

## MELODY

## 1. INTRODUCTION

**H**ITHERTO our attention has been devoted to the problem of harmony. The equally important problems of melody and rhythm remain for consideration. We shall concentrate our attention upon melody, although we shall find (section 13) that rhythm is fundamentally a special kind of melody.

No attempt will be made to state any theory of the structure of actual music as dependent upon combined harmony, melody, and rhythm. The simplest and most natural conjecture would be that the æsthetic measure  $M$  of any musical composition is given by the ratio  $O/C$ , where  $C$  represents the number of notes which enter as melodic constituents and  $O$  is the sum of the elements of order of harmonic, melodic, and rhythmic types, these being appropriately weighted.

## 2. THE PROBLEM OF MELODY

In order to separate melody as much as possible from harmony and rhythm, we shall limit attention to simple melodies in one part, made up of 8, 16, 32, 64 or 128 notes, divided into measures of four notes each. In each measure the first and third notes are to be thought of as theoretically accented. Furthermore we shall exclude the use of the minor mode and of modulation, by requiring that all of the notes are those of the ordinary diatonic scale.

Despite these stringent limitations, the majority of well-known melodies may be reduced to this form, or to a like form in three-part instead of four-part time. In the first place most melodies have one dominant melodic part, usually the soprano, which may be taken as the essential equivalent simple melody. In the second place rhythmic and ornamental effects may be eliminated by retaining only the proper principal notes, although frequently this can be accomplished only with definite diminution of musical effectiveness. Moreover if the melody be longer than 128 notes, there will usually be some shorter characteristic part which underlies the entire melody.

Although we only consider four-part time, it is obvious that the theory here put forth tentatively can easily be extended to other types such as three-part time.

For these reasons the basic problem of melody can be reduced to the following typical simplified form:

*Given a simple melody in four-part time of 8, 16, 32, 64 or 128 equal notes of the diatonic scale, to determine a suitable measure of the complexity  $C$  and of the elements of order  $O$ , such that the ratio  $M = O/C$  furnishes a suitable æsthetic measure of the melody.*

It is obvious that the number of notes of the melody gives a suitable measure of the complexity  $C$ ; of course a note which is held is to be regarded as a single note. Thus the essential difficulty in the problem of melody lies in the effective determination of the elements of order  $O$ .

### 3. THE RÔLE OF HARMONY

A simple (Western) melody is always enjoyed as if it possessed an accompanying harmony, at least to the extent that each of its notes is construed to lie in some definite chord. In case the melody is derived from a musical composi-

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tion this harmonization coincides with that known through the composition.

On the other hand, if the melody is known simply as a tune, a simple automatic mental harmonization will be attached to it. The general law of this automatic harmonization is that it is accomplished by means of chords which are modified as infrequently as possible, with preference for chordal sequences with high æsthetic ratings. Furthermore, in so far as possible the chordal sequences of the alternate accented notes should have the same characteristics; the sequence of these accented notes forms what we shall call the "secondary melody."

In what follows, therefore, we shall assume that the notes of the melody are construed as lying in definite chords.

### 4. THE QUESTION OF PHRASING AND COMPARISON

Just as a poem is expressed in lines, so any melody is arranged in musical phrases, often of equal length, which are terminated by a sustained note or an equivalent rest.

Various types of musical phrasing have come to be used just as various types of linear arrangement and rhyming schemes are used in poetry. The simplest type is that in which the melody is divided into four phrases of equal length. This will be the case in most of the illustrations which we shall use, but the theory applies equally in more complicated cases.

In consequence we shall suppose that any melody under consideration is taken in its usual phrasing. The general character of such phrasing is that the whole melody falls into a certain number of major coordinate parts, each of these in turn falls into a number of smaller coordinate parts, and so on until the individual phrases are reached.

These different types of parts are comparable to the canto, stanza, and line in a poem.

One of the especial advantages of musical phrasing is that it brings out certain comparisons by similarity or contrast between various parts. Presumably the best phrasing and interpretation is that which brings out most clearly the relations of order which exist in the melody.

We shall not attempt to deal with the question as to why one type of arrangement of comparable phrases is more used than another. To do so would be futile, because the accepted forms are largely designated by pure convention; the same kind of arbitrariness prevails in the selection of forms of verse.

One might indeed attempt such an analysis. For example one might ask why the type of phrasing, *AABA*, is preferred to *ABAA*; here *A* represents one phrase and *B* represents a contrasting phrase. Evidently the first arrangement fixes *A* firmly in mind so that the contrast of *B* with *A* is clearly defined. In the second arrangement this contrast is less clearly defined of course and the *two* final *A*'s seem repetitious, since the true function of the concluding *A* is only to effect a return to the starting point. Evidently such an analysis, however valid, falls within the domain of "qualitative" rather than of "quantitative" æsthetics.

For these reasons we shall prescribe the musical form used as being established by convention.

However, there seem to be certain simple principles which may be stated in this connection:

Any part *B* made up of one or more phrases may be compared by similarity or contrast with at most one coordinate preceding part *A*. When, however, the part *B* has been compared with such an earlier phrase, no subsequent phrase which starts with the same first measure as *B* is to be com-

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pared to this same earlier  $A$ . Within the limits of a single phrase, the second measure may be compared with the first, the second two measures with the first two, or the second half of the phrase with the first half.

The types of comparison just mentioned are the ones which we shall take account of. They are the principal types which enter into the simple melodies under consideration here. It is hardly necessary to remark that the trained ear appreciates much more elaborate and subtle forms of comparison in complicated musical structures.

### 5. LIMITATION OF LEAPS

Little effort is required when the voice repeats a note or moves by a step upwards or downwards. In a similar way a sequence of notes in a single consonant chord is readily sung provided that the voice does not leap more than an octave.

There remain, however, certain types of leaps which are generally avoided in simple melodies. The principal rules of limitation will be formulated as follows:

Dissonant leaps other than a diminished fifth and a minor seventh in  $V7$  are forbidden. Two successive leaps in the same direction must either lie in a consonant chord or in a 7th chord; in the latter case the total leap must be a seventh upwards and the following note must fall a step as in a resolution. Any leap of more than a fifth is to be approached and left in the direction opposite to that of the leap. After two or more steps in one direction a further leap in the same direction must be to an accented note.<sup>1</sup>

These rules are essentially empirical ones based on existing practice. Their reasonableness is obvious.

<sup>1</sup>Cf. Prout, *Harmony: Its Theory and Practice*, p. 23.

## 6. PREPARATORY ANALYSIS OF A BEETHOVEN CHORALE

Let us attempt to specify the principal elements of order as they appear in a classical melody, for instance in the chorale from the fourth movement of Beethoven's Ninth Symphony. In the notation already employed the melody may be written in the following form:

3345	5432	1123	$\overset{\cdot}{3}22\overset{\frown}{2}$
3345	5432	1123	$\overset{\cdot}{2}11\overset{\frown}{1}$
2231	23(4)31	23(4)32	$\overset{\cdot}{1}2\underline{55}$
3345	5432	1123	$\overset{\cdot}{2}11\overset{\frown}{1}$

In the above notation the following simple conventions are employed: a parenthesis above a repeated note indicates that the note is held; a period above a note indicates that it is lengthened so that the following note is reduced to an eighth note; a dash above or below indicates that the note is an octave above or below the principal octave; a note in parenthesis is one which appears in the complete melody but is treated as an embellishment in the formal analysis.

Evidently there are four phrases of four measures each in the above melody of which the second is to be compared with the first, the third with the second and the fourth with the second. In fact the form may be indicated as follows:

*A V, A I, B V, A I*

where the first, second and fourth phrases *A* are essentially alike save that the first of these has a dominant close (V) while the other two phrases have a tonic close (I). The sharply contrasting phrase *B* has a dominant close (V).

This is a commonplace type of song form.

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Let us make an analysis of this melody by successive measures. In doing so various types of elements of order will be brought to light. The reader who wishes to see the complete rule for each item counted is referred to the immediately following section 7.

The first note is the mediant which is in the tonic chord. Now it is a wellknown fact that most musical compositions commence in the tonic chord and thus define the tonality. If this expectation of the tonic chord is fulfilled, the fact is appreciated by the ear.<sup>1</sup> In the case at hand this first element of *tonic start* is present and we assign to it a value 1 as far as the first note is concerned.

The second note is also in the tonic chord so that we add 1 more for tonic start on that account: the third note is not in the tonic chord. This gives a complete count of 2 for this element of order.

Furthermore the second note repeats the first. By taking account of this element of *direct repetition*, for which we assign a value 1 also, we obtain then the count of 3 in all for the first two notes.

The last three notes of the first measure form a melodic sequence. Since the sequence is not established for the ear until the second note is heard, we assign a value 1 to each of the last two notes for this element of *melodic sequence*.

We have now exhausted the obvious relationships of order in the first measure. There remains, however, another element of *harmonic contrast* of a more subtle type. When three notes of a measure lie in a consonant chord, while the fourth one does not lie in that chord, a pleasant harmonic

<sup>1</sup> In his interesting thesis *The Rôle of Expectation in Music* (New Haven, 1921) A. D. Bissell has found that in over 96% of a large number of typical cases the melody does commence in the tonic chord.

contrast is secured between the notes of the measure. In the case at hand three of the four notes are in the tonic chord. Hence we assign 2 for this harmonic contrast in the first measure, counted 1 for each of the last two notes.

Thus the total count for the first measure is 7.

The second measure starts off with the repetition of the preceding dominant note, which adds a count of 1. The four notes of this measure form a falling melodic sequence, for which there is a count of 3 (one for each note after the first).

The first three notes of this measure evidently constitute an exact inversion of the last three notes of the first measure. We add a count of 3 for this element of *inversion*, 1 for each note involved.

Moreover the middle two notes of the second measure 5432 stand in *melodic contrast* with the corresponding notes of the comparable first measure 3345 (see section 4) from which they differ by step. Hence we assign a count of 2 for each of these notes.

Consequently there is a count of 11 in all for the second measure.

In proceeding to the third measure we observe first that the first note continues the melodic progression for which we add a count of 1; the second note repeats the first and a preceding accented note is repeated, which gives a further count of 2 for repetition; the last three notes form a melodic sequence for which there is a count of 2; all of the notes but the second are in the tonic chord, for which there is a count of 2 for harmonic contrast; and since the repeated tonic 1 has not appeared before in the first measure (the two halves of the first phrase are comparable) there is a count of 2 for melodic contrast.

The new element entering in the third measure is that of



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*transposition*, for obviously the third measure is a precise transposition of the first and is felt as such. We add a count of 4, since four notes are involved in the comparison.

Hence the total count for the third measure is 13.

In the fourth measure the transposition continues for two notes more (1)<sup>1</sup>; the first note repeats the preceding (1); an accented note of the preceding measure is repeated (1); the first two notes form an inversion of the two preceding notes (2), the last (held) note repeats its predecessor (1); all but one of the notes are in the dominant chord (2); the third note contrasts melodically with the third note of the comparable second measure (1).

Furthermore there is a count for *cadence* of 1 in the final note, since there is a half-cadence in the closing dominant chord of the first phrase.

Accordingly the count for the fourth measure is 10.

In proceeding to the fifth, sixth and seventh measures which repeat the first three, we note first an element of order of 1 for each note because of this exact repetition. Otherwise, aside from the element of tonic start (2) which is lost here, and the element of repetition of a preceding accented note (1) which is gained, the count is the same as before.

Hence the fifth, sixth and seventh measures give a count of 10, 15, and 17, respectively.

The eighth measure yields a count of 13 as follows, without reference to the first four measures: repetition (3); melodic sequence (2); harmonic contrast (2); inversion (2); melodic contrast (4). Moreover there is an authentic cadence (passage from the dominant to the tonic chord) at the end of the second phrase; this is counted as 2 more.

<sup>1</sup>That is, counted as 1. It is not counted as 2 since no transposition counts for more than 4 in all, according to our rule.

Thus the total count for the eighth measure is 18.

In this way we obtain a total count of 41 and 60 in all for the first and second phrases, respectively, and so of 101 for the first half of it, an average of about 3.4 for each of the 30 notes involved.

If we count the third phrase in the same way, we obtain a count of 47 in all. Here the strong melodic contrast with the comparable first phrase is to be observed, for 10 of the 16 notes involved are in such contrast with the corresponding notes of the first phrase. Furthermore the *approximate* direct repetition of the first measure by the second and of the second measure by the third (all but one note the same in each case) yields a count of  $3 + 3 = 6$  for this kind of direct repetition.

The fourth phrase is identically the same as the second with which it is naturally compared, since the first half of the melody is coordinate with the second half. Of course this phrase is not to be contrasted with the third phrase because this has already been used for the same contrast.

The first three measures of this concluding phrase yield the same count as the corresponding measures of the second phrase except that there is a loss of 1 since the first measure contains no accented note of the preceding measure. Thus there is a count of  $9 + 15 + 17$  or 41 for these measures. The fourth measure loses a count of 4 because there is no melodic contrast. But there is a gain of 4 because of repetition, and of 3 for tonic close. Thus there is a count of  $18 + 3$  or 21 for the concluding measure, and so of 62 in all for the last phrase.

This gives a count of 210 in all for the four phrases, an average of 3.5 elements of order for each of the 60 notes involved.

But, according to the theory here advanced, it remains to

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add in the count of the *secondary melody* formed by the 32 accented notes. In my opinion there is little doubt that this somewhat concealed melody forms an effective factor in the æsthetic enjoyment. We shall not require that the rules for limitation of leaps be observed in the secondary melody.

In the particular case under consideration the secondary melody is as follows:

3453	1232	3453	1221
2323	2315	3453	1221

and possesses obvious melodic quality. The harmonization of this brief melody given below has kindly been made by my colleague Professor W. H. Piston.

The image shows two staves of musical notation. The top staff is a single melodic line in G major (one sharp), 4/4 time. The bottom staff is a piano accompaniment with chords and bass line in the same key and time signature. The melody consists of 32 accented notes.

The only new element of order which we find here is that of *harmonic sequence*. Here the harmonic sequences, 531, 531, 3153, 531 in the tonic chord appear, for which we give a count of one for each note after the first and so of  $2 + 2 + 3 + 2 = 9$  in all.

The count is 78 for this melody.

Thus the grand total for this Beethoven chorale is 288 or an average of 4.8 elements of order for each note. In other words the æsthetic measure  $M$  for this Beethoven chorale is 4.8 which turns out to be very high in comparison with that of most other melodies.

#### 7. DEFINITION OF $O$ , $C$ AND $M$ FOR SIMPLE MELODY

The following precise but tentative definitions of the elements of order in simple melody are suggested by the analysis of the above melody and of many others.

##### (1) *Tonic Start and Close*

There is a count of 1 for each note at the beginning as long as these lie in the tonic chord and are in the first measure.

There is a like count of 1 for each note at the end as long as these lie in the tonic chord and are in the last measure; there is a further count of 1 if the last note is the tonic itself.

##### (2) *Cadence*

If there is a passage from dominant to tonic at the close of a phrase (that is, the final change of chord is from dominant to tonic) there is a count of 1 for each note involved and so of 2 in all.

If the final chord is the dominant (half cadence) there is a count of 1 for the final note.

##### (3) *Repetition of Accented Notes*

According as one or both accented notes of a measure reappear in the following measure there is a count of 1 or 2 as the case may be, provided this is not caused by a mere repetition of the first measure. If both accented notes of

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the first measure are the same, there is a count of 1 of course.

### (4) *Direct Repetition*

If a single note, or a pair of two notes of which the first is accented, or a measure, or a larger part, not the half of a phrase, be repeated within a phrase, there is a count of 1 for each note of the first repetition.

Moreover the approximate repetition of parts as large as a measure is counted provided there is at most one exceptional note for each measure, the count being 1 for each non-exceptional note; in this case the second repetition is also counted.

### (5) *Repetition in Comparable Phrases*

If a part of one phrase at least a measure in length is repeated in a later phrase with which it is compared, or if notes in corresponding position are repeated, there is a count of 1 for each repeated note.

### (6) *Transposition*

An exact transposition within a phrase, of at least a measure in length but not all in one direct melodic sequence, counts as 1 for each note of the first transposition up to a count of 4. If the transposition be repeated a second time within a phrase there is a count of 1 for each note of the second transposition, up to a count of 4, provided that the successive transpositions differ by step.

### (7) *Inversion*

In a direct inversion of a rising or falling sequence of at least two notes there is a count of 1 for each repeated note, up to a count of 4.

(8) *Melodic Sequence*

In a rising or falling melodic sequence of at least three notes there is a count of 1 for each note after the first, up to a count of 4.

(9) *Harmonic Sequence*

A harmonic sequence of notes lying in the same consonant chord is counted 1 for each note after the first, up to a count of 4.

(10) *Melodic Contrast*

If a part *B* is compared (cf. section 4) with an earlier part *A*, there is a count of 1 for each note of *B* which either differs by step from the corresponding note of *A*, or which is different from any note found in *A*. A sustained note is counted as double here.

A phrase *B* will only be said to contrast with an earlier comparable phrase *A* in case the count for melodic contrast is at least one half the number of notes in *B* differing from the corresponding notes of *A*.<sup>1</sup>

(11) *Harmonic Contrast*

If all of the notes of a measure but one fall in a consonant major chord there is a count of 1 each for the last two notes.

(12) *Secondary Melody*

A complete count of the elements of order of the above types is to be made for the secondary melody formed by the alternate accented notes.

The order *O* is the total count of all the elements of order of the above types.

<sup>1</sup> Thus the song form *AABA* is only considered to be followed if *B* contrasts with the first phrase *A* in this manner.

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The complexity  $C$  is the total number of notes of the melody.

The æsthetic measure  $M$  is then the ratio  $O/C$ .

### 8. FURTHER CONDITIONS OF SATISFACTORY FORM

There are some further conditions which must be fulfilled if satisfactory form is to be achieved. It is almost certain that the conditions of this kind enumerated below are incomplete.

#### (1) *Ease in Singing*

The rules of section 5 on the limitation of leaps will be adopted.

#### (2) *Regularity of Pattern*

Some established musical pattern must be followed. This implies a suitable distribution of half-cadences and cadences in the successive phrases as well as a tonic close, so that there is a sense of progression throughout.

#### (3) *Continuity*

If one or more notes fail to be connected with *preceding* notes through at least one element of order, there is felt to be discontinuity in the melody inasmuch as this note is not naturally suggested by what precedes. Such notes will not be admitted.

#### (4) *Freedom From Obvious Formal Blemishes*

It must not be possible to increase the total order  $O$  by alterations within a short succession of notes (say not more than four) together with corresponding alterations in its repetitions, transpositions and inversions.

This condition operates to eliminate obvious cacophony because cacophonous repetitions do not lead to counted elements of order according to the rules adopted in section 7.

(5) *Treatment of the Leading Note*

Unless the leading note occurs in a dominant sequence of at least three notes or in a repetition, transposition, or inversion, it must rise to the tonic note directly or *via* the super-tonic, or fall to the submediant note.

In fact the leading note so strongly suggests the tonic that unless it is imbedded in a dominant sequence or in a fixed pattern, it must proceed to the tonic note or to the related minor tonic.

(6) *The Secondary Melody*

The secondary melody, aside from the question of conditions (1)–(4) above, must rate as fair; for definiteness we shall require that  $M$  shall rate at least as high as 1 for the secondary melody.<sup>1</sup>

(7) *Rhythmic and Melodic Embellishment*

If a note be directly repeated it is often desirable to interpret it as a note which is held. Thus an advantageous rhythmic variation may be introduced.

In a similar manner the insertion of half notes may soften gracefully a considerable leap, or introduce valuable elements of contrast, or increase the rhythmic interest.

Wherever possible we shall introduce such slight embellishments in the final interpretation, just as these were eliminated at the outset for purposes of exact formal analysis. By so doing, obvious awkwardness in the narrow type of simple melodies here treated can be mitigated without any fundamental alteration of the melodic relations involved.

<sup>1</sup> In obtaining  $M$  for the secondary melody it will be of course necessary to treat its secondary melody, that is the tertiary melody of the original melody which is formed by the initial notes of the measures.



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### 9. THE APPLICATION TO MELODY

The rules contained in sections 7 and 8 resulted from the study of a variety of simple melodies. In each case I tried to determine the effective elements of order by the method of introspection. In the experimental work I was kindly aided by Dr. C. B. Morrey.

The principal conclusions may be summarized as follows: (1) for good simple melodies the æsthetic measure  $M$  was always high and in general at least 3; (2) it was not possible to find purely mechanical sequences of notes for which  $M$  rates as high as 3; (3) it was not found possible by Mr. Morrey to devise sequences with a fair rating but without a certain degree of melodic quality.

Consequently the theory seems to be fairly satisfactory as far as it goes.

The reader who wishes to examine special melodies can readily verify the first conclusion. In order to verify the other conclusions, let him try to construct sequences of notes without melodic quality and yet with a high æsthetic measure  $M$ .

In partial justification of the third conclusion I propose to turn next to certain experimentally constructed melodies, devised in the light of the theory and not by use of musical imagination. If these seem tolerable to the ear, the theory is justified to that extent. It is to be stressed that these experimental melodies were the *only ones* which I tried to build, and thus do not represent a selection out of a number of attempts. Furthermore these were constructed in the manner indicated below and in very brief time indeed.

Mr. Piston has kindly given these attempts as favorable a setting as possible by supplying an appropriate harmonization.

## 10. FIRST EXPERIMENTAL MELODY

The first experimental melody will be taken to be the secondary melody of the Beethoven chorale (see section 6).

In order to rate it we must add to the count 78 already obtained a further count for the elements of order found in its secondary melody:

3513	3512
2221	2512

The total of these is readily verified to be 37, so that the total count is 115 and the æsthetic measure  $M$  of the first experimental melody is 3.6 as against 4.8 for the Beethoven chorale from which it was derived.

## 11. SECOND EXPERIMENTAL MELODY

The second experimental melody was obtained as follows:

The Westminster chimes melody is essentially the following little melody of 16 notes in four equal phrases:

3125	5231	3215	5231
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in which the last note of each measure is accented. The count for the elements of order is 43. In the secondary melody

3253	3153
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the count is 19.<sup>1</sup> Thus  $O$  is 62 and  $M$  is 3.9.

The first step was to construct an experimental melody of 32 notes of which the secondary melody is precisely the above Westminster chimes melody. This was done in the following manner:

3211	2755	5123	3212
3322	1155	5123	3211

Here the precise selection of the non-accented notes was

<sup>1</sup> This melody of eight notes is taken as made up of two measures, each of which forms a phrase.

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largely determined by the following considerations. The Westminster chimes melody suggests a four-part song form *ABCB* in which *B* and *C* contrast with *A*. In the first measure it was obviously easy to fill up 3\* 1\* to be 3211 and thus introduce the elements of melodic sequence, repetition and harmonic contrast.

In the second measure 2\* 5\* the leap from 2 to 5 was most easily made by introducing an intermediate 7; by completing this measure to be 2755 the four notes were placed in a harmonic sequence within the dominant chord, and the second measure stood in complete melodic contrast to the first since its first note differs by step from that in the first measure, while the other three notes appear in the second measure for the first time.

Similarly if we try to fill out the third measure 5\* 2\* so as to contrast with 3211 (the third note being in contrast) we are naturally led to 5123, especially since 3 is accented in the preceding measure and harmonic contrast is thereby secured.

In the fourth measure 3\* 1\*, there is no possibility of securing effective melodic contrast with the comparable second measure 2755. The obvious completion to 3212 has obvious advantages when taken in conjunction with the first and third measures; in particular the final note gives a half-cadence.

We will not remark upon the second half of the completion except to observe that the third phrase is chosen in as strong melodic contrast to the first as possible, while the fourth phrase is taken as identical with the second in conformity with the song form selected.

In an entirely similar manner the above melody of 16 notes was treated as itself a secondary melody, and expanded once more to the following melody in four phrases:

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$\widehat{3}323$	$1211$	$2176$	$\widehat{5655}$
$\underline{5112}$	2334	$\widehat{3323}$	1122
3236	$2322$	$\underline{1714}$	$\widehat{5655}$
$\underline{5112}$	2334	$\widehat{3323}$	1211

This is given herewith in the key of G major, with the harmonization by Mr. Piston.

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The sustained notes have been introduced so as to relieve the rhythmic flatness.

The total count for the elements of order in this melody is 118, while the count for the secondary melody is 81. Consequently the æsthetic measure is  $199/55 = 3.6$ . Hence this is a fairly satisfactory melody, according to the theory given above. The reader will have to judge for himself whether it actually is so or not for him.

### 12. THIRD EXPERIMENTAL MELODY

The third melody was obtained by taking as starting point the measure 3236 which appears as the ninth measure of the melody devised above. It was constructed purely mechanically and in the course of an hour. As first written the twelfth measure was 1233. Mr. Piston kindly suggested the modification to 1766. This is obviously a definite melodic improvement which not only avoids the leap of an octave at the end of the twelfth measure, but gives the more acceptable phrase form

$$A(V), A(I), B(VI), A(I),$$

whereas the first form

$$A(V), A(I), B(I), A(I)$$

is not usable because all of the three last phrases end in the tonic chord. It may be remarked in passing, however, that, aside from the fact that the original song form is not an acceptable one, the original sequence 1233 yielded a somewhat higher count.

The precise form of the melody so arrived at was the following:

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3236	$\overline{5515}$	4321	$\widehat{7122}$
3236	$\overline{5515}$	$\overline{4321}$	$\widehat{1211}$
4455	$\overline{6677}$	$\overline{1232}$	$\widehat{1766}$
3236	$\overline{5515}$	4321	$\widehat{1211}$

The harmonization by Mr. Piston in the key of G, is given herewith.

First system of musical notation for the melody in G major, 4/4 time. The right hand plays a melody of eighth notes, and the left hand provides a bass line of quarter notes.

Second system of musical notation for the melody in G major, 4/4 time. The right hand continues the melody with eighth notes, and the left hand continues the bass line.

Third system of musical notation for the melody in G major, 4/4 time. The right hand continues the melody with eighth notes, and the left hand continues the bass line.

Fourth system of musical notation for the melody in G major, 4/4 time. The right hand continues the melody with eighth notes, and the left hand continues the bass line, ending with a double bar line.

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The total count for elements of order in the above experimental melody and its secondary melody is  $124 + 68 = 192$  so that the æsthetic measure  $M$  is about 3.2.

### 13. THE PROBLEM OF RHYTHM

The important problem of rhythm is akin in its essential nature to the problem of melody. This fact may be seen as follows:

Let us imagine that only a full beat  $A$  and two half beats  $B$  are to be used as units. Any rhythmic pattern based on these is then defined as a sequence at equal intervals of time such as

*BABA BBBA BABA BBBA* etc.

This may be regarded as a kind of melody in two notes.

We will not develop this analogy further.

The problem of rhythm should be carefully studied from the point of view of the theory of æsthetic measure; it is essentially simpler than the problem of melody which we have considered.

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