADIAL CLEARANCE
AND ROLLER BEARING WITH RELATIVELY LARGE RADIAL CLEARANCE
QUANTITATIVE ANALYSIS OF PHOTOELASTIC EXPERIMENTS

The direct photoelastic method yields the difference between the principal stresses (the maximum shear stress) throughout the model. When it is desirable to obtain the values of the individual principal stresses throughout the model, it becomes necessary to supplement the direct photoelastic method by obtaining an expression involving these stresses which is independent of that obtained photoelastically. A number of different methods are available for the determination of such expressions. They include not only mathematical and mechanical methods, but also optical and electrical methods.

A wise selection of the most suitable method for a particular investigation is highly important. For that reason, the following methods were considered for possible use in this problem. No attempt will be made to describe these methods here, but complete information on them is available in Reference #2. The primary reasons for rejecting the various methods will be given.

1. Slope Equilibrium (Rapid) Method - Frocht illustrates this method for the case of a circular disk with a central hole subjected to diametral compression. However, upon examination of the fringe patterns for the hollow rollers used in this experiment, it was decided that they did not meet the requirements of symmetry imposed by this method.

2. Use of a lateral extensometer together with an application of Hooke's Law - the necessary equipment for this method was not available. Also, because of certain experimental difficulties, this method is not generally used.
3. Relaxation method applied to the solution of LaPlace's differential equation - in this method it is necessary to obtain a sufficiently large and accurate representation of the fringe pattern. This would have been extremely difficult because of the small size of the rollers and the complex stress patterns present in them.

4. Shear difference method - this method, which is based upon the numerical integration of the equations of equilibrium, seemed to be the most suitable one for application to this particular problem.

After carefully considering each of the above methods, the shear difference method was chosen, and work was begun to obtain quantitative values for the individual principal stresses across the section of the rollers perpendicular to the line of load application. One of the first difficulties encountered was that of obtaining a reasonably accurate large scale fringe pattern. This difficulty was largely overcome by enlarging the original fringe photographs considerably, and by using a Brinell microscope to measure accurately the distances between adjacent fringes.

In this method, it is also necessary to have a reasonably accurate representation of the isoclinic lines across the section of the model being studied. At this point, it was realized that the isoclinic patterns which had been obtained experimentally were obviously unsuitable for use in this method. Numerous contradictions were apparent when these patterns were compared with those found in the literature for similar problems. Additional efforts were promptly made to correct this situation, but all such attempts resulted in ultimate failure and the conclusion that the suitable determination of the isoclinic pattern was not possible in the time remaining. Hence, the experimental work yielded no dependable results of a quantitative nature.
Frocht's method of using a solid circular disk as a photoelastic dynamometer was investigated as a possible application for the case of the solid rollers used in these experiments. This idea was abandoned when it was realized that its accuracy depends almost completely on the accurate determination of the fringe order at the center of the disk. This may be seen in the following equation which gives the load \( P \) acting on a disk of diameter \( D \) expressed as a function of the photoelastic constant \( C \) and the fringe order at the center of the disk \( n_c \):

\[
P = \frac{\pi C}{4} n_c D \quad \text{(See Ref. #3, pg. 154)}
\]

Best results are obtained from this method when the fringe order at the center of the disk is quite large, but none of the isochromatic photographs exhibited a fringe order as great as 2, thereby limiting the degree of accuracy which could be expected. In addition to this, no compensator was available for use in determining the fractional values of the fringe order at the center of the disk. It is quite difficult to estimate these fractional values with an appreciable degree of accuracy by experimental observation without the use of a compensator. Hence, no quantitative results were obtained in this manner.

Although it is most unfortunate that the experimental work was a failure in this respect, it is felt that a number of qualitative results were obtained in these experiments which may be of some value to future investigators in this general field of inquiry. These results will be discussed in the next chapter.
QUALITATIVE ANALYSIS OF PHOTOELASTIC EXPERIMENTS

Because of the lack of dependable quantitative results in this investigation, it becomes necessary to place a greater emphasis on the qualitative analysis than would ordinarily be desired. Quite naturally, an accurate determination of the actual stresses present in the particular model under investigation is desirable, but when such results are not obtainable, it is necessary to depend more heavily on results of a qualitative nature, although it is realized that such results may become the subject of considerable controversy unless they can be proved theoretically. For these reasons, the author feels compelled to approach this type of analysis with due caution.

For the sake of convenience, this discussion will be divided into the following cases:

Case I - Plastic cutter shell and standard journal (See Figure 8).

Case II - Steel disk and standard journal (See Figure 9).

Case III - Steel disk and oversize journal (See Figure 10).

The diameter of the standard journal is in proportion to the diameter of the journal in an actual rock bit, and the diameter of the oversize journal is such that the internal radial clearance of the assembled bearing is made negligible. First, each of these cases will be discussed individually, and then a comparison will be made between the three. Wherever possible, an attempt will be made to correlate the results of the mathematical analysis with this analysis, which is based primarily on experimental observations and an examination of the isochromatic photographs.
Analysis of Case I.

Figure 8A approximates the conditions actually encountered in a rock bit butter. The size of the cutter shell, rollers, and journal are all in proportion to the size of the corresponding parts in a particular rock bit. It is apparent in this photograph that only three rollers are supporting the applied load of 85 pounds. It is of further interest to note that although the load on the model was increased to 200 pounds, the same three rollers continued to support the entire load. Based upon the preceding mathematical analysis, this would lead us to believe that this load was not great enough to cause the three loaded rollers to deflect a sufficient amount to allow two additional rollers to come into contact with the inner race. The load was not increased beyond 200 pounds for fear of breaking the model.

During the course of the experiments, the position of the rollers was shifted so that two rollers, rather than one, were centered above the load-bearing tooth. In this case, only these two rollers supported the entire load on the model. In no instance was it possible to obtain a load on more than three rollers, except in the case of the rollers with the 3/4 inch holes, (See Figure 8-D), in which instance five rollers are observed to be supporting the load. It is quite obvious from a consideration of the fringes that, in this case, the three bottom rollers are quite highly stressed and are experiencing appreciable deflections as compared with the other three sets of rollers in Case I. In Figures 8-C and 8-D the effect of the forces acting at the points of contact of adjacent rollers is quite pronounced. As was mentioned in the "Quantitative Analysis", this condition greatly complicated the attempts to obtain quantitative results. The effect of
these forces appears to be nil in Figures 8-A and 8-B. A possible explanation for this may be that the normal deflection of the rollers in these two cases is quite small relative to the normal deflection of the rollers in Figures 8-C and 8-D. This would undoubtedly introduce accelerated bearing wear at these points of contact.

Turning our attention briefly to a consideration of the plastic cutter shell in Case I, we note an increasing tendency for light spots to appear at the roots of the cutter teeth as we move progressively from Figure 8-A to 8-D. When this was first noticed, it was believed that these were possible indications of greater deflections in the cutter shell in Figure 8-D as a result of the load being distributed over a greater arc of the outer bearing race. Without a doubt, the loading of the cutter shell is appreciably altered in this instance as compared with Figure 8-A, but it was subsequently learned that, for the magnitude of loading used in this experiment, the deflection of the cutter shell played no noticeable role as far as the stress patterns in the rollers were concerned. This is based upon experimental observations. It is quite possible that deflection of the cutter shell would play a greater role in cases of greatly increased loading of the model.

Analysis of Case II.

The second case was studied principally to answer the questions raised in Case I regarding possible deflection of the plastic cutter shell and its effect on the load distribution and stresses experienced by the rollers. Also, in the "Mathematical Analysis" the assumption was made that the inner and outer bearing rings remain true circles when loaded, and it was felt that Case II would definitely meet this
requirement. For these reasons, the experiments conducted in Case II were identical to those of Case I except that a steel disk was substituted for the plastic cutter shell. The bore of the steel disk was exactly the same as that of the plastic cutter shell, and the same journal and bearings were used in both cases. The results of these experiments were almost identical to the results obtained in Case I. No appreciable differences were observed in the load distribution or the stress patterns in the rollers.

It should be noted that the load applied to the rollers in Figure 9-A was not completely symmetrical, thereby resulting in a slightly greater load on the roller at the right as compared with the roller at the left. The diameter of the rollers had been carefully measured prior to loading to avoid the possibility of having one roller support more (or less) load than its adjacent roller due to differences in the diameters.

Analysis of Case III.

The third case represents the condition of negligible internal radial clearance in the bearing. Once again the steel disk was used rather than the plastic cutter shell, but in this series of experiments the oversize journal was substituted for the standard journal. All other conditions remained the same as before.

The primary purpose of Case III was to give some experimental confirmation of the mathematical analysis corresponding to this set of conditions. From a consideration of this mathematical analysis, we would expect all rollers whose centers are below the center line of the assembled bearing to come into contact with the inner race as soon as a load is applied to the journal. It is obvious from an examination
of Figure 9-A that this did not occur in the case of the solid rollers. Once again we see that only three bearings are supporting the entire load. The condition is somewhat more favorable for the three sets of rollers with the holes in them. It would be rather unreasonable to expect more than five rollers to support the load in any of these four instances. The reason for this is that angle $38$, (See Figure 11), is equal to $83.1^\circ$ when no external load is acting on the journal. As the load is applied, the journal moves toward the outer bearing race a distance which is dependent upon the deflections of the rollers under load, thereby causing angle $38$ to approach $90^\circ$. Since the load distribution is a function of the cosine of $38$, the net result is that, even if seven rollers originally supported the load, two of these rollers would actually become unloaded as the load on the journal is increased. However, it must be reported that under no conditions of loading were more than five rollers observed to be in contact with the inner race. The only explanation to be offered for this is that the condition of no internal radial clearance was not exactly duplicated, in spite of the fact that the machining tolerances were held quite close in the manufacture of the model. This may be taken as an indication of the precision with which a ball or roller bearing must be manufactured in order to achieve the most favorable load distribution among the bearing elements. It may also point out a discrepancy in the terminology used by Stribeck in his analysis and also used by this author in the preceding pages of this thesis. It was previously assumed that the initial internal radial clearance in the bearing must be "negligible" in order for the theoretical solution to apply. As a result of these experimental observations, the author feels that this
assumption should be modified to state that "no initial internal radial clearance" may exist in the bearing. In fact, we may even be justified in stating that a bearing must be pre-loaded in order for one-half of the rollers to remain in contact with the inner race after the application of an appreciable external load. This is sometimes done in practice, but in many cases it is not advantageous. Hence it is felt that even if quantitative results had been obtained from this experimental work, it is quite likely that they would not be in very close agreement with the theory.

Comparison of the Three Cases

By comparing Case I with Case II, we conclude that deflection of the plastic cutter shell was not an important factor for the magnitude of loading employed in these experiments. A comparison of Case I with Case III indicates a slightly more favorable load distribution for Case III. This was to be expected from a consideration of the mathematical analysis for the two cases. The fact that the journal in a rock bit is stationary makes it highly desirable to distribute the load over a greater arc of contact on the journal. Case III shows some improvement in this respect, but this is an impractical solution to the problem in the case of rock bits because of the reasons given in the "Introduction". Figure 8-D represents a similar improvement over Figure 8-A; however the size of the holes in Figure 8-D is so large that this would also appear to be an impractical solution to the problem. No comparison is necessary between Case II and Case III because of the previously mentioned similarity between Case I and Case II; hence it may be said that any statements which apply to Case I will also apply to Case II.
SUMMARY AND CONCLUSIONS

A two dimensional photoelastic investigation was made primarily to determine experimentally the effect of a relatively large radial clearance on the load distribution among the rollers in a roller bearing. A secondary consideration was the experimental determination of the possible improvement in the load distribution which may result from the use of hollow rollers in the bearing. In the mathematical analysis which was made in conjunction with the experimental work, it was necessary to determine a mathematical expression for the theoretical load distribution in a roller bearing assuming negligible radial clearance. No such analysis could be found in the literature for roller bearings although the corresponding analysis was found for ball bearings. Once this analysis was complete, certain parts of it were used in the mathematical analysis for the case of a roller bearing with a relatively large radial clearance. An attempt was made to correlate the results of these analyses with experimental observations.

It is unfortunate that no dependable quantitative information was obtained from the experimental work; however, it is possible to make a number of conclusions of a qualitative nature. Perhaps the most important such conclusion is that the assumption of "negligible" radial clearance is not entirely satisfactory for a consideration of the optimum load distribution which may be obtained in a ball or roller bearing. Based on experimental observations, the author believes that the assumption of "no" radial clearance would be more correct for this case. It is felt that the use of the oversize journal in these experiments fulfills the requirement of "negligible" radial clearance, but the optimum load distribution was not thus obtained. It is fully
realized that this is simply a matter of the terminology which is used, but it is felt that the use of such terminology may result in a misapprehension concerning the tolerances which may be considered acceptable in the manufacture of bearings in which the optimum load distribution is desired.

In this regard, a further conclusion may be that the optimum load distribution is obtained in practice only through the use of pre-loaded bearings.

Although the use of the oversize journal did not fully confirm the results of the mathematical analysis for that case, it should be noted that a more favorable distribution was observed as compared with the experiments in which the standard journal was used.

The rock bit designer recognizes from past experience that the reduction of the radial clearance in rock bit bearings is an impractical solution to the problem of gaining a more equitable load distribution among the bearings. For that reason, this experimental work attempted to assess the value of using hollow rollers in rock bit bearings. In the course of the mathematical investigation, it was realized that no improvement in load distribution may be achieved through the use of hollow rollers in the case of no radial clearance. This is due to the fact that the load distribution is a function of the external load and the cosines of the angles between the most heavily loaded roller and the remainder of the rollers in the lower half of the bearing. The experimental observations indicate that the use of hollow rollers is also impractical for the case of relatively large radial clearance. This is due to the fact that the hole in the rollers must be quite large in order to distribute the external load.
over a greater number of rollers. The size of such holes would intro-
duce the possibility of crushing the rollers when the necessary load
is applied to the bit.

Another conclusion is that, for the loads used in these experi-
ments, the deflection of the plastic cutter shell was not great enough
to have any noticeable effect on the load distribution or on the stress
patterns in the rollers.

It would also be well at this point to emphasize the fact that
the diffusion type polariscope used in these experiments is not as
well adapted to the accurate determination of isoclinic lines in a
model of complex shape as a polariscope with a light source of rela-
tively high intensity.
Selection of Photoelastic Material

The selection of a suitable photoelastic material must be made at the outset of any photoelastic investigation. Consideration should be given to the following requirements of a good photoelastic material:

1. Optical transparency, free from color tints.
2. Optical sensitivity to deformation, i.e., low stress optical coefficient.
3. Strict adherence to Hooke's Law within the proportional limit. (The proportional limit should be quite high.)
4. Freedom from initial internal stress.
5. Freedom from creep under steadily applied loads.
7. Good Machinability.
8. Freedom from aging effects.
10. Effect of temperature on the physical and optical properties should be nil.

In addition to this, the selection should include a consideration of the size and type of polariscope to be used, size and complexity of the model to be studied, availability of the photoelastic material in sheets of the desired size, and the amount of money available for the construction of the model.

From a consideration of the relatively complex shape of the model to be studied in this investigation, it was apparent that it would be advantageous to make the model quite large. This would minimize certain problems of machining and would result in a more accurate determination of the fringe order in the model, but it made the requirement of low cost
quite an important factor. Also, it was known that the experimental work would extend over a fairly long period of time; hence, no material could be used which develops serious edge stresses at room temperature. These were the primary reasons for choosing MR-310 as the photoelastic material for this experiment. This material had been used quite successfully by W. M. Koch (Ref. #6) and H. W. Murdock (Ref. #8) in photoelastic experiments conducted by them at the Rice Institute in 1953 and 1954.

**Description of MR-310**

MR-310 is a low-pressure, thermosetting resin which may be obtained from the Celanese Corporation in the form of a straw-colored liquid. Its use as a photoelastic material has been established only within the last few years. Its principal use is in the production of laminates and castings of unusually thick cross-section. After being mixed with a catalyst and accelerator, it may be cast at room temperature without the application of pressure. A mild exothermic heat is developed during the curing process, accompanied by a slight shrinkage of the material. Flat sheets with a highly polished surface may be obtained by casting the material between sheets of plate glass. Its cost is quite low compared with the more commonly used photoelastic materials, but its photoelastic sensitivity is not as favorable as some of the more expensive materials. Its low price plus the possibility of casting it in relatively large, highly polished sheets are its chief advantages when large models are to be made.

**Casting the Sheet of Plastic**

A consideration of the size of the model to be made indicated that it would be necessary to cast a sheet of plastic at least fifteen inches square and approximately 1/4 inch thick. A highly polished surface was desired, so the plastic sheet was cast between two pieces of 1/2 inch
thick plate glass, 18 inches square. A thin film of vejin mixed with ethylene chloride was carefully applied to one surface of each glass to serve as a separating agent. This is necessary because of the difficulty which is otherwise encountered in removing the glass from the plastic. A strip of rubber was placed along three edges of one glass plate, and the other glass plate was placed on top of the rubber strip with the vejin-coated surfaces of the glass plates facing one another. The rubber strip served as a dam to contain the liquid plastic in the glass mold, and also served as a spacer between the two glass plates. The glass mold was then placed in a vertical position, being held together by means of a number of C-clamps. The distance between the glass plates was controlled by inserting 0.250 inch thick steel spacers between the glass at various points around the periphery of the mold and adjusting the C-clamps accordingly. The steel spacers were removed before the plastic was poured in order to avoid any interference with the normal shrinkage of the plastic sheet. Due to shrinkage, the actual thickness of the finished plastic sheet was 0.235 inches. The care which was taken in setting up the glass mold resulted in a plastic sheet of extremely uniform thickness.

Next, the required amount of plastic was poured into a beaker, and 2% by weight of MD-1 Paste Catalyst was dissolved in it. Then 2% by weight of Accelerator E was added, and the solution was carefully stirred to give a consistent mixture. The beaker was then placed in a Bell Jar to which a vacuum was applied. This was done in order to remove the air bubbles from the mixture. When this was accomplished, the plastic was poured into the glass mold with due care being exercised to avoid the formation of new air bubbles. The plastic was allowed to cure for 48 hours at 120°F. The purpose of the elevated temperature was to
minimize the problem of separating the glass from the plastic and to obtain maximum clarity in the finished sheet. Little difficulty was actually encountered in the separation process, but it was necessary to be extremely careful to avoid cracking the sheet of plastic. Upon observation in the polariscope, the plastic was seen to be free of residual stresses and contained very few surface defects. The surface of the plastic required no polishing inasmuch as it was as smooth as the plate glass surfaces of the mold.

Description of the Model

When experimental work was begun, the model consisted of the following parts:

1. Plastic cutter shell.
2. Four sets of plastic bearings, 15 bearings per set.
3. One plastic journal.
4. Two plexiglas disks with central holes to accommodate the journal.

These parts are shown in Figure 2.

As the work progressed, it was found advisable to add the following parts:

1. Steel disk with a central hole.
2. One aluminum journal with a larger diameter than the plastic journal mentioned above.

The steel disk with the central hole was substituted for the plastic cutter shell in a series of experiments in an effort to approximate more closely the condition of a rigid outer bearing race. This was done to satisfy one of the conditions imposed by Striebeck in his analysis, and also to determine what effect slight deflections of the cutter shell may have on the load distribution among the bearings.
The aluminum journal was substituted for the plastic one in a series of experiments in an effort to approximate more closely the condition of negligible internal radial clearance which was also assumed by Stribeck in his analysis. The diameter of the plastic journal was such that the internal radial clearance was in proportion to that in an actual rock bit, while the diameter of the aluminum journal was such that the internal radial clearance could be considered negligible. In the analysis of the experimental work, the plastic journal will be referred to as the standard journal, and the aluminum journal as the oversize journal.

MR-31C proved to be a rather unsatisfactory material for the journal because of the problems involved in casting and machining a cylindrical piece of the required size. In addition, such a piece had to be handled with great care during the experimental work because of its fragile nature. The plastic journal which was used possessed excessive residual stresses which resulted in a misleading representation of the stresses present in it when loaded in the polariscope. For that reason, paper was glued to one end of the journal to prevent this misleading stress pattern from appearing in the photographs of the model under load. These factors made it desirable to use aluminum for the second journal.

The plexiglas disks were required to hold the bearings in position as the model was being loaded in the loading frame. They also served to prevent the unloaded bearings from falling out once the model was mounted in the loading frame. It was deemed impractical to attempt to remove them while the model was being photographed; consequently, their outline is visible in the photographs of the isochromatics.

The 15 inch-square sheet of plastic was used in the construction of the plastic cutter shell, the four sets of bearings, and the tensile specimens which were used for the calibration of the material. The
cutter shell represents a section of a rock bit cutter through the middle of the roller race and perpendicular to the center line of the cutter.
The bearings represent a section through a roller bearing perpendicular to its longitudinal axis. Thirteen bearings were made in the form of solid circular disks, while the other three sets of bearings had central holes machined through their longitudinal axis. A 3/8 inch diameter hole was machined through the center of one set of bearings, a 5/8 inch diameter hole through another set, and a 3/4 inch diameter hole through the third set. The outside diameter of all bearings was 1 inch. The purpose of the holes in the bearings was to observe the effect that they would have on the load distribution among the bearings as a result of increased deflections for a given load.

Machining the Model

Two pieces of masonite were cut to the exact outline of the cutter shell, and then were bolted to the two sides of the plastic sheet to serve as templates for machining the cutter shell. Then, the cutter outline was sawed approximately 1/16 inch oversize on a "Do All" band saw. Following the sawing operation, a band file was substituted for the saw blade, and the plastic was filed to the exact outline of the masonite template. This was done to remove the saw marks and any residual stresses which may have been induced in the plastic during the sawing operation. The part was completed by the machining its inside diameter on an engine lathe.

The bearings were sawed roughly to size on the "Do All" saw and then were machined on the outside diameter by being held in compression between steel pads which were mounted on the centers of the head stock and tail stock of an engine lathe. The holes were machined in the bearings while they were being held in a collet chuck on the lathe. Considerable
care is necessary in such machining operations in order to avoid inducing stresses into the material and also to avoid chipping or shattering the plastic. Best results have been obtained by using fairly high cutting speeds, light feeds, and shallow cuts. A cutting tool with a very sharp point is another important requirement. It should be mentioned that severe machining stresses always appear around a hole which has been drilled with a conventional twist drill. There is also an increased tendency for the plastic to chip or shatter. For these reasons, it was necessary to drill a small pilot hole through the bearing and then remove the remainder of the material by a boring operation.

**Calibration of the Loading Frame**

When the model is mounted in the loading frame, a load is applied to it by means of a horizontal lever arm with a steel tank at one end and a counterweight at the opposite end. (See Figure 7). The tank is equipped with a glass tube and scale for measuring the depth of water in the tank. The magnitude of the load acting on the model is proportional to the weight of water added to the tank after the lever arm has been balanced.

A U-shaped steel bar which had previously been calibrated in a Baldwin tensile testing machine was used in the calibration of the loading frame. The calibration bar had a dial indicator mounted between the open ends of the "U" to indicate the deflection of the bar under a given load. A plot of load versus dial indicator readings was made using data obtained by loading the bar in the tensile testing machine. The bar was then loaded in a similar manner in the loading frame, and a plot was made of dial indicator readings versus depth of water in the tank. With this information, it was then possible to make a plot of depth of water in the tank versus force acting on the model. This plot is shown in
In order to obtain quantitative results in any photoelastic investigation, the photoelastic constant of the material must be determined. This constant, which is sometimes called the "stress optical coefficient", may be defined by the following equation:

\[ C = \frac{d}{f} (P - Q) \]

Where:  
- \( C \) = photoelastic constant in lbs. / inch / fringe  
- \( d \) = thickness of model in inches  
- \( f \) = fringe order  
- \( P \) and \( Q \) = principal stresses in lbs. / inch\(^2\)

This determination may be made by loading a specimen of the material in tension in the polariscope and recording the magnitude of load required to produce a number of fringes. In a state of pure tensile stress, one of the principal stresses is equal to zero, hence the above equation may be written:

\[ C = \frac{d}{f} \frac{W}{A} \]

Where:  
- \( W \) = Load on specimen in lbs.  
- \( A \) = cross sectional area of specimen

In this manner, a photoelastic constant of 148 pounds/inch/fringe was determined. A plot of stress versus fringe order is shown in Figure 4, and a photograph of the tensile specimen mounted in the loading frame is shown in Figure 5.

Photographing the Isochromatics

According to the theory of photoelasticity, an isochromatic line (or fringe) is defined as the locus of points along which the difference in principal stresses is constant. It is of utmost importance to make
an accurate determination of these lines and to observe their formation as the load being applied to the model is increased by increments. It is also common practice to take photographs of the isochromatics for use in making a quantitative analysis and to serve as a permanent record of the investigation.

Monochromatic light is most commonly used in a circular polarscope for photographing isochromatics since this gives the most clearly defined lines. In this event, the lines appear as dark interference bands, and the isoclinics, which are lines of constant inclination of the principal stresses, are eliminated through the use of the quarter-wave plates in the circular polarscope. However, if the light source is not truly monochromatic, or if the quarter-wave plates are not made for the exact wave-length of light being used, then isoclinic lines will be superimposed on the pattern of isochromatic lines, thus making good definition of the isochromatics rather difficult.

The light source employed in the polarscope used for this investigation consists of a bank of seventeen 15-vatt standard green fluorescent tubes whose light has a wave-length in the vicinity of mercury green. Since this is not a truly monochromatic source of light, the quarter-wave plates do not effectively eliminate the unwanted isoclinics when the circular polarscope arrangement is used. Consequently, when such conditions exist in a polarscope, it is desirable to make use of some other technique which will result in improved clarity of the isochromatics. Such a technique is presented by Mr. W. M. Koch in Reference 46. It is the double exposure method of photographing isochromatics, and is based upon the fact that there is no variation in the intensity of the isoclinic lines when a double-exposure is made with the polaroids of a plane polarscope rotated 45° in the same direction between exposures.
In this way, the isoclinic lines will be completely eliminated from the resulting photograph, leaving only the isochromatics. This method was employed with considerable success in this experimental work. However, it should be noted that in addition to obtaining good photographs, it was also necessary to exercise great care in counting the isochromatics as they formed under increasing increments of load. This was especially important in the case of the bearings with the larger holes because the isochromatics are so closely spaced that an accurate determination of the fringe order becomes exceedingly difficult by merely examining the finished photographs, even though enlargements are made.

In all cases, load was applied to the model by means of a special bracket which was bolted to the lever arm of the loading frame. (See Figure 7). The photographs were made on Kodolit film which was developed in Dektol. A total exposure time of 4 minutes at 4-45 was used. In all photographs, the axis of the analyzer was crossed with respect to the axis of the polarizer, resulting in what is commonly called the dark-field system. These photographs are shown in Figures 8, 9 and 10.

**Sketching the Isoclinics**

An isoclinic line is defined as the locus of points along which the principal stresses have parallel directions. Their determination is generally required to obtain the individual principal stresses in a photoelastic investigation. When white light is used in the polariscope, the isoclinics appear as black lines, whereas the isochromatics are colored, thus making the isoclinics easily distinguishable. For this reason, the green fluorescent tubes in the light box of the polariscope were replaced with white fluorescent tubes for the determination of the isoclinics.

At first, an effort was made to obtain the isoclinics by mounting
the model in the loading frame in the same way in which it was mounted for photographing the isochromatics. It soon became obvious that satisfactory definition of the isoclinics could not be obtained in this manner because of the small size of the bearings. This problem was particularly acute in the case of the bearings with the larger holes. To overcome this difficulty, additional plastic disks were made which were five times larger than the bearings in the model. Proportionately larger holes were machined in these disks, and then each one was loaded individually in diametral compression in the loading frame. A piece of tracing paper was taped to the ground glass of the camera, and the isoclinics traced for each ten degrees of rotation of the polaroids.

It is fully recognized that this condition does not exactly duplicate the loading which the bearings in the loaded model experience. An examination of the isochromatic photographs shows that the bearings in the model are actually loaded at either three or four points on their circumference, depending on their location in the bearing race. Hence, the isoclinics used in the quantitative analysis are approximations of the actual case and must be recognized as such.

The problem of obtaining a satisfactory representation of the isoclinics was not completely solved by the use of the larger disks. Although extreme care was exercised in this experimental procedure, it was exceptionally difficult to trace the isoclinics with any reasonable degree of accuracy. One of the principal reasons for this was due to the relatively low intensity of light from the light box of the polariscope. This would indicate that the diffusion type polariscope used in these experiments is not as well adapted for the determination of isoclinics as a polariscope with a light source of relatively strong intensity. In addition to this difficulty, the isoclinics appeared as relatively wide
bands which were very closely spaced, making it quite difficult to dis-
tinguish a 10° isoclinic from a 20° isoclinic, and so on.

A possible solution to this problem would have been to take a color
photograph of the loaded disks for each ten degrees of rotation of the
polaroids from 0° to 90°. Black and white photographs would not have
been acceptable because the isochromatics would have been intermingled
with the isoclinics in the finished photograph, resulting in confusion
between the two. The number of such photographs would have been excessive,
the cost would have been quite high, and considerably more time would
have been required. For greatest accuracy, it would also have been
necessary to devise a method of loading the disks to conform with the
loading which the rollers experience in the assembled model. The lack of
sufficient time forbade the execution of such a plan.
BIBLIOGRAPHY


