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MIMO Broadcast Channels with Full-duplex Feedback

by

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Abstract

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In this thesis, I utilize full-duplex radios to increase the downlink spectral efficiency of MIMO broadcast channels by proposing a sequential beamforming strategy. In current half-duplex multiuser MIMO systems, non-trivial time resources is devoted into the collection of channel state information (CSI). In sequential beamforming strategy, instead of waiting for CSI from all users being collected, base station immediately begins to serves users when only partial CSI is available. Such operation, however, brings new inter-node interference. I quantify the impact of this unique new interference and the associated tradeoff in the design of the training control channel with closed-form bounds and simulation results in both finite SNR and high SNR regime. With experimental results collected in both indoor and outdoor environment, I further demonstrate that sequential beamforming achieves significant spectral efficiency improvement for systems with both open-loop training and closed-loop training.

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Contents

List of Figures

Chapter 1

Introduction

In this chapter, I present a high level introduction regarding the utilization of full-duplex radio in Multiuser MIMO training. In Section 1.1, a motivation is provided. A list of the main contribution and the related previous work is discussed in Section 1.2 and Section 1.3, respectively. Finally, I end this chapter in Section 1.4 with the structure of thesis. The majority of this thesis is based on a submitted work [12].

1.1 Motivation

Multiuser MIMO downlink systems have the potential to increase the spectral efficiency by serving multiple users at the same time with a multiple-antenna base station. A base station with M antennas can simultaneously support up to M *half-duplex* single-antenna users at full multiplexing gain, if it has perfect channel information (CSI). Accurate channel knowledge at the transmitter is vital to achieve maximum spectral efficiency. For example, when no CSI is available at the base station, TDMA strategy which can only service one user per time is optimal [2]. Therefore, only one user can be supported with full-multiplexing gain.

To obtain CSI, in transmitter beamforming based systems, either closed or open loop training, defined as below, is used.

- In the *closed loop training* method, each user first estimates CSI by using the training pilots sent out by the base station. Then, the CSI is quantized and sent back to the base station[1].

- In the *open loop training* method, the base station learns downlink CSI by receiving training pilots from users through an uplink channel; channel *reciprocity* is then leveraged to learn downlink CSI from uplink receptions.

In a half-duplex system, uplink training (closed or open loop) consumes time resources, which results in less downlink data transmission time. For time-varying systems with a large number of users, overhead due to CSI acquisition can lead to significant spectral efficiency loss. In Chapter 4, we observe that training can potential take up to half of the duration time. This training overhead can be further translated into the loss of more than half the data transmission time.

The recently developed full-duplex radios [4, 5] allow concurrent uplink and downlink data transmission. However, full-duplex transmission for small form-factor handsets still remains a challenging problem. Thus, only a full-duplex base station is assumed in this thesis. In [6], a full-duplex base station is used to increase spectral efficiency by serving half-duplex downlink and uplink traffic simultaneously. In this thesis, I propose an alternative use of full-duplex, in which full-duplex capability is harnessed to increase downlink spectral efficiency, by saving on the training time.

1.2 Main Contribution

In particular, the key contributions in this thesis are as follows:

[1]Closed loop training is also possible if each user sends channel coefficients through analog signals. Due to its sub-optimality compared to open loop training method [3], I do not consider closed loop analog feedback method in this thesis.

- I propose a sequential beamforming strategy for multiuser downlink transmissions with either closed or open loop training. Instead of waiting to receive all CSI before starting data transmission, the base station now begins transmitting to some users as it receives their CSI. In this scheme, only the base station has to be full-duplex while *all mobiles are half-duplex*.

- The simultaneous transmission of feedback and data creates inter-node interference at receiving downlink users. Due to the relative low user training power[1], I show that inter-node interference only leads to a limited downlink rate reduction during training.

- Compared to its half-duplex counterpart, the proposed scheme is shown to significantly increase the downlink spectral efficiency. For example, in a typical 1.4 MHz LTE system with block length of around 500 symbols, the proposed strategy demonstrates a spectral efficiency improvement of 130% its half-duplex counterpart, for a 8×8 multiuser MIMO system with closed loop training.

- I further derive the spectral efficiency of sequential beamforming in the high SNR regime. For systems with closed loop training, the relative performance of the scheme to half-duplex counterpart is decided by training power. If low training power is used, the proposed scheme can double the performance of half-duplex systems as if no inter-node interference exists. However, if users adopt a higher power than the base station for training, the strong inter-node interference prevents the proposed scheme from achieving any benefit. In open loop systems, the proposed strategy does not bring extra benefit asymptotically.

[1]In current systems, the transmission power of users is usually limited due to lower power budget compared to a base station.

1.3 Related Research

The recently developed full-duplex radios [4, 5] allow concurrent uplink and downlink data transmission. However, full-duplex transmission for small form-factor handsets still remains a challenging problem. Thus, we assume that only the base-station is full-duplex. In [6], a full-duplex base station is used to increase spectral efficiency by serving half-duplex downlink and uplink traffic simultaneously.

The rate loss due to imperfect CSI with different types of training is characterized in [3]. In [7], authors characterize the optimal training duration and its associated spectral efficiency for half-duplex systems. User selection [8, 9] has been proposed to reduce the number of training symbols needed by selecting users with larger distance in channel space. My analysis has two main differences from prior research. First, I study how to utilize the full-duplex operation to obtain gains in spectral efficiency. Second, the influence of limited training power at the mobile user is modeled and emphasized throughout this thesis.

I first proposed to utilize training time in systems composed of both full-duplex base station and *full-duplex* mobile in [10] and [11]. It is then extended to a system comprising a full-duplex base station and only *half-duplex* users in [12]. This thesis is based on results and analysis from [10], [11], [12].

1.4 Thesis Structure

The remainder of this thesis is structured as follows. Chapter 2 describes the system model. In Chapter 3, previous work on continuous adaptive strategy is briefly reviewed with the proposal of the sequential beamforming strategy . The training duration is optimized in Chapter 4 for systems with both closed and open loop training. The associated spectral efficiency is then presented in Chapter 5 with both theoretical analysis and experimental data validation. High SNR analysis is provided in Chapter 6 to evaluate the proposed strategy. I conclude this thesis by summarizing the main results in Chapter 7.

Chapter 2

System Model

I consider a symmetric multiuser MIMO downlink consisting of an M-antenna *full-duplex* base station and M single-antenna *half-duplex* users. The base station aims at delivering downlink data to each user. Albeit sub-optimal,base station adopts zero-forcing (ZF) beamforming [13] for simultaneous transmission to multiple users. In ZF, the base station projects the signal intended to one user on the null space of the others. Thus, if perfect CSI is available, each user only receives the intended signal without interference.

Since CSI is obtained from finite training, it is almost always inaccurate and thus results in inter-beam interference for ZF transmissions. During full-duplex training, the downlink data is communicated in the same band as the training signals sent by users, thus receiving users also suffer from *inter-node interference*. In this thesis, I quantify the impact of inter-node interference on spectral efficiency for training-based ZF strategy.

When User k sends training symbols and base station transmits downlink data to User $1, 2, ..., k-1$, the received signal of User i is immediately captured as

$$y_i = \mathbf{h}_i^* \mathbf{v}\mathbf{s} + h_{ik} x_{\mathrm{tr}_k} + n_i, \quad i = 1, \cdots, k-1. \tag{2.1}$$

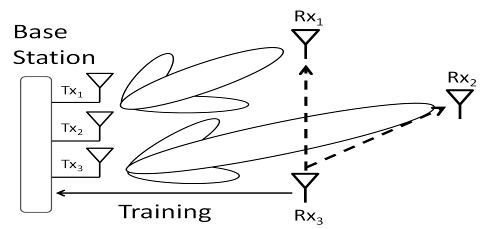


Figure 2.1: A schematic of interference in a 3×3 multiuser MIMO downlink system.

Here $\mathbf{h}_i \in \mathcal{C}^{1\times M}$ and h_{ik} stands for the channel realization between User i to the base station and User k, respectively. In this thesis, block refers to T continuous symbols where the channel state unchanged. I assume a Rayleigh block fading environment, i.e., each element of $\mathbf{h}_i$ and h_{ik} is independently complex Gaussian distributed from block to block.

The term $\mathbf{s} \in \mathcal{C}^{k-1\times 1}$ is the actual signal intended to User $1, 2, \cdots, k-1$ and $\mathbf{v} = [\mathbf{v}_1, ..., \mathbf{v}_{k-1}] \in \mathcal{C}^{M\times k-1}$ represents the precoding matrix generated by ZF method based on the quantized (estimated) CSI of users, which is presented as $\hat{\mathbf{h}}_i, i = 1, 2, .., k-1$. The precoded symbol is then $\mathbf{vs}$, which is constrained to an average power constraint of P. I consider equal power allocation among symbols, i.e., each of the downlink symbols has power P/M, which is mathematically captured as $\mathbb{E}\left[|\mathbf{v}_i s_i|^2\right] = P/M, \forall i$.

If only imperfect CSI is available, inter-beam interference is non-zero. The signal and the inter-beam interference both are contained in term $\mathbf{h}_i^*\mathbf{vs}$. Term x_{tr_k} is the uplink training symbol sent by User k. To account for limitations of both battery and size of user devices, I consider a more strict average power constraint for users, which is described as $\mathbb{E}[|x_{\mathrm{tr}_k}|^2] \leqslant fP,\ f \in (0, 1]$. Term $h_{ik}x_{\mathrm{tr}_k}$ captures the inter-node interference. I assume inter-node interference power to be proportional to the training power fP, i.e., it grows as $|h_{ik}x_{\mathrm{tr}_k}|^2 = \alpha f P$, where $\alpha > 0$. The signal is degraded by an independent unit variance additive white complex Gaussian noise n_i. A high level demonstration of interference can be found from Fig. 2.1 where User

3 sends training and other users receive downlink data. The receiving users suffer inter-beam interference (side lobes) due to imperfect CSI. The receiving users also incur inter-node interference (dashed lines) resulting from User 3's training. Since users are half-duplex nodes, User 3 does not receive while it is sending the training signal.

I assume that the half-duplex Users i, $(i = 1, 2, ..., M)$ have perfect knowledge of their own channels $\mathbf{h}_i$. However, the base station is required to obtain CSI by either closed or open loop training. I assume that self-interference due to the full-duplex operation at base station is reduced to near-noise floor by a combination of both active cancellation [14, 15] and passive suppression [16]. Unlike [10, 11], the proposed sequential beamforming strategy needs only half-duplex users, thus there is no concern of self-interference at end nodes.

Chapter 3

Sequential Beamforming

In this chapter, I propose a sequential beamforming strategy that leverages the full-duplex capability at base station to send downlink data during CSI collection. Previous work [10, 11] of continuous adaptive beamforming with its limitation is first presented in Section 3.1. Section 3.2 is then dedicated to describing the sequential beamforming strategy [12] which is the focus for the rest of this thesis. In Section 3.3, the influence of inter-node and inter-beam interference on downlink rate is characterized for further spectral efficiency analysis.

3.1 Strategy Review: Continuous Adaptive Beamforming

I first describe *Continuous Adaptive Beamforming* strategy for open loop training system. The key idea behind the continuous adaptive beamforming is to send downlink data concurrently with uplink training pilots collection, instead of waiting for all the uplink training pilots to be collected and then starting downlink transmission. For a transmission block with T_{CAB} uplink training pilots, continuous adaptive beamforming with open loop training operates as follows.

1. At the beginning of each block, no CSI is available at the base station. Each of users 1, ..., M sends a training symbol sequentially in a TDMA manner from symbol 1 to symbol M. We refer to such M symbols where each user sends one training pilot as a *cycle*. The first M symbols constitute cycle $j = 1$.

2. For cycle $j + 1$, the base station updates its ZF precoding matrix and transmits downlink data based on all the uplink training pilots received during the whole previous j cycles, i.e., j uplink training pilots from each user. All users decode the received signal by treating interference as noise. An illustration is shown in Fig. 3.1. At the end of cycles, the base station refines its precoding vectors based on the accumulated pilots.

3. Repeat 2) till the end of T_{CAB} symbols.

4. When all T_{CAB} uplink training pilots are collected, all users stop sending training pilots to the base station and transmission continue in the downlink direction only.

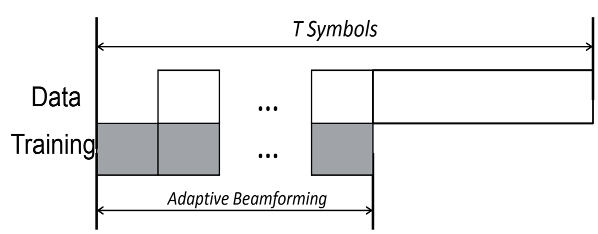


Figure 3.1: A continuous adaptive beamforming with training duration of T_{CAB} symbols.

The continuous adaptive beamforming strategy is shown to provide significant spectral efficiency gain over traditional half-duplex counterparts [10, 11]. In the meantime, there are several limitation to continuous adaptive beamforming.

- Firstly, continuous adaptive beamforming needs full-duplex mobile nodes, which is very challenging due to the size and battle limitation of mobile nodes.
- The feedback message in continuous adaptive beamforming has to be decoded incrementally symbol by symbol. This precoding process makes the digital feedback (in closed loop training) difficult to be implemented.
- Finally, in each of the cycles, each user spends most of the time under inter-node interference from signals intended to all the other $M-1$ users.

In next section, I propose a new sequential beamforming strategy which avoids the drawback of continuous adaptive beamforming while keeping the spectral benefit.

3.2 Strategy Propose: Sequential Beamforming

The proposed strategy is referred to as *sequential beamforming*. In sequential beamforming, users send their channel state information in time orthogonal slots, and as the the base station receives a particular user's information, it starts data transmission to that user. Thus, unlike half-duplex system, the base station does not wait for all the users to send their channel feedback. As noted before, the proposed strategy only requires the base station to be full-duplex and all the mobiles can be half-duplex. A sequential beamforming strategy with total T^{tr} training symbols from all users is described as follows

1. In the beginning of each block, no downlink data transmission is performed due to the lack of CSI knowledge. From Symbol 1 to Symbol $\frac{T^{\text{tr}}}{M}$, User 1 sends[1] training symbols to the base station. I define Symbols $(j-1)\frac{T^{\text{tr}}}{M}+1$ to Symbol $j\frac{T^{\text{tr}}}{M}$ to be Cycle j where User j sends its training symbols.

[1]For a given set of users, the user index $1, 2, ..., M$ may be randomly assigned in every coherence block to achieve fairness among users.

2. In cycle $j+1$, the base station transmits downlink data based on the updated ZF precoding matrix and beamform to User $1, 2, .., j$ that relies on all the received training symbols over the previous j cycles. Users who have finished training, i.e., User $1, 2, \cdots, j$, begin receiving downlink data. All receiving users decode the received signal by treating interference (both inter-beam and inter-node interference) as noise.

3. Repeat 2) till the end of T^{tr} symbols. The above full-duplex training part is referred to as *training phase*.

4. After all training are collected, only downlink data transmission takes place. This part is referred to as *half-duplex phase*. Fig. 3.2 provides an illustration of sequential beamforming. At the end of cycles, the base station updates its precoding vectors and serves all users whose CSI have been collected.

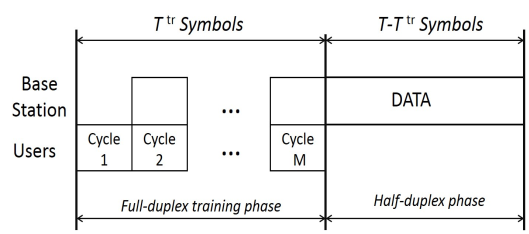


Figure 3.2: A sequential beamforming with training duration of T^{tr} symbols.

3.3 Spectral Efficiency Definition

I will compute the overall *spectral efficiency (*SE*)* for both phases as

$$\mathrm{SE}_{\mathrm{SqBf}} = \frac{T^{\mathrm{tr}}}{MT} \sum_{j=2}^{M} \frac{1}{M} \sum_{i=1}^{j-1} R\left(i, j\right) + \frac{T - T^{\mathrm{tr}}}{T} R^{\mathrm{data}}. \tag{3.1}$$

The first and second terms capture the spectral efficiency achieved during and after training, respectively. Rate expression $R\left(i, j\right)$ stands for downlink rate achieved by User i during cycle j. And R^{data} is the rate achieved after training, i.e., during half-duplex phase. By only considering second term in Eq. (3.1), the spectral efficiency of half-duplex counterpart is immediate as

$$\text{SE}_{\text{Hf}} = \frac{T - T^{\text{tr}}}{T} R^{\text{data}}. \tag{3.2}$$

The objective is to maximize the downlink spectral efficiency. I first quantify the influence of both inter-node and inter-beam interference on $R(i,j)$ and R^{data} in Section 3.4 for further analysis. During training, $i-1$ users are served on downlink in Cycle $i > 1$. Thus, the base station can either use P/M to serve each receiving user or adapt transmit power to each receiving user as $\frac{P}{i-1}$. I first focus on the former situation where no power adaptation is performed during training. In experiment validations in Chapter 5, I will analyze the influence of power adaptation.

In this thesis, performance of the following four systems is examined: sequential beamforming strategy with closed and open loop training, half-duplex with closed and open loop training. I differentiate between sequential beamforming and half-duplex systems through the subscripts SqBf and Hf, respectively. These are further detailed by the training type used by the system through another subscript Cl for closed loop and Op for open loop training. The superscript is used for the specification of system status. For example, $\text{SE}_{\text{SqBf}_{\text{Cl}}}$ stands for the spectral efficiency of system adopting sequential beamforming strategy with closed loop training.

3.4 Rate Performance with Inter-beam and Inter-node Interference

To optimize the spectral efficiency of proposed sequential beamforming strategy, I now quantify the influence of inter-node and inter-beam interference on downlink rate.

In ZF beamforming , $\mathbf{v}_i$ is chosen to be orthogonal to the channel realization of other users, i.e., $|\mathbf{v}_i \mathbf{h}_j| = 0, j \neq i$. In a genie-aided system where perfect CSI is immediately available, the base station beamforms to users without training, each user receives downlink data at rate R^{ZF} as

$$R^{\mathrm{ZF}} = \mathbb{E}\left[\log_2\left(1 + \frac{P}{M}\|\mathbf{h}_i\|^2\right)\right]. \tag{3.3}$$

Rate (3.3) can be viewed as an upper bound for all strategies that employ ZF, since neither training overhead nor inter-beam interference is included. When User k sends the training symbols, the received SINR of User i ($i < k$) is decided by both inter-beam and inter-node interference, which can be mathematically expressed as

$$\mathrm{SINR}_i = \frac{|\mathbf{h}_i^* \mathbf{v}_i|^2 \frac{P}{M}}{1 + \sum_{j \neq i} \frac{P}{M} |\mathbf{h}_i^* \mathbf{v}_j|^2 + |h_{ik} x_{\mathrm{tr}_k}|^2}, \quad i = 1, \cdots, k-1. \tag{3.4}$$

The (ergodic) rate of User i with SINR_i is then described as

$$R_i = \mathbb{E}\left[\log_2\left(1 + \mathrm{SINR}_i\right)\right], \quad i = 1, \cdots, k-1.$$

I now characterize the downlink rate $R(i,j)$ with a unified lower bound that is independent of cycle number. In Cycle k of training phase, the base station beamforms to Users $1, ..., k-1$. Each receiving user suffers inter-beam interference from the signals intended to the other $k-2$ users. The lower bound assumes that users receive additional inter-beam interference from signal intended to other $M-k$ users. Thus, in total, each receiving user suffers inter-beam interference coming from signals to $M-1$ users instead of $k-2$ users, and inter-node interference. The downlink rate of the receiving users in this scenario is denoted as R^{tr}, which is detailed as

$$R\left(i,j\right) \geqslant R^{\mathrm{tr}}\left(T^{\mathrm{tr}}\right), \quad j=2,...,M, \quad i=1,...,j. \tag{3.5}$$

It should be stressed that this lower bound is the same for all the receiving users in all cycles during training phase. Therefore, the rate expression of (3.1) reduces to

$$\mathrm{SE}_{\mathrm{SqBf}} \geqslant \frac{M-1}{2M}\frac{T^{\mathrm{tr}}}{T}R^{\mathrm{tr}}\left(T^{\mathrm{tr}}\right) + \left(1-\frac{T^{\mathrm{tr}}}{T}\right)R^{\mathrm{data}}\left(T^{\mathrm{tr}}\right), \tag{3.6}$$

here T^{tr} is the training duration of sequential beamforming strategy. Comparing to (3.2), I find in sequential beamforming strategy, on average, each user utilizes $\frac{M-1}{2M}$ fraction of training time to also receive downlink data while other users send training signal.

I now present the following lemma that quantifies the influence of interbeam and inter-node interference on downlink rate. The lemma enables further analysis in Chapters 4, 5 and 6. Following the notations used in [3], ΔR^{tr} and ΔR^{data} denote the upper bound of downlink rate gap (compared to perfect zero-forcing) during and after training, respectively.

Lemma 1 *In all cycles of training phase, the downlink data transmission rate of the receiving users, when another user is sending the training symbols are lower bounded* *as*

$$R^{\mathrm{tr}}\left(T^{\mathrm{tr}}\right) \geqslant R^{\mathrm{ZF}} - \log\left(\frac{1+\mathcal{P}_{\mathrm{IBI}}\left(T^{\mathrm{tr}}\right)+\alpha f P}{1+\frac{\alpha f P}{1+\frac{P}{M}}}\right) = R^{\mathrm{ZF}} - \Delta R^{\mathrm{tr}}\left(T^{\mathrm{tr}}\right), \qquad (3.7)$$

where $\mathcal{P}_{\mathrm{IBI}} = P(1+fP)^{-\frac{T^{\mathrm{tr}}}{M(M-1)}}$ *and* $\frac{P}{M}\frac{M-1}{1+\frac{T^{\mathrm{tr}}}{M}fP}$ *for closed and open loop training, respectively.*

Proof. Since perfect CSI is assumed available at each user, the rate loss can be upper bounded by

$$\begin{aligned}\Delta R^{\mathrm{tr}} \overset{a)}{\leqslant} & \mathbb{E}\left[\log\left(1+|\mathbf{h}_i\mathbf{v}_i|^2\frac{P}{M}\right)\right] - \mathbb{E}\left[\log\left(1+\frac{|\mathbf{h}_i\mathbf{v}_i|^2\frac{P}{M}}{1+\sum_{j\neq i}\frac{P}{M}|\mathbf{h}_i\mathbf{v}_j|^2+|\mathbf{h}_{ik}x_{\mathrm{tr}_k}|^2}\right)\right] \\ \leqslant & \mathbb{E}\left[\log\left(1+|\mathbf{h}_i\mathbf{v}_i|^2\frac{P}{M}\right)\right] - \mathbb{E}\left[\log\left(1+|\mathbf{h}_i\mathbf{v}_i|^2\frac{P}{M}+|\mathbf{h}_{ik}x_{\mathrm{tr}_k}|^2\right)\right] \\ & + \mathbb{E}\left[\log\left(1+\sum_{j\neq i}\frac{P}{M}|\mathbf{h}_i\mathbf{v}_j|^2+|\mathbf{h}_{ik}x_{\mathrm{tr}_k}|^2\right)\right]. \end{aligned} \qquad (3.8)$$

The first step is obtained following the same recipe in [3], and ignoring the positive term $\sum_{j\neq i}\frac{P}{M}|\mathbf{h}_i\mathbf{v}_j|^2$ in the minus term leads us to the result above. Term $\sum_{j\neq i}\frac{P}{M}|\mathbf{h}_i\mathbf{v}_j|^2$ and $|\mathbf{h}_{ik}x_{\mathrm{tr}_k}|^2$ stands for the interference due to imperfect precoding and full-duplex training,i.e., inter-beam and inter-node interference respectively. I refer them as $\mathcal{P}_{\mathrm{IBI}}$ and $\mathcal{P}_{\mathrm{INI}}$. Note the concavity of logarithm function, I apply Jensen's inequality and apply the characterization of $\mathcal{P}_{\mathrm{IBI}}$ from Chapter V and Remark 4.2 in [3] to obtain the theorem. ■

In the rate gap term ΔR^{tr}, inter-beam and inter-node interference is reflected through terms $\mathcal{P}_{\mathrm{IBI}}$ and $\alpha f P$, receptively. If more training symbols are sent, $\mathcal{P}_{\mathrm{IBI}}$ decreases. The decrease suggests that by obtaining more training symbols, the base station has better CSI estimates, which leads to less inter-beam interference. The inter-node interference term $\alpha f P$ does not change during the whole training phase.

It is emphasized that the lower bound presented in Lemma 1 is independent to the user index and cycle index, due to the use of Eq. (3.5).

As $T^{\mathrm{tr}} \to \infty$, the rate loss due to inter-beam interference vanishes and rate gap bound becomes $\log\left(\frac{1+\alpha f P}{1+\frac{\alpha f P}{1+\frac{P}{M}}}\right)$, which stands for the influence of inter-node interference and is noted as ΔR^{INI}. Term ΔR^{INI} can be viewed as a constant rate loss caused by inter-node interference during training phase. I will study the impact of this term in following analysis. Even when $\alpha \to \infty$, the rate loss term is still upper bounded by $\log(1 + P/M)$, which is obviously finite. This finite rate loss suggests that positive downlink rate gain can still be achieved asymptotically under the influence of inter-node interference, which is later confirmed in Chapter 6.

After training, each user continues to receive data till the end of the block. Thus, only the effect of inter-beam interference exists. I can conveniently obtain the rate expression R^{data} by setting $\alpha = 0$ in Lemma 1, which characterizes inter-beam interference with the help of [3].

Proposition 1 *The downlink transmission rate of User i after training is lower bounded by*

$$R^{\mathrm{data}} \geqslant R^{\mathrm{ZF}} - \Delta R^{\mathrm{data}} = R^{\mathrm{ZF}} - \log\left(1 + \mathcal{P}_{\mathrm{IBI}}\right), \tag{3.9}$$

where $\mathcal{P}_{\mathrm{IBI}} = P(1+fP)^{-\frac{T^{\mathrm{tr}}}{M(M-1)}}$ and $\frac{P}{M}\frac{M-1}{1+\frac{T^{\mathrm{tr}}}{M}fP}$ for closed and open loop training, respectively.

Similar to the influence of inter-beam interference during training, I find that the influence of inter-beam interference also decreases as training symbols amount increases.

Chapter 4

Training Time Optimization

In sequential beamforming strategy described in Section 3.2, base station obtains CSI from training symbols sent by users. While sending more training symbols helps designing a more accurate precoder and reduce inter-beam interference, longer training also implies higher overall inter-node interference. Thus, to obtain the best spectral efficiency performance, there has to be a balance between inter-beam and inter-node interference achieved by optimizing training duration. In this chapter, I analyze optimal training duration for sequential beamforming and traditional half-duplex strategy with both closed and open loop training. The optimization

$$T_s^{\mathrm{tr}*} = \arg\max \mathrm{SE}_s\left(T^{\mathrm{tr}}\right), \tag{4.1}$$

is implemented for $s = \mathrm{SqBf}_{\mathrm{Cl}}$, $\mathrm{SqBf}_{\mathrm{Op}}$, $\mathrm{Hf}_{\mathrm{Cl}}$ and $\mathrm{Hf}_{\mathrm{Op}}$. I use superscript $*$ to denote optimal solution. The optimality in this thesis is under the criteria of maximum spectral efficiency. I consider T^{tr} to be continuous in the characterization of both training duration and spectral efficiency. The accent $\tilde{}$ is used to represent approximation in Sections 4.1, 4.2, where closed form analytical solutions are not feasible.

4.1 Optimal Training Duration of Sequential Beamforming

In this section, I solve the optimization problem posed in (4.1) by applying a *Marginal Analysis* [18] technique. As shown below, marginal analysis allows accurate closed form approximation for systems with both closed and open loop training.

Proposition 2 *The optimal training duration of sequential beamforming strategy happens at the point where the spectral benefit of adding training symbols equals to loss, i.e.,*

$$\frac{\partial \mathrm{SE}_{\mathrm{SqBf}}\left(T^{\mathrm{tr}*}\right)}{\partial T^{\mathrm{tr}}} = 0. \tag{4.2}$$

Proof. Since the mobile nodes are half-duplex, more training implies less time for downlink data reception. The influence of inter-beam interference on rate in Proposition 1 suggests that the increase in rate with respect to training increase is monotonically decreasing. Combining with the facts above, I conclude that the benefit in spectral efficiency from M additional training symbols is monotonically decreasing as training grows. From Lemma 1, the influence of longer inter-node interference duration is monotonically increasing. Thus, the spectral efficiency $\mathrm{SE}_{\mathrm{SqBf}}$ is concave in T^{tr}. Therefore, a unique $T^{\mathrm{tr}*}_{\mathrm{SqBf}}$ exists to optimize the spectral efficiency. ■

Since solving (4.2) is challenging, I then apply Taylor's expansion to and ignore all the expansion terms, which yields

$$\mathrm{SE}_{\mathrm{SqBf}}\left(\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf}}\right) \approx \mathrm{SE}_{\mathrm{SqBf}}\left(\widetilde{T}^{*}_{\mathrm{SqBf}} + M\right). \tag{4.3}$$

With the help of spectral efficiency characterization provided in Lemma 1, expanding both sides of (4.3) leads to

$$\begin{aligned}&\frac{M-1}{2M}\frac{\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}}{T}\left[R^{\mathrm{tr}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)-R^{\mathrm{tr}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)\right]+\frac{T-\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}}{T}\left[R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)-R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)\right]\\ =&\frac{M-1}{2T}\left[R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)-R^{\mathrm{tr}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)\right]+\frac{M+1}{2T}R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right).\end{aligned} \tag{4.4}$$

The left side in (4.4) is the benefit obtained in spectral efficiency by adding M training symbols. I note this benefit as *Marginal Utility* (MU). The MU comes from two facts: more training can reduce inter-beam interference both during and after training, which corresponds to the first and second term on left side of (4.4), respectively.

More training symbols results in lower inter-beam interference in half-duplex phase. By using Proposition 1, it can be expressed as rate increase of

$$R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)-R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)=\log\left(1+\frac{\mathcal{P}_{\mathrm{IBI}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)-\mathcal{P}_{\mathrm{IBI}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)}{1+\mathcal{P}_{\mathrm{IBI}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)}\right). \tag{4.5}$$

I refer the rate improvement due to less inter-beam interference as $\delta R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)$. In the same spirit, the rate increase of R^{tr} by lower inter-beam interference during training phase is

$$R^{\mathrm{tr}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}+M\right)-R^{\mathrm{tr}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)\approx\delta R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right).$$

I find that, the rate improvement due to less inter-beam interference is almost constant during and after training. Applying the two results above, the marginal utility is then

$$MU = \left(1 - \frac{M+1}{2M}\frac{\widetilde{T}^{\text{tr}}_{\text{SqBf}}}{T}\right)\delta R^{\text{data}}\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}}\right) \approx \left(1 - \frac{1}{2}\frac{\widetilde{T}^{\text{tr}}_{\text{SqBf}}}{T}\right)\delta R^{\text{data}}\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}}\right), \quad (4.6)$$

which suggests a rate increase of $\delta R^{\text{data}}\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}}\right)$ in $1 - \frac{1}{2}\frac{\widetilde{T}^{\text{tr}}_{\text{SqBf}}}{T}$ fraction of the whole block is achieved by adding M training symbols. Later I further obtain the marginal utility of half-duplex counterparts by the same process with SE_{SqBf} substituted as SE_{Hf}.

On the right side of (4.4) is the loss of spectral efficiency, referred as *Marginal Cost* (MC), due to longer training and comprises two parts. The first term corresponds to the fact that additional inter-node interference is suffered in $\frac{M-1}{2}$ of the M symbols suffer . The second term reflects that the rest $\frac{M+1}{2}$ training symbols are still not able to be utilized for downlink. With the help of Lemma 1 and Proposition 1, the rate loss due to additional inter-node interference is

$$R^{\text{data}}\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}} + M\right) - R^{\text{tr}}\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}} + M\right) = \log\left(\frac{1 + \frac{\alpha f P}{1+\mathcal{P}_{\text{IBI}}\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}}+M\right)}}{1 + \frac{\alpha f P}{1+\frac{P}{M}}}\right) \approx \Delta R^{\text{INI}}.$$

The downlink rate loss due to inter-node interference in training phased is almost independent of training duration. The downlink rate is immediate as R^{ZF}. The marginal cost is then

$$MC = \frac{M-1}{2T}\Delta R^{\text{INI}} + \frac{M+1}{2T}R^{\text{ZF}} \approx \frac{M}{2T}\left[\Delta R^{\text{INI}} + R^{\text{ZF}}\right], \quad (4.7)$$

which is independent of training symbol amount. The approximation made in Eq. (4.7) holds for large T. In training phase, Eq. (4.7) suggests that, on average, each user receives downlink data during half of the training time.

The unique optimal point $T_{\mathrm{SqBf}}^{\mathrm{tr}*}$ happens at the point where the spectral efficiency benefit (marginal utility) and cost (marginal cost) break even, i.e., $MU = MC$. Using (4.6) and (4.7), it is mathematically captured as

$$\left(1 - \frac{M+1}{2M}\frac{\widetilde{T}_{\mathrm{SqBf}}^{\mathrm{tr}}}{T}\right)\delta R^{\mathrm{data}}\left(\widetilde{T}_{\mathrm{SqBf}}^{\mathrm{tr}}\right) = \frac{M-1}{2T}\Delta R^{\mathrm{INI}} + \frac{M+1}{2T}R^{\mathrm{ZF}}. \qquad (4.8)$$

The optimal training duration of sequential beamforming strategy with closed and open loop training is then obtained by the inter-beam interference characterization provided in Proposition 1.

Theorem 1 *The approximation of optimal training duration* $T_{\mathrm{SqBf_{Cl}}}^{\mathrm{tr}*}$ *of sequential beamforming strategy with closed loop training is*

$$\widetilde{T}_{\mathrm{SqBf_{Cl}}}^{\mathrm{tr}*} = M\left(M-1\right)\frac{\log\left(TP\right) - \log\left(c\right) + \log\left(\left(1+fP\right)^{\frac{1}{M-1}} - 1\right)}{\log\left(1+fP\right)}, \qquad (4.9)$$

where $c = \frac{M-1}{2}\Delta R^{\mathrm{INI}} + \frac{M+1}{2}R^{\mathrm{ZF}}$.

Proof. The rate increase term $\delta R^{\mathrm{data}}\left(T^{\mathrm{tr}}\right)$ of (4.8) is characterized by applying Proposition 1 as

$$\delta R^{\mathrm{data}}\left(T^{\mathrm{tr}}\right) \approx P\left(1+fP\right)^{-\frac{T^{\mathrm{tr}}}{M(M-1)}}\left[\log\left(1+fP\right)^{\frac{1}{M-1}} - 1\right].$$

Here the last step is directly obtained by using Taylor expansion. Substituting into (4.8), I have

$$\left(1 - \frac{M+1}{2M}\frac{T^{\mathrm{tr}}}{T}\right)P\left(1+fP\right)^{-\frac{T^{\mathrm{tr}}}{M(M-1)}}\left[\left(1+fP\right)^{\frac{1}{M-1}} - 1\right] = \frac{M-1}{2T}\Delta R^{\mathrm{INI}} + \frac{M+1}{2T}R^{\mathrm{ZF}}.$$

Noticing that it is an transcendental equation which is challenging to solve. Then omitting the $\frac{M+1}{2M}\frac{T^{\mathrm{tr}}}{T}$ term leads us to the theorem. This approximation is valid for large T. ■

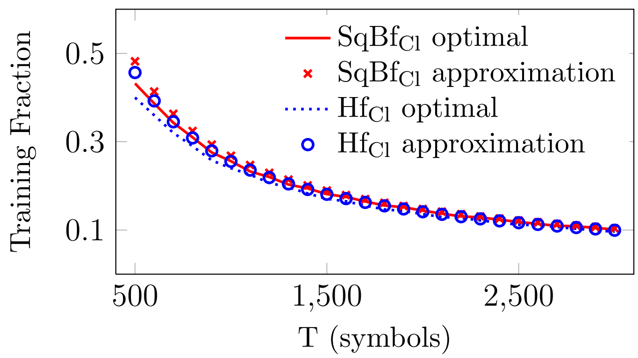
Several interesting observations are made here. First, as T grows, for closed loop training based sequential beamforming strategy, the optimal training duration scales as $\log T$. I later observe similar scaling law for its half-duplex counterpart. This scaling law, to my best knowledge, has not been reported before. Second, as inter-node interference becomes stronger, less training is sent to account for the higher *cost* of training. Third, The optimal training symbols amount scales as $\frac{\log P}{\log(1+fP)}$ with respect to P, which is less than $\log P$. From Theorem 4 in [19], I conclude that full multiplexing is not obtained as P grows. Fourth, the number of training symbols increases almost quadratically with respect to the number of users M, which lies well with the intuition that training symbols number scales with the number of total channel coefficient.

Fig. 4.1a provides both optimal training duration and its approximation of sequential beamforming strategy with closed loop training at $P = 15$dB with $f = 0.1$ and $\alpha = 0.3$. Since optimal training duration scales as $\log T$, the fraction of training duration actually scales as $\frac{\log T}{T}$ and is further confirmed numerically.

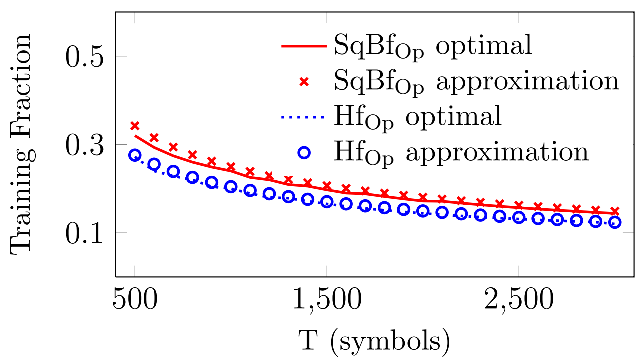
In the same spirit, the optimal training duration of sequential beamforming strategy with open loop training is obtained as below.

Theorem 2 *The approximation of optimal training duration $T^{\mathrm{tr}*}_{\mathrm{SqBf}_{\mathrm{Op}}}$ of sequential beamforming strategy with open loop training is*

$$\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf}_{\mathrm{Op}}} = \sqrt{\frac{(M-1)\,T}{f\,\frac{(M-1)\Delta R^{\mathrm{INI}}+(M+1)R^{\mathrm{ZF}}}{2M}}} \approx \sqrt{\frac{(M-1)\,T}{f\,\frac{\Delta R^{\mathrm{INI}}+R^{\mathrm{ZF}}}{2}}}.$$



(a) Closed Loop Systems



(b) Open Loop Systems

Figure 4.1: Fraction of optimal training duration fraction of 8×8 sequential beamforming and half-duplex strategy.

Proof. For open loop training based systems, the rate improvement due to more training symbols can be obtained by using Proposition 1 as

$$\delta R^{\text{data}}\left(T^{\text{tr}}\right) \approx \frac{(M-1)M}{f\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}}\right)^2},$$

where the last step is the direct result of Maclaurin series. Combining with (4.8) leads to

$$\left(1-\frac{M+1}{2M}\frac{\widetilde{T}^{\text{tr}}_{\text{SqBf}}}{T}\right)\frac{(M-1)M}{f\left(\widetilde{T}^{\text{tr}}_{\text{SqBf}}\right)^2} = \frac{M-1}{2T}\Delta R^{\text{INI}} + \frac{M+1}{2T}R^{\text{ZF}},$$

whose solution leads to the theorem. ■

In Theorem 2, I observe that for large T, the optimal training duration scales as $\sqrt{T}$. The optimal fraction of time resource devoted into training then decreases as $\frac{1}{\sqrt{T}}$, which is slower than that of closed loop training systems. The scaling rate of $\sqrt{T}$ has been observed in various open loop training based systems. For example, similar scaling has been observed for both half-duplex MIMO broadcast channels with analog feedback[7] and point-to-point MIMO[20]. Such scaling rate has also been observed in MIMO downlink with full-duplex base station and full-duplex node [11]. I find an identical scaling law is shared by open loop training based systems. In Chapter 4.2, I find that the half-duplex counterparts also follow the respective scaling laws. Numerical results presented in Fig. 4.1b confirm the observation at $P = 15$dB with $f = 0.1$ and $\alpha = 0.3$.

For sequential beamforming strategy with open loop training, as the number of transmitting antennas M increases, the optimal training duration scales as $\sqrt{M}$, unlike M^2 scaling in closed loop training based systems. The slower scaling rate in open loop training systems suggest a lower overhead cost in systems with large number of users. Further analysis results in Chapter 6 confirm this observation.

Similar to closed loop systems, as inter-node interference increases, larger rate loss during training is expected in open loop training systems. Thus, one should use fewer symbols for training to account for this effect. Another interesting finding is that even when no inter-node interference exists, the optimal training duration is not T. The reason is that sequential beamforming strategy is only able to partially recover the training overhead.

4.2 Optimal Training Duration of Half-duplex

In this section, I apply the marginal analysis method developed in Chapter 4.1 to obtain approximations of optimal training duration of half-duplex systems. As a by-product of analysis in Chapter 4.1, I find the marginal utility, which stands for the gain in spectral efficiency of adding M more training symbols, for half-duplex systems is

$$MU = \frac{T - \widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf}}}{T}\left[R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf}} + M\right) - R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf}}\right)\right] = \frac{T - \widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf}}}{T}\delta R^{\mathrm{data}}\left(\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf}}\right). \tag{4.10}$$

The marginal cost of half-duplex strategy is conveniently obtained by ignoring inter-beam interference after training as

$$MC = \frac{M}{T}R^{\mathrm{ZF}}. \tag{4.11}$$

The approximation is obtained by letting marginal cost and utility be equal in half-duplex systems. I further proceed by applying the rate characterization provided in Proposition 1. The result regarding closed loop training based systems is first presented with open loop result as follows.

Theorem 3 *The approximation of optimal training duration* $T^{\mathrm{tr}*}_{\mathrm{Hf}_{\mathrm{Cl}}}$ *of half-duplex* *strategy with closed loop training is*

$$\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}} = M\left(M-1\right)\frac{\log\left(TP\right)-\log\left(c\right)+\log\left(\left(1+fP\right)^{\frac{1}{M-1}}-1\right)}{\log\left(1+fP\right)}, \tag{4.12}$$

where $c = MR^{\mathrm{ZF}}$.

Proof. The rate increase term $\delta R^{\mathrm{data}}\left(T^{\mathrm{tr}}\right)$ is immediately approximated by using results from the proof of Theorem 1. Applying this rate characterization term to evaluate (4.10) gives

$$\left(1-\frac{T^{\mathrm{tr}}}{T}\right)\frac{\left(1+fP\right)^{\frac{1}{M-1}}-1}{P\left(1+fP\right)^{-\frac{T^{\mathrm{tr}}}{M(M-1)}}} = \frac{M}{T}R^{\mathrm{ZF}},$$

which is an transcendental equation. Following the same step of the proof of Theorem 1, I omit the $\frac{T^{\mathrm{tr}}}{T}$ to obtain the theorem. ■

I observe the optimal training duration $\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}$ and fraction $\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}}{T}$ of half-duplex counterpart to share the same scaling law as sequential beamforming strategy in Theorem 1. It should also be noted that as the number of antenna M increases, the optimal number of training symbols also increases quadratically. Comparing to Theorem 1, the only difference lies in the $\log(c)$ term in the numerator, which can be viewed as the normalized marginal cost of the strategy. This finding also suggests that the optimal training duration difference between sequential beamforming strategy and half-duplex is a constant gap which is independent of block length. Therefore, the difference in the fraction of training time decreases as T increases.

Theorem 4 *The approximation of training duration* $T^{\mathrm{tr}*}_{\mathrm{Hf_{Op}}}$ *that optimizes spectral efficiency of open loop training half-duplex systems is*

$$\widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf_{Op}}} = \sqrt{\frac{\left(M-1\right)T}{fR^{\mathrm{ZF}}}}. \tag{4.13}$$

Proof. Using the rate increase characterization term $\delta R^{\mathrm{data}}\left(T^{\mathrm{tr}}\right)$ from the proof of Theorem 2 and further applying (4.10), (4.11) gives

$$\left(1-\frac{T^{\mathrm{tr}}}{T}\right)\frac{(M-1)M}{f\left(\widetilde{T}^{\mathrm{tr}}_{\mathrm{SqBf}}\right)^{2}}=\frac{M}{T}R^{\mathrm{ZF}},$$

whose solution is the theorem. ■

Similar to sequential beamforming system with open loop training, the optimal training duration scales with T and M at the rate of $\sqrt{T}$ and $\sqrt{M-1}$, respectively, as block length and number of antennas grows. It should also be noted that by substituting the normalized marginal cost term $\frac{\Delta R^{\mathrm{INI}}+R^{\mathrm{ZF}}}{2}$ as the half-duplex system's normalized marginal cost term R^{ZF}, I can also obtain Theorem 4. Instead of assuming each user has the same power constraint P of the base station [7], this approximation results further take the limitation of user power into consideration.

Remark 1 *By comparing the training duration of closed and open loop systems, I find that training type dominates the scaling of optimal training time with respect to both block length and number of antennas. For example, as block length T grows, optimal training duration for both sequential beamforming and half-duplex systems with open loop training scale as $\sqrt{T}$ while the closed loop training counterparts scale as* $\log T$.

The closed-form approximations are further applied in Chapter 5 to characterize the spectral efficiency of sequential beamforming and half-duplex strategy with optimal training duration.

Chapter 5

Spectral Efficiency Evaluation

In multiuser MIMO downlink systems where base station obtains CSI through training, spectral efficiency is reduced due to imperfect CSI and training overhead resulting from its acquisition. To quantify the spectral efficiency loss of different systems, I compare the spectral efficiency of different systems with optimal training duration to a system where perfect CSI is available for base station at no cost. It can be visualized as a genie provides perfect CSI to base station at the beginning of each block, thus it serves as an upper bound for systems' performance with ZF; I label the perfect CSI system as *genie-aided system*. The spectral efficiency achieved is $\mathrm{SE}^{\mathrm{ZF}} = R^{\mathrm{ZF}}$, which is presented in (3.3). The rate loss due to training overhead is then

$$\Delta\mathrm{SE}_s = \mathrm{SE}^{\mathrm{ZF}} - \mathrm{SE}_s, \quad s \in \{\mathrm{SqBf}_{\mathrm{Cl}}, \mathrm{SqBf}_{\mathrm{Op}}, \mathrm{Hf}_{\mathrm{Cl}}, \mathrm{Hf}_{\mathrm{Op}}\}. \tag{5.1}$$

In the investigation, I also consider inter-node interference free scenario to gain further insights on the sequential beamforming performance. I use notation $\cancel{\mathrm{INI}}$ to describe inter-node interference free systems in figure legends. The spectral efficiency of half-duplex counterparts are also analyzed for comparison.

5.1 Spectral Efficiency of Sequential Beamforming

Theorem 5 *The spectral efficiency loss of closed loop training based sequential beamforming system with respect to genie-aided system is upper-bounded as*

$$\Delta\mathrm{SE}_{\mathrm{SqBf_{Cl}}}\left(T^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}\right) \leqslant \left(\frac{M-1}{2M}\Delta R^{\mathrm{INI}} + \frac{M+1}{2M}R^{\mathrm{ZF}}\right)\frac{M\left(M-1\right)}{\log\left(1+fP\right)}\frac{\log T}{T} + o\left(\frac{\log T}{T}\right). \tag{5.2}$$

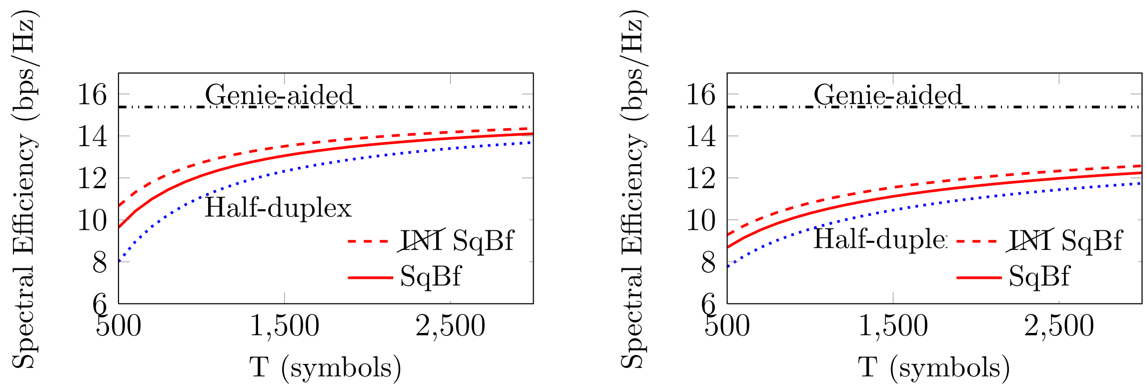
Proof. Evaluating the achieved spectral efficiency of sequential beamforming strategy with approximated optimal training duration $\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}$ obtained in Theorem 1 gives upper bound

$$\begin{aligned}\Delta\mathrm{SE}_{\mathrm{SqBf_{Cl}}}\left(T^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}\right) \leqslant & R^{\mathrm{ZF}} + \frac{M-1}{2M}\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}}{T}\Delta R^{\mathrm{INI}} \\ & - \left(1 - \frac{M+1}{2M}\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}}{T}\right)\left[R^{\mathrm{ZF}} - \log\left(1 + P\left(1+fP\right)^{-\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}}{M(M-1)}}\right)\right].\end{aligned}$$

Omitting negative term $-\frac{M+1}{2M}\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}}{T}\log\left(1 + P\left(1+fP\right)^{-\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Cl}}}}{M(M-1)}}\right)$ and sorting the small term with respect to $\frac{1}{\sqrt{T}}$ lead to the theorem. ■

Here $o\left(\frac{\log T}{T}\right)$ is a term that vanishes as T increases, i.e., $\lim_{T\to\infty}\frac{o(\log T/T)}{\log T/T} = 0$. It can be observed that by employing higher training power f, or by using longer block length T, the spectral efficiency overhead decreases. However, in a more realistic scenario where user power and block length are inherently limited, the spectral efficiency loss cannot be neglected. Based on expression (5.2), some observations are made for sequential beamforming strategy with closed loop training. i) The spectral efficiency loss scales quadratically as M increases, which indicates sequential beamforming strategy with closed loop training is not a good choice for systems with large number of antennas. ii) The spectral efficiency loss decreases rapidly as $\frac{\log T}{T}$ as T increases. Fig. 5.1 presents the spectral efficiency policy for different strategy versus

T. I observe that as T grows, spectral efficiency loss drops rapidly for systems with closed loop training, which agrees with analysis. iii) As inter-node interference level decreases, smaller term ΔR^{INI} suggests less spectral efficiency loss which is confirmed in Fig. 5.1.



(a) Closed Loop Systems

(b) Open Loop Systems

Figure 5.1: Spectral efficiency of 8×8 systems of sequential beamforming and half-duplex counterparts.

Theorem 6 *The spectral efficiency loss of open loop training based sequential beamforming system with respect to genie-aided system is upper-bounded as*

$$\Delta \mathrm{SE}_{\mathrm{SqBf_{Op}}}\left(T^{\mathrm{tr}*}_{\mathrm{SqBf_{Op}}}\right) \leqslant 2\sqrt{\frac{(M-1)\left[\frac{M-1}{2M}\Delta R^{\mathrm{INI}} + \frac{M+1}{2M} R^{\mathrm{ZF}}\right]}{fT}} + o\left(\frac{1}{\sqrt{T}}\right). \tag{5.3}$$

Proof. Following similar analysis as that of the proof of Theorem 5, I substitute approximation of optimal training duration from Theorem 2 into (3.6) to characterize the spectral efficiency loss as

$$\begin{aligned}\Delta \mathrm{SE}_{\mathrm{SqBf_{Op}}}\left(T^{\mathrm{tr}*}_{\mathrm{SqBf_{Op}}}\right) \leqslant & R^{\mathrm{ZF}} + \frac{M-1}{2M}\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Op}}}}{T}\Delta R^{\mathrm{INI}} \\ & - \left(1 - \frac{M+1}{2M}\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Op}}}}{T}\right)\left[R^{\mathrm{ZF}} - \log\left(1 + \frac{(M-1)\frac{P}{M}}{1 + f\widetilde{T}^{*}_{\mathrm{SqBf_{Op}}} P/M}\right)\right].\end{aligned}$$

Then dropping negative term $-\frac{M+1}{2M}\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{SqBf_{Op}}}}{T}\log\left(1 + \frac{(M-1)\frac{P}{M}}{1+f\widetilde{T}^{*}_{\mathrm{SqBf_{Op}}} P/M}\right)$ and sorting small term with respect to $\frac{\log T}{T}$ lead to the theorem. ■

In (5.3), the term $o(\frac{1}{\sqrt{T}})$ shows that the additional spectral efficiency loss term vanishes in systems with large T. Interestingly, I observe a different scaling law with respect to both block length and antenna number. For sequential beamforming with open loop training, the spectral efficiency loss grows only at the rate of $\sqrt{M-1}$, which is slower than $M(M-1)$ in Theorem 5. Thus, for systems with large number of users, sequential beamforming with open loop training is advisable.

On the other hand, the spectral efficiency loss decreases as $\frac{1}{\sqrt{T}}$, which is confirmed from Fig. 5.1 at $P = 15$dB for $f = 0.1$. It should be further noted that the decreasing rate (w.r.t. T) is slower than that of systems with closed loop training ($\log T/T$). From Fig. 5.1, an increase spectral efficiency is achieved by sequential beamforming strategy at low inter-node interference level.

Fig. 5.1 plots closed and open loop training based system with power controlled sequential beamforming, sequential beamforming and half-duplex strategy. I observe a further spectral efficiency increase by allowing power adaptation during training. Having established performance bounds for the spectral efficiency loss of sequential beamforming policy, I now investigate the performance of the half-duplex counterpart to compute the gains of proposed sequential beamforming strategy.

5.2 Spectral Efficiency of Half-duplex

Theorem 7 (Half-duplex system) *The spectral efficiency loss of half-duplex systems with closed loop training respect to genie-aided system is upper bounded as*

$$\Delta \mathrm{SE}_{\mathrm{Hf_{Cl}}}\left(T^*_{\mathrm{Hf_{Cl}}}\right) \leqslant R^{\mathrm{ZF}} \frac{M\,(M-1)}{\log\left(1+fP\right)} \frac{\log T}{T} + o\left(\frac{\log T}{T}\right). \tag{5.4}$$

Proof. Similar to systems adopting sequential beamforming strategy, the spectral efficiency gap of the half-duplex counterparts with respect to the genie-aided scenario can be immediately upper bounded by evaluating the sub-optimal scheme $\mathrm{SE}_{\mathrm{Hf}_{\mathrm{Cl}}}\left(\widetilde{T}^{*}_{\mathrm{Hf}_{\mathrm{Cl}}}\right)$

$$\Delta\mathrm{SE}_{\mathrm{Hf}_{\mathrm{Cl}}}\left(T^{\mathrm{tr}*}_{\mathrm{Hf}_{\mathrm{Cl}}}\right) \overset{(a)}{\leqslant} \frac{T^{\mathrm{tr}*}_{\mathrm{Hf}_{\mathrm{Cl}}}}{T} R^{\mathrm{ZF}} + \log\left(1 + P\left(1 + fP\right)^{-\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf}_{\mathrm{Cl}}}}{M(M-1)}}\right) = R^{\mathrm{ZF}} \frac{M\left(M-1\right)}{\log\left(1+fP\right)} \frac{\log T}{T} + o\left(\frac{\log T}{T}\right).$$

Inequality (a) is the result of dropping negative term $-\frac{\widetilde{T}^{*}_{\mathrm{Hf}_{\mathrm{Cl}}}}{T}\log\left(1 + P\left(1+fP\right)^{-\frac{\widetilde{T}^{\mathrm{tr}*}_{\mathrm{Hf}_{\mathrm{Cl}}}}{M(M-1)}}\right)$. Applying training time approximation in Theorem 3 gives the final step. ■

Here I observe the same scaling of spectral efficiency loss with respect to number of antennas M and block length T as in sequential beamforming strategy with closed loop training. Actually, I can obtain Theorem 7 by replacing the normalized marginal cost term $\frac{M-1}{2M}\Delta R^{\mathrm{INI}} + \frac{M+1}{2M} R^{\mathrm{ZF}}$ in Theorem 5 with R^{ZF}. The main reason is the similarity between the marginal utility term in sequential beamforming and half-duplex system.

Theorem 8 *The spectral efficiency loss of half-duplex systems with open loop training respect to genie-aided system is upper bounded as*

$$\Delta\mathrm{SE}_{\mathrm{Hf}_{\mathrm{Op}}}\left(T^{*}_{\mathrm{Hf}_{\mathrm{Op}}}\right) \leqslant 2\sqrt{\frac{(M-1)R_{\mathrm{ZF}}}{fT}}. \tag{5.5}$$

Proof. Inspired by [7], spectral efficiency gap with respect to genie-aided situation can be immediately upper bounded by evaluating $\mathrm{SE}_{\mathrm{Hf}_{\mathrm{Op}}}\left(\widetilde{T}^{*}_{\mathrm{Hf}_{\mathrm{Op}}}\right)$

$$\begin{aligned}\Delta\mathrm{SE}_{\mathrm{Hf}_{\mathrm{Op}}}\left(T^{*}_{\mathrm{Hf}_{\mathrm{Op}}}\right) \leqslant & \mathrm{SE}^{\mathrm{ZF}} - \mathrm{SE}_{\mathrm{Hf}}\left(\widetilde{T}^{*}_{\mathrm{Hf}_{\mathrm{Op}}}\right) \overset{(a)}{\leqslant} \frac{\widetilde{T}^{*}_{\mathrm{Hf}}}{T} R^{\mathrm{ZF}} + \log\left(1 + \frac{(M-1)\frac{P}{M}}{1 + f\widetilde{T}^{*}_{\mathrm{Hf}_{\mathrm{Op}}} P/M}\right)\\ & \leqslant \sqrt{\frac{(M-1)R^{\mathrm{ZF}}}{fT}} + \sqrt{\frac{R^{\mathrm{ZF}}(M-1)}{fT}} = 2\sqrt{\frac{(M-1)R^{\mathrm{ZF}}}{fT}}.\end{aligned}$$

Inequality (a) is obtained by dropping negative term $-\frac{\widetilde{T}^*_{\mathrm{Hf}_{\mathrm{Op}}}}{T}\log\left(1+\frac{(M-1)\frac{P}{M}}{1+f\widetilde{T}^*_{\mathrm{Hf}}P/M}\right)$. The next step is the result of Maclaurin expansion of the logarithm term, which is tight for large T. ■

The spectral efficiency loss scaling with both block length T and number of antennas M is identical to that of sequential beamforming system with open loop training. Theorem 8 can be viewed as changing the normalized marginal cost of sequential beamforming strategy into its half-duplex counterpart.

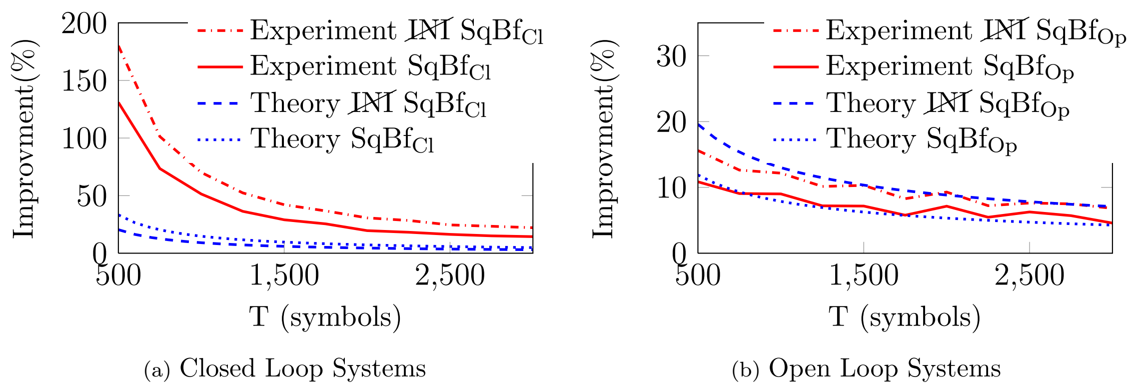


Figure 5.2: Spectral efficiency improvement percentage of 8×8 sequential beamforming strategy with experimental data validation.

Comparing Theorem 7 and Theorem 8 to their sequential beamforming counterparts, spectral efficiency loss is substantially reduced by adopting sequential beamforming strategy. Sequential beamforming strategy improves spectral efficiency performance significantly.

I now further validate sequential beamforming strategy for a 8×8 systems with experimental data from [1]. The base station antenna number is chosen to be 8, which is the maximal number currently supported by LTE. In the experiment, the authors measure the channel realization between a 8×9 two-dimensional antenna array and 12 randomly located users. The measurement is conducted in both indoor and outdoor environment.

The spectral efficiency of both sequential beamforming and half-duplex counterparts are evaluated through Monte Carlo method with 3000 iterations for each systems. In each iteration, 8 random users and first 8 antenna in a random horizontal antenna array is selected. In systems with closed loop training, feedback bits are equally divided to real and imaginary part with 3 bits for integer part and rest for fractional part. In the simulation, I further assumed that the base station will adapt the downlink transmit power in training phase, which is not allowed in previous theoretical analysis. Thus, in cycle $i > 1$, when Users $1, .., i-1$ receive data on downlink, each receiving user signal will be precoded with power constraint $\frac{P}{i-1}$.

Fig. 5.2 confirms the spectral efficiency improvement achieved by sequential beamforming $P = 15$dB with $f = 0.1$ and $\alpha = 0.3$. The experiment in figure refers to the simulation results obtained with experimental data. Theory refers to results in Fig. 5.1. The spectral efficiency improvement achieved in Fig. 5.1 is also shown for reference. Similar to results in Fig. 5.1, sequential beamforming demonstrates a significant spectral efficiency improvement.

For example, in a typical LTE system, there are around 500 to 2100 symbols in each slot depending on the available bandwidth (1.4 MHz to 5 MHz). When the block length equals 500 symbols, proposed sequential beamforming strategy attains a over 130% and 12% spectral improvement under the influence of inter-node interference for closed and open loop training systems, respectively. As T grows, the performance of half-duplex counter parts grows, thus the improvement by sequential efficiency decreases. From Fig. 5.1, I conclude that a notable spectral efficiency improvement is still observed even for systems with long block length ($T = 3000$). Lower inter-node level does show a better spectral efficiency improvement in Fig. 5.2.

Remark 2 *For closed loop systems, sequential beamforming demonstrates a higher spectral efficiency compared to the results in Fig. 5.1a, where the downlink power for each user is fixed to be P/M in training phase. This suggests that proper power* *adaptation can increase the performance of sequential beamforming dramatically. On the other hand, I find power adaptation does not influence the spectral efficiency improvement of open loop system.*

In this chapter, a significant spectral efficiency improvement by adopting sequential beamforming is demonstrated. Power adaptation influences the spectral efficiency of closed loop systems significantly. In Chapter 6, I further compare the spectral efficiency asymptotically to remove the influence of power adaptation and obtain more general results. As a byproduct of the analysis, a comparison between closed and open loop training method in high SNR regime is also presented.

Chapter 6

High SNR Analysis

In Chapter 4 and Chapter5, with optimized training duration, sequential beamforming strategy exhibits significant spectral efficiency improvement in finite SNR regime. In this chapter, I further continue the investigation of sequential beamforming strategy in the high SNR regime. Notation $\doteq$ is used to denote exponential equality, i.e.,

$$g(P) \doteq P^{\zeta} \Leftrightarrow \lim_{\mathrm{P}\to\infty} \frac{\log g(P)}{\log P} = \zeta.$$

Since $fP \doteq P$, I now assume the power constraint for training is P^{ζ} to account for the limitation of training power. In order to capture the spectral efficiency asymptotically, I use the multiplexing gain metric r, which can be mathematically captured as

$$\lim_{P\to\infty} \frac{\mathrm{SE}_{S}(\zeta, T^{\mathrm{tr}})}{\log P} \doteq r_s, \quad s \in \{\mathrm{SqBf}_{\mathrm{Cl}}, \mathrm{SqBf}_{\mathrm{Op}}, \mathrm{Hf}_{\mathrm{Cl}}, \mathrm{Hf}_{\mathrm{Op}}\}. \tag{6.1}$$

The objective is to maximize the spectral efficiency asymptotically under certain training power constraint, which is mathematically captured as, for $s = \mathrm{SqBf}_{\mathrm{Cl}}$, $\mathrm{SqBf}_{\mathrm{Op}}$, $\mathrm{Hf}_{\mathrm{Cl}}$ and $\mathrm{Hf}_{\mathrm{Op}}$,

$$\max_{T^{\mathrm{tr}}} \quad r_s\left(\zeta, T^{\mathrm{tr}}\right). \tag{6.2}$$

I first present the results regarding sequential beamforming system with closed loop training. The results for sequential beamforming strategy with open loop training then follows. In the asymptotic characterization of sequential beamforming strategy, for mathematical concision, I consider the fraction of full-duplex transmission term $\frac{M-1}{2M}$ in (3.6) to be $\frac{1}{2}$. This approximation is validate for systems with large numbers of antennas.

6.1 Sequential Beamforming with Closed Loop Training

In this section, I consider the relationship between multiplexing gain r and training power constraint ζ. Similar to the approach in finite SNR regime, I first present a lemma capturing the influence of inter-beam and inter-node interference in high SNR regime, then the spectral efficiency will be characterized. I define $\theta = \left(M\left(M-1\right)\right)/T$, which is useful in analysis.

Lemma 2 *In closed loop systems, the downlink data transmission rate during training phase, under the influence of inter-beam and inter-node interference, is*

$$\lim_{P\to\infty} \frac{R^{\mathrm{tr}}\left(T^{\mathrm{tr}}\right)}{\log P} = \max\left(\min\left(\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T}, 1-\zeta\right), 0\right). \tag{6.3}$$

Proof. The proof is obtained by substituting the training power in the proof of Lemma 1 fP with P^{ζ}. ■

Interestingly, I observe the impact of inter-node interference in high SNR regime to be divided into two scenarios. If only coarse CSI is available, the influence of inter-beam interference dominates the rate performance during training, i.e., there is no impact of inter-node interference on performance. Otherwise, the influence of inter-node interference dominates the rate performance during training. Now I present the maximal multiplexing gain as a function of training power constraint ζ for different closed loop training systems.

Applying Lemma 2 to characterization (3.6), I have

$$\lim_{P\to\infty}\frac{\mathrm{SE}_{\mathrm{SqBf_{Cl}}}}{\log P}=\frac{1}{2}\frac{T^{\mathrm{tr}}}{T}\max\left(\min\left(\frac{T^{\mathrm{tr}}}{T}\frac{\zeta}{\theta},1-\zeta\right),0\right)+\left(1-\frac{T^{\mathrm{tr}}}{T}\right)\min\left(\frac{T^{\mathrm{tr}}}{T}\frac{\zeta}{\theta},1\right). \tag{6.4}$$

The results regarding Sequential Beamforming system without inter-node interference are first presented as an upper bound for the performance of proposed strategy. Then the results regarding the half-duplex systems are presented for comparison. Finally, the performance of sequential beamforming system with inter-node interference is presented.

Theorem 9 (Inter-node interference free sequential beamforming strategy) *The maximal multiplexing gain of sequential beamforming strategy with closed loop training, without inter-node interference, under training power constraint ζ is*

$$r^*_{\mathrm{SqBf_{Cl}}\cancel{\mathrm{INI}}}(\zeta)=\begin{cases}\frac{1}{2}\frac{\zeta}{\theta}, & \zeta<\theta\\ 1-\frac{1}{2}\frac{\theta}{\zeta}, & \zeta\geqslant\theta\end{cases}.$$

Proof. The multiplexing gain of sequential beamforming without inter-node interference is

$$r_{\mathrm{SqBf_{Cl}}\cancel{\mathrm{INI}}}(\zeta,T^{\mathrm{tr}})=\left(1-\frac{1}{2}\frac{T^{\mathrm{tr}}}{T}\right)\min(\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T},1).$$

By maximizing the multiplexing gain in the cases of $\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T}\geqslant 1$ and $\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T}<1$ by choosing the optimal training duration, the theorem is directly obtained. ■

The multiplexing gain is composed of two regimes. When ζ is small, spectral efficiency increases linearly as training power increases. In this regime, the increase of rate performance during and after training is the major reason. As more training power is allowed, users send training symbols until no spectral efficiency loss is observed due to inter-beam interference after training. The spectral efficiency improvement now attributes to use less time to send the same amount of training information. Thus, lower spectral efficiency performance increase as training power grows is achieved. Now the asymptotic performance of half-duplex counterpart is presented for comparison.

Theorem 10 (Half-duplex system) *The maximal multiplexing gain of closed loop training half-duplex system under training power constraint ζ is*

$$r^*_{\mathrm{Hf_{Cl}}}(\zeta) = \begin{cases} \frac{1}{4}\frac{\zeta}{\theta}, & \zeta < 2\theta \\ 1 - \frac{\theta}{\zeta}, & \zeta \geqslant 2\theta \end{cases}.$$

Proof. Similar to inter-node interference free sequential beamforming strategy, omitting the extra spectral obtained during full-duplex training (6.4), I first express the multiplexing gain of half-duplex system as

$$r_{\mathrm{Hf_{Cl}}}(\zeta, T^{\mathrm{tr}}) = \left(1 - \frac{T^{\mathrm{tr}}}{T}\right) \min\left(\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T}, 1\right).$$

Directly optimizing training duration in two cases of $\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T} \geqslant 1$ and $\frac{\zeta}{\theta}\frac{T^{\mathrm{tr}}}{T} < 1$ leads to the proof. ■

It should be noted that, similar to sequential beamforming strategy with closed loop training, its half-duplex counterpart's spectral efficiency is consisted with two regimes. Actually compared to Theorem 9, a significant multiplexing gain improvement is observed. Thus, the proposed sequential beamforming strategy doubles spectral efficiency of a unidirectional downlink communication asymptotically when $\zeta < \theta$. Finally, I look at the influence of inter-node interference on the asymptotic spectral efficiency of sequential beamforming strategy.

Theorem 11 (Sequential beamforming strategy with inter-node interference) *The maximal multiplexing gain of closed loop training sequential beamforming strategy under training power constraint ζ is*

$$r^*_{\mathrm{SqBf_{Cl}}}(\zeta) = \begin{cases} r^*_{\mathrm{SqBf_{Cl}}\cancel{\mathrm{INI}}}(\zeta), & \zeta \leqslant \frac{3\theta}{2+3\theta} \\ r^*_{\mathrm{SqBf_{Cl}INI}}(\zeta), & \frac{3\theta}{2+3\theta} < \zeta < \min\left(1, \max\left(\frac{3\theta}{2+3\theta}, \frac{3\theta}{2-\theta}\right)\right) \\ r^*_{\mathrm{Hf_{Cl}}}(\zeta), & \min\left(1, \max\left(\frac{3\theta}{2+3\theta}, \frac{3\theta}{2-\theta}\right)\right) \leqslant \zeta \end{cases},$$

where

$$r^*_{\mathrm{SqBf_{Cl}INI}}(\zeta) = \begin{cases} \frac{((2-\theta)\zeta+\theta)^2}{16\zeta\theta}, & \zeta < \frac{3\theta}{2-\theta} \\ 1 - \frac{\theta}{2} - \frac{\theta}{2\zeta}, & \frac{3\theta}{2-\theta} \leqslant \zeta \end{cases}.$$

Proof. Studying (6.4) in different regimes of operation gives the following.

1. When $\zeta \geqslant 1$, the multiplexing gain is the same as the multiplex gain of half-duplex systems. Thus,

$$r^*_{\mathrm{SqBf_{Cl}}}(\zeta) = r^*(\zeta)_{\mathrm{Hf_{Cl}}}, \quad \zeta \geqslant 1.$$

2. $\zeta < 1$

   - $\frac{T^{\mathrm{tr}}}{T} \leqslant \theta\frac{1-\zeta}{\zeta}$: $r_{\mathrm{SqBf_{Cl}}}(\zeta, T^{tr}) = \frac{T^{\mathrm{tr}}}{T}\frac{\zeta}{\theta} - \frac{1}{2}\left(\frac{T^{\mathrm{tr}}}{T}\right)^2\frac{\zeta}{\theta} = r_{\mathrm{SqBf_{Cl}}\cancel{\mathrm{INI}}}(\zeta, T^{tr})$
   - $\theta\frac{1-\zeta}{\zeta} < \frac{T^{\mathrm{tr}}}{T} < \frac{\theta}{\zeta}$: $r_{\mathrm{SqBf_{Cl}}}(\zeta, T^{tr}) = \frac{1}{2}\frac{T^{\mathrm{tr}}}{T}(1-\zeta) + \left(1 - \frac{T^{\mathrm{tr}}}{T}\right)\frac{T^{\mathrm{tr}}}{T}\frac{\zeta}{\theta}$
   - $\frac{\theta}{\zeta} \leqslant \frac{T^{\mathrm{tr}}}{T}$: $r_{\mathrm{SqBf_{Cl}}}(\zeta, T^{tr}) = \frac{1}{2}\frac{T^{\mathrm{tr}}}{T}(1-\zeta) + \left(1 - \frac{T^{\mathrm{tr}}}{T}\right)$

   By carefully evaluating the derivative in different regimes and applying the optimized training duration into equation (6.4) lead to the theorem.

∎

The influence of inter-node interference on the spectral efficiency, interestingly, can be divided into three regimes. For systems targeting small multiplexing gain, only small amount of training power is needed. In this regime, inter-beam interference dominates the downlink performance during full-duplex training and no inter-node interference penalty is observed. However, if higher multiplexing gain is targeted, the inter-node interference dominates the downlink performance during full-duplex training. In this case, inter-node interference will reduce the potential benefit obtained from sequential beamforming strategy. Finally, if a really high training power ($\zeta > 1$) is used to achieve high spectral efficiency. The high inter-node interference level leads to no benefit from the downlink transmission during training phase. This observation is confirmed by simulation shown in Fig. 6.1.

Remark 3 *As the number of antennas increases (with respect to block length), θ increases. Interestingly, I observe that higher θ actually increases the regime that sequential beamforming strategy does not suffer from inter-node interference. As $\theta \to \infty$, sequential beamforming strategy suffers no inter-node interference as long as training power constraint is smaller than 1. The proposed strategy is very useful in systems with large number of users.*

6.2 Sequential Beamforming with Open Loop Training

Following the same approach in Chapter 6.1, I now investigate the spectral efficiency for different open loop training based systems. The influence of inter-beam interference in high SNR regime is first characterized, which is followed by the multiplexing gain analysis.

Lemma 3 *For open loop training based systems, the rate performance under inter-beam interference after training is*

$$\lim_{P\to\infty}\frac{R^{\text{data}}}{\log P}=\lim_{P\to\infty}\frac{R^{\text{ZF}}-\log\left[1+\frac{P}{M}\frac{M-1}{1+\frac{T^{\text{tr}}}{M}P^{\zeta}}\right]}{\log P}=\max\left(1-\zeta,0\right).$$

Proof. Substituting the training power to P^{ζ} in Proposition 1 leads to the theorem. ■

The rate performance achieved after training, surprisingly, is only decided by the training power constraint ζ. More training symbols does not help to deduce inter-beam interference after training. Thus, the optimal training duration goes to zero in high SNR regime. Therefore, the maximal multiplexing gain performance can be easily obtained as follows.

Theorem 12 *For open loop training based systems, the multiplexing gain of both sequential beamforming and half-duplex strategy is only decided by training power constraint as*

$$r^{*}_{\text{SqBf}_{\text{Op}}}=r^{*}_{\text{Hf}_{\text{Op}}}=\zeta. \tag{6.5}$$

Special attention should be given to the fact that this theorem is valid for both half-duplex and sequential beamforming strategy with open loop training. The proposed sequential beamforming strategy does not provide extra spectral benefit in high SNR regime. It should be emphasized that in low-to-moderate SNR regime, from analysis in Chapter 5, sequential beamforming does obtain significant spectral efficiency gain. This is different for closed loop training systems, where significant spectral efficiency improvement is observed in all SNR regime.

Before comparing the spectral efficiency performance of closed and open loop system asymptotically, I first validate the influence of the assumption that each user has a perfect knowledge of its own channel on both closed and open loop training.

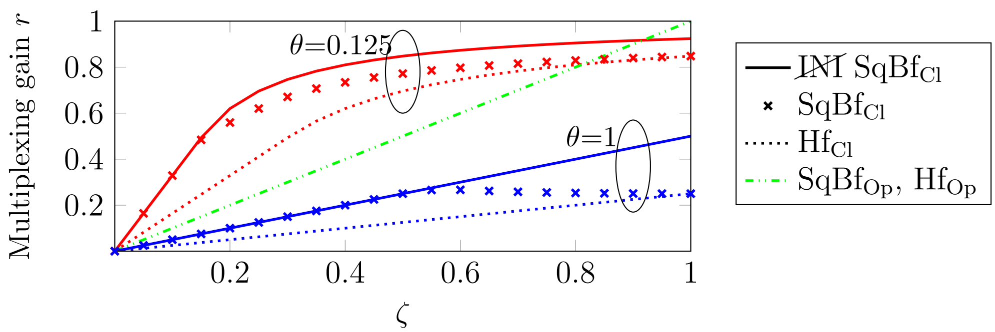


Figure 6.1: Multiplexing gain r as a function of training power constraint ζ.

This is crucial for the decoding at user side and for closed training's quantization. It has been shown [3] that, asymptotically, 1 training pilots from each base station antenna is both necessary and good enough for the influence of imperfect CSI on downlink rate to vanish for both closed and open loop training.

With the help of maximal multiplexing gain characterization obtained in this chapter, I compare the spectral efficiency of systems with different types of training. For systems with longer block length and less users, in general, closed loop training outperforms open loop training. The major reason is that the closed loop training significantly reduces inter-beam interference by learning from more training symbols. Despite the longer training duration, closed loop training is still more advantageous. However, if there are many antennas and block length is short, then it is better to use open loop training, whose training duration is asymptotically short.

Remark 4 *Based on the analysis, I conclude the following about half-duplex systems. For systems with large numbers of antennas and short block length, which can be mathematically captured as $\theta \geqslant \frac{1}{2}$, open loop training outperforms closed loop training systems despite the choice of training power. Otherwise, training method should be picked based on the training power. When a strict training power constraint is imposed on users, closed loop training is more favorable by leveraging the training time. Evaluating Theorems 10 and 12 gives the decision region as $\zeta < \frac{1-\sqrt{1-2\theta}}{2}$. However,* *if more training power is available, open loop training becomes more favorable due to shorter available training time. In general, training method should be chosen wisely to maximize the spectral efficiency.*

Chapter 7

Conclusion

The multiuser MIMO downlink has the potential to increase spectral efficiency linearly as user number grows with the help of accurate CSI. However, in systems with many users, CSI acquisition leads to unavoidable training overhead. In this thesis, I achieved reducing the overhead of multiuser MIMO downlink systems by utilizing full-duplex radios. Instead of requiring both base station and mobile users to be full-duplex capable [10] [11], I propose a sequential beamforming strategy that requires only half-duplex users and less precoder updating.

With characterization of inter-node interference due to full-duplex training, I optimize the training duration of sequential beamforming strategy with both closed and open loop training. The proposed sequential beamforming strategy demonstrates spectral efficiency improvement compared to its half-duplex counterpart for both closed and open training. The sequential beamforming strategy can also be applied in frequency-division duplex systems where uplink and downlink are orthogonal by nature. The orthogonality will also prevent the generation of inter-node interference.

The closed and open loop training methods exhibit distinct spectral efficiency performance. Asymptotically, I find that the number of users, block length and training power jointly decide which type of training should be adopted. It has been observed in [21] that closed loop training is more favorable than open loop training to reduce estimation error, while common sense suggests that open loop training is preferable for large systems. My results quantify the decision region of training method and bridge these two observations and provide key insights to the performance in moderate regimes.

I close this thesis by noting some possible extensions. Perfect CSI is assumed instantaneously available at each user for closed loop training. To obtain CSI, in general, each user needs to estimate pilots sent by the base station, which consumes time. However, since the power limitation at the base station is less severe, high quality CSI is much easier to achieve at the user side. The base station can also increase the quality of CSI by increasing the pilot power. In [3], the authors demonstrate that even without high power pilots, one pilot sent by each base station antenna is enough for the system to obtain full-multiplexing gain. This suggests the more availability of good CSI at users side.

Another assumption I made is that the number of training symbols is symmetric among users. Since the downlink receiving time is decreasing from User 1 to User M, it is clear that extra spectral efficiency can be obtained by allocating training symbols decreasingly from User 1 to User M.

In addition, I consider a $M \times M$ system where users are pre-selected without CSI input. Past study has revealed that picking users wisely from a large sets of users helps yield better spectral efficiency [8] [22]. This scheme can also be easily extended to asymmetric system where more base station antenna are available. Finally, I consider channel realization to be independent and identically distributed Rayleigh fading, which could be viewed as a worst case and serves as performance lower bounded. Determining how to efficiently utilize the spatial information and the channel correlation between users is still an open question.

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