Modeling flow and sediment transport dynamics in the lowermost Mississippi River, Louisiana, USA, with an upstream alluvial-bedrock transition and a downstream bedrock-alluvial transition: Implications for land building using engineered diversions

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Abstract The lowermost Mississippi River, defined herein as the river segment downstream of the Old River Control Structure and hydrodynamically influenced by the Gulf of Mexico, extends for approximately 500 km. This segment includes a bedrock (or more precisely, mixed bedrock-alluvial) reach that is bounded by an upstream alluvial-bedrock transition and a downstream bedrock-alluvial transition. Here we present a one-dimensional mathematical formulation for the long-term evolution of lowland rivers that is able to reproduce the morphodynamics of both the alluvial-bedrock and the bedrock-alluvial transitions. Model results show that the magnitude of the alluvial equilibrium bed slope relative to the bedrock surface slope and the depth of bedrock surface relative to the water surface base level strongly influence the mobile bed equilibrium of low-sloping river channels. Using data from the lowermost Mississippi River, the model is zeroed and validated at field scale by comparing the numerical results with field measurements. The model is then applied to predict the influence on the stability of channel bed elevation in response to delta restoration projects. In particular, the response of the river bed to the implementation of two examples of land-building diversions to extract water and sediment from the main channel is studied. In this regard, our model results show that engineered land-building diversions along the lowermost Mississippi River are capable of producing equilibrated bed profiles with only modest shoaling or erosion, and therefore, such diversions are a sustainable strategy for mitigating land loss within the Mississippi River Delta.

1. Introduction

The lowermost Mississippi River is defined herein as the segment of river downstream of the Old River Control Structure, which is located ~ 500 km upstream of the outlet at the Gulf of Mexico. In spite of its low slope, the lowermost Mississippi River is characterized by an alluvial-bedrock transition at roughly ~ 230 km upstream of the outlet [Nittrouer and Petter, 2012]. Downstream of this transition, the river is only discontinuously covered by modern, actively transported sand; otherwise, the bed consists of exposed bedrock—the highly consolidated, Quaternary-aged fluvial-deltaic stratum that underlies the modern channel. The distribution of the relatively thin alluvial cover is related to flow depth and channel planform (i.e., radius of curvature), and the fraction of river bed covered with alluvium typically ranges between 30\% and 70\% [Nittrouer et al., 2011a]. At ~ 40 km upstream of the outlet, sediment deposition arising from the spatial deceleration of the flow leads to complete alluvial sediment cover of the channel bed. This region corresponds to a bedrock-alluvial transition, coinciding with the present-day birdfoot delta at the Gulf of Mexico. Thus, the lowermost Mississippi River is characterized by two transitions: an upstream alluvial-bedrock transition and a downstream bedrock-alluvial transition (Figure 1).

To the best of our knowledge, within lowland and low-slope rivers (slope $= 10^{-4} - 10^{-5}$), there has been only recent recognition of the existence of bedrock channels, only a few attempts to quantify bedrock incision, and no research regarding the dynamics of transitions between alluvial reaches and reaches with exposed bedrock. For example, Shaw et al. [2013] recognized that the fraction of areal alluvial cover of the primary...
distributary channels of the Wax Lake Delta—a subdelta of the Mississippi River Delta—varies between 0 and 0.5, and they applied an incisional model [Lamb et al., 2008] to estimate rates of channel erosion. Furthermore, in their Figure 2 Shaw and Mohrig [2014] documented the temporal evolution of a bedrock-alluvial transition in Gadwall Pass of Wax Lake Delta from July 2010 to February 2012. As another example, the Gota River (Sweden) has been recognized as a low-slope bedrock channel near its marine outlet [Kolb, 1963], where underlying glacial clay deposits behave as active bedrock into which the channel is incised. The channel possesses a narrow and deep cross section and contains actively migrating dunes within alluvial segments. However, the Gota River is a bedrock river for its entire length, and so once again, there are no alluvial-bedrock or bedrock-alluvial transition between the headwaters and the ocean outlet [Millet, 2011].

Little is known about the dynamics of an alluvial-bedrock or a bedrock-alluvial transition within low-slope rivers. These transitions are thought to be transient [e.g., Parker, 2004; Nittrouer et al., 2012a], particularly within large rivers like the Mississippi River where a significant upstream alluvial sediment feed promotes active morphodynamic change. However, it is difficult to measure the spatial evolution of these transitions because the time scale of development is relatively long (decades to centuries), and there are no systematic observational data available for analysis.

In the case of the Mississippi River, the existence of a bedrock-alluvial and an alluvial-bedrock transition has critical implications in regard to the design of engineered diversions that build new deltaic land to protect human society and infrastructure. Land-building engineered diversions are delta restoration projects designed to extract water and sediment from the river main channel and transport them into drowned areas, where a deltaic lobe forms under quasi-natural conditions [Paola et al., 2011]. Predicting the response of the Mississippi River to engineered diversions is critical to evaluate the project feasibility at engineering time scales, i.e., from several decades up to few centuries. In other words, a numerical model is needed to

Figure 1. Alluvial-bedrock and bedrock-alluvial transitions on the lowermost Mississippi River. Images are from Google Earth.
predict if an upstream-migrating wave of degradation will form, or if a diversion project will result in a downstream-migrating wave of deposition, and if these sediment waves will represent a threat for existing infrastructures, navigation, and flood control. The implementation of such a numerical model is not trivial for a fully alluvial river, and it becomes even more difficult in the case of the Mississippi River due to the lack of information on the morphodynamics of bedrock-alluvial and alluvial-bedrock transitions.

Within the lowermost Mississippi River channel, Nittrouer et al. [2012a] and Lamb et al. [2012] showed that during floods, the cross sectionally averaged flow velocity can increase in the streamwise direction (M2 profile), thereby producing erosion of alluvium within the lower 100 km of the river channel ahead of the Gulf of Mexico outlet. Additionally, Chanatanantavet et al. [2012] used a similar flow model to document that during low and moderate water discharge the cross sectionally averaged flow velocity decreases in the streamwise direction (M1 profile) producing in-channel sediment deposition over the lower 300–500 km of the river. However, these models do not address the decadal- to centennial-scale, as well as the broad spatial-scale (tens to hundreds of kilometers), dynamics of both the alluvial-bedrock and bedrock-alluvial transitions now recognized within the lowermost Mississippi River.

In this study, we present a one-dimensional model for the long-term evolution of a low-slope mixed bedrock-alluvial river that is able to capture the spatiotemporal evolution of both the alluvial-bedrock and bedrock-alluvial transitions. We first implement it as a generic model to study the general morphodynamics of such rivers. We demonstrate that in low-slope rivers the transitions can characterize the mobile bed equilibrium condition. We then apply the model to the lowermost Mississippi River, because new research and associated data for this reach make it possible to constrain model input parameters and validate model output. Lastly, the validated model is used to evaluate the impacts of delta restoration projects with engineered land-building diversions [Paola et al., 2011] on the Mississippi River main channel. In particular, we show how the extraction of water and sediment via engineered channel diversions influences the upstream and downstream development of the channel bed (i.e., shoaling and/or erosion).

2. Model Formulation

Models of previous fluvial-deltaic studies, such as those of Swenson et al. [2000], Parker et al. [2008a], and Trueba et al. [2009], have considered the case of an alluvial reach bounded by a bedrock-alluvial transition at the upstream and a delta at the downstream end. These models treat the bedrock-alluvial transition and the delta shoreline as internal, sharp, moving boundaries, whose position is determined by solving the model governing equations.

Moving boundary models, however, are inadequate to describe alluvial-bedrock and bedrock-alluvial transitions in low-slope rivers because sediment transport processes show a gradual rather than sudden change at these transitions. To overcome this difficulty, we use the formulation of Zhang et al. [2014], which is allied to the “mushy layer” formulation for moving liquid-solid phase transitions of, e.g., Voller and Prakash [1987]. In such a formulation, any moving boundaries are captured in the course of the calculation and need not be followed explicitly as shock conditions.
In particular, Zhang et al. [2014] relaxed the assumption of many previous models of bedrock incision that the bed material transport rate is equal to the supply rate. The Zhang et al. [2014] model accounts for the morphodynamics of both bedrock and purely alluvial reaches, by coupling an alluvial sediment routing formulation to the Sklar and Dietrich [2004] formulation. This approach describes the gradual variation of alluvial transport rates within the bedrock reach, while reproducing the spatiotemporal changes of channel bed elevation so as to naturally include transitions between zones of alluvial sediment cover and zones of bedrock exposure.

2.1. Assumptions and Approximations

The model governing equations are the one-dimensional shallow water equations of mass and momentum conservation for open-channel flow and the equation for sediment mass conservation. The general model formulation is designed to be exportable to other systems and allows for field-scale applications elsewhere. For example, basic assumptions are introduced to simplify a general river reach, but these may be relaxed for the purpose of field validation, as we describe below for the lowermost Mississippi River. Other basic assumptions and approximations included are as follows.

1. While the model describes the long-term evolution of a river channel, it does not account for the exchange of sediment between the river and its floodplain. That is, erosion and deposition of floodplain material due to channel migration are assumed to be equal, and the overbank deposition of fine sediment on the floodplain during floods is taken as minimal compared to the channel sediment flux or is otherwise not accounted for [e.g., Viparelli et al., 2011].

2. Sediment is modeled in terms of a single characteristic grain size, \( D_{\text{f}} \), representing the channel bed material as commonly done in simplified morphodynamic models [e.g., Parker et al., 2008b]. This simplification is strongly justified by numerous field data pertaining to sand bed rivers and rather more loosely justified as a first approximation for gravel bed rivers [e.g., Dietrich et al., 1999; Parker, 2004; Lauer and Parker, 2008a, 2008b, Nittouer et al., 2011b, 2012b; Viparelli et al., 2011]. The equations are solved for the case of a sand bed stream, but the application to the case of a gravel bed river is relatively straightforward.

3. The volumetric bed material load is assumed to be orders of magnitude smaller than the flow rate so that the quasi-steady approximation [de Vries, 1965] holds for the flow.

4. Bedrock elevation is fixed in time and incision is not modeled. This condition could easily be relaxed in terms of the formulations of Lague (2010) or of Zhang et al. (2014) and the erosion relation of Lamb et al. (2008). The implementation of bedrock incision is one of the necessary future steps in modeling low-slope bedrock rivers.

5. The simplified channel cross section is assumed to be rectangular with specified effective width \( B \) [e.g., Lauer and Parker, 2008b, 2008c].

6. Flow is always assumed to be Froude subcritical, as is appropriate for low-slope rivers such as the lower Mississippi River [e.g., Parker, 2004; Nittouer et al., 2011b].

2.2. Model Geometry

A simplified longitudinal geometry of the modeled river reach, with the appropriate parameters labeled, is presented in Figure 2. The channel bed is divided into a lower bedrock region and an upper alluvial region. In Figure 2 the topmost part of the bedrock region is represented at elevation \( \eta_{\text{b}} \) above the datum, and the bedrock slope is assumed to be constant and equal to \( S_{\text{b}} \), whereby \( S_{\text{b}} = -\partial \eta_{\text{b}}/\partial x \), with \( x \) denoting the down-channel coordinate. The channel bed is defined as the topmost part of the alluvial region, and its elevation above the datum is denoted with \( \eta_{\text{fl}} \), whereby the channel slope, \( S_{\text{fl}} \), is defined as \( S = -\partial \eta_{\text{fl}}/\partial x \). The dotted gray line at distance \( L_{\text{ac}} \) above the bedrock surface represents the minimum thickness of alluvial cover for complete channel bed alluviation and is analogous to the macroroughness height of Zhang et al. [2014], i.e., \( \eta_{\text{b}} + L_{\text{ac}} \) represents the minimum channel bed elevation for complete alluviation. Here, however, we substitute the macroroughness height for the minimum thickness of alluvial cover, because it is possible to estimate this parameter using field data from low-slope rivers. For our efforts here, we define a reach to be alluvial when \( \eta > \eta_{\text{b}} + L_{\text{ac}} \), and to be bedrock when \( \eta < \eta_{\text{b}} + L_{\text{ac}} \) [Zhang et al., 2014]. Therefore, the condition for which \( \eta \) passes through \( \eta_{\text{b}} + L_{\text{ac}} \) represents an alluvial-bedrock transition (where \( \eta \) drops from above to below \( \eta_{\text{b}} + L_{\text{ac}} \)) or a bedrock-to-alluvial transition (where \( \eta \) increases from below to above \( \eta_{\text{b}} + L_{\text{ac}} \)).

Additionally, the water surface resides at elevation \( \zeta \) above the datum, and the water depth is denoted as \( H \) (Figure 2). For a condition of sea level rise, a user-specified rate of rise \( \zeta \) may be applied at the downstream end of the modeled domain.
The elevation $\eta_{\text{base}}$ is a material boundary (i.e., zero sediment flux) at which the subsidence rate $\sigma$ is applied. The relative elevations denoted with the prime superscript are introduced to simplify the calculations. For example, the elevation of the alluvial bed above the material boundary is denoted as $\eta'$, where $\eta' = \eta - \eta_{\text{base}}$ and the subsidence rate is expressed as the time rate of change of the elevation of the material boundary above the datum $\sigma = -\frac{\partial \eta_{\text{base}}}{\partial t}$, where $t$ denotes a temporal coordinate. Such a formulation captures tectonic subsidence and can be used as a proxy for subsidence associated with compaction.

2.3. Flow Equations

The governing equations for the flow are the shallow water equations of mass and momentum balance

$$\frac{\partial H}{\partial t} + \frac{\partial UH}{\partial x} = 0$$ (1)

$$\frac{\partial UH}{\partial t} + \frac{\partial U^2H}{\partial x} = -gH \frac{\partial H}{\partial x} + gHS + C_f U^2$$ (2)

where $U$ denotes the mean flow velocity, $C_f$ is a constant nondimensional friction coefficient, and $g$ is the acceleration of gravity. Equations (1) and (2) are simplified with the quasi-steady approximation [de Vries, 1965].

Dropping the time dependence in equation (1), the conservation of water mass takes the form

$$q = UH$$ (3)

where $q$ denotes the volumetric flow rate per unit channel width, $q = Q/B$, with $Q$ being the flow discharge. Dropping the time dependence in equation (2), the momentum balance equation becomes

$$\frac{U}{g} \frac{\partial U}{\partial x} + \frac{\partial H}{\partial x} + \frac{\partial \eta}{\partial x} = \frac{C_f U^2}{gH}$$ (4)

Recalling that the Froude number squared $Fr^2$ is equal to $U^2/(gH)$ and that the friction slope $S_f$ is defined as $C_f Fr^2$, equation (4) is rewritten as

$$\frac{\partial E}{\partial x} = -S_f$$ (5)

where $E$ is the total specific energy defined in equation (6) below as

$$E = \eta + H + \frac{U^2}{2g}$$ (6)

in which the Coriolis coefficient in the energy balance equation of open-channel flow has been taken equal to 1 [Chaudhry, 2008].

In low-slope rivers, i.e., those with slope $S \leq 10^{-4}$, the flow is most likely to be Froude subcritical. Thus, equation (5) is integrated in the upstream direction with the downstream boundary condition

$$\zeta_{\text{in}} = \zeta_d(t)$$ (7)

where $x_d$ denotes the streamwise coordinate of the downstream end of the modeled domain.

Equation (4) could have been reduced to the classical backwater form. However, we prefer to cast it in the form of equation (5) because it allows for relatively straightforward modifications of the bed resistance formulation and for future model applications to river-floodplain systems.

2.4. Bed Material Equations

The time rate of change of channel bed elevation is computed with the equation of conservation of channel bed material for mixed bedrock-alluvial channels [Zhang et al., 2014]

$$\left(1 - \lambda_p \right) B_p \frac{\partial \eta}{\partial t} + \sigma = -\frac{\partial p_c Q_{\text{bmc}}}{\partial x}$$ (8)

where $\lambda_p$ denotes the overall porosity of the alluvial deposit, $Q_{\text{bmc}}$ is the volumetric bed material load at the capacity condition (when the bed is fully alluviated), $p_c$ represents the areal fraction of bed covered with alluvium, and $\sigma$ denotes the subsidence rate. When $p_c = 1$, the channel bed is fully alluviated and equation (8) reduces to the standard Exner equation (with the inclusion of subsidence).
The fraction of bed covered with alluvium is computed herein with a slightly simplified version of the Zhang et al. [2014] formulation. For the case of low-slope rivers we use the minimum thickness of alluvial cover, \( L_{ac} \), to determine if the channel bed is bedrock or fully alluviated. We assume that when the channel bed elevation is equal to the elevation of the bedrock, a residual value of 5% of the channel bed is covered with alluvium [Zhang et al., 2014]. As shown in Figure 2, at the transitions between partially and fully alluviated reaches, i.e., where \( \eta = \eta_b + L_{ac} \), the bed is barely alluviated and \( p_c = 1 \). Assuming that in the mixed bedrock-alluvial reach the cover fraction linearly increases with the channel bed elevation, the following closure relation is obtained:

\[
P_c = \begin{cases} 
0.05 + 0.95 \frac{\eta - \eta_b}{L_{ac}} & \text{if } 0 \leq \frac{\eta - \eta_b}{L_{ac}} \leq 1 \\
1 & \text{if } \frac{\eta - \eta_b}{L_{ac}} > 1 
\end{cases}
\]  

(9)

The above relation, which is analogous to the Zhang et al. [2014] closure for mixed bedrock-alluvial rivers, states simply that the areal cover fraction increases with the thickness of the alluvial deposit, up to the point of complete coverage.

The capacity volumetric bed material load is computed with the Engelund and Hansen [1967] relation for total load (i.e., suspended load plus bed load) in the form

\[
Q_{bmc} = \sqrt{R \rho_g D_b} \frac{\alpha_{EH}}{C_f} (\tau^*)^n
\]  

(10)

where \( R \) is the submerged specific gravity of the bed material (~1.65 for quartz), \( \alpha_{EH} \) and \( n \) are model parameters respectively equal to 0.05 and 2.5, and \( \tau^* \) is the Shields number, i.e., the nondimensional bed shear stress, defined as

\[
\tau^* = \frac{\tau}{\rho \rho_g D_b} = \frac{C_f U^2}{\rho \rho_g D_b}
\]  

(11)

with \( \tau \) being the bed shear stress and \( \rho \) denoting the water density.

The Engelund and Hansen relation is multiplied by the cover fraction \( p_c \) to account for the limited availability of alluvium on the channel bed in a mixed bedrock-alluvial reach. Equation (10) should be substituted with an appropriate bed load relation to apply the model to gravel bed rivers.

2.5. The Flow of the Calculations

Here we are interested in the long-term evolution of a river reach; thus, we model the river as a sediment feed flume analog, whereby the total amount of sediment stored in the channel bed is allowed to change in time [e.g., Viparelli et al., 2011].

The longitudinal profiles of channel bed elevation \( \eta(x,t) \) at \( t = 0 \) defines the model initial condition. The bedrock surface elevation profile \( \eta_b(x,t) \) is a prescribed function; in so far as incision is not included here, it may change in time only in response to subsidence (Figure 2). Water and bed material are fed at a specified rate at the upstream end of the modeled domain. In particular, the bed material is fed in at a ghost node, where the channel bed elevation and the flow parameters are not computed [Parker, 2004]. The downstream boundary condition is given in terms of specified water surface elevation with equation (7).

The model domain is divided in \( N - 1 \) intervals bounded by \( N \) computational nodes, and the initial alluvial cover fraction, \( p_c(x,0) \), is determined with equation (9) using the initial profiles \( \eta(x,0) \) and \( \eta_b(x,0) \). Equation (5) is integrated from downstream to compute the water depth \( H \) and the mean flow velocity \( U \) at all the computational nodes. Equations (11) and (10) are then solved to obtain the Shields number and the bed material load in the computational domain. The time rate of change of channel bed elevation is determined with equation (8), and the channel bed elevation is updated at each computational node.

In the case of a subsiding system (see Figure 2), the elevation of the bedrock is updated to account for subsidence as

\[
\eta_b(x, t + \Delta t) = \eta_b(x, t) + \sigma(x, t) \Delta t
\]  

(12)

where \( \Delta t \) represents the time increment of the calculations. The cover fraction is updated with equation (9), and the loop starts again by solving equation (5).
The differential equations (5) and (8) are integrated with a first-order finite difference scheme, i.e., the Euler method, both in space and time. The model is embedded in a Microsoft Excel file and is written in Visual Basic for Applications.

3. Mobile Equilibrium of the Alluvial-to-Bedrock and Bedrock-to-Alluvial Transitions

In this section we present calculations for a generic reach to discuss the conditions that are responsible for the formation of alluvial-bedrock and bedrock-alluvial transitions in low-slope rivers and to investigate if these transitions are stable or transient features of the river longitudinal profiles. The numerical model presented in section 2 is here applied to a simplified low-slope sand bed river to study the mobile bed equilibrium of alluvial-bedrock and bedrock-alluvial transitions, such as those of the lowermost Mississippi River, for the case of constant base level. The effects of relative base level change on the morphodynamics of alluvial-bedrock and bedrock-alluvial transitions are discussed in subsequent sections of the paper. The model parameters used in the numerical runs, while intended to be applicable to most other low-sloping rivers, are representative of a simplified lowermost Mississippi River because there are abundant data to help constrain the model [e.g., Nittrouer et al., 2011a, 2011b, 2012a; Nittrouer and Viparelli, 2014].

As such, we set the model domain as 500 km long, roughly corresponding to the reach of the Mississippi River which extends from the Old River Control Structure to the Gulf of Mexico (Figure 1). The water discharge and feed rate of bed material are representative of the present lowermost Mississippi River as \( Q = 25,000 \text{ m}^3/\text{s} \) and \( bm, \text{feed} = 27.5 \text{ Mt/yr} \) [Nittrouer and Viparelli, 2014]. To account for the interannual variability of flow discharge, it is assumed that the river is in flood and thus morphologically active a fraction \( f_L \) of the year [Paola et al., 1992]. The parameter \( f_L \) is called the flood intermittency; it multiplies the right-hand side of equation (8) in the calculations; for the lowermost Mississippi River it is equal to 0.34 [Wright and Parker, 2004a, 2004b]. The characteristic grain size of the bed material \( D \) is 0.3 mm [Nittrouer et al., 2012b], and the overall porosity of the alluvial deposit, \( p_c \), is assumed to be equal to 0.4 [Nittrouer and Viparelli, 2014]. The friction coefficient \( C_f \) is equal to 0.003, corresponding to a nondimensional Chezy coefficient of \( \sim 18 \), which is within the reported ranges for low-slope large sand bed rivers [Wilkerson and Parker, 2011]. The effective channel width \( B \) for sediment transport calculations is assumed to be somewhat less than the top width of the channel [e.g., Mooney, 2008]. Here this width is taken to be constant in space and equal to 520 m, \( \sim 70\% \) of the mean top channel widths of the surveyed cross sections considered by Nittrouer et al. [2012b]. The minimum thickness of alluvial cover \( L_{ac} \) is equal to 15 m, which is about \( \sim 1.5 \) times the maximum dune height measured in the mixed bedrock-alluvial reach during high flow conditions [Nittrouer et al., 2008]. Since \( L_{ac} \) is one of the most important model parameters, the assumption \( L_{ac} = 15 \text{ m} \) requires justification. In the case of a fully alluvial bed covered with dunes the average depth of scour below the mean bed elevation is on the order of half of the dune height. Noting that (1) bed form height varies with flow discharge and within a train of bed forms and (2) we have one data set of bed form heights at high flow (i.e., 23,500 \text{ m}^3/\text{s}) for the 5 km reach at Audubon Park [Nittrouer et al., 2008], we assume that in the lowermost Mississippi River the height of the large dunes during floods is similar to the maximum dune height measured at Audubon Park. We further assume that an alluvial layer roughly equal to one large dune height is necessary to guarantee that the migrating large bed forms do not feel the effect of the underlying bedrock surface, so an alluvial cover roughly equal to 1.5 times the dune height during flood flow is needed to guarantee complete alluviation of the channel bed. Laboratory experiments on bed forms in low-slope bedrock rivers may be performed to better constrain this parameter. The input parameters for the runs are summarized in Table 1.

Streamwise distances are measured in kilometers upstream of the downstream end of the modeled reach: thus, \( x = 0 \) corresponds to the downstream end of the modeled reach, i.e., the river outlet, and \( x = 500 \text{ km} \) corresponds to the upstream end of the modeled domain.

The water depth, \( H_{or} \) and the alluvial bed slope, \( S_{or} \), characterizing the alluvial mobile bed equilibrium (\( p_c = 1 \)) are determined from equations (3), (4), (10), and (11). In a fully alluvial reach, at mobile bed equilibrium the net channel bed erosion is equal to zero [e.g., Parker, 2004]. Thus, the channel bed elevation \( H \) does not change in time, and the bed material load, \( Q_{bm} \), which is equal to the feed rate \( Q_{bm, \text{feed}} \), the Shields number \( \tau^* \), and the mean flow velocity \( U \) are constant in space and time. To satisfy water mass conservation, i.e., equation (3), the water depth \( H \) is also taken to be constant in the flow direction and equal to \( H_{or} \) and the
In the elevation plots, the red lines denote the channel bed profile at mobile bed equilibrium. For reference, the longitudinal profile of the bedrock surface, the minimum channel bed elevation for complete alluviation, and the water surface elevation at the downstream end of the modeled reach, \( \zeta_d = 0 \), are respectively represented with the continuous gray lines, the dashed gray lines, and the dashed blue lines, respectively. When the channel bed elevation is above the dashed grey line, the channel bed is fully alluviated and the cover fraction \( \rho_c \) is equal to 1—see equation (9). When the channel bed elevation is between the dashed and the continuous grey lines, the channel bed is partially covered with alluvium, i.e., a bedrock reach, and the

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Definition</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>Bankfull water discharge</td>
<td>25,000 m³/s</td>
</tr>
<tr>
<td>( G_{\text{bm,feed}} )</td>
<td>Mean annual feed of bed material load under present-day conditions.</td>
<td>27.5 Mtyr</td>
</tr>
<tr>
<td>( f_l )</td>
<td>Flood intermittency</td>
<td>0.34</td>
</tr>
<tr>
<td>( D )</td>
<td>Characteristic grain size</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \lambda_d )</td>
<td>Porosity of alluvial deposit</td>
<td>0.4</td>
</tr>
<tr>
<td>( R )</td>
<td>Submerged specific gravity of sediment</td>
<td>1.65</td>
</tr>
<tr>
<td>( C_f )</td>
<td>Friction coefficient</td>
<td>0.003</td>
</tr>
<tr>
<td>( B )</td>
<td>Effective bankfull width used in the equilibrium runs</td>
<td>520 m</td>
</tr>
<tr>
<td>( L_{\text{bc}} )</td>
<td>Minimum thickness of alluvial cover for complete alluviation</td>
<td>15 m</td>
</tr>
<tr>
<td>( S_f )</td>
<td>Alluvial equilibrium channel slope in the equilibrium runs</td>
<td>( 1.9 \times 10^{-5} ) m/m</td>
</tr>
<tr>
<td>( H_o )</td>
<td>Alluvial equilibrium bankfull depth in the equilibrium runs</td>
<td>33.5 m</td>
</tr>
<tr>
<td>( \zeta_d )</td>
<td>Elevation of downstream water surface</td>
<td>0 m</td>
</tr>
</tbody>
</table>
The alluvial cover fraction $p_c$ varies between 0.05 and 1. In particular, as the vertical distance between the channel bed profile (red line) and the bedrock surface (continuous grey line) decreases, the alluvial cover fraction tends to the lower limit of 0.05. If the channel bed elevation is equal to the bedrock elevation, $p_c = 0.05$ and minimal alluvial cover should be expected.

The water depths at mobile bed equilibrium are represented with the continuous blue lines in the plots on the right side of Figure 3 (water depth plots). For reference, the water depth $H_o$ that pertains to the case of alluvial mobile bed equilibrium is represented with the black dashed line.

The case of a bedrock surface with slope $S_b = 1 \times 10^{-5} < S_o$ and elevation equal to $-45$ m below mean sea level is represented in Figure 3a. Under these conditions, the equilibrium channel is divided in two reaches, an upstream fully alluvial reach and a downstream bedrock reach, separated by a stable alluvial-bedrock transition (blue circle in the elevation plot of Figure 3a). In the fully alluvial reach the water depth is equal to $H_o$ and the channel slope is equal to $S_o$ (water depth plot of Figure 3a). As shown in the water depth plot of Figure 3a, a stable M2 (drawdown) backwater curve characterizes the water surface profile downstream of the alluvial-bedrock transition. In the bedrock reach the water depth decreases in the streamwise direction with a consequent increase in mean flow velocity and bed material transport capacity, $Q_{bmc}$—equations (3), (10), and (11). At mobile bed equilibrium the net channel erosion or deposition is equal to zero; thus, the divergence of the bed material load, i.e., the right-hand side of equation (8) $\partial p_c Q_{bmc}/\partial x$, must be equal to zero. Due to the streamwise increase in bed material transport capacity, the alluvial cover fraction $p_c$ has to decrease in the streamwise direction (elevation plot of Figure 3a).

Figure 3. Comparison between channel bed elevations (plots on the left) and water depths (plots on the right) with stable (a) alluvial-bedrock, $S_b < S_o$, and (b) bedrock-alluvial, $S_b > S_o$, transitions. Streamwise distances are measured in kilometers from the downstream end of the modeled reach, and the blue arrows denote the flow direction. In the elevation plots the red line represents the elevation of the channel bed, $\eta$; the continuous grey line is the elevation of the bedrock surface, $\eta_b$; the dashed grey line denotes the minimum thickness of alluvial cover for complete alluviation, $\eta_b = L_{ac}$; and the dashed blue line is the downstream water surface elevation, $\xi_d$. In the water depth plots the continuous blue line represents the streamwise variation of water depth at equilibrium and the dashed black line denotes the water depth that would prevail for purely alluvial equilibrium, i.e., $H_o = 33.5$ m.
The case of a bedrock surface with slope $S_b = 3 \times 10^{-5} < S_o$ and elevation equal to $-35$ m below mean sea level at the downstream end is presented in Figure 3b. At mobile bed equilibrium a stable bedrock-alluvial transition (blue circle in Figure 3b) separates the upstream bedrock reach from the downstream alluvial reach. The mobile bed equilibrium in the alluvial reach is an equilibrium characterized by a constant water depth equal to $H_o$ and a constant channel bed slope equal to $S_o$. A stable M1 backwater curve characterizes the mobile bed equilibrium in the bedrock reach, where the water depth increases in the direction of the flow (water depth plot of Figure 3b). Due to the M1 backwater curve in the bedrock reach, the bed material transport capacity decreases in the downstream direction. Thus, at mobile bed equilibrium, when the divergence of the bed material load must be equal to zero, the streamwise-decreasing bed material transport capacity is balanced by a streamwise-increasing alluvial cover fraction in the bedrock reach (elevation plot of Figure 3b).

The two runs presented in Figure 3 clearly demonstrate that when the bedrock surface is relatively shallow with respect to the downstream water surface elevation, alluvial-bedrock and bedrock-alluvial transitions can be stable features in low-slope rivers depending on the relative magnitude of the bedrock surface slope, $S_b$, and the alluvial equilibrium slope, $S_o$. If the bedrock surface slope is milder than the alluvial equilibrium slope, i.e., $S_b < S_o$, a stable alluvial-bedrock transition may form (Figure 3a). Water depth and channel slope in the upstream alluvial reach are equal to their alluvial equilibrium values, and the bedrock surface forces a stable M2 backwater profile in the downstream bedrock reach. If the bedrock surface is steeper than the alluvial equilibrium slope, i.e., $S_b > S_o$, the mobile bed equilibrium may be characterized by a stable bedrock-alluvial transition (Figure 3b). Water depth and channel slope in the alluvial reach are equal to the alluvial equilibrium values, and in the upstream bedrock reach the shallow bedrock surface forces a stable M1 backwater curve.

For shallower bedrock surfaces relative to the downstream water surface elevation compared to those of Figure 3, the equilibrium channel bed is characterized by exposed bedrock, which forces an M1 or an M2 backwater curve depending on the relative magnitude of $S_b$ and $S_o$. In particular, if $S_b < S_o$, in the upstream alluvial reach the water depth and channel slope are equal to their alluvial equilibrium values, and the bedrock surface forces a stable M2 backwater profile. If $S_b > S_o$, the mobile bed equilibrium may be characterized by a stable M2 backwater curve. For deeper bedrock surfaces relative to the downstream water surface elevation than those considered in Figure 3 the equilibrium channel is fully alluvial with slope equal to $S_o$ and with steady and uniform flow with depth equal to $H_o$.

4. Model Validation on the Lowermost 500 km of the Mississippi River

Field-scale validation of morphodynamic models is best done in consecutive phases. Models are first zeroed to (1) calibrate parameters that cannot be estimated from the available data, (2) verify that they can predict reasonable conditions of mobile bed equilibrium, and (3) determine relatively reasonable initial conditions for the application runs, which often describe contemporary and/or future scenarios. These latter runs depend on the application in question. They are generally several decades long, focusing, for example, on how rivers respond to changes in flow conditions, sediment supply, and base level. Due to the lack of historical data, a predisturbance mobile bed equilibrium condition is commonly assumed as a starting point [e.g., Viparelli et al., 2011; Nittrouer and Viparelli, 2014].

In the case of the lowermost Mississippi River, however, the problem is more complex because an initial condition of mobile bed equilibrium cannot be strictly defined. This is because (1) the river ends in an actively prograding delta, (2) subsidence rates may have influenced the long-term evolution of the system, and (3) since the 1500s the Atchafalaya River has been an active distributary channel for the Mississippi River, and (4) the lower Mississippi River has undergone periodic avulsions in the Holocene [e.g., Fisk, 1944].

Due to a shorter distance to the Gulf of Mexico, the Atchafalaya River maintains a slope roughly 2–3 times greater than that of the modern lowermost Mississippi River. Therefore, there is a strong likelihood that in time much of the current main stem Mississippi River discharge will be captured by the Atchafalaya River via an avulsion. However, from the 1500s to the early 1900s, massive logjams intermittently prohibited much of the flow from avulsing into the Atchafalaya River, but in the early 1900s, after a long campaign to remove the logjam via cutting and burning so as to open the river for shipping navigation, the Atchafalaya River...
started capturing an ever-increasing amount of flow and sediment from the Mississippi River. Thus, the Atchafalaya River distributary is a relatively new and rapidly evolving phenomenon [Roberts, 1998] that can be simplified and modeled in terms of a step function, as further illustrated below.

In the 1960s, the U.S. Army Corps of Engineers constructed the Old River Control Structure in order to prevent an avulsion of the Mississippi River into the Atchafalaya River. For the past 50 years, the structure has regulated the partition of water and sediment load between the two rivers so that approximately two thirds of the water is presently discharged in the lowermost Mississippi River and approximately one third is diverted to the Atchafalaya River [e.g., Roberts, 1998]. We eschew using more precise numbers than this due to the inherent uncertainty of structure operation. Horowitz et al. [2001] estimated a split such that between 25% and 27% of the sediment enters the Atchafalaya. Based on 3 years of measurement, Allison et al. [2012] suggested that the fractional split of sediment load can be even more biased toward the downstream Mississippi River than the water. Nittrouer et al. [2012b] and Roberts [1998], however, showed that at a diversion farther downstream, the fractional split between sediment can be more biased toward the diversion than the water. We assume, in the absence of more precise information that the split of sediment (both sand and mud) is in the same proportion as the water. This assumption can be amended as more data become available.

For large-scale modeling purposes, we simplify the problem as follows. We zero the model with runs of 600 year length under the assumption that previous to the construction of the Old River Control Structure, the amount of water and sediment diverted to the Atchafalaya River was negligible compared to the flow and the sediment transport in the lowermost Mississippi River main channel. This allows the characterization of an antecedent, approximately equilibrium condition before the Old River Control Structure was built. To account for the effects of the Atchafalaya River diversion and of the regulated flow regime at the Old River Control Structure on the channel morphology, we continue the simulations for a further 70 years beyond this zeroing period (1940 to 2010), now imposing the condition that one third of the flow and the sediment transport is diverted to the Atchafalaya River.

In subsequent sections of the paper we refer to a specific 70 yearlong run from 1940 to 2010 as the “Atchafalaya diversion run,” where the word diversion indicates a controlled partition of the flow and of the bed material load between the Mississippi River channel and a distributary channel by means of an engineered structure, as described further in section 6.

In order to simplify the calculations, delta growth into the Gulf of Mexico is not explicitly modeled, and the computations are performed on a fixed grid corresponding to the present Mississippi River channel. In addition, the effects of the spillways to control flood flow are not accounted for in the numerical runs.

The numerical simulations presented in this section pertain to the case of a nonsubsiding system. The effects of different subsidence rates on the morphology of the lowermost Mississippi River channel are explored and presented in the next section. The overall porosity of the alluvial bed, \( \lambda_p \), the minimum thickness of alluvium for complete alluviation, \( L_{ac} \), and the submerged specific gravity of the bed material, \( R \), are reported in Table 1, i.e., respectively 0.4, 15 m, and 1.65.

The spatial distance between two computational nodes is kept constant and equal to 2 km. The model runs with a time step of 1 year. To guarantee the numerical stability of the calculations, each year is divided into 30 shorter temporal intervals.

### 4.1. Modification of the Model

The general model presented in section 2 is modified with appropriate formulations to compute the bed material load and the frictional resistance in a large, low-slope sand bed river. In particular, suspended bed material load and frictional resistances are now computed with the Wright and Parker [2004a, 2004b] formulation. The transport rate associated with bed load transport is estimated with the relation of Ashida and Michiue [1972], which is appropriate for the bed material transport conditions of the modeled reach [Nittrouer et al., 2008]. Herein the formulation is implemented for the case of uniform bed material, but it is easily generalized to nonuniform sediment in accordance with Wright and Parker [2004a, 2004b].

#### 4.1.1. Flow Resistance Formualtion

The Wright and Parker [2004b] formulation for flow resistance is based on the assumption that the shape of the velocity profile averaged over a dune-covered bed has roughly the same shape as in the case of a flat but hydraulically rough bed. That is, dunes can be treated as roughness units. Under this assumption, the
roughness height due to skin friction $k_s$ can be replaced by a composite roughness height $k_c$ that accounts for both skin friction and form drag, and the following relation between mean flow velocity $U$ and water depth $H$ holds [Wright and Parker, 2004b]

$$\frac{U}{u_*} = 8.32 \left( \frac{H}{k_c} \right)^{\frac{1}{6}}$$

(13)

where $u_*$ denotes the shear velocity defined as $(\tau/\rho)^{0.5}$ and $\alpha$ is a parameter that accounts for stratification effects due to suspended sediment transport.

In the original formulation, $\alpha$ was defined as a function of the channel bed slope $S$ because the authors assumed alluvial mobile bed equilibrium conditions [Wright and Parker, 2004b]. Here the flow is assumed steady but not uniform; thus, the channel bed slope $S$ is substituted with the friction slope $S_f$ [e.g., Parker, 2004]:

$$\alpha = \begin{cases} 
1 - 0.06 \left( \frac{C_S}{S_f} \right)^{0.77} & \text{for } \frac{C_S}{S_f} \leq 10 \\
0.67 - 0.0025 \left( \frac{C_S}{S_f} \right) & \text{for } \frac{C_S}{S_f} > 10 
\end{cases}$$

(14)

where $C_S$ denotes the near-bed volumetric suspended sediment concentration, i.e., at a distance above the channel bed equal to 5% of the water depth.

For equilibrium conditions, $C_S = E_b$, where $E_b$ denotes the nondimensional entrainment rate of sediment in suspension [Wright and Parker, 2004b]. As commonly done in large-scale models of river morphodynamics, we further assume that the relation $C_S = E_b$ holds under slowly varying nonequilibrium conditions [Parker, 2004].

The Wright and Parker [2004b] formulation was derived under the assumption of alluvial channel morphology. In a mixed bedrock-alluvial reach, $E_b$ must be multiplied by the cover fraction $p_c$ to account for the limited availability of alluvium on the channel bed. The nondimensional entrainment rate of uniform sediment in suspension is estimated as

$$E_b = p_c \frac{7.8 \times 10^{-7} X^5}{1 + \frac{7.8 \times 10^{-7} X^5}{0.3}}$$

(15)

where $X$ is defined as

$$X = \left( \frac{u_* S_f}{\nu} \right)^{0.08}$$

(16)

Here $u_*$ is the shear velocity associated with skin friction, $\nu$ is the particle settling velocity and $R_p$ is the particle Reynolds number defined as $(RgD)^{0.5}/\nu$, with $\nu$ denoting the kinematic viscosity of water.

Recalling that the shear velocity $u_*$ is defined as $(\tau/\rho)^{0.5}$, the relation between $u_*$ and the shear velocity due to skin friction $u_s$ is given in terms of Shields numbers as

$$\tau_s^* = 0.05 + 0.7 \left( \frac{\tau_s}{\tau_s^*} \right)^{0.8}$$

(17)

where $\tau_s^*$ is the Shields number associated with skin friction [Wright and Parker, 2004b].

The relation between the roughness height due to skin friction $k_s$ and the composite roughness height $k_c$ is obtained from equations (13) and (14) as

$$k_c = k_s \left( \frac{\tau_s^*}{\tau_s} \right)$$

(18)

where $k_s$ is equal to $3D_{50}$, where $D_{50}$ denotes the diameter such that 90% of the bed material is finer. In the numerical runs discussed below, $D_{50} = 0.6 \text{ mm}$ [Nittrouer et al., 2011b].

The Wright and Parker [2004b] formulation for frictional resistance requires an iterative solution of the equations presented above. Equation (5) is solved to determine the total specific energy in the generic computational node. Equations (3) and (6) are solved to determine the water depth $H$ and the mean flow velocity $U$. A first estimate of the friction slope, $S_f$, is obtained substituting equation (18) in equation (13) under the assumption that stratification effects are negligible, i.e., $\alpha = 1$. The friction slope $S_f$ the shear
velocity \( u_r \), the parameter \( \alpha \), the composite roughness height \( k_c \), and the Shields number associated with skin friction \( \tau^* \), are iteratively determined by solving equations (17), (13), and (14) with the aid of equations (11) and (18).

### 4.1.2. Bed Material Load Calculations

The volumetric suspended bed material load \( Q_{bm,s} \) is computed with the Wright and Parker [2004b] formulation for large low-slope sand bed rivers as

\[
Q_{bm,s} = B \frac{9.7 C_s u_H}{\alpha} \left( \frac{H}{k_c} \right)^{\frac{5}{6}} I
\]

In equation (19) \( C_s \) is computed with equation (15) and \( I \) is the integral

\[
I = \int_{y_5}^{1} \left( 1 - \frac{y}{y_s} \right) \frac{\alpha k_c}{u_H} dy
\]

where \( \alpha \) denotes the von Karman constant, equal to 0.4 in the numerical runs presented below, \( y = z / H \), with \( z \) being an upward oriented vertical coordinate with origin on the channel bed, and \( y_s \) is the value of \( y \) where the volumetric suspended sediment concentration is equal to \( C_s \), i.e., \( y_s = 0.05 \).

The bed material transport rate associated with bed load transport is computed as a function of the Shields number associated with skin friction with the Ashida and Michiue [1972] load relation for uniform material:

\[
Q_{bm,b} = 17 p_s \sqrt{gDDB} (\tau^*_s - 0.05) \left( \sqrt{\tau^*_s} - \sqrt{0.05} \right)
\]

In the above relation the cover fraction \( p_s \) corrects for the limited availability of alluvium in the mixed bedrock-alluvial reaches.

### 4.1.3. Calculation of the Mean Annual Bed Material Load

To account for the interannual variability of the flow discharge and of the bed material load, the flow regime is specified in terms of a flow duration curve. Flow rates are grouped in \( M \) discharge bins with a characteristic flow rate \( Q_j \). The average fraction of the year in which the flow is in a given discharge bin is denoted as \( \rho_j \). For each characteristic flow discharge \( Q_j \) the flow and the bed material calculations are performed. The mean annual bed material load \( Q_{bm} \) is computed as a weighted average over the flow duration curve:

\[
Q_{bm} = \sum_{j=1}^{M} (Q_{bm,s,j} + Q_{bm,b,j}) \rho_j
\]

where \( Q_{bm,s,j} \) and \( Q_{bm,b,j} \) respectively denote the suspended bed material load and the bed material load associated with bed load transport, respectively, computed with equations (19) and (21). The equation of conservation of alluvial sediment, equation (8), is solved as a function of the spatial variation of the mean annual bed material load computed with equation (22) [Viparelli et al., 2013].

### 4.2. Model Boundary and Initial Conditions

Model boundary conditions are the flow and sediment transport at the upstream end of the modeled reach, the water surface elevation at the downstream end of the modeled domain, and the elevation of the bedrock surface.

#### 4.2.1. Flow Regime and Bed Material Load

The flow and the sediment transport at the upstream end of the modeled reach are specified in terms of the flow duration curve and mean annual bed material load at the upstream end of the modeled reach, i.e., the U.S. Army Corp of Engineers gaging station at Tarbert Landing (Figure 4).

Since the overall channel geometry, as well as the flow regime, of the Mississippi River has remained roughly constant in the last ~ 2000 years [Fisk, 1944], it seems reasonable to assume that the bed material (i.e., sand) supply to the lowermost Mississippi River has also remained constant over the same time. In addition, Nittrouer and Viparelli [2014] recently demonstrated that the average flow regime and bed material supply to the lowermost Mississippi River has likely remained constant over the past century. Their analysis of river morphodynamics of the Mississippi River channel from Cairo, Illinois, to the Old River Control Structure further shows that the effects of the construction of dams in the upper Mississippi River basin most likely will not affect the morphology of the lowermost Mississippi River for the next several centuries.
As in Nittrouer and Viparelli [2014], we compute the mean annual bed material load at Tarbert Landing with the suspended sediment bed material rating curve and the flow duration curves. We further assume that transport rates associated with bed load and bed form migration are approximately equal to the 25% of the suspended bed material load [Nittrouer et al., 2011b]. The resulting mean annual suspended bed material load is 22 Mt/yr, and the total mean annual bed material load is 27.5 Mt/yr (Table 1).

To estimate the flow duration curve and the mean annual bed material load for the 600 years long zeroing run, i.e., pre-1940 Mississippi River, we multiply the present flow rates and bed material supply at Tarbert Landing by 1.5, to account for the one third of water and sediment currently diverted into the Atchafalaya River [Nittrouer and Viparelli, 2014]. The flow duration curve, specified in terms of 18 characteristic flow discharge intervals, is reported in Table 2.

4.2.2. Sea Level and Rates of Sea Level Rise
The downstream boundary condition of the model is expressed in terms of water surface elevation. In the 600 years long zeroing run, i.e., pre-1940 Mississippi River, we assume that sea level was constant and at elevation equal 0 [Fisk, 1944]. In the Atchafalaya diversion run, i.e., from 1940 to 2010, a conservative rate of sea level rise of 2 mm/yr is imposed (base case in Kim et al. [2009]).

4.2.3. Elevation of the Bedrock Surface
The downstream part of the lowermost Mississippi River channel is incised in Holocene and Pleistocene clay deposits [Kolb, 1963; Hudson and Kesel, 2000; Nittrouer et al., 2011a] and is modeled as a nonerodible bedrock substrate in the numerical runs. In particular, Kolb [1962] reported that from College Point at Rkm 257 to Rkm 130 the river channel is incised into stiff Pleistocene clays, i.e., what we model as bedrock. The surface

Figure 4. Map with the relevant locations for the field-scale model validation: (1) USACE gaging station at Tarbert Landing; (2) Angola; (3) USGS gaging station at St. Francisville; (4) USGS gaging station at Baton Rouge; (5) Donaldsonville; (6) College Point; (7) alluvial-bedrock transition; (8) Rkm 200; (9) Carrolton; (10) Rkm 130; (11) USGS gaging station at Belle Chasse; (12) Rkm 100; (13) bedrock-alluvial transition; and (14) Head of Passes. The image is from Google Earth.
slopes of the Pleistocene clays depend on the ancient history of the Mississippi River Delta plain prior to the development of entrenched valleys during the Holocene sea level rise [Fisk, 1944; Kolb, 1962]. Downstream of Rkm 130 most of the river channel is incised into Holocene prodelta clays, whose surface slopes are again not controlled by river channel processes [Kolb, 1962].

Fisk [1944] and Kolb [1963] also noted that the lowermost Mississippi River has occupied its present course from Donaldsonville, Rkm 304, to Rkm 130 for the past 2500 years (see locations in Figure 4). We thus assume in the zeroing run that most of the lowermost Mississippi River channel was already carved approximately seven centuries ago, i.e., at the beginning of the model zeroing (or pre-1940) run presented below.

This notwithstanding, modeling channel incision in the Holocene and Pleistocene clays of the Mississippi River Delta is an interesting problem that goes well beyond the scope of the present paper.

The longitudinal profile of the bedrock surface represented with the black line in Figure 5 is used in the numerical runs. It is based on the data reported by Kolb [1962] on the elevation of the Pleistocene clay (squares) and Holocene clay (circles), by Stanley et al. [1996] for the Pleistocene clay (squares) and by Nittrouer et al. [2011a, 2012a] for the Mississippi River thalweg (grey dots) and for the elevation of the bedrock substrate (triangles). Due to the paucity of data upstream of New Orleans, ~ Rkm 165, the bedrock surface elevation in this portion of the modeled domain is constrained by comparing contour lines of the Kolb [1962] map of Pleistocene clay elevation and of the Fisk [1944] map of gravel substrate elevation. The bedrock surface slopes, $S_b$, are $\sim 7 \times 10^{-6}$ in the New Orleans area and they increase to $\sim 1.2 \times 10^{-4}$ downstream of Rkm 100. Recalling that the alluvial equilibrium slope for the simplified lowermost Mississippi River of section 3 is $S_o = 1.9 \times 10^{-5}$, a stable bedrock-alluvial transition may form where $S_b < S_o$, and a stable bedrock-alluvial transition may form where $S_b > S_o$ under conditions of constant base level, flow, and sediment supply.

4.2.4. Initial Longitudinal Profile of Channel Bed Elevation

The initial longitudinal profile of channel bed elevation, i.e., the initial condition of the zeroing runs, is defined based on the profiles of the elevation of the Mississippi River channel levees in Fisk’s plate 23 (1944), and on the channel bed elevation data by Nittrouer et al. [2012a].
In particular, the initial condition for the model is roughly built from Fisk's Course 15, which corresponds to the elevation of the natural levees of the Mississippi River ~ 600–700 years ago. We acknowledge that it would be best to determine our model initial condition from a later profile, i.e., when the Mississippi River started forming the present Balize subdelta, but unfortunately, no data are available. The other available longitudinal profile of natural levee elevation, i.e., Fisk's Course 19, is dated 1874, which is too recent to be used as initial condition to zero our morphodynamic model.

Although the lowermost Mississippi River has changed its course over the past ~ 700 years, the Balize lobe has remained the most active of the dispersal channels, particularly after the near-complete abandonment of the Barataria subdelta ~ 450 years ago [Fisk, 1944]. Here we do not account for all these processes, because we are interested in modeling the long-term evolution of the present river channel and not the dynamics of meander migration and channel avulsion. Furthermore, not enough information is available to reliably determine initial and boundary conditions for a more complex morphodynamic model over a 500 km long reach of the Mississippi River.

We thus assume that the lowermost Mississippi River has occupied its present course for the past ~ 700 years, which is an acceptable assumption at least for the ~ 340 km long reach from the Old River Control Structure to New Orleans, as shown in Fisk's plate 15-4.

The procedure to determine the initial longitudinal profile of channel bed elevation is summarized in Figure 6. Given the present course of the lowermost Mississippi River, the levee elevations of Fisk's Course 15 are reported at Angola: Rkm 475, Baton Rouge: Rkm 367, Donaldsonville: Rkm 304, and Carrollton: Rkm 165 (locations in Figure 4). The longitudinal profile of the levee elevation is then inferred by joining the points at known elevation (dashed grey line with diamonds in Figure 6).

The average depth of the cross section can be estimated from the data reported by Nittrouer et al. [2012a] in terms of the difference $\eta_{90} - \eta_{10}$, where $\eta_{90}$ and $\eta_{10}$ respectively denote the 90th and 10th percentiles of the distribution of channel bed elevations in a given cross section. The average depth of the cross section is ~ 25 m between Angola and Donaldsonville and ~ 30 m at Carrollton. The points of the initial channel bed are determined by subtracting the average

![Figure 6](image)

![Figure 7](image)
depth of the cross sections from the levee elevations (squares in Figure 6). A simplified linear initial profile is then fitted to the data (continuous black line). For reference, the elevation of the bedrock substrate is denoted with an orange line.

The streamwise changes in channel width is modeled with the synthetic relation presented in Figure 7 (black line) fitted to the Nittrouer et al. [2012a] top channel width, $B_t$, data for flow rates larger than 35,000 m$^3$/s (black points) and smaller than 5000 m$^3$/s (grey points). In particular, the modeled top channel width mildly decreases in the streamwise direction from 820 m at Rkm 500 to 720 m at Rkm 250. The top channel width remains constant and equal to 720 m from Rkm 250 to Rkm 40.

The flow expansion at the river mouth is modeled with a linear increase of $B_t$ from 720 m at Rkm 40 to 1110 m at Head of Passes. As commonly done in the engineering literature, we further assume that the effective channel width to model sediment transport processes $B$ is approximated by the ~70% of $B_t$ given by the synthetic relation [e.g., Mooney, 2008]. The streamwise change of the effective channel width $B$ is represented with the grey line in Figure 7.

**4.3. Validation Runs**

The model is zeroed using a run of 600 years to establish the zeroed graded state for the lowermost Mississippi River up to 1940. It was about this time that Atchafalaya River started receiving significant flow from the upstream Mississippi River [Roberts, 1989]. The Old River Control Structure was built around 1963, at which time the inflowing water and sediment were regulated so that two thirds continued down the Mississippi River and one third diverted to the Atchafalaya River. For simplicity, we assume that the same split prevailed between 1940 and 1963 as well. The Atchafalaya diversion run is thus performed for 70 years after 1940, i.e., so as to end in 2010, the year in which the data shown in Figure 8a were collected [Nittrouer et al., 2012a]. The initial condition for the Atchafalaya diversion run is the numerical channel bed profile at the end of the zeroing run, i.e., 1940.

Simulation results are presented in Figures 8 and 9. The comparison between the numerical longitudinal profiles of channel bed elevation and the field data is presented in Figure 8a. In Figure 8a, the field data representing current conditions are represented as follows: the yellow dots represent the 40th percentile of channel bed elevation, $\eta_{40}$, and the green dots are the 90th percentile of channel bed elevation $\eta_{90}$.

![Figure 8](image-url)
Nittrouer et al., 2012a. Here $\eta_{40}$ is chosen to be representative of the average channel bed elevation for sediment transport calculations, as indicated in Nittrouer et al. [2012a]. A comparison of the field data for $\eta_{40}$ against the predicted present bed profile in Figure 8a clearly shows that the model is able to reasonably reproduce the longitudinal profile of the present lowermost Mississippi River, notwithstanding the model simplifications.

Of interest in Figure 8a is the front of a wave of bed aggradation that is located at Rkm 356 in 1973 and Rkm 232 in 2010. This result is a consequence of the strongly nonlinear dependence between flow discharge and total bed material transport capacity. That is, the diversion of one third of both the flow and the bed material supply at the Old River Control Structure results in a decreased bed material transport capacity in the Mississippi River below so inducing a downstream-migrating wave of channel bed aggradation in the alluvial reach.

The black line in Figure 8b shows that at the end of the zeroing run (1940), the bed material load has reached the condition of mobile bed equilibrium associated with a bed material supply rate of 40 Mt/yr. Also shown in Figure 8b are the profiles of bed material load as predicted (a) right after the Atchafalaya River started receiving significant sediment from the Mississippi River upstream (grey line, just after 1940), (b) 33 years later (blue line, 1973), and (c) present day (red line, 2010). The profiles show a downstream-migrating front of increased bed material load that corresponds to the wave of bed aggradation shown in Figure 8a. The presence of this front in 2010 reveals that the lowermost Mississippi River is still adjusting to the controlled flow and sediment diversion to the Atchafalaya River.

It is important to note here that Galler et al. [2003] documented significant in-channel deposition between Baton Rouge (Rkm 367) and New Orleans (Rkm 165) via a comparison of Mississippi River historical longitudinal profiles. From their Figure 2 it is possible to estimate a total deposition rate between 1921 and 1992 of ~3.5 m in the Baton Rouge area and of ~0.5 m in the New Orleans area. This corresponds to average deposition rates of ~0.05 m/yr and ~0.007 m/yr from 1921 to 1992 in the Baton Rouge and in the New Orleans area, respectively. These average rates of in-channel deposition are of the same order of magnitude as those predicted by our numerical model and reported in Figure 8a: ~0.04 m/yr from 1940 to 1973 and ~0.05 m/yr from 1973 to 2010 at Rkm 366 (Baton Rouge area) and ~0.079 m/yr from 1940 to 1973 and ~0.0074 m/yr from 1973 to 2010 at Rkm 166 (New Orleans area).

Figure 8c shows the long profile of the fraction of alluvial cover at the end of the zeroing run (1940; black line) and in 2010 (red line). Both an upstream alluvial-bedrock transition and a downstream bedrock-alluvial transition can be seen in 1940 and 2010. In 1940 just after diversion, bedrock is exposed between approximately Rkm 240 and Rkm 32. In 2010, bedrock is exposed between Rkm 232 and Rkm 36. That is, the upstream values correspond to an alluvial-bedrock transition and the downstream values correspond to a bedrock-alluvial transition. The model is thus able to capture the two transitions discussed in section 1 and illustrated in Figure 1.

Figure 8c also includes field estimates of cover fraction (two orange dots, ~2010) presented in Nittrouer et al. [2011a]. Although the field data are sparse, comparison between predicted pre-Atchafalaya diversion (black line, 1940), post-Atchafalaya diversion (red line, 2010) cover fractions, and the modern field estimates...
suggests that the model reasonably captures the magnitude and the streamwise variation of the alluvial cover in the mixed bedrock-alluvial reach. The effect of the controlled flow regime is a mild increase in the cover fraction in the mixed bedrock-alluvial reach.

In interpreting these numerical results it must be kept in mind that we are modeling the Atchafalaya diversion with a step function reduction in 1940 so that its effects on the alluvial morphodynamics of the lowermost Mississippi River are likely amplified compared to the gradually increasing diversion of water and bed material that may have occurred between 1940 and 1963. Additionally, as noted above, the rate of sea level rise in the Atchafalaya diversion run has been set equal to 2 mm/yr, so as to correspond with the base case of Kim et al. [2009]. Thus, part of the channel bed aggradation seen in Figure 8a is driven by sea level rise. Higher rates of sea level rise result in stronger aggradation rates in the modeled domain.

The comparison between numerical and measured suspended bed material (sand) rating curves at St. Francisville (Rkm 425), Baton Rouge (Rkm 367), and Belle Chasse (Rkm 120) (see locations in Figure 6) is presented in Figure 9. The red dots in Figure 9 represent the U.S. Geological Survey (USGS) measurements, and the blue squares denote the numerical predictions, with error bars indicating ±50% of the computed value, i.e., a reasonable error for a 1-D morphodynamic model. The plots in Figure 9 show that the model is able to properly reproduce the suspended bed material transport in both the alluvial—St. Francisville and Baton Rouge—and the bedrock—Belle Chasse—reaches during floods, i.e., when the river is morphologically active. Due to the simplification of the cross-sectional geometry, however, the model tends to somewhat underestimate the suspended bed material transport at low flow in the downstream part of the modeled domain, where the backwater effects are strongest. Sand transport rates at low flow, however, have negligible effects on river morphodynamics [Nittrouer et al., 2011b].

5. The Effect of Subsidence on the Morphodynamics of the Lowermost Mississippi River Channel

In the absence of channel avulsion, the ultimate effect of sea level rise and subsidence on the alluvial morphodynamics of a low-slope mixed bedrock-alluvial river is the complete alluviation of the channel bed. In the modeling results presented above, sea level rise was included but subsidence was neglected. Here we investigate if and how the predicted locations of the alluvial-bedrock and bedrock-alluvial transitions significantly change when subsidence is accounted for in the calculations.

In the system of Figure 2, the elevation of the bedrock surface changes in time because it is assumed that the subsidence rate $\sigma$ is applied at an appropriately deep material boundary with elevation $\eta_{\text{base}}$ above the datum. The position of this material boundary can be chosen so as to approximate tectonic subsidence, subsidence due to compaction, or some combination thereof. If the available field data provide information on the present elevation of the bedrock surface, we must account for the subsidence rate to specify the initial bedrock surface elevation for the numerical simulation $\eta_{b}(x,0)$. This initial state corresponds to the starting point of the zeroing run, i.e., 670 years before 2010. This must be done so that at the end of the numerical runs, the estimated bedrock surface reasonably approximates the field data, in accordance with equation (12).

Estimates of subsidence rates in the lowermost Mississippi River and in the Mississippi Delta differ by 2 orders of magnitude depending on the data and on the study site. Straub et al. [2009] estimated a long-term subsidence rate of 0.26 mm/yr in Breton Sound, East of the Mississippi River in Louisiana, from data collected by the petroleum industry. This estimate is in agreement with other long-term estimates in the Mississippi Delta based on stratigraphic data [Blum and Roberts, 2009]. On the other hand, Gagliano et al. [2003] and Blum and Roberts [2009] reported on estimates of Holocene subsidence rates as high as 3–8 mm/yr. Blum and Roberts [2009] also noted that subsidence rates of 6–8 mm/yr estimated from tidal gauges can characterize the present delta, but most likely, they decrease landward reaching values of about 1–3 mm/yr in the Baton Rouge area, Louisiana. In addition, Morton et al. [2005] showed that due to the extraction of subsurface fluids, subsidence rates increased of about 1 order of magnitude from the 1960s to the 1980s. Due to the decline in subsurface fluid withdrawal over the past two decades, subsidence rates have already started to decline, and this can be expected to continue in the near future [Morton et al., 2005; Kolker et al., 2011].

To investigate the effects of subsidence rate on the alluvial morphodynamics of the lowermost Mississippi River, we repeat the validation run under six different subsidence cases: constant subsidence rates of
to the Atchafalaya diversion of Figure 8a is, however, only partially due to the regulated generation of a downstream-migrating wave of alluviation, as shown in Figure 8a. The channel aggradation due to sea level rise further reduces the transport capacity of the material load is equal to that of the present rates of sea level rise and subsidence, land-building diversions are a sustainable way to build new alluvial-bedrock and bedrock-alluvial transitions, respectively, with \( \sigma_{upstream} \) equal to 0.3 mm/yr, 1.8 mm/yr, and 8 mm/yr, respectively, with \( \sigma_{downstream} \) corresponding to \( \sigma_{upstream} \) and \( \sigma_{downstream} \), respectively.

To investigate the effects of historical accelerated subsidence rates, we repeat the postdiversion runs, i.e., from 1940 to 2010, for the cases of twofold and fivefold increases in subsidence rates, which correspond to a maximum historical subsidence rate at Head of Passes of 40 mm/yr. The subsidence runs are summarized in Table 3 in terms of \( \sigma_{upstream} \) and \( \sigma_{downstream} \), the subsidence acceleration factors described above, and the down-channel position of the alluvial-bedrock and bedrock-alluvial transitions, respectively, \( X_{AB} \) and \( X_{BA} \).

At the end of all the simulations summarized in Table 3 the alluvial-bedrock transition is located around Rkm 232 and the bedrock-alluvial transition is found around Rkm 34, i.e., \( \pm 4 \) km from the locations found in the validation runs. A maximum difference of 4 km in the predicted location of a transition is well within the errors of a one-dimensional model of river morphodynamics. The results of the validation runs can thus be considered representative of the present lowermost Mississippi River.

If subsidence rates are neglected, future predictions of channel bed elevation represent an upper limit, and thus a conservative estimate, of possible longitudinal river channel profiles, as can be deduced from equation (8) and Figure 2. These predictions are crucial to determine the impact of restoration projects such as land-building diversions on navigation and flood control in the main channel of the Mississippi River both upstream and downstream of the diversion.

### 6. Implications for the Restoration of the Mississippi River Delta by Means of Land-Building Diversions

Land-building diversions are one of the restoration strategies proposed for the Mississippi River Delta in the 2012 Louisiana Coastal Master Plan \[Coastal Protection and Restoration Authority, 2012\]. In a land-building diversion project, a diversion structure guides some fraction of river water and bed material load (sand) into an engineered distributary channel. The distributary channel delivers the diverted water and sediment into a drowned area where a quasi-natural delta lobe forms and grows [Paola et al., 2011]. Kim et al. [2009] demonstrated that under present rates of sea level rise and subsidence, land-building diversions are a sustainable way to build new deltaic land in coastal Louisiana. However, the prediction of the impacts of land-building diversions on the Mississippi River main channel is still a matter of debate [e.g., Blum and Roberts, 2009].

Our validation run shows that at engineering time scales (70 years), when the fraction of diverted bed material load is equal to that of flow discharge, the resulting reduction in bed material transport capacity generates a downstream-migrating wave of alluviation, as shown in Figure 8a. The channel aggradation due to the Atchafalaya diversion of Figure 8a is, however, only partially due to the regulated flow regime. It is also driven by sea level rise that further reduces the transport capacity of the flow.

Here we discuss the response of the lowermost Mississippi River to an ideal diversion project in which one structure captures 20% of the flow and bed material load, and we then compare our results with the ideal cases...
of diversion projects that capture 10% and 30% of the flow and of the bed material. More specific discussion of diversion scenarios is not attempted herein because (1) the operational procedures of a land-building project are very sensitive matters that require consensus among the agencies that are responsible for managing the river and the nearby infrastructures and (2) a more realistic scenario must depend on the characteristics of the engineered distributary channel that connects the diversion structure to the drowned area, and this is beyond the scope of the present paper.

We perform four runs, one “regulated mobile equilibrium run” to characterize the mobile bed equilibrium of the Lowermost Mississippi River under the flow regime subsequent to 1940 and regulated at the Old River Control Structure and three 150 yearlong runs, starting from 2010, to compare the responses of the lowermost Mississippi River to a land-building diversion located upstream or downstream of New Orleans against the case of no diversion. For brevity, these runs are termed the “150 year runs”.

The regulated mobile bed equilibrium run must be performed under the unrealistic assumption that sea level was constant from 1940 to 2010 and will remain constant in the future, because it is impossible to strictly reach the conditions of mobile bed equilibrium illustrated in section 3 in case of variable base level [see, e.g., Parker, 2004]. The initial condition for the regulated mobile bed equilibrium run is taken to be the channel bed profile at the end of the zeroing runs, i.e., in 1940. Under the post-1940 flow regime, the flow and the bed material load reach conditions of mobile bed equilibrium in approximately 400 years.

In the 150 year runs, we impose an overall rate of sea level rise of 5 mm/yr, corresponding to the middle range of the most recent Intergovernmental Panel on Climate Change (IPCC) [2013] forecasts. A higher rate yields results that are strongly affected by sea level rise and thus cannot give clear insight on the impact of the land-building diversion structure itself on the river channel.

We assume that the land-building diversion structure is located either at Rkm 200 or at Rkm 100, as shown by the red arrows in Figure 4, i.e., respectively ~130 km and ~30 km upstream of the diversion sites proposed by Kim et al. [2009]. To simplify the model, we do not link the morphodynamics of the land-building diversion itself to channel morphodynamics. Instead, we simply study the response of the Mississippi River to the loss of 20% of flood discharge and bed material load.

The land-building diversion results are compared with the case of the undisturbed flow condition, i.e., no land-building diversion with flow regime subsequent to 1940, and regulated by the Old River Control Structure since 1963. The initial conditions for all the 150 year runs (i.e., land-building diversion upstream of New Orleans, i.e., Rkm 200, land-building diversion downstream of New Orleans, i.e., Rkm 100, and no land-building diversion) are taken to be the results of the calculations outlined above for 2010.

The results of the regulated mobile bed equilibrium run and of the 150 year runs are presented in Figure 10, which is analogous to Figure 8, i.e., model results are presented in terms of channel bed elevation (Figure 10a), total mean annual bed material load (Figure 10b), and alluvial cover fraction (Figure 10c). The black lines in Figure 10 pertain to the regulated mobile bed equilibrium run (label “regulated equilibrium”). The green lines in Figure 10 pertain to the 150 year run under undisturbed conditions, i.e., without a land-building diversion (label “no diversion”) but with a rate of sea level rise of 5 mm/yr. The blue and the red lines respectively refer to the land-building diversion runs with a diversion structure respectively at Rkm 100 and Rkm 200. The continuous and dashed grey lines in Figure 10a respectively denote the bedrock elevation and the minimum channel bed elevation for complete alluviation.

Under the flow regime regulated at the Old River Control Structure, the wave of alluviation documented in Figure 8 continues to migrate downstream, causing channel bed aggradation in the alluvial reach and an increasing cover fraction in the mixed bedrock-alluvial reach. At mobile bed equilibrium, i.e., in the case of constant base level (continuous black lines in Figure 10), the length of the bedrock reach is reduced to ~ 40 km with the alluvial-bedrock transition at Rkm 110 and the bedrock-alluvial transition at Rkm 62. The fraction of exposed bedrock is everywhere smaller than 10% (Figure 10c). Thus, under the present regulated flow regime the lowermost Mississippi River channel tends to transition in the direction of a purely alluvial river. This tendency is expected to be amplified by the rate of sea level rise predicted for the next century by the IPCC [2013], i.e., a middle value of 5 mm/yr.

At the end of the undisturbed (i.e., no land-building diversion) 150 year run with a constant rate of relative base level rise of 5 mm/yr and flow regime regulated at the Old River Control Structure (green lines in
Figure 10), the wave of alluviation generated at the Old River Control Structure (Figure 8a) is predicted to reach the Gulf of Mexico (Figure 8b). The alluvial-bedrock transition is at Rkm 138, the bedrock-alluvial transition is at Rkm 54, and the fraction of exposed bedrock in the mixed bedrock-alluvial reach is everywhere smaller than 20% (Figure 10c). Notwithstanding the rate of sea level rise, the channel bed elevation at the end of the 150 year run is everywhere lower than in the mobile equilibrium case because the flow and the bed material load reach conditions of mobile bed equilibrium in ~ 400 years.

The streamwise variation of mean annual bed material load is presented in Figure 10b. In the regulated mobile bed equilibrium case (black line), the mean annual bed material load is constant in space and time and it is equal to the bed material load at Tarbert Landing, i.e., 27.5 Mt/yr. At the end of the undisturbed 150 year run (green line in Figure 10b), i.e., 150 years after 2010, the bed material load is decreasing in the streamwise direction because (1) the modeled reach is trying to reach the condition of mobile bed equilibrium and (2) sea level is rising at a constant rate of 5 mm/yr.

When a land-building diversion structure diverts 20% of the flow and of the bed material (red and blue lines in Figure 10), a new downstream-migrating wave of alluviation forms downstream of the diversion site and reaches the Gulf of Mexico in less than 150 years. These waves of alluviation are analogous to the wave of alluviation discussed for Atchafalaya diversion and generated at the Old River Control Structure. They are due to the reduced transport capacity of the flow downstream of a diversion site and are responsible for (1) the channel bed aggradation observed in Figure 10a downstream of the diversion site and are responsible for (1) the channel bed aggradation observed in Figure 10a downstream of a diversion site and are responsible for (1) the channel bed aggradation observed in Figure 10a downstream of a diversion site and (2) the reduction of exposed bedrock compared to the undisturbed case (green lines), i.e., no land-building diversion, represented in Figure 10c. The sudden drop in bed material load that characterizes the 150 year runs with a land-building diversion (red and blue lines in Figure 10b) corresponds to the modeled 20% diversion of the bed material load at the land-building diversion structure.

Figure 10. Results of prediction runs for the response of the main stem of the Mississippi River to diversion at the end of the 150 year runs, including comparisons between (a) channel bed profiles, (b) mean annual bed material loads, and (c) alluvial cover fraction. The continuous black lines represent the equilibrium condition for the flow regime subsequent to 1940 and regulated at the Old River Control Structure, under the assumption of constant base level. The green lines refer to the simulation with a constant sea level rise of 5 mm/yr, and the continuous blue and red lines represent the combined effects of a constant sea level rise of 5 mm/yr and a land-building diversion located at Rkm 200 and at Rkm 100, respectively. The continuous and dashed grey lines in Figure 10a respectively represent the bedrock elevation and the minimum channel bed elevation for complete alluviation.
The downstream-migrating waves of alluviation are described in Figure 11 in terms of spatial and temporal evolution of the bed material load in the 150 year runs with a land-building diversion structure at Rkm 200 (Figure 11a) or at Rkm 100 (Figure 11b). In Figure 11 the streamwise variation of bed material load is presented just after the beginning of the operations at the land-building diversion structure (black line) and after 15 years, 60 years, and 150 years of operation (red, green, and blue lines, respectively).

In the case of a land-building diversion located at Rkm 200 (i.e., upstream of New Orleans), the wave of alluviation generated at the Old River Control Structure reaches the land-building diversion structure within the first 15 years of the simulation (red line in Figure 11a), with a significant increase of the bed material load at the land-building diversion site. Downstream of the land-building diversion, a wave of alluviation forms (red and green lines in Figure 11a) and reaches the Gulf of Mexico within 150 years.

When the land-building diversion is located at Rkm 100 (i.e., downstream of New Orleans), the wave of alluviation generated at the Old River Control Structure reaches the diversion site in ~ 40 years. Thus, the bed material load at the diversion site is considerably smaller than in the case of the land-building diversion at Rkm 200. Downstream of the land-building diversion site, a downstream-migrating wave of alluviation forms and reaches the Gulf of Mexico in ~ 60 years (Figure 11b).

Due to the predicted streamwise declining bed material load in the Mississippi River channel, and to the downstream-migrating alluvial waves, the land-building potential, i.e., the amount of sand potentially diverted from the Mississippi River, at a diversion site located upstream of New Orleans is higher than the land-building potential of a land-building diversion located further downstream in the river, as shown in Figure 12 in terms of mean annual bed material load diverted during the 150 year runs. For reference, the rate of diverted bed material considered by Kim et al. [2009], i.e., ~ 5.2 Mt/yr in 100 years, is represented with the dashed black line. In the idealized scenarios considered herein the diverted bed material load is always smaller than that of Kim et al. [2009].

In the case of a land-building diversion structure that diverts 20% of the flow and of the bed material, the diverted mean annual bed material load at Rkm 200 (grey line in Figure 12) increases...
as the wave of alluviation generated at the Old River Control Structure approaches and reaches the diversion site. It then remains nearly constant at ~ 4 Mt/yr. Conversely, when the land-building diversion is at Rkm 100, the mean annual diverted bed material load is lower than 3 Mt/yr for the first ~ 40 years of operation, i.e., before the wave of alluviation generated at the Old River Control Structure reaches the diversion site (black line in Figure 12). In the following ~110 years, the mean annual diverted bed material load increases in time, but it consistently remains smaller than the mean annual bed material load diverted in the case of a diversion site at Rkm 200 due to the imposed rate of sea level rise.

As a consequence of the predicted streamwise declining bed material load in the Mississippi River, the channel tends to aggrade everywhere in the modeled domain (Figure 10a). The differences between the channel bed elevations at the end of the 150 yearlong land-building diversion runs and at the end of the 150 yearlong undisturbed flow regime run, i.e., the case of no land-building diversion structure, are presented in Figure 13, where the grey line refers to the case of a land-building diversion at Rkm 200. The black line pertains to the case of a land-building diversion at Rkm 100.

Downstream of the diversion structure at Rkm 200, the difference in elevation between the predicted channel bed elevation and the channel bed elevation at the end of the 150 year undisturbed flow regime run is ~ 3.5 m (grey line in Figure 13); this value is up to ~ 4 m in the case of a diversion structure at Rkm 100 (black line in Figure 13). These numbers compare to a bankfull channel depth of ~ 30 m under present conditions. Such diversions thus might cause a mild impediment to navigation that could easily be remedied by appropriate dredging.

More specifically, noting that Nittrouer et al. [2008] measured bed form heights at low flow varying between 0.5 m and 3 m and that the average channel depth between New Orleans (Rkm 165) and the bedrock-alluvial transition (Rkm 40) is ~ 30 m, the land-building diversions induced deposition in the lowermost Mississippi River relative to the undisturbed channel bed profile in the case of a constant base level rise of 5 mm/yr is of the same order of magnitude of the bed forms at low flow and equal to ~ 1/10 of the average channel depth. Higher aggradation rates, however, may be expected under the condition of an accelerated rate of sea level rise.

The reduced flow rate downstream of the land-building diversion sites causes a backwater effect in the upstream portion of the modeled reach stronger than in the undisturbed case, with the consequent mild decrease of the bed material load (Figure 10b) and a slightly higher channel aggradation rate upstream of the land-building diversions compared to the undisturbed 150 year run (Figure 13.).

As shown in Figure 13, the difference in elevation between the undisturbed 150 year run and the land-building diversion runs upstream of the diversion structures is on the order of ~ 0.5 m. Noting that the bankfull channel depth of the Mississippi River upstream of New Orleans is ~ 20–30 m, half a meter of deposition does not represent a significant threat for navigation and flood control.

The comparison between the backwater curves during flood flows, i.e., when the river channel is morphologically active, at the end of the 150 year runs is shown in Figure 14a for a flood flow of 25,000 m³/s, which corresponds to the bankfull flow rate in the lowermost Mississippi River [Nittrouer and Viparelli, 2014], and for the maximum flood flow of the flow duration curve used in the post-1940 simulations, i.e., 39,000 m³/s in Figure 14b.

In interpreting the backwater curves of Figure 14 it must be borne in mind that the effective channel width B is not constant in the modeled domain. It is indeed modeled with the relation of Figure 7. The effective channel width linearly decreases in the streamwise direction from Rkm 500 to Rkm 250, remains constant between Rkm 250 and Rkm 40, and then linearly increases from Rkm 40 to Head of Passes (Rkm 0). Thus, the
breaks in slope at Rkm 250 and at Rkm 40 in the backwater curves of Figure 14 are due to the changes in the synthetic relation to model the effective channel width at Rkm 250 and at Rkm 40.

Figure 14a clearly shows that for a flow rate of 25,000 m$^3$/s in the run with the land-building diversion structure located at Rkm 200 (grey line), the water depth upstream of the diversion site is consistently deeper than in the case of a land-building diversion at Rkm 100 (continuous black line). Further, the water depth upstream of the diversion structures in the undisturbed case (dashed back line), i.e., no diversion, is consistently shallower than in the runs with land-building diversion. For a flood flow of 39,000 m$^3$/s, the differences between the backwater curves upstream of the land-building diversion and the backwater curve of the undisturbed 150 year run become negligible. Thus, the mild increase in aggradation rates observed in Figure 13 upstream of the land-building diversion structures is related to a decreased bed material transport capacity at moderate flood flows.

It is worth noting here that the land-building diversion runs are performed under the assumption that 20% of the flow and of the bed material are diverted at low, medium, and high flow. The backwater curves of Figure 14 show that by diverting a different fraction of the flow and of the bed material at low, medium, and high flow, the backwater effect and thus the aggradation rate upstream of the land-building diversion structures may be reduced.

At the end of the 150 year runs, downstream of the land-building diversion structures the water depth is shallower compared to the undisturbed case (no diversion) due to the reduction of the flow rate and of the transport capacity of the flow, which results in higher aggradation rates compared to the undisturbed case. However, for a flood flow of 25,000 m$^3$/s, the difference between the water depth in the undisturbed case and the land-building diversion runs is on the order of 3 m. This difference increases up to 4 m for the maximum flow rate considered in the numerical runs, i.e., 39,000 m$^3$/s. As expected, these changes in water depth are of the same order of magnitude as the differences in channel bed elevation of Figure 13 and of the low-flow bed forms measured by Nittouer et al. [2008], and thus, they likely do not represent a significant hazard for navigation and flood control, particularly if an appropriate remediation-dredging program is implemented. If dredging is necessary, it may be sufficient to concentrate on zones near crossings between bends.

It can be argued that 3–5 m of aggradation represents a significant threat for navigation in the crossings and near Head of Passes and that dredging costs to mitigate these effects might be high. Even if this is the case, however, relatively high dredging costs will likely be a small part of the total costs of the large civil engineering project discussed herein, and in particular when they are amortized over the design life of the project. In addition, dredging costs will very likely be negligible compared to the socioeconomic costs that will result from not performing any delta restoration project. Finally, the Mississippi River-Gulf Outlet Channel offers a potential route for shipping to and from the Gulf of Mexico that bypasses the entire length of the Mississippi River below New Orleans.
It is of value to ask if and how the response of the lowermost Mississippi River would change for different percentages of flow and bed material diverted from the main channel. To answer this question, we have repeated the 150 year diversion runs for the case of two diversion projects that capture 10% and 30% of the flow and the bed material, located either at Rkm 100 or at Rkm 200. Under these conditions, the overall response of the system is very similar to the response described above for the 20% diversion case: very small in-channel deposition upstream of the diversion structure and higher in-channel deposition downstream of the diversion site. In particular, regardless of the fraction of water and sediment diverted and of the location of the diversion project, i.e., Rkm 100 or Rkm 200, the in-channel deposition after 150 years upstream of the diversion site is always predicted to be ~ 0.5 m higher than in the undisturbed 150 year run. Downstream of the diversion sites, however, the difference in channel bed elevation between the 150 year runs and the undisturbed case increases as the fraction of diverted water and bed material increases, i.e., ~ 2 m in the case of the diversion of 10% of water and bed material and ~ 5.5 m in the case of the diversion of 30% of water and bed material.

### 7. Conclusions

A mathematical formulation to model low-slope mixed bedrock-alluvial rivers is derived and implemented in a one-dimensional model of river morphodynamics. The formulation is able to reproduce the alluvial morphodynamics of mixed bedrock-alluvial rivers and to track the long-term evolution of both alluvial-bedrock and bedrock-alluvial transitions. A generic version of the model is first applied over a 500 km reach extending upstream from a river mouth, to study how the mobile bed equilibrium of low-slope rivers varies with bed material load supply and the geometry of the underlying profile of the top of the bedrock. This model reveals that alluvial-bedrock and bedrock-alluvial transitions can be stable features in low-slope, sand bed rivers. The model is adapted and validated up to year 2010 in a field-scale study of the lowermost 500 km of the Mississippi River and is then applied to predict the morphodynamic response of the lowermost Mississippi River to land-building diversions. The main results of the study are summarized below.

1. A bedrock channel morphology can characterize the mobile bed equilibrium of a low-slope river. Given the flow regime and the bed material load, the characteristics of the mobile bed equilibrium depend on (a) the relative magnitude of the alluvial equilibrium slope, $S_o$, with respect to the slope of the bedrock surface, $S_b$, and (b) the elevation of the bedrock surface relative to water surface base level. When the bedrock surface is deep enough not to influence in-channel sediment transport processes, mobile bed equilibrium is independent of the underlying bedrock. Under this condition, the equilibrium river channel is fully alluviated with constant slope $S_o$ and normal flow conditions. However, when the bedrock surface elevation relative to water surface base level is not sufficiently deep, the bedrock surface can influence in-channel sediment transport processes. In this case, two different cases have to be considered:

   $S_o > S_b$. Mobile bed equilibrium is characterized by a bedrock reach with a stable M2 backwater curve downstream and an alluvial cover fraction that decreases in the flow direction. Depending on the depth of the bedrock surface relative to the downstream water surface base level, a stable alluvial-bedrock transition may form. In this case steady and uniform flow develops on the alluvial reach, and an M2 backwater curve characterizes the equilibrium of the bedrock reach.

   $S_o < S_b$. Mobile bed equilibrium is characterized by a bedrock morphology with a M1 backwater curve, and the cover fraction increases in the flow direction. Depending on the depth of the bedrock surface relative to the downstream water surface base level, a stable bedrock-alluvial transition may form with normal flow on the alluvial reach downstream and an M1 backwater curve on the bedrock reach upstream.

2. The long-term response of a bedrock reach to sea level rise and subsidence rates is, in the absence of channel avulsion, complete alluviation due to the increasing depth of the bedrock surface.

3. We successfully validated the version of the model for large low-slope sand bed rivers, based on results for lowermost Mississippi River. The model is able to reasonably capture (a) the locations of both the observed alluvial-bedrock transition upstream and the observed bedrock-alluvial transition downstream and (b) the pattern of variation of the alluvial cover fraction in the bedrock reach in between. The comparison between measured and numerical suspended sediment rating curves in both the alluvial and
the mixed bedrock-alluvial reaches demonstrates that the model properly reproduces sediment transport capacities at both low and high flows.

4. The site-specific version of the model is applied to predict the effects of land-building diversions on the lowermost Mississippi River channel. One diversion is located at Rkm 100, i.e., downstream of New Orleans, and the other is located at Rkm 200, i.e., upstream of New Orleans. The simulations show that for the period from 2010 to 150 years into the future, the aggradation downstream of a diversion structure, i.e., ~ 3.5 m, is of the same order of magnitude of the bed forms at low flow and is on the order of 1/10 of the average channel depth in the downstream part of the modeled domain. It thus likely does not present a serious impediment to navigation and flood control. Higher long-term channel aggradation rates, however, can be induced by rates of sea level rise that are higher than the value of 5 mm/yr used here.

Future research is needed to design the land-building diversion structures. A potential design of the diversion structures, based on culverts, is presented in Kenney et al. [2013]. Furthermore, Nittrouer et al. [2012a] and Meselhe et al. [2012] demonstrated that significant amount of sand can be extracted from the Mississippi River main channel if a spillway is adequately designed and positioned along the inner back of a meander bend. However, the discussion as to whether a spillway or a system of culverts are appropriate land-building diversion structures goes well beyond the scope of the present paper.

In the present model the expansion of flow at the mouth of the Mississippi River [Lamb et al., 2012] is accounted for in terms of an increase in channel width, but not in terms of the much larger expansion as the river flow enters the Gulf of Mexico. In the future the model could be adapted to consider the M2 backwater curve induced by channel expansion when the angle of expansion is better constrained.

Notation

\[ B \] effective channel width for sediment transport, m;
\[ B_t \] top channel width, m;
\[ C_f \] nondimensional friction coefficient;
\[ C_s \] volumetric suspended sediment concentration at 5% of the water depth, nondimensional;
\[ D \] characteristic grain size of the bed material, mm;
\[ D_{90} \] diameter such that 90% of the bed material is finer, mm;
\[ E \] total specific energy, m;
\[ E_s \] nondimensional entrainment rate of sediment in suspension;
\[ Fr \] Froude number, nondimensional;
\[ Fr_b \] Froude number at bankfull flow, nondimensional;
\[ g \] acceleration of gravity, m/s^2;
\[ G_{bm,feed} \] bed material feed rate to the modeled reach, Mt/yr;
\[ H \] water depth, m;
\[ H_o \] water depth at mobile bed equilibrium in an alluvial reach, m;
\[ I \] integral in the Wright and Parker suspended load relation, nondimensional;
\[ I_f \] flood intermittency, nondimensional;
\[ k_e \] composite roughness height, mm;
\[ k_s \] roughness height due to skin friction, mm;
\[ L \] length of the modeled reach, km;
\[ L_{ac} \] minimum thickness of the alluvial cover for complete alluviation, m;
\[ n \] exponent in the Engelund and Hansen load relation, nondimensional;
\[ N \] number of computational nodes;
\[ M \] number of characteristic flow rates in the flow duration curve;
\[ p_c \] fraction of the bed covered with alluvium, nondimensional;
\[ p_i \] average fraction of the year that the flow is within the jth discharge bin of the flow duration curve;
\[ Q \] flow discharge, m^3/s;
\[ Q_{bmc} \] volumetric bed material transport capacity, m^3/s;
\[ Q_{bm} \] volumetric bed material load, m^3/s;
\[ Q_{bm,b} \] volumetric bed material load associated with bed load and bed form transport, m^3/s;
\[ Q_{bm,feed} \] volumetric bed material feed rate to the modeled reach, m^3/s;
Q_{bsm} \text{ suspended volumetric bed material load, m}^3/\text{s};
Q_i \text{ characteristic flow discharge of the } i\text{th} \text{ discharge bin of the flow duration curve, m}^3/\text{s};
q \text{ flow discharge per unit channel width, m}^2/\text{s};
R \text{ submerged specific gravity of the bed material, nondimensional;}
R_p \text{ particle Reynolds number, nondimensional;}
S \text{ channel slope, m/m;}
S_b \text{ slope of the topmost part of the bedrock, m/m;}
S_f \text{ friction slope, m/m;}
S_i \text{ initial slope of the channel bed, m/m;}
S_o \text{ channel slope at mobile bed equilibrium in an alluvial reach, m/m;}
U \text{ mean flow velocity, m/s;}
u_\tau \text{ shear velocity, m/s;}
u_\tau^* \text{ shear velocity associated with skin friction, m/s;}
t \text{ temporal coordinate, yr;}
x \text{ down-channel coordinate, km;}
x_d \text{ down-channel coordinate of the downstream end of the modeled reach, km;}
X \text{ parameter of the entrainment relation for uniform sediment, nondimensional;}
y \text{ nondimensional elevation in the water column;}
y_5 \text{ nondimensional elevation in the water column where the volumetric concentration of suspended sediment is equal to } C_5, \text{ i.e., 0.05;}
v_\tau \text{ particle settling velocity, m/s;}
z \text{ upward oriented vertical coordinate with origin on the datum, m;}
\alpha \text{ parameter that accounts for stratification effects due to suspended load, nondimensional;}
\alpha_{EH} \text{ coefficient of the Engelund and Hansen load relation, nondimensional;}
\Delta t \text{ temporal interval, s;}
\zeta \text{ rate of sea level rise, mm/yr;}
\eta \text{ channel bed elevation above the datum, m;}
\eta^* \text{ relative channel bed elevation, m;}
\eta_b \text{ elevation of the topmost part of the bedrock above the datum, m;}
\eta_{base} \text{ elevation of the material boundary where subsidence is applied above the datum, m;}
\eta_{bd} \text{ elevation of the bedrock at the downstream end of the modeled reach, m;}
\eta_d \text{ initial channel bed elevation at the downstream end of the modeled reach, m;}
\eta_{10} \text{ elevation above 10% of the cross section, m;}
\eta_{40} \text{ elevation above 40% of the cross section, m;}
\eta_{90} \text{ elevation above 90% of the cross section, m;}
\kappa \text{ Von Karman constant, nondimensional;}
\lambda_p \text{ overall porosity of the alluvial deposit, nondimensional;}
\nu \text{ kinematic viscosity of water, m}^2/\text{s;}
\bar{\zeta} \text{ water surface elevation above the datum, m;}
\bar{\zeta}_b \text{ water surface elevation at the downstream end of the modeled reach, m;}
\rho \text{ water density, kg/m}^3;\n\sigma \text{ subsidence rate, mm/yr;}
\sigma_{upstream} \text{ subsidence rate at the upstream end of the modeled reach, mm/yr;}
\sigma_{downstream} \text{ subsidence rate at the downstream end of the modeled reach, mm/yr;}
\tau \text{ bed shear stress, Pa;}
\tau^* \text{ Shields number, nondimensional;}
\tau^*_s \text{ Shields number associated with skin friction, nondimensional.}

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