RICE UNIVERSITY

Modeling and Analysis of an In-line Pump Jet Thruster for Swimming Robots

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Master of Thesis

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April, 2015
ABSTRACT

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Nowadays, underwater robots play a significant role in many aspects of industry such as mine hunting, sea floor mapping, sub-sea pipeline construction and pipe surveys. Ever increasing demand on better control systems of underwater robots ignites engineers on modeling the systems for perfection. Since they are the lowest chain of the control problem, yet the most important ones. Thrusters are the most crucial parts of the underwater robot actuation. The need for maneuvering precision, for small and fast dynamic positioning, for precise motion and position control and for station keeping made developments of thruster modeling inevitable.

The fundamental difficulty in modeling of a thruster is that axial flow velocity is an unmeasurable state, yet the model depends on that variable. This work addresses this difficulty. It explores the dynamics of in-line pump jet thrusters which are used for underwater robotic actuation. This work uses impeller geometry characterization, fluid flow analysis through impeller blades and through pump jet, thrust modeling and DC motor modeling. The main contribution of this work is incorporation of axial flow velocity into thruster modeling in order to resolve the unmeasurable state problem, thanks to the dynamical properties of impeller geometry and flow conditions through impeller blades.
Acknowledgments

I dedicate this work to my wife, Seda. None of this study would have been possible without her sincerity, patience and love.

I would like to show my deepest gratitude to my advisor, Professor Fathi Ghorbel, whose guidance, patience and understanding have never ended since the first day of my studies at Rice Robotics and Intelligent Systems Laboratory (RiSYS Lab). His insightful comments and suggestions always inspired me to overcome every difficulty. Also, I would like to thank Dr. James Dabney for his numerous assistance, guidance and contribution to my research. Special thanks to my thesis committee, Dr. John E. Akin and Dr. Andrew J. Dick, for being supportive and constructive throughout thesis defense.

A special thanks to my labmate David Trevino for his countless help and guiding me for success as a senior graduate student and to Issam Ben Moallem for his friendship and calm helps. They have been more than friends. Also, I would like to thank my colleagues Lt.J.G. Ali Karalomlu and Lt.J.G. Aykut Ersen for their friendship and companionship.

I am grateful to my mother Emine Kirmizi, to my father Ilyas Kirmizi, to my elder brother Muammer Kirmizi and to my mother-in-law Fatma Tufan for their support and love.

I gratefully acknowledge Turkish Naval Forces for supporting this work financially.
Nomenclature

A  Duct\shroud cross-sectional area  $m^2$
$C_D$  Drag coefficient  -
$C_L$  Lift coefficient  -
$C_M$  Motor-shaft viscous friction coefficient  N-m s
$cv$  Control Volume  -
$cs$  Control surface  -
e  Back emf  V
$gz$  Potential energy per unit mass  $J/kg$
$\dot{h}$  Specific enthalpy  $(m/s)^2$
i  Armature current  A
j  Blade index  -
$J_M$  Mass moment of inertia of motor  $kg \, m^2$
$J_P$  Mass moment of inertia of propeller nimpeller  $kg \, m^2$
K  Exiting kinetic energy per volume  $kg/m \, s^2$
$K_M$  Back emf constant  Volt/rad/s
$K_T$  Motor torque constant  $N - m/amp$
L  Duct\shroud length  m
$L_a$  Inductance  H
n  Number of impeller blades  -
$P$  Propeller pitch  rad
Q  Volume flow rate  $m^3/s$
$\dot{Q}$  Rate of heat energy transfer from \to control volume  watt
R  Armature resistance  ohm
r  Propeller\impeller radius  m
s  Flow cross-sectional area  $m^2$
T  Kinetic energy  $J$
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<td>J</td>
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<tr>
<td>$\Omega_P$</td>
<td>Propeller\impeller angular velocity</td>
<td>rad/s</td>
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Chapter 1

Introduction

Since the beginning of the first sail experience, people have always been seeking better ways to propel sea vehicles. The idea of propulsion of ships started with inventing oars, namely, human propulsion. Using wind power for sails has followed human propulsion until the invention of coal-fired steam engines. When the mechanization period started in the early nineteenth century, people started to use machines in every part of life including sailing industry. After machines started to find places in ships, people started to integrate those machines as propulsion systems of ships such as reciprocating engines, water jets, nuclear powered steam turbines and electric motors. However, these machines are not always sufficient by themselves for creating thrust in order to move the sea vehicle. Thereafter, propellers were started to be used to create a propulsive force or thrust. Thus, many different types of propellers were invented for specific purposes of the sea vehicles.

During the early time of propellers and propulsion systems, inventors recognized more significance on the modeling of vehicles rather than the thrusters. In early stage of the mechanization period, the technological race was mostly focused on monetary purposes. The needs of people and technological developments depending on these needs guided inventors to clarify the dynamics of the thrusters. The need for maneuvering precision, for small, fast dynamic positioning, for precise motion and position control and for station keeping made developments of thruster modeling inevitable. Many of the aforementioned tasks are currently being performed by small underwater
vehicles. In order to achieve an improved vehicle positioning system, modeling of a thruster has paramount significance since it is the lowest part of the control system.

In this thesis, in-line pump modeling is successfully constructed by three governing system differential equations with the state variables of impeller angular velocity, DC motor current and volume flow rate. The challenge in modeling is to construct and incorporate axial flow velocity into the complete model. Thanks to the dynamical properties of impeller geometry and flow conditions through impeller blades, this work addresses this issue by confirming that volume flow rate is proportional to the angular velocity of the impeller.

1.1 Motivation

It has been almost thirty years since underwater robots have been essentially in use for operations that require precision and safety such as mine hunting, sea floor mapping, sub-sea pipeline constructions and pipe surveys. In order to sustain the precision and safety, vehicle dynamics and thruster dynamics need to be carefully described. That is why studies of Yoerger et al. [5] and Cooke [6] are very crucial for defining the dynamics of propeller thrusters on an energy-based modeling. In a similar manner, McLean [7] proposed a hydrodynamic model regarding thrust and torque. Further and more included study was presented by Cody and Healey [8] where DC motor dynamics were included in a two state model. Eventually, Fossen and Blanke [9] presented a three state model with unmeasurable state axial flow velocity. However, they derived a nonlinear state observer for unmeasurable state. More comprehensive and comparative experimental results about aforementioned models were emerged from studies of Whitcomb and Yoerger [10], [11], [12]. Besides, discussions and studies on thrusters with propellers clearly indicates that models are lacking of explaining
the response completely. Additionally, these three models do not clearly reflect the axial flow velocity which contributes to the thrust.

Although there is a wide study of propeller thrusters in the literature, yet many unanswered questions arise about pump jet thrusters with impellers. A pump jet is a marine propulsion system which produces a jet of water. It contains a propeller with a nozzle or an impeller with a nozzle which is the main focus of this study. Pump jets have certain advantages over conventional propeller applications such as low cavitation, reduced noise and safety. Lately, these advantages of pump jets have gained importance while requirements of some operations have been changing accordingly. When these requirements necessitate pump jets to be used, new design applications emerged from different ideas one of which is in-line pump jet with impeller. Eventually, lack of theoretical and experimental study of in-line pump jet has guided this work. Therefore, the first objective of this study is to extensively comprehend, examine and characterize in-line pump jets. The second objective is to design and construct an experimental setup to verify the accuracy of the model and measurements.

1.2 Problem Definition

In order to fully comprehend the dynamics of pump jets from inlet to outlet, it is crucial to describe every physical parameter which contributes to the flow. These physical parameters which contribute to flow can be listed as:

- Pump jet and impeller geometry,
- Hydrodynamic torque on the impeller,
- Constructing and incorporating axial flow velocity into complete model and resolve the unmeasurable state,
. Linear momentum of the flow through pump jet,

. DC motor dynamics.

While these contributors of the flow require a great care separately, the connections between them need to be established properly, as well. Therefore, each element listed above are broken into its smallest pieces for analysis. For instance, DC motor should be analyzed for both electrical and mechanical components. Hydrodynamic torque should be carefully determined by using geometric parameters of impeller and flow conditions through impeller blades. It is very significant to comprehend the flow since it has considerable effect on DC motor modeling. Besides, the fundamental difficulty in modeling of a thruster is the axial flow velocity which is analyzed and resolved thanks to the dynamical properties of the impeller. In light of aforementioned contributors, this study will focus on describing impeller geometry, hydrodynamic torque, linear momentum of the flow through pump jet and DC motor dynamics.

1.3 Contributions

Section 1.1 presented current studies about modeling of thrusters with propeller. This section presents the contributions of this study by proposing a new model of in-line pump jet thrusters. The contributions of this study are as follows:

1. The first contribution of this study is to analyze the models of thrusters with propeller and characterize the current deficiencies (Chapter 2).

2. The second contribution of this study is to introduce the pump theory and in-line pump concept. Chapter 3 will present a comprehensive analysis of an in-line pump.
3 The third contribution is to obtain a comprehensive model for the in-line pump jet which include every system parameter. However, most importantly, analyzing flow through impeller blades in order to construct an expression for volume flow rate. The incorporation of unmeasured state axial flow velocity is achieved thanks to the dynamical properties of impeller.

1.4 Outline of this thesis

Chapter 2 presents different types of propulsion systems and introduces two models for ducted and tunnel thrusters. Chapter 3 presents an in-line pump jet modeling which includes DC motor modeling, impeller geometry, hydrodynamic load modeling through impeller blades and thrust modeling as well pump terminology and pump theory. Chapter 4 presents a comprehensive comparison between experimental results and model response. Furthermore, experimental setup and instruments which are used for experiments are presented. Finally, Chapter 5 presents a brief conclusion of this thesis.
Chapter 2

Marine Propulsion Types and Modeling of Ducted Propellers

Propulsion systems in aircrafts/ships are used to generate a force to move forward. In general, propellers are in use for boats and ships as a propulsion system. Lately, however, pump jets or water jets find their places in some applications for certain advantages such as reduced noise and cavitation. In this chapter, several propulsion systems are covered and two ducted propeller modeling which are commonly used in underwater robots are analyzed.

2.1 Non-ducted Thrusters

2.1.1 Fixed Pitched Propellers

The dictionary definition of propeller is that “a device with a central hub and radiating blades placed so that each forms part of a helical (spiral) surface. By its rotation in water or air, a propeller produces thrust owing to aerodynamic or fluid forces acting upon the blades and gives forward motion to a ship or aircraft”[13]. In the application of marine vessels, it is known as screw propellers. Fixed pitch propellers shown in Figure 2.1 are the most common type of propulsion systems. They have certain applications from small pleasure boats to larger merchant ships.
For fixed pitched propellers, the choice of the number of blade varies depending on the application. Typically the number of blades changes from two to seven. In the earlier times, design of a propeller mainly focused on the performance and energy consumption but afterwards other aspects of optimization such as vibration and noise came into prominence for torpedos and warships.

2.1.2 Azimuth Thrusters

Azimuth thrusters have the configuration which can be rotated to a horizontal angle and it delivers two thrust components in the horizontal plane. This configuration
provides navigational ability by changing the thrust direction and it also improves the efficiency by eliminating rudder uses. When rudder is not used, designers’ need of occupying either one or two compartments for lying the rudder mechanism can be eliminated as well as rudder resistance. The motor is located inside the hull either horizontally or vertically which is defined as Z-shape and L-shape, respectively. Mechanical losses are more for Z-shape configuration due to the power transmission losses when compared to L-shape configuration. In order to reduce the power losses, L-shape configuration can be chosen if the engine room has enough height [14].

2.1.3 Podded Thrusters

Podded thrusters are very similar to azimuth thrusters and thrust can be rotated to any direction as in the azimuth thrusters. The only difference between these two thrusters is the motor configuration. In a podded thruster, motor is located inside the pod and directly coupled to the propeller. When compared to azimuth thrusters, poded thrusters have less transmission losses. Podded thruster can have either pusher or tractor design configuration. Hence, this design configuration reduces the cavitation and vibration [14]. Furthermore, it provides good maneuverability to the ship and reduces the fuel consumption which provides up to 2%-4% more efficiency over conventional propellers [15].

2.1.4 Contra-Rotating Propellers

In contra-rotating propeller configuration, two propellers sited coaxially along the shaft and rotate in opposite directions. Due to the opposite rotation of two propellers, torque effect is improved and the balance of the naval platform is achieved. This property is very important especially for small platforms such as torpedos. Besides,
having contra rotating propellers enables two propellers to share the load compared to a single propeller. Therefore, propellers can have an improved cavitation margin according to the design [16].

2.1.5 Overlapping Propellers

Overlapping propellers have two propellers sited on separate shaft systems but the distance between two shafts is smaller than the propellers’ diameter. This kind of propeller arrangement seems like contra-rotating propellers. But it produces higher performance by recovering the wake behind the ship. Besides it provides a simple shaft and engine room arrangement.

2.2 Ducted Thrusters

Ducted propellers generally consist of two components as shown in Figure 2.2: One of which is called the propeller, and the second component is called the shroud or the duct, which has an aerofoil cross-section for regulating the fluid flow for better efficiency.

The duct can be designed for either accelerating or decelerating purposes which affect the ahead and astern operations. As it can be seen from Figure 2.3(a), leading edge section of duct has a larger cross-sectional area than trailing edge cross-sectional area. Hence, according to the continuity equation, which Carlton [17] pointed out, the water column velocity is increased with increased thrust and this type of duct shape is called as accelerating duct. The same line of logic holds for the decelerating duct which is shown in Figure 2.3(b). The water column velocity is decreased in trailing edge with an increased cross-sectional area rather than in leading edge. Hence, the thrust is decreased accordingly. The accelerating duct shape has a better efficiency in ahead
operations such as towing and trawling than astern operations while decelerating duct shape has better efficiency in astern operations.

Besides, decelerating ducts are widely used for decreasing the noise due to the cavitation and are used in submarines, torpedos and research ships which need to be quiet during operation. Moreover ducted propellers may have fixed or controllable pitch blades for different purposes and they can be either attached to the body or
steerable around a pin for eliminating the need of rudder to navigate the ship. By means of this design, thrust can be steered any direction which is needed. For larger naval platforms, the duct size in diameter can vary from 0.5 m to 8.0 m. Therefore, for larger diameters it can be very difficult to maintain the circularity of the duct. Additionally, ducted propellers can be beneficial when high thrust is required [18].

2.3 Modeling of Ducted Propellers

In general, many of the underwater robots use ducted conventional propellers as a thruster whose advantages are covered in Section 2.2. This section briefly describes two different models which are developed by Cooke [6], McLean [7], Cody and Healey [8]. In Cooke’s model, thrust and torque equations are achieved by using kinetic-energy based model and in McLean’s, Cody’s and Healey’s model, electrical and mechanical motor dynamics are included in the system as well as hydrodynamic properties.
2.3.1 Model 1: Ducted Propeller

In this model Cooke [6] developed a kinetic energy based approach for thrust and torque modeling. The thruster used in the model simply consists of a shroud or a duct, a propeller and an electric motor which drives the propeller via shaft and it can be defined with the parameters; motor torque ($\tau_M$), propeller angular velocity ($\Omega_p$), duct cross-sectional area ($A$), volume surrounded by duct ($V$), ambient fluid density ($\rho$) and volume flow rate inside the thruster ($Q$). The thruster schematic is shown in Figure 2.4.

![Thruster Schematic](image)

Figure 2.4: Thruster Schematic

While modeling the thruster, for simplicity, there needs to be made the following assumptions:

- The energy which is stored inside the duct is only the kinetic energy,
- Kinetic energy of ambient fluid is negligible,
Friction losses are negligible,

Ambient fluid is incompressible,

Fluid flow at the inlet and the outlet of the thruster is parallel and at ambient pressure,

Gravity effects are negligible.

According to the constitutive law [19], the relation between flow \((Q)\) and generalized momentum \((\Gamma)\) expresses the relation between kinetic energy and kinetic co-energy. Upper side and lower side of the curve in Figure 2.5 represent kinetic co-energy and kinetic energy, respectively. The kinetic energy of the fluid inside the thruster can be stated as:

\[
T(Q) = \frac{1}{2} \rho V \left[ \frac{Q^2}{A} \right].
\]
From the definition of energy in the constitutive relation, generalized momentum$(\Gamma)$-in other words pressure momentum, can be expressed as:

$$\Gamma = \frac{dT^*}{dQ} = \rho V \frac{Q}{A^2}.$$ 

Since the relation is linear between volume flow($Q$) and pressure momentum($\Gamma$) kinetic energy and kinetic co-energy are equal. Hence kinetic energy can be defined as the function of pressure momentum:

$$T(\Gamma) = \frac{A^2}{2\rho V} \Gamma^2. \quad (2.1)$$

A power balance equation is introduced as the difference between the power which is applied by the thruster propeller and exiting mass flow power- in other words power balance is the time rate of change of kinetic energy of the fluid inside the thruster. Power balance equation:

$$\frac{dT}{dt} = \frac{A^2}{\rho V} \hat{\Gamma} \hat{\Gamma} = \Omega_p \tau_M - K Q, \quad (2.2)$$

where $\Omega_p$, $\tau_M$, $K$ and $\hat{\Gamma}$ represent the applied power by means of thruster propeller, exiting kinetic energy per volume and time rate of change of pressure momentum. Exiting kinetic energy per volume is defined as $K \triangleq \frac{A^2 \Gamma^2}{2\rho V^2}$.

The thruster and the fluid are linked together by linear momentum and thrust force can be expressed as:

$$Thrust = \rho Q^2 A. \quad (2.3)$$

The expression pitch($p$) is the distance advanced by a propeller in one revolution in axial direction as shown in Figure 2.6 (a). However in a fluid, the advance of a propeller will be less than its actual magnitude. The difference between unyielding
advance and yielding advance is defined as slip [20] which is shown in Figure 2.6 (b) and the ratio is slip ratio \((\sigma)\) which is expressed as follows:

\[
\sigma = \frac{\Omega_p \, pA - Q}{\Omega_p \, pA}.
\]

![Figure 2.6: (a) Pitch \((p)\) (b) Slip Ratio \((\sigma)\)](image)

Propeller efficiency is defined as \(\eta = 1 - \sigma\). Hence volume flow rate \((Q)\) can be expressed as follows:

\[
Q = \eta p A \Omega_p. \tag{2.4}
\]

From the power balance equation and volume flow rate definition, thruster angular velocity\((\Omega_p)\) and motor torque\((\tau_M)\) relation is represented as a first order differential equation:
Consequently, steady-state thrust force is derived and observed that it is proportional to torque, in other words, square of propeller angular velocity. However, according to Equation (2.3), thrust force is only depending on volume flow rate, namely, axial flow velocity. Since axial flow velocity is an unmeasurable state, volume flow rate is approximated by Equation (2.4) proportional to propeller angular velocity. But, this approximation is unable to produce the actual volume flow rate.

2.3.2 Model 2: Tunnel Thruster

In this model, McLean [7] developed a hydrodynamic model by making use of linear momentum theory and conservation of energy in order for modeling thrust and torque. Besides, Cody and Healey [8] included electrical and mechanical motor dynamics.

Electrical Model

A mathematical model of a DC motor [21] is demonstrated in Figure 2.7. The armature is shown as a resistance ($R$) and in a series connection with inductance ($L_a$). Model parameters are input voltage ($V_a$), back emf ($e$), motor angular velocity ($\Omega_p$), motor torque ($\tau_m$), torque constant ($K_T$), armature current ($i$) and back emf constant ($K_M$).
The torque is developed by the motor and the back emf can be expressed as:

\[ \tau_M = K_T i, \]

\[ e = K_M \Omega_p. \]

Additionally, voltage-current relation can be defined using Kirchoff’s voltage law as follows:

\[
\frac{d i}{dt} = \frac{1}{L_a} V_a - \frac{R}{L_a} i - \frac{1}{L_a} e. \tag{2.7}
\]

Mechanical Model

Mechanical part consists of DC motor and propeller/impeller. Therefore, mechanical torque can be assessed as the sum of inertial, frictional and hydrodynamic loads with the parameters: mass moment of inertia of motor \((J_M)\), of propeller \((J_P)\), motor-shaft viscous friction coefficient \((C_M)\), coulomb friction \((\tau_F)\), hydrodynamic torque
(τ_H) and propeller angular velocity (Ω_P). Therefore, the governing equation for mechanical model can be expressed as [22]:

\[
\frac{dΩ_P}{dt} = \frac{1}{J_M + J_P} (τ_M - C_M Ω_P - τ_H - τ_F).
\] (2.8)

Eventually, DC motor modeling can be defined with two first order differential equations as expressed in (2.7) and (2.8).

**Thrust Modeling**

Before deriving the thrust force and hydrodynamic torque equations, the following assumptions for the hydrodynamic phenomenon which is shown in Figure 2.8, needs to be made:

1. Adiabatic control volume (No heat transfer),
2. Inviscid flow,
3. Incompressible flow (ρ = constant),
4. Potential and internal energy are negligible,
5. Enthalpy is negligible,
6. \(\dot{W}_v\) is negligible [3],
7. Fluid flow enters and exits normal to the cross-sectional area,
8. Inlet and outlet fluid pressure are constant.

According to Newton’s second law, the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system. From
McLean [7], Cody and Healey [8], in order to derive the thrust equation, conservation of momentum theory is applied to the fluid by using Reynold’s transport theorem and following integral is derived:

\[
\sum F = \frac{d}{dt} \int \int \int_{cv} \vartheta \rho dV + \int \int \vartheta \rho (\vartheta \cdot n) dA. \tag{2.9}
\]

By dividing the Equation (2.9) into two components; first integral- in other words unsteady term is defined as the time rate of change of linear momentum of the contents of the control volume and the second integral- in other words steady term, is defined as the net flow rate of the linear momentum out of the control surface [23].

As long as the flow is incompressible, fluid density can be extracted out of integral and first part can be denoted as:

\[
\frac{d}{dt} \int \int \int_{cv} \vartheta \rho dV = \rho \frac{d}{dt} \int \int \int_{cv} \vartheta dV,
\]
where \( \dot{\vartheta} \) is axial flow velocity. Volume flow rate \( (Q) \) can be defined as follows:

\[
Q = \int \int_s \dot{\vartheta} dA. \tag{2.10}
\]

For simplicity, the triple integral over control volume can be transformed into double integral over flow cross-sectional area by extracting length out of the integral as follows:

\[
\rho L \frac{d}{dt} \int_s \dot{\vartheta} dA = \rho L \dot{Q}. \tag{2.11}
\]

The net flow rate of the linear momentum out of the control surface- in other words, steady term can be defined with momentum-flux correction factor \( (\beta) \) in order to transform the control surface integral into an algebraic form. Momentum-flux correction factor \( (\beta_O \) for outflow and \( \beta_i \) for inflow) can be defined as follows:

\[
\beta_O = \frac{\int \int \dot{\vartheta}^2 dA}{A \dot{\vartheta}_{avg}^2}, \quad \beta_i = \frac{\int \int \dot{\vartheta}^2 dA}{A \dot{\vartheta}_{avg}^2}, \tag{2.12}
\]

where average velocity can be defined as \( \dot{\vartheta}_{avg} = \frac{Q}{A} \).

Substituting Equation (2.12) into second component of thrust Equation (2.9) yields the following expression:

\[
\int \int_{cs} \dot{\vartheta} \rho (\dot{\vartheta} \cdot n) dA = \frac{\rho}{A} (\beta_O - \beta_i) Q |Q|. \tag{2.13}
\]

Combining Equations (2.11) and (2.13) yields to the thrust equation as follows:

\[
Thrust = \rho L \dot{Q} + \frac{\rho}{A} \Delta \beta Q |Q|. \tag{2.14}
\]

**Torque Modeling**

In order to model torque , conservation of energy law is applied by means of Reynold’s transport theorem as follows [3]:

\[
\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{d}{dt} \int \int_{cv} (\dot{u} + \frac{1}{2} |\dot{\vartheta}|^2 + gz) \rho dV + \int \int_{cs} (\dot{h} + \frac{1}{2} |\dot{\vartheta}|^2 + gz) \rho (\dot{\vartheta} \cdot n) dA. \tag{2.15}
\]
where shaft work can be defined as $\dot{W}_s = -\tau_p \Omega_p$. Hereby negative sign represents that a work is done on control volume. Therefore, Equation (2.15) is simplified according to the aforementioned assumptions:

$$
\tau_p \Omega_p = \frac{d}{dt} \iiint_{cv} \frac{1}{2} |\vartheta|^2 \rho dV + \iint_{cs} \frac{1}{2} |\vartheta|^2 \rho (\vartheta \cdot n) dA. \quad (2.16)
$$

From the derivation of thrust equation, same steps can be followed. Firstly Equation (2.16) is divided into two components. For the first component—in other words, time rate of change of the energy content of the control volume, triple integral can be transformed into double integral over flow cross-sectional area by extracting length out of the integral and fluid density ($\rho$) is removed from the inside of integral regarding of incompressibility.

$$
\frac{\partial}{\partial t} \iiint_{cv} \frac{1}{2} |\vartheta|^2 \rho dV = \frac{L}{A} \frac{\partial}{\partial t} \iint_s \vartheta^2 dA.
$$

Substituting double integral with Equation (2.12), first part can be transformed into an algebraic expression as follows:

$$
\frac{d}{dt} \iiint_{cv} \frac{1}{2} |\vartheta|^2 \rho dV = \frac{\rho L}{A} \beta Q \dot{Q}. \quad (2.17)
$$

As for the second part, in other words, the net flow rate of energy out of the control surface can be transformed into an algebraic equation by means of kinetic energy correction factor ($\alpha$):

$$
\alpha_O = \frac{\int \vartheta^3 dA}{A \vartheta_{avg}^3}, \quad \alpha_i = \frac{\int \vartheta^3 dA}{A \vartheta_{avg}^3}. \quad (2.18)
$$

Substituting the integral with Equation (2.18) yields to:

$$
\iint_{cs} \frac{1}{2} |\vartheta|^2 \rho (\vartheta \cdot n) dA = \frac{\rho}{2A^2} \left( \alpha_O - \alpha_i \right) Q^3. \quad (2.19)
$$
Combining Equation (2.17) and (2.19) yields to the hydrodynamic torque as follows:

\[
\tau_H = \frac{\rho L \beta}{A} Q \dot{Q} + \frac{\rho}{2A^2} (\alpha_O - \alpha_i) Q^3. \tag{2.20}
\]

For completeness of the modeling of a tunnel thruster, governing system equations can be stated as follows:

\[
\frac{d\Omega_p}{dt} = \frac{1}{K_0} (K_T i - C_m \Omega_p - K_2 \Omega_p^2 \Omega_p - \tau_F), \tag{2.21}
\]

\[
\frac{di}{dt} = \frac{1}{L_a} V_a - \frac{R}{L_a} i - \frac{K_M \Omega_p}{L_a}. \tag{2.22}
\]

\[
\text{Thrust} = \rho L \dot{Q} + \frac{\rho}{A} \Delta \beta Q |Q|. \tag{2.23}
\]

where;

\[
K_0 = J_m + J_p + K_1,
\]

\[
K_1 = (\eta p)^2 (\rho A) L \Delta \beta,
\]

\[
K_2 = \frac{(\eta p) \rho A}{2} \Delta \alpha.
\]

It is convenient to restate Equations (2.21) and (2.22) in state space representation as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
\frac{K_T}{K_0} & -\frac{C_m}{K_0} \\
-\frac{K_M}{L_a} & -\frac{R}{L_a}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
-\frac{K_2 \Omega_p}{K_0} \\
0
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{V_a}{L_a}
\end{bmatrix}
\]

where \(x_1=\Omega_p\) and \(x_2=i\).

In this two state model, thrust is only depending on the volume flow rate, namely, axial flow velocity. Thus, this model also expresses volume flow rate is proportional to
propeller angular velocity as stated in Equation (2.4). However, this approximation for volume flow rate is unable to represent the actual amount.

2.4 Conclusions

In this chapter, different types of propulsion systems for naval vehicles are introduced and explained for their certain uses and advantages. Thereafter, two different ducted thruster models are introduced: Ducted thruster developed by Cooke [6] and tunnel thruster developed by McLean [7]. The first model is derived by using kinetic energy and power relations where the input is the torque and the output is the thrust force. Although this one state model shows consistency under state conditions, transient condition is not stated in modeling. Therefore, Cooke’s model does not reflect the complete response of the thruster. In the second model, McLean introduced hydrodynamic relations for thrust force and load on the propeller. Additionally, Cody and Healey included DC motor dynamics and presented a two state model. Even though this model shows more comprehensive explanations, it is still lacking of explaining the transient response where it requires a great care on the geometry of the propeller and velocity analysis on propeller blades. Therefore, Whitcomb and Yoerger [10], [11], [12] studied these two models intensively through experimentation and concluded that: Two of the models are capable of explaining steady state conditions however, they are lacking of explaining transient conditions. In addition to one state and two state models, Fossen and Blanke [9] presented a three state model whose variables are propeller angular velocity, axial flow velocity and surge speed of vehicle.

Furthermore, first model presents one state model of propeller angular velocity and second model presents two state model of propeller angular velocity and axial flow velocity. However, axial flow velocity is approximated by propeller angular velocity.
Yet, this approximation still incapable of explaining actual volume flow rate value. In three state model, axial flow velocity is also introduced as an unmeasurable state and a nonlinear state observer is derived for unmeasurable state axial flow velocity. In the next chapter, incorporation of axial flow velocity into the complete model for in-line pump jet thrusters will be fundamentally covered in order to resolve that unmeasurable state.
Chapter 3

Modeling and Analysis of an In-line Pump as Thruster Generator

It is crucial to analyze every parameter which contributes to the system in order to model a physical instrument. Accordingly, this chapter analyzes an in-line pump by decomposing it into constituent parameters, such as electro-mechanical DC motor modeling, geometric impeller modeling and hydrodynamic modeling. Moreover, volume flow rate is characterized by analyzing flow through impeller blades and incorporated into complete model. Besides, pump terminology, pump types and pump theory are briefly explained to sustain the completeness of the chapter.

3.1 Pumps

A pump is a mechanical device which is used to transfer, deliver or compress the fluid. Besides, pumps add energy to the system in order to pressurize the fluid. They can be classified under turbo-machinery. There are two types of pumps: Positive displacement and dynamic pumps.

3.1.1 Positive Displacement Pumps

Positive displacement pumps drag the fluid from the inlet, trap it inside the pump body, squeeze and discharge it from the outlet. This type of pumps can be used where accurate and constant flow rate is required such as medical applications. Positive displacement pumps produce same volume flow rate at given angular velocities. However,
fluid leakage is required to be carefully observed in order to prevent pressure losses, consequently volume flow rate losses. Moreover, this type of pumps requires more care in use. When there is a closed valve on the pump line, pump continues to pressurize the line since it does not have a shut off valve. This situation can cause a damage either on the line or on the pump. There are many types of positive displacement pumps currently in use in many fields and these types can be classified as shown in Table 3.1. Besides, they can be used for any fluid without considering viscosity. They produce a great amount of pressure in the inlet and can suck water from an enough distance, namely they don’t require priming. Furthermore, their lifetime is longer compared to other pumps since it is running at moderate speeds. However this type of pumps produce periodic flow since it is not practical for some applications.

3.1.2 Dynamic Pumps

Dynamic pumps add energy to the fluid by increasing its momentum via propeller or impeller blades. They are also called velocity pumps since flow velocity is increased by adding kinetic energy. This type of pumps are used where high flow rate

<table>
<thead>
<tr>
<th>Positive Displacement Pump Classification [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reciprocating Pumps</strong></td>
</tr>
<tr>
<td>Piston or plunger</td>
</tr>
<tr>
<td>Diaphragm</td>
</tr>
<tr>
<td><strong>Rotary Pumps</strong></td>
</tr>
<tr>
<td>Single rotor</td>
</tr>
<tr>
<td>Sliding vane</td>
</tr>
<tr>
<td>Flexible tube or lining</td>
</tr>
<tr>
<td>Screw</td>
</tr>
<tr>
<td>Peristaltic(wave contraction)</td>
</tr>
<tr>
<td>Multiple Rotor</td>
</tr>
</tbody>
</table>
is required. However, dynamic pumps produce low pressure rise and require priming before operation. Their performance is also sensitive to high viscous fluids as well. Unlike positive displacement pumps, this type of pumps can be operated when there is a closed valve on the pipe line. Dynamic pumps can be classified as follows:

Table 3.2 : Dynamic Pump Classification [3]

<table>
<thead>
<tr>
<th>Rotary Pumps</th>
<th>Centrifugal or radial exit flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial flow</td>
</tr>
<tr>
<td></td>
<td>Mixed flow</td>
</tr>
<tr>
<td>Special Designs</td>
<td>Jet pump or ejector</td>
</tr>
<tr>
<td></td>
<td>Electromagnetic pumps for liquid metals</td>
</tr>
<tr>
<td></td>
<td>Fluid actuated: gas lift or hydraulic-ram</td>
</tr>
</tbody>
</table>

**Axial Pumps**

Axial pumps consist of a propeller, a casing and a motor which can be either inside or outside of the casing. In the case of axial pumps, flow enters and leaves the pump axially parallel to the shaft of the propeller. Axial pumps find many area for use such as handling sewage from tanks, transferring ballast in ships, pumping water from sources. The main advantage of this pump is that they produce high flow rate compared to the centrifugal pumps.

**Centrifugal Pumps**

This type of pumps consist of an impeller and a casing which encapsulates the fluid and impeller. Fluid enters axially through the eye of the casing, gains both velocity
and pressure through the impeller blades and leaves the impeller radially through the
diffuser part of the casing. Centrifugal pumps convert rotational kinetic energy into
flow hydrodynamic energy in order to transport fluids and they required to be filled
with fluid before operation, otherwise they become incapable of pumping. Centrifugal
pumps are found in many areas such as dryers, vacuum cleaner, ventilation systems,
water pumps of cars. A centrifugal pump is sketched in Figure 3.1.

![Centrifugal Pump](image)

Figure 3.1 : Centrifugal Pump

### 3.2 In-line Pump Thruster: The Rule iL500K Pump

Pump jet thrusters are preferred for underwater robots which are used in highly sensi-
tive operations for their certain advantages such as reduced noise, reduced cavitation
and turbulence. Therefore, in-line pump thrusters are very feasible for underwater
robots. In addition, flow characteristics through impeller blades and impeller geom-
etry are very practical in order to address the unmeasurable state problem in ducted thrusters. In-line pump is combination of an axial flow pump and a centrifugal pump where flow enters and leaves the pump axially although it has an impeller for flow excitation. In this thesis, Rule iL500K submersible and in-line pump is analyzed. It consists of 5 components: an impeller, water resistant DC motor protection case, case cap, a DC motor and the outer shell as illustrated in Figure 3.2, 3.3 and 3.4 and the design characteristic of the pump is tabulated in Table 3.3.

Table 3.3 : Rule iL500PK Submersible In-line Pump Specifications [4]

<table>
<thead>
<tr>
<th>Part No.</th>
<th>iL500PK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>12Vdc(6amp)</td>
</tr>
<tr>
<td>Head</td>
<td>9.7 m</td>
</tr>
<tr>
<td>Pressure</td>
<td>0.96 bar</td>
</tr>
<tr>
<td>Flow Rate</td>
<td>1920 lph</td>
</tr>
<tr>
<td>Length</td>
<td>165 mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>38 mm</td>
</tr>
<tr>
<td>Outlet</td>
<td>19mm</td>
</tr>
<tr>
<td>Weight</td>
<td>500 g</td>
</tr>
</tbody>
</table>

It has a centrifugal type, equally spaced and equiangular 3-blades, 25.8 mm diameter impeller which is designed for changing the direction of the fluid flow. However, according to the internal dynamics and geometry, fluid flows axially. The power is transferred to the impeller through 6 amp, 12 V DC motor. The DC motor is located within the water-resistant protection case and closed by a case cap which provides
water resistance by means of shaft seal. The DC motor is encapsulated and its shaft pierced out of protection case through the case cap. The case cap also mounts the impeller and motor shaft is connected to the impeller on the case cap. The outer shell which has one inlet and one outlet encapsulates all other components. There are eight channels lie through the inner side of the outer shell which direct fluid in the axial way.

The fluid is dragged from the inlet and its pressure and velocity are increased through impeller blades. In principle, centrifugal type pumps are designed to drag the fluid axially and discharge it radially through volute. In this specific design,
after the fluid leaves the tip of impeller blades, it hits to the outer shell’s wall and moves ahead axially through the space between outer shell and protection case up to the outlet. When the fluid leaves impeller blades, guiding rails on the case cap in Figure 3.5 and flow channels on the inner side of the outer shell in Figure 3.6 guide fluid to flow smoothly through pump. Besides, side view of the in-line pump shown in Figure 3.7 where fluid flow path is designated within the dashed volume. From inlet to the outlet, fluid flows through the dashed volume.

The impeller geometry is a very important tool to analyze fluid flow from inner tip to outer tip of the impeller. According to [23], there are three types of impeller blade geometry as shown in Figure 3.8: backward-inclined blades, forward inclined blades
and straight blades. Backward inclined blades produce highest efficiency among others and its pressure rise is between them. Straight blades give the highest pressure rise over a range of volume flow rate. Forward inclined blades produce almost constant pressure rise. It depends only on the design requirements to decide on impeller blade geometry.

Rule iL500K in-line pump has equiangular bladed impellers where blade curve can be defined as follows [24]:

Figure 3.4: The Rule iL500K Pump In-line Pump Design Drawing
where $\theta_b$ is the blade angle, $j$ is the blade index $\{j \in \mathbb{N} \mid 1 \leq j \leq n \}$, $n$ is the number of
blades and $\theta_{o1}$ is the offset angle. Blade angle for equiangular bladed impellers shown in Figure 3.9 has the same angle between radius and tangent of the curve at that
radius. Hence impeller geometry related to blade angle can be expressed as follows:

\[ \text{Impeller blade} = \begin{cases} 
\text{Backward inclined,} & \text{if } 0 < \theta < \frac{\pi}{2}; \\
\text{Straight,} & \text{if } \theta = 0; \\
\text{Forward inclined,} & \text{if } \frac{\pi}{2} < \theta < \pi.
\end{cases} \]

Figure 3.9 : Blade Angle ($\theta_b$)

Impeller blade curve is defined with the Equation (3.2), however blade width ($b$) is still unknown. Blade width decreases linearly from inner tip to outer tip as shown in Figure 3.10 and the inclination is expressed as:

\[ b = C_1 + C_2 r, \quad (3.2) \]

where $C_1 = \frac{(b_1-b_2)}{r_2-r_1} + b_2$ and $C_2 = -\frac{b_1-b_2}{r_2-r_1}$. 


3.2.1 Normal Flow Velocity Characterization

It is of importance to analyze the fluid flow and velocity components through impeller blades. There are two velocity vectors: normal velocity vector and tangential velocity vector denoted by subscripts $n$ and $t$, respectively. Therefore these velocity components can be denoted as $V_{1,n}$ and $V_{1,t}$ at radius $r_1$ and $V_{2,n}$, $V_{2,t}$ at radius $r_2$. The fluid flows through the circumferential cross-sectional area from the eye of the
impeller to the outer tip of the impeller as shown in Figure 3.11. Hence volume flow rate \( Q \) can be defined as:

\[
Q = 2\pi rbV_n.
\]  

(3.3)

According to the conservation of mass, volume flow rate is same at every cross-sectional area through the impeller blade and Equation (3.3) can be written as:

\[
Q = 2\pi r_1b_1V_{1,n} = 2\pi r_2b_2V_{2,n}.
\]  

(3.4)

![Figure 3.11 : Volume Flow Rate](image)

The fluid velocity components are sketched on a backward-inclined impeller blade in Figure 3.12 and the same analysis holds for other impeller blade geometries. The inlet and the outlet of the impeller blade rotate with tangential velocities \( \Omega r_1 \) and \( \Omega r_2 \), respectively. These velocities are different from fluid tangential velocity components in both direction and magnitude because of the impeller blade geometry. Under the assumption of that flow is tangent to the blade surface in a moving reference frame,
relative velocity vector \( (V_r) \) becomes parallel to the blade surface. Besides, the angles between relative velocity vectors and reverse tangential velocity vectors are defined as leading edge angle \( (\Phi_1) \) and trailing edge angle \( (\Phi_2) \) at the inlet and outlet of the impeller blade, respectively.

\[ V_t = \Omega_p r - \frac{V_n}{\tan \Phi}. \]  

(3.5)

The control volume of the system is defined from \( r_1 \) to \( r_2 \) surrounding impeller
blades as shown in Figure 3.13. Hence differential form of control volume and circumferential cross-sectional area can be expressed as:

\[ dV = 2\pi b r dr, \quad (3.6) \]
\[ dA = 2\pi b dr. \quad (3.7) \]

Figure 3.13 : System Control Volume

Velocity analysis through impeller blades plays a significant role in order to characterize flow conditions developed by angular velocity of the motor. Axial flow velocity is an unmeasurable state for ducted propeller thruster models. Similarly, normal velocity component \( V_n \) is an unmeasurable state of in-line pump thrusters and a concrete expression is required to address this issue by incorporating unmeasurable state into the complete model. Therefore, velocity analysis of the flow through impeller blades and geometric characterization of impeller enable this study to incorporate unmeasurable state normal flow velocity into complete model thanks to the dynamical properties of incompressible flow conditions and geometric properties of impeller
3.2.2 Hydrodynamic Load Modeling of Impeller of the In-line Pump

In order to calculate the hydrodynamic load on the impeller, angular momentum equation can be used as follows:

$$\sum \vec{M} = \frac{d}{dt} \iint_{cv} (\vec{r} \times \vec{V}_A) \rho dV + \iint_{cs} (\vec{r} \times \vec{V}_A) \rho (\vec{V}_A \cdot \vec{n}) dA.$$  \hspace{1cm} (3.8)

where $\sum \vec{M}$ denotes the sum of all external moments acting on the control volume. The triple integral over control volume denotes the time rate of change of the angular momentum of the contents of the control volume and the double integral over circumferential cross-sectional area denotes the net flow rate of the angular momentum out of the control surface by mass flow ([23]).

Vector products in Equation (3.8) can be reduced to $rV_t$ and substituting differential volume ($dV$), differential area ($dA$), width ($b$) and tangential velocity component ($V_t$) into Equation 3.8 yields to:

$$\sum \vec{M} = \frac{d}{dt} \int_{r_1}^{r_2} \rho r \left[ \Omega_p r - \frac{V_n}{\tan \Phi} \right] 2\pi (C_1 + C_2 r) dr + \int_{r_1}^{r_2} \rho \left[ \Omega_p r - \frac{V_n}{\tan \Phi} \right] V_n 2\pi (C_1 + C_2 r) dr.$$  \hspace{1cm} (3.9)

Using Equation (3.3) and solving the integral within limits yields to;

$$\tau_H = \left[ \frac{\rho \pi C_1 (r_2^4 - r_1^4)}{2} \right] + \left[ \frac{2 \rho \pi C_2 (r_2^3 - r_1^3)}{5} \right] \Omega_p - \left[ \frac{\rho (r_2^2 - r_1^2)}{2 \tan \Phi} \right] \dot{Q} + \left[ \frac{\rho (r_2^2 - r_1^2)}{2} \right] \Omega_p Q - \left[ \frac{\rho}{2 \pi \tan \phi} \ln \frac{r_2}{C_1} - \ln (C_1 + C_2 r_2) - \ln r_1 - \ln (C_1 + C_2 r_1) \right] Q Q.$$  \hspace{1cm} (3.10)
DC Motor Modeling of the In-line Pump

Modeling of a DC Motor is achieved in Chapter 2 with Equations (2.7)–(2.8) and it is convenient to repeat the equations:

$$\frac{di}{dt} = \frac{1}{L_a} V_a - \frac{R}{L_a} i - \frac{1}{L_a} e. \quad (3.11)$$

$$\frac{d\Omega_p}{dt} = \frac{1}{J_M + J_p} (\tau_M - C_M \Omega_p - \tau_H - \tau_F). \quad (3.12)$$

3.2.3 Thrust Modeling of the In-line Pump

According to the linear momentum theory, sum of all external forces acting on a control volume is equal to the time rate of change of linear momentum of control volume plus the net flow rate of linear momentum out of the control surface. Therefore, it is applicable to use linear momentum equation in order to derive an expression of thrust created by in-line pump and control volume of the system is shown in Figure 3.14. However, following assumptions are needed to be made to model thrust:

- Adiabatic control volume (No heat transfer),
- Inviscid flow,
- Incompressible flow ($\rho = constant$),
- Irrotational flow,
- Flow enters and leaves normal to the cross-sectional area.

Under above assumptions, linear momentum equation can be written as follows:

$$\sum F_x = \frac{d}{dt} \int_{cv} \rho \frac{\partial u}{\partial x} dV + \int_{cs} \rho (\mathbf{u} \cdot \mathbf{n}) dA. \quad (3.13)$$
Figure 3.14: In-line Pump Control Volume

Triple integral and double integral describe unsteady part and steady part of the flow, respectively. It is convenient to divide Equation (3.13) into two parts. First part of the equation as previously described in Chapter 2 can be stated with Equation (2.11). For completeness, it is appropriate to repeat the equation.

\[ \rho L \frac{d}{dt} \int_s \vartheta dA = \rho L \dot{Q}. \quad (3.14) \]

Momentum-flux correction factor (\( \beta \)) can be used to transform second part of the equation into an algebraic equation. Momentum-flux correction factor (\( \beta_o \) for outflow and \( \beta_i \) for inflow) is defined in Equation (2.12). Therefore second part of the linear momentum equation yields to:
\[
\int_{cs} \vartheta \rho (\vartheta \cdot n) dA = \rho \Delta \beta Q \left[ \vartheta_{out} - \vartheta_{in} \right],
\]

(3.15)

where \( \vartheta_{out} = \frac{Q}{A_{out}} \) and \( \vartheta_{in} = \frac{Q}{A_{in}} \). Eventually, thrust modeling is obtained by combining Equations (3.14) and (3.15) as follows:

\[
Thrust = \rho L \dot{Q} + \rho \Delta \beta \ Q \left[ \frac{1}{A_{out}} - \frac{1}{A_{in}} \right].
\]

(3.16)

Therefore, in this study, thrust modeling is depending only on the volume flow rate. As previously described, volume flow rate is proportional to normal flow velocity component \( (V_n) \) through impeller blades. Thus, normal velocity is the unmeasured state of the in-line pump modeling. In order to analyze the system carefully, volume flow rate equation can be expanded by substituting normal velocity component \( (V_n) \) in Equation (3.5) into Equation (3.3) as follows:

\[
Q = 2\pi r^2 b \Omega_p \tan \Phi - 2\pi r b V_i \tan \Phi.
\]

(3.17)

Thus, velocity analysis at the inner tip of the impeller can be achieved with the assumption that flow enters to the impeller blades normal to the cross-sectional area as shown in Figure 3.15. In other words, absolute velocity vector \( (V_{1,A}) \) is in the direction of normal velocity component \( (V_{1,n}) \) and there is no tangential velocity component \( (V_{1,t}) \). Therefore, normal velocity component and volume flow rate can be defined as follows:

\[
V_{1,n} = r_1 \tan \Phi_1 \Omega_p.
\]

(3.18)

\[
Q = 2\pi b_1 r_1^2 \tan \Phi_1 \Omega_p.
\]

(3.19)
Furthermore, this characterization will be verified experimentally in Chapter 4 and leading edge angle ($\Phi_1$) will be determined by using experimental results. Therefore, time rate of change of volume flow rate can be expressed as:

$$\dot{Q} = 2\pi b_1 r_1^2 \tan \Phi_1 \Omega_p.$$  \hspace{1cm} (3.20)

For completeness of modeling, hydrodynamic torque and motor modeling equations can be combined into one equation. Therefore, governing system equations can be explained with three first order differential equations with the state variables of angular velocity of the impeller, motor current and volume flow rate as follows:
\[ \dot{\Omega}_p = \frac{1}{K_5 K_9 - K_8} \left( C_m \Omega_p - K_T i + K_7 \Omega_p \right) Q - K_6 Q \dot{Q} + \tau_F \], \quad (3.21)  \\
\[ \frac{d i}{dt} = \frac{1}{L_a} \left( V_a - R i - K_M \Omega_p \right). \]  \\
\[ \dot{Q} = \frac{K_9}{K_5 K_9 - K_8} \left( C_m \Omega_p - K_T i + K_7 \Omega_p \right) Q - K_6 Q \dot{Q} + \tau_F \], \quad (3.23)  \\
\[ \text{Thrust} = \rho L \dot{Q} + \rho \Delta \beta Q \left[ \frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} \right]. \] \quad (3.24)

where;

\[ K_3 = 2\pi \rho C_1 \frac{(r_2^4 - r_1^4)}{4} \]
\[ K_4 = 2\pi \rho C_2 \frac{(r_2^5 - r_1^5)}{5} \]
\[ K_5 = \frac{\rho}{\tan \Phi} \left( \frac{r_2^2 - r_1^2}{2} \right) \]
\[ K_6 = \frac{\rho}{2\pi \tan \Phi} \left[ \frac{\ln r_2 - \ln (C_1 + C_2 r_2)}{C_1} - \frac{\ln r_1 - \ln (C_1 + C_2 r_1)}{C_1} \right] \]
\[ K_7 = \frac{\rho}{2} (r_2^2 - r_1^2) \]
\[ K_8 = J_m + J_p + K_3 + K_4 \]
\[ K_9 = 2\pi b_1 r_1^2 \tan \Phi_1. \]

It is convenient to restate Equations (3.21), (3.22) and (3.23) in state space representation as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
\frac{C_M}{K_5 K_9 - K_8} & -K_T & 0 \\
-K_M & -R & 0 \\
\frac{K_3 C_M}{K_5 K_9 - K_8} & -K_9 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
+
\begin{bmatrix}
\frac{K_7 x_1 x_3 - K_6 x_3}{K_5 K_9 - K_8} \\
0 \\
K_9 |K_7 x_1, x_3 - K_6 x_3, x_3 + \tau_F| - \frac{K_5 K_9 - K_8}{K_5 K_9 - K_8}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]
where $x_1 = \Omega_p$, $x_2 = i$ and $x_3 = Q$.

Furthermore, the block diagram of governing system equations can be shown as in Figure 3.16.
Figure 3.16 : Block Diagram of Governing System Equations
Chapter 4

Experiment and Simulation

This chapter includes three sections: Experimental setup, validation and simulation. First section briefly introduces the instruments which are used to build experimental setup and indicates the experimentation process. Second section presents DC motor parameter identification, volume flow rate analysis and thrust force analysis. In this section, a comparison between experimental values and simulated values is presented. In the third section, a proposed model which includes the ambient flow effects is presented and thrust force changes in disturbed water are examined.

4.1 Experimental Setup

4.1.1 Laboratory Equipment

Test Box

The test box is made of plexiglass material and each surface sheet is bonded by using acrylic glue in order to maintain water resistance. In addition, silicone glue sticks are applied on intersecting edges for increasing resistance. Test box is used to contain fluid as well as being support for the instruments. Its dimensions are 49(l)x49(w)x47(h) cm\(^3\).
Load Cell

FX1901 compression load cell which has a range of 0-10 lbf is used for measuring output thrust force. Specifications to load cell as shown in Figure 4.1 can be seen in the data sheet [1]. Besides INA125 instrumentation amplifier is used to amplify the sensor output. The gain of the amplifier is calculated as follows:

\[ \text{Gain} = 4 + \frac{60k\Omega}{R_G}, \]

where \( R_G \) is the external resistance and set to 320 ohms. As in its nature, load cells respond linearly with applied weight. Therefore, the load cell is calibrated with known weights and sensor output-known weight linear relation is shown in Figure 4.2 and expressed as follows:

\[ \text{Load} = a \times \text{SensorOutput} + b. \]

where unitless coefficients are defined as \( a=0.0114 \) and \( b=2.432 \).
Liquid Flow Meter

Liquid flow meter has a working range of 0-30 liters/min and sends 450 pulses per liter. Besides, its pulse characteristic is $Frequency(Hz) = 7.5 \times FlowRate(l/min)[2]$. It has 1/2" NPS nominal pipe connections, 0.78" outer diameter, 1/2" of thread and 2.5” x 1.4” x 1.4” of dimensions which are shown in Figure 4.3.
DC Regulated Power Supply

TENMA 72 7655 DC regulated power supply shown in Figure 4.4 is used to supply pump with different voltages ranging from 2 to 12 volts. Specifications for the power supply are tabulated in Table 4.1:
Table 4.1: TENMA 72 7655 DC Regulated Power Supply Specifications

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Voltage</td>
<td>115 Volt</td>
</tr>
<tr>
<td>Output Current</td>
<td>60 A</td>
</tr>
<tr>
<td>Load Regulation</td>
<td>0.1% +5mV</td>
</tr>
<tr>
<td>Line Regulation</td>
<td>0.05% +3mV</td>
</tr>
<tr>
<td>Ripple and Noise</td>
<td>40mV (p-p)</td>
</tr>
<tr>
<td>Efficiency</td>
<td>85%</td>
</tr>
<tr>
<td>Meter Accuracy</td>
<td>1% +1 count</td>
</tr>
<tr>
<td>Power requirements</td>
<td>120VAC, 60Hz</td>
</tr>
<tr>
<td>Dimensions</td>
<td>4-1/4” (H) x 8-3/4” (W) x 14-1/4” (D)</td>
</tr>
<tr>
<td>Weight</td>
<td>12.8 lbs.</td>
</tr>
</tbody>
</table>

**Micro Controller**

Arduino Uno microcontroller shown in Figure 4.5 is used during the experiments in order to collect data from sensors. Arduino Uno has digital and analog pins and works with a computer connection through a USB cable or externally powered by a DC supply (7-12V) [25]. It also has power pins such as Vin, 5V, 3.3V and GND which enable users to supply voltage through these pins.
4.1.2 Laboratory Procedure

Laboratory equipments are described in details in the previous section. This section expresses the test bed construction and experimentation.

Test Bed

Test box is filled with water up to half, however the depth of the pump was not taken into account for modeling. Main support is placed on the top of the test box and supports the main rod, sensor mount and pump. Main rod is fixed at the main support with two male ball joint rod ends. Main rod composes of two M10 threaded rode and formed in T-shape. Horizontal main rod is carried by the ball joints and vertical rod is connected to the horizontal rode in the middle. A 3D printed collar is suited for pump and placed at the bottom of vertical rod. T-shape rods behave like pendulum. The output force is intended to be amplified mechanically by placing the
sensor and pump with a distance ratio from the rotating point. The reason behind this idea is to increase the accuracy of the sensor. Besides, sensor mount is fixed to the main support and sensor is placed on the sensor mount with a distance which gives $\frac{1}{3}$ ratio with the pump distance from the rotating point. Therefore resulting force amplified three times mechanically. Eventually, test bed design drawings and actual setup are shown in Figures 4.6, 4.7, 4.8 and 4.9.

**Experimentation**

Test setup is built with a pendulum idea where pump is attached to a pivotal point with a rod. Force sensor is attached to the setup with a distance of $\frac{1}{3}$ length of the rod from the pivotal point. The main reason to apply this idea is that pump jet produces relatively small forces compared to the range of the force sensor. Therefore, it is aimed to increase the resolution of the sensor since the error of the measurement is higher with very small values of applied force. Hence, the force produced by pump jet is increased three times for measurements. During experimentation, volume flow rate sensor is attached to the pump in order to do real time measurements. Besides, a nozzle attachment which has smaller cross-sectional area than pump outlet is connected to the outlet of the pump in order to increase the outlet fluid velocity. Accordingly, increased thrust response is obtained.

### 4.2 MODEL VALIDATION

The aim of this section is to make observations regarding simulated and experimental results of DC motor response, volume flow rate response and thrust force response. Experiments are achieved by running the pump-jet through a range of DC voltages and measurements are collected for thrust force and volume flow rate. Therefore,
Figure 4.6: Testbed Design Drawings: (a) Right View (b) Front View (c) Top View (d) Isometric View
Figure 4.7: In-line Pump Configuration
Figure 4.8: Experimental Set-up 1
Figure 4.9 : Experimental Set-up 2
experimental results are compared to the simulated results.

**DC Motor Parameter Identification**

Mabuchi FS-390-PH3752 permanent magnet DC motor is used to run this particular pump however design characteristics are unknown. The very first part of the experimentation is to identify the model parameters and clearly observe the motor response with certain volt inputs since it is a starting point for accurate modeling. According to the DC motor modeling in Chapter 2, the parameters needed to be identified are resistance ($R$), back-EMF constant ($K_M$), torque constant ($K_T$), inductance ($L_a$), mass moment of inertia of motor ($J_m$), motor friction constant ($C_m$) and coulomb friction ($\tau_F$). Besides, all the experiments for model parameter identification are done with no applied load on the shaft. At first, mass moment of inertia of DC motor is identified by Solidworks mass properties analysis. Secondly, DC motor resistance measured by Fluke 117 multimeter. Another parameter which is back-EMF constant is defined by using Equation (2.7) under steady state conditions. Measurements for current and impeller velocity are collected during steady state for different voltages and evaluated in the equation. Therefore, back-EMF constant is obtained. Furthermore, motor torque constant is equal in magnitude when:

$$K_m(Volt/rad/sec) = K_T(N - m/Amp).$$

Another two parameters which are motor friction constant and coulomb friction can be identified by using Equation (2.8) under steady state conditions as follows:

$$K_T i = C_M \Omega_p + \tau_F.$$

The last parameter, inductance is tuned according to the experimental results. Eventually, model parameters are tabulated in Table 4.2. DC motor simulated time re-
Table 4.2 : DC Motor Parameters

<table>
<thead>
<tr>
<th>DC Motor Parameters</th>
<th>Magnitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor friction constant</td>
<td>0.000004 N-m sec</td>
</tr>
<tr>
<td>DC motor inertia</td>
<td>$5.5 \times 10^{-6} \text{ kg/m}^2$</td>
</tr>
<tr>
<td>Back-emf constant</td>
<td>0.0134 N/rad/sec</td>
</tr>
<tr>
<td>Torque constant</td>
<td>0.0134 N-m/amps</td>
</tr>
<tr>
<td>Inductance</td>
<td>0.07 H</td>
</tr>
<tr>
<td>Resistance</td>
<td>1.8 ohms</td>
</tr>
<tr>
<td>Coulomb friction</td>
<td>0.0048 N-m</td>
</tr>
</tbody>
</table>

response and measured time response are shown in Figure 4.10. Besides steady state values of the velocity with different voltages are presented in Figure 4.11.

Figure 4.10 : Simulated and Measured Time Response of DC Motor
Volume Flow Rate Analysis

Volume flow rates for each input voltages are measured by a flow meter which is previously introduced in this chapter. This experiment is required in order to understand the nature of the flow velocity analysis at the inner radius of the impeller. It is convenient to repeat the assumption about the flow rate definition in Chapter 3. The fluid flow enters to the impeller blades radially. Therefore, leading edge angle ($\Phi_1$) is required to be determined with respect to this assumption since the model produces the same volume flow rate with sensor measurements. Eventually, leading edge angle ($\Phi_1$) is set to 39° for backward inclined impeller configuration and 38° for forward inclined impeller configuration. Hence, results are shown in Figure 4.12 where negative voltages represent forward inclined impeller configuration and positive voltages represent backward inclined impeller configuration.
Thrust Force Analysis

Experiments are achieved in test conditions that include calm water and no tide. In other words, ambient water at the inlet is not disturbed. However, boundary effects still exist inside the box. Measurements of thrust response are collected by force sensor during experimentation within a range of input voltages. Besides, flow meter is attached to the pump during force measurements. The reason for this configuration is that volume flow rate data sets are collected with the same configuration. So, control volume of the system is reconfigured according to the flow meter attached configuration. Hence, it is aimed to keep the same configuration during the experimentation. Thereafter, model equations are simulated in order to make a comparison with experimental results. In the system equations, momentum flux correction factor ($\Delta \beta$) shown in Equation (3.24) needs to determined experimentally since it
affects the steady state results of the simulation. At a first step, it is set to $\Delta\beta = 1$. Therefore, the results showed that simulation lacks of explaining experimental results under steady state conditions. Eventually, momentum flux correction factor is set to a value $\Delta\beta = 1.02$ where simulation results match with the experimental results within a range of different input voltages. Momentum flux correction factor values for each measurement input are shown in Figure 4.13 and Figure 4.14.

![Momentum Flux Correction Factor vs Input](image)

**Figure 4.13 :** Backward Inclined Impeller Momentum Flux Correction Factor ($\Delta\beta$) vs Input Voltages

After momentum flux correction factor is identified, all the parameters in the system equations are known. The system equations are simulated and results are compared with the experimental results as shown in Figure 4.15 and Figure 4.16. It can be seen from the figures that model response is able to match with the experimental result under both transient and steady state conditions.

Therefore, simulated results show an excellent relation between square of angular
Figure 4.14: Forward Inclined Impeller Momentum Flux Correction Factor ($\Delta\beta$) vs Input Voltages

Figure 4.15: Backward Inclined Impeller Thrust Time Response
velocity and steady state thrust as illustrated in Figure 4.17 and Figure 4.18.

Figure 4.16: Forward Inclined Impeller Thrust Time Response

Figure 4.17: Backward Inclined Impeller Squared Angular Velocity vs. Thrust
4.3 Simulation Study

In the previous section, experiments are achieved in a test environment which is calm water and no tide. However, when test environment is disturbed, flow conditions change. Therefore, experimentally measured force in a disturbed environment shows differences compared to the calm water. According to the thrust equation (3.24) derived in Chapter 3, thrust force depends on the volume flow rate. The experimental test bed is disturbed by a secondary pump by placing its inlet facing to the primary pump’s inlet in the same direction as shown in Figure 4.20 and Figure 4.21. In this configuration, secondary pump is run and it creates a disturbance inside the test box. This experiment is achieved to observe the effects of the environment. Since underwater robots can be operated around oil tank walls or sea bed which affect the environment flow. While primary pump is off, load cell does not read any applied force. Afterward, primary pump is run and load cell data is collected. It is observed
that the disturbed water behaves like small waves inside the box. According to the measurements, while two pumps are in operation, measured thrust force shows an increased and a sinusoidal response compared to the calm water measurements. Since thrust equation depends on the volume flow rate, an arbitrary change on the flow affects the thrust force. In such a case where ambient flow is disturbed, it is proposed that if volume flow rate is disturbed by a constant plus sinusoidal disturbance, thus, the model is capable of showing consistency with the measured thrust response. Thus, volume flow rate in a disturbed case can be expressed as follows:

\[ Q_d = Q + Q_0 + Q_1 \sin (wt), \]

where \( Q_d \) denotes the volume flow rate state in a disturbed water, \( Q \) denotes the volume flow rate state in calm water, \( Q_0 \) and \( Q_1 \sin (wt) \) denote constant and sinusoidal disturbance, respectively. The block diagram of the system is shown in Figure 4.19.

In Figure 4.22, experimental calm water thrust force response, experimental disturbed water thrust force response and flow rate disturbed model simulated thrust force response are shown. From the figure, it can be observed that the proposed model is able to show the experimental response. However, the initial peak of the response come from the experimental environment where the test bed is relatively small for this experiment. According to the proposed model and the experiment, disturbance in the flow affects thrust. Therefore, volume flow rate and impeller angular velocity are affected by the disturbance. Figure 4.23 and Figure 4.24 show the simulated response of the volume flow rate and impeller angular velocity, respectively.
Figure 4.19: Flow Disturbed Model Block Diagram
Figure 4.20: Disturbed Water Experimental Set-up 1
Figure 4.21: Disturbed Water Experimental Set-up 2

Figure 4.22: Disturbed Water Experimental and Simulated Force Response
Figure 4.23: Simulated Volume Flow Rate Response

Figure 4.24: Simulated Impeller Angular Velocity Response
From the simulation study, it is clearly seen that thrust force, volume flow rate and impeller angular velocity are very sensitive to the disturbance in the flow. According to the Equation (3.19) and Equation (3.24), volume flow rate is proportional to the impeller angular velocity and thrust force depends on the volume flow rate. Therefore, any change in impeller angular velocity affects thrust force response. Therefore, it is proposed that if DC motor is controlled at a given angular velocity, then volume flow rate can be controlled. Accordingly, thrust force response can be controlled. Controlling the DC motor or impeller angular velocity makes the model insensitive to the ambient flow effects. In Figure 4.25, impeller angular velocity simulated response of undisturbed case and volume flow rate disturbed case are shown. Besides, it can be observed that if DC motor is controlled, even in flow rate disturbed case, impeller angular velocity can be adjusted to the impeller angular velocity of undisturbed case. So, volume flow rate and thrust force response also can be adjusted to the undisturbed case as shown in Figure 4.26 and Figure 4.27.
Figure 4.25 : Simulated Impeller Velocity Response

Figure 4.26 : Simulated Volume Flow Rate Response
Figure 4.27: Simulated Thrust Force Response
Chapter 5

Conclusions

Dynamical modeling of a thruster has been always the main concern of naval vehicle propulsion, especially for underwater robot propulsion. For many years, underwater robotic applications suffered from complete thruster modeling, therefore, most of the effort and enthusiasm have been concentrated on compensation of thrust losses by applying control methods due to the lack of completeness in modeling.

This work has presented an overall view of propulsion types for naval vehicles and narrowed the scope to underwater robotic thrusters. In Section 2, ducted propeller thruster and tunnel thruster modelings are analyzed. Major deficiencies of the models are recognized carefully where they should make contributions for developing the thrust response. It has been observed from the previous studies that generally thruster modeling suffered from lack of axial flow velocity definition, propeller geometry and flow analysis on propeller blades. One-state model which is explained in Chapter 2 is not a realistic model since the model only depends on the input torque and angular velocity. Two-state model presents an improved modeling by including hydrodynamic relations and DC motor dynamics. Yet, axial flow velocity still remains obscure. Three-state model presents more accurate modeling where a nonlinear observer is derived for axial flow velocity. However, this state is still unmeasurable.

In chapter 3, in-line pump application for underwater robotic actuation carefully analyzed through every constituent of the pump. The analyses are divided into sub-systems such as hydrodynamic torque modeling, thrust modeling, impeller geometry
characterization and DC motor modeling. Eventually, these sub-systems combined into each other in order to complete modeling. However, the challenges and at the same time the deficiencies of the previous studies to modeling are geometry of the impeller, flow development through impeller blades, flow velocity analysis and, most importantly, incorporating axial flow velocity expression into the complete model. Since axial flow velocity can be attributed as an unmeasurable state of governing system equations, this study has contributed to address this issue by validating that axial flow velocity can be expressed by volume flow rate definition and volume flow rate is proportional to the impeller angular velocity. Consequently, volume flow rate definition leads this study to improve the accuracy of the modeling thanks to the dynamical properties of impeller geometry and flow conditions through impeller blades.

Furthermore, thrust modeling of an in-line pump is modified by adding disturbance in volume flow rate in order to emulate the ambient flow effects in simulation. It is observed from the experiment and simulation, disturbance in volume flow rate affects the thrust force. Besides, according to the simulation results, it also affects the states of governing system equations such as impeller angular velocity and volume flow rate. Therefore, these states are very sensitive to the ambient flow effects. It is proposed that if impeller angular velocity is controlled at a required velocity, then ambient flow effects are avoided since volume flow rate and thrust response become unaffected from ambient flow changes. This simulation study opens a future research area where in-line pump jet thruster modeling can be improved according to the ambient flow changes. A DC motor controller can be applied in order to make the system insensitive to ambient flow affects. In simulation study, it is shown that adjusting the impeller angular velocity can make the volume flow rate and the thrust force responses insensitive to ambient flow effects.
Appendix A

System Parameters

Table A.1: Parameters of DC Motor and Impeller Geometry

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor friction constant</td>
<td>0.000004 N-m sec</td>
</tr>
<tr>
<td>DC motor inertia</td>
<td>$5.5 \times 10^{-6} , kg/m^2$</td>
</tr>
<tr>
<td>Back-emf constant</td>
<td>0.0134 N/rad/sec</td>
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<tr>
<td>Torque constant</td>
<td>0.0134 N-m/amps</td>
</tr>
<tr>
<td>Inductance</td>
<td>0.07 H</td>
</tr>
<tr>
<td>Resistance</td>
<td>1.8 ohms</td>
</tr>
<tr>
<td>Coulomb friction</td>
<td>0.0048 N-m</td>
</tr>
<tr>
<td>Length (L)</td>
<td>0.227 m</td>
</tr>
<tr>
<td>Pump Inlet Area</td>
<td>$7.8540 \times 10^{-5} , m^2$</td>
</tr>
<tr>
<td>Pump Outlet Area</td>
<td>$3.3183 \times 10^{-5} , m^2$</td>
</tr>
<tr>
<td>Impeller Inner Radius</td>
<td>0.00383 m</td>
</tr>
<tr>
<td>Impeller Outer Radius</td>
<td>0.0129 m</td>
</tr>
<tr>
<td>Blade Angle ($\theta_b$)</td>
<td>$\pi/3$</td>
</tr>
</tbody>
</table>
Bibliography

[1] *FX1901 Compressible Load Cell.*


