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Rendezvous and Proximity Operations at the Earth-Moon L2 Lagrange Point: Navigation Analysis for Preliminary Trajectory Design

by

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Abstract

The Earth-Moon L2 point has attracted considerable attention as a potential location for future missions that enable spacecraft to explore beyond the far-side of the Moon, and other deep space celestial bodies. The capability to rendezvous and dock is required for multiple mission objectives. A critical element associated with the trajectory design involves determining the navigation requirements, which are indirectly affected by operating in the contemplated environment.

Three preliminary trajectory designs are proposed employing an innovative targeting algorithm that accounts for multi-body affects at L2. The trajectory designs range from replicating traditional rendezvous profiles to introducing strategies that capitalize on the unique relative motion at L2. Navigation requirements necessary for rendezvous and docking are then derived and validated for each trajectory. These requirements are subsequently utilized for a sensor suite analysis to identify an optimal one for each trajectory within an efficient and automated framework.
Acknowledgements

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1 Introduction

As science and exploration initiatives consider operating multiple spacecraft near the Earth-Moon L2 point, the conventional rendezvous and docking techniques and trajectories used for decades must be revisited to address this unique environment. Conventional trajectories are designed based upon the relative motion experienced in Low-Earth Orbit (LEO) or similar locations, that have a less significant effect at L2. During close proximity operations at this location, the navigation system plays a critical role to ensure mission success. At L2, the relative dynamics between two spacecraft changes noticeably and future rendezvous profiles must adapt to these new conditions. An automated navigation technique can rapidly derive and validate the necessary navigation requirements and subsequently determine an optimal sensor suite for potential rendezvous and docking trajectories at the Earth-Moon L2 point. This chapter introduces the Lagrange points and their potential relevance to the aerospace community, outlines the necessity for rendezvous and docking operations, and summarizes the current research related to this field.

1.1 The Earth-Moon Lagrange Points

![Image of Lagrange Points]

Figure 1: The Lagrange Points in the Restricted Three-Body Problem [1]

In 1767, Leonard Euler published *De motu rectilineo trium corporum se mutuo attrahentium* [2], after studying the orbital mechanics regarding the restricted three-body problem. He detailed his
discovery of three collinear points with a notable property: the combined gravitational force that two massive bodies would exert on a third body at these points equaled the exact centripetal force required for all three to stay in a constant, rotating formation. Later, in 1772, Joseph Louis Lagrange discovered another two points, this time not collinear but instead forming equilateral triangles with the two bodies, that had the same property. These points have become known as the Lagrange, or libration, points, and are shown with their appropriate labels in Figure (1).

Several centuries later, NASA’s manned space program and the aerospace community are shifting focus from LEO back to deep space. This can be seen from the retirement of the low Earth orbital Space Shuttle and work on the deep-space capable Orion Multi-Purpose Crew Vehicle, along with interest regarding asteroid mining and Mars exploration from private companies. As a result, the Lagrange points have become of particular interest due to their unique properties. In particular, the Earth-Moon L2 point, referred to here simply as L2, has become a potential location for spacecraft such as a space station or propellant depot. This can be completed as a relatively short term goal and act as a precursor to farther destinations.

Numerous reasons exist to place a spacecraft at this point. The L2 point, along with L1, is part of an Interplanetary Superhighway System, allowing travel by low energy pathways and greatly decreasing propulsion costs for deep space travel [3]. It is an optimal location for refueling as it is nearly at Earth escape energy, and as stated in Evolving to a Depot-Based Space Transportation Architecture by Zegler and Kutter [4]:

Just as importantly L2 is in deep space far away from any planetary surface and hence the thermal, micrometeoroid, and atomic oxygen environments are vastly superior to those in LEO. Thermodynamic stasis and extended hardware life are far easier to obtain without these punishing conditions seen in LEO. L2 is not just a great gateway-it is a great place to store propellants. (p. 4)

But beyond being a precursor or depot stop for further travel, the L2 spot holds merits by itself. Specifically, because of its location behind the Moon, a spacecraft orbiting near the L2 point can either be placed within a radio quiet zone, for radio astronomy [3], or act as a communications satellite for Moon far-side based exploration or colonization [8]. Such a case is demonstrated in Figure (2). It would also allow one to evaluate the physiological and psychological effects that deep space may have on humans, as the L2 point is over four hundred thousand kilometers away from the Earth, order of magnitudes farther than the current placement of the International Space Station (ISS), at three hundred and seventy kilometers [5].
1.2 Rendezvous Overview

Rendezvous consists of a series of controlled maneuvers by an active chaser vehicle to bring it in the near vicinity to a passive target vehicle. Rendezvous was first successfully performed on March 16, 1966, between the Gemini 6 and Gemini 7 [8]. Over the last half century, rendezvous has been successfully achieved numerous times, from low lunar orbit (LLO) to geostationary orbit (GEO) to LEO.

For deep space exploration, rendezvous and docking serve as a critical capability to satisfy basic mission objectives. These operations become necessary for assembling larger units in orbit, resupplying and refueling, exchanging crew, repairing spacecraft, or rejoining an orbiting vehicle with a lander or other auxiliary vehicle [8]. For spacecraft at L2, all of these would be applicable, especially for refueling in the case of a propellant depot or resupplying and exchanging of crew at a space station.

The trajectory design for rendezvous and docking at the L2 environment is necessary in order to support missions at this location, yet the relative motion experienced during standard rendezvous operations in LEO are different from the relative motion at L2. Consequently, the rendezvous techniques developed over the last half century must be re-evaluated and adjusted. The trajectory design and on-orbit operations must take into account an assortment of random errors experienced while on-orbit such as initial condition dispersions, sensor noise, inaccurate thrust velocities, modeling errors, disturbance accelerations and other random effects introduce deviations to a desired path and must be accounted during initial stages of mission planning.

The conventional approach to model trajectories influenced by random, or stochastic, processes can involve heavily computational and time consuming Monte Carlo simulations. These simulations can repeat the same trajectory hundreds or thousands of times, with each run generating new random
variables to produce possible outcomes for a single trajectory. However, an alternative method exists to model the dispersion and navigation error in the preliminary design stages of a rendezvous trajectory, known as linear covariance analysis (LinCov). LinCov can determine the equivalent statistical information as Monte Carlo analysis in a fraction of the time, by linearizing the governing systems about nominal points for small step sizes. These linearized equations can be used to directly propagate a covariance matrix through time, even as the nominal state continues to be propagated through non-linear dynamics. This greatly decreases the time required to test multiple combinations of trajectories and the complex interaction of the guidance, navigation and control (GN&C) system, to find an optimal solution. Once the design is complete, the linearization assumptions used to propagate the covariance matrix can be validated through Monte Carlo simulations, such as shown in Figure (3).

1.3 L2 Literature Review

Significant work has been conducted for transfer trajectories to various Lagrange points such as the Earth-Moon L2 location, L2 halo orbit determination and insertion, as well as for station keeping
and formation flying. The author of this thesis has not found in the public literature documentation regarding proximity operations at the Lagrange points, particularly at L2.

Articles mentioning rendezvous at Lagrange points have an alternative notion of rendezvous, as those found were published in 1970. This includes *Libration Point Rendezvous*, by Edelbaum [10], and *Rendezvous Equations in the Vicinity of the Second Libration Point*, by Gerding [11]. However, in the first article, the author spoke not of rendezvous between two vehicles, but instead focused on transfer trajectories, such as from L1 to lunar orbits. The latter focused on rendezvous as just defined, but attempted to make the target vehicle the L2 point instead of a spacecraft, so that one could derive orbits around the L2 point.

Considerable work has also been completed in recent years regarding the orbits around L2, known as halo orbits. Recent articles such as *Station-Keeping Strategies for Translunar Libration Point Orbits*, by Gomez, Howell and Simo [12], *Generic Halo Orbit Insertion and Dispersion Error Analysis*, by Jenkin and Campbell [13], and *Halo Orbit Mission Correction Maneuvers Using Optimal Control*, by Serban et al. [14], contain several techniques for arriving at and maintaining a halo orbit. However, none of these subjects are concerned with rendezvousing and docking spacecraft at these points. As such, they do not use the pertinent relative equations or derive targeting maneuvers required for rendezvous.

Formation flying control techniques for multiple spacecraft has also received noticeable attention. The dissertation *Nonlinear Control Design Techniques for Precision Formation Flying at Lagrange Points*, by Luquette [15], contains the linearized equations of motion for a leader and follower vehicles. However, formation flying has different objectives than rendezvous, as the intent is to keep a follower vehicle in the same orientation and distance away from a leader vehicle instead of safely bringing them to close proximity in a controlled manner. The challenges associated with manned spaceflight during the rendezvous and approach phase have not been addressed.

1.4 Contributions

Although the concepts and procedures associated with rendezvous and docking have been demonstrated and repeated over the past half century in both lunar and Earth orbits, it has not yet been explored how this practice can be accomplished in halo orbits about the various Lagrange points. Can traditional techniques be utilized or must the basics in rendezvous and docking be thoroughly recreated to adapt to the new relative motion dynamics at L2?

To answer this question, this thesis begins by focusing on the targeting algorithms and trajectory
design that can support rendezvous and docking at L2. Several contributions of this research include
the development of a novel linear targeting algorithm that efficiently and accurately accounts for
multi-body dynamics, and the proposal of multiple rendezvous trajectory designs that considers
multiple mission operation constraints while reducing fuel usage.

Subsequently, conventional methods for determining the effects of random processes can be com-
putationally expensive, greatly increasing development time. For a chosen trajectory, how can the
navigation requirements be derived and optimal sensor suite selected, while not expending hours of
simulation to test each possible combination?

This thesis also includes the validation of a new technique to derive navigation requirements
at the L2 environment in an automated fashion, and an extensive navigation analysis that derives
the navigation performance requirements for multiple trajectories at multiple locations along each
trajectory. This analysis concludes by rapidly evaluating several hundreds of potential sensor suites
and identifying one that can optimally support such mission requirements.

1.5 Thesis Outline

To demonstrate the automated navigation analysis techniques applied for rendezvous trajectories at
L2, three trajectories are designed and analyzed that satisfy safety requirements and limit propellant
usage. For each trajectory, the navigation requirements are derived, followed by an optimal selection
of sensors to fulfill the requirements.

In Chapter 2, basic L2 properties, halo orbits, and relevant reference frames are introduced. The
LVLH reference frame found prominently in rendezvous scenarios is introduced. The potential
targeting algorithms and their accuracies are also addressed. It is found through preliminary analysis
that a selection of linear targeting algorithms appear to perform reliably at L2, including a new
targeting algorithm based upon the linearized equations of motion at L2.

In Chapter 3, three trajectories are designed utilizing the proposed linear targeting algorithms
adapted for L2 relative motion dynamics. Multiple constraints are described, including the safety
regions and their relevance to free-drift trajectories. The first rendezvous profile is based upon a
common trajectory currently performed in LEO, but modified for L2. The final two are developed to
improve piloting factors, facilitation of sensor operations, and fuel efficiency given the circumstances
unique to L2.

In Chapter 4, stochastic analysis is then introduced. Linear covariance analysis provides a quick
and reliable approach for modeling uncertainties and untangling the intricacies associated with a
complex GN&C system. By extending the capabilities of linear covariance with stochastic navigation, it is possible to model the navigation error directly, instead of requiring a specific sensor suite and onboard navigation filter. This is then further extended with sensitivity analysis and scaling, which leads to requirement derivation.

In Chapter 5, the automated process of deriving navigation requirements is then implemented to all three trajectories. Several navigation regions are defined for each trajectory, based upon the time instants of key impulsive maneuvers of each trajectory. The navigation requirements for each region of each trajectory are derived using an automated approach and are validated through conventional Monte Carlo simulations. The dispersions are displayed both for the full trajectories and for the partially completed free-drift cases. Once the navigation requirements are derived, a sensor suite analysis utilizing LinCov and pre-determined sensor weights determine the most effective sensor suites to satisfy the navigation requirements.

The thesis is completed in Chapter 6, the conclusion and summary of methodology and results. In addition, the work completed in this thesis is chosen to be expandable. As such, several topics are mentioned where future research could be conducted, in order to increase the number of relevant applications this research would influence.
2 Deterministic Mechanics at L2

As a prerequisite to introducing the nominal trajectories and stochastic considerations to the rendezvous problem, this section details the reference frames, basic L2 properties, and the halo orbits a potential target vehicle may be placed on. It also introduces targeting algorithms based upon the relative mechanics present at the L2 point.

2.1 Reference Frames

The inertial and relative trajectories introduced in this thesis use several frames of reference, three of which are pictured in Figure 4. The non-rotating inertial frame quantifies absolute reference states, such as the inertial position and velocity of the vehicles and celestial bodies. The origin is located at the center of the Earth, and the $X_{ECI}$-$Y_{ECI}$ plane is defined by the J2000 equatorial plane, with $X_{ECI}$ determined through the intersection of the equatorial and ecliptic planes, and $Z_{ECI}$ determined as perpendicular to the J2000 equatorial plane. The picture on the left is not to scale, as $r_{target} >> \rho$ in practice.

![Image: Inertial, Earth/Moon Rotating and LVLH Reference Frames]

Figure 4: The Inertial, Earth/Moon Rotating and LVLH Reference Frames

Displayed on the top-right, the local vertical, local horizontal (LVLH) reference frame has its
origin placed at the target vehicle. This reference frame has been used extensively for rendezvous, and is the basis for several targeting algorithms. The x-axis is known as downrange and points in the general direction of the target’s velocity vector. As a result, it is designated as the V-bar. The z-axis, or R-bar, is the altitude, describing the radial direction pointing towards the Earth or central body. Note that positive altitude is defined as ‘down’. Cross-track, or the direction perpendicular to both downrange and altitude, is the y-axis or H-bar.

On the bottom right is the Earth/Moon rotating pulsating frame. Here, $x_{EM}$ is pointed from the Earth to the Moon, $z_{EM}$ is perpendicular to the plane of the Moon’s orbit, and $y_{EM}$ completes the triad pointing in the velocity of the Moon. The frame is pulsating because the distance between the Earth and the Moon is normalized to the same value, regardless of position of the Moon on its elliptical path. This frame of reference is used to display the halo orbits around L2.

2.2 L2 Properties and Halo Orbits

The L2 point is a function of the distance between the Moon and the Earth. The distance beyond the Moon, $x_{L2}$ as shown in Figure (4), can be determined iteratively by the equation

$$x_{L2} = \frac{[(m_m/m_e) \cdot R_m \cdot R^2 \cdot (R + x_{L2})^2]/[(R_m + x_{L2})(R + x_{L2})^2 - R_m \cdot R^2]}{\text{1}}$$

where $m_m$ and $m_e$ are the mass of the Moon and Earth, respectively, $R_m$ is the distance from the Earth-Moon barycenter to the Moon, and $R$ is the distance between the Moon and the Earth. Using the average value of the Earth to Moon of 384,401 km, this results in $x_{L2} = 58,006$ km, or the distance from the Earth to the L2 point equal to 442,407 km. Both values will vary approximately +/− 10% as the Earth-to-Moon distance changes.

The orbital angular velocity is orders of magnitude smaller at L2 than in LEO. The period of a spacecraft in LEO ranges from 88 minutes to 127 minutes, depending on the chosen altitude. For a spacecraft at L2, the orbital period will equal that of the Moon, approximately 27.5 days when measured from perigee to perigee. Consequently, the approximate orbital angular velocity for LEO is $\omega \approx 0.0114$ rad/s, while for L2, the average orbital angular velocity is $\bar{\omega} = 2.66 \cdot 10^{-6}$ rad/s. This difference will have a noticeable effect on the relative motion.

While the Earth-Moon L4 and L5 points are stable, the collinear points are not. However, certain halo orbits around the L2 point can be maintained through minimal station-keeping maneuvers. Data were gathered from one such halo orbit where the target is on a 14 day halo orbit, as shown in Figure (5). This information was provided by Juan Senent, from NASA-JSC [18], and serves as
the initialization point of the target vehicle.

2.3 Targeting Algorithms at L2

Navigation requirements are directly affected by the targeting algorithms chosen, as they determine what relative and absolute states are required to perform a maneuver. A variety of targeting algorithms are currently implemented for rendezvous in LEO or GEO. However, many of these traditional algorithms are not applicable. For example, a Hohmann transfer burn commonly used for altitude adjustment maneuvers would take 14 days to complete at L2.

Even though the dynamics at L2 is different, the required complexity for rendezvous targeting algorithms may decrease due to the simplicity of the resulting relative motion. Linear targeting algorithms are shown to be simple and reliable, and adapt well to L2 dynamics. Non-linear targeting algorithms, such as Lambert targeting, were also considered. However, they did not easily lend themselves to multi-body dynamics, and only increased complexity. Therefore, three linear targeting methods are considered to create several preliminary trajectories. In this section, Clohessy-Wiltshire (C-W) targeting, Straight-Line targeting, and L2 Linearized Relative (LR) targeting are detailed and examined for their reliability at L2.

All three require varying amounts of information, and preliminary results demonstrate the likely differences in the accuracy and precision of each targeting algorithm. C-W targeting is a commonly used linear algorithm for typical rendezvous objectives, that accounts for only one gravitational body. Straight-Line targeting assumes no gravitational forces and can be seen as a simplification of the C-W targeting where the angular rate approaches zero. LR targeting requires the most information, accounting for gravitational forces from multiple bodies. The accuracy of the targeting algorithms are seen to increase as additional information is utilized.

2.3.1 Linearized Relative Motion and Targeting

If the relative equations of motion can be linearized to the state space representation form of

\[
\dot{x} = A \cdot x, \tag{2}
\]

where \(A\) is the system matrix and \(x\) is the relative state, then the state transition matrix can be solved using the equation

\[
\Phi(\Delta t) = e^{A\Delta t}, \tag{3}
\]
Figure 5: The Halo Orbit, within the Earth-Moon Rotating, Pulsating Reference Frame. The top view represents the trajectory as viewed from Earth looking towards the Moon. The bottom view is along the axis towards the Moon, within the same plane as the Moon’s orbit.
where \( \Delta t \) is the transfer time. Then, the state transition matrix and relative state vector can be decomposed into

\[
\Phi(\Delta t) = \begin{bmatrix}
\Phi_{rr} & \Phi_{rv} \\
\Phi_{rv} & \Phi_{vv}
\end{bmatrix},
\]

and

\[
x = \begin{bmatrix}
x \\
y \\
z \\
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}^T = \begin{bmatrix}
r \\
v
\end{bmatrix},
\]

where \( x, y, \) and \( z \) are defined as downrange, cross-track, and altitude. The change in velocity required at the point of the burn, \( \Delta v \), can be determined to achieve any three of the six states of the final relative state, \( x_f \), beginning with the initial relative state, \( x_i \), as

\[
x_f = \Phi(\Delta t) \cdot x_i.
\]

To target a desired position, \( r_f \), after a transfer time, \( \Delta t \), substitute the required initial velocity, \( v_{i,req} \), for the current initial velocity

\[
\begin{bmatrix}
r_f \\
v_f
\end{bmatrix} = \begin{bmatrix}
\Phi_{rr} & \Phi_{rv} \\
\Phi_{rv} & \Phi_{vv}
\end{bmatrix} \cdot \begin{bmatrix}
r_i \\
v_{i,req}
\end{bmatrix}.
\]

Then, \( v_{i,req} \) can be determined with respect to \( r_f \) as

\[
v_{i,req} = \Phi_{rv}^{-1} \cdot (r_f - \Phi_{rr} \cdot r_i).
\]

The desired \( \Delta v \) can be determined by taking the difference between the vehicle’s required velocity and its current velocity

\[
\Delta v = v_{i,req} - v_i.
\]

### 2.3.2 Clohessy-Wiltshire Targeting

For close proximity operations, two assumptions are often made to simplify the relative equations of motions. First, it is assumed that the distance between the target and the center of mass of the central body is much larger than the distance between the target and the chaser. Second,
it is assumed that the target is moving on a circular orbit around the central body, so that the
distance to the central body from the target is constant, \( r = r_0 \). Under these conditions, the relative
equations of motion can be linearized to the C-W equations, which can be written in the state space
representation as \([20]\):

\[
\dot{x} = A_{CW} \cdot x,
\]

(10)

with

\[
n = \frac{2\pi}{T} = \sqrt{\frac{\mu}{(r_0^3)}},
\]

(11)

where the variable \( n \) is the mean motion for a circular orbit, the average of the orbital angular
velocity, \( \omega \). The variables \( T \) and \( \mu \) are the orbital period and the standard gravitational parameter.

Several perturbations are required to use the C-W equations at L2. The cumulative gravitational
forces from the Moon and the Earth must be approximated as a force from a single point. Choosing
the Earth-Moon barycenter, which can be approximated as the center of the Earth, allows a strategic
selection of the mean motion parameter. By also assuming small transfer times, the mean motion,
\( n \), can be replaced by the magnitude of the target’s instantaneous angular rate with respect to the
Earth at the moment of the maneuver:

\[
\omega_i = \left| \frac{\mathbf{r}_{\text{target}} \times \mathbf{v}_{\text{target}}}{|\mathbf{r}_{\text{target}}|^2} \right|.
\]

(12)

The instantaneous velocity of the target with respect to the Earth, \( \mathbf{v}_{\text{target}} \), is a function of the
gravitational pull of both the Moon and the Earth. This substitution captures the dynamics at the
start of a maneuver, and remains accurate for the short durations expected of targeting maneuvers.
The effects of the approximations and assumptions on the precision and accuracy of the targeting
algorithm can be seen through the simulation results in “Subsection 2.3.6 - Targeting Performance
Evaluation.”
2.3.3 Straight-Line Targeting

Straight-Line targeting is the simplest and most intuitive approach. The desired $\Delta v$ aims the velocity vector towards the targeted point at all times, disregarding gravitational effects. That is

$$\Delta v = \frac{r_f - r_i}{\Delta t} - v_i.$$  \hspace{1cm} (13)

From Equation (11), the orbital angular rate approaches zero as the distance from the central body, $r_0$, increases. As a result, C-W targeting approaches Straight-Line targeting:

$$\dot{x} = \lim_{n \to 0} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2n \\ 0 & -n^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3n^2 & -2n & 0 & 0 \end{bmatrix} \cdot x = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot A_{S-L} \cdot x \hspace{1cm} (14)$$

In 1965, the Gemini 4 vehicle attempted the first rendezvous in history, utilizing Straight-Line targeting due to the limited knowledge of the orbital mechanics involved. It failed to rendezvous with its spent Titan II launch vehicle’s upper stage, as both target and chaser were still in LEO. However, since the magnitude of the mean motion, $n$, at L2 is approximately one-fifth hundredth of that at LEO, the Straight-Line targeting assumptions are more relevant at L2 compared to LEO. Due to the algorithms simplicity and independence of inertial state knowledge, it has potential that is worth investigating for applications at L2.

2.3.4 L2 Linearized Relative Targeting

In the doctoral dissertation Nonlinear Control Design Techniques for Precision Formation Flying at Lagrange Points [15], by Luquette, the equations of motion at any Lagrange point are linearized for two spacecraft using the variables displayed in Figure (6). In the IVLH frame used here, the state space representation is:

$$\dot{x} = \begin{bmatrix} 0 & I_3 \\ \{\Xi(t) - [n \times] [n \times]\} & -2[n \times] \end{bmatrix} \cdot x = A_{LR} \cdot x, \hspace{1cm} (15)$$

where
\[ \Xi(t) = -\left( \frac{\mu_1}{|r_{1L}|^3} + \frac{\mu_2}{|r_{2L}|^3} \right) \mathbf{I}_3 + \frac{3\mu_1}{|r_{1L}|^3} [\mathbf{e}_{1L} \cdot \mathbf{e}_{1L}^\top] + \frac{3\mu_2}{|r_{2L}|^3} [\mathbf{e}_{2L} \cdot \mathbf{e}_{2L}^\top], \]  

(16)

\[ [n \times] = \begin{bmatrix} 0 & 0 & -n \\ 0 & 0 & 0 \\ n & 0 & 0 \end{bmatrix}, \]  

(17)

and

\[ [n \times][n \times] = \begin{bmatrix} -n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix}. \]  

(18)

Here, \( n \) is the angular rate of the LVLH frame with respect to the barycenter of the two masses. If the location of the Earth-Moon barycenter is approximated as the center of the Earth, then \( n = \omega_i \), the instantaneous orbital angular velocity from Equation (12). Also, \( r_{1L} \) and \( r_{2L} \) are the vectors in the LVLH reference frame from the Earth and the Moon, respectively, to the target, and \( \mathbf{e}_{1L} \) and \( \mathbf{e}_{2L} \) are the unit vectors of \( r_{1L} \) and \( r_{2L} \), respectively. The constant \( \mu_1 \) equals the gravitational parameter for the Earth, while \( \mu_2 \) is the same parameter for the Moon.

Furthermore, if \( \mu_2 \) is set to zero, and the mean motion set to \( n = \sqrt{\mu/(r_{1L}^3)} \), the state space representation of Equation (15) reduces to the C-W representation found in Equation (10). In comparison to C-W targeting, the Linearized Relative (LR) targeting algorithm requires the distance vector from the Earth to the Moon, as well as the data originally required for C-W targeting. As the linearized equations of motion are derived for any Lagrange point, LR targeting should be applicable for proximity operations at any Lagrange point within the restricted three-body problem.

### 2.3.5 Improving Linear Targeting Algorithms with Internal State Prediction

This thesis utilizes targeting algorithms that do not require an internal state propagation or prediction capability. With internal propagation routines, targeting algorithms could be further improved. In the targeting algorithms described above, the state transition matrix is evaluated once, from the states at maneuver initiation, allowing it only to capture the relative dynamics at the initial time instant. Alternatively, the transfer time can possibly be divided into several time intervals. If the absolute position data as well as the instantaneous orbital angular velocity at the start of each interval could be accurately predicted by propagating the inertial states forward in time by an in-
ternal algorithm, the state transition matrix could be calculated for each time interval. Multiplying the individual state transition matrices could compose a total state transition matrix that would increase accuracy for transfer times of multiple days.

Further, the capabilities required to gather such data could also facilitate shooting methods that utilized LR targeting to solve for the $\Delta v$ required with a minimum number of steps. Such methods would propagate the states reflecting the proposed maneuver, calculate the position error at the end of the transfer time, and then propose a new maneuver to correct for this error, iteratively converging to the required burn. However, these methods would be computationally costly and require more input information than the linear targeting algorithms, when linear targeting algorithms are shown to be sufficient.

### 2.3.6 Targeting Performance Evaluation

Several targeting algorithms were tested to evaluate their performance in the L2 environment with multi-body effects. Although an exhaustive evaluation of each targeting algorithm was not performed, a healthy examination of each one has provided insight to their level of execution. The true states are propagated through time by the non-linear equations of motion incorporating ephemeris
data of both the Earth and Moon. The simulation truth models can be found in *Linear Covariance Techniques for Orbital Rendezvous Analysis and Autonomous Onboard Mission Planning*, by Geller [21]. Further details can be found in *Multiple Event Triggers in Linear Covariance Analysis for Spacecraft Rendezvous*, by Zanetti, Woffinden, and Sievers [22].

Basic trends and observations revealed during this exercise are summarized below. For ease of demonstration, only in-plane ($y = 0$ and $\dot{y} = 0$) motion is considered. However all techniques can be applied to the general case, even if these values were non-zero.

Not surprisingly, linear targeting algorithms requiring more input data generally performed better than their counterparts. C-W targeting outperformed Straight-Line targeting, but performed worse than LR targeting. One such example is in Figure (7). Here, two maneuvers are completed using all three targeting algorithms. The two maneuvers have transfer times totaling 95 minutes and cause the vehicle to travel 60 km downrange and about 12 km in altitude. The C-W and LR trajectories each reached within a meter of the desired position. Straight-Line targeting is approximately 200 m away from the targeted destination.

C-W targeting was never significantly far from the desired position, for the maneuver time and distances typically used for rendezvous trajectories. If transfer times were kept short, C-W targeting was found to be rather accurate for large distances. Even while targeting points 100 km away from the chaser in both the downrange and altitude, maneuvers with transfer times of an hour landed within 10 m of their intended target. This reflects even higher accuracy than expected at LEO over a comparable transfer range.

However, for time durations greater than a few hours, C-W targeting began to show limited performance and an error pattern. This is likely a result of using the instantaneous orbital angular velocity, capturing the dynamics at the start of a maneuver, while not considering the multi-body effects. Figure (8) is a loop demonstrating the difference between using 8 hour transfer times going to the top-left, versus 1 hour transfer times to the bottom-right. C-W targeting is shown on the left. A full trajectory has a maximum duration of 8 hours, and thus this time is used as an upper bound. Although the 1 hour transfer time maneuvers always return to the targeted point with tolerable accuracy, within 3 m (0.03%), errors for an 8 hour transfer time can grow as large as 250 m (2.45%). These errors consistently change in direction and magnitude as the target continues on its halo orbit and the L2 point travels slightly closer to or farther from the Earth. Straight-Line targeting fit the definition of precise but inaccurate. Although it consistently hit the same point, it was inaccurate by approximately 600 m (5.88%).

For large transfer times, LR targeting provides a more accurate solution. Both the 8 hour and 1
Figure 7: Comparison Between Targeting Algorithms. For this example, results for LR targeting are visually identical to C-W targeting.
Figure 8: C-W, Straight-Line, and Linearized Relative Targeting for an 8 and 1 Hour Loop
hour duration maneuvers reach the intended target with higher precision and accuracy. The 8 hour transfers were within 20 m (0.20%), while the 1 hour transfers were within a meter (0.01%). If C-W or Straight-Line targeting maneuvers at L2 yields unsatisfactory results, switching to LR targeting will certainly show a desired improvement.
3 Preliminary Trajectory Design

Several preliminary trajectory designs are proposed that utilize the linear targeting algorithms. These preliminary trajectories are used to demonstrate the anticipated navigation requirements and sensor suites for close proximity operations at L2. In particular, three trajectories are developed: the Double Co-Elliptic (DC), the Line-of-Sight Corridor (LoS-C), and the Line-of-Sight Glide (LoS-G). The first trajectory is adapted from profiles already flown in LEO, while the final two are partially inspired from Edwin Eugene Aldrin, Jr.’s Line-of-Sight Guidance Techniques for Manned Orbital Rendezvous [23]. The Line-of-Sight Corridor and the Line-of-Sight Glide both take advantage of the near rectilinear motion of the chaser at L2. All trajectories are developed with several factors taken into consideration, such as fuel consumption, ease of piloting, and safety constraints. Several alternate trajectories, such as the Stable Orbit Rendezvous or a parallel approach corridor, are described but not developed due to not adequately satisfying one of these factors.

For on-orbit operations, safety becomes a high priority to protect critical assets and crew members. In regards to rendezvous trajectories, the chaser vehicle must not enter particular safety regions or constraint regions surrounding the target until permission is granted, including dispersions. Not only must the complete, or nominal end-to-end, trajectory be designed to not enter these areas until approval is received, but partially completed trajectories where thrusters possibly cease to operate, called free-drift trajectories, must also not enter these areas. For example, all commercial vehicles visiting the ISS must receive an ‘authority-to-proceed’ prior to entering the 2 km by 1 km Approach Ellipsoid (AE) or the 200 m radius Keep Out Sphere (KOS) [21], pictured in Figure (9). Also illustrated is a free-drift condition in LEO demonstrating how a drift case is handled. Assuming similar safety constraints are adopted at L2 for future NASA missions, the three proposed trajectory designs conform to this safety requirement.

Further considerations include:

I. The orientation of the target vehicle is not fixed with regards to the LVLH reference frame, and has multiple docking ports. Rendezvous trajectories must be able to adapt to mission specifications required, and be able to dock along any axis based upon the orientation of the target.

II. Close proximity operations must be completed in a single operating day for the pilots, an 8 hour limitation, but a shorter duration is preferred.

III. Minimal fuel usage must be pursued. A full rendezvous trajectory using a $\Delta v$ of more than 10
m/s is considered high.

IV. Rendezvous trajectories start when relative sensors become active. This will require that all trajectories begin a minimum of 40 km away.

V. Rendezvous trajectories end when the chaser has docked with the target at the docking port. Various vehicles may have docking port offsets, so for this generic study the docking port is considered to be at the origin of the LVLH frame.

The scope of this thesis does not cover the departure phase. It is anticipated that the navigation requirements for departing in the near rectilinear motion at L2 will not be stringent unless fly-arounds or other various inspection profiles of the target vehicle are desired prior to departure.

3.1 Adapting Heritage: Double Co-Elliptic

For the Orion program, NASA adopted the Double Co-Elliptic trajectory for rendezvous and docking in LEO. To preserve the same concept of operations and time-line procedures for close proximity at L2 as in LEO, the first proposed trajectory is a modified Double Co-Elliptic. Astronauts who have become familiar with piloting the DC will already be familiar with the positions and angles of
In general, a co-elliptical profile is often used to decrease the downrange distance between the target and the chaser while requiring no additional fuel, as seen in Figure (10). The rate of change in downrange, $\dot{x}_0$, is related to altitude, $z_0$, by the equation

$$\dot{x}_0 = \frac{3}{2} \omega z_0, \tag{19}$$

where $\omega$ is the orbital angular rate and the altitude remains constant.

For LEO, the Orion capsule would approach the target on two different co-elliptic altitudes causing the trajectory to be known as the Double Co-Elliptic [26], and is shown in Figure (11). A similar trajectory was also used by Space-X’s Dragon C2/3 for their first rendezvous with the ISS in May, 2012 [27], and Orbital Sciences Corporation’s Cygnus rendezvous with the ISS [28] in September, 2013. In addition, this trajectory is currently planned for Sierra Nevada’s Dream Chaser rendezvous with the ISS [29].

For this study, the rendezvous scenario starts with the chaser initially on a co-elliptic orbit 4 km
Figure 11: Full Double Co-Elliptic Trajectory with Free-Drifts in LEO. Nominal is in blue, with free-drifts in grey. All free-drifts avoid the Approach Ellipsoid (AE) or the Keep Out Sphere (KOS) as necessary. Maneuver times can be seen on the bottom time-line, which sets the rendezvous Start on the far right.
below and 40 km behind the target. Maneuvers M1 and M2 allow the chaser to slow its approach rate, shortly after entering communication range (CommRng). The M1 maneuver transfers the chaser from a 4 km to 1.4 km co-elliptic profile. The chaser enters the second co-elliptic orbit with the M2 maneuver. After this, the chaser drifts along the 1.4 km co-elliptic until permission is granted to perform the Terminal Phase Initiation (TPI) maneuver that causes it to enter the AE. The chaser first aligns just below the target with a series of maneuvers known as R-bar Acquisition. Once permission is granted to enter the KOS, docking is completed using a sequence of glideslope maneuvers (GS or GLS) that gradually brings the chaser to the target.

This trajectory allows for docking in the positive R-bar or V-bar directions, with Figure (11) demonstrating docking along the positive R-bar. If a positive V-bar docking is desired, the chaser uses a TORVA (Twice-Orbital-Rate V-bar-Acquisition) maneuver to place itself on the V-bar, and then continues with the GLS. Also not shown in this particular image of the Double Co-Elliptic are the correction maneuvers that may be employed on the M1 transfer to reduce dispersions. In the nominal trajectory, these would have \( \Delta v = 0 \).

When attempting to adapt a similar trajectory to L2, similar positions and times for maneuvers are used, but for the chaser to arrive at the target within a similar time frame, the trajectory cannot rely on traveling on co-elliptical paths. This is because it must travel the same distances downrange in LVLH, but the downrange rate is proportional to the angular rate. This would require an unreasonable duration time on the co-elliptic trajectory to travel the necessary distances. Instead, a pseudo co-elliptic trajectory is generated using LR targeting where the approach rates comparable to those in LEO are replicated.

The nominal trajectory at L2, with free-drifts, is shown in Figure (12). Due to the free-drift after M1, towards the M2 point, both M1 and M2 maneuvers at L2 are moved farther behind the target while holding the relative locations between the maneuver points the same. Both maneuvers still take place within communication range. The M3 point behaves as the TPI maneuver, where permission is granted to enter the AE.

If the docking axis was along the positive R-bar, the M4 maneuver would act as the holding maneuver, acquiring the docking axis at 500 m below the target. Here, it acts as a correction maneuver to the positive V-bar. In this instance, the Hold Point maneuver, HP1, acquires and maintains the docking axis at 500 m downrange. This is the first maneuver that sets the chaser to the docking diamond, a strategy used for all the L2 trajectories presented in this thesis, and displayed in Figure (13). By approaching the target at a 45° angle, the chaser nominally approaches no closer than 350 m to the target, which provides a margin from the 200 m KOS. Adapting the
Figure 12: Double Co-Elliptic Trajectory at L2, with Free-Drifts and Time-Line
diamond profile allows docking on any axis, as it can also be rotated 90° to allow for docking on the H-bar. After acquiring the docking axis, docking is again completed using the glideslope (GLS1) maneuvers.

The range-rate profile can be seen in Figure (14), with each maneuver decreasing the approach rate until the Hold Point. The M1 and M2 $\Delta v$ vectors are approximately parallel with the z-axis, with the spacecraft significantly slowing only in the downrange direction at the M3 maneuver, with only 34 minutes remaining until GLS1. Delta-v usage remains low, despite using pre-defined transfer times. The initial conditions place an initial velocity of 5.74 m/s, while the full trajectory has a $\Delta v$ of 8.84 m/s. The entire flight duration is 3.38 hours.

Also briefly considered was the Stable Orbit Rendezvous, flown frequently by the Space Shuttle [30]. An image of the trajectory can be seen in Figure (15). However, two key points make adapting this trajectory at L2 difficult. First, in order to reach the same relative positions at required times, a large amount of fuel is consumed in order to reach the proper altitudes. The orbital features present in LEO that allow the chaser to reach 12 km below the V-bar are not present at L2. Altering this altitude would violate the desire to keep a consistent concept of operations at both LEO and L2.

Second, the original benefits of this trajectory will not be present at L2. If the maneuvers are completed in LEO, this trajectory is inherently safe, as nominally the chaser will never drift above
Figure 14: Range-Rate vs. Time for the Double Co-Elliptic Trajectory
Figure 15: Stable Orbit Rendezvous. All free-drifts never cross the x-axis. Approach Initiation (AI) has the same role as the TPI of the DC maneuver, allowing entrance to the AE.
the V-bar, which will not be the case at L2. Also, by consistently performing maneuvers along
the V-bar, if the mission must be paused for any reason, the chaser can do a holding maneuver
as opposed to an approaching maneuver. The chaser and the target can stay at the same relative
distance, as can be seen from Equation (19) with $z_0 = 0$. This is vital for LEO, but for L2, the
chaser can stop at any location in close proximity to the target, and the drift that would result
would be minimal due to the decreased orbital angular rate.

3.2 Line-of-Sight Corridor

At L2, the relative dynamics support the possibility of approaching the target directly along the
docking port. This allows the final destination to always be within sight and simplifies the approach
trajectory toward the target. To ensure it still possesses sufficient safety characteristics, several
additional features are added, such as never approaching the target directly, but instead between a
set of user defined angle.

In total, the Line-of-Sight Corridor trajectory is defined by four angles: an approach angle, $\theta$;
two trigger angles, $\alpha$ and $\beta$; as well as an offset angle less than either trigger angle, $\phi < \alpha$ and $\phi < \beta$.
These are illustrated in Figure (16). This strategy assumes that the chaser approaches the target a
small distance below and parallel to the docking axis, which can be rotated from the positive $x$-axis
of the LVLH frame by the approach angle, $\theta$. For simplicity of demonstration, the approach angle
is set to zero, though it can be accounted for through a rotation matrix. The chaser continues on
this profile until it reaches the first trigger angle, $\alpha$, defined counter-clockwise from the docking axis.
The chaser then uses a burn perpendicular to the $x$-axis in order to rotate the velocity vector a total
of $(\alpha + \phi)$ counterclockwise.

The chaser continues on its altered trajectory until reaching the second trigger angle, $\beta$. This
angle is defined as clockwise from the docking axis, and defines the upper boundary of the approach
corridor. At this point, the chaser once again burns perpendicular to the docking axis until it has
rotated an angle $(\beta + \phi)$ clockwise. It then continues on this new profile until returning to the first
trigger angle, where it repeats the first burn. This process repeats until reaching the desired range.
At all times, with possible exception of the rendezvous initiation conditions which are previously
determined as safe, the chaser never approaches the target at any angle closer than $\phi$, thus making
the offset angle a safety angle.

Assuming rectilinear motion, these maneuvers create a series of similar triangles, seen in Figure
(17). An application of the law of sines and mathematical induction allows one to solve for the $n$th
Figure 16: Derivation of the Line-of-Sight Corridor. The top image (a) displays the tilt of the target with respect to the LVLH axis. The bottom image (b) displays the chaser arriving at either trigger angle, and the resulting rotation in the tilted target axis frame. The dashed lines represent the approach corridor of angle \((\alpha + \beta)\).
Figure 17: Geometry of the LoS-Corridor. By odd (a) or even (b) steps, $n$. 
targeting point as

$$r_f(n) = \begin{cases} A_n \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \end{bmatrix}^\top & \text{if } n \text{ is odd} \\ A_n \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \end{bmatrix}^\top & \text{if } n \text{ is even} \end{cases} = \begin{bmatrix} x_n & y_n & z_n \end{bmatrix}^\top, \quad (20)$$

where

$$A_n = A_0 \left[ \frac{\sin(\phi)}{\sin(\alpha + \beta + \phi)} \right]^n, \quad (21)$$

and $A_0$ is the range that the chaser first arrives at the corridor.

In order to keep the downrange velocity component constant, for the purpose of conserving fuel, the transfer time is proportional to the distance traveled downrange. That is,

$$\Delta t_n = \frac{\Delta x_n}{\Delta X_{corridor}} \cdot T_{corridor} = \begin{cases} \frac{B_n \cdot \cos(\alpha + \phi) \cdot T_{corridor}}{\Delta X_{corridor}} & \text{if } n \text{ is odd} \\ \frac{B_n \cdot \cos(\beta + \phi) \cdot T_{corridor}}{\Delta X_{corridor}} & \text{if } n \text{ is even} \end{cases}, \quad (22)$$

where

$$B_n = A_0 \left[ \frac{\sin(\beta + \alpha)}{\sin(\phi)} \right] \left[ \frac{\sin(\phi)}{\sin(\alpha + \beta + \phi)} \right]^n, \quad (23)$$

and $T_{corridor}$ is the total time traveled within the corridor, with the total downrange distance $\Delta X_{corridor}$. With the computed target position and the transfer time, LR targeting determines the required thruster burns for all maneuvers.

The LoS-C grants three possible benefits:

I. It allows ease of piloting for an LVLH or docking axis pointing chaser.

II. It keeps the chaser within a given field-of-view, $(\alpha + \beta)$, of the target until entering the docking diamond, allowing it to use sensors with a restricted field-of-view, or use reflectors on the target that are only visible within a restricted field-of-view, defined from the docking axis.

III. It facilitates the use of angles-only navigation, by constantly and predictably changing the bearing of the target to the chaser.

The second trigger angle $\beta$ does not have to be positive, as long as $(\alpha + \beta) > 0$. If $\beta = -\phi$, the parallel approach is the result, where half of the trajectory is parallel to the docking axis, as shown in Figure (18). This trajectory would be an intuitive approach to pilot, as it would allow the chaser the
desired velocity direction for docking while constantly approaching the docking axis. Any deviations would be easily detectable and alert the user to abort. However, permission to enter the approach ellipsoid must be granted early in the trajectory time-line, especially if the vehicle is not approaching along the V-bar.

Another option is the symmetric approach, where $\beta = \alpha$. This would allow full range of motion for any angles-only-navigation or reflectors that are symmetric about the docking axis. Such an approach was used for the chosen trajectory shown in Figure (19). The trajectory starts 4 km below and 50 km ahead of the target, with an initial speed to reach the z-axis in approximately 3 hours. Here, $\alpha = \beta = 10^\circ$, and $\phi = 7.5^\circ$. These values allow the first maneuver, M1, to be taken inside radar range, and still be able to safely avoid the approach ellipsoid at any approach angle $\theta$. M2 acts as the TPI maneuver, entering the AE while avoiding the KOS. The chaser drifts to the M3 point, where it is set to the docking diamond. After this, all further maneuvers are similar to the DC trajectory.

The initial conditions place a downrange velocity of $-4.63$ m/s. The spacecraft is allowed to continue with constant downrange velocity until the M2 maneuver, which decreases it considerably, as shown in Figure (20). The amount of maneuvers are minimized, reducing $\Delta v$ to a total of 9.36 m/s.
Figure 19: The Line-of-Sight Corridor Trajectory
Figure 20: Range-Rate vs. Time for the LoS-C Trajectory
3.3 Line-of-Sight Glide

Similar to the Line-of-Sight Corridor, the Line-of-Sight Glide assumes rectilinear motion to allow the chaser to approach with the target always within sight, but safely between a trigger angle and an offset angle. However, in an effort to save fuel, the offset angle is not set across the line-of-sight between the chaser and the target, but on the same side as the trigger angle. Although this increases the number of maneuvers, the ∆v required per maneuver is drastically reduced.

The Line-of-Sight Glide is defined by three angles: an approach angle, θ, a trigger angle, α, and an offset angle, φ < α. Unlike the LoS-C, β is the angle of rotation, determined to be (α − φ). This can be seen in Figure [21]. Once again the chaser approaches the target a small distance below and parallel to the x-axis, although this time the docking axis is the positive z-axis as opposed to the x-axis. The approach angle can be defined as earlier, although for this scenario it is still set to zero. The chaser continues on this profile until it reaches the trigger angle, α, now defined as the angle between the velocity vector and position vector of the chaser. However, instead of rotating (α + φ), it rotates (α − φ) = β counter-clockwise, and continues on. The chaser continues to move on this segment, until it hits the trigger angle α, rotates β, and repeats this until it reaches the desired range.

These maneuvers create one set of similar triangles, each of which rotates depending on the step number, n. The nth targeting point is

\[ r_f(n) = A_n \begin{bmatrix} \cos(\alpha + n\beta) & 0 & \sin(\alpha + n\beta) \end{bmatrix}^T = \begin{bmatrix} x_n & y_n & z_n \end{bmatrix}^T, \]

\[ (24) \]

where

\[ A_n = A_0 \left( \frac{\sin(\phi)}{\sin(\alpha)} \right)^n = A_0 \left( \frac{\sin(\alpha - \beta)}{\sin(\alpha)} \right)^n, \]

\[ (25) \]

and A₀ is the range that the chaser first reaches the trigger angle. If given an initial downrange value, z₀, with the chaser traveling parallel to the x-axis, then \( A_0 = \frac{z_0}{\sin(\alpha)} \). To land directly on the docking diamond, the final step, \( n_f = 45^\circ/\beta \), must align precisely on the line formed by the docking diamond, \( x_{n_f} = z_{n_f} - 500 \) m. If \( z_0 = 4000 \) m, solving for the trigger angles based upon the chosen \( n_f \) or \( \beta \) results in the values displayed by Table [1]. Increasing the number of maneuvers beyond 15 was not worth considering due to thruster inaccuracy per maneuver, while larger rotation angles caused significant free-drift dispersions.
Figure 21: Derivation of the Line-of-Sight Glide. The top image (a) displays the chaser arriving at the trigger angle twice, and the resulting rotation. The bottom (b) displays the similar triangles made by step $n$. $\beta = \alpha - \phi$.
Eventually $\beta = 5^\circ$ was chosen to allow for a sizable amount of maneuvers to correct free-drifts, without unnecessarily increasing the maneuver count. The completed trajectory is shown in Figure (22). M1 is a correction maneuver once inside communication range, while GC1 is a correction maneuver after the LoS maneuvers are finished and the spacecraft is drifting on the docking diamond to the z-axis. Either M2 or G1 can act as the TPI maneuver.

To minimize $\Delta v$, all maneuvers after M2 strictly decrease the approach rate, which must be neutralized by the HP1 maneuver regardless. This can be seen in Figure (23). This requires the transfer times to be proportional to the altitude traveled per maneuver, or

$$\Delta t_n = \frac{\Delta z_n \cdot T_{glide}}{\Delta Z_{glide}} = \frac{B_n \cdot \sin(n\beta) \cdot T_{glide}}{\Delta Z_{glide}},$$

where

$$B_n = A_0 \left[ \frac{\sin(\beta)}{\sin(\phi)} \right] \left[ \frac{\sin(\phi)}{\sin(\alpha)} \right]^n,$$

and $T_{glide}$ is the total time to travel the altitude $\Delta Z_{glide}$ on the glide. LR targeting is once again used for all maneuvers up to the glideslope.

The LoS-G benefits include:

I. Minimal $\Delta v$, reducing fuel costs.

II. Ease of piloting, requiring tracking only the difference between the LVLH position and velocity vectors.

III. Minimal attitude adjustment for a maneuver pointing chaser, as all burns on the glide are in a uniform direction.

As highlighted with the range-rate profile in Figure (24), the initial conditions place a downrange velocity of -4.60 m/s, along with an altitude rate of 0.021 m/s to account for orbital mechanics over 39 km. The total $\Delta v$ is equal to 6.00 m/s. The drawback is the time length of the rendezvous trajectory, a total of 5.67 hours. This could be reduced by reducing $T_{glide}$, which was set to approximately 3
Figure 22: The Line-of-Sight Glide Trajectory
Figure 23: The Δv's of the LoS-G. Delta-v vectors are shown in red, for M2 to GC1.

hours, but at the cost of additional Δv. For example, if $T_{glide}$ is instead set to 1 hour, the total time length would be reduced by slightly less than two hours, at the additional Δv of approximately 1.5 m/s. This alternate time-line would be closer in comparison to the DC and LoS-C trajectories, but would not take advantage of the Line-of-Sight Glide’s ability to minimize fuel consumption.

Figure 24: Range-Rate vs. Time for the LoS-G Trajectory
3.4 Trajectory Summary

Table (2) summarizes the maneuvers performed for the Double Co-Elliptic, Line-of-Sight Corridor and the Line-of-Sight Glide. The maximum $\Delta v$ for each scenario, along with the total number of impulsive maneuvers, is included. The Double Co-Elliptic takes the shortest time, has the lowest maximum $\Delta v$, and has the closest TPI maneuver. The Line-of-Sight Corridor has the fewest maneuvers, while the Line-of-Sight Glide has the lowest fuel usage. All trajectories were designed with a focus for ease of piloting, either through a heritage concept of operations or constant angle triggers.

To justify a decision among the possible trajectories, evaluating the trajectories in a quantitative manner can increase objectivity. To this end, an application of multiattribute utility theory would allow a systems engineer to create a utility function and derive weighting factors to select a suitable trajectory. For more information, please refer to “Chapter 9: Decision Analysis and Support” of Systems Engineering: Principles and Practice, by Kossiakoff, Sweet, Seymour, and Biemer [31].

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Time (hours)</th>
<th>Max $\Delta v$ (m/s)</th>
<th>Total $\Delta v$ (m/s)</th>
<th>Maneuvers</th>
<th>Range of TPI (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>3.38</td>
<td>2.94</td>
<td>8.84</td>
<td>4</td>
<td>1.664</td>
</tr>
<tr>
<td>LoS-C</td>
<td>3.96</td>
<td>3.03</td>
<td>9.36</td>
<td>3</td>
<td>6.072</td>
</tr>
<tr>
<td>LoS-G</td>
<td>5.67</td>
<td>2.13</td>
<td>6.00</td>
<td>11</td>
<td>8.702 to 11.395</td>
</tr>
</tbody>
</table>

Table 2: Nominal Trajectory Comparison
4 Stochastic Considerations

With the trajectory designs complete in a deterministic sense, random aspects of the system are now considered. The desire is to predict the potential difference between the intended and actual states of the spacecraft and contain these trajectory dispersions. Trajectory dispersions throughout the duration of the profile are directly caused by sources such as the initial trajectory dispersions at the start of proximity operations, imperfect thrusters, modeling errors, and disturbance accelerations. In this thesis, the error distributions for each of these sources are fixed according to the pre-determined value for each scenario.

The trajectory dispersions caused by these parameters can be counteracted by determining the type, quality, and quantity of sensors the spacecraft is using, or the spacecraft’s sensor suite. Ultimately, the top level mission requirements specifying constraints on the trajectory dispersions indirectly determines how accurately the position and velocity must be known by the navigation system. By combining an accurate navigation system with targeting maneuvers developed in the previous sections, dispersions can be contained. This allows a chaser vehicle more direct approaches towards the target, while maintaining a safe trajectory.

Therefore, the cost and mass of higher quality and quantity of sensors must be factored against the improved performance to find an optimal sensor suite. To capitalize on the speed of linear covariance analysis and the robustness of Monte Carlo analysis, this research employs both techniques to evaluate the effects of stochastic processes on the trajectory design. This thesis uses LinCov simulations for solving the required accuracy of the sensor suite, and Monte Carlo for verification. The fundamental derivation of a rendezvous LinCov simulation tool can be found in Linear Covariance Techniques for Orbital Rendezvous Analysis and Autonomous Onboard Mission Planning, by Geller [21], and the basic concepts utilized in this work are summarized in “Appendix: Linear Covariance”. The Monte Carlo simulation does not linearize the equations of motion for any run, and can thus verify that the LinCov simulation linearization used for propagating the covariance matrix during the rendezvous and close proximity operations does not affect the accuracy of its results.

First, the assumptions that allow the use of the implemented stochastic techniques are listed and the stochastic variables defined. These principles provide the foundation for deriving a fast and reliable approach of analyzing the complex inter-dependency of the GN&C system to the proposed rendezvous trajectory. By quantifying the difference between the desired location, the estimate of the navigation system, and the true location; it provides the framework to quickly identify the effects of stochastic processes on the trajectory design, minimize them, and ensure a safe rendezvous
Second, stochastic navigation is introduced. Stochastic navigation provides an effective way of representing the navigation requirements or anticipated system performance without having to model particular sensors and implementing navigation filters. During the initial phases of the trajectory design, this feature enables the designer to see the impact the navigation system has on the trajectory dispersions even though many aspects of the navigation system have not been designed or developed yet. This allows the specification of multiple navigation regions at different locations along trajectory, corresponding to the times of key maneuvers. Each navigation region has a specific and constant navigation error bound. Both the Monte Carlo and LinCov simulations determine the resulting trajectory dispersions that result from utilizing stochastic navigation as a model of the navigation system.

Third, requirement derivation is applied. Alongside stochastic navigation, it is possible to determine how each error source affects the total dispersion. The navigation error of the selected navigation region is then linearly scaled until the free-drift dispersions approach the corresponding safety region, but do not intersect it. This scaled navigation error is the maximum allowable error for maneuvers within the navigation region. Scaling is completed without re-running the simulation, through sensitivity analysis. Sensitivity analysis allows the designer to determine the contributions individual error sources have upon the total dispersions at any time instant, and, specifically, to determine the effects of the navigation error. These results are verified through Monte Carlo simulations.

The final section is the sensor suite analysis. After solving for the allowable navigation error for each navigation region, sensors that satisfy the requirements are determined through simulating different potential combinations. The sensor suites are the final product of the navigation analysis. The sensor and actuator models can also be found in Multiple Event Triggers in Linear Covariance Analysis for Spacecraft Rendezvous [22].

The basic techniques of incorporating stochastic navigation in linear covariance analysis to derive navigation requirements through sensitivity analysis and identify optimal sensor suites are introduced by David Woffinden and Louis Breger in Automated Derivation and Verification Requirements for On-Orbit Rendezvous [32]. Although there are several alternative techniques that could adequately model stochastic processes, these techniques implemented within LinCov allow one to rapidly determine the necessary navigation requirements. For the derivation of these techniques in full, refer to the aforementioned article. However, due to their relevance to this research, the key results are highlighted below.
4.1 Stochastic Variables

There are several assumptions conventionally used in Kalman filters, which serve as foundations for linear covariance analysis. This includes that all modeled noise consists of zero-mean, white, Gaussian random processes. Generally, the noise is assumed to be driven by broadband sources, justifying modeling it as white noise. The amplitude of the noise is also assumed to be collectively caused by a number of smaller sources. Theoretically, the resultant summed effect is approximated by a Gaussian distribution, as stated by the central limit theorem.

In general, the stochastic vectors consist of $n$ states in an $n \times 1$ column vector, unless the navigation states are not one-to-one with the true states. In this case the navigation state consists of $\hat{n}$ states in an $\hat{n} \times 1$ column vector, and $N$ is the $\hat{n} \times n$ mapping matrix, which maps the navigation states onto the true or design states. All states, dispersions, and errors are shown in Figure 25.

![Figure 25: Stochastic Variables for Linear Covariance and Monte Carlo](image)

To introduce the fundamental concepts of linear covariance analysis, several key GN&C state variables must be defined. The actual location of the spacecraft is the true state, denoted by $x$ and shown in red. The desired location along the designed trajectory is the nominal state, denoted by $\bar{x}$ and shown in blue. The estimated location of the spacecraft by the navigation system is the navigation state, denoted by $\hat{x}$ and shown in green. The design state used to derive the navigation filter equations is denoted by $x$, and shown in violet.

Next, the dispersion and error deltas, or perturbations, between the states are defined. The true dispersion, $\delta x \triangleq x - \bar{x}$, indicates how far the spacecraft may deviate from the desired location along the trajectory. The navigation dispersion, $\delta \hat{x} \triangleq \hat{x} - \bar{x}$, indicates how far the spacecraft’s navigation estimate is from the desired location along the trajectory. The true navigation error, $\delta e \triangleq x - N\hat{x}$, indicates how far away the navigation estimate is from the true state.

The onboard navigation error, $\delta e \triangleq x - N\hat{x}$, estimates how accurately the onboard filter can determine the spacecraft’s location. The onboard navigation error is never computed, but its covariance is used by the navigation filter to optimally update the estimate of the vehicle’s state. The true
state is only known in simulations, whereas in actual flight the true state is unknown. Consequently, the onboard navigation error is not defined with respect to the true state, but the design state which approximates the true state when deriving the onboard filter equations.

Overall, it is desired that all perturbations are kept as low as possible, but the effects of random processes naturally drive them to greater values. In a linear covariance simulation, the perturbations are not propagated directly, but through their corresponding covariance matrices. Specifically, this includes \( D \triangleq E[\delta x \delta x^\top] \) the true dispersion covariance, \( \hat{D} \triangleq E[\delta \hat{x} \delta \hat{x}^\top] \) the navigation dispersion covariance, \( P \triangleq E[\delta e \delta e^\top] \) the true navigation covariance, and \( \hat{P} \triangleq E[\delta \hat{e} \delta \hat{e}^\top] \) the onboard navigation covariance.

Finally, \( \delta X \triangleq \begin{bmatrix} \delta x \\ \delta \hat{x} \end{bmatrix} \) is the augmented state, including the true dispersion and navigation dispersion, and \( C \triangleq E[\delta X \delta X^\top] \) is the corresponding augmented state covariance. By taking note of the matrix equations

\[
D = [I_{n\times n}, 0_{n\times n}N^\top] \ C \ [I_{n\times n}, 0_{n\times n}N^\top]^\top,
\]

\[
\hat{D} = [N0_{n\times n}, I_{n\times n}] \ C \ [N0_{n\times n}, I_{n\times n}]^\top,
\]

\[
P = [NI_{n\times n}, -I_{n\times n}] \ C \ [NI_{n\times n}, -I_{n\times n}]^\top,
\]

\[
\hat{P} = \hat{P},
\]

for linear covariance only the augmented state covariance and onboard navigation covariance must be propagated. All the stochastic information needed to create the probability distribution of the dispersions and navigation error of the trajectory at any instant can be obtained from these two covariance matrices. The onboard navigation covariance is constantly re-calculated by the onboard navigation filter.

For comparison, to gather these data from a series of Monte Carlo runs, the equations

\[
D = \frac{1}{N-1} \sum \delta x \delta x^\top,
\]

\[
\hat{D} = \frac{1}{N-1} \sum \delta \hat{x} \delta \hat{x}^\top,
\]

\[
P = \frac{1}{N-1} \sum \delta e \delta e^\top,
\]
are used. The onboard navigation covariance, $\hat{P}$, is compared on a run by run basis to the true navigation covariance, $P$, to ensure the onboard filter is properly tuned.

4.2 Stochastic Navigation

Instead of fully incorporating sensor models and filter algorithms to capture the effects of the navigation system, it is perhaps useful to directly perturb the true states by an error bound. This concept is called stochastic navigation, and it is central to the derivation of navigation requirements. In this context, the navigation state is modeled as the true state plus a bounded error, denoted as $\delta \bar{e}$. Then,

$$\dot{x} = x + \delta \bar{e}. \quad (30)$$

The 3-$\sigma$ error bound of $\delta \bar{e}$ can be derived from multiple sources. For this research, it will be determined through requirement derivation. However, it could also be arbitrarily specified by the analyst, it could represent the prior performance of a particular navigation filter, or it could simply represent proposed navigation requirements. Stochastic navigation is implemented differently in Monte Carlo and linear covariance, and is outlined as follows.

4.2.1 Stochastic Navigation with Monte Carlo Simulations

For Monte Carlo simulation, stochastic navigation can be implemented through a first order Markov Process, or an Exponentially Correlated Random Variable (ECRV). The bounded error $\delta \bar{e}$ at any time instant is obtained by sampling from the probability distribution corresponding to the bounded error covariance matrix $\bar{P} \triangleq E[\delta \bar{e} \delta \bar{e}^\top]$. Even though the distribution of the error is defined at any time instant, how one time instant correlates to another during a single run can be specified.

If $z_k$ is an ECRV at time $k$, $\Delta t$ is the step size between the intervals $k$ and $k + 1$, $\tau$ is the ECRV time constant, and $\eta_k$ is white noise with the power spectral density, $Q_k$, where $Q_k \delta_{kk'} = E[\eta_k \eta_{k'}^\top]$, then the ECRV at the time $k + 1$ can be determined as

$$z_{k+1} = z_k e^{-\Delta t/\tau} + \eta_k. \quad (31)$$

The noise strength is selected so the steady state variance of the ECRV remains constant and has a value of one. That is,
Figure 26: The ECRV with Differing $\tau$ Values. The violet line shows a single sample, while the grey lines are the culmination of all samples. The two blue and black lines show the expected and actual 3-$\sigma$ values, respectively. [32]

$$Q_k = (1 - e^{-2\Delta t/\tau})^2.$$

With a large $\tau$ value, the ECRV approaches a constant value. With $\tau = 0$, the ECRV behaves as a white noise. This can be seen in Figure (26).

Finally the stochastic navigation error at time $k$ can be obtained by utilizing a vector of ECRV’s, $z_k$, and $\bar{P}$ as

$$\delta\bar{e}_k = \sqrt{\bar{P}} \cdot z_k.$$

With the initial value of the navigation chosen simply as a random Gaussian variable, the time evolution of $z_k$ can be determined. Equation (33) can then be used to sample the bounded error covariance matrix to solve for the bounded error at any time instant during the trajectory.

4.2.2 Stochastic Navigation with Linear Covariance Overview

Applying stochastic navigation within LinCov is a four step process: initialization, propagation, update, and correction. These correspond to the same steps taken by LinCov while simulating navigation errors with typical sensors, and occur after the LinCov simulation has taken other dispersion sources into account. For the stochastic navigation implementation in this research, it is desired that the navigation error does not change during time propagation or maneuver correction; it changes only when the vehicle enters a new stochastic navigation region, replacing typical sensor updates.
Initialize the Covariance Matrix for Initial Dispersion and Navigation Error

First, the initial augmented state covariance matrix, $C_0$, and initial onboard navigation covariance, $\hat{P}_0$, must be determined through the initial true dispersion, $\delta x_0$, and bounded error $\delta e_0$. Assuming the initial trajectory dispersions and initial navigation error are independent yields

\[
C_0 = \begin{bmatrix}
D_0 & D_0 N^\top \\
ND_0 & ND_0 N^\top + \bar{P}_0
\end{bmatrix},
\]  

(34)

and

\[
\hat{P}_0 = \bar{P}_0,
\]  

(35)

where

\[
D_0 = E[\delta x_0 \delta x_0^\top],
\]  

(36)

and

\[
\bar{P}_0 = E[\delta e_0 \delta e_0^\top].
\]  

(37)

All values are derived from the initial dispersions and navigation error.

Unlike rendezvous scenarios in LEO, for close proximity operations at L2, the correlation values between downrange-rate and altitude are near zero due to the linear, uncoupled relative dynamics. This observed trend is adopted for this research.

It is noted that the augmented state covariance matrix must be positive definite and Equation (34) ensures this unique property is satisfied during initialization. During initialization and the subsequent phases of propagation, update, and correction; checks are also included in the simulation to ensure the user has properly incorporated the stochastic navigation error to preserve this property. Otherwise, the fundamental linear operations corresponding to each of these segments preserves the positive definite attribute of the augmented state covariance matrix.

Propagate the Covariance Matrix through Time

After initialization, the next step is to propagate the covariance matrices forward through time. The states that are under stochastic navigation must be rewritten with their previous values, while those that are not are allowed to propagate normally. This involves altering the time derivative of
the covariance matrices and defining a new matrix specifying which states will be under stochastic
navigation. These equations will be re-applied after every time-step. Matrices with an asterisk
(Example: $Y^*$) are matrices after stochastic navigation has overwritten the new propagated values
with their previous values. Specifically,

$$\dot{C}^* = A\dot{C}A^\top,$$

(38)

and

$$\dot{P}^* = \hat{A}\dot{P}\hat{A}^\top,$$

(39)

where $A$ is the augmented navigation reset matrix, and $\hat{A}$ is the onboard navigation reset matrix.
Both matrices set the derivative of the selected navigation states to zero, and are defined as

$$A = \begin{bmatrix} I & 0N^\top \\ HN & (I - H) \end{bmatrix},$$

(40)

and

$$\hat{A} = (I - H).$$

(41)

Here, $H$ is the selector matrix, specifying which states are under stochastic navigation. For instance,
if all states were to be specified through stochastic navigation, $H$ would be the identity matrix of
size $\hat{n} \times \hat{n}$, while if all states were to ignore stochastic navigation, $H$ would be the zero matrix of the
same dimension.

**Update the Covariance Matrix for New Navigation Regions**

If during a time-step a new navigation region is entered, the navigation error must be updated
and rewritten again. Instead of being rewritten to its previous time-step value, however, it is now
rewritten to the new desired value, $\delta e^+$. That is,

$$\delta e^+ = \delta \bar{e},$$

(42)

with
The post-updated augmented state covariance and onboard navigation covariance matrices ($C^+$ and $\hat{P}^+$) are then given in relationship to their pre-updated values ($C^-$ and $\hat{P}^-$) by the equations

$C^+ = AC^-A^\top + B\bar{P}B^\top,$  \hspace{1cm} (44)  

and

$\hat{P}^+ = \hat{A}\hat{P}^-\hat{A}^\top + \hat{B}\bar{P}\hat{B}^\top,$  \hspace{1cm} (45)  

where $A$ and $\hat{A}$ null out the selected stochastic navigation states. Simultaneously, $B$, the augmented navigation inclusion matrix, and $\hat{B}$, the onboard navigation inclusion matrix, defined as

$B = \begin{bmatrix} 0N^\top \\ -H \end{bmatrix},$  \hspace{1cm} (46)  

and

$\hat{B} = H,$  \hspace{1cm} (47)  

incorporate the new values of $\bar{P}$ into the post-updated augmented state covariance and the onboard navigation covariance matrices.

**Correct the Covariance for Impulsive Maneuvers**

When a maneuver occurs, both the trajectory dispersions and navigation errors are altered. Similar to propagation, this change to both the navigation errors and trajectory dispersions is undone and reset to previous values. If a superscript $-c$ represents pre-maneuver, $+c$ represents post-maneuver before correction, and $+c^*$ represents post-maneuver after correction, correct the covariance matrix as

$C^{+c^*} = AC^{+c}A^\top + FC^{-c}F^\top,$  \hspace{1cm} (48)  

and
\[ \hat{P}^{+c^*} = \hat{A} \hat{P}^{+c} \hat{A}^T + \hat{F} \hat{P}^{-c} \hat{F}^T, \]

where \( F = (I - A) \) is the augmented covariance maneuver nulling matrix, and \( \hat{F} = (I - \hat{A}) \) is the onboard navigation maneuver nulling matrix. These matrices are determined to be

\[ F = \begin{bmatrix} 0 & 0N^T \\ -HN & H \end{bmatrix}, \]

and

\[ \hat{F} = H. \]

In this manner, \( C^{+c^*} \) and \( \hat{P}^{+c^*} \) obtain values from their corresponding pre-maneuver matrices, \( C^{-c} \) and \( \hat{P}^{-c} \), through the maneuver nulling matrices, \( F \) and \( \hat{F} \). Otherwise, they obtain values from their corresponding post-maneuver before correction covariance matrices, \( C^{+c} \) and \( \hat{P}^{+c} \), through the navigation reset matrices, \( A \) and \( \hat{A} \). Consistently, all maneuvers are assumed to be instantaneous. These equations are applied after every maneuver.

### 4.3 Sensitivity and Requirement Derivation

With stochastic navigation implemented, the focus is now to scale the navigation error to determine the navigation error bound, and ultimately the navigation requirements. Given the linear nature of linear covariance, linear superposition can be applied to the variance of any element of the true state \([33, 32]\). The total dispersion, \( \sigma_p^2 \), can be expressed as

\[ \sigma_p^2 = \sum_{i=1}^{n} \sigma_{p|\delta_i}^2. \]

Here, \( p \) represents any performance index such as the navigation error, arrival time uncertainty, maneuver execution error, range-rate dispersions, etc.; but for the ensuing developments it designates the total relative trajectory dispersions. For example, \( \sigma_{p|\delta_1} \) could be the total dispersion due to the initial dispersions, \( \sigma_{p|\delta_2} \) could be the total dispersion due to process noise, etc. With this equation, sensitivity analysis is implemented, where the total dispersion is divided into a root-sum-square (RSS) of its individual sources at any point during its trajectory, as shown in Figure \([27]\). In this example, the free-drift dispersions due to the navigation error at the time of the M1 maneuver, ‘Region 1 Sensor Error’, is compared to the dispersions due to all other sources. In addition, the dispersions of the root-sum-square of the individual sources can be seen to match the total dispersions.
of all sources enacting simultaneously.

Requirement derivation follows easily from sensitivity analysis. By adding scaling coefficients, a larger or smaller total dispersion, $\sigma^2_{p^*}$, can be obtained as

$$\sigma^2_{p^*} = \sum_{i=1}^{n} \alpha_i^2 \sigma^2_{p_i|\delta_i}. \quad (53)$$

In this application, only one source is scaled: the navigation error based on the final relevant navigation region. This coefficient is solved through a bi-sectional search, such that the 3-$\sigma$ dispersions of the trajectory do not intersect with a pre-determined safety region but do come within a specified tolerance. An example of this can be seen for the first maneuver of the Double Co-Elliptic trajectory in Figure (28). The scaling factor, $\alpha$, corresponds to the navigation requirement of the first maneuver. The blue ellipses represent the 3-$\sigma$ trajectory dispersions that result from the current scaling factor. The full process of scaling takes significantly less time than a full LinCov run; each step of the scaling sequence below is completed in only a fraction of a second.

### 4.4 Sensor Suite Analysis

After solving for navigation requirements, or the error bound for each navigation region of a trajectory, a sensor suite analysis can be conducted. The sensor suite analysis considers a full combination of available sensors, and simulates how each would affect the navigation states throughout an entire trajectory, by determining the navigation error that would occur from that specific combination. This is completed by linear covariance through a Kalman filter, with processes introduced in Stochastic Models, Estimation, and Control, Volume I by Maybeck [33], and expanded for rendezvous applications in Linear Covariance Techniques for Orbital Rendezvous Analysis and Autonomous Onboard Mission Planning by Geller [21].

The resulting navigation performance generated by each sensor suite are then compared to the navigation requirements found through the requirement derivation, determining if the sensor suite is acceptable or not. An example of this can be seen in Figure (29). A sensor suite satisfies all requirements if the navigation accuracy provided is greater than the allowable navigation error for all navigation regions in both position and velocity, for all three axes. This can be seen visually in Figure (29); if the color corresponding to a certain sensor suite is above the black constant values for any time duration in any graph, the sensor suite fails. Sensors are not determined for the final, straight approach from five-hundred to ten meters; the sensor suite analysis only considers the phase of flight starting at close proximity operations until the final approach.
Figure 27: Sensitivity of Dispersions due to Navigation Errors. Although at first dispersions are primarily a result of ‘Others’, which includes initial dispersions, dispersions due to sensor error at the time of M1, ‘Region 1 Sensor Error’, grow after the targeted maneuver. Note how the ‘All’ error ellipse matches the ‘RSS’ error ellipse, showing that the dispersion due to all the sources is equal to the root-sum-square of the individual sources.
Figure 28: Requirement Derivation through Bi-Sectional Search. Nine steps of a bi-sectional search for the error bound, the navigation requirements for the navigation region containing the first maneuver of the Double Co-Elliptic trajectory. Ends with a scaling factor of 3.1086, with position dispersions approximately tangent to the Approach Ellipsoid.
The sensors will also be weighted, for the spacecraft’s limiting factors such as monetary cost or physical mass, so that an optimal sensor suite can be selected from all passable combinations. This is another opportunity to implement multiattribute utility theory to aid in the decision making process. For ease of demonstration, in this thesis high quality, medium quality, or low quality sensors are given weights of three, two, or one, respectively. If a particular sensor is not used, it is given a weight of zero.
Figure 29: A Sensor Suite Analysis. The constant black lines represent the maximum allowable navigation errors for downrange (drng), cross-track (ctrk), and altitude (alt) in both position (r) or velocity (v). Each color corresponds to a separate sensor suite. Six sensor suites fail in the downrange position requirement. Practically instantaneous decreases in navigation error can be seen when a particular sensor of a suite enters its operational range.
5 Navigation Analysis

This section utilizes the developed stochastic methods to analyze the three previously designed L2 rendezvous trajectories. The uncertainty parameters assumed are generic values used for LEO trajectories, and are listed in Table 3. Stochastic variables are all modeled as zero-mean Gaussian distributions, and are thus determined completely by their $3\sigma$ values. All initial trajectory dispersions are placed on the target. Process noise is determined through simulation, such that if all other stochastic processes are excluded, either vehicle has a $3\sigma$ dispersion in any direction of 50 m over 90 minutes. The time constant for stochastic navigation used in Monte Carlo runs is chosen to model white noise, to match with LinCov. For this thesis, all the scenarios utilize the same parameters.

<table>
<thead>
<tr>
<th>Stochastic Variable</th>
<th>$3\sigma$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Position Dispersion</td>
<td>$[1500 500 500]$ m</td>
</tr>
<tr>
<td>Initial Velocity Dispersion</td>
<td>$[0.3 0.1 0.1]$ m/s</td>
</tr>
<tr>
<td>Process Noise</td>
<td>$[0.006 0.006 0.006]$ mm/(s/s)</td>
</tr>
<tr>
<td>Thruster Maneuver Execution Errors</td>
<td></td>
</tr>
<tr>
<td>Misalignment</td>
<td>$[1 1 1]$ mrad</td>
</tr>
<tr>
<td>Bias</td>
<td>$[0.40 0.40 0.40]$ mm/s</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>1600 ppm</td>
</tr>
<tr>
<td>Noise</td>
<td>$[1.5 1.5 1.5]$ mm/s</td>
</tr>
<tr>
<td>Markov Time Constant, $\tau$</td>
<td>0.36 s</td>
</tr>
</tbody>
</table>

Table 3: Uncertainty Parameters for LinCov and Monte Carlo Simulations. All vectors in [down-range, cross-track, altitude] format.

Navigation requirements are first derived using the automated approach introduced in Chapter 4, and then implemented and verified through 250 Monte Carlo runs. A sensor suite analysis is applied to determine which combination of sensors will optimally satisfy the requirements.

5.1 Requirement Derivation and Verification

This section derives the navigation requirements for each time interval, or stochastic region, of all three trajectories. Not all maneuvers define a stochastic region, but every maneuver satisfies the free drift safety requirements using the navigation error derived for the specified region. For each maneuver that determines the navigation requirements for a particular segment of the trajectory, a figure is displayed. The figures corresponding to the Double Co-Elliptic (DC) trajectory include (31) to (34). Those corresponding to the Line-of-Sight Corridor (LoS-C) include (37) to (39). Finally, the figures corresponding to the Line-of-Sight Glide (LoS-G) include (41) to (44).

All figures contain three sub-figures. Sub-figure (a) demonstrates the expected trajectory dispersions given the final scaling value of the automated requirement derivation. The derivation of the
Table 4: Percent Probabilities of a Random Variable Landing within an n-σ Ellipsoid

<table>
<thead>
<tr>
<th></th>
<th>1-Dimension</th>
<th>2-Dimension</th>
<th>3-Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-σ</td>
<td>68.3%</td>
<td>39.4%</td>
<td>19.9%</td>
</tr>
<tr>
<td>2-σ</td>
<td>95.4%</td>
<td>86.5%</td>
<td>73.9%</td>
</tr>
<tr>
<td>3-σ</td>
<td>99.7%</td>
<td>98.9%</td>
<td>97.1%</td>
</tr>
</tbody>
</table>

navigation requirements scales the default stochastic navigation 3-σ error of 50 m and 0.057 m/s in downrange, cross-track, and altitude. The LVLH relative position and velocity navigation errors are scaled until the 3-σ dispersion ellipse lies tangent to the pre-defined constraint region.

The resulting derived navigation requirement is subsequently implemented within simulations as a stochastic navigation error. Sub-figure (b) demonstrates the implementation in both the LinCov and Monte Carlo simulations. Each grey line is the navigation error for an individual Monte Carlo run, with the sampled 3-σ bound displayed in black. The navigation error for a single Monte Carlo run is highlighted, to demonstrate how it is correlated in time. The maroon colored 3-σ bound represents the simulated navigation error in LinCov. The selection of 250 Monte Carlo runs ensures the resulting sample covariances can support the initial validation phase, but the exact value is arbitrarily selected to balance computational resources and statistical accuracy.

Finally, the validation of the derived navigation results is completed by comparing the trajectory dispersions of the Monte Carlo simulation to the dispersions predicted by LinCov. Sub-figure (c) shows the trajectory dispersions resulting from the scaled navigation requirements. Each grey line is an individual Monte Carlo run, and the sampled 3-σ ellipses are displayed in black. The red ellipses show the 3-σ bounds predicted from LinCov. For a two dimensional, 3-σ ellipse, 98.9% of the samples are expected to be contained, as shown in Table 4. If linear covariance properly predicts dispersions, rounding to the nearest integer, no more than 3 Monte Carlo runs are expected to lie outside of the red ellipse.

The navigation errors for the preceding maneuver are then used to determine the navigation errors for the following maneuvers. The previous accuracy of the navigation system determines the resulting dispersions of a targeted maneuver, which subsequently affects navigation requirements for the current maneuver. The initial navigation region assumes a scaling factor of 5, or a 3-σ navigation error of 250 m, but becomes irrelevant since no maneuvers are performed in this initial navigation region.

The completed trajectory, with Monte Carlo runs, LinCov analysis, and table summarizing the navigation regions and requirements, is found at the end of each trajectory subsection. For the full trajectory run where the derived navigation requirements for each region are simulated, the
navigation errors for the last 500 m are proportionally reduced from the requirements at 500 m to the requirements specified for docking. The requirements for docking are assumed to have a 3-σ navigation error of 10 cm.

5.1.1 Double Co-Elliptic Requirements

Before the TPI maneuver, the Approach Ellipsoid (AE) is used as the constraint region. The initial conditions for the DC trajectory are selected to ensure the free drift trajectory and the anticipated 3-σ dispersion cases do not enter this space. Through a LinCov simulation based upon the initial velocity and position, no free-drifts are seen to approach the constraint region, as shown in Figure (30).

The first navigation region of the double co-elliptic trajectory begins with the M1 maneuver. The accuracy of the chaser's position and velocity state estimate onboard the spacecraft when executing this first maneuver must ensure that the free-drift trajectory and the corresponding 3-σ trajectory dispersions avoid the AE, as shown in Figure (31a). It is found that a 3-σ navigation requirement of 156 m, corresponding to a scaling factor of 3.1086, ensures the M1 free-drifts do not enter the AE.

To begin validating this derived navigation requirement, it is first implemented as the stochastic navigation error for the first navigation region in the Monte Carlo simulation as shown in Figure (31b), from the 53rd to 87th minute. These times correspond to a short time interval before the M1 and M2 maneuvers, respectively, to provide time for the onboard GN&C system to adequately target the maneuver prior to its execution. The 3-σ navigation error bounds generated from 250 Monte Carlo samples (black) overlays the derived navigation requirement (red). The individual errors for each run (grey) are seen to approximate white noise. Both the error bounds and the white noise approximation demonstrate that the implemented stochastic navigation errors in the Monte Carlo simulation replicate the derived navigation performance requirements.

Figure (31c) compares the LinCov to Monte Carlo trajectory dispersions due to the derived navigation requirement associated with the 3-σ navigation requirement of 156 m. Of the 250 Monte Carlo runs, 3 lie outside of the LinCov 3-σ ellipse near the AE, as expected. One outlier sample clearly enters the AE, but the sampled 3-σ ellipse is consistent with the LinCov prediction.

The navigation requirements for the M2 maneuver are also derived to refrain from entering the AE, as shown in Figure (32a). The derived 3-σ navigation requirement for the second navigation region is 107 m. Figure (32b) shows this derived value implemented in the Monte Carlo simulation starting from the 87th minute to the 134th minute. Figure (32c) shows that both the Monte Carlo and LinCov predictions share a consistent result that the safety condition is satisfied.
Figure (33) demonstrates the derivation of the navigation requirements for the third phase of the trajectory starting with the M3 maneuver. M3 acts as the TPI maneuver, and the M3 free-drift is shown to be safe with respect to the Keep Out Sphere (KOS) with a 3-σ navigation requirement of 90 m. No Monte Carlo run appears to leave the LinCov 3-σ ellipse at the KOS in Figure (33c), although more typical deviations appear to be prevalent before the M3 maneuver or after the chaser drifts past the KOS.

M4 is a pre-docking correction maneuver, and an extended approach corridor acts as a constraint region to restrict dispersions at this time. The V-bar corridor extends a 16° opening in the KOS along the docking axis to the holding point 500 m away from the target. This can be seen in Figure (34a) and (34c). A 3-σ navigation requirement of 24 m is required for the M4 maneuver, as shown in (34a), and completes the final navigation region in Figure (34b). With the high navigation accuracy, M4 greatly decreases any deviations from the nominal, as can be seen in Figure (34c). Dispensions of approximately 200 m are decreased to 10 m as the chaser nears the V-bar corridor. Monte Carlo and LinCov ellipses closely overlap each other as the chaser nears docking. Three runs lie outside of either, by a few meters. However, only one enters the constraint region.
Figure 30: LinCov Simulation using the Initial Conditions of the Double Co-Elliptic. Each red ellipse is the 3-σ predicted dispersion of that time instant.
(a) Requirement Analysis with the Approach Ellipsoid

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs

Figure 31: M1 Free-Drift Trajectory of the Double Co-Elliptic
(a) Requirement Analysis with the Approach Ellipsoid

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs

Figure 32: M2 Free-Drift Trajectory of the Double Co-Elliptic
(a) Requirement Analysis with the Keep Out Sphere

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs

Figure 33: M3 Free-Drift Trajectory of the Double Co-Elliptic
(a) Requirement Analysis with the Extended V-Bar Corridor

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs

Figure 34: M4 Free-Drift Trajectory of the Double Co-Elliptic
<table>
<thead>
<tr>
<th>Navigation Region</th>
<th>Maneuvers</th>
<th>Start Time (min)</th>
<th>Safety Region</th>
<th>Scaling</th>
<th>Position 3-(\sigma) (m)</th>
<th>Velocity 3-(\sigma) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M1</td>
<td>53.0</td>
<td>AE</td>
<td>3.1086</td>
<td>155.43</td>
<td>0.177</td>
</tr>
<tr>
<td>2</td>
<td>M2</td>
<td>87.0</td>
<td>AE</td>
<td>2.1316</td>
<td>106.58</td>
<td>0.122</td>
</tr>
<tr>
<td>3</td>
<td>M3</td>
<td>134.0</td>
<td>KOS</td>
<td>1.7844</td>
<td>89.220</td>
<td>0.102</td>
</tr>
<tr>
<td>4</td>
<td>M4 to Docking</td>
<td>149.4</td>
<td>V-bar Extended Corridor</td>
<td>0.46508</td>
<td>23.254</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Figure 35: Completed Linear Covariance Analysis, Monte Carlo, and Summary Table for the Double Co-Elliptic
By combining the derived navigation requirements for each trajectory segment as developed in Figures (31) through (34), the complete end-to-end rendezvous trajectory simulating the navigation requirements is shown in Figure (35). The summary table in Figure (35) highlights the natural trend where the navigation requirements become more stringent as the distance to the target decreases.

The Double Co-Elliptic was undemanding in mid-range, requiring only a 3-σ navigation accuracy of 89.2 m up to the 500 m correction maneuver, M4. This is a result of the AE only extending one kilometer in the altitude direction for the M2 maneuver, and allowing M4 to greatly decrease dispersions at the hold point, as opposed to attempting to do so from M3.

The LinCov 3-σ ellipses did not enter the safety region for all figures, displaying the effectiveness of the scaling method. Almost all of the Monte Carlo runs also kept outside of the safety regions as well, barring the expected amount of outliers. LinCov and Monte Carlo 3-σ values were generally consistent, with LinCov predicting slightly lower dispersions than the Monte Carlo results indicated for individual maneuvers. This does not hold true for the full trajectory of Figure (35), where they overlap completely.

5.1.2 Line-of-Sight Corridor Requirements

In order to allow the LoS trajectories to be capable of docking at any approach angle, a 2 km radius Approach Sphere (AS) is used as the constraint region where the AE otherwise would. This can first be seen in Figure (36), which displays that the initial conditions for the LoS trajectories are safe with respect to the AS. Although this increases versatility, it also forces higher navigation accuracy.

For the LoS-Corridor, the AS is only used to derive navigation requirements for the M1 maneuver, shown in Figure (37a). However, this navigation region spans to the 152.7th minute, as seen in Figure (37b). The 3-σ navigation requirement of 136 m is implemented for the first navigation region, and resulting Monte Carlo and LinCov comparison is shown in Figure (37c). The Monte Carlo and LinCov 3-σ ellipses overlap closely. Although a pair of Monte Carlo runs lie just outside of either ellipse at and slightly after the M1 maneuver, both are safe with respect to the AS.

For M2, the TPI maneuver, free-drifts need to be constrained to avoid the KOS, and do so comfortably, as shown in Figure (38a). The scaling factor is found to be 4.4566, corresponding to a 3-σ navigation error of 223 meters. This is especially high considering the range of the chaser at M2, only 6.07 km away and acting as the TPI point. The sudden increase in navigation error is also obvious when implemented in Figure (38b). However, Monte Carlo runs validate the large scaling value, as the Monte Carlo and LinCov 3-σ again repeatedly overlap, as seen in Figure (38c). No Monte Carlo run enters the KOS, with only a single run approaching it.
Because navigation requirements for M2 are found to be much less restrictive than M1, they are set to the scaled values of the M1 maneuver in order to lower dispersions inside of the AS. This is justifiable because relative navigation error typically decreases as range decreases. This value is used to determine the M3 navigation requirements, which aligns the target to dock using the V-bar extended approach corridor in Figure (39a) and Figure (39c). The navigation requirement scaled to 23 m, 3-\( \sigma \), as shown in Figure (39a). It is implemented, along with the extended M1 navigation error, in Figure (39c). Monte Carlo runs validate the LinCov results in Figure (39c); no run enters the safety region, and 3-\( \sigma \) ellipses align almost perfectly.

Figure 36: Free-drift of LoS-Corridor and LoS-Glide with Initial Conditions
(a) Requirement Analysis with the Approach Sphere

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs

Figure 37: M1 Free-Drift Trajectory of the LoS-Corridor
Figure 38: M2 Free-Drift Trajectory of the LoS-Corridor
Figure 39: M3 Free-Drift Trajectory of the LoS-Corridor
<table>
<thead>
<tr>
<th>Navigation Region</th>
<th>Maneuvers</th>
<th>Start Time (min)</th>
<th>Safety Region</th>
<th>Scaling</th>
<th>Position 3-σ (m)</th>
<th>Velocity 3-σ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M1</td>
<td>99.0</td>
<td>AS</td>
<td>2.7101</td>
<td>135.51</td>
<td>0.154</td>
</tr>
<tr>
<td>2* (see caption)</td>
<td>M2</td>
<td>152.7</td>
<td>KOS</td>
<td>4.4566</td>
<td>222.83</td>
<td>0.254</td>
</tr>
<tr>
<td>3</td>
<td>M3 to Docking</td>
<td>182.7</td>
<td>V-bar Extended Corridor</td>
<td>0.45104</td>
<td>22.552</td>
<td>0.0257</td>
</tr>
</tbody>
</table>

Figure 40: Completed Linear Covariance Analysis, Monte Carlo, and Summary Table for LoS-Corridor. *The second navigation region is set to the values of the first for determination of the third, as well as the sensor suite analysis.
The summary table and full trajectory comparison for the LoS-Corridor can be seen in Figure (40). For the navigation requirements for the LoS-Corridor trajectory, the required navigation scaling did not, at first, decrease as range decreased. This resulted in excessively large dispersions inside the AS. Attempting to scale dispersions of this magnitude while also attempting to determine navigation requirements for the next maneuver, which used the V-bar extended approach corridor to prepare for docking, was impossible regardless of how accurate the navigation was. Instead, the scaling values of the first navigation region were also used for the second, and it will be seen that this does not reduce the number of sensor suites that satisfy the navigation requirements for this trajectory.

The low number of maneuvers for this trajectory decreased the number of navigation regions required, but the LoS-C M3 point is more than 600 m farther than the DC M4 point, yet required approximately the same navigation requirement. The LoS-Corridor will thus require higher quality sensors during the close-range segment of the rendezvous.

Overall, no LinCov 3-σ ellipses entered any of the safety regions, again demonstrating the effectiveness of the automated approach of deriving navigation requirements. This is initially validated by the Monte Carlo runs, as no Monte Carlo run drifted through any safety regions for all of the maneuvers analyzed. All LinCov 3-σ ellipses match well with their Monte Carlo counterparts, including the complete trajectory of Figure (40).

5.1.3 Line-of-Sight Glide Requirements

For the LoS-Glide, because of the high amount of maneuvers, not every maneuver has a corresponding navigation region. The reason for this was discussed earlier in the navigation analysis of the LoS-C M2 and M3 maneuvers; a large navigation scaling factor for a preceding maneuver can greatly increase the dispersions at the point of the subsequent maneuver. This can increase dispersions to a magnitude where unrealistic navigation accuracy is required to scale down dispersion for the corresponding safety region, if it is at all possible. The likelihood of this situation occurring increases with a high amount of navigation regions.

This is counteracted by including multiple maneuvers onto the same navigation region, that can be assumed to have similar navigation accuracy and the same constraint or safety region. The allowable navigation errors for the total navigation region is equal to the minimum of the allowable navigation error of the individual maneuvers contained within it. In the case of the LoS-Glide, the minimum occurs at the last maneuver within the navigation region. It is these last, key maneuvers that are analyzed below. The free-drifts of other maneuvers within the same navigation region are
not designed to approach the safety region.

As was the case with the LoS-Corridor, the AS is used in place of the AE to allow for any approach angle. The initial drift conditions are similar to those of the LoS-Corridor and are not repeated. The first navigation region contains M1 and M2, which share the AS safety region shown in Figure (41b), as G1 acts as the TPI maneuver. The scaled 3-σ navigation requirement is determined by the M2 free-drift as 120 m, and is implemented in Figure (41b). The first navigation region extends from a short time interval prior to M1, 101.9 minutes, to preceding the first glide maneuver G1, 148.7 minutes. The Monte Carlo and LinCov results implementing this navigation region and scaling can be seen in Figure (41b). The Monte Carlo 3-σ ellipses closely match their LinCov counterparts. At the AS, one Monte Carlo run crosses just below the 3-σ ellipses. However, none approach the AS.

The remaining glide maneuvers, G1 through G8, are not allowed to free-drift into the KOS. These maneuvers are split into two navigation regions expected to have similar navigation accuracy: those inside and outside of a 2 km range, the expected operational range of a lidar, to be discussed in “Subsection 5.2.1 - Sensor Types”. This places G2 through G6 within the second navigation region, while G7 and G8 are within the third. The navigation requirements for the second navigation region are determined from the G6 maneuver in Figure (42a), with a 3-σ value of 59 m. This is only a slight change from the default values, as seen in Figure (42b). The third navigation region contains only G7 and G8, with the navigation requirement determined from the G8 free-drift in Figure (43a) as 26 m. This is implemented in Figure (43b). The Monte Carlo simulation predicted trajectory dispersions can be compared to the LinCov predictions in Figure (42c) and Figure (43c). For both cases, Monte Carlo and LinCov 3-σ ellipses overlay one another, and no Monte Carlo run enters the KOS.

Finally, the fourth navigation region is determined so that the correction maneuver, GC1, does not leave the R-bar extended approach corridor, shown in Figure (44a) and Figure (44b). The navigation requirements are scaled to 18 m. Implemented in Figure (44b), it can be seen that the selected mapping of maneuvers to navigation regions creates a desired increase in navigation accuracy as range decreases. Due to high navigation accuracy required for the previous maneuvers G7 and G8, the magnitude of the dispersion reduction is not to the same extent of the DC M4 correction maneuver. No Monte Carlo runs lie exterior to the LinCov 3-σ ellipses, nor enter the safety region.
Figure 41: M2 Free-Drift Trajectory of the LoS-Glide

(a) Requirement Analysis with the Approach Sphere  
(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs
(a) Requirement Analysis with the Keep Out Sphere

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs.

Figure 42: G6 Free-Drift Trajectory of the LoS-Glide
(a) Requirement Analysis with the Keep Out Sphere

(b) Relative Position LVLH 3-σ Errors

(c) LinCo v and 250 Monte Carlo Runs

Figure 43: G8 Free-Drift Trajectory of the LoS-Glide
(a) Requirement Analysis with the Extended R-Bar Corridor

(b) Relative Position LVLH 3-σ Errors

(c) LinCov and 250 Monte Carlo Runs

Figure 44: GC1 Free-Drift Trajectory of the LoS-Glide
<table>
<thead>
<tr>
<th>Navigation Region</th>
<th>Maneuvers</th>
<th>Start Time (min)</th>
<th>Safety Region</th>
<th>Scaling</th>
<th>Position 3-σ (m)</th>
<th>Velocity 3-σ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M1 and M2</td>
<td>101.9</td>
<td>AS</td>
<td>2.3989</td>
<td>119.95</td>
<td>0.137</td>
</tr>
<tr>
<td>2</td>
<td>G1 through G6</td>
<td>148.7</td>
<td>KOS</td>
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<td>58.035</td>
<td>0.0662</td>
</tr>
<tr>
<td>3</td>
<td>G7 and G8</td>
<td>253.5</td>
<td>KOS</td>
<td>0.51085</td>
<td>25.543</td>
<td>0.0291</td>
</tr>
<tr>
<td>4</td>
<td>GC1 to Docking</td>
<td>276.3</td>
<td>R-bar Extended Corridor</td>
<td>0.34120</td>
<td>17.060</td>
<td>0.0194</td>
</tr>
</tbody>
</table>

Figure 45: Completed Linear Covariance Analysis, Monte Carlo, and Summary Table for LoS-Glide
The navigation requirements for the LoS-Glide were more restrictive than the previous trajectories, as can be seen in the summary table of Figure (45). This is due to the constant re-adjustment of the chaser's velocity vector towards the target coupled with its low range-rate profile, which allows process noise to greatly increase dispersions. Although at farther range the LoS-Glide has comparable requirements, the mid-range G6 maneuver required a $3\sigma$ navigation requirement of 58.0 m to avoid the KOS. This is a requirement that is not matched by the LoS-Corridor or the Double Co-Elliptic until their final navigation regions, preparing for docking. At close-range, the navigation requirements narrowed further. The last maneuvers of the glide required a $3\sigma$ navigation requirement of 25.5 m, while the final correction maneuver required 17.1 m.

Finally, none of the free-drift cases entered any of the safety regions through LinCov or Monte Carlo. Monte Carlo results once again validated the LinCov predictions.

5.2 Sensor Suite Analysis Results

Given the various derived requirements, the next step is to select a combination of sensors that satisfy them and is optimal in terms of overall cost, mass, and availability to the program. This section performs an extensive sensor suite analysis that first identifies all the potential sensor suites, out of several hundreds of possible combinations, and then selects a sensor suite based on what the mission objectives weigh as important. The method of selecting the ideal sensor suite is automated and capitalizes on utilizing the linear covariance analysis techniques to quickly and reliably converge on a solution.

All available sensor suites are run for each trajectory through LinCov, and compared to all navigation requirements derived for each trajectory. Of those that satisfy the navigation requirements for downrange, cross-track, and altitude, for both position and velocity, the sensor suites of interest with the lowest weight or cost are tabulated. The remainder of this section will first outline the sensor types being considered, quantify the performance parameters for different quality of sensors, and then evaluate each combination to the three trajectory profiles introduced previously.

5.2.1 Sensor Types

There are multiple sensors that are available at L2, and each can have different generic quality levels, corresponding to different accuracies for the navigation states they are designed to measure. In this study, five sensors are analyzed: a lidar, an optical sensor, a target based ground updated available at rendezvous initiation, a chaser based ground update also available at rendezvous initia-
tion, and an RF (radio frequency) sensor. From NASA JSC's article *A Comparison Between Orion Automated and Space Shuttle Rendezvous Techniques* [35], by Jose P. Ruiz and Jeremy Hart, the Orion capsule plans to be equipped with these sensors. While the Orion capsule is GPS enabled, GPS is only available to support LEO operations. The accuracies of different qualities of sensors used for this study are shown in Table [5].

A lidar is a laser sensor used at close proximity, approximately a few kilometers from the target [26], with high accuracy. The maximum range the lidar measurements are available changes with the lidar quality, in addition to the accuracy of the measurements. In this study, the lidar also has two operational regions; the closer region allows higher accuracy in range, but the farther region allows higher accuracy in bearing.

Ground updates are the only sensors that update the inertial state, relative to Earth. As such, they are only considered at rendezvous initiation, where they are of primary importance, before the navigation can depend solely on relative sensors. Ground updates are available to both the target and chaser. If the target or the chaser is hidden from the Earth’s line-of-sight, it is assumed other modes of communication and navigation will be put in place to convey the same data.

The optical sensor is a modified star-tracker. During rendezvous, it switches from measuring the spacecraft’s inertial attitude by detecting star patterns to solely identifying the elevation and azimuth of the target. While measuring the relative angles between the chaser and target, it can do so at ranges up to 200 km [26]. However, the optical sensor becomes ineffective when the chaser is within close range of the target. Similar to the lidar, the operational range changes with quality, but in this instance it is only the lower end of the operational range that varies.

Finally, the RF ranging sensor provides range and range-rate measurements using the communication system of the spacecraft. It operates at larger distances relative to the target compared to a lidar, and it can provide range, bearing, and range-rate information.

It is anticipated that ground updates of the target and chaser vehicles will be required to initialize the onboard navigation filter. To represent the different quality of ground updates that may be experienced, three options are investigated. However, sensor suites are considered having no lidar, RF sensor, or optical sensor, giving a total of four options for those three sensors. In total, there are 576 possible sensor combinations. The sensors considered ensure that all typical measurements types are provided, yet are unique and dissimilar. For example, a potential radar sensor that could provide bearing, range, and range-rate measurements at the same distance as the RF sensor is not included.
<table>
<thead>
<tr>
<th>Sensor</th>
<th>Quality</th>
<th>Operational Range</th>
<th>Type A</th>
<th>3 − σ Error</th>
<th>Type B</th>
<th>3 − σ Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lidar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td></td>
<td>1.5 km to 750m</td>
<td>Range</td>
<td>20 m</td>
<td>Bearing</td>
<td>0.1°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>750m to 0</td>
<td></td>
<td>1 m</td>
<td></td>
<td>0.2°</td>
</tr>
<tr>
<td>Med.</td>
<td></td>
<td>3.0 km to 1.5 km</td>
<td></td>
<td>10 m</td>
<td></td>
<td>0.05°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5 km to 0</td>
<td></td>
<td>0.1 m</td>
<td></td>
<td>0.1°</td>
</tr>
<tr>
<td>High</td>
<td></td>
<td>5.0 km to 2.5 km</td>
<td></td>
<td>5 m</td>
<td></td>
<td>0.01°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.5 km to 0</td>
<td></td>
<td>0.05 m</td>
<td></td>
<td>0.05°</td>
</tr>
<tr>
<td>Ground Update (Target)</td>
<td>Low</td>
<td>All Applicable</td>
<td>Abs. Position</td>
<td>[375, 75, 75] m</td>
<td>Abs. Velocity</td>
<td>[0.06, 0.06, 0.3]m/s</td>
</tr>
<tr>
<td></td>
<td>Med.</td>
<td></td>
<td></td>
<td>[250, 50, 50] m</td>
<td></td>
<td>[0.04, 0.04, 0.2]m/s</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
<td>[125, 25, 25] m</td>
<td></td>
<td>[0.02, 0.02, 0.1]m/s</td>
</tr>
<tr>
<td>Ground Update (Chaser)</td>
<td>Low</td>
<td>All Applicable</td>
<td>Abs. Position</td>
<td>[375, 75, 75] m</td>
<td>Abs. Velocity</td>
<td>[0.06, 0.06, 0.3]m/s</td>
</tr>
<tr>
<td></td>
<td>Med.</td>
<td></td>
<td></td>
<td>[250, 50, 50] m</td>
<td></td>
<td>[0.04, 0.04, 0.2]m/s</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
<td>[125, 25, 25] m</td>
<td></td>
<td>[0.02, 0.02, 0.1]m/s</td>
</tr>
<tr>
<td>Optical Sensor</td>
<td>Low</td>
<td>200 km to 5.0 km</td>
<td>Bearing</td>
<td>0.1°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Med.</td>
<td>200 km to 3.5 km</td>
<td></td>
<td>0.05°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>200 km to 1.5 km</td>
<td></td>
<td>0.01°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RF Sensor</td>
<td>Low</td>
<td>30 km to 200 m</td>
<td>Range</td>
<td>45 m</td>
<td>Range Rate</td>
<td>.5 m/s</td>
</tr>
<tr>
<td></td>
<td>Med.</td>
<td></td>
<td></td>
<td>15 m</td>
<td></td>
<td>.35 m/s</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
<td>5 m</td>
<td></td>
<td>.2 m/s</td>
</tr>
</tbody>
</table>

Table 5: Sensors for Sensor Suite Analysis. All vectors in [downrange, cross-track, altitude] format.
5.2.2 Double Co-Elliptic Sensor Suites

The results of the DC sensor suite analysis can be seen in Figure (46). The constant black lines represent the derived navigation requirements from Figure (35). For the Double Co-Elliptic, 255 of the 576 sensor suites satisfied the navigation requirements. Of those, the lowest weighted are shown in Table (6).

The ground updates are abbreviated as ITGU for initial target ground update and ICGU for initial chaser ground update. For the individual sensors, a low quality sensor is represented as (L), medium as (M), and high as (H). If the sensor was not included within a suite, it is indicated by (N). Non-trivial suites, where no sensor could be reduced in quality and still satisfy the navigation requirements, are **bolded**.

The DC trajectory did not require as accurate of a navigation state estimate as the other two trajectories presented. Consequently, a full suite of low quality sensors fulfilled all the requirements. Because measurement types were unique and dissimilar, every sensor provided a different role in the navigation system, and all had to be active to fulfill the full navigation requirements with a sensor suite of low overall weight. As a total of 255 sensor suites fulfilled all requirements, and there are only \(3^5 = 243\) sensor suites that utilize all sensors, the remaining 12 sensor suites must have fulfilled the DC navigation requirements while excluding a sensor. However, as none are shown in the lowest weighted sensor suites of Table (6), excluding one sensor must come at the cost of including several high quality sensors of other measurement types.

Including sensors in the analysis with redundant measurement types would result in a larger variety of sensor qualities, if resources are available to test all possible combinations. This scenario is also likely in practice, as redundant measurement types are often included to ensure that any single sensor type is not malfunctioning.

5.2.3 Line-of-Sight Corridor Sensor Suites

The results of the LoS-Corridor sensor suite analysis can be seen in Figure (47), with 234 suites satisfying all requirements. Although there are only two navigation regions, many suites failed the second. The second region corresponds to the M3 maneuver, which required higher navigation accuracy at a range of 1.12 km from the target. As a result, a full suite of low quality sensors did not satisfy all navigation requirements.

The lowest weighted sensor suite that satisfied all navigation requirements are shown in Table (7). Of those, there are three sensor suite combinations that satisfy the navigation requirements with the
same lowest total weight score of 6. Similar to the DC analysis results, all sensors were necessary to fulfill requirements with an overall low sensor suite weight. The top three possible selections also required a medium quality lidar, RF ranging sensor, or optical camera. The advantages of an improved ground update only benefited navigation accuracy at farther ranges, and did not have any effect on whether a low weight suite passed or failed.

5.2.4 Line-of-Sight Glide Sensor Suites

The results of the LoS-Glide sensor suite analysis can be seen in Figure (48). For the LoS-Glide, 207 of the 576 satisfied the requirements. The LoS-Glide had the strictest navigation requirements at close-range, and this is reflected in the total number of suites that passed all navigation requirements.

The lowest weighted sensor suites are shown in Table (8). Similar to the LoS Corridor, the lowest sensor suite weight is 6. Only one suite of this weight passed, which required a medium quality lidar. The final two glide maneuvers required a 3-σ navigation error of less than 25.5 m and 0.0291 m/s. However, these maneuvers were farther than two kilometers from the target, outside the range of the low quality lidar. As a result, a medium quality lidar was required to reduce navigation errors at this distance.

However, with the extended optical sensor operational range at medium quality, the close-range requirements can also be fulfilled with the use of a medium quality optical sensor and RF-sensor. Although the measurements from the medium quality optical sensor are not useful within the range of 3.5 km, the navigation errors do not grow at a fast enough rate to require the use of a high quality optical sensor. If the weights were significantly different and a medium quality lidar was found to be cost ineffective, this would be a viable option.
<table>
<thead>
<tr>
<th>Sensor Suite ID</th>
<th>Weight/Cost</th>
<th>ITGU</th>
<th>ICGU</th>
<th>Optical Sensor</th>
<th>Lidar</th>
<th>RF Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>555</td>
<td>5</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>363</td>
<td>6</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>491</td>
<td>6</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>539</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>551</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
</tr>
<tr>
<td>554</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
</tbody>
</table>

Table 6: Optimal Sensor Suites for the Double Co-Elliptic

<table>
<thead>
<tr>
<th>Sensor Suite ID</th>
<th>Weight/Cost</th>
<th>ITGU</th>
<th>ICGU</th>
<th>Optical Sensor</th>
<th>Lidar</th>
<th>RF Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>539</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>551</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
</tr>
<tr>
<td>554</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
</tbody>
</table>

Table 7: Optimal Sensor Suites for the LoS-Corridor

<table>
<thead>
<tr>
<th>Sensor Suite ID</th>
<th>Weight/Cost</th>
<th>ITGU</th>
<th>ICGU</th>
<th>Optical Sensor</th>
<th>Lidar</th>
<th>RF Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>551</td>
<td>6</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
</tr>
<tr>
<td>359</td>
<td>7</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
</tr>
<tr>
<td>487</td>
<td>7</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>L</td>
</tr>
<tr>
<td>535</td>
<td>7</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>L</td>
</tr>
<tr>
<td>538</td>
<td>7</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>547</td>
<td>7</td>
<td>L</td>
<td>L</td>
<td>L</td>
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<td>L</td>
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<td>7</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
</tbody>
</table>

Table 8: Optimal Sensor Suites for the LoS-Glide
Figure 46: The Double Co-Elliptic Sensor Suite Analysis Results

Sensor Suite Analysis Results:
- Sensor suites that satisfied all navigation requirements: 255/576
- Minimum weighted sensor suites that satisfied all navigation requirements: 1/255
Figure 47: The LoS-Corridor Sensor Suite Analysis Results.
Figure 48: The LoS-Glide Sensor Suite Analysis Results

Sensor Suite Analysis Results:
- Sensor suites that satisfied all navigation requirements: 207/576
- Minimum weighted sensor suites that satisfied all navigation requirements: 1/207
6 Concluding Remarks

The Earth-Moon L2 Lagrange point is a potential deep-space destination for a space station or propellant depot. Possible reasons include its location in an Interplanetary Superhighway System, optimal environment for fuel storage, and ability to act as a radio quiet zone or communications satellite to the far side of the Moon. However, for the Earth-Moon L2 point to fulfill this potential, rendezvous capabilities between a target vehicle in a halo orbit around L2 and a chaser must be developed. Several trajectories and navigation systems currently used for rendezvous at LEO or other destinations with comparable relative motion dynamics cannot be directly applied.

Three linear targeting algorithms have been described, within an L2 local vertical, local horizontal frame of reference. Straight-Line targeting provides a simplistic solution, if transfer times and distances are small. If planetary ephemeris data are limited and the targeting performance requirements are not stringent, Straight-Line targeting is a viable option at L2. Clohessy-Wiltshire targeting has found frequent use in LEO, and lends itself well to L2 rendezvous if slightly modified and given reasonable transfer times. L2 Linearized Relative targeting has higher accuracy and precision for long transfer times, by taking multi-body effects into account.

Utilizing L2 Linearized Relative targeting, three preliminary trajectories were developed: the L2 Double Co-Elliptic, the LoS-Corridor, and the LoS-Glide. Although the Double Co-Elliptic trajectory does not exhibit all of the benefits it has in LEO, adapting it for L2 was possible and did not incur heavy fuel usage, while preserving the concept of operations for all pilots and systems. The LoS-Corridor and LoS-Glide utilized the rectilinear motion at L2 to create trajectories with recognizable trigger angles and offset angles for safety. The LoS-Corridor can act within the bounds of sensors or reflectors constrained with limited field-of-views, while the LoS-Glide minimizes fuel consumption required to arrive at the target.

To ensure the trajectory design is robust to potential contingency scenarios, free-drift trajectories associated with each maneuver are required to remain outside predesignated safety regions. To determine the navigation requirements that assured this to a 3-$\sigma$ confidence level, stochastic navigation was implemented to both Monte Carlo and LinCov simulations. Sensitivity and scaling were then introduced to create an automated requirement derivation capability. By deriving requirements for relevant navigation regions, that corresponded to key maneuvers and constraint regions, navigation requirements for an entire rendezvous trajectory could be determined.

Navigation requirement derivation was performed for all three trajectories. The Double Co-Elliptic was found to be the most lax in its requirements, due to using the Approach Ellipsoid instead
of Approach Sphere for its constraint region and allowing for a low range correction maneuver to
decrease dispersions at the hold point. The LoS-Glide required the highest navigation accuracy at
close-range, a result of its low range-rate profile and dispersion requirements. The LoS-Corridor
required an intermediate navigation accuracy with a maneuver barely within the Approach Sphere
necessary to greatly reduce dispersions at the hold point. Furthermore, Monte Carlo simulations
for every maneuver successfully validated the linearization and stochastic navigation employed by
LinCov.

These navigation requirements were contrasted versus the navigation performances of 576 dif-
derent sensor suites. The Double Co-Elliptic trajectory required only sensors of low quality. The
intermediate LoS-Corridor needed an increase in quality in one of three close to mid-range sensors.
Finally, the LoS-Glide required a higher quality lidar, or a combination of a higher quality RF-sensor
and optical sensor, to satisfy its close-range navigation requirements.

6.1 Future Work

Several possible future uses of this material can be explored. This thesis focused on close proximity
operations. Work to transfer a chaser to an L2 halo orbit in the vicinity of the target may be required.
Even though much work has been completed for transfers to halo orbits, the orbit insertion of the
chaser at a particular time and position relative to the target may prove more complex. Also, studies
have generally excluded the effects of dispersions. Both LR targeting and navigation analysis could
help complete these objectives.

This study was restricted to one halo orbit within a specific 14 day time period. Targeting
algorithms were tested for different periods along the halo orbit, yet multiple families of halo orbits
exist. If investigating other halo orbits becomes a priority for future missions, re-verifying these
results and gaining additional insights will be necessary.

The three trajectories evaluated in this thesis represent only a small portion of possible trajecto-
ries, all using LR targeting for significant maneuvers. It would be of particular interest to see
the effect of using only Straight-Line targeting, so a chaser could potentially require no information
on absolute reference states of either the target or itself. Navigation requirements could be derived
for such a trajectory, and compared and contrasted for consistency against the requirements for the
same trajectory with the target initialized at a different position along the halo orbit and the Moon
at a different position along its path. If consistency was found, and one sensor suite could satisfy
the requirements for the trajectory for all situations, Straight-Line targeting could present a simple
solution to a complex problem.

All maneuvers in this study were time triggered events, even though they can be initiated on other parameters such as the chaser’s relative position or angle to the target. Although some missions are sensitive to the time dispersions created by such triggers, they potentially reduce dispersions in position. In particular regard to the Line-of-Sight trajectories and Straight-Line targeting, angle triggers could be used to reduce dispersions in position, thus reducing the navigation requirements.

Linear covariance stochastic navigation was conducted assuming white noise, yet Monte Carlo allowed correlation through time using a first order Markov process. This process modeled white noise by reducing $\tau$ to a low value. A non-zero time constant that correspond to navigation errors can be identified and incorporated to yield more representative results.

Only the navigation error corresponding to the most recent maneuver was optimized in the requirement derivation. As shown in Equation (52), multiple dispersion sources could be scaled. Theoretically, a multivariable optimization could be performed, to solve for requirements across an entire guidance, navigation, and control system. This is another area of research that warrants further investigation.

The sensor suite analysis used a limited number of sensors, each with a specific role. If more sensors were used with some redundancy, perhaps more insightful combinations would result. For example, if a relative optic sensor was able to operate at all ranges, it could replicate angles-only-navigation. This could possibly differentiate the LoS-Corridor trajectory from the other two.

Sensor suites utilized for a specific vehicle at L2 will likely be relied upon for rendezvous operations in LEO. A common set of sensors that would satisfy rendezvous navigation requirements for both locations would be necessary. If this was the case, a sensor suite analysis could be conducted for trajectories in LEO and contrasted with the results of a trajectory at L2.

Currently, automated requirement derivation and sensor suite analysis can be performed for any preliminary trajectory with well defined constraint regions. At L2 where many design parameters are in flux and undetermined; maneuver times, triggers, distances, targeting algorithms, approach angles, or other parameters can be adjusted for any unique situation. The resulting plausibility and cost of such adjustments for the navigation system can be found precisely and efficiently.
References


Appendix: Linear Covariance

This appendix will demonstrate the fundamental concepts and equations for a simple linear covariance simulation tool (LinCov). LinCov can determine the state dispersions and navigation errors of a spacecraft for any point during its trajectory, if the random processes involved can be assumed to be zero-mean and Gaussian in regard to amplitude. Further, they must be white in regard to frequency. These assumptions also justify use of the Kalman filter.

Several state vectors consisting of $n$ states in an $n \times 1$ column vector are first defined. This includes the true state, $x$, the actual state of the spacecraft; the nominal state, $\bar{x}$, which specifies the desired location of the spacecraft; and the navigation state, $\hat{x}$, which represents the onboard estimate of the spacecraft’s state given sensor measurements and dynamic models.

Three perturbations or deltas are defined based on the three state vectors defined above. The true dispersion is defined as $\delta x \triangleq x - \bar{x}$, and quantifies how accurately the spacecraft is actually following the desired profile. The navigation dispersion is defined as $\delta \hat{x} \triangleq \hat{x} - \bar{x}$, and quantifies how far the navigation system approximates the spacecraft to be from the desired profile. Finally, the true navigation error is defined as $\delta e \triangleq x - \hat{x}$, and quantifies how well the navigation system can properly determine its true state. The covariance of these errors are defined to provide performance bounds of the overall GNC system: $D \triangleq E[\delta x \delta x^\top]$, the true dispersion covariance; $\hat{D} \triangleq E[\delta \hat{x} \delta \hat{x}^\top]$, the navigation dispersion covariance; and $P \triangleq E[\delta e \delta e^\top]$, the true navigation error covariance.

By combining the true and navigation dispersions, the augmented state dispersion is defined as

$$\delta X \triangleq \begin{bmatrix} \delta x \\ \delta \hat{x} \end{bmatrix}.$$  \hspace{1cm} (A.1)

The augmented state dispersion covariance can now be formed, and ultimately provides the mathematical framework to generate both the trajectory dispersions and navigation errors simultaneously. It is defined from the augmented state dispersion as

$$C \triangleq E[\delta X \delta X^\top] = \begin{bmatrix} D & D_{xx} \\ D_{\hat{x}x} & \hat{D} \end{bmatrix},$$ \hspace{1cm} (A.2)

where $D_{xx} \triangleq E[\delta x \delta x^\top]$ and $D_{\hat{x}x} \triangleq E[\delta \hat{x} \delta x^\top]$. If given initial conditions for the trajectory dispersions, $D_0$ and navigation error, $P_0$, the initial augmented state dispersion covariance is defined as
as

\[
C_0 \triangleq \begin{bmatrix}
D_0 & D_0 \\
D_0 & D_0 + P_0
\end{bmatrix}.
\]  

This allows the initial navigation error and dispersion to be set independently.

The true navigation error covariance can be derived from the augmented state dispersion. If \(J = [I_n, -I_n]\), then

\[
P = JCJ^\top.
\]

This, by propagating the augmented state dispersion covariance through the specified time-steps, the true navigation error, true dispersion, and navigation dispersion are readily available. Along with the nominal trajectory, all the stochastic information required to create the probability density of the trajectory at any instant is available.

After initialization and propagation of the nominal state, the augmented state covariance matrix evolves over time through propagation of the state dynamics, inputs from impulsive maneuvers, and updates with discrete sensor measurements. Derivations of each step will be explained in sequence.

**Propagate through Time**

In general, the non-linear dynamics function, \(a\), is dependent on the true state, \(x\), and process noise, \(w\). That is

\[
\dot{x} = a(x, w).
\]

The process noise has power spectral density \(Q\), evaluated as \(Q_i (i' - i) = E[w_i w_i']\). Given a nominal trajectory, Equation (A.5) can be linearized about the nominal state, \(\bar{x}\):

\[
\dot{\bar{x}} = a(\bar{x}, 0) + \left. \left[ \frac{\delta a}{\delta x} \right] \right|_{x=\bar{x}} \delta x + H.O.T. + \left[ \frac{\delta a}{\delta w} \right]_{w=0} w + H.O.T.,
\]

If the higher order terms (\(H.O.T\)) are set to zero, then

\[
\delta \dot{x} = A_x \delta x + A_w w,
\]

where \(A_x = \left. \frac{\delta a}{\delta x} \right|_{x=\bar{x}}\) and \(A_w = \left. \frac{\delta a}{\delta w} \right|_{w=0}\).
The navigation state dynamics is only a function of the onboard navigation state, \( \hat{x} \), as

\[
\dot{\hat{x}} = \hat{a}(\hat{x}).
\]  

(A.8)

Linearizing the navigation state equations of motion about the nominal state yields the equation for the navigation state as

\[
\delta\dot{\hat{x}} = \hat{A}_\delta \delta\hat{x},
\]  

(A.9)

where \( \hat{A}_\delta = \frac{\delta\hat{a}}{\delta\hat{x}} |_{\hat{x} = \bar{x}} \). Using the definition of augmented state dispersion, Equation (A.1), the augmented state dispersions are propagated with the following equations:

\[
\delta\dot{X} = A_{aug} \delta X + G_{aug} \mathbf{w},
\]  

(A.10)

where

\[
A_{aug} = \begin{bmatrix} A_x & 0 \\ 0 & \hat{A}_\delta \end{bmatrix},
\]  

(A.11)

and

\[
G_{aug} = \begin{bmatrix} A_w \\ 0 \end{bmatrix}.
\]  

(A.12)

The differential equation for the augmented state dispersion covariance becomes

\[
\dot{C} = A_{aug} C + C A_{aug}^T + G_{aug} Q G_{aug}^T.
\]  

(A.13)

**Correct for Impulsive Maneuvers**

For this thesis, all maneuvers are modeled as impulses that instantaneously update the true velocity state. If a prolonged thruster burn is needed, it can be closely approximated by a succession of instantaneous maneuvers.

The non-linear impulsive maneuver function, \( \mathbf{m} \), is a function of the true state, navigation state, and thruster noise, \( \eta \), with power spectral density, \( \mathbf{S}_\delta \), where \( \mathbf{S}_\delta \delta_i \delta_j = E[\eta_i \eta_j^T] \). The impulsive maneuver and the uncorrected true state vector, \( \mathbf{x}^{-c} \), are related to the corrected true state vector, \( \mathbf{x}^{+c} \), by the equation

\[
\mathbf{x}^{+c} = \mathbf{x}^{-c} + \mathbf{m}(\mathbf{x}, \hat{x}, \eta).
\]  

(A.14)
The navigation impulsive maneuver function, $\hat{m}$, is a function solely of the navigation state. The estimated impulsive maneuver, uncorrected navigation state vector, $\hat{x}^{-c}$, and the corrected navigation state vector, $\hat{x}^{+c}$, are related as

$$\hat{x}^{+c} = \hat{x}^{-c} + \hat{m}$$  \hspace{1cm} (A.15)$$

By linearizing about the nominal state, the true dispersion and the navigation dispersion correction can be simplified to

$$\delta x^{+c} = \delta x^{-c} + M_x \delta x^{-c} + M_\delta \delta x^{-c} + M_\eta \eta, \hspace{1cm} (A.16)$$

and

$$\delta \hat{x}^{+c} = \delta \hat{x}^{-c} + \hat{M}_x \delta \hat{x}^{-c}, \hspace{1cm} (A.17)$$

where $M_x = \frac{\delta m}{\delta x}|_{\hat{x}=\bar{x}}$, $M_\delta = \frac{\delta m}{\delta \delta x}|_{\hat{x}=\bar{x}}$, $M_\eta = \frac{\delta m}{\delta \eta}|_{\eta=0}$, and $\hat{M}_x = \frac{\delta \hat{m}}{\delta \hat{x}}|_{\hat{x}=\bar{x}}$.

Combining Equations (A.16) and (A.17) produces the expression to correct the augmented state given an impulsive maneuver:

$$\delta X^{+c} = M_{aug} \delta X^{-c} + T_{aug} \eta, \hspace{1cm} (A.18)$$

where

$$M_{aug} = \begin{bmatrix}
I + M_x & M_\delta \\
0 & I + \hat{M}_x
\end{bmatrix}, \hspace{1cm} (A.19)$$

and

$$T_{aug} = \begin{bmatrix}
M_\eta \\
0
\end{bmatrix}. \hspace{1cm} (A.20)$$

Finally, the relation for the covariance after correction, $C^{+c}$, to the covariance before the correction, $C^{-c}$, is

$$C^{+c} = M_{aug} C^{-c} M_{aug}^T + T_{aug} S T_{aug}^T, \hspace{1cm} (A.21)$$
**Update for Sensor Measurements**

The navigation state is dependent on the sensors onboard the spacecraft and the modeled dynamics. A Kalman filter optimally weights the importance of the sensor measurement versus the modeled prediction. Large errors in the sensor measurements due to sensor noise or other sources produce a low gain, $K$, resulting in the filter utilizing the predicted measurement more than the actual measurement.

Discrete sensor measurements, $z$, are modeled as a function of the true state and sensor noise, $\nu$:

$$z = h(x, \nu).$$  \hfill (A.22)

The sensor noise is white with power spectral density $R$, where $R \delta_{ii'} = E[\nu_i \nu_{i'}^\top]$. Meanwhile, the onboard predicted measurement, $\hat{z}$, is a function solely of the navigation state, as

$$\hat{z} = \hat{h}(\hat{x}).$$  \hfill (A.23)

Linearizing Equations (A.22) and (A.23) about the nominal state yields

$$z = h(\bar{x}, 0) + H_x \delta x + H_\nu \nu,$$  \hfill (A.24)

and

$$\hat{z} = \hat{h}(\bar{x}) + \hat{H}_x \delta \hat{x},$$  \hfill (A.25)

where $H_x = \frac{\delta h}{\delta x} |_{x = \bar{x}}$, $H_\nu = \frac{\delta h}{\delta \nu} |_{\nu = 0}$, and $\hat{H}_x = \frac{\delta \hat{h}}{\delta x} |_{\hat{x} = \bar{x}}$.

Since sensor measurements do not directly affect the true state, the true dispersion after update, $\delta x^+$, is equal to that before the update, $\delta x^-$:

$$\delta x^+ = \delta x^- = \delta x.$$  \hfill (A.26)

Implementing the gain, the navigation dispersion becomes

$$\delta \hat{x}^+ = \delta \hat{x}^- + K[z - \hat{z}].$$  \hfill (A.27)

Assuming $h(\bar{x}, 0) = \hat{h}(\bar{x})$, substituting Equations (A.24) and (A.25) into Equation (A.27) yields

$$\delta \hat{x}^+ = \delta \hat{x}^- + K[H_x \delta x + H_\nu \nu - \hat{H}_x \delta \hat{x}],$$  \hfill (A.28)
and thus

$$
\delta \hat{x}^+ = [I - K\hat{H}_k] \delta \hat{x}^- + [KH_x] \delta x^- + KH_{\nu} \nu.
$$

(A.29)

Combining Equations (A.26) and (A.29) produces the augmented state update equation as

$$
\delta X^+ = H_{aug} \delta X^- + L_{aug} \nu,
$$

(A.30)

where

$$
H_{aug} = \begin{bmatrix}
I & 0 \\
KH_x & I - K\hat{H}_k
\end{bmatrix},
$$

(A.31)

and

$$
L_{aug} = \begin{bmatrix}
0 \\
KH_{\nu}
\end{bmatrix}.
$$

(A.32)

The augmented covariance after the update, $C^+$, is updated with discrete measurements from its previous values, $C^-$, as

$$
C^+ = H_{aug} C^- H_{aug}^\top + L_{aug} R L_{aug}^\top.
$$

(A.33)

After propagating over a given time interval, Equation (A.13), correcting the covariance for impulsive maneuvers, Equation (A.21), and updating the covariance with a sensor measurement, Equation (A.33), the simulation continues the process of propagating, correcting, and updating the augmented state covariance matrix for the selected trajectory profile.