RICE UNIVERSITY

Essays Investigating Extreme Events in Financial Markets

by

James N. Gualtieri II

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

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ABSTRACT

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This thesis, through three empirical applications, provides an analysis of extreme events in financial markets. Robust growth in financial markets has greatly increased the ability of economic agents to share risk according to their preferences or tastes. Despite this, many markets have demonstrated extreme instability at times. These events have the potential to shake the confidence of investors and this fear can lead to inefficient outcomes with respect to risk sharing and resource allocation. By investigating the dynamics of securities during extreme events one can gain intuition as to their root causes and a better understanding of the inherent risk.

The first chapter analyzes how international equity markets interact during extreme events. Using a novel set of high-frequency data on exchange traded funds (ETFs), designed to track international equity markets, I examine the dynamics of intra-day returns between 11 countries. Using non-parametric tests designed to identify jumps in the price process I examine the dynamics across markets during jumps, as well as continuous movements. Contrary to other literature that uses coarser data, I find a high-degree of commonality in the jump components. Specifically, there are many instances when different markets co-jump and returns are significantly more correlated on jump days. I also find substantial evidence of self and cross excitation across markets and that international markets respond to US macroeconomic...
news announcements. These findings suggest that international financial markets are heavily intertwined and that shocks propagate across markets. This information is valuable from a modeling perspective as it provides evidence of channels through which economies are linked that must be accounted for. Further, it provides valuable information to investors into the benefits and risks associated with international diversification that allows them to take a more proactive, rather than a reactionary, approach to risk management.

The second chapter, based on Gualtieri and Sizova (2015), investigates the joint dynamics of portfolios considered to represent priced risk in asset markets. Specifically, it considers the joint modeling of the market return, and two zero net cost portfolios that are used as proxies for systematic risk factors: Value and Momentum. As in the case of chapter 1, we allow for a separation between continuous and jump dynamics. We find a number of interesting relationships between factor dynamics that have implications for risk-based explanations of factor risk premia as well as factor investing. Specifically, we find that although volatilities are highly correlated, the orthogonal (to the Market volatility) component of Momentum volatility contains information about the Market’s dynamics. With respect to extreme events, we find that volatility co-jumps are present in both-return pairs (Market-Momentum and Market-Value). We find that Value does not jump independent of the Market, whereas Momentum does. We also find that a number of the Momentum jumps occur in bear markets, which is consistent with documented Momentum crashes (see for example Daniel and Moskowitz (2013)). We also use the model output to investigate the merits of factor investing. We estimate a variety of metrics on jump days to analyze the benefits of diversifying away from the market and into additional stylized portfolios. We find that the a combination position in the Market and Value significantly improves performance during extreme events in terms of average loss, volatility and value-at-risk.

Aside from the empirical analysis we also provide a generalization of the univariate
stochastic volatility conditional jump (SVCJ) model of Eraker et al. (2003) to the multivariate case. We provide a detailed appendix documenting the sampling scheme that can be used to investigate joint dynamics in extreme events.

The third chapter, based on Bada et al. (2015), examines whether algorithmic trading (AT) has a time varying effect on measures of liquidity such as bid-ask spreads and volatility. Specifically, in the context of a panel model with individual and time fixed effects we allow for structural breaks in the slope parameters at an unknown number of times and automatically detect the break points. The model is free from any ad-hoc identification of break points or restrictions on the number of breaks imposed by the econometrician a priori. The study is the first to use this estimator (in any context) and the results show clear evidence of breaks in the relationship between AT and liquidity during the financial crisis. These results are in contrast to prior literature that demonstrates a clear positive relationship between AT and market liquidity. The timing of the breaks is important as the merits of added liquidity during relatively stable periods versus its withdrawal during periods when it is in high demand are somewhat ambiguous and may possibly present a net welfare loss to society. The results indicate the presence of a state contingent relationship between AT and liquidity.
Acknowledgements

First and foremost, I would like to dedicate this thesis to my mother, whom I miss dearly and whose time in this world I will always cherish and remember. She was an incredible woman and although my time with her was cut short, I wouldn’t trade it for anything in the world. I would also, like to dedicate this thesis to my father for his unconditional support throughout my entire life. I owe him a lot and I hope this accomplishment serves as a form of validation for all the effort he put in. I am grateful for the contributions both have made molding me into the man I am today.

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Chapter 1

How Do International Equities Jump?

1.1 Introduction

Although somewhat of a simplification, stock returns can generally be modeled as having two distinct components: (1) smooth continuous variation that is approximately conditionally Gaussian and (2) infrequent large fluctuations. The second component, typically referred to as jumps, are clearly the more salient of the two. Events such as the stock market crash of 1987, the “bursting of the dot-com bubble” in 2001 and the onset of the financial crisis in 2008 stand out as particularly memorable examples of large market fluctuations.

An open question, to which this chapter makes a contribution, is whether jumps are systematic. If investors dislike jumps then the inability to mitigate their impact on portfolio returns through diversification implies that investors may assign a high price of risk to jump exposure. On the other hand, if the effect of jumps can be eliminated, or substantially reduced, through diversification then their presence can be considered more of a peculiarity rather than a key factor driving the risk-return relationship.

To this date much of the literature has focused on only one market (typically the US). Systematic jump exposure is commonly modeled in one of two ways. One line of research supposes that jumps are systematic via a common component in time-varying jump intensities. Examples of this include Chan and Maheu (2002),
who examine whether co-movement with the time varying market jump arrival rate earns a premium. In a similar spirit, Kelly and Jiang (2014) derive a time-varying measure of tail activity and show that exposure to it similarly earns a premium. As an alternative, Bollerslev et al. (2014) investigate covariance with the actual instances of jumps and find that stocks with high “jump betas” earn a significantly larger premium to compensate for this risk.

In contrast to the above research, this chapter considers the nature of jump across international markets. The question of how jumps propagate across countries is of interest for many reasons. As world economies become more intertwined through globalization and as trade linkages stretch the production process across national borders, a natural question is whether and to what extent large shocks in one market occur concurrently in the others. In addition, do these large shocks transmit across markets, i.e. is there contagion in extreme events? With the rise in accessible (to the average investor) investment vehicles that offer access to international equity markets, the question of whether a globally diversified portfolio can smooth performance over time is also of interest. To the extent that jumps are less systematic across countries, international diversification could present something of a free lunch.

Only a handful of papers have examined the role of jumps in international markets and this chapter differs in a number of substantial ways. First, I use recent high-frequency return data to detect the presence of jumps. This allows for more accurate jump detection as test statistics constructed using finer return intervals are closer to their limiting distributions. Second, the use of more recent data also allows for the analysis to concentrate on the current state of the world, when international economic linkages are certainly stronger as many companies are now spread out across the production chain. Lastly, I use, as a proxy, exchange traded funds (ETFs) designed
to mimic the returns to international equity markets. From the perspective of a US investor seeking to diversify across markets, these represent the least costly and most convenient investment vehicles to do so. Thus, from a practical standpoint, the analysis is certainly of use.

In order to investigate the dynamics of jumps across international markets I model portfolio returns in a continuous time framework. The above two empirically observed features of stock returns can be conveniently modeled within the setting of jump-diffusion models. There have been a large number of recent advances in econometric methods concerning the design of non-parametric tests and methods to estimate components of jump-diffusion models from high-frequency intra-day return data based on realized power variations.\(^1\) I apply a number of these methods to a sample of 11 different ETFs that are designed to match the performance of equity markets in 11 different countries. In contrast to most of the previous literature on international correlation this is the first to investigate jump dynamics at frequencies higher than daily or semi-daily (i.e., open to close and close to open) intervals on a broad scale.\(^2\) The use of ETFs is somewhat of a departure from the standard practice, however one key benefit is that these investment vehicles present the most accessible options for international investment available to an investor in the US.\(^3\) Their use also resolves a number of the simultaneity issues that plague other studies that attempt to examine

\(^1\)See Andersen et al. (2003), Andersen et al. (2009), Barndorff-Nielsen (2002), Barndorff-Nielsen et al. (2003), Barndorff-Nielsen and Shephard (2004), to name a few.

\(^2\)King and Wadhwani (1990) analyze hourly return data between the US and UK markets in a short period around the 1987 stock market crash. They are of course limited to the time period when the two markets are open simultaneously.

\(^3\)Li et al. (2014) investigate co-jump dynamics using the SPDR sector ETFs managed by State Street Global Advisors.
common movement at relatively high frequencies across international markets.\textsuperscript{4}

The contributions are three-fold: I first examine whether international markets jump together, that is are jumps common across markets? I find that although some of the jumps in various markets are purely idiosyncratic or region specific, a non-trivial percentage of jumps are systematic. To further enhance this point I also show that a globally diversified portfolio exhibits substantial evidence of jumps. Second, I examine the determinants of jumps. Specifically, I test if jumps self and cross excite and whether jumps in one market are related to fundamental economic information released in other markets. I find that there is ample evidence of both self and cross excitation and that jumps in many international markets are related to the release of US macroeconomic data. Lastly, I examine the role sensitivity to jumps in a global factor plays in explaining cross-sectional differences in the returns across markets. Although I find distinct time periods for many countries where jump sensitivities depart from continuous sensitivities, over the sample considered I find no evidence that these exposures are related to risk premia.

The chapter proceeds as follows: Section 1.2 briefly reviews the related literature. Section 1.3 presents the econometric methods. Section 1.4 describes the data. Section 1.5 presents the main results and Section 1.6 concludes.

1.2 Related Literature

This chapter is at the intersection of two strands of literature: the analysis of jump-diffusion models from high-frequency return data and the analysis of spillover between international equity markets. I provide a brief discussion of each below.

\textsuperscript{4}Namely, that international markets are not open at the same time.
1.2.1 Testing for jumps using high-frequency data

Jump-diffusion models provide a very convenient model for security prices. They are able to accommodate a number of the empirically observed aspects of security prices (or equivalently security returns). Of particular interest is that many prices changes are too large, relative to spot volatility estimates, to be explained by a continuous diffusion process. The jump component allows for the modeling of such departures from the instantaneous Gaussian world.

Since jumps are rare,\(^5\) large relative to the conditional volatility and unexpected, they have a number of asset pricing implications. As a result a number of tests have been proposed in the literature to both confirm their existence and analyze their behavior.

Barndorff-Nielsen and Shephard (2006) provide feasible tests for the presence of jumps over a return interval that are derived from various measures of power variation. Much of the theory behind these results is laid out in Barndorff-Nielsen et al. (2003) and Barndorff-Nielsen et al. (2006). They construct three test statistics \((G, H\) and \(J)\), all essentially based on the difference or ratio of bipower variation to realized variance, scaled by some function of higher power variations.\(^6\) Andersen et al. (2007) derive a similar test statistic, replacing the quadpower variation in the denominator with tripower quarticity. Huang and Tauchen (2005) examine the accuracy of these

---

\(^5\)For the purpose of this chapter I assume that the process driving the jumps is a Poisson process with finite activity. That is over a time interval \([0, T]\) the process can only jump a countable number of times. Jumps with infinite activity (see for example, Madan et al. (1998) and Carr et al. (2002)) allow for an infinite number of jumps over any interval.

\(^6\)The \(G\) statistic was chosen, after a simulation study, to examine jumps in international equities using daily data by Pukthuanthong and Roll (2014).
statistics, as well as some additional variants, and find that they perform quite well in both a simulation study and an empirical analysis. Aït-Sahalia et al. (2009) derive a test statistic based on the ratio of powers of absolute returns, sampled over the same time interval but at different frequencies. Aït-Sahalia et al. (2012) extend the basic idea to the case when prices are observed with microstructure noise.

A limitation of all the statistics mentioned above is that they can only be computed for the whole of some time interval $[0, T]$. That is, the statistics can estimate whether there was a jump within that time interval, but not when. If one could sample at finer and finer frequencies then the interval can be made arbitrarily small. However, for all but the most liquid of securities such a sampling frequency is not available. A typical choice is to set $T = 1$ day and compute a series of these statistics for each trading day. Even then, for the statistics to have desirable properties returns must be sampled fairly frequently (around 5-minute intervals). Huang and Tauchen (2005) find evidence of size and power distortions as the sampling frequency decreases. Furthermore, as one aggregates across a longer time frame ($T$), the ability to examine the simultaneity of events on any meaningful scale is lost.

Lee and Mykland (2008) derive a test, which I use in the analysis below, that allows for the spot detection, if you will, of jumps. The test statistic is a quite natural choice in that it is the return normalized by an appropriate estimate of the conditional (or spot) volatility. In their construction Lee and Mykland (2008) use a trailing estimate of bipower variation whereas Lee and Hannig (2010) use the spot volatility estimate
from truncated power variation.\footnote{A related statistic is derived in Lee and Mykland (2012) that also accommodates the presence of microstructure noise. Although noise is essentially unavoidable in except the most liquid of securities, in the analysis below I instead choose to sample at relatively larger return intervals (30 minutes) in order to mitigate these effects.} Lee and Mykland (2008) compare the performance of their statistic to the Barndorff-Nielsen and Shephard (2006) statistics mentioned above as well as the swap variance statistic of Jiang and Oomen (2008) and find that it proves superior. They also show that at sampling frequencies around the level considered in this chapter (30 minutes) the probabilities of detecting a jump are quite high for even relatively small jump sizes (on the order of $1/2$ the spot volatility level). I also draw attention to the precipitous drop off in detection rates as one moves from intra-day to the daily level. Table 2 of Lee and Mykland (2008) shows that jump detection rates for moderate jump sizes are approximately 0.96 ($1/2$ times spot volatility jump size) and 0.98 (1 times spot volatility jump size) for the 1-hour return frequency but only 0.57 and 0.85 for the daily frequencies considered in other studies. Additionally, Lee (2011) shows that the jump time estimates from this test statistic can be used as a proxy for the true unknown jump times and used to make inference about the intensity process that drives them.

The above mentioned statistics are certainly not exhaustive. However, they provide a brief background on the econometrics of testing for jumps and the sheer volume of work in this area highlights its importance.

\footnote{In untabulated results the Lee and Hannig (2010) was also considered. While the results were qualitatively similar, I found a significantly larger number of jumps using Lee and Hannig (2010) statistic. To err on the side of caution against erroneous jump detection, the Lee and Mykland (2008) statistic was used.}
1.2.2 International equity market correlation

While the modeling of jump dynamics between international equity markets is fairly uninvestigated in the literature, it can be considered a special case of time varying correlation. Increases in correlation after some inciting event have often been termed financial contagion and naturally have received a fair amount of attention.

In one of the earliest examinations, King and Wadhwani (1990) posit that contagion occurs as a result of asymmetric information. In such a model price changes can be informative as they contain private information. In an empirical investigation they examine the behavior of the US, UK and Japanese markets and find an increase in correlation after the 1987 stock market crash. In light of these and other results, Forbes and Rigobon (2002) argue that not accounting for changes in volatility biases the results of these tests and show that applying a heteroskedasticity correction eliminates much of the evidence of contagion.\(^9\)

Other studies have attempted to explicitly account for time varying volatility, for example, Longin and Solnik (1995) examine the changes in the correlation matrix of international equity returns using a bivariate GARCH model (Bollerslev (1986)) to explicitly account for time varying volatility. They further test for the presence of a threshold type effect for the correlation coefficient; do international returns correlate more when they are more extreme? They find evidence of this effect in a number of return pairs and reject the hypothesis of constant correlations. While not related to

\(^9\)I note that the test statistics I use to test correlation explicitly account for spot volatility. Furthermore, the setting of Forbes and Rigobon involves the spillover from a jump in one market to another, in this chapter we examine simultaneity in jumps and measure spillover in terms of cross excitation. Thus, the results should be largely immune from their critique. In addition, I also examine sensitivity in a regression context, which Forbes and Rigobon show is free from this bias.
testing for contagion effects, Cappiello et al. (2006) model international equity and bond returns using the dynamic conditional correlation (DCC) model of Engle (2002). They find substantial variation in correlation coefficients over time.

Of the papers that do consider the effect of jumps in international equity returns, Asgharian and Bengtsson (2006) use the SVCJ model of Eraker et al. (2003) to identify jumps in a number of international markets using daily return data. They find significant evidence of jump spillover, particularly between countries with clear economic linkages.10 Das and Uppal (2004) consider the benefits of international diversification when jumps occur concurrently across markets. They show that this induces systematic risk and reduces the benefits of diversification. Aït-Sahalia et al. (2013) consider a Hawkes Process specification for jump intensity and provide evidence that jumps propagate both within markets over time and across markets (i.e. self and cross excitation). Pukthuanthong and Roll (2014) is likely the most similar to this chapter and they consider a broad sample of 82 countries over 40 years and find little evidence of correlation in jumps. Despite the findings in Pukthuanthong and Roll, this chapter differs in both time period and frequency of return observations. Both are key differences in that (1) international economies are far more intertwined in the recent decade than in the previous and (2) the use of higher frequency return data increases the power of the tests used below. In addition, I focus on a smaller set of countries and remain agnostic to whether the results would hold with a broader set.

10It should be noted that a more thorough analysis would consider multivariate modeling of returns of the type considered in Gualtieri and Sizova (2015), on which chapter 2 of this dissertation is based.
1.3 Econometric Framework

In this section I describe the econometric tests that underlie the empirical results presented in section 5. I first describe the continuous time dynamics assumed for security returns and then describe the tests used to estimate the jump incidences, jump sizes and jump sensitivity.

1.3.1 General continuous time framework

I assume that the instantaneous change in the log-price of an asset \( p_t \), in this case ETFs that proxy for international indices, is governed by the following stochastic differential equation,

\[
dp_t = \mu_t dt + \sigma_t dB_t + \xi_t dN_t,
\]

where \( dB_t \) is a standard Brownian motion, \( \mu_t \) a possibly time varying drift, and \( \sigma_t \) a possibly time varying instantaneous (or spot) volatility. The third term in equation (1.1) is the jump component. \( dN_t \) is the instantaneous change in a Poisson process with a possibly time varying intensity \( \lambda_t \). Lastly, \( \xi_t \) is the jump size, which is assumed to be drawn from a distribution with possibly time varying mean \( \mu_{J,t} \) and variance \( \sigma_{J,t}^2 \).

The specification above provides a convenient representation for asset returns that accommodates two empirically observed components. Smooth, Gaussian like innovations with time varying volatility are modeled by the diffusion terms and large, infrequent fluctuations are modeled by the jump term. A number of tests have been developed recently to infer the presence and timing of jumps as well as to model dependence in the jump components. Although the model above is a characterization of the continuous sample path of an asset, in practice asset prices can only be ob-
served at discrete intervals. However, similar to the case for more standard limiting arguments, the tests used in this analysis are based on the idea that prices can be observed at increasingly higher frequencies. The availability of Trade and Quotation (TAQ) data and a general increase in both the volume and frequency of trading has made approximations to this continuous limit feasible. It is assumed that \( N \) prices are observed over a period of \( T \) days. As a result the return interval is of length \( \Delta = t_i - t_{i-1} \). The limiting behavior of the test statistics is obtained as the ratio \( T/N \to 0 \), so that the return intervals become arbitrarily small.

In a series of papers, Barndorff-Nielsen and Sheppard show that as returns are sampled at increasingly finer intervals the following statistics converge as follows,

\[
RV_t = \sum_{j=1}^{M} r_{t,j}^2 \to \int_{t-1}^{t} \sigma_s^2 ds + \int_{t-1}^{t} \xi_s^2 dN_s + \int_{t-1}^{t} \sigma_s^2 ds,
\]

\[
BV_t = \frac{\pi}{2} \sum_{j=2}^{M} |r_{t,j}| |r_{t,j-1}| \to \int_{t-1}^{t} \sigma_s^2,
\]

where \( r_{t,j} = p_{t,j} - p_{t,j-1} \) and \( M \) denotes the number of return intervals per time unit \( T \) (i.e., \( M = N/T \)). The statistic in (1.2), realized variance, is a measure of the total integrated variation of the asset over the time interval considered. That is, it includes both the continuous diffusive variation and the variation of any jumps over the time interval. The statistic in (1.3), bipower variation, is a measure of only the integrated variation over the time interval. These statistics, and some of their variants, form the basis of many tests for jumps.

The main statistic I use for jump detection is that of Lee and Mykland (2008). The Lee and Mykland statistic can be used to identify jumps at the intra-day level and thus provides a means to examine co-incidences and correlations of jumps free from contamination by continuous returns. I additionally consider the continuous and discontinuous covariance statistics used in Bollerslev et al. (2014) and derived in
Bollerslev et al. (2013b). The following two subsections briefly present the statistics used.

### 1.3.2 Lee and Mykland jump statistic

Lee and Mykland propose the following statistic to estimate the presence of a jump over a return interval $j$,

\[
L_j = \frac{r_{t,j}}{\hat{\sigma}_{t,j}},
\]

where,

\[
\hat{\sigma}_{t,j}^2 = \frac{\pi}{2 \ K - 2} \sum_{i=j-K+2}^{j-1} |r_{t,i}| |r_{t,i-1}|.
\]

The use of bipower variation in the denominator mitigates (and asymptotically eliminates) the effect of return jumps in constructing a measure of spot variance. The statistic $L_j$, under the null hypothesis of no jump over return interval $j$, converges to a standard Normal random variable. Since this test can be applied to every data point for which there is a return observation, it allows for tests of jumps to be constructed at every interval. LM suggest that the number of trailing returns to include in the bipower variation estimate should be the smallest integer greater than $\sqrt{252 \ \text{nobs}}$, where nobs is the number of return observations per day. For the purpose of the analysis below, since I use 30-minute returns and thus have 11 observations per day and I set $K = 53$ (approximately 4 days).

Since the statistic can be applied to every data point and a large number of data

---

11As is discussed in section 1.4, I remove the first and last return of the day to avoid possible micro-structure effects related to the open and close. This results in a reduction of the number of 30-minute return observations over the trading day from 13 to 11.
points are examined, using the standard Normal critical values would result in over-
rejection due to the repeated sampling. However, under the null hypothesis of no
jumps Lee and Mykland construct the following statistic,

\[
\max_{l \in \{1, \ldots, N\}} \frac{|L_l| - C_N}{S_N} \rightarrow \zeta
\]

(1.6)

where,\(^{12}\)

\[
C_N = \left(2 \log(N)\right)^{1/2} - \frac{\log(\pi) + \log(\log(N))}{2(2 \log(N))^{1/2}}
\]

(1.7)

\[
S_N = \frac{1}{(2 \log(N))^{1/2}},
\]

(1.8)

which converges to a standard Gumbel random variable with cumulative distribution
function (CDF) \(P(\zeta \leq X) = \exp(\exp(-X))\). Thus, for an appropriately chosen
significance level, \(\alpha\), a critical value can be constructed as \(CV_{1-\alpha} = -\log(-\log(1-\alpha))\). For the purpose of the analysis below, I classify an observation as a jump using
the 5\% significance level.

1.3.3 Bollerslev et al. continuous and discontinuous betas

Bollerslev et al. consider the possibility that assets have different sensitivities to
continuous and discontinuous shocks to an asset pricing factor (in their case the S&P
500 index). To allow for this difference they use two separate estimators for continuous
and discontinuous betas. The continuous beta is estimated as,

\[
\beta_{i,c,t} = \frac{\sum_{s=t-l}^{t} \sum_{j=1}^{M} (r_{i,s,j} + r_{f,s,j})^2 I(+) - (r_{i,s,j} - r_{f,s,j})^2 I(-)}{4 \sum_{s=t-l}^{t} \sum_{j=1}^{M} r_{f,s,j}^2 I(f)}. \tag{1.9}
\]

\(^{12}\)Note that I remove the \(\pi/2\) term from the construction of the maximum statistic due to its
inclusion in the bipower variation estimate in the test statistic above. The combination of two is
equivalent to the presentation in Lee and Mykland.
where \( r_{i,s,j} \) refers to the high-frequency return of asset \( i \) and \( r_{f,s,j} \) refers to the high-frequency return in the asset pricing factor. The indicator \( I(\cdot) \) takes a value of one if the absolute value of the return preceding it is below some threshold \( k_{s,j} \).\(^{13}\) \(^{14}\) \( l \) is the trailing window of days over which beta is estimated.\(^{15}\) The numerator is a “polarization” of the covariance term and conveniently represents each component as a return itself. The truncation serves the purpose of removing jumps and thus the estimator only picks up the continuous movement. The discontinuous beta is estimated as,\(^{16}\)

\[
\beta_{i,d,t} = \text{sign} \left( \sum_{s=t-l}^{t} \sum_{j=1}^{M} r_{i,s,j} r_{f,s,j} \right) \sqrt{\frac{\sum_{s=t-l}^{t} \sum_{j=1}^{M} (r_{i,s,j} r_{f,s,j})^2}{\sum_{s=t-l}^{t} \sum_{j=1}^{M} r_{f,s,j}^4}}.
\]

(1.10)

Raising the covariance term to a higher power marginalizes the effect of the continuous components and allows the jump components to dominate. In addition to applying this estimator to intra-day returns I also follow Bollerslev et al. and apply it to overnight returns which may also be considered jumps.

### 1.4 Data

The data used in this chapter are high-frequency returns to 10 iShares ETFs designed and managed by BlackRock that attempt to mimic the returns to equities in 10 different countries. In addition to the iShares ETFs I use the SPDR S&P 500 ETF

\(^{13}\)For the purpose of the analysis below I set \( k_{s,j} = 2.5(13)^{-0.49} \sqrt{\min(BV_t, RV_t)} \).

\(^{14}\)The notation +, − and \( f \) denote sum of the two returns, the difference of the two returns and factor return respectively.

\(^{15}\)For the purpose of this analysis I set \( l \) equal to 120 days, corresponding to approximately 6 months in order to ensure a sufficient number of jumps in the discontinuous beta estimation.

\(^{16}\)In the analysis below all the covariances are positive and thus the signing of the discontinuous beta is unnecessary.
designed and Managed by State Street Global Advisors as a proxy for the US market. Although there exist many ETFs designed to mimic returns in countries other than the 11 used in this chapter, a number of these were filtered out due to concerns over their liquidity and the possibility of infrequent trading.

The time period consists of every trading day from 2 January 2007 through 31 December 2013. The data was collected from the TAQ database available through the Wharton Research Data Services (WRDS). Only trades made within the hours the market was open, 9:30am to 4:00pm\textsuperscript{17}, were included. Trades listed with a zero or negative price or size were removed from the sample. Trades with correction indicator other than 0, 1 or 2 were also removed. Trades with sale conditions O, Z, B, T, L, G, W, J or K were also removed. In addition, trades in the TAQ database are only time stamped to the second and, as such, there are times when two or more trades occur with the same time stamp. On these occasions the price is set to the average price over all those observations.

With these prices, 30-minute log-returns were constructed by taking the closest valid trade price before each 30 minute interval.\textsuperscript{18} If no trade occurred during any interval, the price was set to the value at the end of the previous interval. In order to ensure that the results were not driven by an abnormal amount of stale prices, the data set was checked for the percentage of prices which occurred within 5 minutes of the desired time interval.\textsuperscript{19} For each stock and in any month, less than 5% of the price

\textsuperscript{17}Observations later than 2:30pm were removed on days in which the NYSE closed early.

\textsuperscript{18}As an example, if the last trade before 11:30:00am occurred at 11:29:57am, then the price at 11:30:00am is set to that value.

\textsuperscript{19}As an example, for the return over the time period 10:00:00am to 10:30:00am, the prices used to construct this return were typically those from trades between the 9:55:00am to 10:00:00am interval and the 10:25:00 to 10:30:00am interval.
observations were stale by this definition. Overnight returns were also removed for
the portion of the analysis concerning jump detection.²⁰ Lastly, I remove observations
from the first 30-minute interval of the trading day and the last 30-minute interval of
the trading day. This filtering removes any possible micro-structure effects associated
with the open and close of the trading day (such as the bounce back effect) and
mitigates the effects of the observed U-shaped pattern in intra-day volatility that
may contaminate the jump tests.²¹ The results are broadly similar when these end
points are included.

The 30-minute return interval is large compared to the standard 5-minute return
intervals used for extremely liquid securities, such as the S&P 500 index futures
or heavily traded blue-chip stocks. The choice was made in order to remove any
abnormalities caused by micro-structure effects or infrequent trading over periods of
the day. Table 1.1 lists the ticker symbols for the 11 ETFs and the international equity
market whose performance they seek to mimic. It also lists the average daily volume
and the market capitalization. These ETFs are consistently among the top 100 ETFs
in terms of volume. Their average daily volume over the time period considered is
comparable to, and in some cases more favorable than, a substantial portion of the
S&P 500 with roughly 2-3 million shares traded per day.²²

One would expect competitive forces in the ETF market would ensure these prox-
ies do a good job of matching the indices they seek to replicate. There are also addi-
tional reasons why their use may be desirable even if one could obtain high-frequency

²⁰ Including overnight returns was considered but for certain ETFs a relatively large percentage,
on the order of 20%, were classified as jumps.
²¹ See, for example, Andersen et al. (2012).
²² A notable exception is the SPDR S&P 500 ETF which is extremely heavily traded.
data from the actual markets in question. First, as one expands to markets all over
the globe, the times in which all markets are simultaneously open is essentially zero.
Second, these indices would then be quoted in local currencies which would have to
be converted to a common currency at the same frequency in order to remove pure
exchange rate effects. Lastly, continuous trading in a large number of markets is
completely infeasible for a typical investor in any market. ETFs on the other hand
are quite readily accessible.

1.5 Results

In this section I first discuss the results from the intra-day jump tests. I then analyze
to what extent jumps are predictable from past jumps and macroeconomic news
announcements. I finish by constructing a global factor and examining the sensitivity
of each individual ETF to its continuous and discontinuous components. With brevity,
for the rest of this chapter I will refer to each ETF by the country of the market it
seeks to represent, when appropriate.

1.5.1 Intra-day jumps

Table 1.2 presents summary statistics for the intra-day return observations for each
market. I first note that three of the countries considered (Canada, Brazil and Mexico)
have negative intra-day returns on average. This is likely the result of a combination
of factors: (1) overnight returns and the first and last 30-minute return of the trading
day are removed. To the extent these returns are a net positive for investors, their
removal lowers the average return. (2) Dividends, which all of the ETFs pay, are also
not included. (3) The sample period induces a bit of a selection bias by beginning
with the onset of the financial crisis. Interestingly, over the full sample, I find positive skewness for all 11 markets considered, in contrast to the negative skewness typically observed at daily and lower frequencies. The excess kurtosis of the intra-day returns is large. This is in line with results in other studies and is emblematic of the non-normality of stock returns. The standard deviation of the intra-day returns is roughly consistent across markets with Brazil (s.d. 0.50%) being a noticeable outlier.

The returns for all markets are plotted in Figures 1.1 and 1.2 and Figures 1.3 and 1.4 depict the square root of the trailing bipower variation that is used in the construction of the Lee and Mykland jump statistics. Large fluctuations, that from casual inspection, seem to clearly indicate the presence of jumps are readily apparent in Figures 1.1 and 1.2. Some of these spikes are quite large, on both the positive and negative side. Both the largest positive and negative return out of all markets considered occurred in the Brazilian market at 3:30:00pm on October 10, 2008 (7.7%) and October 16, 2008 (-5.25%). The first return was classified as a jump based on the Lee and Mykland test and coincided with jumps in both the US and Japanese markets as well. However, the second large negative return a few days later was not classified as a jump, likely a result of the dramatic increase in volatility across all markets during that time period. Inspection of the volatility plots also indicates a substantial amount of correlation in equity volatilities across markets. The lowest correlation among the estimated volatility paths is between the German and the Brazilian markets (0.83), while the rest are on average approximately 0.93.

The choice of sample period was dictated primarily by a desire to include a diverse enough group of countries while maintaining sufficient liquidity in those chosen. Although all these ETFs were introduced pre-2001, reasonably sufficient trading volume did not seem to exist prior to late 2006/early 2007.
Table 1.3 presents the results of the Lee and Mykland jump statistics across markets. All markets exhibit substantial evidence of jump with smallest number of jumps occurring in the Japanese market (36, or approximately 1 jump every 50 days) and the largest number occurring in the German market (79, or approximately 1 jump every 22 days). With the exception of South Korea, the Asian markets tend to have a larger percentage of positive jumps relative to the rest of the markets considered. In fact, (slightly) over 50% of the identified jumps in the Taiwanese and Japanese markets are positive. The $Z_{ww}$ column in Table 1.3 shows the test statistic from the Wald and Wolfowitz (1940) runs test that examines whether the sequence of jumps are independent over time. For every market considered the null hypothesis of independence is rejected, and in most cases strongly. The implication is that there is ample evidence of jump clustering or time varying intensity (which I explore later) which is in line with results of Lee (2011) and Aït-Sahalia et al. (2013). Although the mean jump sizes (estimated as the return over an interval in which a jump was identified, under the assumption that the jump component dominates the return) are not very large, once jumps are separated into positive and negative jumps, the estimated means are quite large in magnitude. Relative to the full sample standard deviations in in Table 1.1 jumps are approximately 3-4 times larger in magnitude than a typical return.\footnote{This is to be expected as the jump statistic is constructed roughly as the absolute value of the return divided by an estimate of the spot volatility.} This underlies the notion that jumps are salient events and, assuming investors dislike large shocks, investors would like to avoid them if possible.

Table 1.4 depicts the number of jumps each country-pair has in common as well as that number as a percentage of total jumps in each market. What stands out is the fact that, contrary to the results of Pukthuanthong and Roll, I find that co-incidences
of jumps are quite common (as a percentage of total jumps in one market). For example, out of all jumps in the non-US markets, on average 41% occur at the same time as a jump in the US market. I find similar effects among some regional pairings as well, with 50% of Japanese jumps occurring at the same time as an Australian jump and 69% of UK jumps occurring at the same time as a German jump. Contrary to the belief that jumps are largely idiosyncratic, I find a relatively large degree of co-jumps at the 30-minute time interval.

Having presented evidence that international markets do in fact jump together, Table 1.5 looks at the extent to which returns are correlated on days in which both markets jumped. Specifically, it presents the correlation of the signed Lee and Mykland jump statistic \( r_{t,j}/\hat{\sigma}_{t,j} \) over days in which both equities jumped (not necessarily at the same exact time) and the rest of the days on which either one market jumped and the other did not or neither jumped. I use the statistic over the full day so that (1) the correlation statistics are not based on an extremely small number of returns and (2) to account for possible jump spillover (as investigated later) and for the fact that there may be relatively large values of the statistic that are nonetheless not classified as jumps.

Universally, across all markets considered, the correlation among signed Lee and Mykland jump statistics is significantly larger on days in which both markets jumped than on other days. The average correlation in Lee and Mykland jump statistic across country-pairs on common jump days is 0.86, while on non-common jump days it is 0.68. The average t-statistic from a test of equality of correlation based on the Fisher Transformation\(^{25}\) of the correlation coefficients across country-pairs is 8.59. The bivariate normality assumption, while seemingly skeptical, is not as implausible.

\(^{25}\)The Fisher Transformation, assuming Normality for the bivariate distribution of \( x \) and \( y \), is
based on the limiting distribution of the Lee and Mykland test statistic. Under the null of no jumps the statistic converges to a standard normal random variable. Also, under the alternative hypothesis the statistic is approximately equal to that same standard normal random variable, plus the jump size scaled by the spot volatility.\textsuperscript{26} Under the assumption that the instances of jumps themselves are normal random variables, which has been used in Eraker et al. (2003) and Tauchen and Zhou (2011) among others, then the assumption of bivariate normality is at least plausible.\textsuperscript{27} As a sanity check, I also examine the unsigned Lee and Mykland jump statistic ($|r_{t,j}|/\hat{\sigma}_{t,j}$) to safeguard against the possibility that these results are driven by general return correlation and spot volatility estimates that do not perfectly match the instantaneous spot volatility. It can also be considered a test of whether jumps occur concurrently, with no statement in terms of direction. These results are presented in Table 1.6 and confirm all the prior results.

### 1.5.2 Jump spillover

Figures 1.5 and 1.6 plot the estimated jump times for each market considered. The plots validate the results of the runs test discussed above. It appears that jumps cluster in groups and thus that the intensity driving the jump process is not likely to be constant.\textsuperscript{28} In light of this, I investigate the extent to which jumps self and cross excite via Logistic regression.

\[ Z_{x,y} = \text{arctanh}(\hat{\rho}_{x,y}), \] which is distributed approximately Normal with mean \( \text{arctanh}(\rho_{x,y}) \) and standard error \( 1/\sqrt{N - 3} \).

\textsuperscript{26}See Theorem 1 and 2 of Lee and Mykland (2008).

\textsuperscript{27}However, 30 minutes is a far cry from 30 milliseconds.

\textsuperscript{28}The thick lines do not actually represent long runs of consecutive jumps, which are quite rare in the sample. One notable exception is the “Flash Crash” of May 6, 2010.
Lee (2011) shows that using the estimated jump times instead of the unobservable true jump times to estimate jump arrival is asymptotically equivalent to observing the true jump times. In a similar manner, I investigate the extent to which prior jump arrivals induce future jump arrivals. These results can be seen in the left column of Table 1.7. As predictor variables in the left column I include an exponential moving average of past jumps, similar in spirit to the Hawkes process model of Aït-Sahalia et al.. Intuitively, I ask whether past jumps induce future jumps without restricting the response to occur consecutively. The aim of this section is not to deduce the optimal jump predictor but instead to identify a relationship between past jumps and the future probability of jumps. As such, I set the smoothing parameter ($\alpha$) equal to 0.2, which is within the range of 0.1 to 0.3 generally accepted for forecasting. The results show that, with the exception of the Mexican market, there is strong evidence of self-excitation. That is, over the period considered, jumps appear to propagate across time.

The middle column of Table 1.7 includes both the exponential moving average of jumps in the US market and the exponential moving average of jumps in each respective country’s own market. This specification asks whether jumps propagate across markets and specifically whether there is jump spillover from the US to other markets. The choice of using only the US market is in line with the results of Aït-Sahalia et al. who find much stronger evidence of transmission from the US market to other markets than vice versa. The results from the middle column show that, for the majority of markets, self-excitation maintains after including recent jumps in the US market as a predictor. Additionally, for two of the international markets we find evidence of spillover from the US (UK and Mexico). This result is not too surprising given the economic linkages (especially Mexico). It is also somewhat in
line with the results from Aït-Sahalia et al. who find spillover from the US to to all the international markets they consider (UK, Mexico, Hong Kong and Japan) over a much longer time period.

The right column of Table 1.7 tests whether US macroeconomic news announcements increase the probability of jumps in other markets. The first of the two news announcements I consider is the release of non-farm payroll employment figures by the Bureau of Labor and Statistics (BLS) which is released monthly on Fridays at 8:30am during the first week of the month that includes the 12th of the month (the so-called “reference week”). The second is the weekly release of initial jobless claims by the US Department of Labor. This data is released at 8:30am on Thursdays.

To account for the release of this data I include indicator variables for the first two return intervals considered each day the data is released (10:00:00am to 10:30:00am and 10:30:00am to 11:00:00am). Similar indicator variables were used in Lee, who examined the response to macroeconomic news announcements using a sample of US firms. Similarly, Andersen et al. (2002b) examine the impact of US macroeconomic news data on US dollar spot exchange rates. As may be expected, I find a link between US macroeconomic news releases and jumps in the US market. I additionally find evidence that US macroeconomic news is related to jumps in many of the other markets. Specifically, the German, UK, Australian, Brazilian, Mexican, Hong Kong and Taiwanese markets all have significant increases in jump probabilities with respect to one or both of the US macroeconomic news announcements. The evidence

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29 When that Friday happens to be a holiday the data is released the preceding non-holiday.
30 When that Thursday happens to be a holiday the data is released the preceding non-holiday.
31 However, they use the surprise component of the release (i.e. the forecast error) rather than an indicator variable. While this is certainly desirable, the forecast database they use provided by International Money Market Services is proprietary.
indicates a relationship between shocks to the US market and global expectations of economic performance.

1.5.3 Global portfolio and continuous vs. discontinuous sensitivity

Having presented evidence that jumps are correlated across markets, both in timing and in direction, and that jumps propagate both across time and across markets, a natural question is to what extent does a global portfolio diversify jumps (if at all) and what reward does an investor get for bearing global jump risk. To do so I form an equal weighted portfolio of the intra-day returns to all 11 markets. Table 1.8 presents the jump statistics for this global portfolio. Surprisingly the global portfolio jumps more frequently than a number of the individual portfolios. This, as noted by Lee who finds a similar result between the US market portfolio and individual equities, can happen if small jumps are not detected at the individual market level but accumulate in the global portfolio. The global jumps are also typically negative (56%) but also less severe in magnitude when compared to the individual market results in Table 1.3.

Given that the global portfolio exhibits jumps, I use the estimators of Bollerslev et al., in order to examine whether particular markets load more heavily on continuous or discontinuous global returns. For this part of the analysis I follow Bollerslev et al. and also include the overnight returns separately and consider them to be a different form of a jump or discontinuity. Table 1.9 presents the time series average of continuous, discontinuous and overnight betas on the global portfolio for each country as well as their time series standard deviations. On average, the continuous betas are quite similar to the discontinuous betas, while the overnight betas are often quite different. The time series plots of the three betas in Figures 1.9 and 1.10
tell a different story. There is substantial separation in the betas over time. For example, there is a substantial increase in the Japanese market’s discontinuous beta around the time of the Fukushima Daiichi nuclear disaster in 2011. Similar evidence of separation is also found in most other markets to varying degrees. That is, there are substantial departures over time when countries seem to load more heavily on discontinuous movements. Given the relative paucity of jumps in all markets (on an absolute scale), it is perhaps unsurprising that there are frequent periods when the two estimators coincide. The overnight betas also show a large degree of difference from the continuous betas, which supports the hypothesis that overnight movement in markets is different from that of the typical continuous movement within the trading day.

Despite the extremely short sample, I also test to what extent exposure to different types of movements (continuous, discontinuous and overnight) earns investors additional premia. To do so I use the estimates of the three betas, as of the month’s end in Fama-MacBeth (Fama and MacBeth (1973)) regressions on each market’s excess returns over the next month. That is, for each month \( t \) I estimate a cross sectional regression of the excess returns on the three beta measures. This results in 78 total coefficient estimates for each beta measure (corresponding to the full sample period, less the 6 month period used for the initial estimation). A t-test is performed on the time series average of each coefficient in order to determine whether the beta measure has predictive power in explaining the cross-sectional spread of country returns. The results of the Fama-MacBeth regressions are shown in Table 1.10. Despite the fact that one might expect exposure to jumps to earn a premium, I find no evidence that any of the beta measures explain differences in country returns. However, 7 years is a particularly short time period to test an asset pricing model. Furthermore, the coun-
tries chosen and their weights in the global portfolio maybe far from mean-variance efficient. To the extent reliable estimates can be constructed from lower frequency returns, this presents an interesting avenue for future research.

1.6 Conclusion

Sudden, extreme changes in security returns are particularly salient events to investors. The rapid depreciation in the value of an investor’s portfolio over the matter of a few days, or even a few hours, represents a key risk factor. This chapter has examined the extent to which international diversification can mitigate this risk as well as analyzed the extent to which international economies are linked.

International diversification has been greatly facilitated by recent advances in the design of investment vehicles intended to mimic the performance of equity markets in various countries. ETFs are an example of one such vehicle that enable any individual with a computer and an internet connection (and of course capital) to quickly and easily make such an investment. Exploiting the fact that ETFs allow for an analysis, in a common currency and on a common time scale, I examine commonality in extreme price movements.

The findings above shed light on a number of areas of interest. First, the extent to which jump risk can be mitigated by international diversification is limited. For many of the country pairs considered I find substantial evidence of co-jumps. In addition, I find that jumps self and cross excite and that jumps coincide with US macroeconomic news releases. Second, I find that in addition to jumping together there is a directional component to the relationship, i.e. negative jumps in one market coincide with negative jumps in another. Lastly, I examine the sensitivity of country returns to different types of variation (continuous, discontinuous and overnight) in a
global factor and find that the relationship varies according to the shock. The sum of the evidence points to (1) a systematic component to jumps that affects all markets and (2) spillover effects, where large shocks in one market have implications for the future return distribution of that market and others.

A further implication comes from the fact that variation in security prices (particularly equities) can be thought of as an update to the average investor’s expectation of future performance. If the quality or importance of information varies over time then jumps can be thought of as a response to the arrival of far-reaching, salient information. Within this context, the results above imply that these large information shocks have broad implications for global economic expectations and that large shocks in one market lead to subsequent revisions in the others.
This table presents the information on the ETFs used in the analysis. SPY is designed and managed by State Street Global Advisors. All other ETFs are designed and managed by BlackRock. Volume figures are averages over the sample period 2007 - 2013 in millions of shares traded per day. Market capitalization figures are averages over the sample period 2007 - 2013 in millions of US dollars.
Table 1.2 : Intra-day return summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Moments</th>
<th>Auto-correlation</th>
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<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
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</table>

This table presents summary statistics for each of the 11 ETFs considered. The left panel presents the first four moments, in percentage terms, of the intra-day returns (10:00:00am to 15:30:00am) over the full sample (2007-2013). The right panel presents the first 5 auto-correlations of these returns.
<table>
<thead>
<tr>
<th># of jumps per day</th>
<th>% (+)</th>
<th>% (-)</th>
<th>$Z_{WW}$</th>
<th>Mean (+)</th>
<th>Mean (-)</th>
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This table presents the intra-day jump statistics from the LM jump test. The left panel presents the number of jumps and their breakdown by whether they were positive or negative. It also include the test statistic from the Wald-Wolfowitz runs test. The right hand panel presents estimates of the jump means, in aggregate and broken down by whether they were positive or negative.
Table 1.4 : Intra-day co-jumps among country pairs

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This table presents the number of common intra-day jumps between country pairs (top figure within each box) and the percentage of each country’s jumps in common (bottom figure within each box). The percentage correspond to the country on each row. As an example, for the bottom left box, South Korea (EWY) and US (SPY) have 29 jumps in common and that number corresponds to 51% of the total jumps for South Korea.
This table presents an analysis of the correlation coefficients between signed LM jump statistics. Panel A presents the correlation coefficients between signed LM statistics on days in which both markets jumped at least once (top figure) and all other days (bottom figure). Panel B presents the average, across all 52 country pairs, of the correlation coefficients in Panel A as well as the average t-statistic, based on the Fisher Transformation of the correlation coefficient, across all country pairs testing whether the correlation is greater on jumps days as well as the percentage of country pairs for which the null hypothesis was rejected.
Table 1.6: Intra-day unsigned LM jump statistic correlation

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<th>EWJ</th>
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Average correlation statistics

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This table presents an analysis of the correlation coefficients between unsigned LM jump statistics. Panel A presents the correlation coefficients between signed LM statistics on days in which both markets jumped at least once (top figure) and all other days (bottom figure). Panel B presents the average, across all 52 country pairs, of the correlation coefficients in Panel A as well as the average t-statistic, based on the Fisher Transformation of the correlation coefficient, across all country pairs testing whether the correlation is greater on jumps days as well as the percentage of country pairs for which the null hypothesis was rejected.
Table 1.7 : Intra-day jump prediction

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<td>2.684</td>
<td>0.072</td>
<td>0.786</td>
</tr>
<tr>
<td>EWT</td>
<td>12.589</td>
<td>8.420</td>
<td>6.102</td>
<td>1.150</td>
</tr>
<tr>
<td></td>
<td>4.330</td>
<td>1.785</td>
<td>1.266</td>
<td>1.131</td>
</tr>
<tr>
<td>EWY</td>
<td>12.079</td>
<td>9.627</td>
<td>3.859</td>
<td>1.275</td>
</tr>
<tr>
<td></td>
<td>5.699</td>
<td>2.668</td>
<td>0.928</td>
<td>0.025</td>
</tr>
</tbody>
</table>

This table presents the coefficient estimates from a Logistic regression. The dependent variable is the estimate of a time $t$ jump in the respective market. The independent variable in Model 1 is an exponential moving average of prior jumps for each respective country. The independent variables in Model 2 are exponential moving averages of prior jumps for each respective country AND for the US. For both cases the smoothing parameter $\alpha$ is set to 0.2. The independent variables in Model 3 are indicator variables set to 1 for the first two period of each day (10:00:00am to 10:30:00am and 10:30:00am to 11:00:00am) on which non-farm payroll economic data was released (NFP) or weekly unemployment claims data were released (JOBS). The top number in each box is the coefficient estimate and the bottom number is the associated t-statistic.
Table 1.8: Intra-day jumps in EW global equity portfolio

<table>
<thead>
<tr>
<th>Jumps</th>
<th>Jump sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td># of jumps</td>
<td>per day</td>
</tr>
<tr>
<td>60.00</td>
<td>0.03</td>
</tr>
</tbody>
</table>

This table presents the intra-day jump statistics from the LM jump test for the EW global portfolio. The left panel presents the number of jumps and their breakdown by whether they were positive or negative. It also includes the test statistic from the Wald-Wolfowitz runs test. The right hand panel presents estimates of the jump means, in aggregate and broken down by whether they were positive or negative.
This table presents the time series mean and standard deviation of estimates of continuous, discontinuous and overnight betas. The continuous and discontinuous betas are estimated from intra-day returns over the prior 60 days. The overnight betas are estimated from overnight returns over the prior 60 days. The time period covered is July 2007 through December 2013.
Table 1.10: Fama-MacBeth regressions

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\beta}_{c,t}$</th>
<th>$\hat{\beta}_{d,t}$</th>
<th>$\hat{\beta}_{on,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>3.90</td>
<td>-3.63</td>
<td>0.46</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>21.45</td>
<td>22.06</td>
<td>7.79</td>
</tr>
<tr>
<td>$t$</td>
<td>1.61</td>
<td>-1.45</td>
<td>0.52</td>
</tr>
</tbody>
</table>

This table presents estimates from Fama-MacBeth regressions of the estimated betas on monthly returns. Each month a cross sectional regression is run with the dependent variable as the monthly excess return on the 11 markets. The independent variables are the continuous, discontinuous and overnight betas estimated over the prior 6 months. This table presents the time series average, standard deviation and t-statistic of these estimated coefficients. The total number of months is 78.
Figure 1.1: Intra-day returns - Developed markets
Figure 1.2: Intra-day returns - Emerging markets
Figure 1.3: Intra-day volatility - Developed markets
Figure 1.4: Intra-day volatility - Emerging markets

- **EWZ**
- **EWW**
- **EWH**
- **EWT**
- **EWY**

Date and volatility (%) trends for the mentioned Emerging markets.
Figure 1.5: Intra-day jumps - Developed markets

![Diagram showing intra-day jumps for developed markets]

- **SPY**
- **EWC**
- **EWG**
- **EWU**
- **EWA**
- **EWJ**
Figure 1.6: Intra-day jumps - Emerging markets

EWZ

EWW

EWH

EWT

EWY
Figure 1.7: Intra-day jump sizes - Developed markets
Figure 1.8: Intra-day jump sizes - Emerging markets
Figure 1.9: Daily beta estimates - Developed markets

**SPY**

**EWC**

**EWG**

**EWU**

**EWA**

**EWJ**
Figure 1.10: Daily beta estimates - Emerging markets

 EWZ

 EWW

 EWH

 EWT

 EWY
Chapter 2

Extreme Events in Stock Market Factors

2.1 Introduction

The market return is universally regarded as the primary driver of systematic risk in asset prices. In light of this particular role, it is not surprising that the dynamics of the market return, as approximated by returns on broad stock market indices, have been extensively studied in the literature. At the daily frequency, various characteristics of the market return dynamics have important asset pricing and risk management implications. First, market returns are largely unpredictable, see Kendall and Hill (1953) and Fama (1965, 1970, 1991). Second, the volatility of the market return is not constant, see the reviews by Bollerslev et al. (1992) and Andersen et al. (2003). Third, shocks to return volatility are negatively correlated with shocks to returns (the phenomenon known as the leverage effect), see Black (1976) and Christie (1982). Fourth, the presence of infrequent, large negative returns can be conveniently captured by models with jumps, e.g., Jarrow and Rosenfeld (1984), Jorion (1988), Andersen et al. (2002a), Chernov et al. (2003).

The voluminous literature on cross sectional asset pricing suggests that, in addition to the market return, there are likely additional sources of priced risk. Indirect evidence of this fact is the success of the Fama and French (1993) factors in capturing the cross-section of expected stock returns. A multi-factor structure for systematic risk provides an incentive for active portfolio management that seeks to capture extra
premia or, conversely, to reduce exposure to excess risk by deviating from the market portfolio. The sources of risk that explain the premia earned by many of these strategies are ambiguous, however, they may be due to correlations between returns and macroeconomic variables (Winkelmann et al. (2013)) or increased volatility of the portfolios. The focus here is on the latter definition of the risk and we attempt to fully characterize the volatility dynamics of three proposed stock market factors: the market return, the value premium, and the momentum premium (hereafter, referred to as the Market, Value and Momentum).

We begin by pointing out that these factors are constructed as excess returns on portfolios of stocks, and therefore, their dynamics can be modeled in a similar manner to the Market. We then provide a complete characterization of their systematic risk by estimating the factors’ co-interaction. Does the timing of extreme events in the additional factors coincide with extreme events in the Market? Do the additional factors have different volatility dynamics? Do they respond to changes in Market volatility, and does the Market respond to changes in the volatility of the other factors? All of these questions ultimately relate to the complete characterization of priced risk in the portfolios based on the values of their factor loadings.

To answer these questions we analyze the series of factor returns through the lens of jump-diffusion models. The jump-diffusion model allows for a decomposition of the innovations to returns into a continuous (diffusion) part and a discontinuous (jump) part. Likewise, volatility innovations can also be decomposed into diffusion and jump components. Modeling the fundamental factors in pairs (i.e. the market return plus a secondary factor) provides a characterization of the joint dynamics in both components and thus allows for an evaluation of their “typical” dynamics (as proxied by the diffusion component) and their dynamics during extreme events (as
proxied by the jump components).

For diffusion components, we find that the factors’ volatilities co-move but differ substantially. The Market and Momentum volatilities share the same slow-decaying component, while Momentum volatility does not include the fast-decaying component present in the Market volatility. The Market and Value volatilities share a common component and the Value volatility deviates from the Market volatility by an independent process. From this analysis we conclude that Momentum volatility carries additional information about the Market volatility dynamics, while Value volatility does not. Furthermore, we find that Momentum volatility leads to a separate leverage effect in the Market. In general, our results point to an advantage from modeling the the Market and Momentum jointly.

The novel contribution of this chapter is the description of co-interaction between tail events in factor returns. With our model, we are able to differentiate between co-jumps and individual jumps. We find statistical evidence for two distinct types of co-jumps. First, a jump in the Market increases the probability of jumps in the other factors. The estimated probability of these co-jumps conditional on the market jump is close to one. Second, volatility jumps are correlated among the factors. Such volatility co-jumps are characterized by the same timings and correlated magnitudes across factors. Additionally, we find some evidence for individual jumps in Momentum. This evidence points to additional sources of risk in this strategy. Circumstantial evidence presented in Figure 5 shows that the recent individual Momentum jumps normally occurred during bear markets. This is consistent with the findings of Daniel and Moskowitz (2013).

The first application of our empirical results is towards establishing a link between structural asset pricing models and the estimated reduced-form model of factor dy-
namics. We analyze our results in conjunction with insights from the long-run risk literature (see, Bansal and Yaron (2004)). We note that co-interactions between the factors, as well as co-interactions between the factor returns and factor volatilities, have implications for possible sources of risk in factor portfolios. In particular, we conclude that Value is not explained by tail risk, and Momentum likely has a complicated structure and comes from several sources. For example, we show that Momentum responds to the changes in the fast-decaying component of the Market volatility. Additionally, we find indirect evidence that the individual jumps in Momentum may be priced.

The second application of our empirical results involves establishing properties of the factor portfolios during extreme events. That is, we consider portfolios that are customized to include a combination of the Market and the other factors. Such portfolios are the basis of the investing style known as factor investing. The study of the properties of these portfolios during extreme events is impeded by their rarity. Our estimation results allow us to calculate a variety of summary statistics for the returns on jump days. We find that adding Value to the Market improves the performance during the extreme events in terms of the average loss, volatility, and the value-at-risk (VaR). Similarly, adding Momentum reduces volatility and VaR. Momentum has historically led to larger, albeit statistically indistinguishable, average extreme losses. In light of this we again posit that only Momentum can be associated with some additional tail risk, while Value cannot.

The estimation methods in this chapter are most closely related to those in Eraker et al. (2003). Eraker et al. derive an algorithm to perform Markov Chain Monte Carlo (MCMC) estimation for univariate jump-diffusion models. We extend this method to the bivariate case and provide a detailed description of the estimation
steps. Other papers that estimate multivariate jump-diffusion models for equity data include Johannes and Polson (2010), Aït-Sahalia et al. (2013). Johannes and Polson estimate the multivariate Merton model with constant volatility. Aït-Sahalia et al. assume only one volatility component but allow for extra features in the dynamics of the jump occurrences (self and cross excitation). Our model puts the least restrictions on the volatility dynamics and, therefore, is one of the most general among these models.

The chapter proceeds as follows. Section 2.2 describes the data. Section 2.3 summarizes the properties of the data by fitting univariate jump-diffusion models. Section 2.4 motivates the elements of the multivariate model that is introduced in Section 2.5. Section 2.6 reports the estimation results. Sections 2.7 discusses the implications of these results for factor investing and Section 2.8 concludes.

2.2 Data

We consider stock-market factors over the sample time period of January 1, 1980 through March 31, 2013 using daily excess returns for a total of 8385 observations. We extend the period studied by Eraker et al. to include the recent decade and also the years from 1980 to 1990. Therefore, our sample includes three extremely turbulent periods: 1987-1988, 1997-2002, and 2007-2009. The data is from Kenneth French’s data library. The Market (MKT) is constructed as the value weighted return of the common shares of all firms incorporated in the US in the CRSP database that are listed on the NYSE, AMEX or NASDAQ less the daily return from holding a one-month treasury bill. Value (HML) is constructed as the returns to a zero net cost

\footnote{We thank Kenneth French for making this data available on his website.}
portfolio that takes a long position in two value portfolios and a short position in two growth portfolios. Momentum (UMD) is constructed as the returns to a zero net cost portfolio that takes a long position in two portfolios of cumulative winners over the past two months to one year and a short position in two portfolios of losers over the same time period (i.e. long two portfolios of firms with the highest cumulative return and short two portfolios of firms with the lowest cumulative return over the formation period). A month is waited between formation period and holding period in order to avoid any short term mean reversion. For more information on the factor construction the reader is referred to the notes on Kenneth French’s data library.

Figure 2.1 plots the factor series and Table 2.1 reports summary statistics. Apparent similarities in the dynamics of the returns include small averages that are virtually zero in comparison with volatilities; co-movements in long-run volatilities, with the most recent increase starting around September 2008; and high kurtosis and negative skewness in the Market and Momentum. In order to gain a better understanding of the time series properties of daily factor returns, we begin with a univariate analysis of each within a jump-diffusion framework.

### 2.3 Univariate Properties of Factor Series

The stock market factors in our data set are constructed as weighted averages of the same set of returns. It is reasonable to assume that they may exhibit similar time series dynamics, in a very general sense. We select a modeling framework previously used in the literature that is known to fit the dynamics of one of these factors: the Market.

Stock market returns, as approximated by the returns to stock market indices, have been the subject of many econometric studies. Chernov et al. (2003) compare
a comprehensive list of models and find that, within the affine family, the jump-diffusion model is best supported by the data. Although jumps are formally defined as discontinuities in the price path, the inclusion of jumps in the daily return model can be thought of as a way of incorporating large price movements that would be inconsistent with typical Gaussian shocks. Within this framework we can explain the high kurtosis and negative skewness apparent in stock returns by the existence of infrequent, large negative fluctuations.

MCMC estimation, suggested by Eraker et al., provides a convenient method for estimating jump-diffusion models. The reasons, as noted by Eraker et al. and Johannes and Polson is that it avoids numerical integration of the state variables (in most cases) in the objective function. Additionally, we are able to jointly estimate both the parameters of the model and the state variables in a unified procedure. Among the models considered in Eraker et al. (Heston (1993), Bates (1996), Duffie et al. (2000)) they find that the model with contemporaneous jumps in returns and volatility (Stochastic Volatility Conditional Jump, SVCJ) fits the data most closely.

We apply the SVCJ model\(^2\) to all of the factor prices, \(F_t\), and corresponding spot variances, \(V_t\):

\[
\frac{dF_t}{F_t} = \mu dt + \sqrt{V_t-}dB_{1,t} + \xi^f_t dN_t, \tag{2.1}
\]

\[
dV_t = \kappa(\theta - V_t-)dt + \sigma_v \sqrt{V_t-}d(B_{1,t} + \sqrt{1 - \rho^2}d(B_{2,t}) + \xi^v_t dN_t,
\]

where \(V_{t-} = \lim_{s\downarrow t} V_s\) is the left limit of spot volatility, \(B_{1,t}\) and \(B_{2,t}\) are independent Brownian motions, \(N_t\) is a Poisson process with intensity \(\lambda\), and \(\xi^f_t\) and \(\xi^v_t\) are the

\(^2\)In Eraker et al. the model is applied to log-returns, however we apply the model to simple returns. Although this creates a minor issue with respect to the distributional assumptions (i.e., simple returns cannot be less than -1), at the daily frequency the two are very similar.

\(^3\)With brevity, we omit factor-specific subscripts
jump sizes of returns and variance. The diffusion components consist of the shocks \( \sqrt{V_t} dB_{1,t} \) and \( \sigma_v \sqrt{V_t} (\rho dB_{1,t} + \sqrt{1-\rho^2} dB_{2,t}) \). The jump components are given by \( \xi^f_t dN_t \) and \( \xi^v_t dN_t \). The parameters \( \mu, \kappa, \theta, \sigma_v \) and \( \rho \) represent the average instantaneous return in the absence of jumps, the rate of mean reversion of volatility, the normal level of volatility, the volatility-of-volatility and the correlation between continuous shocks to variance and returns, respectively. The processes \( \xi^f_t \) and \( \xi^v_t \) are the jump sizes. To ensure positivity of \( V_t \), \( \xi^v_t \) is modeled as exponential random variable, \( \xi^v_t \sim \exp(\mu^{v,J}) \), with the density \( f_{\xi^v}(x) = \mathbb{I}(x > 0)\mu^{v,J} \exp(-\mu^{v,J} x) \). The size of the jumps in prices are allowed to be correlated with the size of the jumps in volatility. This is accomplished by modeling the return jump size conditional on the volatility jump size, \( \xi^f_t | \xi^v_t \sim N(\mu^j + \rho^j \xi^v_t, (\sigma^j)^2) \).

The model above allows for a decomposition of expected returns and expected variance into two components: one related to the diffusion terms and one related to jumps. The expected instantaneous return from the above model is,

\[
\frac{1}{dt} E_t \left( \frac{dF_t}{F_t} \right) = \mu + \lambda \left( \mu^j + \frac{\rho^j}{\mu^{v,J}} \right).
\]

The contribution of the diffusion component is given by \( \mu \) and the contribution of the jump component is given by \( \lambda \left( \mu^j + \frac{\rho^j}{\mu^{v,J}} \right) \). Similarly, the expected conditional variance of returns is,

\[
\frac{1}{dt} \text{Var}_t \left( \frac{dF_t}{F_t} \right) = V_t + \lambda \left( \xi^f_t \right)^2 = V_t + \lambda \left( \mu^j \right)^2 + 2 \left( \frac{\rho^j}{\mu^{v,J}} \right)^2 + 2 \frac{\mu^j \rho^j}{\mu^{v,J}} + \left( \sigma^j \right)^2 \right).
\]

Replacing \( V_t \) by its expectation \( \theta + \frac{\lambda}{\mu^{v,J}} \), we obtain that the contribution of the diffusion component to the variance is \( \theta \) and the contribution of the jump component
is $\lambda \left( \frac{1}{\kappa \mu - \gamma} + E \left( \xi_t^f \right)^2 \right)^4$.

It should be noted that aside from jumps in returns, the SVCJ model also accommodates other types of return dynamics. First, continuous return shocks (dB$_{1,t}$) can be negatively related to continuous volatility shocks due to the leverage effect ($\rho < 0$). That is, prices adjust downwards in response to higher volatility. Also, an increase in the level of $V_t$ naturally increases the occurrence of extreme positive and negative returns.

To estimate the model we use an Euler discretization scheme (on daily data in percentage form) and apply the same estimation steps as in Eraker et al. Table 2.2 reports the parameter estimates and Table 2.3 provides a decomposition of average returns and average variances into diffusion and jump components. Figure 2.2 plots the estimated latent variance series$^5$, $V_t$, together with the estimated volatility jump markings and Figure 2.3 plots the estimated jump probabilities.$^6$

**Market.** We begin by discussing the results for the Market, which serves as a reference point for the other factors. The implied daily return for the Market, given the estimated parameters, is approximately 0.023% daily (5.8% annualized). Similar to prior studies, we find evidence of a strong leverage effect between returns and volatility shocks with a negative correlation coefficient $\rho = -0.619$. The continuous

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$^4$The contribution of the jumps to volatility is often measured by contribution of the jumps in returns only, here $\lambda E \left( \xi_t^f \right)^2$. For example, Huang and Tauchen (2005) find that this component constitutes 7.5% of the market volatility. In our estimation, jumps in market returns contribute 0.07 out of the total 1.09 variance, i.e., the estimate is in line with the prior literature.

$^5$The reported variance series correspond to $E(V_t | F_{t-1} = F_{t-1}^{\tau}, i = 1, ..., T)$, i.e., estimates of $V_t$ conditional on the observed returns.

$^6$Note that discontinuities in $F_t$ (i.e., jump occurrences) are unobserved in discrete data. Therefore, we report probabilities of jumps occurrences $\mathbb{P}(N_t - N_{t-1} > 0 | \frac{F_t - F_{t-1}}{F_{t-1}}, i = 1, ..., T)$ for each day $t$. 

mean of volatility is $\theta = 1.09$, which corresponds to a daily long-run Value-at-Risk (VaR) of around -1.71%. The decomposition in Table 2.3 complements previous findings that the Market is subject to sporadic, large changes in value that are generally negative. Given the estimate for the parameter $\lambda$, we should expect to see around 66 jumps in our sample. The average loss, conditional on a jump, is approximately 2.4%, which substantially reduces the annual premium given by the continuous component of 4.6% annualized. Jumps also constitute a large portion (41%) of total volatility.

For the Market we have a total of 12 days over the sample in which the probability of a jump is over 80%. They include the stock market crash of 1987 (October 19), the mini-crash of 1989 (October 13), the Asian Crisis of 1997 (October 27), as well as dates in 2000 (January 4) and 2007 (February 27, Chinese correction) marking the first crack in the dot-com bubble and the start of the global financial crisis, respectively. Less apparent crash days with jump probabilities of over 80% include the market correction on July 7, 1986, “Black Thursday” (September 11, 1986) - subsequently attributed to news about rising inflation, the market liquidity break (Christie and Schultz (1998)) on November 15, 1991, and the market reaction to news about the federal funds rate (Pakko and Wheelock (1996)) on February 4, 1994.

Our estimates of the market variance are plotted in the top panel of Figure 2.2. We find increases in variance in the late 1990’s through the early 2000’s and at the onset of the financial crisis in 2008. On top of the volatility dynamics, Figure 2.2 marks days with the jumps in volatility, i.e., days on which the estimated probability of a jump exceeds 20% and the estimated value of $\xi$ is not zero. As follows from the figure, almost all of the high-volatility episodes begin with jumps in volatility. However, not all of the detected jumps are accompanied by an increase in volatility. This follows from a comparison between the jump marks in Figure 2.2 and the estimated jump
probabilities in Figure 2.3. For example, during the period of “great moderation” in the 1990’s jumps were not accompanied by increases in volatility.

To further clarify the interpretation of jump terms, $\xi_t^f dN_t$, Figure 2.4 compares the estimated jump probabilities to the ratios, $\left(\frac{F_t/F_{t-h} - 1}{\sqrt{\hat{V}_{t-h}}}\right)$, $h = 1$ day. These ratios are similar to the Z-statistics of Lee and Mykland (2008) that enter the non-parametric Lee-Mykland test for jumps. The Z-statistics are approximately normally distributed in the absence of jumps as $h \to 0$. Figure 2.4 plots the Z-statistics and marks days with the estimated probability of jumps above 20%. It follows from Figure 2.4 that the large negative values of the Z-statistics, below the 0.5th percentile of the standard normal distribution (-2.57), always correspond to estimated jump probabilities of more than 20%. The opposite is also true: days on which we estimate the probability of a jump to exceed 20% typically correspond to Z-statistics that are around or less than $-2.57$. Note that the ratios $\left(\frac{F_t/F_{t-h} - 1}{\sqrt{\hat{V}_{t-h}}}\right)$ compare stock price movements to the current level of volatility. Therefore, for the Market, extreme events captured by the terms $\xi_t^f dN_t$ in the SVCJ model correspond to abrupt price movements that are inconsistent with the current level of volatility. For example, in the period after the recent financial crisis, large absolute returns were primarily driven by high levels of volatility. We note that these extreme returns are not detected as jumps, and are best explained by continuous dynamics.

**Value.** Perhaps the most interesting feature of our analysis of Value is that the jumps in returns are typically positive and even make up a substantial contribution to its average returns. We estimate the total return to the value factor to be approximately 0.01% per day (2.52% annualized) of which we find approximately 30% attributable to rare, large positive jumps. Jumps seem to occur less frequently in Value than they do in the Market. There are only two days with estimated jump
probabilities greater than 80%. One such episode, when Value increases by 1.60%, coincides with the (negative) market correction on July 7, 1986. Hence there is some, albeit informal, evidence of co-jumps in the Market and Value.

In addition to typically positive jump sizes for Value, we also find no evidence of a leverage effect for the continuous shocks. The corresponding correlation is slightly positive but is very close to zero ($\rho = 0.037$). Since Value is constructed as a zero net cost position, this suggests that any correlation between continuous shocks to variance and continuous shocks to returns in the long end of the portfolio are offset by an equivalent relationship in the short end. Value is also less volatile, with a total variance of 0.29 compared to 1.09 for the market return. The same holds true for jump days: we estimate the variance of jump sizes in Value to be 1.9 compared to 3.7 for the Market. The volatility dynamics for Value and the Market are similar, as follows from Figure 2.2, but do not completely coincide. For example, the rise in Value volatility during the period 1998-2002 was almost as high as in 2008-2009. For the Market, on the other hand, the volatility in the latter period greatly surpasses that during 1998-2002.

We investigate the nature of jumps in Value by comparing the volatility jump markings in Figure 2.2, the estimated jump probabilities in Figure 2.3, and the Z-statistics in Figure 2.4. We conclude that the estimated jumps in Value are typically associated with an abnormal rise in volatility rather than with abnormal returns.

**Momentum.** Momentum, much like the Market, is subject to infrequent ($\lambda = 0.0046$) but large ($-1.68\%$ on average) negative jumps. The total excess return of Momentum is 10.9% annualized, after accounting for average annual losses due to jumps of $-2.3\%$. There are five trading days for which the probability of a jump in Momentum exceeds 0.8. One such instance is on January 3, 2001, when the Federal
reserve announced a cut in the federal funds rate and technology stocks (NASDAQ) surged by a record 14% amid a prolonged bear market for the technology sector. On this date Momentum lost 7.49%. Interestingly, we do not find evidence of a jump in the market return on this date. In general, our univariate analysis yields no evidence of co-jumps in the Market and Momentum.

With regard to the interpretation of jumps in Momentum, we find that many but not all of the jumps are associated with an increase in volatility. Similar to the Market, we observe a correlation between large negative (below $-2.57$) Z-statistics and high (above 20%) estimated probabilities of jumps. We also find that jumps make up the largest fraction of total variance for Momentum. As was the case for Value, we find no evidence of a leverage effect in Momentum. In fact, our estimates of $\rho$ indicate a positive relationship between continuous variance shocks and shocks to Momentum. Comparing the volatilities for the factors in Figure 2.2, we again find similarities in the dynamics of Momentum and the Market. However, the dynamics clearly do not coincide. For example, Momentum volatility was largely unresponsive to the Asian crisis.

The univariate analysis in this section confirms similarities in the dynamics of the factors. The SVCJ model captures the presence of extreme returns and extreme volatility changes in the Market, Value, and Momentum. The aim of the next two sections is to generalize these univariate SVCJ models to the multivariate case in order to analyze the co-interactions among the factor series.

### 2.4 Building Blocks of the Multivariate Model

Before we formulate a reduced-form model for the joint dynamics of the factors, we discuss the components that we wish to examine with our multivariate analysis. We
focus on three components: standard correlations, as measured by the covariance matrix of continuous shocks; extreme-event correlations, as measured by the characteristics of co-jumps, and the role of volatilities in predicting factor returns. In the next subsections we consider these blocks in turns.

2.4.1 Correlation of Continuous Shocks

It is important to note that the univariate jump-diffusion models allow for a division of shocks into four categories: moderate-size shocks to volatility, moderate-size shocks to returns that are not associated with the shocks to volatility, extreme shocks to volatility, and extreme shocks to returns that are not associated with extreme shocks to volatility. In contrast to the univariate case, the joint modeling of the Market with the other factors allows for a further decomposition.

Long-run risk models based on the ideas of Bansal and Yaron (2004) provide some intuition as to the determinants of continuous shocks. In such models, all priced factors are driven by the same set of underlying shocks to the economy, albeit with different loadings. The underlying shocks can represent fluctuation in aggregate consumption as well as fluctuation in the expectation of future consumption. Additionally, the volatility associated with each of the above quantities is also time varying and covariation with shocks to their respective volatilities can also earn a premium. As such, return volatility is a mixture of shocks to aggregate consumption, expected aggregate consumption as well shocks to their volatility.\(^7\) If the factors correspond to different loadings on particular shocks, then their joint modeling allows us to decompose common and independent components. The joint analysis allows us to

\(^7\)Shocks to higher order terms, such as a time varying volatility of volatility, are also considered in Bollerslev et al. (2011) and Amaya et al. (2011).
obtain cleaner measures of shocks related to volatility and thus we can examine the
dynamics between components related to volatility and the remaining parts.

2.4.2 Jump dynamics

We rely on insights from the asset pricing literature to choose an appropriate joint
model for the jump components of the factors. One explanation for the presence of
jumps in prices is to assume large changes in macroeconomic fundamentals (see Drech-
sler and Yaron (2011)) or quick revisions in subjective quantities, such as macroeco-
nomic forecasts or uncertainty (Shaliastovich (2011)). The model of Drechsler and
Yaron (2011) introduces two sources of jumps: jumps in expected growth components,
and jumps in macroeconomic uncertainty. The first type of jump is present only in
returns whereas the second is present in both returns and volatilities. For this latter
case, there is a perfect correlation between jump sizes in returns and volatilities for
each separate jump component.

Based on these models, we can expect both (i) co-jumps among the factors and (ii)
differences in the occurrence of jumps for the factors if they load with zero, or nearly
zero, on some systematic jump processes. We introduce the co-jumps through the
presence of jump betas. Furthermore, based on the fact that some jumps may affect
both returns and volatility and some may affect only returns, we allow for differences
in return and volatility jump betas.

We further extend the set of possible jump dynamics by considering the dynamics
specified by Bansal and Shaliastovich (2011). They suggest that jumps in financial
markets come from an endogenous learning process. When market participants re-
vise their expectations due to increased uncertainty, sharp changes can occur. The
implication from their model is that jumps in the market are caused by a common pro-
cess. The revisions in prices, however, can be uncorrelated. Even though the factors may jump simultaneously, each factor’s jump size can be independent. Our reduced-form model accommodates both types of co-jumps: jump betas capture Drechsler and Yaron’s co-jumps, and a common jump process drives Bansal and Shaliastovich’s co-jumps.

2.4.3 Premia

Within univariate models, we could estimate the risk premium associated with each of the factors and their dependence on corresponding variances. Multivariate (pairwise) analysis allows for additional cross-terms in the definition of risk premia. For example, Value’s premium may be a function of both Market volatility and its own volatility. Bollerslev et al. (2009) using monthly data and Bollerslev et al. (2013a) using intra-day data show that a large portion of the Market return at short to medium horizons is explained by a higher-order uncertainty process - volatility premium, rather than by the Market volatility itself. In this chapter we deal with several systematic volatility processes, one for each factor. As a result we can comment on whether the separation of systematic volatility processes improves the fit of return premia compared to exclusively using the factor’s own volatility.

2.5 Bivariate Model

Taking into account the discussion in the previous section, we consider a straightforward generalization of the model by Eraker et al. to a bivariate process \((F_{1,t}, F_{2,t})\):

\[
\frac{dF_{i,t}}{F_{i,t}} = \mu_{i,t} dt + \sqrt{V_{i,t}} dB_{i,t} + \xi_{i,t}^f dN_{i,t},
\]

\[
dV_{i,t} = \mu_{i,t}^v dt + \sigma_i \sqrt{V_{i,t}} dB_{i,t}^v + \xi_{i,t}^v dN_{i,t}^v, \quad i = 1, 2
\]
where \( N_{i,t} \) and \( N^v_{i,t} \) are counting processes, \( \mu_{i,t} \) and \( \mu^v_{i,t} \) are predictable means of returns and volatilities, respectively. The subsections below discuss how the above model accommodates the features described in Section 2.4. Throughout the chapter, the index \( i = 1 \) denotes the market return.

**Model for systematic volatilities.** Systematic volatility in this model is driven by two processes \( V_{1,t} \) and \( V_{2,t} \), where for simplicity we define \( V_{1,t} \) as the spot volatility of the first factor (the Market) and \( V_{2,t} \) as that of the second factor. As in the univariate case, we model expected changes in volatility as linear functions:

\[
\mu^v_{i,t} = \kappa_{i,1} (\theta_1 - V_{1,t-}) + \kappa_{i,2} (\theta_2 - V_{2,t-}), \quad i = 1, 2.
\]

We allow each factor’s volatility to be driven by two processes. In the preceding discussion, this assumption pertains to the presence of two systematic volatility components. Both factors depend on these components, possibly with different coefficients.

**Model for premia.** We allow the systematic volatilities to drive each factors’ risk premia,

\[
\mu_{i,t-} = \alpha_i + \beta^{f}_{i,1} V_{1,t-} + \beta^{f}_{i,2} V_{2,t-}.
\]

**Model for diffusion shocks.** We allow the covariance matrix of Brownian shocks \( (dB_{1,t}, dB_{2,t}, dB^v_{1,t}, dB^v_{2,t}) \) to take the most general form. Corresponding correlations are denoted by \( \rho_{i,j}, i, j = 1, \ldots, 4 \). For example, \( \rho_{1,4} \) is the correlation between the first shock, \( dB_{1,t} \), and the fourth shock, \( dB^v_{2,t} \).

**Model for jumps.** Because the jumps in this model are by definition rare events, estimation of the most general model for co-jumps is impossible without imposing strong priors.\(^8\) The approach we take in this chapter is to deduce possible relationships

\(^8\)We have also estimated a model with an unrestricted covariance structure between the jumps. The resulting estimates of all cross-correlations were unstable.
between the jump components of the factors from the relationships between diffusion components while relying on intuition from the structural models discussed in Section 2.4.

Our first step involved estimating the model without co-jumps and analyzing the variance-covariance structure of the continuous shocks. Jump sizes \((\xi_i, \xi_{iv})\) for each \(i = 1, 2\) are modeled as in the univariate case, by a conditional normal, \(N(\mu_i^J + \rho_i^J \xi_{iv}, (\sigma_i^J)^2)\), and exponential \(\exp(\mu_i^{v,J})\) random variables. The estimates are reported in Table 2.4.

We start by discussing the estimated correlations, \(\rho_{i,j}\). The Value and Momentum volatilities are highly correlated with the Market’s volatility. This finding suggests a common volatility component. We expect Value and Momentum to respond to the extreme spikes in market volatility, i.e., a successful model should include non-zero jump betas in volatilities. We also find a negative \((\rho_{1,2} = -0.48)\) correlation between the Market and Value. Therefore, we should allow for a non-zero jump beta for the market-value pair.

In addition to these findings, we also note the difference in the estimated jump intensities, \(\lambda_i\), for the Market and Momentum. This finding implies that the occurrences of jumps do not perfectly coincide, and therefore, two jump processes will be needed in the joint model. Finally, using insight from Bansal and Shaliastovich (2011), we introduce co-jumps in the factors with independent jump sizes. That is, we allow jumps to occur concurrently but their correlation is not restricted to be unity.

Our final model accommodates all of these features. Following Eraker et al. we

---

\(^9\)The estimation steps for this model follow from those of the more general model discussed in Appendix A.
restrict \( N_{1,t} = N_{1,t}^v \) to be a Poisson process with intensity \( \lambda_1 \). We model \( \xi_{1,t}^f \) and \( \xi_{1,t}^v \) as conditional Normal and Exponential random variables, respectively. The model for the jump process in the second factor is composed as follows:

\[
\xi_{2,t}^f dN_{2,t} = \beta_{f,J} \xi_{1,t}^f dN_{1,t} + \xi_{2,t}^c d\tilde{N}_{2,t},
\]

\[
\xi_{2,t}^v dN_{2,t}^v = \beta_{v,J} \xi_{1,t}^v dN_{1,t} + \xi_{2,t}^c d\tilde{N}_{2,t},
\]

where \( \tilde{N}_{2,t} \) is a Poisson process with intensity \( \lambda_2 \), independent of \( N_{1,t} \). The first components \( \beta_{f,J} \xi_{1,t}^f dN_{1,t} \) and \( \beta_{v,J} \xi_{1,t}^v dN_{1,t} \), are co-jumps with the Market due to jump betas. That is, these first components measure the sensitivity of the additional factor to a jump in the Market. The second components, \( \xi_{2,t}^c dN_{1,t} \) and \( \xi_{2,t}^c d\tilde{N}_{2,t} \) are co-jumps with the Market due to a common process that causes jumps in all factors. That is, the while the occurrence of jumps coincide, the actual effect on the price process is independent. In the context of structural models, the first components are in line with the model of Drechsler and Yaron (2011) and the second components can be explained by the model of Bansal and Shaliastovich (2011). The third components are individual jumps with sizes \( (\xi_{2,t}^f, \xi_{2,t}^v) \) that are modeled by conditional normal and exponential variables, respectively.

We retain the possibility that individual jumps in the Market are not associated with jumps in the other factors by setting \( (\xi_{2,t}^{f,c}, \xi_{2,t}^{v,c}) \) to zeros with probability \( 1 - \lambda_c \). For parsimony, we set the distribution of \( (\xi_{2,t}^{f,c}, \xi_{2,t}^{v,c}) \) to that of \( (\xi_{2,t}^f, \xi_{2,t}^v) \) in the rest of the cases. That is, the distribution of \( (\xi_{2,t}^{f,c}, \xi_{2,t}^{v,c}) \) is a mixture of the distribution of \( (\xi_{2,t}^f, \xi_{2,t}^v) \) with probability \( 1 - \lambda_c \) and the distribution of the constant \((0,0)\) with probability \( \lambda_c \), \( \lambda_c \in [0,1] \). Thus, \( \lambda_c \) is the probability of a co-jump, apart from those

\[\text{Note that we redefined } (\xi_{2,t}^f, \xi_{2,t}^v) \text{ on the right-hand side of the system to include only individual jumps.}\]
due to jump betas \((\beta^f,J, \beta^v,J)\).

To summarize, jump dynamics can be described as follows. (1) The Market and its volatility jump simultaneously. (2) A jump in the Market can affect the additional factor with sensitivity determined by the jump beta. (3) A jump in the Market volatility can affect the volatility of the additional factor with sensitivity determined by the volatility jump beta. (4) Jumps in the Market can trigger jumps in the other factor with probability \(\lambda_c\). (5) The other factor can jump independent of the Market.

The above characterization is quite exhaustive and captures the main components of tail events one is concerned with, that being whether there is covariance in jumps and/or coincidence in jumps.

The estimation is implemented using the following Euler discretization scheme:

\[
\begin{align*}
\frac{F_1,t - F_1,t-h}{F_1,t-h} &= \mu_{1,t-h}h + \sqrt{V_1,t-h}\varepsilon_{1,t} + \xi^f_{1,t}J_{1,t}, \\
\frac{F_2,t - F_2,t-h}{F_1,t-h} &= \mu_{2,t-h}h + \sqrt{V_2,t-h}\varepsilon_{2,t} + \beta^{Jf}\xi^f_{1,t}J_{1,t} + \xi^f_{2,t}J_{2,t}, \\
V_1,t - V_1,t-h &= \mu_{1,t-h}h + \sigma_{1,v}\sqrt{V_1,t-h}\varepsilon_{1,t} + \xi^v_{1,t}J_{1,t}, \\
V_2,t - V_2,t-h &= \mu_{2,t-h}h + \sigma_{2,v}\sqrt{V_2,t-h}\varepsilon_{2,t} + \beta^{Jv}\xi_{1,t}J_{1,t} + \xi^v_{2,t}J_{2,t},
\end{align*}
\]

where \((\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon^v_{1,t}, \varepsilon^v_{2,t})\) are zero-mean, jointly Normal, with unit variances on the diagonal of the covariance matrix. \(J_{1,t}\) and \(J_{2,t}\) are Bernoulli random variables with parameters \(\lambda_1h\) and \(1 - (1 - \lambda_2h)(1 - \lambda_cJ_{1,t})\), respectively. Note that the proposed discretization scheme excludes the probability of a double jump in the second factor due to components \(\xi^f_{2,t}dN_{1,t}\) and \(\xi^f_{2,t}d\tilde{N}_{2,t}\). The estimation is carried out using the MCMC algorithm described in Appendix A with \(h = 1\) day.
2.6 Estimation Results

Estimates for the Market-Momentum and Market-Value models are presented in Tables 2.5 and 2.6. For the Market-Momentum model, we first set jump betas to zero. The corresponding results are reported in the first column of Table 2.5. In the second column, we report the results for the final model with a non-zero volatility jump beta that allows for coordination in volatility jumps. The comparison of the estimates shows that a failure to account for the volatility channel in co-jumps results in the following estimation biases. First, we obtain an upward biased estimate of $\lambda_c$. If $\beta^{v,J}$ is set to 0, then two different types of co-jumps are described by one jump process in Momentum. The higher total probability of this combined co-jump results in a higher estimate of $\lambda_c$. Second, because this combined jump process now includes co-jumps with the Market volatility, then the corresponding average Momentum volatility jump increases, i.e., $1/\mu^v_2$ is upward biased. Third, the intensity of the individual momentum jumps, $\lambda_2$, is underestimated. This bias occurs because the individual jumps in Momentum that are not accompanied by increases in volatility are now less likely to be flagged as jumps. This is a consequence of the upward bias in $1/\mu^v_2$, which results in smaller volatility jumps being mistaken for continuous shocks. We conclude that the volatility channel of co-jumps is an important feature and we further analyze only the results of the models with $\beta^{v,J} \neq 0$ for both Momentum and Value.

For the Market-Value model, we begin by estimating a simpler model with a non-zero volatility jump beta, $\beta^{v,J} \neq 0$, but with a zero return jump beta, $\beta^{f,J} = 0$. The resulting estimated parameters are reported in the first column of Table 2.6. The second column of Table 2.6 shows the estimated parameters for the full model. The comparison of the first and the second columns in Table 2.6 shows that the estimates do not change much between the two specifications. The estimated
\( \beta^{f,J} \) is also small and statistically insignificant. The point estimates for all of the parameters and the corresponding standard errors are about the same. Therefore, our analysis is not sensitive to the restriction on \( \beta^{f,J} \). We analyze the final results in the second column. As discussed earlier in Section 2.5, we conjectured there would be return jump correlation (\( \beta^{f,J} \neq 0 \)) for the market-value pair due to the statistically significant diffusion correlation, \( \rho_{1,2} = -0.48 \).

**Implications for premia and volatility.** Table 2.7 calculates the total premium in annual units. The Market premium is given by,

\[
\alpha_1 + \beta_{1,1}^f E[V_{1,t}] + \beta_{1,2}^f E[V_{2,t}] + \lambda_1 E[\xi_{1,t}^f]
\]

and the premia for the additional factors are given by,

\[
\alpha_2 + \beta_{2,1}^f E[V_{1,t}] + \beta_{2,2}^f E[V_{2,t}] + \lambda_1 \beta^{f,J} E[\xi_{1,t}^f] + (\lambda_c \lambda_1 + \lambda_2) E[\xi_{2,t}^f]
\]

We separate the components that are due to volatilities, \( \beta_{1,1}^f E[V_{1,t}] + \beta_{1,2}^f E[V_{2,t}] \) and \( \beta_{2,1}^f E[V_{1,t}] + \beta_{2,2}^f E[V_{2,t}] \), and those due to jumps, \( \lambda_1 E[\xi_{1,t}^f] \) and \( \lambda_1 \beta^{f,J} E[\xi_{1,t}^f] + (\lambda_c \lambda_1 + \lambda_2) E[\xi_{2,t}^f] \). We also separately estimate the contribution of independent jumps to Momentum and Value, \( \lambda_2 E[\xi_{2,t}^f] \). We find, over the sample we consider, that the total premium earned by the Market and Momentum are roughly equal and are around 8% annualized. The value premium is smaller, at 3.5% annually. The components of the premia that are explained by volatilities are small and statistically insignificant. That is, the two systematic volatility components do not help to predict the dynamics of excess returns at daily horizons. This is in line with results from prior literature (see Bollerslev et al. (2009)) that Market volatility does not predict returns at short to medium horizons and other measures of risk are needed to extract the risk-return relationships.
Note that the Market loses on average 1.2% - 2.8% annually due to the existence of rare negative corrections. A similar effect is observed for Momentum; 2.6% loss annually out of which 1% is attributed to the individual jumps. The jump losses for Value are small, positive, and statistically insignificant.

**Implications for the interrelation of volatilities.** Correlations between continuous shocks to volatilities are estimated at 0.72 with a standard error (s.e.) of 0.06 and at 0.81 (s.e. 0.04) for the Market-Momentum and for the Market-Value models, respectively. We confirm that, as in Figure 2.2, factor volatilities are correlated but do not entirely coincide. Thus, a model with several positively correlated volatility components is more appropriate.

Consider, for example, the Market-Momentum pair. We apply a spectral decomposition to the matrix $\begin{pmatrix} 1 - \kappa_{1,1} & -\kappa_{1,2} \\ -\kappa_{2,1} & 1 - \kappa_{2,2} \end{pmatrix}$ to obtain the first auto-correlations of the two underlying components, 0.991 and 0.947, in daily units. That is, the half-life of shocks to the slower component is 77 days, and the half-life of the shocks to the faster component is 13 days. The unconditional continuous correlation between the two underlying components is 0.28 (s.e. 0.09) – much smaller than the original correlation, 0.72. The variance decomposition of the continuous shocks to volatilities shows that 95% of the variance of Momentum is due to the slow-decaying component (s.e. 3%). For the Market volatility the division is less polarized: 55% is due to the fast-decaying volatility and 30% is due to the slow-decaying volatility, the rest is due to covariance between the components. These findings imply that Momentum volatility contains information about the slow-decaying component of the market volatility. Bivariate modeling of the the Market and Momentum series allows us to better estimate the dynamics of the Market volatility.
The variance decomposition is different for the Market-Value pair. We find that the Market and Value volatilities are driven by a slow-decaying component, with an autoregressive parameter of 0.990 (69-day half-life), and a fast-decaying component, with an autoregressive parameter of 0.975 (27-day half-life). In contrast to the Market-Momentum case, the fast-decaying component almost entirely (93%) coincides with the Market volatility. The volatility of Value is a combination of the two, with 69% attributed to the slow-decaying component and 29% attributed to the fast-decaying component. We find that with respect to the multivariate volatility dynamics, Value volatility revolves around the Market volatility but contains no additional information about it.

**Implications for the diffusion leverage effect and volatility-unrelated co-movement between returns.** It is common to refer to the negative correlation between returns and shocks to volatility as the *leverage effect*. Though, initially explained by the increased leverage of companies with decreasing stock prices (see, Black (1976), and Christie (1982)), it is now more prevalent to interpret the leverage effect as coming from an increase in risk premia demanded by investors when volatility is high (see, French et al. (1987) and Campbell and Hentschel (1992)). A more nuanced picture of the effect for individual assets emerges from the models based on Bansal and Yaron’s (2004) representative agent model (e.g., Bollerslev et al. (2011)). In these models the leverage effect can be positive for assets whose pay outs are only moderately correlated with aggregate consumption.

We interpret the two volatility components in our jump-diffusion model as two different sources of risk and analyze the leverage effect for each factor with respect to each volatility component. We also ask, how much of the correlation between the market and the other factors is explained by common correlations with the same
volatility components?

Our analysis depends on the order in which we orthogonalize shocks \((dB_{1,t}, dB_{2,t}, dB_{v1,t}, dB_{v2,t})\). Here we present the results based on two distinct sets of structural assumptions. **Hypothesis 1:** The Market is driven only by shocks to its own volatility and a volatility-unrelated Market shock according to the SVCJ model of Eraker et al.. Under this hypothesis, we orthogonalize the shocks in the order \((dB_{v1,t}, dB_{v2,t}, dB_{1,t}, dB_{2,t})\) and expect that the Market is not correlated with \(dB_{v2,t}\) controlling for the effect of \(dB_{v1,t}\). **Hypothesis 2:** The Market is driven by shocks to the slow-decaying volatility component, the fast-decaying volatility component and a volatility-unrelated Market shock. For the Market-Momentum pair, because the volatility of Momentum is almost entirely explained by the slow-decaying volatility component, we orthogonalize the shocks in the order \((dB_{v2,t}, dB_{v1,t}, dB_{1,t}, dB_{2,t})\). For the Market-Value pair, the correlation between slow-decaying and fast-decaying components is virtually zero, 0.05 (s.e. 0.23). As a result, we can switch the order of the volatility shocks. And because the Market volatility is almost entirely driven by the fast-decaying component, the shocks of the Market-Value pair are orthogonalized in the order \((dB_{v1,t}, dB_{v2,t}, dB_{1,t}, dB_{2,t})\), which is the same as for the first hypothesis.

Panels C and D in Table 2.7 show leverage effect estimates under the two hypotheses. The estimates of the total leverage effect do not depend on the order of the shocks and, therefore, are reported only once in Panel C. The Market-Momentum leverage effects explain 31% of the continuous shocks to the Market and 8% to Momentum. The Market-Value leverage effects explain 21% of the shocks to market returns and 3% to value returns. These results imply that the leverage effect is less important for the other factors than for the Market, perhaps due to the fact that leverage is approximately equivalent in both the long and short ends of the portfolios. We also
find that bivariate modeling of the Market and Momentum helps to uncover more of the Market leverage effect.

Next, we consider the first hypothesis (Panel C). For the Market-Momentum model, the results are consistent with Momentum being informative about the dynamics of the Market volatility. We estimate the partial correlation between the Market and Momentum volatilities to be 0.16 (s.e. 0.06). Therefore, we interpret only the estimates under the second hypothesis. In particular, we identify two sources of the leverage effect for the Market. One is with respect to the slow-decaying volatility component, which explains 8.7% of the continuous shocks to the Market. Another is with respect to the fast-decaying volatility component, which explains 21.8% of continuous shocks to the Market. The fast-decaying component has a noticeably larger effect, which is in line with the results of Bollerslev et al., who find that the faster-decaying component of the Market volatility is the main driver of the leverage effect at 5-minute to 5-day frequencies. For Momentum, the fast-decaying component results in a negative leverage effect and explains 5.2% of Momentum shocks. In contrast, the slow-decaying component results in a small positive leverage effect.

For the Market-Value model, the estimates in Panels C and D naturally coincide. In accordance with hypothesis 1, the Market is uncorrelated with Value volatility after controlling for the effect of its own volatility. Thus, not only does Value volatility carry no additional information about the Market volatility, but it also carries no information about the Market returns. The leverage effect for the Market, with respect to its own volatility, is large (−0.447) and negative. Value, on the other hand, does not respond to individual shocks to its own volatility, although it exhibits a small positive leverage effect with respect to the Market variance (0.156). The latter could stem from the negative correlation between the Market and Value returns,
\( \rho_{1,2} = -0.476, \) and the negative leverage coefficient for the Market. Therefore, we also find that Value returns contain a small Market volatility component while the rest is uncorrelated with volatilities, i.e. it is Value specific. We subtract the Market component from Value returns and obtain that the leverage coefficient for the refined Value factor is -0.056 (s.e. 0.028) with respect to the Market volatility and is 0.012 (s.e. 0.047) with respect to its own volatility. The total variation of Value explained by volatilities reduces to 0.6% (s.e. 0.5%). That is, once we remove the part of Value that is negatively correlated with the Market, the rest of Value is uncorrelated with volatilities.

Finally, Panel C of Table 2.7 reports the correlations between factors controlling for volatilities. That is, we remove leverage effects and compute correlations for the residuals. As expected, the correlation between the Market and Value changes by very little and is now \(-0.453\) (s.e. 0.017). On the other hand, the small correlation between the continuous shocks to the Market and Momentum returns (0.097) reduces even further to 0.039 when we subtract the effect of common volatility shocks. The residual correlation is statistically insignificant.

**Jump leverage effect** In our model, a jump leverage effect would be captured by a negative \( \rho_i^J \). As follows from the estimation results in Table 2.7, the estimated \( \rho_i^J \) for the Market and Momentum are negative, but only the estimate for the individual Momentum jump is significant at the 10% level. Somewhat similarly, the estimates of \( \rho_i^J \) for the Market and Value, in Table 2.7 are statistically insignificant and positive. Thus, the estimation results lack accuracy due to the rarity of jumps. Eraker et al. find a similar result in their simulation study.

Due to the lack of accuracy and because all volatility jumps are restricted to be positive and occur concurrently, we propose to infer the sign of the jump leverage...
effect by the sign of the expected jump returns. These results are given in Panel E of Table 2.7. As follows from the table, both the Market and Momentum exhibit negative skew during the extreme events. Depending on the model considered, we estimate the expected jump size of the Market to be $-1.45\%$ and $-3.7\%$. For Momentum we estimate the expected jump size to be $-2\%$. The expectation of the jump size in Value is small, positive, and statistically insignificant. We conclude that there is some evidence for a negative jump leverage effect in both the Market and Momentum and no evidence for jump leverage effect in Value.

**Implications for co-jumps.** Next we revisit the empirical findings regarding co-jumps. In the prior univariate analysis of Section 2.3, we found only one co-jump for the Market-Value pair and none for the Market-Momentum pair. In our multivariate analysis, we now find that most of the Market jumps are associated with the jumps in Momentum. This finding is inferred from the estimate of $\lambda_c = 0.865$ (s.e. 0.104). Similar results are obtained for Value, evidenced by the estimate of $\lambda_c = 0.726$ (s.e. 0.161). In support of this conclusion, we also add the evidence of a statistically significant volatility jump beta for Momentum, $\beta^{v,J} = 0.154$ (s.e. 0.033). This evidence is consistent with the two-component volatility story, with one affecting both the Market and Momentum.

**Implications for individual jumps in Momentum and Value.** Finally, we answer the question: do we observe jumps in Momentum and Value that are not associated with a jump in the Market? These individual jumps occur with intensity $\lambda_2$. For Momentum, the estimate of $\lambda_2 = 0.0022$ (s.e. 0.0011) implies that there are around 18 individual Momentum jumps in the sample. The same estimate for the Value is both small ($\lambda_2 = 0.0007$) and statistically insignificant (s.e. 0.0005). Thus, we do not find conclusive evidence for independent jumps in Value in the absence of
Market jumps.

**Market-Momentum: what do extreme events look like?** The model in Section 2.5 allows for two types of extreme events which can be classified as Market-originated and Momentum-originated. In the first scenario, the Market Value-at-Risk (VaR) sharply increases from around $-1.64\sqrt{V_{1,t-}}$, which is $-1.59\%$ (s.e. 0.05%) at normal volatility levels ($V_{1,t-} = E[V_{1,t}]$), to the 5th percentile of the mixture of normal distributions $N(\mu_I^j + \rho_I^j \xi_v^{v,t},V_{1,t-} + (\sigma_I^j)^2)$, where $\xi_v^{v,t}$ is exponentially distributed ($\exp(\mu_v,I^j)$). At normal volatility levels, the new Market VaR is $-8\%$ and can be estimated by simulation. This estimate is roughly the same as $\mu_I^j + \rho_I^j \mu_v^v - 1.64\sqrt{E[V_{1,t}]} + (\sigma_I^j)^2 = -7.98\%$ (s.e. 1.43%). The post-event VaR also increases due to the jump in volatility ($\xi_v^{v,t}$) to $-1.64\sqrt{V_{1,t-} + \xi_v^{v,t}}$ and gradually reverts to the usual level in the absence of further extreme events.

Meanwhile, the risk of Momentum also rises: the VaR increases from $-1.64\sqrt{V_{2,t-}}$, which is $-1.08\%$ (s.e. 0.06%) at normal volatility levels $V_{2,t-} = E[V_{2,t}]$, to around $\mu_J^j + \rho_J^j \mu_v^v + q_{0.05}/\lambda_c \sqrt{EV_{2,t} + (\sigma_J^j)^2} = -4.77\%$ (s.e. 0.61%). The subsequent Momentum VaR increases to $-1.64\sqrt{V_{2,t-} + \beta v,J^v \xi_v^{v,t}}$ if Momentum returns were not affected by the event or to $-1.64\sqrt{V_{2,t-} + \beta v,J^v \xi_v^{v,t} + \xi_v^{v,t}}$, otherwise.

**Examples of co-jumps.** We use as examples recent “likely” co-jumps in the Market and Momentum. There are six days with $\hat{J}_{1,t} > 0.8$ and $\hat{J}_{2,t} > 0.8$ in the sample after 2000. For illustration, 2.5 shows the following examples. (i) The dot-com bubble starts to crack on January 4, 2000 with the news on the federal rates hike. (ii) The

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$q_{0.05}/\lambda_c$ is the 0.05/\lambda_c-th percentile of a standard normal variable and is adjusted for a small probability $1 - \lambda_c = 0.135$ of an extreme event in the Market that does not affect Momentum.

The series $\hat{J}_{i,t}$ are estimated probabilities of jump occurrences, conditional on the observed returns.
troubles in the sub-prime mortgage sector spill over into equity strategies during the week of August 6, 2007, see Khandani and Lo (2011). (iii) A day during the turbulent recovery (“double-dip fear”) of August 2, 2011 when a disappointing U.S. consumer spending report wiped out the year-to-date gains in the stock market. As follows from the figure, all of these events begin with a large absolute returns for at least one of the factors, although the signs and magnitudes vary widely. For all of these cases, the subsequent volatilities substantially increase. For the second example covering the period of July - August 2007, the exact date on which the extreme event occurred is not visually obvious. Note that the Market was already volatile in the weeks leading up to August 6, 2007. Our estimation helps to pinpoint that event.

In the second scenario, only Momentum is affected. Momentum VaR increases to around $\mu_2^f + \rho_2^f/\mu_2^v + q_{0.05}\sqrt{EV_{2,t}} + (\sigma_2^f)^2 = -4.90\%$ (s.e. 0.63\%) on the day of the event, and to $-1.64\sqrt{EV_{2,t-} + \xi_{2,t}^v}$ on the subsequent day.

**Examples of individual jumps.** There are four days with $\hat{J}_{2,t} > 0.8$ and $\hat{J}_{1,t} \approx 0$ in the sample after 2000 that are “likely” individual jumps in Momentum. Three of them are during the bear market after the dot-com bubble, and one is during the bear market after the housing bubble. (i) An unexpected federal rate cut leads to the NASDAQ soaring a record 14\% in one day on January 3, 2001 amid the continuing deflation of the dot-com bubble. (ii) A similar upward correction in technology stocks on May 8, 2002 during a bear market; (iii) Momentum reversal during the bear market of 2008 on July 8. All of these episodes are characterized by a sharp reversal in Momentum with negative returns of -7.22\%, -4.52\%, and -2.95\%, respectively.

**Market-Value: what do extreme events look like?** We perform the same analysis for the Market-Value pair. There are no “likely” individual jumps in Value with $\hat{J}_{2,t} > 0.8$ and $\hat{J}_{1,t} \approx 0$. Therefore, we only consider examples of co-jumps.
Examples of co-jumps. There are three “likely” co-jumps in the period after 2000 with $\hat{J}_{1,t} > 0.8$ and $\hat{J}_{2,t} > 0.8$. (i) The bursting of the dot-com bubble on April 14, 2000 attributed to news about higher inflation and the prospect of the federal funds rate increase. (ii) A market rally (4.3% for S&P 500) on September 18, 2008, attributed to rumors about a “bad bank” creation. News were circulated that the U.S. government would create a “bad bank” to absorb bad debt from bank’s balance sheets. Small stocks rallied even stronger, with the Russell 2000 index increasing by 7%. (iii) The market sell-off on November 7, 2012, following the re-election of President Obama, which became the worst trading day of 2012. Following the re-election, the market moved on the worries about the U.S. “fiscal cliff”: mandatory sharp tax increases and reductions in government spending which could come into effect in January, 2013. The dynamics of returns for these days are shown in the right panels of 2.5. Again, we observe an increase in volatility following each of these events. The first event is associated with an abnormal negative (-6.73%) Market return. The second event is associated with abnormal positive Market (4.41%) and Value (3.95%) returns. In the last example, the Market lost 2.31% and Value lost a modest 1.31%.

Note that the examples presented in this section are chosen based on the statistical likelihood that these events constituted jumps, i.e., based on high values of $\hat{J}_{2,t}$ and/or $\hat{J}_{1,t}$. The selected events are neither a representative sample of possible extreme scenarios, nor the sample of the most extreme events.

2.7 Implications for Factor Investing

Suppose the dynamics of the Market and the other factors are exactly as in (2.2) with the model details specified in Section 2.5. Clearly, any portfolio of the factors will
exhibit price dynamics that follow a jump-diffusion process with stochastic volatility that by itself is subject to jumps. Portfolio optimization problems under jump-diffusion dynamics of prices have been extensively studied in the last few decades. For example, the seminal paper by Merton (1971) considers dynamic portfolio optimization with prices described by a simpler continuous diffusion process. Other studies include, Liu and Pan (2003) and Liu et al. (2003) who work with diffusions with stochastic volatility, and Branger et al. (2008) who extend these methods to the case of volatility jumps. For most of such models, the optimization problem cannot be solved in closed form, but numerical solutions can be obtained. Simulation-based approaches offer yet another route to optimize asset portfolios, see Brandt et al. (2005).

However, the goal of this section is not to demonstrate the methods from the above studies, but rather to discuss the general implications of adding Momentum or Value to the Market portfolio. Our analysis focuses on extreme event risk. Note that due to the rarity of salient jump events, such as in October 1987, one cannot directly assess extreme-event properties of the portfolio returns. On contrary, the jump-diffusion model in this chapter readily produces estimates of average returns and other return moments on the “jump” days and afterwards.

First, we analyze the behavior of the mixed portfolios on the exact days of extreme shocks, i.e., on the days with $N_{1,t+1} - N_{1,t-} > 0$ or $\tilde{N}_{2,t+1} - \tilde{N}_{2,t-} > 0$. Consider, first, the Market-Momentum case. As discussed in Section 2.3, jump days for the Market and Momentum are normally associated with negative returns that are abnormally large relative to the current level of volatility. We note that such days are often followed by a string of extreme returns triggered by increased volatility, see examples in Figure 2.5. On the one hand, adding Momentum to the Market increases the number of jump days to the total frequency of $\lambda_1 + \lambda_2 = 0.0058$, while holding only
the Market results in a frequency of $\lambda_1 = 0.0036$. On the other hand, due to the low correlation between Market and Momentum jump magnitudes, i.e., due to $\beta_{f,J} = 0$, Momentum diversifies the portfolio. To analyze this trade-off between increasing the number of extreme events and diversifying the portfolio across types of extreme events, we calculate the following moments: mean returns, variances and VaRs during the market and momentum jump days. Table 2.8 reports the results for different choices of $w_2 \in [0,1]$, the portfolio’s weight on the other factor, Momentum in this case. For mean returns we find that including Momentum increases average losses during extreme events from $-0.85\%$ per event for $w_2 = 0$ to $-1.78\%$ per event for $w_2 = 1$. The black frame in the first column of the table indicates that these numbers are statistically the same at the 5% significance level. Therefore, despite the economic significance of the reported difference, one cannot rule out that the difference in expected losses is due to sampling error.

For the return variances, we find that adding Momentum reduces the risk of the portfolio during the jump days as $w_2$ increases from 0 to 0.5. The variance of the portfolio does not significantly change for $w_2$ from 0.5 to 0.9, and increases as we go from $w_2 = 0.9$ to 1.0. Finally, the VaRs repeat the pattern of the variances: the risk decreases from $w_2 = 0$ (VaR $= -6.63\%$) to $w_2 = 0.5$ (VaR $= -4.45\%$), remains statistically the same for $w_2 = 0.5 - 0.9$ (VaR between $= -4.14\%$ and $= -4.48\%$), and increases for $w_2 = 0.9 - 1.0$ (VaR $= -4.84\%$). We conclude that adding Momentum to the Market historically reduced the overall jump risk, but leads to more negative returns on average.

In contrast to Momentum, Value does not exhibit individual jumps, i.e., $\lambda_2 \approx 0$.

---

\(^{13}\)We find that the model restricts kurtosis of returns during the extreme events. The obtained kurtosis estimates appear statistically the same across portfolios.
Therefore, the number of extreme events is the same for all of the Market-Value portfolios. That is, adding Value to the Market portfolio simply brings diversification gains: the average losses per extreme event, variance, and the Value-at-Risk are all reduced, see Table 2.8.

Next, we analyze the behavior of the mixed portfolios immediately after jump days. As illustrated by Figure 2.2, in addition to days with \( N_{1,t+1} - N_{1,t} > 0 \) or \( \tilde{N}_{2,t+1} - \tilde{N}_{2,t} > 0 \), the investors are faced with prolonged periods of extreme returns, such as in the years 1988-1989, 1997-2003, and 2007-2010. These periods start with jumps in volatility, i.e., with \( N_{1,t+1} - N_{1,t} > 0 \) or \( \tilde{N}_{2,t+1} - \tilde{N}_{2,t} > 0 \), and continue due to volatility persistence. We next analyze how adding Momentum to the Market changes the frequency, severity, and length of such periods.

It follows from the estimation results that during the individual Momentum jumps, the volatility jumps on average by only \( 1/5.404 \approx 0.185 \), which is a moderate change in comparison to the base value of both Momentum and the Market, 0.283 and 0.756, respectively. The same is confirmed by the analysis in Figure 2.2: the prolonged extreme periods coincide for the two factors. Thus, we conclude that adding Momentum to the Market does not affect the frequency of the extreme periods.

The severity of the extreme period can be compared based on the value of the initial jump in volatility, \( (1-\omega_2)^2 \Delta V_{1,t} + \omega_2^2 \Delta V_{2,t} + 2(1-\omega_2)\omega_2 \rho_{1,2} \Delta \sqrt{V_{1,t}V_{2,t}} \) for \( N_{1,t} - N_{1,t} = 1 \). The deviation in the Market volatility \( \Delta V_{1,t} \) is approximately distributed as \( \xi_{1,v}^{v} \). The deviation in Momentum volatility \( \Delta V_{2,t} \) is approximately the sum of \( \beta_{J,v}^{v} \xi_{1,v}^{v} \) and a mixture distribution that yields 0 with probability \( 1 - \lambda_c \) and \( \xi_{2,v}^{v} \) with probability \( \lambda_c \). Because \( \rho_{1,2} \approx 0 \), we ignore the cross-term. Table 2.9 shows that adding the momentum strategy to the portfolio reduces the overall variance of the

\[14\text{Due to the high level of persistence of in volatilities.}\]
portfolio with the most significant reduction occurring in the range \( w_2 = 0.1 - 0.4 \).

The length of the extreme periods can be compared by the decay rate of the shocks. Because Momentum volatility is more persistent, the mixed Market-Momentum portfolio may have a higher volatility than the market return after a period of time. With this intuition in mind, we compare the effect of jump days on volatility after one month (22 business days), where the effect on volatility is calculated using the formula 
\[
\omega'_v L \Lambda^{22} L^{-1} E[\Delta V_t].
\]
The vector \( \omega'_v \) consists of the volatility loadings 
\[
\begin{pmatrix}
(1 - \omega_2)^2 & \omega_2^2
\end{pmatrix},
\]
The matrix combination \( L \Lambda L^{-1} \) is the spectral decomposition of the daily autocovariance matrix of \( (V_{1,t}, V_{2,t}) \), and the expectation \( E[\Delta V_t] \) is the vector of expected volatility jumps: \( (E[\Delta V_{1,t}], E[\Delta V_{2,t}]) \). We find that the pattern of volatilities after one month repeats the pattern of initial volatility jumps across different weights, \( w_2 \). Therefore, we conclude that the length of extreme variance periods are defined mainly by the shock to the Market, and, therefore, are roughly the same for all of the mixed portfolios.

In contrast to Momentum, Value exhibits a negative correlation with the Market. Therefore, we take into account the term 
\[
2(1 - \omega_2)\omega_2 \rho_{1,2} \Delta \sqrt{V_{1,t}V_{2,t}}
\]
when evaluating jumps in volatilities. As before, we find that mixing the factors in one portfolio reduces volatility during extreme periods with the maximum effect observed for \( w_2 = 0.5 - 0.9 \). As in the case with Momentum, this effect is not overshadowed by the increased persistence of Value volatility, as follows from the last column of Table 2.9.

Our overall conclusion is that combining factors in one portfolio historically reduced the extreme-event risk. Therefore, factor investing offered advantages for managing risk during past extreme events and extreme periods.
2.8 Conclusion

This chapter proposes and estimates joint jump-diffusion models for pairs of fundamental stock market factors: Market-Value and Market-Momentum. Our results strongly support the presence of co-jumps in the factor series. We find that extreme negative Market returns greatly increase the odds of observing extreme Momentum and Value returns. However, the model with only one type of co-jumps is mis-specified, because it does not account for the volatility channel of co-jumps. This chapter suggests how to introduce the latter type of extreme co-movements in the model.

Perhaps the most intriguing finding is that Momentum returns carry additional information about dynamics of the Market. We observe an additional leverage effect in the Market with respect to Momentum volatility. We also find that Momentum volatility contains information about the slow-varying component of Market volatility. Thus, our results indicate potential advantages from modeling the Market and Momentum series jointly.

Our results also shed some light on the origins of the Value and Momentum premia. We find that Value exhibits no leverage effect: neither in the diffusion or jump components. Therefore, we conclude that Value cannot be explained by increased tail risk. Similarly, Momentum exhibits only a modest leverage effect. Thus, at least part of the Momentum premium should arise due to causes other than the increased tail risk, e.g., due to amplification during business cycles. Surprisingly, the small leverage component in Momentum does not originate from its own volatility, but rather from the fast component of the Market volatility. Similarly, the Market leverage effect comes mostly from the fast volatility component. Both results are consistent with the new risk-return relationships examined by Bollerslev et al. (2009). Finally, the
presence of a jump leverage effect in Momentum implies that part of the Momentum premium is due to jump risk.

We additionally investigate the gains from diversifying away from the Market portfolio into a combination of the Market and an additional factor. We specifically investigate the behavior of these combination portfolios during periods of extreme stress (on jump days and in the subsequent period). We find that diversifying away from the Market brings gains in terms of reduced risk both on jump days and in the turbulent (high volatility) periods afterwards.
<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Volatility</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKT</td>
<td>0.0236</td>
<td>1.1091</td>
<td>-0.9074</td>
<td>21.5029</td>
<td>-19.1645</td>
<td>10.7508</td>
</tr>
<tr>
<td>HML</td>
<td>0.0149</td>
<td>0.5440</td>
<td>0.0511</td>
<td>9.8998</td>
<td>-5.0241</td>
<td>3.8740</td>
</tr>
<tr>
<td>UMD</td>
<td>0.0273</td>
<td>0.7914</td>
<td>-1.1957</td>
<td>17.7388</td>
<td>-8.6648</td>
<td>6.8126</td>
</tr>
</tbody>
</table>

This table reports summary statistics for the log daily excess returns of the three stock market factor portfolios. The time period runs from January 1, 1980 through March 31, 2013. The daily returns are in percentage form.
Table 2.2: Univariate Model (SVCJ) Parameters

<table>
<thead>
<tr>
<th></th>
<th>MKT</th>
<th>HML</th>
<th>UMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.0409 (0.0084)</td>
<td>0.0069 (0.0040)</td>
<td>0.0522 (0.0047)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0250 (0.0075)</td>
<td>0.0198 (0.0027)</td>
<td>0.0194 (0.0155)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.6398 (0.0704)</td>
<td>0.1261 (0.0132)</td>
<td>0.2022 (0.0364)</td>
</tr>
<tr>
<td>$\sigma_v^2$</td>
<td>0.0132 (0.0027)</td>
<td>0.0018 (0.0003)</td>
<td>0.0040 (0.0009)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0079 (0.0025)</td>
<td>0.0046 (0.0012)</td>
<td>0.0058 (0.0032)</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>-1.8815 (0.8058)</td>
<td>0.4814 (0.5174)</td>
<td>-1.3342 (0.6980)</td>
</tr>
<tr>
<td>$\rho_J$</td>
<td>-0.5045 (0.6178)</td>
<td>0.2073 (0.6286)</td>
<td>-0.3685 (0.6255)</td>
</tr>
<tr>
<td>$\sigma_y^2$</td>
<td>3.6664 (1.2821)</td>
<td>1.9197 (0.5532)</td>
<td>2.8064 (0.9309)</td>
</tr>
<tr>
<td>$\mu_v$</td>
<td>0.8769 (0.2049)</td>
<td>1.5875 (0.2770)</td>
<td>1.0101 (0.2262)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.6188 (0.0362)</td>
<td>0.0367 (0.0490)</td>
<td>0.1855 (0.0466)</td>
</tr>
</tbody>
</table>

This table reports posterior means for the univariate SVCJ specification for log daily excess returns of the three stock market factor portfolios. The parameter estimates correspond to percentage daily excess returns. The time period considered runs from January 1, 1980 through March 31, 2013. Posterior standard deviations are reported in parentheses.
Table 2.3: Jump-Diffusion Decomposition

### Panel A: Expected Return Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Continuous</th>
<th>Jump</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKT</td>
<td>0.0409</td>
<td>-0.0183</td>
<td>0.0225</td>
</tr>
<tr>
<td></td>
<td>(0.0084)</td>
<td>(0.0045)</td>
<td>(0.0089)</td>
</tr>
<tr>
<td>HML</td>
<td>0.0069</td>
<td>0.0028</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td>(0.0040)</td>
<td>(0.0016)</td>
<td>(0.0042)</td>
</tr>
<tr>
<td>UMD</td>
<td>0.0522</td>
<td>-0.0091</td>
<td>0.0431</td>
</tr>
<tr>
<td></td>
<td>(0.0047)</td>
<td>(0.0034)</td>
<td>(0.0052)</td>
</tr>
</tbody>
</table>

### Panel B: Variance Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Continuous</th>
<th>Jump</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKT</td>
<td>0.6398</td>
<td>0.4518</td>
<td>1.0916</td>
</tr>
<tr>
<td></td>
<td>(0.0704)</td>
<td>(0.1005)</td>
<td>(0.0917)</td>
</tr>
<tr>
<td>HML</td>
<td>0.1261</td>
<td>0.1605</td>
<td>0.2865</td>
</tr>
<tr>
<td></td>
<td>(0.0132)</td>
<td>(0.0381)</td>
<td>(0.0366)</td>
</tr>
<tr>
<td>UMD</td>
<td>0.2022</td>
<td>0.3509</td>
<td>0.5531</td>
</tr>
<tr>
<td></td>
<td>(0.0364)</td>
<td>(0.0858)</td>
<td>(0.0790)</td>
</tr>
</tbody>
</table>

This table reports decomposition of expected log daily excess returns (in %) and variances into continuous and jump components for each of the three stock market factor portfolios. The formulas are given in Section 2.3. Posterior standard deviations for each component are given in parentheses.
Table 2.4: Estimated Parameters: Multivariate Model Without Co-jumps

<table>
<thead>
<tr>
<th></th>
<th>MKT-UMD</th>
<th></th>
<th>MKT-HML</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MKT</td>
<td>UMD</td>
<td>MKT</td>
<td>HML</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.043 (0.012)</td>
<td>0.044 (0.007)</td>
<td>0.042 (0.011)</td>
<td>0.022 (0.006)</td>
</tr>
<tr>
<td>$\beta_{f,1}$</td>
<td>0.015 (0.026)</td>
<td>0.051 (0.017)</td>
<td>0.009 (0.021)</td>
<td>-0.045 (0.010)</td>
</tr>
<tr>
<td>$\beta_{f,2}$</td>
<td>-0.031 (0.039)</td>
<td>-0.116 (0.032)</td>
<td>-0.008 (0.014)</td>
<td>0.034 (0.009)</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.743 (0.032)</td>
<td>0.272 (0.015)</td>
<td>0.890 (0.055)</td>
<td>0.203 (0.010)</td>
</tr>
<tr>
<td>$\kappa_{i,1}$</td>
<td>0.055 (0.006)</td>
<td>0.001 (0.001)</td>
<td>0.027 (0.004)</td>
<td>0.002 (0.001)</td>
</tr>
<tr>
<td>$\kappa_{i,2}$</td>
<td>-0.046 (0.009)</td>
<td>0.010 (0.003)</td>
<td>-0.024 (0.013)</td>
<td>0.008 (0.002)</td>
</tr>
<tr>
<td>$\rho_{1,2}$</td>
<td>0.999 (0.013)</td>
<td>-0.48 (0.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{1,3}$</td>
<td>-0.546 (0.033)</td>
<td>-0.030 (0.031)</td>
<td>-0.441 (0.037)</td>
<td>0.146 (0.031)</td>
</tr>
<tr>
<td>$\rho_{1,4}$</td>
<td>-0.250 (0.045)</td>
<td>0.148 (0.036)</td>
<td>-0.353 (0.044)</td>
<td>0.130 (0.036)</td>
</tr>
<tr>
<td>$\rho_{3,4}$</td>
<td>0.670 (0.060)</td>
<td>0.790 (0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^v_i$</td>
<td>0.167 (0.008)</td>
<td>0.075 (0.004)</td>
<td>0.151 (0.009)</td>
<td>0.058 (0.004)</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>0.0023 (0.0006)</td>
<td>0.0052 (0.0013)</td>
<td>0.0022 (0.0006)</td>
<td>0.0018 (0.0008)</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{f,J}$</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{v,J}$</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{i}^J$</td>
<td>-5.095 (1.308)</td>
<td>-1.024 (0.683)</td>
<td>-4.670 (1.627)</td>
<td>0.701 (1.047)</td>
</tr>
<tr>
<td>$\mu_{i}^J$</td>
<td>0.447 (0.344)</td>
<td>-2.332 (1.890)</td>
<td>0.904 (0.890)</td>
<td>0.055 (0.889)</td>
</tr>
<tr>
<td>$\sigma_i^J$</td>
<td>2.849 (0.627)</td>
<td>1.654 (0.285)</td>
<td>3.112 (0.690)</td>
<td>1.749 (0.690)</td>
</tr>
<tr>
<td>$\mu_{\tau}^J$</td>
<td>1.159 (0.052)</td>
<td>3.207 (1.364)</td>
<td>1.122 (0.070)</td>
<td>4.482 (1.125)</td>
</tr>
</tbody>
</table>

This table reports posterior means for the multivariate SVCJ specification for the pairs of fundamental factor portfolios: MKT-HML and MKT-UMD. The parameter estimates correspond to percentage daily returns. Posterior standard deviations are reported in parentheses. The time period considered runs from January 1, 1980 through March 31, 2013.
Table 2.5: Estimated Parameters: Multivariate Model MKT-UMD

<table>
<thead>
<tr>
<th></th>
<th>MKT-UMD</th>
<th>MKT-UMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MKT UMD</td>
<td>MKT UMD</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.047 (0.012)</td>
<td>0.045 (0.008)</td>
</tr>
<tr>
<td>$\beta_{f,i}^{1}$</td>
<td>0.005 (0.020)</td>
<td>0.035 (0.015)</td>
</tr>
<tr>
<td>$\beta_{f,i}^{2}$</td>
<td>-0.007 (0.011)</td>
<td>-0.030 (0.010)</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.745 (0.044)</td>
<td>0.285 (0.029)</td>
</tr>
<tr>
<td>$\kappa_{i,1}$</td>
<td>0.067 (0.011)</td>
<td>0.005 (0.002)</td>
</tr>
<tr>
<td>$\kappa_{i,2}$</td>
<td>-0.057 (0.013)</td>
<td>0.006 (0.003)</td>
</tr>
<tr>
<td>$\rho_{1,2}$</td>
<td>0.996 (0.013)</td>
<td>0.997 (0.013)</td>
</tr>
<tr>
<td>$\rho_{1,3}$</td>
<td>-0.530(0.037)</td>
<td>-0.038 (0.030)</td>
</tr>
<tr>
<td>$\rho_{1,4}$</td>
<td>-0.279(0.045)</td>
<td>0.144 (0.035)</td>
</tr>
<tr>
<td>$\rho_{3,4}$</td>
<td>0.718 (0.058)</td>
<td>0.752 (0.052)</td>
</tr>
<tr>
<td>$\sigma_{i}^v$</td>
<td>0.182 (0.015)</td>
<td>0.076 (0.005)</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>0.0040 (0.0009)</td>
<td>0.0010 (0.0007)</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>0.920 (0.069)</td>
<td>0.865 (0.104)</td>
</tr>
<tr>
<td>$\beta^{f,J}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta^{v,J}$</td>
<td>0</td>
<td>0.154 (0.033)</td>
</tr>
<tr>
<td>$\mu_{i}^v$</td>
<td>-0.502(1.102)</td>
<td>-1.900 (0.801)</td>
</tr>
<tr>
<td>$\mu_{i}^{f}$</td>
<td>-0.473(0.334)</td>
<td>-0.229 (1.427)</td>
</tr>
<tr>
<td>$\sigma_{i}^{f}$</td>
<td>3.925(0.630)</td>
<td>1.850(0.310)</td>
</tr>
<tr>
<td>$\mu_{i}^{v,J}$</td>
<td>1.159(0.072)</td>
<td>2.473 (0.618)</td>
</tr>
</tbody>
</table>

This table reports posterior means for the multivariate SVCJ specification for the MKT (market returns) - UMD (momentum) pair. The parameter estimates correspond to percentage daily returns. Posterior standard deviations are reported in parentheses. The time period considered runs from January 1, 1980 through March 31, 2013.
This table reports posterior means for the multivariate SVCJ specification for the MKT (market returns) - HML (value) pair. The parameter estimates correspond to percentage daily returns. Posterior standard deviations are reported in parentheses. The time period considered runs from January 1, 1980 through March 31, 2013.
Table 2.7: Interpretation of Estimated Parameters: Multivariate Model

<table>
<thead>
<tr>
<th></th>
<th>MKT-UMD</th>
<th></th>
<th>MKT-HML</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MKT</td>
<td>UMD</td>
<td>MKT</td>
<td>HML</td>
</tr>
<tr>
<td><strong>A: Premium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8.765 (2.620)</td>
<td>8.299 (1.866)</td>
<td>8.384 (2.722)</td>
<td>3.479 (1.651)</td>
</tr>
<tr>
<td>Due to vols</td>
<td>-0.896 (3.838)</td>
<td>-0.257 (2.619)</td>
<td>0.949 (3.791)</td>
<td>-2.554 (2.202)</td>
</tr>
<tr>
<td>Loss from jumps</td>
<td>-1.211 (1.010)</td>
<td>-2.646 (0.734)</td>
<td>-2.770 (0.973)</td>
<td>0.430 (0.444)</td>
</tr>
<tr>
<td>... of which ind. jump</td>
<td>-1.080 (0.535)</td>
<td></td>
<td>0.127 (0.156)</td>
<td></td>
</tr>
<tr>
<td><strong>B: Vol Interrelations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{3,4}$</td>
<td>0.752 (0.052)</td>
<td></td>
<td>0.811 (0.040)</td>
<td></td>
</tr>
<tr>
<td>Mean-reversion slow</td>
<td>0.991 (0.002)</td>
<td></td>
<td>0.990 (0.002)</td>
<td></td>
</tr>
<tr>
<td>Mean-reversion fast</td>
<td>0.947 (0.006)</td>
<td></td>
<td>0.975 (0.005)</td>
<td></td>
</tr>
<tr>
<td>Var. decomp. (slow)</td>
<td>0.302 (0.067)</td>
<td>0.945 (0.028)</td>
<td>0.073 (0.216)</td>
<td>0.693 (0.192)</td>
</tr>
<tr>
<td>Var. decomp. (fast)</td>
<td>0.546 (0.055)</td>
<td>0.014 (0.010)</td>
<td>0.928 (0.072)</td>
<td>0.292 (0.235)</td>
</tr>
<tr>
<td>$\rho_{\text{slow,fast}}$</td>
<td>0.281 (0.090)</td>
<td></td>
<td>0.054 (0.231)</td>
<td></td>
</tr>
<tr>
<td><strong>C: Cont. Leverage: Hyp. I</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>... wrt MKT vol</td>
<td>-0.530 (0.038)</td>
<td>-0.042 (0.031)</td>
<td>-0.447 (0.039)</td>
<td>0.156 (0.031)</td>
</tr>
<tr>
<td>... ind. factor vol</td>
<td>0.157 (0.057)</td>
<td>0.268 (0.048)</td>
<td>0.006 (0.065)</td>
<td>0.009 (0.050)</td>
</tr>
<tr>
<td>... total vol $R^2$</td>
<td>0.310 (0.045)</td>
<td>0.077 (0.027)</td>
<td>0.205 (0.035)</td>
<td>0.028 (0.010)</td>
</tr>
<tr>
<td>residual factor corr.</td>
<td>0.039 (0.033)</td>
<td></td>
<td>-0.453 (0.017)</td>
<td></td>
</tr>
<tr>
<td><strong>D: Cont. Leverage: Hyp. II</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>... wrt slow vol</td>
<td>-0.295 (0.045)</td>
<td>0.143 (0.035)</td>
<td>0.006 (0.065)</td>
<td>0.009 (0.050)</td>
</tr>
<tr>
<td>... wrt fast vol</td>
<td>-0.467 (0.056)</td>
<td>-0.229 (0.050)</td>
<td>-0.447 (0.039)</td>
<td>0.156 (0.031)</td>
</tr>
<tr>
<td><strong>E: Jump Leverage</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \xi_{t,i}$</td>
<td>-1.446 (1.260)</td>
<td>-1.989 (0.445)</td>
<td>-3.678 (1.083)</td>
<td>0.527 (0.528)</td>
</tr>
</tbody>
</table>

This table reports posterior means for derived parameters based on the multivariate SVCJ specification for the pairs of fundamental factor portfolios: MKT-HML and MKT-UMD. The parameter estimates correspond to percentage daily returns. Posterior standard deviations are reported in parentheses. The time period considered runs from January 1, 1980 through March 31, 2013.
Table 2.8: Factor Portfolios: Performance on Jump Days

<table>
<thead>
<tr>
<th>$w_2$</th>
<th>MKT-UMD</th>
<th></th>
<th></th>
<th>MKT-HML</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
<td>VaR</td>
<td>Mean</td>
<td>Variance</td>
<td>VaR</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.85%</td>
<td>-6.63% ↓*</td>
<td>-3.04% ↓*</td>
<td>9.71 ↓*</td>
<td>-8.36% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>-0.94%</td>
<td>-6.15% ↓*</td>
<td>-2.76% ↓*</td>
<td>8.06 ↓*</td>
<td>-7.53% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>-1.04%</td>
<td>-5.66% ↓*</td>
<td>-2.35% ↓*</td>
<td>6.63 ↓*</td>
<td>-6.74% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>-1.12%</td>
<td>-5.21% ↓*</td>
<td>-2.01% ↓*</td>
<td>5.42 ↓*</td>
<td>-5.97% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>-1.22%</td>
<td>-4.80% ↓*</td>
<td>-1.67% ↓*</td>
<td>4.42 ↓*</td>
<td>-5.24% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-1.32%</td>
<td>-4.45%</td>
<td>-1.32% ↓*</td>
<td>3.63</td>
<td>-4.53% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>-1.41%</td>
<td>-4.21%</td>
<td>-0.98% ↓*</td>
<td>3.07</td>
<td>-3.87% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>-1.50%</td>
<td>-4.14%</td>
<td>-0.64% ↓*</td>
<td>2.71</td>
<td>-3.27% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>-1.60%</td>
<td>-4.24%</td>
<td>-0.30% ↓*</td>
<td>2.57</td>
<td>-2.76% ↓*</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>-1.67%</td>
<td>-4.48%</td>
<td>-0.04% ↓*</td>
<td>2.65</td>
<td>-2.43% ↓*</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>-1.78%</td>
<td>-4.84% ↑*</td>
<td>0.38%</td>
<td>2.95 ↑*</td>
<td>-2.56% ↑*</td>
<td></td>
</tr>
</tbody>
</table>

This table reports mean returns, variances, and the Value-at-Risk (VaR) on jump days for factor portfolios. The factor portfolios are daily re-balanced and include $(1 - w_2)$ share of the market return and $w_2$ share of the momentum return (left columns) or value return (right columns). The entries are estimated based on the parameters from Table 5 (second column) for the momentum return and Table 6 (second column) for the value return. Frames indicate entries that are statistically equal at 5% significance level. Arrows with asterisks indicate directions in which the changes in entries are statistically significant.
Table 2.9: Factor Portfolios: Risk during Extreme Volatility Periods

<table>
<thead>
<tr>
<th>$w_2$</th>
<th>MKT-UMD</th>
<th>MKT-HML</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excess Vol (initial)</td>
<td>Excess Vol (after month)</td>
</tr>
<tr>
<td>0.0</td>
<td>0.88 ↓*</td>
<td>0.39 ↓*</td>
</tr>
<tr>
<td>0.1</td>
<td>0.71 ↓*</td>
<td>0.32 ↓*</td>
</tr>
<tr>
<td>0.2</td>
<td>0.57 ↓*</td>
<td>0.26 ↓*</td>
</tr>
<tr>
<td>0.3</td>
<td>0.46 ↓*</td>
<td>0.21 ↓*</td>
</tr>
<tr>
<td>0.4</td>
<td>0.37 ↓*</td>
<td>0.17 ↓*</td>
</tr>
<tr>
<td>0.5</td>
<td>0.30 ↓*</td>
<td>0.15 ↓*</td>
</tr>
<tr>
<td>0.6</td>
<td>0.25 ↓*</td>
<td>0.14</td>
</tr>
<tr>
<td>0.7</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td>0.8</td>
<td>0.24</td>
<td>0.15 ↑*</td>
</tr>
<tr>
<td>0.9</td>
<td>0.26 ↑*</td>
<td>0.18 ↑*</td>
</tr>
<tr>
<td>1.0</td>
<td>0.31 ↑*</td>
<td>0.21 ↑*</td>
</tr>
</tbody>
</table>

This table reports average increases in volatilities (initial excess volatility) on jump days and the residual effect of these changes for factor portfolios. The factor portfolios are daily re-balanced and include $(1-w_2)$ share of the market return and $w_2$ share of the momentum return (left columns) or value return (right columns). The entries are estimated based on the parameters from Table 5 (second column) for the momentum return and Table 6 (second column) for the value return. Frames indicate entries that are statistically equal at 5% significance level. Arrows with asterisks indicate directions in which the changes in entries are statistically significant.
Figure 2.1: Daily returns
Figure 2.2: Daily spot volatilities and volatility jumps
Figure 2.3: Estimated jump probabilities

- Market
- HML
- Momentum
Figure 2.4: Jump probabilities and Z-statistics
Figure 2.5: Co-jumps

Crack in .com bubble: Jan 2000

End of .com bubble: Apr 2000

Mortgage crisis spill-over: 2007

"Bad bank" creation: Sep 2000

Volatile recovery: Aug 2011

Obama re-election: Nov 2012

Return, %

Dec-1 Jan-4 Jan-31

Mar-15 Apr-14 May-15

Jul-2 Aug-6 Aug-31

Aug-15 Sep-18 Oct-15

Jul-1 Aug-2 Aug-31

Oct-1 Nov-7 Nov-30

MKT --- UMD

MKT --- HML
Chapter 3

The Time Varying Effects of Algorithmic Trading on Market Liquidity

3.1 Introduction

An issue of increasing debate, both academically and politically, is the impact of algorithmic trading (AT) on standard measures of market quality such as liquidity and volatility. Proponents, including many of the exchanges themselves, argue that AT provides added liquidity to markets and is beneficial to investors. Opponents instead caution that AT increases an investor’s perception that an AT partner possesses an informational advantage and thus may undermine investor’s belief that markets are in fact “fair”. Additionally, there are concerns that the benefits from AT are transient. That is, AT provides “phantom” liquidity that may disappear at a moment’s notice. Incidents such as the ”flash crash” of May, 2010, although circumstantial in nature, do nothing to alleviate these fears.

From a regulatory standpoint AT has received a plethora of attention, both in US markets and abroad. A number of initiatives have been put forth by regulatory agencies. In 2012 the SEC\(^1\) put forth rule 613 (“The Consolidated Audit Trail”), which requires exchanges to essentially track the footprint of every order put into the system. Similarly, the “Large Trader Reporting Rule”, put forth in 2011, imposes certain reporting requirements on large traders in order for the SEC to monitor their

\(^1\)The US Securities and Exchange Commission.
trading patterns. Similar legislation has been proposed in the European markets in the context of MiFiD II.\textsuperscript{2} An extensive discussion is presented Shorter and Miller (2014).

Given the intense scrutiny placed on AT, along with the uncertain effects, thorough empirical analysis is required. Recent work examining the effects of AT on market quality have generally found its presence to be beneficial in the sense that standard measures of liquidity such as bid-ask spreads and price-impact are reduced as a consequence of the increase in AT. For example Hendershott et al. (2011) find that, with the exception of the smallest quintile of NYSE stocks, AT almost universally reduces quoted and effective spreads in the remaining quintiles. Hasbrouck and Saar (2013) find similarly compelling evidence using a measure of AT constructed from order level data. A drawback of both approaches and more specifically of the standard panel regression approach is that estimates of the marginal effects of AT on spreads are necessarily averaged over all possible states of the market. This is problematic from an asset pricing perspective.

Of particular importance to the concept of liquidity is the timing of its provision. The merits of added liquidity during stable market periods at the expense of its drawback during periods of higher uncertainty are ambiguous without a valid welfare analysis and can potentially leave investors worse off. The issue of timing is particularly important for empirical work examining the effects of AT on market quality. Samples are often constrained in size due to limitations on the availability of data and computational concerns. As noted by Hendershott et al. (2011), it may be because samples often used do not cover large enough periods of market turbulence that detection of possible negative effects of AT on market quality have not been empirically docu-

\textsuperscript{2}The Markets in Financial Instruments Directive.
mented. Additionally, standard sub-sample analysis requires the econometrician to
diagnose market conditions as well their start and end dates, in effect imposing their
own prior beliefs on the factors that might cause variation in the marginal effects.

Because of the above, I propose the use of a new estimator to automatically detect
jumps in slope parameters. Specifically, in the context of a two-way panel model, I
provide estimates of the marginal effects of AT on various liquidity measures while
allowing those effects to change at unknown times over the sample period. The
estimation procedure automatically detects the unknown breakpoints and provides
point estimates of marginal effects at all breakpoints. This methodology alleviates
concerns about ad-hoc sub-sample selection. It also provides key insight into the
dynamics between AT and liquidity through the analysis of periods where the effects
vary. Furthermore, a more careful evaluation of the breakpoints may provide valuable
insight for future studies (both theoretical and empirical) and policy recommendations
regarding the regulation of trading in financial markets.

The chapter proceeds as follows: Section 3.2 gives a brief exposition on how time
varying liquidity effects the cost of capital. Section 3.3 introduces the estimation
procedure used to allow for multiple, unknown structural breaks. Section 3.4 discusses
our measures of market quality, our proxy for algorithmic trading and our control
variables. Section 3.5 presents the specific model considered and discusses a potential
endogeneity issue and our choice of instrument. Section 3.6 presents the results and
Section 3.7 concludes.
3.2 Liquidity and the Cost of Capital

A general result in asset pricing that is a consequence of no arbitrage is that there exists a strictly positive stochastic discount factor (SDF) such that,

\[ 1 = E_t(M_{t+1}R_{t+1}), \]

where \( M_{t+1} \) is the SDF and \( R_{t+1} \) is the return on a security. This expression can be expanded and rewritten as,

\[ E_t(R_{t+1}) = \frac{1}{E_t(M_{t+1})} - \frac{1}{E_t(M_{t+1})} \text{cov}_t(M_{t+1}, R_{t+1}). \]

Expected excess returns to an asset (i.e., its risk premium) are a function of covariance with the SDF. While the form of the SDF depends on the asset pricing model one is considering, it can in general be thought of as the ratio of the marginal value of wealth between time \( t + 1 \) and \( t \). Therefore, holding the expectation of \( M_{t+1} \) constant, if an asset pays off more in states in which the marginal value of wealth is relatively low and less in states where the marginal value of wealth is relatively high (\( \text{cov}_t(M_{t+1}, R_{t+1}) < 0 \)) then a rational investor would discount the price of that asset more heavily (generating higher expected returns). Thus, if an asset’s return contain a stochastic liquidity component then its covariance with the SDF can have a substantial impact on pricing.

The model of Acharya and Pedersen (2005) is particularly relevant as it exemplifies the many avenues through which time varying liquidity can affect expected returns. Using an overlapping generations model they decompose conditional security returns into five components: one related to the expected level of illiquidity and four others related to terms involving the covariances between market return, market illiquidity, asset returns and asset illiquidity. They show that asset returns are increasing in the
covariance between portfolio illiquidity and market illiquidity and decreasing in the
covariance between asset illiquidity and the market return. A consequence of this
is that if AT intensifies these liquidity dynamics for a particular security then the
effect will be to increase the risk premium associated with that security. Increased
risk premiums represent higher costs of capital for firms and thus increased AT can
potentially decrease firm investment (relative to a market with no AT) through its
effect on liquidity dynamics.

3.3 A panel data model with multiple unknown structural
breaks

This section introduces and discusses the estimator proposed in Bada et al. (2015),
to which the reader is referred for a thorough presentation, proofs of consistency and
monte carlo analysis under a variety of data generating processes. I first motivate the
model. I then introduce a simplified version for ease of exposition. The more general
model is covered in detail Bada et al. (2015).

3.3.1 Motivation

A standard two-way panel data model takes the form,

$$ y_{it} = \mu + \sum_{p=1}^{P} X_{it,p} \beta_p + \alpha_i + \theta_t + \epsilon_{it} $$

where $\mu$ is an intercept term, $\alpha_i$ some individual (or firm, country, etc.) level fixed
effect and $\theta_t$ some time period fixed effect. A number of methods for estimating such
a model under a variety of assumptions concerning the distribution of the error term,
$\epsilon_{it}$, have been developed in the literature. A key assumption in the above model is
the stability of the marginal effects ($\beta_p$) over time. If $\beta_p$ is not constant over time
then estimation of the above model produces only an estimate of the average effect. One can easily think of examples where the average effect is not nearly as important as the conditional effect. This analysis considers one such case.

After acknowledging the possible instability of marginal effects a difficult question is how to account for and estimate them. In a general form, one can think of each parameter \((\beta_p)\) as jumping at various points in time \((S_p)\) such that it is piece-wise constant over the time intervals defined by, \(\{1 = \tau_0, ..., \tau_{S_p+1} = T\} \subseteq \{1, ..., T\}\). Under these assumptions the general model can be written as,

\[
y_{it} = \mu + \sum_{p=1}^{P} \sum_{j=1}^{S_p+1} X_{it,p} \beta_{pt} I(\tau_{j-1,p} < t \leq \tau_{j,p}) + \alpha_i + \theta_t + \epsilon_{it}, \tag{3.2}
\]

where \(I(\tau_{j-1,p} < t \leq \tau_{j,p})\) is an indicator function equal to 1 if time \(t\) is in the interval and 0 otherwise. Obviously if one knew the exact breakpoints estimation is quite simple. However, that is almost never the case. Instead one is interested in estimating, along with the parameters of the model, the points of each structural break. To do so, I use the estimator of Bada et al. (2015), which consists of expressing \(\beta_{pt}\) in terms of its Haar wavelet expansion.

### 3.3.2 Univariate example

To present the estimator, the case of a univariate centered panel data model provides a convenient mechanism. Specifically, assume,

\[
Y_{it} = X_{it} \beta_t + e_{it} \quad \text{for} \quad i \in \{1, \ldots, n\} \quad \text{and} \quad t \in \{1, \ldots, T\}. \tag{3.3}
\]

The coefficient, \(\beta_t\), is allowed to jump at \(S\) unknown points in time. As a result, it is piece-wise constant over the intervals defined by \(\tau_0 = 1 < \ldots < \tau_{S+1} = T\) and can be
written generally as,

$$\beta_t = \sum_{j=1}^{s+1} I(\tau_{j-1} < t \leq \tau_j) \beta_{\tau_j}, \quad t = 1, \ldots, T$$  \hfill (3.4)

where $I(\tau_{j-1} < t < \tau_j)$ is an indicator variable that takes the value 1 if $t$ is within the interval and 0 elsewhere.

As mentioned above, the estimator relies on expressing the coefficient $\beta_t$ in terms of its Haar wavelet expansion. Since wavelet functions are constructed as dyadic dilations of order $2^l$, we assume that the length of the sample in the time dimension is dyadic as well (i.e., $T = 2^{L-1}$ for some $L \geq 2$).\(^3\) The Haar wavelet expansion of $\beta_t$ is defined by a two parameter system: (i) the dilation level ($l \in \{1, \ldots, L\}$) and (ii) the translation index ($k \leq 2^{l-2}$). These parameters define a collection of scaling ($\phi_{l_0,k}, k = 1, \ldots, K_{l_0}$) and wavelet ($\psi_{l,k}, l = l_0 + 1, \ldots, L; k = 1, \ldots, 2^{l-2}$) functions on the interval $\{1, \ldots, 2^{L-1}\}$.

$$\varphi_{l_0,k}(t) = a_{l_0}^\varphi I_{l+1,2k-1}(t) + a_{l_0}^\varphi I_{l+1,2k}(t),$$ \hfill (3.5)

$$\psi_{l,k}(t) = a_l^\psi I_{l,2k-1}(t) - a_l^\psi I_{l,2k}(t),$$ \hfill (3.6)

where $a_{l_0}^\varphi = \sqrt{2^{l_0-1}}$ and $a_l^\psi = \sqrt{2^{l-2}}$. The function $I_{l,m}(t)$ is an indicator function that is equal to 1 if the $t \in \{2^{l-1}(m-1) + 1, \ldots, 2^{l-1}m\}$ and 0 elsewhere. Thus, the coefficient $\beta_t$ can be expressed as,

$$\beta_t = \sum_{k=1}^{K_{l_0}} \phi_{l_0,k}(t)d_{l_0,k} + \sum_{l=l_0+1}^{L} \sum_{k=1}^{K_l} \psi_{l,k}(t)c_{l,k},$$ \hfill (3.7)

\(^3\)The restriction that the time dimension is dyadic may seem strong, however, one can always split the sample into subsets such that each satisfies this criteria. Moreover, the estimation procedure presented here is used to identify the break points. Once the breakpoints are known, we can estimate
where $K_l = 2^{l-2}$ if $l > 1$ and $K_1 = 1$. $d_{l_0,k}$ and $c_{l,k}$ are scaling and wavelet coefficients respectively. Thus the problem of estimating the unknown breakpoints and value of the parameter over the various intervals can be transformed into a problem of solving for the scaling and wavelet coefficients.

The collection of functions in (3.7) are not unique. This can be overcome by fixing $l_0 = 1$, which implies we have only one scaling coefficient, $d_{1,1}$, that represents the overall mean of the parameter $\beta_t$. Furthermore, in order to facilitate estimation, the Haar wavelet expansion in (3.7) is modified slightly,

$$\beta_t = \sum_{l=1}^{L} \sum_{k=1}^{K_l} w_{l,k}(t) b_{l,k}$$

(3.8)

where,

$$w_{l,k} = \begin{cases} a_{1,1} = a_{2,1} h_{2,1}(t) + a_{2,2} h_{2,2}(t) & \text{if } l = 1, \\
 a_{l,2k-1} h_{l,2k-1}(t) + a_{l,2k} h_{l,2k}(t) & \text{if } l > 1 \end{cases}$$

The function $h_{l,m}(t) = \sqrt{2^{l-2}} I_{l,m}(t)$, where $I_{l,m}(t)$ is defined as above. The exact form of the scaling constants, $a_{l,m}$, which are functions of the data and thus known, is provided in Bada et al. (2015). The key insight from the modification in 3.7 is that it guarantees that at most $(S + 1)L$ wavelet coefficients $(b_{l,k})$ are non-zero.\(^4\)

Given the Haar wavelet expansion of $\beta_t$, the model in (3.3) can be re-written as,

$$y_{i,t} = \sum_{l=1}^{L} \sum_{k=1}^{K_l} \mathcal{X}_{l,k,it} b_{l,k} + \epsilon_{it}$$

(3.9)

$$\mathcal{X}_{l,k,it} = X_{it} w_{l,k}(t).$$

Equivalently (3.9) can be expressed in vector notation as,

$$y_{it} = \mathcal{X}_{it}^\prime b + \epsilon_{it},$$

(3.10)

the parameter over each piece-wise interval using a simple post-estimation procedure for which there are no restrictions on the size of $T$.

\(^4\)See proposition 1 of Bada et al. (2015).
where $X_{it} = (X_{1,1,it}, ..., X_{L,K_{L,it}})'$ and $b = (b_{1,1}, ..., b_{L,K_L})'$. (3.10) is recognizable in that it is of the same form as a standard multiple linear regression. As such, an initial estimate of $b$ can be obtained by minimizing the sum of squares,

$$
\tilde{b} = \min_b \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - X_{it}'b)'(y_{it} - X_{it}'b) \quad (3.11)
$$

Moreover, Bada et al. (2015) prove the consistency of the estimator in the case when $X_{it}$ is endogenous. That is, supposing the econometrician has a valid instrument, $Z_{it}$, an instrumental variable (IV) estimator of $b$ can be obtained as,

$$
\tilde{b} = \min_b \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - X_{it}'b)'Z_{it}'Z_{it}(y_{it} - X_{it}'b) \quad (3.12)
$$

where $Z_{it}$ is defined analogously. With this initial estimate of the wavelet coefficients, an estimate of the coefficient of interest (the structure adapted wavelet or SAW estimator) can be constructed by applying a thresholding scheme to the wavelet coefficients in the expansion of $\beta_t$,

$$
\hat{\beta}_t = \sum_{l=1}^{L} \sum_{k=1}^{K_l} w_{l,k}(t)\tilde{b}_{l,k}I(\tilde{b}_{l,k} > \lambda_{N,T}), \quad (3.13)
$$

where $I(\tilde{b}_{l,k} > \lambda_{N,T})$ is an indicator equal to 1 if $\tilde{b}_{l,k}$ is greater than some threshold, $\lambda_{N,T}$, that depends on $N$ and $T$.\footnote{Z_{it} correlated with X_{it} and E[Z_{it} \epsilon_{it}].} While the above estimator is shown to be consistent for $\beta_t$, efficiency gains can be achieved by applying a post-SAW procedure. That is, we treat the detected jump times as the true jump times, and re-estimate the model conditional on those jump times. The efficiency gains come from a reduction in the number of parameters that need to be estimated.\footnote{The universal thresholding scheme of Donoho and Johnstone (1994) is one such example.}
3.4 Data and variables

Our sample consists of a balanced panel of stocks whose primary exchange is the New York Stock Exchange (NYSE) and covers the calendar period 2003 – 2008. To build measures of market quality, we use the NYSE Trade and Quotation Database (TAQ) provided by Wharton Research Data Services (WRDS) to collect intra-day data on securities. We construct various daily liquidity measures at the daily level and then average those over the course of the month to construct our sample. The full sample consists of 378 firms over 71 months for each firm.\(^7\) We merge the TAQ data with information on price and shares outstanding from the Center for Research in Security Prices (CRSP). The choice of this sample period reflects our desire to include both relatively stable and turbulent market regimes. We are limited in our choice of sample periods by the fact that AT is a recent phenomenon and that our estimation procedure requires a balanced panel. We additionally filter out a number of firms due to infrequent intra-day data.

3.4.1 The Algorithmic Trading Proxy

Our AT proxy is motivated by Hendershott et al. (2011) and Boehmer et al. (2014), who note that AT is generally associated with an increase in order activity at smaller dollar volumes. Thus the proxy we consider is the negative of dollar volume (in hundreds of dollars, \(\$\text{Vol}_t\)) over time period \(t\) divided by total order activity over

\(^7\)One month is lost due to the use of lagged market quality measures in the regression specification. Furthermore, due to the sample size being very close to a dyadic integer we utilize only the final 64 months when applying the SAW estimator to our data set. As is abundantly clear in the analysis, all evidence of structural breaks occurs toward the end of the sample and thus we view this a reasonable decision.
time period $t$. We define order activity as the sum of trades ($\text{Tr}_{it}$) and updates to the best prevailing bid and offer ($q_{it}$) on the security’s primary exchange:

$$\text{AT}_{it} = -\frac{\$\text{Vol}_{it}}{\text{Tr}_{it} + q_{it}}.$$  

An increase in $\text{AT}_{it}$ represents a decrease in the average dollar volume per instance of order activity and represents an increase in the AT in the particular security. For example, an increase of 1 unit of $\text{AT}_{it}$ represents a decrease of $100 of trading volume associated with each instance of order activity (trade or quote update).

Our proxy, like that in Boehmer et al. (2014), differs from the proxy in Hendershott et al. (2011) since the latter have access to the full order book of market makers whereas we only have access to the trades and the best prevailing bid and offers of market makers through TAQ. We appeal to the same argument as Boehmer et al. (2014) in that many AT strategies are generally executed at the best bid and offer rather than behind it. Therefore, we feel our proxy is in general representative of the full order book.

### 3.4.2 Market Quality Measures

We consider several common measures of market quality to assess the impact of AT on markets for individual securities.

#### Proportional Quoted Spread

The proportional quoted spread ($\text{PQS}_{it}$) measures the quoted cost as a percentage of price (Bid-Offer midpoint) of executing a trade in security $i$ and is defined as,

$$\text{PQS}_{it} = 100 \left( \frac{\text{Ofr}_{it} - \text{Bid}_{it}}{0.5(\text{Ofr}_{it} + \text{Bid}_{it})} \right).$$
We multiply by 100 in order to place this metric in terms of percentage points. We aggregate this metric to a monthly quantity by computing a share volume-weighted average over the course of each month. An increase in PQt represents a decrease in the amount of liquidity in the market for security i due to increased execution costs.

**Proportional Effective Spread**

The proportional effective spread (PESit) is quite similar to (PQSit) but accommodates potentially hidden liquidity or stale quotes by evaluating the actual execution costs of a trade. It is defined as,

\[
PES_{it} = 100 \left( \frac{|P_{it} - M_{it}|}{M_{it}} \right),
\]

where \( P_{it} \) is the price paid for security i at time t and \( M_{it} \) is the midpoint of the prevailing bid and ask quotes for security i at time t. Thus, PESit is the actual execution cost associated with every trade. We again aggregate this measure up to a monthly quantity in the same way as we do for quoted spreads. Like PQt, PESit is also in terms of percentage points. An increase in PESit represents a decrease in the amount of liquidity in the market for security i due to increased execution costs.

**Measures of Volatility**

We also consider two different measures of price volatility in security i over time period t. The first is the daily high-low price range given by,

\[
H-\text{L}_{it} = 100 \left( \frac{\max_{\tau \in t}(P_{it}) - \min_{\tau \in t}(P_{it})}{P_{it}} \right),
\]

which represents the extreme price disparity over the course of a trading day. We also consider the realized variance of returns over each day computed using log percentage
(i.e. \( \ln(p_{i,t}/p_{i,t-1}) \times 100 \)) returns over 5-minute intervals:

\[
RV_{it} = \left( \sum_{\tau \in t} r_{i\tau}^2 \right) .
\]

Realized variance is a nonparametric estimator of the integrated variance over the course of a trading day (see, for example, Barndorff-Nielsen (2002)). We aggregate both measures up to a monthly level by averaging over the entire month. We additionally represent each in terms of percentages, i.e. \( H - L_{it} \) is the price range as a percentage of the daily closing price and \( RV_{it} \) is an estimate of the integrated variance of log returns in percentages. Both measures represent a measure of the price dispersion over the course of the trading month.

### 3.4.3 Additional Control Variables

While we attempt to determine the effect our AT proxy has on measures of market quality we include in all our regressions a vector of control variables to isolate the effects of AT independent of the state of the market. We lag the control variables by one month so they represent the state of the market at the beginning of the trading month in question. The control variables are: (1) Share Turnover (\( ST_{it} \)), which is the number of shares traded over the course of a day in a particular stock relative to the total amount of shares outstanding; (2) Inverse price, which represents transaction costs due to the fact that the minimum tick size is 1 cent; (3) Log of market value of equity to accommodate effects associated with smaller securities; (4) Daily price range to accommodate any effects from large price swings in the previous month. To avoid adding lagged dependent variables in the model, for regressions where the daily price range is the dependent variable we replace it in the vector of controls with the previous month’s realized variance. We additionally include security and time period
fixed effects to proxy for any time period or security related effects not captured by our included variables.

### 3.5 Specific application and instrumentation

We assume a linear relationship between our measures of market quality, our proxy for algorithmic trading and our control variables,

\[
MQ_{it} = \mu + \alpha_i + \gamma_t + AT_{it} \beta_t + X'_{it}\delta + \epsilon_{it}. \tag{3.14}
\]

The key distinction between the model considered here and others in the literature is that we allow the marginal effect AT has on market quality to be time varying. Ideally one could apply this model directly and, given our limited knowledge of AT strategies, this may in fact be the case. However, absent a theoretical model of AT, an issue on which the literature is still somewhat agnostic, it is uncertain whether AT strategies attempt to time shocks to market quality. This creates a potential problem of endogeneity with our AT proxy. That is, when estimating the regression equation (3.14) our estimates may be biased \(E(AT_{it}\epsilon_{it}) \neq 0\) and inconsistent.

To overcome this potential issue we use the approach of Hasbrouck and Saar (2013) (albeit with different variables) and choose as an instrument the average value of algorithmic trading over all other firms not in the same industry as firm \(i\). To this end, we define industry groups using 4-digit SIC codes and define these new variables \(AT_{-IND, it}\). The use of this IV assumes that there is some commonality in the level of AT across all stocks that is sufficient to pick up some exogenous variation. It further rules out trading strategies by ATs across firms in different industry groups. Lacking much knowledge of the algorithms used by AT firms we view this assumption to be reasonable. As noted by Hasbrouck and Saar (2013), it is unlikely that AT firms
implement cross-stock trading strategies for a particular firm with the entire universe of firms (or in our case the other 377). To the extent that AT firms do implement these cross-stock strategies across industries, their effect on the average is likely to be marginal.

To estimate the model we use a two-stage approach and first fit the regression model,

\[ AT_{it} = a_i + g_t + bAT_{-IND,it} + dW_{it} + \epsilon_{it} \]  \hspace{1cm} (3.15)

to obtain an instrument, \( Z_{it} \), for \( AT_{it} \) given by the fitted values from (3.15), i.e.,

\[ Z_{it} := \hat{AT}_{it} = \hat{a}_i + \hat{g}_t + \hat{b}AT_{-IND,it} + \hat{d}W_{it}, \]

where \( \hat{a}_i, \hat{g}_t, \hat{b}, \) and \( \hat{d} \) are the conventional estimates of \( a_i, g_t, b, \) and \( d \). We then carry out the second stage regression using equation (3.14) using the estimator discussed in Section 3.3. For comparison purposes, we additionally apply the conventional panel data model assuming a constant slope parameter, i.e., \( \beta_1 = \beta_2 = \cdots = \beta_T \).

### 3.6 Empirical results

Table 3.1 presents the results from a baseline model that assumes the slope parameters are constant over time.\(^8\) These results are largely consistent with previous studies that find a positive (in terms of welfare) average relationship between AT and measures of market quality over the time period considered. The coefficient estimates on the AT proxy are negative and significant for all four measures of market quality that we consider. That is, increases in AT generally reduce both of the spread measures and both of the variance measures we consider. As far as the direction of the effect,

\[^8\text{For the purpose of readability we divide the AT variable by 100 to reduce trailing zeros after the decimal.}\]
differences in our proxies and choice of instruments do not seem to reach different conclusions than the prior literature.

To gauge the size of this effect we note that the within-standard deviation of our AT proxy, after being scaled by 100, is 0.18. Combining this with the coefficient estimates from Table 3.1 implies that a one standard deviation increase in AT results in quoted spreads (effective spreads) being lowered by approximately 0.002% (0.001%). On an absolute level these effects are small. For example, given a hypothetical stock with an initial price of $100, a one standard deviation increase in AT would reduce the quoted spread by less than a penny.\(^9\) These results differ from those in Hendershott et al. (2011). We attribute this to a combination of the differences in our AT proxies and instrument as well as our inclusion of a more recent sample period. One possible explanation is that the initial increase in AT during its inception has been far larger in terms of effects than subsequent increases. For the variance measures, a one standard deviation increase in our AT proxy results in a decrease in the proportional daily high-low spread of approximately 0.25% and a decrease in realized variance associated with percentage log returns of approximately 0.12 (or equivalently a reduction in realized daily volatility of approximately 0.35%).

From a welfare perspective the magnitude of the effect is important. As mentioned above and further investigated below, if AT amplifies variation in liquidity this is likely to demand a premium from investors and increase the cost of a capital for firms using markets in which AT is present. Because of this, any benefits in terms of increased liquidity on average, needs to be evaluated against the costs associated with increased variation.

The coefficients on the control variables are also, generally, in line with what we

\(^9\)It should be noted that this is technically impossible.
would expect. The log of market equity is negatively related to both spreads and the high-low price range variable. This is expected given that smaller firms typically have a smaller group of potential investors and are likely less liquid. It is also in line with previous results. We do note that we find a positive and statistically significant relationship between market equity and realized variance. However, variance is typically found to be highly persistent. Although ideally we would include the lag of realized variance as a regressor, as mentioned above, we attempt to avoid a dynamic panel setting. We proxy for this using the lagged value of the price range variable, but we note that this may not fully compensate for the exclusion. The rest of the results for the control variables fit with prior research and theory. Higher share turnover, smaller inverse prices (i.e., higher prices) and lower variance all increase liquidity.

Tables 3.2 through 3.5 present the results when we allow the parameter to jump discretely over time. The coefficient estimates in Tables 3.2 through 3.5 represent the size of the estimated jump in the coefficient and a Chow (Chow (1960)) type test of its significance. Figures 3.1 through 3.4 plot both the estimated post-SAW coefficients and the results from period by period cross-sectional regressions.

The first takeaway is that we find, unanimously, that the effect of AT on our measures of market quality is constant for the majority of the sample (the period prior to the financial crisis). This was a relatively placid period for equity markets, and the lack of time varying effects accords with our prior belief that structural breaks in the marginal effect are likely to occur during periods of turmoil. Of note is that the estimated coefficients of the marginal effect of AT on the two spread measures (PES and PQS) over this stable period are positive, however they are both extremely small in terms of economic magnitude and insignificant.

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10 See post-SAW estimation discussion in Bada et. al.
The 2007-2008 period covers the financial crisis, a time during which liquidity in many markets tightened substantially. During the financial crisis period we find significant evidence of both positive and negative jumps in the coefficient on AT. For the two spread measures we find evidence of two large positive jumps in the coefficients in April and September/October of 2008 and other smaller jumps around those two time periods. A positive jump in the coefficient represents a reduction in the benefit of AT on spreads and, potentially, a reversal in its effects on spreads. Such is the case for the two large positive jumps mentioned above. We find that during these two months increases in AT lead to an increase in spreads and thus transacting in the securities with high AT is, other things being equal, costlier than in low AT securities. April and September/October of 2008 represent two particularly volatile periods for equity markets (and markets in general) in the US. In April markets were still rebounding from the bailout of Bear Stearns and its eventual sale to JP Morgan. This all occurred during a period when the exposure of many banks to US housing markets through various structured financial products was beginning to be understood by investors. Similarly, the failure of Lehman Brothers in September was another event that rattled financial markets.

The results for our variance measures are similar, as we also find evidence of both positive and negative jumps during the 2007-2008 period. Of note is that for realized variance we find the jumps to be, in general, beneficial for investors. That is, we find that increases in AT cause a larger reduction in realized variance. As mentioned above, some caution should be taken with respect to the interpretation of these results due to the fact that variance is generally found to be strongly auto-correlated.

A potential explanation for the variation in the marginal effect of AT is the presence of increased uncertainty. From both a valuation and a regulatory/policy per-
spective, the periods following large, unpredictable shocks to asset markets can be associated with heightened uncertainty among investors. If investors fear that algorithmic traders possess an informational advantage then it would be precisely during these periods when an increase in AT would cause investors to be most at risk. Although a model of the dynamic effects of AT and uncertainty is beyond the scope of this chapter, the above results clearly point to a time varying relationship between the effects of AT on various measures of market quality.

3.7 Conclusion

This chapter investigates the relationship between algorithmic trading and four measures of market quality. We allow for the marginal effect of algorithmic trading to undergo multiple structural breaks at unknown points in time and use an estimator based on the expansion of the coefficient via the Haar wavelet to automatically detect this breakpoints. By implementing this estimator we avoid any bias introduced when an econometrician is forced to identify potential break points a priori.

We find strong evidence of structural breaks during a particularly turbulent period in the market: the financial crisis. The results suggest that the relationship between AT and market quality is possibly state contingent. We also find that the breaks are generally short lived; typically on the order of a month or two. We believe these results provide guidance for future work. Specifically, drilling down and investigating the dynamics between some measure of AT and various liquidity at finer frequencies (e.g. daily or intra-day) during the time periods in which the effects vary may shed light on the true nature of the breaks.
Table 3.1: Instrumental variable panel data model with constant parameters

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( \hat{AT}_{it} )</th>
<th>( \ln(ME)_{it-1} )</th>
<th>( T/O_{it-1} )</th>
<th>( 1/P_{it-1} )</th>
<th>H-L(_{it-1} )</th>
<th>RV(_{it-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PQS(_{it} )</td>
<td>Coef. -0.013</td>
<td>-0.003</td>
<td>0.027</td>
<td>0.619</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>( t )-value</td>
<td>-3.61</td>
<td>-2.65</td>
<td>0.73</td>
<td>15.38</td>
<td>5.67</td>
<td></td>
</tr>
<tr>
<td>PES(_{it} )</td>
<td>Coef. -0.006</td>
<td>-0.001</td>
<td>-0.077</td>
<td>0.517</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>( t )-value</td>
<td>-3.62</td>
<td>-1.83</td>
<td>-3.42</td>
<td>18.51</td>
<td>16.96</td>
<td></td>
</tr>
<tr>
<td>RV(_{it} )</td>
<td>Coef. -0.691</td>
<td>0.046</td>
<td>-1.575</td>
<td>7.25</td>
<td>0.415</td>
<td></td>
</tr>
<tr>
<td>( t )-value</td>
<td>-12.73</td>
<td>2.39</td>
<td>-2.45</td>
<td>10.19</td>
<td>44.46</td>
<td></td>
</tr>
<tr>
<td>H-L(_{it} )</td>
<td>Coef. -1.404</td>
<td>-0.038</td>
<td>-6.15</td>
<td>7.88</td>
<td>1.151</td>
<td></td>
</tr>
<tr>
<td>( t )-value</td>
<td>-11.6</td>
<td>-1.04</td>
<td>-4.71</td>
<td>6.04</td>
<td>35.78</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the results of the 2SLS panel regression of our measures of market quality on our \( AT \) proxy. The dependent variables are proportional quoted spread, proportional effective spread, daily high-low price range and daily realized variance. In addition to \( AT \), additional regressors included as control variables are the previous month’s log of market Cap (\( \ln(ME) \)), share turnover (\( T/O \)), inverse price (\( 1/P \)) and high-low price range (H-L). When the dependent variable is the current month’s high-low price range, last month’s value of realized variance (RV) is used instead to avoid a dynamic panel model. Standard errors are corrected for heteroskedasticity.
<table>
<thead>
<tr>
<th>Period</th>
<th>Coef.</th>
<th>Z-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 2003-09-01 to 2008-02-01</td>
<td>6.49e-05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>from 2008-03-01 to 2008-03-01</td>
<td>6.51e-04</td>
<td>1.650</td>
<td>0.0998</td>
</tr>
<tr>
<td>from 2008-04-01 to 2008-04-01</td>
<td>4.13e-03</td>
<td>6.420</td>
<td>1.36e-10</td>
</tr>
<tr>
<td>from 2008-05-01 to 2008-08-01</td>
<td>7.66e-04</td>
<td>-7.420</td>
<td>1.16e-13</td>
</tr>
<tr>
<td>from 2008-09-01 to 2008-10-01</td>
<td>1.03e-03</td>
<td>0.932</td>
<td>0.3510</td>
</tr>
<tr>
<td>from 2008-11-01 to 2008-12-01</td>
<td>-1.46e-04</td>
<td>-4.620</td>
<td>3.83e-06</td>
</tr>
</tbody>
</table>

Table 3.2: Post SAW estimates for PQS

This table presents the Post-SAW estimates for the parameters and the results of tests for jump significance for the coefficient of AT when the dependent variable is PQS. The column labeled estimate is the Post-SAW estimate for the parameter and the Z statistic represents a test of the significance of the change from the previous time period (set equal to 0 for the first period). All tests are asymptotic. *** denotes significance at the 0.1% level, ** denotes significance at the 1% level, * denotes significance at the 5% level and . denotes significance at the 10% level.
<table>
<thead>
<tr>
<th>Period</th>
<th>Coef.</th>
<th>Z-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 2003-09-01 to 2007-08-01</td>
<td>9.06e-06</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>from 2007-09-01 to 2007-12-01</td>
<td>6.73e-04</td>
<td>3.750</td>
<td>0.000176 ***</td>
</tr>
<tr>
<td>from 2008-01-01 to 2008-02-01</td>
<td>1.35e-04</td>
<td>-2.000</td>
<td>0.045800 *</td>
</tr>
<tr>
<td>from 2008-03-01 to 2008-03-01</td>
<td>5.31e-04</td>
<td>1.160</td>
<td>0.248000</td>
</tr>
<tr>
<td>from 2008-04-01 to 2008-04-01</td>
<td>4.19e-03</td>
<td>10.700</td>
<td>&lt; 2.2e-16 ***</td>
</tr>
<tr>
<td>from 2008-05-01 to 2008-08-01</td>
<td>4.15e-04</td>
<td>-15.600</td>
<td>&lt; 2.2e-16 ***</td>
</tr>
<tr>
<td>from 2008-09-01 to 2008-09-01</td>
<td>-1.74e-03</td>
<td>-7.540</td>
<td>4.77e-14 ***</td>
</tr>
<tr>
<td>from 2008-10-01 to 2008-10-01</td>
<td>1.79e-03</td>
<td>11.700</td>
<td>&lt; 2.2e-16 ***</td>
</tr>
<tr>
<td>from 2008-11-01 to 2008-12-01</td>
<td>3.55e-06</td>
<td>-9.610</td>
<td>&lt; 2.2e-16 ***</td>
</tr>
</tbody>
</table>

Table 3.3 : Post SAW estimates for PES

This table presents the Post-SAW estimates for the parameters and the results of tests for jump significance for the coefficient of AT when the dependent variable is PES. The column labeled estimate is the Post-SAW estimate for the parameter and the Z statistic represents a test of the significance of the change from the previous time period (set equal to 0 for the first period). All tests are asymptotic. *** denotes significance at the 0.1% level, ** denotes significance at the 1% level, * denotes significance at the 5% level and . denotes significance at the 10% level.
This table presents the Post-SAW estimates for the parameters and the results of tests for jump significance for the coefficient of $AT$ when the dependent variable is $H - L$. The column labeled estimate is the Post-SAW estimate for the parameter and the Z statistic represents a test of the significance of the change from the previous time period (set equal to 0 for the first period). All tests are asymptotic. *** denotes significance at the 0.1% level, ** denotes significance at the 1% level, * denotes significance at the 5% level and . denotes significance at the 10% level.
<table>
<thead>
<tr>
<th>Period</th>
<th>Coef.</th>
<th>Z-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 2003-09-01 to 2008-08-01</td>
<td>-0.008080</td>
<td>-14.20</td>
<td>&lt; 2.2e-16 ***</td>
</tr>
<tr>
<td>from 2008-09-01 to 2008-09-01</td>
<td>-0.063100</td>
<td>-5.09</td>
<td>3.57e-07 ***</td>
</tr>
<tr>
<td>from 2008-10-01 to 2008-10-01</td>
<td>0.000888</td>
<td>5.29</td>
<td>1.21e-07 ***</td>
</tr>
<tr>
<td>from 2008-11-01 to 2008-12-01</td>
<td>-0.007830</td>
<td>-1.26</td>
<td>0.208</td>
</tr>
</tbody>
</table>

Table 3.5: Post SAW estimates for RV.

This table presents the Post-SAW estimates for the parameters and the results of tests for jump significance for the coefficient of AT when the dependent variable is RV. The column labeled estimate is the Post-SAW estimate for the parameter and the Z statistic represents a test of the significance of the change from the previous time period (set equal to 0 for the first period). All tests are asymptotic. *** denotes significance at the 0.1% level, ** denotes significance at the 1% level, * denotes significance at the 5% level and . denotes significance at the 10% level.
Figure 3.1: Post SAW estimates of the effect of AT on PES

Pointwise effect of the algorithmic trading on PESVW

The effect of AT on the Proportional Effective Spread (PESVW) from 2003-08-01 to 2008-12-01

Results obtained using wavelet shrinkage method on a monthly data set ($T = 64$, $n = 378$)
Figure 3.2: Post SAW estimates of the effect of AT on PQS

Pointwise effect of the algorithmic trading on PQS/VW

The effect of AT on the Proportional Quoted Spread (PQSVW) from 2003-08-01 to 2008-12-01

Results obtained using wavelet shrinkage method on a monthly data set (T = 64, n = 508)

empirical threshold = 0.11
Figure 3.3: Post SAW estimates of the effect of AT on RNG

Pointwise effect of the algorithmic trading on RNG_PRC

The effect of AT on the Price Range (RNG_PRC) from 2003-06-01 to 2008-12-01

Results obtained by using wavelet shrinkage method on a monthly dataset ($T = 64$, $n = 378$)
Figure 3.4: Post SAW estimates of the effect of AT on RV

Pointwise effect of the algorithmic trading on RV

The effect of AT on the Realized Variance (RV) from 2003-06-01 to 2008-12-01

Results obtained by using wavelet shrinkage method on a monthly data set ($T = 64, n = 378$)
Bibliography


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Appendix A

Estimation of the Multivariate SVCJ model

A.1 Re-parameterization

We start by re-parameterizing the model described in section 2.5,

\[ f_{1,t} = F_{1,t}/F_{1,t-1} - 1 = \alpha_1 + \beta_{11}^f V_{1,t-1} + \beta_{12}^f V_{2,t-1} + \sqrt{V_{1,t-1}}\epsilon_{1,t} + \xi_{1,t}^f J_{1,t} \]

\[ f_{2,t} = F_{2,t}/F_{2,t-1} - 1 = \alpha_2 + \beta_{21}^f V_{1,t-1} + \beta_{22}^f V_{2,t-1} + \sqrt{V_{2,t-1}}\epsilon_{2,t} + \beta^{1,f} \epsilon_{1,t}^f J_{1,t} + \xi_{2,t}^f J_{2,t} \]

\[ V_{1,t} - 1 = \beta_{11}^v (V_{1,t-1} - 1) + \beta_{12}^v (V_{2,t-1} - 1) + \sqrt{V_{1,t-1}}\epsilon_{1,t}^v + \xi_{1,t}^v J_{1,t} \]

\[ V_{2,t} - 1 = \beta_{21}^v (V_{1,t-1} - 1) + \beta_{22}^v (V_{2,t-1} - 1) + \sqrt{V_{2,t-1}}\epsilon_{2,t}^v + \beta^{1,v} \epsilon_{1,t}^v J_{1,t} + \xi_{2,t}^v J_{2,t} \]

In order to facilitate block sampling of the covariance matrix of fundamental shocks, \((\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{1,t}^v, \epsilon_{2,t}^v)\), we remove the restriction that each has a unit variance. If we were to additionally estimate a long-run continuous mean for each volatility we would suffer from an identification problem. As such, we fix the long-run continuous mean of volatilities to be 1. We note that this is simply a re-parameterization as the new long-run mean can be easily recovered as the variance of the transformed shock. Additionally, the volatility of volatility parameters can also be recovered as the variance of their respective transformed shocks. The covariance matrix of the transformed shocks is written as \(\Omega\). We also replace the \(\kappa_{i,j}\) parameters with \(\beta_{i,j}^v\), where \(\beta_{i,j}^v = 1 - \kappa_{i,j}\) for \(i = j\) and \(\beta_{i,j}^v = -\kappa_{i,j}\) for \(i \neq j\). This allows for estimation in the more standard vector autoregression implementation.
A.2 Notation

Before proceeding we first define some notation that will be used throughout. We use the notation $X^T = (X_1, ..., X_T)$ and $X^{T\setminus t} = (X_1, ..., X_{t-1}, X_{t+1}, ..., X_T)$. We further collect the returns, volatilities, jumps and jump sizes into vectors, $f_t = (f_{1,t}, f_{2,t})'$, $V_t = (V_{1,t}, V_{2,t})'$, $J_t = (J_{1,t}, J_{2,t})'$, $\xi^f_t = (\xi^f_{1,t}, \xi^f_{2,t})'$ and $\xi^v_t = (\xi^v_{1,t}, \xi^v_{2,t})'$. We will also write the vector of jumps for a particular factor as $\xi_{i,t} = (\xi^f_{i,t}, \xi^v_{i,t})'$. We further write the collection of all the parameters and latent states as $\Theta$ and we use $\Theta_p$ to refer strictly to the parameters.

In addition to the above, the following notation is used in order to save space when deriving the full conditional distributions,

$$Y^{(1)}_t \equiv \begin{bmatrix} f_{1,t} - \xi^f_{1,t}J_{1,t} \\ f_{2,t} - \beta^{j,f}\xi^f_{1,t}J_{1,t} - \xi^f_{2,t}J_{2,t} \\ V_{1,t} - 1 - \xi^f_{1,t}J_{1,t} \\ V_{2,t} - 1 - \beta^{j,v}\xi^v_{1,t}J_{1,t} - \xi^v_{2,t}J_{2,t} \end{bmatrix}$$

$$Y^{(2)}_t \equiv \begin{bmatrix} f_{2,t} - \alpha_2 - \beta^{f}_{21}V_{1,t} - \beta^{f}_{22}V_{2,t} - \xi^f_{2,t}J_{2,t} - \sqrt{V_{2,t-1}\mu_{\epsilon_{2,t}|\epsilon_{1,t},\epsilon^v_{1,t}}} \\ V_{2,t} - 1 - \beta^{v}_{21}(V_{1,t} - 1) - \beta^{v}_{22}(V_{2,t} - 1) - \xi^v_{2,t}J_{2,t} - \sqrt{V_{2,t-1}\mu_{\epsilon^v_{2,t}|\epsilon_{1,t},\epsilon^v_{1,t}}} \end{bmatrix}$$

where $\mu_{\epsilon_{2,t}|\epsilon_{1,t},\epsilon^v_{1,t}}$ denotes the mean of the conditional distribution of the fundamental shock, conditional on $\epsilon_{1,t}$ and $\epsilon^v_{1,t}$, and likewise for $\epsilon^v_{2,t}$. Furthermore, $\Omega_{\epsilon_{2,t}\epsilon_{1,t}}$ denotes the conditional covariance matrix of shocks $\epsilon_{2,t}$ and $\epsilon^v_{2,t}$ conditional on $\epsilon_{1,t}$ and $\epsilon^v_{1,t}$.

$$Y^{(3)}_t \equiv \begin{bmatrix} f_{2,t} - \alpha_2 - \beta^{f}_{21}V_{1,t} - \beta^{f}_{22}V_{2,t} - \beta^{j,f}\xi^f_{1,t}J_{1,t} - \sqrt{V_{2,t-1}\mu_{\epsilon_{2,t}|\epsilon_{1,t},\epsilon^v_{1,t}}} \\ V_{2,t} - 1 - \beta^{v}_{21}(V_{1,t} - 1) - \beta^{v}_{22}(V_{2,t} - 1) - \beta^{v,j}\xi^v_{1,t}J_{1,t} - \sqrt{V_{2,t-1}\mu_{\epsilon^v_{2,t}|\epsilon_{1,t},\epsilon^v_{1,t}}} \end{bmatrix}$$
\[ Y_t^{(4)} \equiv \begin{bmatrix} f_{1,t} - \alpha_1 - \beta_{11} f_{1,t} - \beta_{12} V_{2,t} \\ f_{2,t} - \alpha_2 - \beta_{21} f_{1,t} - \beta_{22} V_{2,t} - \xi_{2,t} J_{2,t} \\ V_{1,t} - 1 - \beta_{11} (V_{1,t} - 1) - \beta_{12} (V_{2,t} - 1) \\ V_{2,t} - 1 - \beta_{21} (V_{1,t} - 1) - \beta_{22} (V_{2,t} - 1) - \xi_{2,t} J_{2,t} \end{bmatrix} \]

\[ Y_t^{(5)} \equiv \begin{bmatrix} f_{1,t} - \alpha_1 - \beta_{11} f_{1,t} - \beta_{12} V_{2,t} - \xi_{1,t} J_{1,t} \\ f_{2,t} - \alpha_2 - \beta_{21} f_{1,t} - \beta_{22} V_{2,t} - \beta_{22} f_{1,t} J_{1,t} \\ V_{1,t} - 1 - \beta_{11} (V_{1,t} - 1) - \beta_{12} (V_{2,t} - 1) - \xi_{1,t} J_{1,t} \\ V_{2,t} - 1 - \beta_{21} (V_{1,t} - 1) - \beta_{22} (V_{2,t} - 1) - \beta_{22} J_{1,t} \end{bmatrix} \]

\[ W_t^{(1)'} \equiv \begin{bmatrix} 1 & V_{1,t} & V_{2,t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & V_{1,t} & V_{2,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & V_{1,t} - 1 & V_{2,t} - 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{1,t} - 1 & V_{2,t} - 1 \end{bmatrix}. \]

\[ W_t^{(2)} \equiv \begin{bmatrix} \xi_{1,t} J_{1,t} & 0 \\ 0 & \xi_{1,t} J_{1,t} \end{bmatrix}. \]

\[ W_t^{(3)} \equiv \begin{bmatrix} \xi_{2,t} \\ \xi_{2,t} \end{bmatrix}. \]

\[ W_t^{(4)} \equiv \begin{bmatrix} \xi_{1,t} \\ \beta_{22} f_{1,t} \xi_{1,t} \\ \xi_{1,t} \\ \beta_{22} v_{1,t} \xi_{1,t} \end{bmatrix}. \]

\[ W_t^{(5)} \equiv J_{1,t} \begin{bmatrix} 1 & \beta_{22} & 0 & 0 \\ 0 & 0 & 1 & \beta_{22} \end{bmatrix}. \]
Lastly, we write,

$$W_t^{(6)} \equiv J_{2,t} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  

$$R_i^J \equiv \begin{bmatrix} 1 & -\rho_i^J \\ -\rho_i^J & (\rho_i^J)^2 \end{bmatrix}.$$  

$$M_i^J \equiv \begin{bmatrix} \frac{\mu_i^J}{(\sigma_i^J)^2} \\ -\mu_i^J - \rho_i^J \frac{\mu_i^J}{(\sigma_i^J)^2} \end{bmatrix}.$$  

which allows us to express the conditional covariance matrix of the factors and volatilities as \( \Omega_{t-1} = H_{t-1} \Omega H_{t-1}. \)

### A.3 Algorithm

The goal of the MCMC algorithm is to make inference on the joint posterior distribution of the model parameters and latent states, \( \Theta \). We can do so by applying Bayes theorem to the desired posterior distribution,

$$p(\Theta|f^T) \propto p(f^T|\Theta)p(\Theta).$$  

It is obviously impossible to draw directly from this distribution as it is not of any known form and is high dimensional. We rely on the Hammersley-Clifford theorem and note that an approximation to the above distribution can be obtained by iteratively sampling from the full conditional distributions. Specifically, assume that we
have initial estimates of $\Theta$, $\Theta^{(0)}$. Then, for some element $\theta_i \in \Theta$, we apply Bayes' theorem to obtain the full conditional distribution,

$$p(\theta_i | \Theta^{(0)} \setminus \theta_i, f^T) \propto p(\Theta^{(0)} \setminus \theta_i, f^T | \theta_i) p(\theta_i).$$

In many cases we can express the first term on the right hand side in much simpler form and thus are able to draw quite easily (in terms of computation time) from known distributions given the appropriate choice of priors. From this full conditional distribution we obtain a new sample of $\theta_i$, $\theta_i^{(1)}$. With this sample in tow, we move to the next element of $\Theta$, $\theta_{i+1}$, which we sample from,

$$p(\theta_{i+1} | \Theta^{(0)} \setminus \theta_i, \theta_i^{(1)}, f^T) \propto p(\Theta^{(0)} \setminus \theta_i, \theta_i^{(1)}, f^T | \theta_{i+1}) p(\theta_{i+1}).$$

We continue this procedure cycling through all the elements of $\Theta$ to obtain a draw, $\Theta^{(1)}$. This procedure is then repeated a number of times, $G$, where $G$ is typically large. The Hammersley-Clifford theorem proves that, under mild conditions, the sequence of estimates, $(\Theta^{(1)}, ..., \Theta^{(G)})$, converges to a draw from the initial posterior distribution we targeted. Furthermore, we can calculate various statistics using the elements of $\Theta$ which again converge to posterior averages. For example, we may compute the mean of some parameter or state $\theta_i$ as $\frac{1}{G} \sum_{g=1}^{G} \theta_i^{(g)} \rightarrow E[\theta_i | f^T]$ or we may compute some function of the parameter or state as $\frac{1}{G} \sum_{g=1}^{G} f(\theta_i^{(g)}) \rightarrow E[f(\theta_i) | f^T]$. As is typically the case, we exclude the first $B$ draws as a burn-in period. For a more thorough treatment on MCMC estimation the reader is referred to Johannes and Polson (2010) and the references therein.

### A.4 Priors

We split the parameter vector, $\Theta_p$ into blocks and use semi-conjugate priors when appropriate in order to facilitate efficient sampling from the full conditional distribu-
tions. We choose the following prior distributions for the various blocks:

1. We first separate the elements of Θp that include all the parameters that are present in the conditional means of returns and volatilities,

\[ B = (\alpha_1, \beta_{11}^f, \beta_{12}^f, \alpha_2, \beta_{21}^f, \beta_{22}^f, \beta_{11}^v, \beta_{12}^v, \beta_{21}^v, \beta_{22}^v)'. \]

One can think of these as the “regression parameters”. As such we choose a multivariate Normal prior for B with the vector of means set equal to zero. We model the covariance matrix as diagonal and set the variances equal to ∞, thus the prior is uninformative (and improper).

2. We then separate the elements of Θp that include the covariance matrix (Ω) of the transformed shocks, \( \epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{1,v,t}, \epsilon_{2,v,t})' \). We choose an inverse-Wishart prior for Ω with degrees of freedom, \( \nu_\Omega = 7 \) and scale matrix \( \Psi_\Omega = 0_{4\times4} \), which correspond to a diffuse (and improper) prior over Ω.

3. We separate the elements of Θp that include the jump probabilities, \( \lambda_1 \), \( \lambda_2 \) and \( \lambda_c \). We follow Eraker et al. and choose a Beta prior with shape parameters, \( a_\lambda = 2 \) and \( b_\lambda = 40 \), for \( \lambda_1 \) and \( \lambda_2 \). For the co-jump probability, \( \lambda_c \), we choose a flat Uniform prior which is uniformly distributed over the interval [0, 1].

4. We separate the elements of Θp that include the parameters that determine the return jump sizes, \( \mu_i^J \) and \( \rho_i^J \), for \( i = 1, 2 \), which we write in vector form as \( \gamma_i = (\mu_i^J, \rho_i^J)' \). We can think of these parameters as representing those from a regression of return jump size on volatility jump size. As such, we choose multivariate Normal priors with vector of means set equal to zero and the covariance matrix diagonal with variances \( s_\mu = 100 \) and \( s_\rho = 4 \) (\( S_{\gamma_i} \)).

5. We separate the elements of Θp that include the parameters that determine the
variance of return jump sizes, \((\sigma_i^J)^2\), for \(i = 1, 2\). Given the multivariate Normal prior for \(\gamma_i\) above, we follow Eraker et al. and choose inverse-Gamma priors for the variances with shape parameter \(a_\sigma = 5\) and scale parameter \(b_\sigma = 20\).

6. We separate the elements of \(\Theta_p\) that determine the volatility jump size (and all other moments given the Exponential distribution), \(\mu_i^{\nu,J}\), for \(i = 1, 2\). We choose a Gamma distribution with shape parameter \(a_{\mu_i} = 20\) and scale parameter \(b_{\mu_i} = 0.1\), i.e.,

\[
p(\mu_i^{\nu,J}) \propto (\mu_i^{\nu,J})^{a_{\mu_i}-1} \exp \left( - \frac{\mu_i^{\nu,J}}{b_{\mu_i}} \right),
\]

for \(\mu_i^{\nu,J} > 0\).

7. Lastly, we separate the jump betas \(B^J = (\beta^{f,J}, \beta^{\nu,J})'\) from \(\Theta_p\). We again choose a multivariate Normal prior for the jump betas with the vector of means set to \(M_{B^J} = (0, 1)'\) and the covariance matrix diagonal with variances set to 100, i.e. \(S_{B^J} = \text{diag}(100, 100)\).

### A.5 Sampling model parameters

#### A.5.1 Sampling the diffusion parameters \((B, \Omega)\)

To derive the full conditional distribution for the drift parameters, \(B\), we apply Bayes theorem as follows:

\[
p(B|\Theta_{-B}, f^T) \propto p(f^T, \Theta_{-B}|B)p(B).
\]

We can then re-express the first term on the right hand side of the above equation as the product of a conditional density and a marginal density,

\[
p(B|\Theta_{-B}, f^T) \propto p(f^T, V^T|\Theta_{-f^T,V^T})p(\Theta_{-f^T,V^T,B}|B)p(B)
\]

\[
\propto p(f^T, V^T|\Theta_{-f^T,V^T})p(B).
\]
Notice that the middle term drops out because the jump sizes, jump times and other parameters do not depend on \(B\) and thus the term can be treated as a constant. We then exploit the conditional independence of each observation,

\[
p(B|\Theta_{-B}, f^T) \propto \left( \prod_{t=1}^{T} p(f_t, V_t|V_{t-1}, \Theta_{-f^T,Y^T}) \right) p(B).
\]

Since the prior, \(p(B)\), is uninformative it also drops out and expanding the first term we have,

\[
p(B|\Theta_{-B}, f^T) \propto \exp \left( -\frac{1}{2} \sum_{t=1}^{T} (Y_t^{(1)} - W_{t-1}^{(1)'B})\Omega_{t-1}^{-1} (Y_t^{(1)} - W_{t-1}^{(1)'B})' \right).
\]

Collecting the terms involving \(B\) and completing the matrix square gives,

\[
p(B|\Theta_{-B}, f^T) \propto \exp \left( -\frac{1}{2} \sum_{t=1}^{T} (B - M_{B,T})' \Sigma_{B,T}^{-1} (B - M_{B,T}) \right),
\]

where \(\Sigma_{B,T} = \left( \sum_{t=1}^{T} W_{t-1}^{(1)'} \Omega_{t-1}^{-1} W_{t-1}^{(1)} \right)^{-1}\) and \(M_{B,T} = \Sigma_{B,T} \left( \sum_{t=1}^{T} W_{t-1}^{(1)'} \Omega_{t-1}^{-1} Y_t^{(1)} \right)\).

This is easily recognized as the kernel of a multivariate Normal distribution and thus we sample \(B\) from \(N(M_{B,T}, \Sigma_{B,T})\).

The full conditional distribution for \(\Omega\) is derived similarly,

\[
p(\Omega|\Theta_{-\Omega}, f^T) \propto \left( \prod_{t=1}^{T} p(f_t, V_t|\Theta_{-f^T,Y^T}) \right) p(\Omega).
\]

Again expanding the first term we have,

\[
p(\Omega|\Theta_{-\Omega}, f^T) \propto |\Omega|^{-\frac{T + \nu_{\Omega} + 5}{2}} \exp \left( -\frac{1}{2} \text{tr} \left( \sum_{t=1}^{T} H_{t-1}^{-1} (Y_t^{(1)} - W_{t-1}^{(1)'B})(Y_t^{(1)} - W_{t-1}^{(1)'B})' H_{t-1}^{-1} \right) \Omega^{-1} \right)
\]

\[
\propto |\Omega|^{-\frac{T + \nu_{\Omega} + 5}{2}} \exp \left( -\frac{1}{2} \text{tr} \left( \Psi_{\Omega,T} \Omega^{-1} \right) \right),
\]

where \(\nu_{\Omega,T} = T + \nu_\Omega\) and \(\Psi_{\Omega,T} = \sum_{t=1}^{T} H_{t-1}^{-1} (Y_t^{(1)} - W_{t-1}^{(1)'B})(Y_t^{(1)} - W_{t-1}^{(1)'B})' H_{t-1}^{-1}\). Note that the choice of an improper prior for the scale matrix in the inverse-Wishart density causes its second term to drop out. The above expression is easily recognized as the kernel of an inverse-Wishart distribution and thus we sample \(\Omega\) from \(W^{-1}(\nu_{\Omega,T}, \Psi_{\Omega,T})\).
A.5.2 Sampling the jump probability parameters \((\lambda_1, \lambda_2, \lambda_c)\)

We again derive the full conditional distribution of jump intensity parameters by an application of Bayes theorem. We consider first the case of \(\lambda_1\),

\[
p(\lambda_1|\Theta_{-\lambda_1}, f^T) \propto p(J_1^T|f^T, \Theta_{-J_1^T})p(f^T, \Theta_{-J_1^T, \lambda_1}|\lambda_1)p(\lambda_1).
\]

As before the middle term drops out because the density does not depend on \(\lambda_1\).

Exploiting the independence of jumps we re-write this as,

\[
p(\lambda_1|\Theta_{-\lambda_1}, f^T) \propto \left( \prod_{t=1}^T p(J_{1,t}|f^T, \Theta_{-J_1^T}) \right) p(\lambda_1).
\]

The first density inside the parameters is a Bernoulli density, thus for days on which there are jumps it takes the value \(\lambda_1\) and on days when there are no jumps it takes the value \((1 - \lambda_1)\). As such, we expand the full conditional distribution as,

\[
p(\lambda_1|\Theta_{-\lambda_1}, f^T, V^T, \xi^T, J^T) \propto \lambda_1^{\sum_{t=1}^T J_{1,t}} (1 - \lambda_1)^{T - \sum_{t=1}^T J_{1,t}} \lambda_1^{a_{\lambda_1} - 1} (1 - \lambda_1)^{b_{\lambda_1} - 1} \propto \lambda_1^{a_{\lambda_1,T} - 1} (1 - \lambda_1)^{b_{\lambda_1,T} - 1},
\]

where \(a_{\lambda_1,T} = \sum_{t=1}^T J_{1,t} + a_{\lambda_1}\) and \(b_{\lambda_1,T} = T - \sum_{t=1}^T J_{1,t} + b_{\lambda_1,T}\). This is easily recognized as the kernel of a Beta distribution and thus we sample \(\lambda_1\) from \(\text{Beta}(a_{\lambda_1,T}, b_{\lambda_1,T})\).

In the model with \(\lambda_c = 0\) (i.e., no co-jumps) the full conditional distribution for \(\lambda_2\) is derived in exactly the same way with a suitable interchange of notation. However, for \(\lambda_c \neq 0\), the full conditional distribution of \(\lambda_2\) and \(\lambda_c\) do not take known forms. Specifically, we can re-write their joint density as,

\[
p(\lambda_2, \lambda_c|\Theta_{-\lambda_2, \lambda_c}, f^T) \propto \left( \prod_{t=1}^T p(J_{2,t}|f^T, \Theta_{-J_2^T}) \right) p(\lambda_2)p(\lambda_c)
\]

\[
\propto (1 - (1 - \lambda_2)(1 - \lambda_c J_{1,t}))^{\sum_{t=1}^T J_{2,t}} ((1 - \lambda_2)(1 - \lambda_c J_{1,t}))^{T - \sum_{t=1}^T J_{2,t}} \lambda_2^{a_{\lambda_2} - 1} (1 - \lambda_2)^{b_{\lambda_2} - 1}
\]
In this case we implement a random walk Metropolis-Hastings step in the algorithm. Specifically, conditional on the previous draws, $\lambda_2^{(g-1)}$ and $\lambda_c^{(g-1)}$, we obtain prospective draws of the two parameters from a proposal distribution as $\lambda_2^{*g} = \lambda_2^{(g-1)} + 0.0001e_1$ and $\lambda_c^{*g} = \lambda_c^{(g-1)} + 0.01e_2$, where $e_1$ and $e_2$ are drawn from a standard Normal distribution. We accept these prospective draws with probability $A(\lambda_2^{*g}, \lambda_c^{*g}, \lambda_2^{(g-1)}, \lambda_c^{(g-1)})$, which is computed as,

$$A(\lambda_2^{*g}, \lambda_c^{*g}, \lambda_2^{(g-1)}, \lambda_c^{(g-1)}) = \frac{p(\lambda_2^{*g}, \lambda_c^{*g}|\Theta_{-\lambda_2, \lambda_c}, f^T)}{p(\lambda_2^{(g-1)}, \lambda_c^{(g-1)}|\Theta_{-\lambda_2, \lambda_c}, f^T)}.$$ 

Note that the additional term typically present due to the proposal distribution drops out due to its symmetry.

### A.5.3 Sampling the jump size parameters $(\mu_i^J, \rho_i^J, \sigma_i^J, \mu_i^{v,J})$

We start by deriving the full conditional distribution for the return jump size parameters, $\mu_i^J$ and $\rho_i^J$. The full conditional distribution is derived in exactly the same way for $i = 1, 2$, thus we present it in the general case. We begin, as usual, by applying Bayes theorem,

$$p(\mu_i^J, \rho_i^J|\Theta_{-\mu_i^J, \rho_i^J}, f^T) \propto p((\xi_i^f)^T|f^T, \Theta_{-(\xi_i^f)^T})p(f^T, \Theta_{-\mu_i^J, \rho_i^J, (\xi_i^f)^T}|\mu_i^J, \rho_i^J)p(\mu_i^J, \rho_i^J).$$

As before, the middle density drops out because it does not depend on $\mu_i^J$ or $\rho_i^J$. As discussed above, these parameters are essentially the parameters from a linear regression of return jump size on volatility jump size (including an intercept). As such, the above density can be expanded as follows,

$$p(\mu_i^J, \rho_i^J|\Theta_{-\mu_i^J, \rho_i^J}, f^T) \propto \left( \prod_{t=1}^T p(\xi_{i,t}^f|f^T, \Theta_{-(\xi_i^f)^T}) \right) p(\mu_i^J, \rho_i^J)$$

$$\propto \exp\left( -\frac{1}{2(\sigma_i^J)^2} \sum_{t=1}^T (\xi_{i,t}^f - \gamma^f_i\tilde{\xi}_{i,t})^2 \right)$$

$$\exp\left( -\frac{1}{2}(\gamma_i - M_{\gamma_i})'S_{\gamma_i}^{-1}(\gamma_i - M_{\gamma_i}) \right),$$
where $\tilde{\xi}_{i,t}^v = (1, \xi_{i,t}^v)'$. Dropping the terms that do not depend on $\gamma_i$ and completing the matrix square yields,

$$
p(\mu_i^J, \rho_i^J|\Theta_{-\mu_i^J, \rho_i^J}, f^T) \propto \exp \left( -\frac{1}{2} (\gamma_i - M_{\gamma_i, T})' S_{\gamma_i}^{-1} (\gamma_i - M_{\gamma_i, T}) \right),
$$

where $S_{\gamma_i, T} = \left( \sum_{t=1}^{T} \frac{1}{(\sigma_i^v)^2} \tilde{\xi}_{i,t}^v S_{\gamma_i}^{-1} + S_{\gamma_i}^{-1} \right)^{-1}$ and $M_{\gamma_i, T} = S_{\gamma_i, T} \left( \sum_{t=1}^{T} \frac{1}{(\sigma_i^v)^2} \tilde{\xi}_{i,t}^v f \right)^T$. This is again easily recognizable as the kernel of a multivariate Normal distribution and thus we draw $\gamma_i$ from $\mathcal{N}(M_{\gamma_i, T}, S_{\gamma_i, T})$ and set $\mu_i^J$ equal to the first element of $\gamma_i$ and $\rho_i^J$ equal to the second element.

The return jump variance is derived similarly. Specifically,

$$
p((\sigma_i^J)^2|\Theta_{-(\sigma_i^J)^2}, f^T) \propto ((\sigma_i^J)^2)^{-T} \exp \left( -\frac{1}{2(\sigma_i^J)^2} \sum_{t=1}^{T} (\xi_{i,t}^v - \gamma_i \tilde{\xi}_{i,t}^v)^2 \right)
$$

$$
\times ((\sigma_i^J)^2)^{-a_{\sigma} - 1} \exp \left( -\frac{b_{\sigma}}{(\sigma_i^J)^2} \right)
$$

$$
\times ((\sigma_i^J)^2)^{-a_{\sigma,T} - 1} \exp \left( -\frac{b_{\sigma,T}}{(\sigma_i^J)^2} \right),
$$

where $a_{\sigma,T} = T/2 + a_\sigma$ and $b_{\sigma,T} = \left( \frac{1}{2} \sum_{t=1}^{T} (\xi_{i,t}^v - \gamma_i \tilde{\xi}_{i,t}^v)^2 \right) + b_\sigma$. This is easily recognizable as the kernel of an inverse-Gamma distribution, thus we draw $(\sigma_i^J)^2$ from $\text{Inv-Gamma}(a_{\sigma,T}, b_{\sigma,T})$.

The full conditional distribution of $\mu_i^{v,J}$ is also derived in a similar manner by noticing that only the density of the volatility jump sizes depend on it. Therefore,

$$
p(\mu_i^{v,J}|\Theta_{-\mu_i^{v,J}, f^T}) \propto \left( \prod_{t=1}^{T} p(\xi_{i,t}^v|f_T, \Theta_{(\xi_{i,t}^v)'}) \right) p(\mu_i^{v,J})
$$

$$
\propto (\mu_i^{v,J})^T \exp \left( -\mu_i^{v,J} \sum_{t=1}^{T} \xi_{i,t}^v \right) a_{\mu}^{-1} \exp \left( -\frac{\mu_i^{v,J}}{b_{\mu}} \right)
$$

$$
\times (\mu_i^{v,J})^{a_{\mu,T} - 1} \exp \left( -b_{\mu,T} \mu_i^{v,J} \right),
$$

where, $a_{\mu,T} = T + a_{\mu}$ and $b_{\mu,T} = \left( \sum_{t=1}^{T} \xi_{i,t}^v \right) + 1/b_{\mu}$. This can be easily recognized as the kernel of a Gamma distribution and thus we sample $\mu_i^{v,J}$ from $\text{Gamma}(a_{\mu,T}, 1/b_{\mu,T})$. 

A.5.4 Sampling the jump betas ($B^J$)

We again start by applying Bayes theorem,

$$p(B^J|\Theta_{-B^J}, f^T) \propto p(f^T, \Theta_{-B^J}|B^J)p(B^J).$$

First we note that the jump betas affect only the distribution of $f_{2,t}$ and $V_{2,t}$, conditional on the time $t - 1$ information set. Furthermore, conditional on the rest of the time $t$ state variables and parameters, we can reconstruct the continuous shocks $\epsilon_{1,t}$ and $\epsilon^v_{1,t}$. Thus to derive the full conditional distribution of the $B^J$, we start by re-expressing the first density as the product of the conditional density of $f_{2,t}$ and $V_{2,t}$ multiplied by another density that does not depend on $B^J$. Specifically,

$$p(B^J|\Theta_{-B^J}, f^T) \propto \left( \prod_{t=1}^{T} p(f_{2,t}, V_{2,t}|f^T_1, V^T_1, \Theta_{-V^T_1}, V^T_2) \right) p(B^J)$$

$$\propto \exp \left( -\frac{1}{2} \sum_{t=1}^{T} \frac{1}{V_{2,t}} (Y^{(2)}_t - W^{(2)}_t B^J)^\prime \Omega^{-1}_{\epsilon_2|\epsilon_1} (Y^{(2)}_t - W^{(2)}_t B^J) \right)$$

$$\exp \left( -\frac{1}{2} (B^J - M_{B^J})^\prime S_{B^J}^{-1} (B^J - M_{B^J}) \right).$$

We then collect only terms involving $B^J$, since the others can be disregarded as constants, and complete the matrix square to obtain,

$$p(B^J|\Theta_{-B^J}, f^T) \propto \exp \left( -\frac{1}{2} (B^J - M_{B^J,T})^\prime S_{B^J,T}^{-1} (B^J - M_{B^J,T}) \right),$$

where $S_{B^J,T} = \left( \sum_{t=1}^{T} \frac{1}{V_{2,t}} W^{(2)}_t \Omega^{-1}_{\epsilon_2|\epsilon_1} W^{(2)}_t + S^{-1}_{B^J} \right)^{-1}$ and $M_{B^J,T} = S_{B^J,T} \left( \sum_{t=1}^{T} \frac{1}{V_{2,t}} W^{(2)}_t \Omega^{-1}_{\epsilon_2|\epsilon_1} Y^{(2)}_t + S^{-1}_{B^J} M_{B^J} \right)$. Again, this is easily recognized as the kernel of a multivariate Normal distribution and thus we sample $B^J$ from $\mathcal{N}(M_{B^J,T}, S_{B^J,T})$. 
A.6 Sampling latent state variables

A.6.1 Sampling the jump occurrences \((J_{1,t}, J_{2,t})\)

Since the jump occurrences can take only values of 0 or 1, they are necessarily distributed Bernoulli. Thus, to sample the jump occurrences we must only find the appropriate probability \(\pi_{i,t} = P(J_{i,t} = 1|\ldots).\) Like the case for the jump betas, only the distribution of \(f_{2,t}\) and \(V_{2,t}\) are affected by \(J_{2,t}.\) We thus evaluate the full conditional density, for the case where \(J_{2,t} = 0\) and \(J_{2,t} = 1,\)

\[
p_{J_{2,t}=0} = \exp\left(-\frac{1}{2V_{2,t}}(Y_t^{(3)})^\prime \Omega_{t2|t}\Omega_{t2|t}^{-1}(Y_t^{(3)})\right) (1 - \lambda_2)(1 - \lambda_c J_{1,t})
\]

\[
p_{J_{2,t}=1} = \exp\left(-\frac{1}{2V_{2,t}}(Y_t^{(3)} - J_{2,t} W_t^{(3)})^\prime \Omega_{t2|t}^{-1}(Y_t^{(3)} - J_{2,t} W_t^{(3)})\right) (1 - (1 - \lambda_2)(1 - \lambda_c J_{1,t})).
\]

Since the above expressions do not necessarily sum to one, we compute the full conditional probability of a jump as \(\pi_{2,t} = p_{J_{2,t}=1}/(p_{J_{2,t}=1} + p_{J_{2,t}=0}).\) We therefore sample \(J_{2,t} \sim \text{Bern}(\pi_{2,t}).\)

While conceptually similar, the full conditional distribution for \(J_{1,t}\) affects the time \(t\) distribution of all factor returns and volatilities as well as the conditional distribution of \(J_{2,t}.\) As such, we derive the following expressions,

\[
p_{J_{1,t}=0} = \exp\left(-\frac{1}{2}(Y_t^{(4)})^\prime \Omega_{t-1}(Y_t^{(4)})\right) (1 - \lambda_1)
\]

\[
\lambda_2^{J_{2,t}} (1 - \lambda_2)^{1-J_{2,t}}
\]

\[
p_{J_{1,t}=1} = \exp\left(-\frac{1}{2}(Y_t^{(4)} - J_{1,t} W_t^{(4)})^\prime \Omega_{t-1}(Y_t^{(4)} - J_{1,t} W_t^{(4)})\right) \lambda_1
\]

\[
(1 - (1 - \lambda_2)(1 - \lambda_c J_{1,t}))^{J_{2,t}} (1 - \lambda_2)(1 - \lambda_c J_{1,t})^{1-J_{2,t}}.
\]

Again, we compute the full conditional probability of a jump as \(\pi_{1,t} = p_{J_{1,t}=1}/(p_{J_{1,t}=1} + p_{J_{1,t}=0}).\) We therefore sample \(J_{1,t} \sim \text{Bern}(\pi_{1,t}).\)
A.6.2 Sampling the jump sizes \((\xi^{f}_{i,t}, \xi^{v}_{i,t})\)

We sample the jump sizes for each factor jointly. With a slight change of notation the derivation is exactly the same for \(i = 1, 2\) and thus we present the case for \(i = 1\). Specifically for the case of \(i = 1\) we use the matrix \(W^{(5)}_{t}\) and vector \(Y^{(4)}_{t}\) and for the case of \(i = 2\) we use the matrix \(W^{(6)}_{t}\) and vector \(Y^{(5)}_{t}\). To start note that the jump size terms, \(\xi_{i,t}\), are present in the time \(t\) distribution of factor returns and volatilities as well as the conditional distribution of return jumps (conditional on volatility jumps) and the marginal distribution of volatility jumps. Thus we can write the full conditional distribution of jump sizes as,

\[
p(\xi_{i,t}|\Theta_{-\xi_{i,t}}, f^{T}) \propto \exp \left( -\frac{1}{2} \left( Y^{(4)}_{t} - W^{(5)'}_{t} \xi^{f}_{i,t} \right) (\Omega_{t}^{-1} \left( Y^{(4)}_{t} - W^{(5)'}_{t} \xi^{f}_{i,t} \right)) \right) \exp \left( -\mu^{v,J}_{i} \xi^{v}_{i,t} \right) \exp \left( -\frac{1}{2} \left( \xi^{f}_{i,t} - \mu^{f}_{i} + \rho^{f,v}_{i} \xi^{v}_{i,t} \right)^{2} + \sigma^{2}_{f} \right) - \mu^{v,J}_{i} \xi^{v}_{i,t} \right) .
\]

We can then expand the terms and drop those that don’t depend on \(\xi_{i,t}\) as follows,

\[
p(\xi_{i,t}|\Theta_{-\xi_{i,t}}, f^{T}) \propto \exp \left( -\frac{1}{2} \left( \xi^{f'}_{i,t} W^{(5)'}_{t} \Omega_{t}^{-1} W^{(5)}_{t} \xi^{f}_{i,t} - 2\xi^{f'}_{i,t} W^{(5)'}_{t} \Omega_{t}^{-1} Y^{(4)}_{t} \right) \right) \exp \left( -\frac{1}{2 \sigma^{2}_{f}} \left( \xi^{f'}_{i,t} + \rho^{f,v}_{i} \xi^{v}_{i,t} \right)^{2} - 2 \rho^{f,v}_{i} \xi^{f'}_{i,t} \xi^{v}_{i,t} - 2 \mu^{f}_{i} \xi^{f'}_{i,t} + 2 \mu^{f}_{i} \xi^{v}_{i,t} \right) - \mu^{v,J}_{i} \xi^{v}_{i,t} \right) .
\]

The joint sampling is obtained by writing the second exponential term in matrix form as, \(\xi^{f'}_{i,t} R^{f} \xi_{i,t} + 2 \xi^{f'}_{i,t} M^{f}\), where \(R^{f}\) and \(M^{f}\) are as defined in the notation section. Using this notation, the full conditional distribution can be written as,

\[
p(\xi_{i,t}|\Theta_{-\xi_{i,t}}, f^{T}) \propto \exp \left( -\frac{1}{2} \left( \xi^{f'}_{i,t} \left( W^{(5)'}_{t} \Omega_{t}^{-1} W^{(5)}_{t} + \frac{1}{\sigma^{2}_{f}} R^{f} \right) \xi^{f}_{i,t} \right) - 2 \xi^{f'}_{i,t} \left( W^{(5)'}_{t} \Omega_{t}^{-1} Y^{(4)}_{t} + M^{f} \right) \right) \exp \left( -\frac{1}{2} \left( \xi^{f}_{i,t} - M^{f}_{i} \xi^{f}_{i,t} \right) S^{f}_{i,t} \left( \xi_{i,t} - M^{f}_{i} \xi_{i,t} \right) \right) .
\]
where \( S_{\xi,i,t} = \left( W_t^{(5)'} \Omega_t^{-1} W_t^{(5)} + \frac{1}{(\sigma_t')^2} R^j \right)^{-1} \) and \( M_{\xi,i,t} = S_{\xi,i,t} \left( W_t^{(5)'} \Omega_t^{-1} Y_t^{(4)} + M^j \right) \).

The key distinction here is that the volatility jump size is restricted to be positive. Thus, the above expression is the kernel of a multivariate truncated Normal distribution and we sample from \( \mathcal{N}(M_{\xi,i,t}, S_{\xi,i,t}) \mathbb{I}(\xi_{i,t}^{\nu} > 0) \). Also note that in the case where there is no estimated jump at time \( t \) the part of the distribution that comes from the returns and volatilities drops out. That is, when there is no jump the return and volatility observations provide no information about jump size. This relative paucity of information is the reason estimates of the jump size parameters are considerably less precise.

### A.6.3 Sampling the volatilities \((V_{1,t}, V_{2,t})\)

The full conditional distribution of the volatilities, \( V_{i,t} \), cannot be expressed in any known form. We implement the multivariate analog of the method in used Eraker et al.. As was the case above, we introduce a random walk Metropolis-Hastings step into the algorithm and we sample prospective draws of the volatilities as \( V_{i,t}^{g*} = V_{i,t}^{(g-1)} + 0.25 e_i \), where the \( e_i \) are again drawn from a standard Normal distribution. We again accept these draws with probability \( A(V_{i,t}^{g*}, V_{i,t}^{(g-1)}) \), where the acceptance probability is computed as,

\[
A(V_{i,t}^{g*}, V_{i,t}^{(g-1)}) = \frac{p(V_{i,t}^{g*} | f^T, \Theta_{-V_i})}{p(V_{i,t}^{(g-1)} | f^T, \Theta_{-V_i})},
\]

where,

\[
p(V_{i,t}^{g*} | f^T, \Theta_{-V_i}) \propto p(f_{t+1}, V_{t+1} | V_t, \Theta_{-V^T}) p(V_t | V_{t-1}, \Theta_{-V^T}).
\]

Of note is that the current draw of volatilities affect not only themselves, but also the next period’s return and volatility due to their affect on the conditional variance and the conditional mean.