I. INTRODUCTION

The standard model (SM) of particle physics [1–5] describes very successfully the electroweak and strong interactions of elementary particles over a wide range of energies. In the SM, the massive mediators of the electroweak force, the W and Z bosons, acquire mass through the mechanism of spontaneous symmetry breaking [6–11]. This mechanism introduces a complex scalar field with four degrees of freedom, three of which lead to the W and Z bosons acquiring mass, while the fourth gives rise to a physical particle, the scalar Higgs boson $H$. The masses of the fermions arise through Yukawa interactions between the fermions and the scalar field $H$. The mass of the Higgs boson $m_H$ is a free parameter of the model and has to be determined experimentally. General theoretical considerations on the unitarity of the SM [14–17] suggest that $m_H$ should be smaller than $\approx 1$ TeV, while precision electroweak measurements imply that $m_H < 152$ GeV at the 95% confidence level (C.L.) [18]. Using about 5 fb$^{-1}$ of data collected at $\sqrt{s} = 7$ TeV in 2011 and about 5 fb$^{-1}$ of additional data collected in the first half of 2012 at $\sqrt{s} = 8$ TeV, the ATLAS and CMS experiments have reported the discovery of a new boson at a mass around 125 GeV, with properties compatible with those of the SM Higgs boson [19–21]. Previously, direct searches for the Higgs boson have been carried out at the LEP collider, leading to a lower bound of $m_H > 114.4$ GeV at the 95% C.L. [22], and at the Tevatron proton-antiproton collider, excluding the mass ranges 90–109 GeV and 149–182 GeV at the 95% C.L. and indicating a broad excess of events in the range 120–135 GeV [23,24].

Searches for the SM Higgs boson in the $H \rightarrow ZZ \rightarrow 4\ell$ ($\ell = e, \mu$) channel at the Large Hadron Collider (LHC) have been previously performed using a sample corresponding to an integrated luminosity of about 5 fb$^{-1}$ of 2011 data by the ATLAS [25–27] and Compact Muon Solenoid (CMS) [28–30] collaborations. After the new boson discovery, the spin-parity properties have been further studied by both experiments, using more data. The pseudoscalar hypothesis is excluded by CMS [31] and ATLAS experiments [32,33] at the 95% C.L. or higher. ATLAS has also excluded at the 99% C.L. the hypotheses of vector, pseudovector, and graviton-like spin-2 bosons, under certain assumptions on their production mechanisms [33].

In this paper, the analysis of the $H \rightarrow ZZ \rightarrow 4\ell$ channel is presented using the entire data set collected by the CMS experiment during the 2011–2012 LHC running period. The data correspond to an integrated luminosity of 5.1 fb$^{-1}$ of $pp$ collisions at a center-of-mass energy of $\sqrt{s} = 7$ TeV, and 19.7 fb$^{-1}$ at $\sqrt{s} = 8$ TeV. The search looks for a signal consisting of two pairs of same-flavor, opposite-charge, well-identified and isolated leptons, $e^+e^−, \mu^+\mu^−$, compatible with a ZZ system, where one or both of the Z bosons can be off shell, appearing as a narrow resonance on top of a smooth background in the four-lepton invariant mass distribution. Improved calibrations and alignment constants with respect to those used in Refs. [20,21,31], based on the full data set, are used in the reconstruction of the events considered for this paper. The statistical significance of the observation of the new boson in the four-lepton decay mode is reported, together with measurements of the boson’s mass and its cross section times its branching fraction with respect to the SM prediction, an upper limit on the boson’s width, and the
compatibility of the boson with nine alternative spin-parity hypotheses. The compatibility of the data with a mixed scalar/pseudoscalar state is also assessed. A search is also conducted for additional resonances compatible with the SM Higgs boson in the \( H \to ZZ \to 4\ell \) channel in the mass range 110–1000 GeV.

The paper is organized as follows: The apparatus, the data samples, and the online selection are described in Secs. II through IV. Sections V through VII describe the reconstruction and identification algorithms used in this analysis for leptons, photons, and jets. The event selection and categorization are discussed in Sec. VIII. The background estimation is described in Sec. IX. Kinematic discriminants used to further improve the separation between signal and background and to test the spin and parity of the new boson are presented in Sec. X. The event yields, kinematic distributions, and measured properties are discussed in Secs. XI through XIII.

II. THE CMS DETECTOR

The central feature of the CMS apparatus is a superconducting solenoid of 6 m internal diameter, providing a 3.8 T field. Within the superconducting solenoid volume are a silicon pixel and strip tracker, a lead tungstate crystal electromagnetic calorimeter (ECAL), and a brass/scintillator hadron calorimeter (HCAL). Muons are detected in gas-ionization detectors embedded in the iron flux return placed outside the solenoid. Extensive forward calorimetry complements the coverage provided by the barrel and end-cap detectors. The CMS detector is described in detail in Ref. [34].

The CMS experiment uses a coordinate system with the origin at the nominal interaction point, the \( x \) axis pointing to the center of the LHC ring, the \( y \) axis pointing up (perpendicular to the LHC ring), and the resulting \( z \) axis along the beam direction using a right-handed convention. The polar angle \( \theta \) is measured from the positive \( z \) axis, and the azimuthal angle \( \phi \) is measured in the \( x-y \) plane in radians. The pseudorapidity is defined as \( \eta = -\ln(\tan(\theta/2)) \).

The inner tracker measures charged particle trajectories within the range \( |\eta| < 2.5 \). It consists of 1440 silicon pixel and 15 148 silicon strip detector modules and is immersed in the magnetic field. It provides an impact parameter resolution of \( \approx 15 \mu \text{m} \) and a transverse momentum (\( p_T \)) resolution of about 1.5% for 100 GeV particles [35,36].

The ECAL consists of 75 848 lead tungstate crystals and provides coverage of \( |\eta| < 1.479 \) in the barrel region (EB), and \( 1.479 < |\eta| < 3.0 \) in the two end-cap regions (EE). The EB uses 23 cm long crystals with front-face cross sections of around 2.2 cm \( \times \) 2.2 cm, while the EE comprises 22 cm long crystals with front-face cross sections of 2.86 cm \( \times \) 2.86 cm. A preshower detector consisting of two planes of silicon sensors interleaved with a total of 3 radiation lengths of lead is located in front of the EE. The ECAL energy resolution for electrons with transverse energy \( E_T \approx 45 \text{ GeV} \) from the \( Z \to e^+e^- \) decays is better than 2% in the central region of the EB (\( |\eta| < 0.8 \)), and is between 2% and 5% elsewhere. For low-bremsstrahlung electrons that have 94% or more of their energy contained within a 3 \( \times \) 3 array of crystals, the energy resolution improves to 1.5% for \( |\eta| < 0.8 \) [37]. The Gaussian resolution of the dielectron mass distribution for a Z-boson sample, when both electrons belong to this class, is \( 0.97 \pm 0.01 \text{ GeV} \) in \( \sqrt{s} = 7 \text{ TeV} \).

The HCAL is a sampling calorimeter with brass as the passive material and plastic scintillator tiles serving as active material, providing coverage of \( |\eta| < 2.9 \). The calorimeter cells are grouped in projective towers of granularity \( \Delta \eta \times \Delta \phi = 0.087 \times 0.087 \) in the HB (covering \( |\eta| < 1.3 \)) and \( \Delta \eta \times \Delta \phi \approx 0.17 \times 0.17 \) in the HE (covering \( 1.3 < |\eta| < 2.9 \)), the exact granularity depending on \( |\eta| \). A hadron forward calorimeter extends the coverage up to \( |\eta| < 5.2 \).

Muons are detected in the pseudorapidity range \( |\eta| < 2.4 \), with detection planes made using three technologies: drift tubes, cathode-strip chambers, and resistive-plate chambers. The global fit of the muon tracks matched to the tracks reconstructed in the silicon tracker results in a transverse momentum resolution, averaged over \( \phi^\mu \) and \( \eta^\mu \), from 1.8% at \( p_T^\mu = 30 \text{ GeV} \) to 2.3% at \( p_T^\mu = 50 \text{ GeV} \) [36].

III. SIMULATED DATA SAMPLES

The Monte Carlo (MC) simulated samples, generated with programs based on state-of-the-art theoretical calculations for both the SM Higgs boson signal and relevant background processes, are used to optimize the event selection and to evaluate the acceptance and systematic uncertainties. The samples of Higgs boson signal events produced in either gluon fusion (\( gg \to H \)) or vector-boson fusion (\( qq \to qgH \)) processes are generated with the POWHEG [38–40] generator at next-to-leading-order (NLO) QCD accuracy. The Higgs boson decay is modeled with JHUGEN 3.1.8 [41–43] and includes proper treatment of interference effects associated with permutations of identical leptons in the four-electron and four-muon final states. Alternative spin-parity states are also modeled with JHUGEN, where production of the spin-0 states is modeled with JHUGEN, where production of the spin-0 states is modeled with POWHEG at NLO QCD accuracy. It is also found that NLO QCD effects relevant for this analysis are approximated well with the combination of leading-order (LO) QCD matrix elements and parton showering. Therefore, simulation of spin-1 and spin-2 resonances is performed in quark-antiquark and gluon fusion production at LO QCD accuracy, followed by parton showering generated with PYTHIA 6.4.24 [44].

For low-mass Higgs boson hypotheses (\( m_H < 400 \text{ GeV} \)), the Higgs boson line shape is described with a Breit-Wigner (BW) distribution. At high mass (\( m_H > 400 \text{ GeV} \)), because of the very large Higgs boson width (\( \Gamma_H > 70 \text{ GeV} \)), the
line shape is described using the complex pole scheme (CPS) [45–47]. The inclusive cross section for every $m_H$ is computed including corrections due to the CPS [48]. The interference between the Higgs boson signal produced by gluon fusion and the background from $gg \rightarrow ZZ$ is taken into account, as suggested in Ref. [49]. The theoretical uncertainty in the shape of the resonance due to missing NLO corrections in the interference between background and signal is considered, as well as the uncertainties due to electroweak corrections [46,49,50]. Samples of $WH$, $ZH$, and $t\bar{t}H$ events are generated with PYTHIA. Higgs boson signal events for all the production mechanisms are reweighted using the generator-level invariant mass, to include contributions from gluon fusion up to next-to-next-to-leading order (NNLO) and next-to-next-to-leading logarithm (NNLL) [51–63], and from the vector-boson fusion (VBF) contribution computed at NNLO in Refs. [55,64–68].

The dominant background to the Higgs signal in this channel is the SM $ZZ$ or $Z\gamma^* \rightarrow \mu^+\mu^-$ production via $q\bar{q}$ annihilation and gluon fusion, which is referred to as $ZZ$ in what follows. Smaller contributions arise from $Z$ + jets and $t\bar{t}$ production where the final states contain two isolated leptons and two heavy-flavor jets producing secondary leptons. Additional backgrounds arise from $Z$ + jets, $Z\gamma$ + jets, $WW$ + jets, and $WZ$ + jets events, where misidentified leptons can arise from decays of heavy-flavor hadrons, in-flight decays of light mesons within jets, and, in the case of electrons, overlaps of $x^0$ decays with charged hadrons. The $ZZ$ production via $q\bar{q}$ is generated at NLO with POWHEG [69], while the $WW$, $WZ$ processes are generated with MADGRAPH [70] and normalized to cross sections computed at NLO. The $gg \rightarrow ZZ$ contribution is generated with GG2ZZ [71]. The $Z\bar{b}b$, $Zc\bar{c}$, $Z\gamma$, and $Z$ + light jets samples (referred to as $Z$ + jets in the following) are generated with MADGRAPH, comprising inclusive $Z$ production of up to four additional partons at the matrix-element level, which is normalized to the cross section computed at NNLO. The $t\bar{t}$ events are generated at NLO with POWHEG. The event generator takes into account the internal initial-state and final-state radiation effects which can lead to the presence of additional hard photons in an event. In the case of LO generators, the CTEQ6L [72] set of parton distribution functions (PDFs) is used, while the CT10 [73] set is used for the NLO and higher-order generators.

All generated samples are processed with PYTHIA for jet fragmentation and showering. For the underlying event, the PYTHIA 6.4.24 tunes Z2 and Z2*, which rely on $p_T$-ordered showers, are used for 7 and 8 TeV MC samples, respectively [74]. Events are processed through the detailed simulation of the CMS detector based on GEANT4 [75,76] and are reconstructed with the same algorithms as used for data. The simulations include overlapping $pp$ interactions (pileup) matching the distribution of the number of interactions per LHC beam crossing observed in data. The average number of measured pileup interactions is approximatively 9 and 21 in the 7 and 8 TeV data sets, respectively.

IV. ONLINE EVENT SELECTION

The first level (L1) of the CMS trigger system, composed of custom hardware processors, uses information from the calorimeters and muon detectors to select the most interesting events in a time interval of less than 4 μs. The L1 trigger rate of 100 kHz is further reduced by the high-level trigger (HLT) processor farm to around 300 Hz before data storage.

Collision events analyzed in this paper are selected by the trigger system, requiring the presence of two leptons: electrons or muons. The minimal transverse momenta of the leading and subleading leptons are 17 and 8 GeV, respectively, for both electrons and muons. The online selection includes double-electron, double-muon and mixed electron-muon triggers. In the case of the $4\ell$ final state, a triple-electron trigger is added with thresholds of 15, 8, and 5 GeV to increase the efficiency for low-$p_T$ electrons. The trigger efficiency for events within the geometrical acceptance of this analysis is greater than 98% for a Higgs boson signal with $m_H > 110$ GeV. The same trigger paths are applied on the 7 and 8 TeV data, whereas different identification criteria are applied on the HLT lepton candidates to account for the different LHC conditions.

In addition to the events selected to form the four-lepton sample, dedicated triggers are used for lepton calibration and efficiency measurements. In the case of dimuon events, the online trigger algorithms used to select the signal events are sufficiently loose that they can also be used to measure the selection efficiency with the $Z \rightarrow \mu^+\mu^-$ events. In order to measure the selection efficiency of events with low-$p_T$ leptons, low-mass resonances are used. Events corresponding to these low-mass resonances are collected in the dimuon case using dedicated triggers that require an opposite-sign muon pair, with dedicated kinematic conditions on the dimuon system. In the case of electrons, low-mass resonances are collected, with a smaller rate, with standard dielectron triggers. Two specialized triggers are introduced to maximize the number of $Z \rightarrow e^+e^-$ events covering both high- and low-$p_T$ ranges. The one having the most stringent (relaxed) identification and isolation requirement on one electron requires the presence of a cluster in the electromagnetic calorimeter with $p_T > 8(17)$ GeV, forming an invariant mass with the other electron exceeding 50 GeV.

V. LEPTON RECONSTRUCTION AND SELECTION

The analysis is performed by reconstructing a $ZZ$ system composed of two pairs of same-flavor and opposite-charge
isolated leptons, $e^+ e^-$ or $\mu^+ \mu^-$. The main background sources, described in Sec. III, are the SM ZZ production, with smaller contributions from other diboson ($WW$, $WZ$) processes, single bosons with hadronic activity that can mimic lepton signatures, and top-quark-pair events. Given the very low branching fraction of the $H \rightarrow ZZ \rightarrow 4\ell$ decay, of $\mathcal{O}(10^{-4})$ [$\mathcal{O}(10^{-3})$] for $m_H = 125(200)$ GeV [77], it is important to maintain a very high lepton selection efficiency in a wide range of momenta, to maximize the sensitivity for a Higgs boson within the mass range 110–1000 GeV.

The signal sensitivity also depends on the $4\ell$ invariant mass resolution. The signal appears as a narrow resonance on top of a smooth background, and therefore it is important to achieve the best possible four-lepton mass resolution. To obtain a precise measurement of the mass of a resonance decaying into four leptons, it is crucial to calibrate the individual lepton momentum scale and resolution to a level such that the systematic uncertainty in the measured value of $m_H$ is substantially smaller than the statistical uncertainty in the current data set. This section describes the techniques used in the analysis to select electrons and muons in order to achieve the best momentum resolution, measure the momentum scale, resolution, and selection efficiency, and derive corrections based on dilepton resonances.

The CMS particle flow (PF) algorithm [78–81], which combines information from all subdetectors, is used to provide an event description in the form of reconstructed particle candidates. The PF candidates are then used to build higher-level objects, such as jets, missing transverse energy, and lepton isolation quantities.

### A. Electron reconstruction and identification

Electron candidates are required to have a transverse momentum $p_T^e > 7$ GeV and be within the geometrical acceptance, defined by $|\eta^e| < 2.5$. The electron reconstruction combines information from the ECAL and the tracker [82–85]. Electron candidates are formed from arrays of energy clusters in the ECAL (called superclusters) along the $\phi$ direction, which are matched to tracks in the silicon tracker. Superclusters, which recover the energy of the bremsstrahlung photons emitted in the tracker material and of some of the nearly collinear final-state radiation (FSR) from the electron, are also used to identify hits in the innermost tracker layers in order to initiate the reconstruction of electron tracks. This track seeding procedure is complemented by an approach based on tracker seeds which improves the reconstruction efficiency at low $p_T^e$ and in the transition between the EB and EE regions. Trajectories, when initiated outside-in from the ECAL superclusters as well as inside-out from the measurements in the innermost tracker layers, are reconstructed using the Gaussian sum filter (GSF) algorithm [86], which accounts for the electron energy loss by bremsstrahlung. Additional requirements [37] are applied in order to reject electrons originating from photon conversions in the tracker material. Electron candidates are selected using loose criteria on track-supercluster matching observables that preserve the highest possible efficiency while removing part of the QCD background.

Electron identification relies on a multivariate discriminant that combines observables sensitive to the bremsstrahlung along the electron trajectory, and the geometrical and momentum-energy matching between the electron trajectory and the associated supercluster, as well as ECAL shower-shape observables. The multivariate discriminant is trained using a sample of $\approx 10^7$ simulated Drell-Yan events for the signal (true electrons) and a high-purity $W + 1$ jet data sample for the background (misidentified electrons from jets). The expected performance is validated using jets misidentified as electrons in a $Z(\rightarrow \mu^+ \mu^-)$ and $Z(\rightarrow e^+ e^-)$ data sample, with exactly one reconstructed electron not originated from the $Z$ boson decay. The sources of prompt electrons, such as dibosons or $t\bar{t}$ decays, are suppressed with appropriate selections on the number of extra leptons and the presence of small missing transverse energy in the event [85]. The selection of the $Z$ boson is the same as the one used in the analysis, so the $\eta^e$ and $p_T^e$ spectrum is similar to the one for the electrons characterizing the reducible background in the analysis. The selection is optimized in six regions of the electron $p_T^e$ and $|\eta^e|$ to maximize the expected sensitivity for a low-mass Higgs boson. These regions correspond to two $p_T^e$ ranges, 7–10 GeV and $> 10$ GeV, and three pseudorapidity regions, corresponding to two regions in the EB with different material in front of the ECAL, the central barrel ($|\eta^e| < 0.8$) and the outer barrel ($0.800 < |\eta^e| < 1.479$), in addition to the EE, $1.479 < |\eta^e| < 2.500$.

Several procedures are used to calibrate the energy response of individual crystals [37,87]. The energy of the ECAL superclusters is corrected for the imperfect containment of the clustering algorithm, the electron energy not deposited in the ECAL, and leakage arising from showers near gaps between crystals or between ECAL modules. This is done using a regression technique based on boosted decision trees (BDT) [88] trained on a simulated dielectron sample with the pileup conditions equivalent to the ones measured on data, covering a flat spectrum in $p_T^e$ from 5 to 100 GeV. The variables include the electron supercluster raw energy, $\eta$ and $\phi$ coordinates, several shower-shape variables of the cluster with largest energy within the supercluster (the seed cluster), the ratio of the energy in the HCAL behind the seed cluster to the seed cluster energy, and the number of clusters in the electron supercluster. In addition, the distance of the seed crystal with respect to the gap between the ECAL modules, the $\eta$ and $\phi$ coordinates of the seed cluster, and the energies of the first three subleading clusters in the supercluster are used. A similar subset of variables is used depending on
whether the electron is detected in the EB or EE. Using this multivariate technique, the effective width and Gaussian resolution of the reconstructed invariant mass are improved by 25% and 30%, respectively, for simulated $H \rightarrow 4e$ decays compared to those obtained with a more traditional approach based on ECAL-only energy measurements and corrections with a parameterized energy response obtained from simulation. The effective width, $\sigma_{\text{eff}}$, is defined as the half-width of the smallest interval that contains 68.3% of the distribution.

The precision of the electron momentum measurement is dominated by the ECAL at high energies, whereas for low-$p_T$ electrons the precision is dominated by the tracker momentum determination. Moreover, for electrons near poorly instrumented regions, such as the crack between the EB and the EE, the intermodule cracks [89], or regions close to dead channels, the measurement accuracy and resolution can also be improved by combining the ECAL energy with the track momentum. To account for biases arising from bremsstrahlung losses in the tracker material, electron categories are defined based on the cluster multiplicity inside the supercluster as well as on the amount of bremsstrahlung as estimated from the GSF. The magnitude of the electron momentum is then determined by combining the two estimates with a multivariate regression function that takes as input the corrected ECAL energy from the supercluster regression, the track momentum estimate, their respective uncertainties, the ratio of the corrected ECAL energy over the track momentum as obtained from the track fit, the uncertainty in this ratio, and the electron category, based on the amount of bremsstrahlung. The direction is taken from the fitted track parameters at the point of closest approach to the nominal beam spot position. Figure 1 (top) shows the reconstructed invariant mass for $H \rightarrow 4e$ decays, compared to the traditional approach for the electron energy estimation. The residual offset in the peak position [\(< 0.2\%\), black histogram in Fig. 1 (top)] is irrelevant for the analysis, because the absolute electron momentum scale is calibrated using known resonances in data, as described in Sec. V.D. Figure 1 (bottom) presents the expected effective resolution of the combined momentum measurement as a function of the electron momentum at the vertex. The expected effective momentum resolution for the ECAL-only and tracker-only estimates is also shown.

### B. Muon reconstruction and identification

Muon candidates are required to have a transverse momentum $p_T^\mu > 5$ GeV and be within the geometrical acceptance, defined by $|\eta^\mu| < 2.4$. The reconstruction combines information from both the silicon tracker and the muon system. The matching between track segments is done either outside-in, starting from a track in the muon system, or inside-out, starting from a track in the silicon tracker. Both these candidates are referred to as global muons. Very low-$p_T$ muons ($p_T^\mu \lesssim 5$ GeV) may not have sufficient energy to penetrate the entire muon system and leave track segments in one or two stations of the muon system, where a station is composed of multiple detection planes between two iron layers. Tracks matched to such segments form so-called tracker muon objects. More details on muon reconstruction in CMS can be found in Ref. [91]. Both global and tracker muons are used in this analysis.
The muons are selected among the reconstructed muon track candidates by applying minimal requirements on the track segments in both the muon system and inner tracker system and taking into account compatibility with small energy deposits in the calorimeters [81,91].

The $p_T$ resolution for muons in the momentum range relevant for this analysis varies between 1.3% and 2.0% in the barrel, and up to 6% in the end caps. The dominant effect determining this resolution is the multiple scattering of muons in the tracker material. The achieved statistical accuracy on the determination of the position of the tracker modules is generally better than 10 μm, reaching a level of ≤ 2 μm in the pixel tracker. Besides cosmic ray tracks, the usage of resonance mass and vertex information in the alignment procedure successfully constrains systematic deformations of the geometry that could bias reconstructed track parameters [92].

The accuracy of the hit measurements in the muon chambers and the overall alignment contribute to a lesser degree to the momentum measurement. This is achieved using several alignment procedures using cosmic muons, optical surveys, a laser system, and, finally, using several alignment procedures using cosmic muons.

C. Lepton isolation and vertex compatibility

Lepton isolation is used to discriminate leptons originating from high-$p_T$ boson decay, as in the case of the signal, from those arising from hadronic processes, which are typically immersed in a jet of other hadrons.

The isolation of individual leptons, measured relative to their transverse momentum $p_T^\ell$, is defined by

$$ R_{iso}^\ell = \left( \sum p_T^{charged} + \max[0, \sum p_T^{neutral}] + \sum p_T^{\mu} - p_T^{PU}(\ell) \right) / p_T^\ell, \quad (1) $$

where the sums are over charged and neutral PF candidates in a cone $\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} < 0.4$ around the lepton direction at the interaction vertex, where $\Delta \eta = \eta^\ell - \eta^i$ and $\Delta \phi = \phi^\ell - \phi^i$ quantify the angular distance of the PF candidate $i$ from the lepton $\ell$ in the $\eta$ and $\phi$ directions, respectively. In Eq. (1), $\sum p_T^{charged}$ is the scalar sum of the transverse momenta of charged hadrons originating from the chosen primary vertex of the event. The primary vertex is selected to be the one with the highest sum of $p_T^2$ of associated tracks. The sums $\sum p_T^{neutral}$ and $\sum p_T^\mu$ are the scalar sums of the transverse momenta for neutral hadrons and photons, respectively. The latter excludes photons that are candidates for final-state radiation from the lepton, as defined in Sec. VI. The contribution from pileup $[p_T^{PU}(\ell)]$ in the isolation cone is subtracted from $R_{iso}^\ell$ with different techniques for electrons and muons. For electrons, the FASTJET technique [93–95] is used, in which $p_T^{PU}(\ell) = \rho \times A_{eff}$, where the effective area, $A_{eff}$, is the geometric area of the isolation cone scaled by a factor that accounts for the residual dependence of the average pileup deposition on the electron $\eta^\ell$. The variable $\rho$ is defined as the median of the energy-density distribution for the neutral particles within the area of any jet in the event, reconstructed using the $k_t$ clustering algorithm [96,97] with distance parameter $D = 0.6$, with $p_T^{PU} > 3$ GeV and $|\eta|$ < 2.5. For muons, $p_T^{PU}(\mu) = 0.5 \times \sum p_T^{PU,i}$, where $i$ runs over the momenta of the charged hadron PF candidates not originating from the primary vertex. The factor 0.5 in the sum corrects for the different fraction of charged and neutral particles in the isolation cone. The electrons or muons are considered isolated if $R_{iso}^\ell < 0.4$. The isolation requirement has been optimized to maximize the discovery potential in the full $m_H$ range of this analysis.

In order to suppress leptons originating from in-flight decays of hadrons and muons from cosmic rays, all leptons are required to come from the same primary vertex.

This is achieved by requiring SIP$_{3D}$ < 4, where SIP$_{3D}$ = IP$_{3D}/\sigma$IP$_{3D}$ is the ratio of the impact parameter of the lepton track (IP$_{3D}$) in three dimensions (3D), with respect to the chosen primary vertex position, and its uncertainty.

D. Lepton momentum scale, resolution and selection efficiency

The determination of the momentum differs for electrons and muons, and it depends on the different CMS subdetectors involved in their reconstruction. The CMS simulation used in this analysis is based on the best knowledge of the detector conditions, as encoded in the ECAL calibrations and tracker and muon system alignment. Nevertheless, small discrepancies between data and simulation remain. In the case of the electron momentum scale and resolution, the main sources of discrepancy are the residual tracker misalignment and the imperfect corrections at the crystal level of the transparency loss due to irradiation, especially in the forward region. The average measured drop in energy response, before the crystal calibrations, is about 2%–3% in the barrel, rising to 20% in the range 2.1 ≤ $|\eta|$ ≤ 2.5 [37], and it is reduced to a subpercent level after the calibrations. In the case of muons, the momentum determination is affected by the tracker and muon system alignment geometry used for the reconstruction. The misalignment of the tracker causes a dependence of the systematic uncertainties in the reconstructed muon momentum on the $\eta^\mu$, $\phi^\mu$, and charge measurements.

The momentum scale and resolution for electrons and muons are studied using different data control samples for different $p_T^\ell$ ranges. In the range of interest for this analysis ($p_T^\ell < 100$ GeV), the dileptons from decays of the $J/\psi$, $\Upsilon(nS)$, and $Z$ resonances are used to calibrate or validate the momentum scale and measure the momentum resolution. The $J/\psi$ and $\Upsilon(nS)$ decays constitute a clean data source of
TABLE I. Number of $Z \rightarrow \ell^{+} \ell^{-}, J/\psi \rightarrow \ell^{+} \ell^{-}$ and $\Upsilon$(nS) → $\ell^{+} \ell^{-}$ [sum of $\Upsilon$(1S), $\Upsilon$(2S) and $\Upsilon$(3S)] used to validate or derive lepton momentum scale and resolution and to measure lepton efficiencies ($Z \rightarrow \ell^{+} \ell^{-}$ only) in 7 and 8 TeV data. Low-mass dimuon resonances are collected with specialized triggers.

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$Z \rightarrow \ell^{+} \ell^{-}$</th>
<th>$J/\psi \rightarrow \ell^{+} \ell^{-}$</th>
<th>$\Upsilon$(nS) → $\ell^{+} \ell^{-}$</th>
</tr>
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<tr>
<td>$e$</td>
<td>$10^7$</td>
<td>$5 \times 10^7$</td>
<td>$2.5 \times 10^7$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$1.4 \times 10^7$</td>
<td>$2.7 \times 10^7$</td>
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low-$p_T$ electrons and muons are used to validate (calibrate) the electron (muon) momentum scale for $p_T^\ell < 20$ GeV. The $Z \rightarrow \ell^{+} \ell^{-}$ decay mode is a copious and pure source of leptons, with a wide momentum range covering the full spectrum of leptons of interest to this analysis. Table I provides the approximate number of dilepton resonance decays reconstructed in the 7 and 8 TeV data used for the calibration of the lepton momentum.

For electrons, the calibration procedure consists of three steps. First, a set of corrections for the momentum scale is obtained by comparing the displacement of the peak position in the distributions of the $Z$-boson mass in the data and in the simulation in different $\eta$ regions and in two categories depending on the amount of bremsstrahlung. The corrections are derived as a function of time in order to account for the time-dependent crystal transparency loss [37]. Second, a linearity correction to the momentum scale is applied to account for the $p_T$-dependent differences between data and simulation by comparing the dielectron mass distributions, binned in $p_T^\ell$ of one of the two electrons, in data and in simulated $Z \rightarrow e^+e^-$ events. The $J/\psi \rightarrow e^+e^-$ and $\Upsilon$(1S) → $e^+e^-$ events are used as validation for electron $p_T^\ell < 20$ GeV. All the corrections on the electron momentum scale from the first two steps are applied to data. Third, the energies of single electrons in the simulation are smeared by applying a random Gaussian multiplicative factor of mean 1 and width $\Delta \sigma$, in order to achieve the resolution observed in the data $Z$-boson sample.

For muons, an absolute measurement of momentum scale and resolution is performed by using a reference model of the $Z$ line shape convolved with a Gaussian function. The bias in the reconstructed muon $p_T^\ell$ is determined from the position of the $Z$ mass peak as a function of muon kinematic variables, and a correction is derived for the data according to the procedure of Ref. [91]. A correction for the resolution is also derived for the simulation from a fit to the $Z \rightarrow \mu^+\mu^-$ mass spectrum. The large event sample based on low-mass dimuon resonances provides an additional calibration source for the momentum resolution in a similar manner.

After this calibration, the lepton momentum scale and resolution are validated in data using dileptons from $J/\psi$, $\Upsilon$(nS) and $Z$ decays in several bins of lepton $|\eta|$ and $p_T^\ell$ in order to cover the full momentum range relevant for the $H \rightarrow ZZ \rightarrow 4\ell$ search. Electrons with $p_T^\ell > 7$ GeV and muons with $p_T^\mu > 5$ GeV are considered. For the selection of $Z \rightarrow \ell^{+}\ell^{-}$ events, all lepton selection criteria are applied as in the $H \rightarrow ZZ \rightarrow 4\ell$ analysis.

The events are separated into categories according to the $p_T^\ell$ and $|\eta|$ of one of the electrons, integrating over the other, while for dimuons, the average $p_T^\mu$ and $|\eta|$ are used. The dilepton mass distributions in each category are fitted with a BW parameterization convolved with a single-sided Crystal-Ball (CB) function [90] [dimuon resonances or dielectron $J/\psi$ and $\Upsilon$(1S)] or with MC templates ($Z \rightarrow ee$). From these fits, the offset in the measured peak position in data with respect to the nominal $Z$ mass, $\Delta m_{data} = m_{peak}^{data} - m_Z$, with respect to that found in the simulation, $\Delta m_{MC} = m_{peak}^{MC} - m_Z$, is extracted. Figure 2 shows the relative difference between data and simulation of the dilepton mass scale. After the electron calibration, the relative momentum scale between data and simulation is consistent within 0.2% in the central barrel and up to $\approx 0.3\%$ in the forward part of the ECAL end caps. The residual dependence at low momentum is due to the use of wide bins in measured electron $p_T^\ell$ in evaluating the $Z$-peak mass shift. The measured $p_T^\ell$ dependence of the momentum scale before the $p_T^\ell$ linearity correction, up to 0.6% in the central barrel and up to 1.5% in the end cap, is propagated to the reconstructed four-lepton mass from simulated Higgs boson events. The resulting shift of 0.3% (0.1%) for the $4e$ ($2e2\mu$) channel is assigned as a systematic uncertainty in the signal mass scale. For muons, the agreement between the observed and simulated mass scales is within 0.1% in the entire pseudorapidity range of interest. A somewhat larger offset is seen for $J/\psi$ events with two high-$p_T^\mu$ muons in the very forward region. However, for these events, the muons are nearly collinear, and such a kinematic configuration is very atypical for the $H \rightarrow ZZ \rightarrow 4\ell$ events. Hence, the observed larger mass scale offset for such events is irrelevant in the context of this analysis.

Similarly, the widths of the peak due to instrumental resolution in data, $\sigma_{data}$, and in the simulation, $\sigma_{MC}$, are compared. For electrons, $\sigma_{eff}$ ranges from 1.2% for the best category, which consists of two central single-cluster electrons with a small amount of bremsstrahlung (“barrel golden” (BG) [98]), to 4% for the worst category, which consists of two electrons either with multiple clusters or with a high amount of bremsstrahlung, one central and one forward (“barrel showering” (BS) and “end-cap showering” (ES) [98]). The amount of energy lost by bremsstrahlung before the electron reaches the ECAL is estimated with the GSF algorithm. The relative difference in $\sigma_{eff}$ between data and simulation is less than 3%, for different electron categories [Fig. 3 (top)]. For the muons, in the whole kinematic range considered for this analysis, the instrumental $Z$-peak mass resolution observed in data is consistent with that in the simulation within about 5%, when not considering $J/\psi$ events with two high-$p_T^\mu$, high-$|\eta|$ muons [Fig. 3 (bottom)].
The combined efficiency for the reconstruction, identification, and isolation (and conversion rejection for electrons) of prompt electrons or muons is measured in data using a "tag and probe" method [99] based on an inclusive sample of Z-boson events, separately for 7 and 8 TeV data. The efficiency is measured from the Z → l⁺l⁻ yields obtained by fitting the Z line shape plus a background model to the dilepton mass distributions in two samples, the first with the probe lepton satisfying the selection criteria, and the second with the probe lepton failing them. The same approach is used in both data and simulation, and the ratio of the efficiency in the different ℓ⁺T and ℓ⁻T bins of the probed lepton is used in the analysis to rescale the selection efficiency in the simulated samples. The efficiencies for reconstructing and selecting electrons and muons in the full ℓ⁺T and ℓ⁻T range exploited in this analysis are shown in Fig. 4. The deviation of the efficiency in simulation relative to data, for the majority of the phase space of the leptons, is less than 3% for both electrons and muons. In the case of electrons with ℓ⁺T < 15 GeV, the deviation is larger, 5%–9%, but still consistent with unity, given the large statistical uncertainty. The dependency of the

FIG. 2 (color online). Relative difference between the dilepton mass peak positions in data and simulation as obtained from Z, J/ψ and Υ(nS) resonances as a function of (top) the transverse momentum of one of the electrons regardless of the second for dielectron events, and (bottom) the average muon ℓ⁺T for dimuon events for the 8 TeV data.

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FIG. 3 (color online). (top) Relative difference between the dielectron σeff in data and simulation, as measured from Z → e⁺e⁻ events, where the electrons are classified into different categories (B: barrel, E: end caps, G: golden, S: showering). (bottom) Relative difference between the dimuon mass resolutions in data and simulation as measured from J/ψ, Υ(nS), and Z decays as functions of the average muon ℓ⁺T. The uncertainties shown are statistical only. Results are presented for data collected at √s = 8 TeV.
reconstructed with the PF reconstruction with a dedicated clustering algorithm, efficient down to an energy of 230 MeV in the EB and 600 MeV in the EE [80]. The determination of the photon energies and directions is monitored in the data with $\pi^0 \rightarrow \gamma \gamma$ decays, and is in agreement with the predictions from simulation.

Final-state radiated photons are mostly produced with a direction nearly collinear with the parent lepton and have a harder spectrum than background photons from initial-state radiation or pileup interactions. Therefore, to be identified as FSR, a reconstructed photon must either have a transverse momentum $p_T > 2$ GeV and be found within a cone of size $\Delta R < 0.07$ from a selected lepton candidate, or have $p_T > 4$ GeV and be found isolated from charged particles and energy deposits and within $0.07 < \Delta R < 0.5$ from a selected lepton candidate.

The photon isolation observable $R_{\text{iso}}$ is the sum of the transverse momenta of charged hadrons, other photons, and neutral hadrons (including the ones originating from other vertices with respect to the primary vertex of the event) identified by the PF reconstruction within $\Delta R = 0.3$ around the candidate photon direction, divided by the photon transverse momentum. Isolated photons must satisfy $R_{\text{iso}} < 1$.

If more than one FSR candidate is associated with a $Z$ candidate, the one with the highest $p_T$ is chosen, if there is at least one with $p_T > 4$ GeV; otherwise, the one closest to any of the individual daughter leptons of the $Z$-boson candidate is chosen. These criteria are chosen to maximize the efficiency of the selection for photon emissions collinear with the lepton direction, while keeping the contribution from background or pileup interactions sufficiently low. The performance of the FSR recovery algorithm on the simulation of signal events is described in Sec. VIII.

VII. JET RECONSTRUCTION AND IDENTIFICATION

In the analysis, the presence of jets is used as an indication of vector-boson fusion (VBF) or associated production with a weak boson, $VH$, with $V = W$ or $Z$, where the $V$ decays hadronically. Jets are reconstructed using the anti-$k_T$ clustering algorithm [100] with distance parameter $D = 0.5$, as implemented in the FASTJET package [95,101], applied to the PF candidates of the event. Jet energy corrections are applied as a function of the jet $p_T$ and $\eta$ [102]. An offset correction is applied to subtract the energy contribution not associated with the high-$p_T$ scattering, such as electronic noise and pileup, based on the jet-area method [93,94,102]. Jets are only considered if they have $p_T > 30$ GeV and $|\eta| < 4.7$. In addition, they are required to be separated from the lepton candidates and from isolated FSR photons by $\Delta R > 0.5$.

Within the tracker acceptance, the jets are reconstructed with the constraint that the charged particles are compatible with the primary vertex. In addition, in the entire


\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Fig4.png}
\caption{(color online) Efficiency, as a function of the lepton $p_T$, for reconstructing and selecting (top) electrons and (bottom) muons, measured with a $Z \rightarrow \ell \ell$ data sample by using a tag-and-probe method.}
\end{figure}

VI. FINAL-STATE RADIATION RECOVERY

A $Z$-boson decay into a lepton pair can be accompanied by final-state radiation, in which case it is desirable to identify and associate the radiated photon to the corresponding lepton to form the $Z$-boson candidate: $Z \rightarrow \ell^+ \ell^- \gamma$. Photons reconstructed within $|\eta| < 2.4$ are possible FSR candidates. Low-energy photons are identified and
acceptance, a multivariate discriminator is used to separate jets arising from the primary interaction from those reconstructed from energy deposits associated with pileup interactions, especially due to neutral particles not associated with the primary vertex of the event. The discrimination is based on the differences in the jet shapes, the relative multiplicity of charged and neutral components, and the fraction of transverse momentum carried by the hardest components.

VIII. SELECTION AND CATEGORIZATION OF FOUR-LEPTON CANDIDATES

The event selection is designed to give a set of signal candidates in the $H \rightarrow ZZ \rightarrow 4\ell$ final state in three mutually exclusive subchannels: $4\ell$, $2\ell 2\mu$, and $4\mu$. Four well-identified and isolated leptons are required to originate from the primary vertex to suppress the $Z + \text{jet}$ and $\bar{t} t$ backgrounds.

A $Z$ candidate formed with a pair of leptons of the same flavor and opposite charge ($\ell^+ \ell^-$) is required. When forming the $Z$-boson candidates, only FSR photon candidates that make the lepton-pair mass closer to the nominal $Z$-boson mass are incorporated. If the mass $m_{\ell^+\ell^-} > 100$ GeV, the photon is not considered, to minimize the fraction of misidentified FSR candidates. With the photon selection requirements described in Sec. VI, about 1.5%, 4.6%, and 9% of the simulated $H \rightarrow 4\ell$, $H \rightarrow 2\ell 2\mu$, and $H \rightarrow 4\mu$ decays, respectively, are affected by the FSR recovery procedure. As the photon emission is most often collinear with one of the leptons, measured electron energies, by construction, include the energy of a large fraction of the emitted photons in the associated ECAL supercluster, while measured muon momenta do not include the emitted photons. Therefore, without photon recovery, FSR is expected to degrade the four-lepton mass resolution for Higgs boson candidates, especially in the $4\mu$ and in the $2\ell 2\mu$ final states and, to a lesser extent, in the $4\ell$ final state. The performance of the FSR recovery algorithm is estimated using simulated samples of $H \rightarrow ZZ \rightarrow 4\ell$, and the rate is verified with inclusive $Z$ and $ZZ$ data events. Genuine FSR photons within the acceptance of the FSR selection are selected with an efficiency of $\approx 50\%$ and with a mean purity of $80\%$. The FSR photons are selected in 5% of inclusive $Z$ events with muon pairs, and in 0.5% of single-$Z$ events with electron pairs. A gain of $\approx 3\%$ (2%, 1%) in efficiency is expected for the selection of $H \rightarrow 4\mu$ ($2\ell 2\mu$, $4\ell$) events in this analysis. The momentum of the selected FSR photon is added to the momentum of the nearest lepton for the computation of every $4\ell$ kinematic variable. Hereafter, $\ell$ denotes a $\ell^+ \gamma$, in the case of a recovered FSR photon.

Among all the possible opposite-charge lepton pairs in the event, the one with an invariant mass closest to the nominal $Z$-boson mass is denoted $Z_1$ and retained if its mass, $m_{Z_1}$, satisfies $40 < m_{Z_1} < 120$ GeV. Then, all remaining leptons are considered and a second $\ell^+ \ell^-$ pair is required ($Z_2$), with the mass denoted $m_{Z_2}$. If more than one $Z_2$ candidate is selected, the ambiguity is resolved by choosing the pair of leptons with the highest scalar sum of $p_T$. Simulation studies demonstrate that this algorithm selects the true $Z_2$ in the majority of cases without sculpting the shape of the $ZZ$ background. The chosen $Z_2$ is required to satisfy $12 < m_{Z_2} < 120$ GeV. For the mass range of $m_{Z_2} < 180$ GeV, at least one of the $Z$ candidates is off shell. The lower bound for $m_{Z_2}$ provides an optimal sensitivity for a Higgs boson mass hypothesis in the range $110 < m_H < 160$ GeV.

Among the four selected leptons forming the $Z_1$ and the $Z_2$, at least one lepton is required to have $p_T^\ell > 20$ GeV, and another one is required to have $p_T^\ell > 10$ GeV. These $p_T^{\ell}$ thresholds ensure that the selected events have leptons on the efficiency plateau of the trigger. To further remove events with leptons originating from hadron decays produced by jet fragmentation or from the decay of low-mass hadron resonances, it is required that any opposite-charge pair of leptons chosen among the four selected leptons (irrespective of flavor) satisfy $m_{\ell^+\ell^-} > 4$ GeV. The phase space for the search of the SM Higgs boson is defined by restricting the measured mass range to $m_{4\ell} > 100$ GeV.

The overall signal detection efficiencies, including geometrical acceptance, for the $4\ell$, $2\ell 2\mu$, and $4\mu$ channels increase as a function of $m_H$ rapidly up to approximately $2m_Z$, where both the $Z$ bosons are on shell, and then flattens. The residual rise for $m_H > 300$ GeV is mostly due to the increased acceptance. The efficiency versus $m_H$ is shown in Fig. 5 for the gluon fusion Higgs boson production mode, and it is very similar for other production modes. The signal events are generated with $|\eta^{\ell}| < 5$ and invariant mass of the dileptons from both the $Z_1$ and the $Z_2$

![FIG. 5](color online). Geometrical acceptance times selection efficiency for the SM Higgs boson signal as a function of $m_H$ in the three final states for gluon fusion production. Points represent efficiency estimated from full CMS simulation; lines represent a smooth polynomial curve interpolating the points, used in the analysis. The vertical dashed line represents $m_H = 126$ GeV.
boson decays $m_{f^- f^-} > 1$ GeV. The efficiency within the geometrical acceptance is $\approx 30\%$ ($58\%$), $43\%$ ($71\%$), and $62\%$ ($87\%$) for the three channels, respectively, for $m_H = 126(200)$ GeV.

For a Higgs boson with $m_H = 126$ GeV, the resolution of the Gaussian core of the mass distribution, estimated from simulated signal samples with a double-sided Crystal-Ball function fit, is about 2.0, 1.6, 1.2 GeV for $4e, 2e2\mu$, and $4\mu$, respectively. The expected signal yield, split by category, about 5% ($20\%$) of the signal events are expected to come from the VBF production mechanism, as estimated from simulation. The full rms of the four-lepton mass distribution, including the asymmetric tails, is estimated to be 2.9, 2.3, 1.7 GeV for the three channels, respectively. For a Higgs boson with $m_H = 600$ GeV, in which the natural width of the resonance contributes most, the double-sided Crystal-Ball-function core width parameter is about 75 GeV.

While in the dominant gluon fusion mechanism the Higgs boson is produced only in association with jets from initial-state radiation of the quarks, in the VBF production the two vector bosons are radiated from the initial-state quarks to produce the Higgs boson. The cross section for VBF production is about 1 order of magnitude smaller than that for the gluon fusion process. In the vector-boson scattering process, the two initial-state quarks deviate at a polar angle large enough such that as final-state quarks they create measurable additional jets in the event. These two jets, being remnants of the incoming proton beams, have typically a large separation in $\eta$ and high momentum. These characteristics are used to distinguish gluon fusion from VBF Higgs boson production in the analysis. Jets in the final state also come from $t\bar{t}H$ and $VH$ production, where the $V$ decays hadronically.

In order to improve the sensitivity to the Higgs boson production mechanisms, the event sample is split into two categories based on the jet multiplicity, where a jet is defined as in Sec. VII. These categories are defined as the 0/1-jet category, containing events with fewer than two jets, and the dijet category, containing events with at least two jets. In the 0/1-jet category, the transverse momentum of the four-lepton system ($p_T^4$) is used to distinguish VBF production and associated production with a weak boson, $VH$, from gluon fusion. In the dijet category, a linear discriminant ($\mathcal{D}_{\text{jet}}$) is formed combining two VBF-sensitive variables, the absolute difference in pseudorapidity ($|\Delta \eta_{jj}|$) and the invariant mass of the two leading jets ($m_{jj}$). The discriminant maximizes the separation between vector-boson and gluon fusion processes. In the 0/1-jet (dijet) category, about 5% (20%) of the signal events are expected to come from the VBF production mechanism, as estimated from simulation. The expected signal yield, split by category and by production mode, is reported in Table V.

A. Per-event mass uncertainties

For the Higgs boson mass and width measurement, the uncertainty in the four-lepton mass, which can be estimated on a per-event basis, is relevant because it varies considerably over the small number of selected events.

Uncertainties in the measured lepton momentum arise from imperfect calibration of the ECAL supercluster and uncertainty in the GSF track fit due to possible high-bremstrahlung emissions in the case of the electrons, and from the uncertainty in the muon track fit due to multiple scattering of the muons in the material of the inner tracker. These uncertainties depend on and are evaluated from the lepton’s direction and transverse momentum, as well as from possible mismeasurements specific to each lepton. In the case of electrons, the momentum uncertainties are assessed from the combination of the quality of the ECAL supercluster and the GSF track fit, through a similar multivariate regression as the one used to refine the estimate of the electron momentum, described in Sec. VI.A. In the case of muons, the momentum uncertainties are assessed from the properties of hits in the tracker and in the muon system, and the quality of the muon candidate fit. If FSR photons are identified and associated with the event, their uncertainty, assessed by the quality of the ECAL clusters, is also accounted for in the event mass uncertainty.

The momentum uncertainties for each of the four leptons in an event are then propagated into a relative uncertainty $\mathcal{D}_m \equiv \sigma_{m_{4\ell}} / m_{4\ell}$ in the four-lepton mass. The per-event mass uncertainty is given as the sum in quadrature of the individual mass uncertainty contributions from each lepton and any identified FSR photon candidate. A calibration of the per-lepton uncertainties is derived using large $J/\psi \rightarrow \mu^+ \mu^-$, $Z \rightarrow \mu^+ \mu^-$, and $Z \rightarrow e^+ e^-$ event samples, both in data (Table I) and in simulation. The line shape of these resonances is modeled, as for the SM Higgs boson, with a BW convolved with a double-sided CB function, where the resolution is estimated as $\lambda \times \sigma(m_{4\ell})$. In this procedure, $\sigma(m_{4\ell})$ is fixed to the value computed using the uncertainties in the individual momenta of the leptons, and $\lambda$, defined as the calibration constant, is a floating parameter. The latter is derived for electrons and muons in several bins of the average $p_T^4$ and $\eta^4$ of the lepton: $J/\psi \rightarrow \mu^+ \mu^-$ is used for muons with $p_T^4 < 20$ GeV, while, for lack of a sufficiently large sample of $J/\psi \rightarrow e^+ e^-$, $Z \rightarrow e^+ e^-$ events are used in the entire $p_T^4$ range. The value of $\lambda$ obtained from the fit is approximately 1.2 for electrons and 1.1 for muons, in the entire kinematic range of the leptons used in this analysis.

As a closure test, the $Z \rightarrow \ell\ell$ events are grouped into subsets based on their per-event predicted dilepton mass resolution and fit to the $Z$ line shape in each subset as described above. A systematic uncertainty of $\pm 20\%$ is assigned to the per-event mass uncertainty for both electrons and muons based on the agreement between per-event computed and observed mass resolutions as shown in Fig. 6 (top). In Fig. 6 (bottom), the comparison between data and simulation of the $\mathcal{D}_m$ observable in the $Z \rightarrow 4\ell$ mass region is shown.
production are calculated with MCFM [104–106]. The relative contribution of LO $gg \to ZZ$ with respect to NLO $q\bar{q} \to ZZ$ is about 2% at four-lepton mass $m_{4\ell} = 126$ GeV and about 6% at 1 TeV. The expected contribution of the ZZ processes to the total background, in the region $100 < m_{4\ell} < 1000$ (121.5 < $m_{4\ell} < 130.5$) GeV, is approximately 91%, 94%, and 97% (58%, 71%, and 86%) in the $4e$, $2e2\mu$, and $4\mu$ channels, respectively. The shape uncertainties arising from imperfect simulation of the $p_T^l$ and $\eta^l$ dependence of the efficiency and other experimental sources are completely overshadowed by the uncertainties from the normalization systematics, such that shape variations have negligible effects compared to the normalization variations.

The irreducible four-lepton background arising from double-parton interactions (DPI), $Z +$ Drell-Yan (DY), is evaluated using PYTHIA 6.4.24 with the overall cross section calculated as $\sigma_{\text{DPI}} = \sigma_Z \cdot \sigma_{\text{DY}} / \sigma_{\text{pheno}}$, where the phenomenological effective cross section, measured at $\sqrt{s} = 7$ TeV, is $\sigma_{\text{pheno}} = 15$ mb [107], and the cross sections $\sigma_Z$ and $\sigma_{\text{DY}}$ are taken from simulation. The DPI $Z +$ DY background is much smaller than normalization uncertainties on either $q\bar{q} \to ZZ$, $gg \to ZZ$ or a reducible background; hence, the DPI $Z +$ DY background is neglected in the analysis.

B. Reducible background

Two independent methods, using dedicated control regions in data, are considered to estimate the reducible background, denoted as $Z + X$ in the following paragraphs because the background is dominated by the $Z + \text{jets}$ process. The control regions are defined by a dilepton pair satisfying all the requirements of a $Z_1$ candidate and two additional leptons, opposite sign (OS) or same sign (SS), satisfying certain relaxed identification requirements when compared to those used in the analysis. The invariant mass of the additional dilepton pair is required to be larger than 12 GeV, in order to be consistent with the criteria imposed on the $Z_2$ candidate in the signal selection.

In both methods, the extrapolation from the control region to the signal region is performed using the lepton misidentification probability, $f(\ell, p_T^\ell, |\eta^\ell|)$, which is defined as the fraction of non-signal leptons identified with the analysis selection criteria, estimated in an enriched sample of nongenuine electrons and muons. This sample is composed of $Z_1 + 1\ell_{\text{loose}}$ events in data consisting of a pair of leptons, both passing the selection requirements used in the analysis, and exactly one additional lepton passing the relaxed selection. The mass of the $Z_1$ candidate is required to satisfy $|m_{4\ell} - m_Z| < 10$ GeV for the OS leptons method. Such a stringent requirement suppresses from the $f(\ell, p_T^\ell, |\eta^\ell|)$ calculation the contribution of events with FSR where the photon converts and one of the conversion products is not reconstructed. For the SS leptons method, a requirement of $|m_{4\ell} - m_Z| < 40$ GeV is

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**FIG. 6** (color online). (top) Measured versus predicted relative mass uncertainties for $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ events in data. The dashed lines represent the ±20% envelope, used as systematic uncertainty in the resolution. (bottom) Relative mass uncertainty distribution for data and simulation in the systematic uncertainty in the resolution.

**IX. BACKGROUND ESTIMATION**

The dominant background contribution in the $H \to ZZ \to 4\ell$ search is irreducible and is due to direct $ZZ$ production via $q\bar{q}$ annihilation and gluon fusion. The remaining subleading contributions arise from reducible multilepton sources, $Z + \text{jets}$, $t\bar{t}$, and $WZ + \text{jets}$.

A. Irreducible background

The expected yield and shape of the ZZ background is evaluated by simulation. The NLO cross section for $q\bar{q} \to ZZ$ production and the LO cross section for $gg \to ZZ$ production are calculated with MCFM [104–106]. The relative contribution of LO $gg \to ZZ$ with respect to NLO $q\bar{q} \to ZZ$ is about 2% at four-lepton mass $m_{4\ell} = 126$ GeV and about 6% at 1 TeV. The expected contribution of the ZZ processes to the total background, in the region $100 < m_{4\ell} < 1000$ (121.5 < $m_{4\ell} < 130.5$) GeV, is approximately 91%, 94%, and 97% (58%, 71%, and 86%) in the $4e$, $2e2\mu$, and $4\mu$ channels, respectively. The shape uncertainties arising from imperfect simulation of the $p_T^l$ and $\eta^l$ dependence of the efficiency and other experimental sources are completely overshadowed by the uncertainties from the normalization systematics, such that shape variations have negligible effects compared to the normalization variations.

The irreducible four-lepton background arising from double-parton interactions (DPI), $Z +$ Drell-Yan (DY), is evaluated using PYTHIA 6.4.24 with the overall cross section calculated as $\sigma_{\text{DPI}} = \sigma_Z \cdot \sigma_{\text{DY}} / \sigma_{\text{pheno}}$, where the phenomenological effective cross section, measured at $\sqrt{s} = 7$ TeV, is $\sigma_{\text{pheno}} = 15$ mb [107], and the cross sections $\sigma_Z$ and $\sigma_{\text{DY}}$ are taken from simulation. The DPI $Z +$ DY background is much smaller than normalization uncertainties on either $q\bar{q} \to ZZ$, $gg \to ZZ$ or a reducible background; hence, the DPI $Z +$ DY background is neglected in the analysis.

B. Reducible background

Two independent methods, using dedicated control regions in data, are considered to estimate the reducible background, denoted as $Z + X$ in the following paragraphs because the background is dominated by the $Z + \text{jets}$ process. The control regions are defined by a dilepton pair satisfying all the requirements of a $Z_1$ candidate and two additional leptons, opposite sign (OS) or same sign (SS), satisfying certain relaxed identification requirements when compared to those used in the analysis. The invariant mass of the additional dilepton pair is required to be larger than 12 GeV, in order to be consistent with the criteria imposed on the $Z_2$ candidate in the signal selection.

In both methods, the extrapolation from the control region to the signal region is performed using the lepton misidentification probability, $f(\ell, p_T^\ell, |\eta^\ell|)$, which is defined as the fraction of non-signal leptons identified with the analysis selection criteria, estimated in an enriched sample of nongenuine electrons and muons. This sample is composed of $Z_1 + 1\ell_{\text{loose}}$ events in data consisting of a pair of leptons, both passing the selection requirements used in the analysis, and exactly one additional lepton passing the relaxed selection. The mass of the $Z_1$ candidate is required to satisfy $|m_{4\ell} - m_Z| < 10$ GeV for the OS leptons method. Such a stringent requirement suppresses from the $f(\ell, p_T^\ell, |\eta^\ell|)$ calculation the contribution of events with FSR where the photon converts and one of the conversion products is not reconstructed. For the SS leptons method, a requirement of $|m_{4\ell} - m_Z| < 40$ GeV is
imposed. In order to suppress the contribution from WZ and t\bar{t} processes, which have a third lepton, the missing transverse energy (E_{T}) is required to be less than 25 GeV. The E_{T} is defined as the modulus of the vector sum of the transverse momenta of all reconstructed PF candidates (charged or neutral) in the event. The invariant mass of the loose lepton and the opposite-sign lepton from the Z_{1} candidate, if they have the same flavor, are required to be greater than 4 GeV to reject contributions from low-mass resonances, such as J/ψ. As a result of these requirements, the control sample largely consists of events with a Z boson and a misidentified additional lepton. Hence, the fraction of these events in which the additional lepton passes the analysis identification and isolation requirements gives a rate \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) that ranges from 1%-15% (5%-10%) depending on the \( p_{T}^{\ell} \) and \( |\eta^{\ell}| \) of the electron (muon).

1. Method using opposite-sign (OS) leptons

In this method, the control region consists of events with a Z_{1} candidate and two additional leptons with the same flavor and opposite charge. Two categories of events are considered in this method.

The category 2P2F is composed of events in which two leptons pass (P) the selection requirements of the analysis and two fail (F), but pass the loose selection. It is used to estimate the contribution from backgrounds that intrinsically have only two prompt leptons (Z + jets, t\bar{t}). To estimate the contribution of these background processes in the signal region, each 2P2F event is weighted by a factor \( f_{j}^{3, 4} \), where \( f_{j}^{3} \) and \( f_{j}^{4} \) are the \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) for the third and fourth lepton. Analogously, the 3P1F category consists of events where exactly one of the two additional leptons passes the analysis selection. It is used to estimate the contribution from backgrounds with three prompt leptons and one misidentified lepton (WZ + jets and Z+ + jets with the photon converting to an e+ e− pair). Each event \( j \) in the 3P1F control region is weighted by a factor \( f_{j}^{3, 4} \), where \( f_{j}^{i} \) is the \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) for the third or fourth lepton to fail the analysis selection. This control region also has contributions from ZZ events where one of the four prompt leptons fails the analysis selection, and from the processes with only two prompt leptons (2P2F type), where one of the two nonprompt leptons passes the selection requirements. The contribution from ZZ events, \( n_{3P1F}^{ZZ} \), is estimated from simulation, and the background estimate is reduced by a factor of \( 1 - n_{3P1F}^{ZZ}/N_{3P1F} \), where \( N_{3P1F} \) is the number of events in the 3P1F control region. The contribution from 2P2F-type processes in the 3P1F region is estimated as \( \sum_{l}(f_{l}^{3} + f_{l}^{4}) \). It contributes to the final weighted sum of the 3P1F events with the component \( \sum_{l}(2 f_{l}^{3} f_{l}^{4} (1 - f_{l}^{4})) \), which has to be subtracted from the background estimate. Therefore, in this method, the expected yield for the reducible background in the signal region, \( N_{SR}^{\text{reducible}} \), becomes

\[
N_{SR}^{\text{reducible}} = \left(1 - \frac{n_{3P1F}^{ZZ}}{N_{3P1F}}\right) \sum_{j} f_{j}^{3} - \sum_{l} f_{l}^{3} (1 - f_{l}^{4}).
\]

2. Method using same-sign (SS) leptons

In this method, the control region consists of events with a Z_{1} candidate and two additional leptons with the same flavor and same charge. The \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) is measured using a Z_{1} + 1\ell_{\text{loose}} sample, which is similar to that used for the OS control region, but with the invariant mass of the Z_{1} candidate, \( m_{\ell\ell} - m_{Z} < 40 \text{ GeV} \), consistent with the requirement on the Z_{1} candidate used in the analysis. Here, the contribution from FSR photons to the electron misidentification probability is much larger and needs to be taken into account. This is done by exploiting the observed linear dependence of the \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) on the fraction of loose electrons with tracks having one missing hit in the pixel detector, \( r_{\text{miss}}(p_{T}^{\ell}, |\eta^{\ell}|) \), which is indicative of a possible conversion. The fraction \( r_{\text{miss}}(p_{T}^{\ell}, |\eta^{\ell}|) \) is estimated using samples with different FSR contributions obtained by varying the requirements on \( m_{\ell\ell} - m_{Z} \) and \( m_{\ell\ell_{\text{loose}} - m_{Z}} \). The corrected \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) is then computed using the value \( r_{\text{miss}}(p_{T}^{\ell}, |\eta^{\ell}|) \) measured in the control sample where the method is applied.

The expected number of reducible background events in the signal region is obtained as

\[
N_{SR}^{\text{reducible}} = r_{\text{OS/SS}} \cdot \sum_{i} j_{i}^{3} \cdot j_{i}^{4},
\]

where \( N_{2P2LSS}^{\text{OS}} \) is the number of observed events in the region 2P2LSS, in which both the additional leptons fulfill the loose selection requirements for leptons, having the same flavor and charge. The ratio \( r_{\text{OS/SS}} \) between the number of events in the 2P2LSS and 2P2LSS control regions is obtained from simulation.

3. Combination of the two methods

The predicted yields of the \( Z + X \) background from the two methods are in agreement within their statistical uncertainties. The dominant sources of these uncertainties are the limited number of events in the 3P1F, 2P2F, and 2P2LSS control regions, as well as in the region where the correction factor for \( f(\ell, p_{T}^{\ell}, |\eta^{\ell}|) \) is computed. Since they are mutually independent, results of the two methods are combined.

The shape of the \( m_{\ell\ell} \) distribution for the reducible background is obtained from the OS method by fitting the \( m_{\ell\ell} \) distributions of 2P2F and 3P1F events separately with empirical functional forms built from Landau [108] and
exponential distributions. The systematic uncertainty in the $m_{4\ell}$ shape is determined by the envelope that covers alternative functional forms or alternative binning for the fit used to determine its parameters. The additional discriminating variables for this background are described by binned templates, as discussed in Sec. XII.

The total systematic uncertainties assigned to the $Z + X$ background estimate take into account the uncertainty in the $m_{4\ell}$ shape. They also account for the difference in the composition of the $Z_1 + V_{\text{loose}}$ sample used to compute $f(\ell_1^e, p_T^\ell_1, |\eta_1^\ell|)$ and the control regions in the two methods used to estimate the $Z + X$ background—in particular, the contribution of the heavy flavor jets and photon conversions. The systematic uncertainty is estimated to be 20%, 25%, and 40% for the $4e$, $2e2\mu$, and $4\mu$ decay channels, respectively. The two methods have been further validated using events that pass the analysis selection with the exception that the $Z_2$ candidate is formed out of a lepton pair with the wrong combination of flavors or charges (control region $Z_1 + e^+e^-/e^+\mu^-/\mu^+\mu^-$). The predicted contribution of the reducible background in this control region is in agreement with the observed number of events within the uncertainties. Figure 7 (top) shows the validation of the OS method.

The prediction for the $Z + X$ background yields with combined statistical and systematic uncertainties is given in Sec. XI and also shown in Fig. 7 (bottom). The expected yields of the $Z + X$ background in the signal region from the 2P2F-like and 3P1F-like sources are estimated separately. The weighted events of the two control regions are also fitted independently and then added together to give the total $Z + X$ $m_{4\ell}$ probability density function used in the fit. The relative contribution of the reducible background to the total background in the region $100 < m_{4\ell} < 1000$ (121.5 $< m_{4\ell} < 130.5$) GeV depends on the final state, being approximately $9\%$ ($42\%$), $6\%$ ($28\%$), and $3\%$ ($14\%$) in the $4e$, $2e2\mu$, and $4\mu$ channels, respectively. The estimated yields of this background are reported in Sec. XI.

X. KINEMATIC DISCRIMINANTS

The four-lepton decay mode has the advantage that the kinematics of the Higgs boson and its decay products are all visible in the detector, providing many independent observables that can be used for different purposes. First, in addition to their invariant mass, the angular distributions of the four leptons and the dilepton pairs’ invariant masses can be used to further discriminate signal from background and thus increase the signal sensitivity and reduce the statistical uncertainty in measurements, including the cross section, the mass, and the width of the resonance. Second, this extra information on angular correlations can be used to experimentally establish the consistency of the spin and parity quantum numbers with respect to the SM. This section describes how the full kinematic information from the production and decay can be encoded in a kinematic discriminant optimized for the separation of two processes, be it signal from background or between different signal hypotheses.

The kinematic properties of the SM Higgs boson or any non-SM exotic boson decay to the four-lepton final state has been extensively studied in Refs. [41–43,109–122]. Five angles $\Omega \equiv (\Omega', \Phi_1, \theta_1, \theta_2, \Phi)$ defined in Fig. 8 [41,123] and the invariant masses of the lepton pairs, $m_{2\ell}$ and $m_{Z_2}$, fully describe the kinematic configuration of a four-lepton system in its center-of-mass frame, up to an
arbitrary rotation around the beam axis. These observables provide significant discriminating power between signal and background, as well as between alternative signal models. A matrix-element likelihood approach is used to construct kinematic discriminants related to the decay observables [20,31].

In addition to the four-lepton center-of-mass-frame observables, the four-lepton transverse momentum and rapidity are needed to completely define the system in the lab frame. The transverse momentum of the four-lepton system is used in the analysis as an independent observable because it is sensitive to the production mechanism of the Higgs boson, but it is not used in the spin-parity analysis. The four-lepton rapidity is not used because the discriminating power of this observable for events within the experimental acceptance is limited.

Kinematic discriminants are defined based on the event probabilities depending on the background ($P_{bkg}$) or signal spin-parity ($J^{P}$) hypotheses under consideration ($P_{J^{P}}$):

$$P_{bkg} = P_{bkg}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega m_{4\ell}) \times P_{bkg}^{\text{mass}}(m_{4\ell}),$$ (4)

$$P_{J^{P}} = P_{J^{P}}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega m_{4\ell}) \times P_{J^{P}}^{\text{mass}}(m_{4\ell} | m_H).$$ (5)

where $P_{\text{kin}}$ is the probability distribution of angular and mass observables ($\Omega, m_{Z_1}, m_{Z_2}$) computed from the LO matrix element squared for signal and ZZ processes, and $P_{\text{mass}}$ is the probability distribution of $m_{4\ell}$ and is calculated using the parameterization described in Sec. XII A. Matrix elements for the signals are calculated with the assumption that $m_H = m_{4\ell}$. The probability distributions for spin-0 resonances are independent of an assumed production mechanism. Only the dominant $q\bar{q} \rightarrow ZZ$ background is considered in the probability parameterization. For the reducible backgrounds, empirical templates derived from the data control samples defined in Sec. IX B are used to model the probability density functions of the kinematic discriminants, as described in Sec. XII.

For the alternative signal hypotheses, nine models have been tested, following the notations from Refs. [41,42]. The most general decay amplitude for a spin-0 boson decaying to two vector bosons can be defined as

$$A(H \rightarrow ZZ) = v^{-1}(a_1 m^{2}_{Z_2} e_i^2 + a_2 f_{\mu\nu}^{(1)} f^{*(i\mu\nu)},$$

$$+ a_3 f_{\mu\nu}^{(1)} f^{*(i\mu\nu)},$$ (6)

where $f^{(i)}_{\mu\nu} = e_i^0 q_i^0 - e_i^1 q_i^1$ is the field-strength tensor of a gauge boson with momentum $q_i$, and polarization vector $e_i$, $f_{\mu\nu}^{(i)} = 1/2 e_{\mu\alpha\beta} f^{(i\alpha\beta)} = e_{\mu\alpha\beta} e_i^\alpha q_i^\beta$ is the conjugate field strength tensor, $f^*$ denotes the complex conjugate field strength tensor, and $v$ is the vacuum expectation value of the SM Higgs field. $e_{\mu\alpha\beta}$ is the Levi-Civita completely antisymmetric tensor. The $a_i$ coefficients generally depend on $q_i^2$. In this analysis, we consider the lowest-dimension operators in the effective Lagrangian corresponding to each of the three unique Lorentz structures, therefore taking $a_1$ to be constant for the relevant range $q_i^2 = m_Z^2 < m_H^2$. The SM Higgs boson decay is dominated by the tree-level coupling $a_1$. The $0^-$ model corresponds to a pseudoscalar (dominated by the $a_3$ coupling), while $0^+$ is a scalar (dominated by the $a_2$ coupling) not participating in the electroweak symmetry breaking, where $h$ refers to higher-dimensional operators in Eq. (6) with respect to the SM Higgs field. The spin-0 signal models are simulated for the gluon fusion production process, and their kinematics in the boson center-of-mass frame is independent of the production mechanism.

The $1^-$ and $1^+$ hypotheses represent a vector and a pseudovector decaying to two $Z$ bosons. The spin-1 resonance models are simulated via the quark-antiquark production mechanism, as the gluon fusion production of such resonances is expected to be strongly suppressed. The spin-1 hypotheses are considered under the assumption that the resonance decaying into $4\ell$ is not necessarily the same resonance observed in the $H \rightarrow \gamma\gamma$ channel [19,20], as $J = 1$ in the latter case is prohibited by the Landau-Yang theorem [124,125]. This also provides a test of the spin-1 hypothesis in an independent way.

The spin-2 model with minimal couplings, $2_{\mu\nu}$, represents a massive graviton-like boson $X$ suggested, for example, in models with warped extra dimensions (ED) [126,127], where gluon fusion is the dominant process. For completeness, 100% quark-antiquark annihilation is also considered, which provides a projection of the spin of the resonance on the parton collision axis equal to 1, instead of 2, as in the case of gluon fusion with minimal couplings.

FIG. 8 (color online). Illustration of the production and decay of a particle $H$, $g(q\bar{q}) \rightarrow H \rightarrow ZZ \rightarrow 4\ell$, with the two production angles $\theta_1$ and $\Phi_1$ shown in the $H$ rest frame and three decay angles $\theta_1$, $\theta_2$, and $\Phi$ shown in the $Z_1$, $Z_2$, and $H$ rest frames, respectively.
A modified minimal coupling model $2_h^+$ is also considered, where the SM fields are allowed to propagate in the bulk of the ED [128], corresponding to $g_1 \ll g_8$ in the $XZZ$ coupling for the $2_h^+$ model, where the $g_1$'s are the couplings in the effective Lagrangian of Ref. [42]. Finally, two spin-2 models with higher-dimension operators are considered with both positive and negative parity, $2_+^+$ and $2_+^-$, corresponding to the $g_4$ and $g_8$ couplings. The $2_+^+$, $2_+^-$, and $2_+^0$ resonances are assumed to be produced in gluon fusion. The above list of the spin-2 models does not exhaust all possible scenarios, nor does it cover possible mixed states.

The discriminant defined this way does not carry direct discrimination power based on the four-lepton mass $m_{4\ell}$ distribution between the signal and the background. Hence, it can be used as a second discriminating observable in addition to the $m_{4\ell}$ distribution. The $\mathcal{P}_l$'s are normalized with additional constant factors for a given value of $m_{4\ell}$, such that the ratio of probabilities is scaled by a constant factor leading to probabilities $P(\mathcal{D} > 0.5|H) = P(\mathcal{D} < 0.5|\text{bkg})$.

In this analysis, the SM Higgs boson signal is distinguished simultaneously from the background and from alternative signal hypotheses. The former is separated with $\mathcal{D}_{\text{bkg}}$, and the latter with $\mathcal{D}_{J^0}$ observables constructed from the background, signal, and the probability of the alternative hypotheses defined in Eqs. (4) and (5). The $\mathcal{D}_{\text{bkg}}$ observable extends $\mathcal{D}_{\text{bkg}}^{\text{kin}}$ defined in Eq. (7) with the four-lepton mass probability for separation at a fixed value of the mass $m_{4\ell}$:

$$
\mathcal{D}_{\text{bkg}} = \left[ 1 + \frac{\mathcal{P}_{\text{bkg}}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell}) \times \mathcal{P}_{\text{bkg}}^{\text{mass}}(m_{4\ell})}{\mathcal{P}_{\text{bkg}}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell})} \right]^{-1}.
$$

The other observable discriminates between the SM Higgs boson and the alternative signal hypothesis:

$$
\mathcal{D}_{J^0} = \left[ 1 + \frac{\mathcal{P}_{J^0}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell}) \times \mathcal{P}_{\text{sig}}^{\text{mass}}(m_{4\ell}|m_{0^+})}{\mathcal{P}_{J^0}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell})} \right]^{-1}.
$$

The spin-0 discriminants $\mathcal{D}_{0^+}$ and $\mathcal{D}_{0^-}$ are independent of any production mechanism, since in the production of a spin-0 particle the angular decay variables are independent of production mechanism. This is not the case for the spin-1 and spin-2 signal hypotheses. Therefore, it is desirable to test the spin-1 and spin-2 hypotheses in a way that does not depend on assumptions about the production mechanism. This is achieved by either averaging over the spin degrees of freedom of the produced boson or, equivalently, integrating the matrix elements squared over the production angles $\cos \theta$ and $\Phi_1$ [48]. With the latter, the discriminants are defined as

$$
\mathcal{D}_{\text{dec}} = \left[ 1 + \frac{\frac{1}{4\pi} \int d\Phi_1 d\cos \theta \mathcal{P}_{\text{bkg}}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell}) \times \mathcal{P}_{\text{bkg}}^{\text{mass}}(m_{4\ell})}{\mathcal{P}_{\text{bkg}}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell})} \right]^{-1},
$$

$$
\mathcal{D}_{J^0}^{\text{dec}} = \left[ 1 + \frac{\frac{1}{4\pi} \int d\Phi_1 d\cos \theta \mathcal{P}_{J^0}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell}) \times \mathcal{P}_{\text{sig}}^{\text{mass}}(m_{4\ell}|m_{0^+})}{\mathcal{P}_{J^0}^{\text{kin}}(m_{Z_1}, m_{Z_2}, \Omega|m_{4\ell})} \right]^{-1}.
$$

The superscript “dec” indicates that these discriminants use decay-only information. The probabilities for spin-0 resonances are already independent of the production mechanism; however, their distributions, for all the $J^0$ hypotheses, do carry some production dependence due to detector and analysis acceptance effects. Such production-dependent variations in the discriminant distribution shapes are found to be small and are treated as systematic uncertainties.

Table II summarizes all kinematic observables used in this analysis, for different purposes. To make an optimal use of the available information, the distribution of these observables is used without any selection in a fit.

This analysis uses the matrix-element likelihood approach (MELA) framework [20,42,43], with the matrix elements for different signal models taken from HUGEN [41–43] and the matrix element for the $q\bar{q} \rightarrow ZZ$ background taken from MCFM [104–106]. Within the MELA framework, an analytical parameterization of matrix elements for signal [41,42] and background [120] was adopted in the previous analyses of CMS data with results reported in Refs. [20,31]. The above matrix-element calculations are validated against each other and also tested with the matrix-element kinematic discriminant (MEKD) framework [121], based on MadGraph [70] and FeynRules [129], and with a stand-alone framework implementation of MadGraph. The inclusion of the lepton interference in the kinematic
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TABLE II. List of observables and kinematic discriminants used for signal versus background separation and studies of the properties of the observed resonance. The alternative hypotheses for $J = 0$ are independent of the production mechanism without the need of integrating out the production angles $\cos \theta^* \Phi_1$.  

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Note</th>
<th>Observables used for the signal-strength measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{4\ell}$</td>
<td></td>
<td>Four-lepton invariant mass, main background discrimination.</td>
</tr>
<tr>
<td>$D_{4\ell}^{\text{kin}}$</td>
<td>Linear discriminant, uses jet information to identify VBF topology.</td>
<td></td>
</tr>
<tr>
<td>$D_{4\ell}^{\text{sig}}$</td>
<td>Discriminates SM Higgs boson against ZZ background.</td>
<td></td>
</tr>
<tr>
<td>$p_T^{4\ell}$</td>
<td>$p_T$ of the $4\ell$ system, discriminates between production mechanisms.</td>
<td></td>
</tr>
</tbody>
</table>

Observables used in the spin-parity hypothesis testing  

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Note</th>
<th>Observables used in the spin-parity hypothesis testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{4\ell}^{\text{bkg}}$</td>
<td>Discriminates SM Higgs boson against ZZ background, includes $m_{4\ell}$.</td>
<td></td>
</tr>
<tr>
<td>$D_{0}$</td>
<td>Exotic vector ($1^+$), $q\bar{q}$ annihilation.</td>
<td></td>
</tr>
<tr>
<td>$D_{1}$</td>
<td>Exotic pseudovector ($1^+$), $q\bar{q}$ annihilation.</td>
<td></td>
</tr>
<tr>
<td>$D_{2}^{\text{g}}$</td>
<td>Graviton-like with minimal couplings ($2^+_m$), gluon fusion.</td>
<td></td>
</tr>
<tr>
<td>$D_{3}^{\text{g}}$</td>
<td>Graviton-like with minimal couplings ($2^+_m$), $q\bar{q}$ annihilation.</td>
<td></td>
</tr>
<tr>
<td>$D_{2}^{\text{h}}$</td>
<td>Graviton-like with SM in the bulk ($2^+_h$), gluon fusion.</td>
<td></td>
</tr>
<tr>
<td>$D_{3}^{\text{h}}$</td>
<td>Tensor with higher-dimension operators ($2^+_h$), gluon fusion.</td>
<td></td>
</tr>
<tr>
<td>$D_{4}^{\text{h}}$</td>
<td>Pseudotensor with higher-dimension operators ($2^+_h$), gluon fusion.</td>
<td></td>
</tr>
</tbody>
</table>

Production-independent observables used in the spin-parity hypothesis testing  

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Note</th>
<th>Observables used in the spin-parity hypothesis testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{0}^{\text{dec}}$</td>
<td>Pseudoscalar ($0^-$), discriminates against SM Higgs boson.</td>
<td></td>
</tr>
<tr>
<td>$D_{0}^{\text{dec}}^{\text{bkg}}$</td>
<td>Non-SM scalar with higher-dimension operators ($0^+_b$).</td>
<td></td>
</tr>
<tr>
<td>$D_{1}^{\text{dec}}$</td>
<td>Discriminates against ZZ background, includes $m_{4\ell}$, excludes $\cos \theta^*, \Phi_1$.</td>
<td></td>
</tr>
<tr>
<td>$D_{1}^{\text{dec}}^{\text{bkg}}$</td>
<td>Exotic vector ($1^+$), decay-only information.</td>
<td></td>
</tr>
<tr>
<td>$D_{1}^{\text{dec}}^{\text{bkg}}$</td>
<td>Exotic pseudovector ($1^+$), decay-only information.</td>
<td></td>
</tr>
<tr>
<td>$D_{2}^{\text{dec}}^{\text{bkg}}$</td>
<td>Graviton-like with minimal couplings ($2^+_m$), decay-only information.</td>
<td></td>
</tr>
</tbody>
</table>

The discriminant parameterization is a small improvement in the expected separation significance of $\sim 3\%$ for spin-0 models with respect to earlier published results [20,31], as indicated by cross-checks with generator-based matrix-element calculations performed in the MELA and MEKD frameworks within studies reported in Ref. [31].  

Detector acceptance effects approximately cancel in the probability ratios, such as those in Eq. (7). In principle, the kinematic discriminants could be modified to account for detector resolution effects. However, the matrix-element approach with detector transfer functions modeling detector resolution effects showed nearly identical performance. This is not unexpected for leptons, as their resolutions are of $\mathcal{O}(1\%)$ and are therefore negligible.  

In order to provide additional validation of the kinematic discriminants, machine-learning techniques have been used to construct discriminants. Two techniques have been used: the Bayesian neural networks (BNN) framework [110,131] and the BDT framework [88,132,133]. In the BNN framework, a Bayesian procedure is used to create a posterior probability density over the space of neural network parameters. This probability density is then used to calculate a BNN. In both frameworks, a discriminant is built using the four-lepton angular and mass variables, and the output is used in the same way as the $D_{4\ell}^{\text{kin}}$ in the analysis described above. The BNN and BDT discriminants are trained using simulated samples to discriminate signatures for signal events from those for background events or to discriminate between different signal hypotheses. The MC samples generated for training are based on the same matrix elements for signal and background as used in the analysis and include the effects of the full detector simulation. The machine-trained discriminants are found to give similar performance to the matrix-element approaches described above.  

XI. YIELDS AND KINEMATIC DISTRIBUTIONS  

The signal and background yields are extracted from a fit to the invariant mass and other kinematic properties, characterizing the decay of the Higgs boson candidate and its production mechanism. The expected distributions of signal and background components are used as probability density functions in the likelihood function. Simulation and control samples from data are used to estimate the initial fit values for the signal and background yields.  

The background from $ZZ$ and $Z+X$ processes dominates after the event selection. The reconstructed four-lepton invariant mass distribution for the combined $4e$, $2e2\mu$, and $4\mu$ channels is shown in Fig. 9 and compared with the expectations from background processes. Here, and in the other figures of this section, the normalization and shape of the $ZZ$ background and the signal
The expected distributions are presented as observed in the
FIG. 9 (color online). Distribution of the four-lepton recon-
structed mass in the full mass range $70 < m_{4\ell} < 1000$ GeV for
the sum of the $4e$, $2e2\mu$, and $4\mu$ channels. Points with error bars
represent the data, shaded histograms represent the backgrounds,
and the unshaded histogram represents the signal expectation for a
mass hypothesis of $m_{H} = 126$ GeV. Signal and the ZZ background
are normalized to the SM expectation; the $Z + X$ background to the
estimation from data. The expected distributions are presented as
stacked histograms. No events are observed with $m_{4\ell} > 800$ GeV.

($m_{H} = 126$ GeV) are obtained from simulation, while the
normalization and shape of the reducible background is
estimated from control samples in data, as described in
Sec. IX B. The error bars on data points are asymmetric
Poisson uncertainties that cover the 68% probability
interval around the central value [134]. A clear peak around
$m_{4\ell} = 126$ GeV is seen, not expected from background
processes, confirming with a larger data sample the results
reported in Refs. [19–21,31]. The observed distribution is
in good agreement with the expected backgrounds and a
narrow resonance compatible with the SM Higgs boson
with $m_{H}$ around 126 GeV. The $Z \rightarrow 4\ell$ resonance peak
at $m_{4\ell} = m_{Z}$ is observed in agreement with simulation.
The measured distribution at masses greater than $2m_{Z}$ is
dominated by the irreducible ZZ background, where the
two $Z$ bosons are produced on shell.

The number of candidates observed in data as well as the
expected yields for background and several SM Higgs
coson mass hypotheses are reported in Table III, for
$m_{4\ell} > 100$ GeV. The observed event rates for the various
channels are compatible with SM background expectations in
the $m_{4\ell}$ region above $2m_{Z}$, while a deviation is observed in the
lower region. Given that the excess of events observed in the $4\ell$
mass spectrum is localized in a narrow region in the vicinity of 126 GeV, the events expected in a
narrower range, $121.5 < m_{4\ell} < 130.5$ GeV, are reported in
Table IV. Table V reports the breakdown of the events
observed in data and the expected background yields in the
same $m_{4\ell}$ region in the two analysis categories, together

with the expected yield for a SM Higgs boson with
$m_{H} = 126$ GeV, split by production mechanism. The
$m_{4\ell}$ distribution for the sum of the $4e$, $2e2\mu$, and $4\mu$
channels, in the mass region $70 < m_{4\ell} < 180$ GeV, is
shown in Fig. 10. Figure 11 shows the reconstructed

\begin{table}[h]
\centering
\caption{The number of observed candidate events compared to the mean expected background and signal rates for each final state. Uncertainties include statistical and systematic sources. The results are given integrated over the full mass measurement range $m_{4\ell} > 100$ GeV and for 7 and 8 TeV data combined.}
\begin{tabular}{|c|c|c|c|c|}
\hline
Channel & $4e$ & $2e2\mu$ & $4\mu$ & $4\ell$ \\
\hline
ZZ background & 77 \pm 10 & 191 \pm 25 & 119 \pm 15 & 387 \pm 31 \\
Z + X background & 7.4 \pm 1.5 & 11.5 \pm 2.9 & 3.6 \pm 1.5 & 22.6 \pm 3.6 \\
All backgrounds & 85 \pm 11 & 202 \pm 25 & 123 \pm 15 & 410 \pm 31 \\
m_{H} = 500$ GeV & 5.2 \pm 0.6 & 12.2 \pm 1.4 & 7.1 \pm 0.8 & 24.5 \pm 1.7 \\
m_{H} = 800$ GeV & 0.7 \pm 0.1 & 1.6 \pm 0.2 & 0.9 \pm 0.1 & 3.1 \pm 0.2 \\
Observed & 89 & 247 & 134 & 470 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{The number of observed candidate events compared to the mean expected background and signal rates for each final state. Uncertainties include statistical and systematic sources. The results are integrated over the mass range from 121.5 to 130.5 GeV and for 7 and 8 TeV data combined.}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Channel & $4e$ & $2e2\mu$ & $4\mu$ & $4\ell$ \\
\hline
ZZ background & 1.1 \pm 0.1 & 3.2 \pm 0.2 & 2.5 \pm 0.2 & 6.8 \pm 0.3 \\
Z + X background & 0.8 \pm 0.2 & 1.3 \pm 0.3 & 0.4 \pm 0.2 & 2.6 \pm 0.4 \\
All backgrounds & 1.9 \pm 0.2 & 4.6 \pm 0.4 & 2.9 \pm 0.2 & 9.4 \pm 0.5 \\
m_{H} = 125$ GeV & 3.0 \pm 0.4 & 7.9 \pm 1.0 & 6.4 \pm 0.7 & 17.3 \pm 1.3 \\
m_{H} = 126$ GeV & 3.4 \pm 0.5 & 9.0 \pm 1.1 & 7.2 \pm 0.8 & 19.6 \pm 1.5 \\
Observed & 4 & 13 & 8 & 25 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{The number of observed candidate events compared to the mean expected background and signal rates for each final state. Uncertainties include statistical and systematic sources. The results are integrated over the mass range from 121.5 to 130.5 GeV and for 7 and 8 TeV data combined.}
\begin{tabular}{|c|c|c|c|c|}
\hline
Category & 0/1-jet & Dijet \\
\hline
ZZ background & 6.4 \pm 0.3 & 0.38 \pm 0.02 \\
Z + X background & 2.0 \pm 0.3 & 0.5 \pm 0.1 \\
All backgrounds & 8.5 \pm 0.5 & 0.9 \pm 0.1 \\
$ggH$ & 15.4 \pm 1.2 & 1.6 \pm 0.3 \\
$tH$ & \ldots & 0.08 \pm 0.01 \\
VBF & 0.70 \pm 0.03 & 0.87 \pm 0.07 \\
WH & 0.28 \pm 0.01 & 0.21 \pm 0.01 \\
ZH & 0.21 \pm 0.01 & 0.16 \pm 0.01 \\
All signal, $m_{H} = 126$ GeV & 16.6 \pm 1.3 & 3.0 \pm 0.4 \\
Observed & 20 & 5 \\
\hline
\end{tabular}
\end{table}
Data

121.5 and 130.5 GeV.

FIG. 11 (color online). Distribution of the four-lepton reconstructed invariant masses of the $Z_1$ and $Z_2$ in a $m_{4\ell}$ range between 121.5 and 130.5 GeV.

The distributions of the $\mathcal{D}_{\text{bkg}}^{\text{kin}}$ versus $m_{4\ell}$ are shown for the selected events and compared to the SM background expectation in Fig. 12. The distribution of events in the $(m_{4\ell}, \mathcal{D}_{\text{bkg}}^{\text{kin}})$ plane agrees well with the SM background expectation in the high-mass range [Fig. 12 (bottom)], while discrepancies in the two-dimensional plane are observed in the low-mass range $110 < m_{4\ell} < 180$ GeV [Fig. 12 (top)], indicative of the presence of a signal. Figure 13 (top) shows the same data points as in Fig. 12 (top), but compared with the expected distribution from SM backgrounds plus the contribution of a Higgs boson with $m_H = 126$ GeV. A signal-like clustering of events is apparent at high values of $\mathcal{D}_{\text{bkg}}^{\text{kin}}$ and for $m_{4\ell} \approx 126$ GeV. Figure 13 (bottom) shows the distribution of the kinematic discriminant $\mathcal{D}_{\text{bkg}}^{\text{kin}}$ in the mass region $121.5 < m_{4\ell} < 130.5$ GeV.

The distribution of the transverse momentum of the $4\ell$ system in the 0/1-jet category and its joint distribution with $m_{4\ell}$ are shown in Fig. 14. The $p_T$ spectrum shows good agreement with a SM Higgs boson hypothesis with $m_H = 126$ GeV in the 0/1-jet category with few events having $p_T > 60$ GeV, where VBF and VH production are relatively more relevant. In order to compare the $p_T$ spectrum in data with the SM Higgs boson distribution more quantitatively, a background subtraction using the $\mathcal{P}$lot weighting technique [135] is performed. The event weights, related to the probability for each event to be signal-like or background-like, are computed according to the one-dimensional likelihood based on the $m_{4\ell}$ distribution, which shows a small correlation with the four-lepton $p_T^{4\ell}$. The weighted distribution has the property that it corresponds to the signal-only distribution and is normalized to the fitted signal yield. The background-subtracted weighted $p_T^{4\ell}$ distribution is shown in Fig. 15.

The distribution of the production mechanism discriminant in the dijet category and its joint distribution with $m_{4\ell}$ are shown in Fig. 16. Good agreement is found with the expectation from simulation, which predicts a negligible background and a fraction of 42% of the signal events.
arising from vector-boson-induced production (VBF and VH). No events with a high rank of the $D_{\text{jet}}$ ($D_{\text{jet}} > 0.5$) discriminant are observed.

**XII. HIGGS BOSON PROPERTIES MEASUREMENT**

In this section, the fit models used to perform the measurements in the $H \rightarrow ZZ \rightarrow 4\ell$ channel, based on the observables defined in the previous sections, are presented. Then, the systematic uncertainties effects considered in the fits for both assessing the presence of a signal and performing the measurement of different properties are described.

**A. Multidimensional likelihoods**

The properties of interest to be measured in this analysis, such as the signal and background yields, the mass and
FIG. 14 (color online). (top) Distribution of $p_T^{4\ell}$ versus $m_{4\ell}$ in the low-mass-range 0/1-jet category with colors shown for the expected relative density in linear scale (in arbitrary units) of background plus the Higgs boson signal for $m_H = 126$ GeV. No events are observed for $p_T > 150$ GeV. The points show the data, and horizontal bars represent the measured mass uncertainties. (bottom) Distribution of $p_T^{4\ell}$ in the 0/1-jet category for events in the mass region $121.5 < m_{4\ell} < 130.5$ GeV. Points with error bars represent the data, shaded histograms represent the backgrounds, and the red histograms represent the signal expectation, broken down by production mechanism. Signal and background histograms are stacked.

width of the resonance, and the spin-parity quantum numbers, are determined with unbinned maximum-likelihood fits performed to the selected events. The fits include probability density functions for five signal components (gluon fusion, VBF, $WH$, $ZH$, and $t\bar{t}H$ productions) and three background processes ($q\bar{q} \rightarrow ZZ$, $gg \rightarrow ZZ$, and $Z + X$). The normalizations of these components and systematic uncertainties are introduced in the fits as nuisance parameters, assuming log-normal $a$ priori probability distributions, and are profiled during the minimization. The shapes of the probability density functions for the event observables are also varied within alternative ones, according to the effect induced by experimental or theoretical systematic uncertainties [30,136]. Depending on the specific result to be extracted, different multidimensional models, using different sets of discriminating variables, are used. The dimension refers to the number of input variables used in the likelihood function. In the cases where one of the discriminants listed in Table II is used, this observable typically combines more than one discriminating variable. Each of these models is outlined below:

(1) For the assessment of exclusion limits as a function of $m_H$, the signal significance, and the measurement of the signal strength, $\mu = \sigma/\sigma_{SM}$, defined as the measured cross section times the branching fraction into $ZZ$, relative to the expectation for the SM Higgs boson, the following 3D likelihood functions are used:

$$L_{3D}^{\mu} = L_{3D}^{0/1\text{-jet}}(m_{4\ell}, \mathcal{D}_{\text{bkg}}^{\text{kin}}, p_T^{4\ell})$$

$$= \mathcal{P}(m_{4\ell}|m_H, \Gamma) \mathcal{P}(\mathcal{D}_{\text{bkg}}^{\text{kin}}|m_{4\ell}) \times \mathcal{P}(p_T^{4\ell}|m_{4\ell}),$$

(12)

$$L_{3D}^{\mu, \text{dijet}}(m_{4\ell}, \mathcal{D}_{\text{bkg}}^{\text{kin}}, \mathcal{D}_{\text{jet}})$$

$$= \mathcal{P}(m_{4\ell}|m_H, \Gamma) \mathcal{P}(\mathcal{D}_{\text{bkg}}^{\text{kin}}|m_{4\ell}) \times \mathcal{P}(\mathcal{D}_{\text{jet}}|m_{4\ell}),$$

(13)
where \( m_H \) and \( \Gamma \) are the mass and the width of the SM Higgs boson. The likelihood \( L_{3D}^\mu \) includes the kinematic discriminant to differentiate the Higgs boson signal from the ZZ background, defined in Eq. (7). As the third dimension of the fit, depending on the category, the production-mode-sensitive discriminant \( p_T^\ell \) of Eq. (12) (0/1-jet category) or the \( \mathcal{D}_{\text{jet}} \) of Eq. (13) (dijet category) is used. These discriminants are defined in Sec. VIII. The template distributions used as probability density functions for \( \mathcal{P}(p_T^\ell|m_4\ell) \) and \( \mathcal{P}(\mathcal{D}_{\text{jet}}|m_4\ell) \) are derived in the same way as for the \( \mathcal{P}(\mathcal{D}_{\text{bkg}}|m_4\ell) \), which is discussed later in this section.

(2) For the measurement of the mass and width of the resonance, we use the following 3D likelihood:

\[
\mathcal{L}_{3D}^{m,\Gamma} \equiv \mathcal{L}_{3D}^{m_4\ell}(m_H, \mathcal{D}_m, \mathcal{D}_{\text{bkg}}) = \mathcal{P}(m_4\ell|m_H, \Gamma, \mathcal{D}_m) \mathcal{P}(\mathcal{D}_m|m_4\ell) \times \mathcal{P}(\mathcal{D}_{\text{bkg}}|m_4\ell).
\]

In this case, the information about the per-event mass uncertainty, \( \mathcal{D}_m \), based on the estimated resolution of the single leptons, as described in Sec. VIII A, is used. The probability density function \( \mathcal{P}(\mathcal{D}_m|m_H) \) is used for the simulated signal, while \( \mathcal{P}(\mathcal{D}_m|m_4\ell) \) is used for backgrounds. The parameterization of the \( \mathcal{P}(\mathcal{D}_m|m_H) \) and \( \mathcal{P}(\mathcal{D}_m|m_4\ell) \) probability density functions is discussed later in Sec. XIII B.

(3) For the spin-parity hypothesis tests, the following two-dimensional (2D) likelihood is used:

\[
\mathcal{L}_{2D}^{J^P} \equiv \mathcal{L}_{2D}^{J^P}(\mathcal{D}_{\text{bkg}}, \mathcal{D}_{\ell})
\]

In this case, as described in Sec. X, the four-lepton invariant mass and the separation of the Higgs boson signal from the ZZ background using angular variables are condensed in a single discriminant, \( \mathcal{D}_{\text{bkg}} \), defined in Eq. (8). The second dimension of the likelihood provides discrimination between the SM Higgs boson \((0^+)\) and the alternative \( J^P \) hypothesis. The discriminant \( \mathcal{D}_{\ell} \) is defined in Eq. (9). In the case of production-independent hypothesis tests, \( \mathcal{D}_{\text{bkg}} \) and \( \mathcal{D}_{\ell} \) are used.

As mentioned in Sec. III, the theoretical line shape is described by the functional form of a relativistic BW function centered at \( m_H \) and with the expected natural width for the SM Higgs boson, \( \Gamma_H \), in the mass region \( m_H < 400 \text{ GeV} \). The BW function is convolved with a double-sided CB function (to account for the core and for the asymmetric non-Gaussian tails of the experimental resolution) to parameterize the reconstructed signal \( m_4\ell \) distributions, \( \mathcal{P}(m_4\ell|m_H, \Gamma) \). The expected four-lepton mass distributions with their parameterizations superimposed for the three final states are shown in Fig. 17 for the SM Higgs boson with \( m_H = 126 \text{ GeV} \). For a SM Higgs boson with mass \( m_H \geq 400 \text{ GeV} \), the total width is much larger than the experimental four-lepton mass resolution, as described in Sec. III. Given the \( m_4\ell \) distribution of the signal in the high-mass (HM) range, the functional form of the theoretical line shape has to be modified as follows:
where the $\Gamma_{HM}$ parameter is left floating in the fit used to determine the signal parameterization. This modified BW function is convolved with a double-sided CB function to account for the experimental resolution as in the low-mass case. In the fit used to determine the $m_{4\ell}$ parameterization for $m_H \geq 400$ GeV, the constraint that the experimental resolution parameter, $\sigma_{\text{CB}}$, must be much smaller than the natural Higgs boson width is imposed. Systematics on the line shape are incorporated by varying the signal weights for the interference effects, as a function of the generated Higgs boson mass, by $\pm 1\sigma$.

The probability distribution $\mathcal{P}(m_{4\ell})$ for the background is parameterized with empirical functions using simulation for the ZZ background and data control regions for the $Z+X$ background.

The correlated three-dimensional likelihood $\mathcal{L}_3^\mu$, defined in Eqs. (12) and (13) for the 0/1-jet and dijet categories, respectively, is described by the one-dimensional (1D) parametric probability distribution $\mathcal{P}(m_{4\ell})$ multiplied by a two-dimensional template distribution of $(m_{4\ell}, \mathcal{D}^{\text{kin}}_{\text{bg}})$, and a two-dimensional $(m_{4\ell}, p_T^Z)$ or $(m_{4\ell}, \mathcal{D}_{\text{jet}})$ template distribution, where $p_T^Z$ is used in the 0/1-jet category and $\mathcal{D}_{\text{jet}}$ is used in the dijet category. The $\mathcal{P}(m_{4\ell}, \mathcal{D}^{\text{kin}}_{\text{bg}})$, $\mathcal{P}(m_{4\ell}, p_T^Z)$, and $\mathcal{P}(m_{4\ell}, \mathcal{D}_{\text{jet}})$ probabilities are normalized to 1 in the second dimension for each bin of $m_{4\ell}$.

For the signal and background, the 2D probability density functions $\mathcal{P}(\mathcal{D}^{\text{kin}}_{\text{bg}}|m_{4\ell})$ are obtained from simulation, for each of the four-lepton final states and two center-of-mass energies. The effect of instrumental uncertainties (lepton reconstruction efficiency and momentum resolution) on the shapes of this parameterization is incorporated using alternative distributions or Gaussian nuisance parameters in the likelihood and is small. For the reducible background, the probability density function is built using the control regions. The reducible background templates are found to be similar to the ones of the $q\bar{q} \rightarrow ZZ$ background. The difference in shapes is taken as a systematic uncertainty in the reducible background templates. The binning used for $\mathcal{P}(\mathcal{D}^{\text{kin}}_{\text{bg}}|m_{4\ell})$ is shown in Figs. 12 (top) and 12 (bottom) for the low- and high-mass regions, respectively.

The template distributions for $\mathcal{P}(p_T^{Z}|m_{4\ell})$ are derived from simulation for both the signal and SM ZZ processes and from control regions for the $Z+X$ background. The Higgs boson $p_T^{Z}$ spectrum for gluon fusion production is obtained by tuning the POWHEG simulation to include contributions up to NNLO and NLLL expectations, including effects from resummation [137–139]. For the $p_T^{Z}$ spectra for VBF production and the ZZ background, POWHEG is used. Several uncertainties are taken into account for the probability density function $\mathcal{P}(p_T^{Z}|m_{4\ell})$: using alternative PDF sets and varying the fixed-order QCD scales produce systematic uncertainties for all the samples. For gluon fusion Higgs boson production, variations of the default scale for NNLL resummation, and of the quark mass effects, are also considered. For the associated production process, the LO spectrum predicted by PYTHIA is used, and the difference due to NLO effects is considered as a systematic uncertainty. For the $q\bar{q} \rightarrow ZZ$ process, a systematic uncertainty is extracted, comparing the $p_T^{Z}$ distribution of the inclusive $Z$-boson production in events simulated with POWHEG and in the data. The binning used for the $\mathcal{P}(p_T^{Z}|m_{4\ell})$ template is shown in Fig. 14 (top) for the low-mass region.

The template distributions for $\mathcal{P}(\mathcal{D}_{\text{jet}}|m_{4\ell})$ are taken from POWHEG simulations for both the signal and SM ZZ processes and from control regions for the $Z+X$ background. Alternative shapes are introduced to account for statistical and systematic uncertainties in these observables. In the dijet category, alternative shapes of $\mathcal{D}_{\text{jet}}$ arise from

![Higgs boson mass, by $\pm 1\sigma$.](http://example.com/higgs_mass.png)

**FIG. 17** (color online). The $H \rightarrow ZZ \rightarrow 4\ell$ invariant mass distribution for $m_H = 126$ GeV in the (left) $4\ell$, (center) $2e2\mu$, and (right) $4\mu$ channels. The distributions are fitted with a double-sided CB function, and the fitted values of the CB width $\sigma_{\text{CB}}$ are indicated. The values of effective resolution, defined as half the smallest width that contains 68.3% of the distribution, are also indicated. The distributions are arbitrarily normalized.
the comparison with different generators and underlying event tunes. The change in the $\mathcal{D}_{\text{jet}}$ shape with variations of the jet energy scale is negligible. The binning used for the $p_T(\mathcal{D}_{\text{jet}}m_{4\ell})$ template is shown in Fig. 16 (top) for the low-mass region.

### B. Systematic uncertainties

Experimental systematic uncertainties in the normalization of the signal and the irreducible background processes are evaluated from data for the trigger, which contributes 1.5%, and for the combined lepton reconstruction, identification, and isolation efficiencies, which vary from 5.5% to 11% in the $4e$ channel, and from 2.9% to 4.3% in the $4\mu$ channel, depending on the considered $m_H$. The theoretical uncertainties in the irreducible background are computed as functions of $m_{4\ell}$, varying both the renormalization and factorization scales and the PDF set following the PDF4LHC recommendations [73,140–143]. Depending on the four-lepton mass range, the theoretical uncertainties for $q\bar{q} \to ZZ$ and $gg \to ZZ$ are 4%–14% and 25%–50%, respectively.

Samples of $Z \to \ell^+\ell^−$, $\Upsilon(nS) \to \ell^+\ell^−$, and $J/\psi \to \ell^+\ell^−$ events are used to set and validate the absolute momentum scale and resolution. For electrons, a $p_T^\ell$ dependence of the momentum scale is observed, but it only marginally affects the four-lepton mass, and the per-electron uncertainty is propagated, accounting for the correlations, to the $4e$ and $2e2\mu$ channels. This dependence is corrected for, but the observed deviation is conservatively used as a systematic uncertainty, resulting in effects of 0.3% and 0.1% on the mass scales of the two channels, respectively. The systematic uncertainty in the muon momentum scale translates into a 0.1% uncertainty in the $4\mu$ mass scale. The effect of the energy resolution uncertainties is taken into account by introducing a 20% uncertainty in the simulated width of the signal mass peak, according to the maximum deviation between data and simulation observed in the $Z \to \ell^+\ell^−$ events, as shown in Fig. 3.

Additional systematic uncertainties arise from the limited statistical precision in the reducible background control regions, as well as from the difference in background composition between the control regions and the sample from which the lepton misidentification probability is derived. As described in Sec. IX, systematic uncertainties of 20%, 25%, and 40% are assigned to the normalization of the reducible background for the $4e$, $2e2\mu$, and $4\mu$ final states, respectively. All reducible background sources are derived from control regions, and the comparison of data with the background expectation in the signal region is independent of the uncertainty in the LHC integrated luminosity of the data sample. The uncertainty in the luminosity measurement (2.2% at 7 TeV and 2.6% at 8 TeV) [144,145] enters the evaluation of the ZZ background and the calculation of the cross-section limit through the normalization of the signal.

Systematic uncertainties in the Higgs boson cross section and branching fraction are taken from Refs. [55,146]. In the 0/1-jet category, an additional systematic uncertainty in the $ZZ$ background normalization comes from the comparison of POWHEG and MADGRAPH. In the dijet category, a 30% normalization uncertainty is taken into account for the $gg \to H + 2 \text{jets}$ signal cross section, while 10% is retained for the VBF production cross section. Table VI shows the summary of the systematic uncertainties in the normalization of the signal and background processes.

Shape uncertainties for both categories are considered, accounting for the lepton scale and resolution variations on the $m_{4\ell}$ line shape, theoretical uncertainties in the $p_T^\ell$ signal and background models, and theoretical and experimental uncertainties (such as the variations on the jet energy scale and resolution) in the $\mathcal{D}_{\text{jet}}$ distribution.

| TABLE VI. Effect of systematic uncertainties on the yields of signal ($m_H = 126$ GeV) and background processes for the 8 TeV data set and the 0/1-jet category. Uncertainties appearing on the same line are 100% correlated, with two exceptions: those related to the missing higher orders are not correlated, and those from the $\alpha_s +$ PDG (gg) in $t\bar{t}H$ are 100% anticorrelated. Uncertainties for the 7 TeV data set are similar. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Source          | Signal ($m_H = 126$ GeV) | Backgrounds     |
|                 | $ggH$            | VBF             | $VH$            | $t\bar{t}H$     | $q\bar{q} \to ZZ$ | $gg \to ZZ$ | $Z + X$         |
| $\alpha_s +$ PDF (gg) | 7.2%            | $\cdots$       | $\cdots$       | $7.8%$          | $\cdots$       | $7.2%$          | $\cdots$       |
| $\alpha_s +$ PDF ($q\bar{q}$) | $\cdots$       | $2.7%$          | $3.5%$          | $\cdots$       | $3.4%$          | $\cdots$       | $\cdots$       |
| Missing higher orders | $7.5%$          | $0.2%$          | $0.4%$, $1.6%$  | $6.6%$          | $2.9%$          | $24%$           | $\cdots$       |
| Signal acceptance | $\cdots$       | $2%$            | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       |
| $BR(H \to ZZ)$ | $\cdots$       | $2%$            | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       |
| Luminosity      | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $2.6%$          | $\cdots$       | $\cdots$       |
| Electron efficiency | $\cdots$       | $10%$ ($4e$), $4.3%$ ($2e2\mu$) | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       |
| Muon efficiency  | $\cdots$       | $4.3%$ ($4\mu$), $2.1%$ ($2e2\mu$) | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       |
| Control region  | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $\cdots$       | $40%$           |
XIII. RESULTS AND INTERPRETATION

The results of the search for a signal consistent with a SM Higgs boson in the \( m_H \) range 110–1000 GeV are described along with the estimation of the significance of the excess observed in the low-mass region. Then, the measurement of the mass of the new boson in the hypothesis of a narrow resonance and limits on its width are reported. For this resonance, the compatibility of the cross section measurement with the SM Higgs boson calculation is given together with constraints on the production mechanisms. Finally, the spin and parity of the boson are tested to check the compatibility with the hypothesis of a \( 0^+ \) resonance as compared with the alternatives, and the measurement of the fraction of a CP-odd contribution to the decay amplitude is reported.

A. Signal significance and exclusion limits

The selected events are split into two subcategories based on the three final states, two data-taking periods (7 and 8 TeV), and two jet categories. These events are examined for 187 hypothetical SM-like Higgs boson masses in a range between 110 and 1000 GeV, where the mass steps are optimized to account for the expected width and resolution [136]. A 3D model, \( \mathcal{L}_{3D}^{0/1\text{-jet}}(m_{4\ell}, \mathcal{D}_\text{bkg}^{\text{kin}}, P_T) \) and \( \mathcal{L}_{3D}^{\text{dijet}}(m_{4\ell}, \mathcal{D}_\text{bkg}^{\text{kin}}, \mathcal{D}_{\text{jett}}) \), defined, respectively, in Eqs. (12) and (13) for the 0/1-jet category and for the dijet category, is used. The statistical approach discussed in Ref. [136] is followed to set exclusion limits and to establish the significance of an excess. The modified frequentist construction CL\text{q} [136,147,148] is adopted as the primary method for reporting limits. As a complementary method to the frequentist construction, a Bayesian approach [149] yields consistent results.

Upper limits on the ratio of the production cross section to the SM expectation are shown in Fig. 18 (top). The results presented in this section make use of asymptotic formulas from Ref. [150]. The SM-like Higgs boson is excluded by the four-lepton channels at the 95% C.L. in a range between 114.5–119.0 GeV and 129.5–832.0 GeV, for an expected exclusion range of 115–740 GeV. The local \( p \) values, representing the significance of a local excess relative to the background expectation, are shown for the full mass range as a function of \( m_H \) in Fig. 18 (bottom). The minimum of the local \( p \) value is reached around \( m_{4\ell} = 125.7 \) GeV, near the mass of the new boson, confirming the result in Ref. [20], and corresponds to a local significance of 6.8\( \sigma \), consistent with the expected sensitivity of 6.7\( \sigma \). As a cross-check, 1D [\( \mathcal{L}_{1D}^{m_{4\ell}} = \mathcal{L}_{1D}(m_{4\ell}) \)] and 2D [\( \mathcal{L}_{2D}^{m_{4\ell}} = \mathcal{L}_{2D}(m_{4\ell}, \mathcal{D}_\text{bkg}^{\text{kin}}) \)] models are also studied, as shown in Figs. 18 (bottom) and 19, resulting in an observed local significance of 5.0\( \sigma \) and 6.9\( \sigma \), for an expectation of 5.6\( \sigma \) and 6.6\( \sigma \), respectively. These results are consistent with the 3D model; however, with a systematically lower expected sensitivity to the signal. No other significant deviations with respect to the expectations is found in the mass range 110–1000 GeV.

The second most significant \( p \)-value minimum is reached around \( m_{4\ell} = 146 \) GeV, with a local significance of 2.7\( \sigma \). This computation does not take into account the look-elsewhere effect [151].

B. Mass and width

In order to measure the mass and width of the new boson precisely and to correctly assign the uncertainties in these measurements, the four-lepton mass uncertainties estimated on a per-event basis, as described in Sec. VIII A, are incorporated into the likelihood. This approach has the largest impact in a context of a low number of events and a wide spread of per-event uncertainties, both of which are present in the \( H \to ZZ \to 4\ell \) analysis. Tests on simulation indicate that, with this approach, the uncertainties in the measured mass and the upper limit on the width of the Higgs boson are expected to improve by about 8\% and 10\%, respectively, with respect to using the average resolution.

The experimental resolution parameter of the double-sided CB function, used to model the \( m_{4\ell} \) line shape, is substituted with the per-event estimation of the mass uncertainty \( \mathcal{D}_m \). The parameters describing the tail of the double-sided CB from simulation are also corrected on a per-event basis.

The likelihood used for the mass and width measurements is defined in Eq. (14). By construction, this likelihood neglects potential correlations between \( \mathcal{D}_\text{bkg}^{\text{kin}} \) and \( \mathcal{D}_m \). Simulated Higgs boson and \( q\bar{q} \to ZZ \) events show no evident correlations between these two observables. The probability density functions \( \mathcal{P}(\mathcal{D}_{m} | m_H) \) of the per-event uncertainty distributions for the signal are obtained from simulation. The probability density functions \( \mathcal{P}(\mathcal{D}_{m} | m_{4\ell}) \) for the ZZ background are obtained from simulation and are cross-checked with data in control regions dominated by the ZZ background events \( (m_{4\ell} > 180 \text{ GeV}) \) and \( Z \to 4\ell \) events \( (80 < m_{4\ell} < 100 \text{ GeV}) \) [152], as shown in Fig. 6 (bottom). The \( \mathcal{P}(\mathcal{D}_{m} | m_{4\ell}) \) for the reducible background is obtained from the control regions in the data with the same technique used to derive the \( m_{4\ell} \) line shapes. The \( \mathcal{P}(\mathcal{D}_{m} | m_{4\ell}) \) is a conditional probability distribution function, where for all the channels and both signal and background components, the probability density functions \( \mathcal{P}(\mathcal{D}_m) \) are parameterized as a sum of a Landau [108] and a Gaussian function.

Figure 20 (top) shows the profile likelihood scan versus the SM Higgs boson mass performed under the assumption that its width is much smaller than the detector resolution, for the single channels, combining 7 and 8 TeV data, and for the combination of all the channels. The Higgs boson cross section is left floating in the fit. To decompose the total mass uncertainty into statistical and systematic components, a fit with all nuisance parameters fixed at their
The best-fit values are performed. The mass uncertainty obtained in this way is purely statistical. The systematic uncertainties account for an effect on the mass scale of the lepton momentum scale and resolution, shape systematics in the background-only model are also shown with green and yellow bands, respectively. (bottom) Significance of the local excess with respect to the SM background expectation as a function of the Higgs boson mass in the full mass range 110–1000 GeV. Results are shown for the 1D fit ($L_{1D}^m$), the 2D fit ($L_{2D}^m$), and the reference 3D fit ($L_{3D}^m$).

Figure 20 (top) also shows likelihood scans separately for the $4\ell$, $2e2\mu$, and $4\mu$ final states when using the 3D model $L_{3D}^{m,\Gamma}$ of Eq. (14). The measurements in the three

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FIG. 19 (color online). Significance of the local excess with respect to the SM background expectation as a function of the Higgs boson mass in the full mass range 110–1000 GeV. Results are shown for the full data sample in the low-mass region only.

The final states are statistically compatible. The best-fit values for each subchannel are also shown in Table VII. The dominant contribution to the systematic uncertainty is the limited knowledge of the lepton momentum scale.

Two more mass measurements are performed with a reduced level of information, by dropping the $\mathcal{P}(D_{bkg}^{m} | m_{4\ell})$ term of the likelihood in Eq. (14), resulting in a 2D model, $\mathcal{L}_{2D}^{m,\Gamma} = \mathcal{L}_{2D}^{m} (m_{4\ell}, \Gamma_m)$, or by performing only a mass shape fit and assuming the average mass resolution is applicable for each channel, resulting in a 1D model, $\mathcal{L}_{1D}^{m,\Gamma} = \mathcal{L}_{1D}^{m} (m_{4\ell})$. The measured central value is the same in all three cases, with an increasing uncertainty, due to the reduced information available to the fit in the case of 2D or 1D models. Figure 20 (right) shows the likelihood scans for the combination of all the final states separately for the $\mathcal{L}_{1D}^{m,\Gamma}$, $\mathcal{L}_{1D}^{m}$, and $\mathcal{L}_{3D}^{m}$ models.

The mass distribution for the $Z \rightarrow 4\ell$ decay exhibits a pronounced resonant peak at $m_{4\ell} = m_Z$ close to the new boson (80 $< m_{4\ell} < 100$ GeV). Hence, the $Z \rightarrow 4\ell$ peak can be used as validation of the measurement of the mass of the new boson using the same techniques as for the Higgs boson. The mass of the reconstructed $Z$ boson in $Z \rightarrow 4\ell$ decays, with the assumption of the Particle Data Group (PDG) [149] value for the $Z$-boson natural width, is consistent in each subchannel. The measured value for the combination of all the $Z \rightarrow 4\ell$ final states is $m_Z = 91.1$ GeV, compatible with the PDG value (91.1876 ± 0.0021 GeV) within the total estimated uncertainty of 0.4 GeV [149].

Figure 21 shows the scan of the 3D likelihood versus the width of the SM-like Higgs boson with an arbitrary width. In this scan, the mass and the signal strength $\mu$ are profiled, as all other nuisance parameters. This shows that the data
FIG. 20 (color online). (top) Scan of the negative log likelihood $-2\Delta \ln L$ versus the SM Higgs boson mass $m_H$, for each of the three channels separately and the combination of the three, where the dashed line represents the scan including only statistical uncertainties when using the 3D model. (bottom) Scan of $-2\Delta \ln L$ versus $m_H$ for the combination of the three channels, and using the 1D fit ($\mathcal{L}_{m1}^{\Gamma}$), 2D fit ($\mathcal{L}_{m2}^{\Gamma}$), and 3D fit ($\mathcal{L}_{m3}^{\Gamma}$). The horizontal lines at $-2\Delta \ln L = 1$ and 3.84 represent the 68% and 95% C.L.’s, respectively.

are compatible with a narrow-width resonance. The measured width is $\Gamma_H = 0.93^{+0.26}_{-0.23}$ (stat) $^{+0.13}_{-0.09}$ (syst) at the best-fit mass ($m_H = 125.6$ GeV) with the models of Eqs. (12) and (13) for the 0/1-jet category and the dijet category, respectively. The median expected signal strength is $\mu = 1.00^{+0.31}_{-0.26}$, for which the total uncertainty agrees with the observed one. The result is $0.83^{+0.31}_{-0.25}$ in the 0/1-jet category and $1.45^{+0.89}_{-0.62}$ in the dijet category. The best-fit values are shown in Fig. 22 (top). For each category, the signal strength is consistent with SM expectations within the uncertainties, which are dominated by the statistical ones with the current data set.

The categorization according to jet multiplicity and the inclusion of VBF-sensitive variables in the likelihood, like $p_T^{4\ell}$ and $D_{\text{jet}}$, used to measure the cross section in the inclusive category, are also used to disentangle the production mechanisms of the observed new state. The production mechanisms are split into two families depending on whether the production is through couplings to fermions (gluon fusion, $t\bar{t}H$) or vector bosons (VBF, $VH$).

For $m_H = 126$ GeV, about 55% of the VBF events are expected to be included in the dijet category, while only 8%...

### TABLE VII. Best-fit values for the mass of the Higgs boson candidate, measured in the $4\ell$, $\ell = e, \mu$ final states using a $\mathcal{L}_{3D}$ model. For the combination of all the final states $H \rightarrow 4\ell$, the separate contributions of the statistical and systematic uncertainty to the total one are given.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Measured mass (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4e$</td>
<td>$126.2^{+1.5}_{-1.8}$</td>
</tr>
<tr>
<td>$2e2\mu$</td>
<td>$126.3^{+0.9}_{-0.7}$</td>
</tr>
<tr>
<td>$4\mu$</td>
<td>$125.1^{+1.0}_{-0.6}$</td>
</tr>
<tr>
<td>$4\ell$</td>
<td>$125.6 \pm 0.4(\text{stat}) \pm 0.2(\text{syst})$</td>
</tr>
</tbody>
</table>

FIG. 21 (color online). Scan of the average expected and observed negative log likelihood $-2\Delta \ln L$ versus the tested SM Higgs boson width $\Gamma_H$ obtained with the 3D fit ($\mathcal{L}_{3D}^{\Gamma}$). The horizontal lines at $-2\Delta \ln L = 1$ and 3.84 represent the 68% and 95% C.L.’s, respectively.
of the gluon fusion events are included in the dijet category. As shown in Table V, a fraction of 43% of WH and ZH production contributes to the dijet category. Events that contribute are those in which the vector boson decays hadronically.

Two signal-strength modifiers ($\mu_{ggH, t\bar{t}}$ and $\mu_{VBF, VH}$) are introduced as scale factors for the fermion and vector-boson induced contribution to the expected SM cross section. A two-dimensional fit is performed for the two signal-strength modifiers assuming a mass hypothesis of $m_H = 125.6$ GeV. The likelihood is profiled for all nuisance parameters and 68% and 95% C.L. contours in the ($\mu_{ggH, t\bar{t}}, \mu_{VBF, VH}$) plane are obtained. Figure 22 (bottom) shows the result of the fit leading to the measurements of $\mu_{ggH, t\bar{t}} = 0.80^{+0.46}_{-0.36}$ and $\mu_{VBF, VH} = 1.7^{+2.2}_{-1.1}$. The measured values are consistent with the expectations for the SM Higgs boson, ($\mu_{ggH, t\bar{t}}, \mu_{VBF, VH}$) = (1, 1). With the current limited statistics, we cannot establish yet the presence of VBF and VH production, since $\mu_{VBF, VH} = 0$ is also compatible with the data. Since the decay (into ZZ) is vector-boson mediated, it is necessary that such a coupling must exist in the production side and that the SM VBF and SM VH production mechanisms must be present. The fitted value of $\mu_{VBF, VH}$ larger than 1 is driven partly by the hard $p_T$ spectrum of the events observed in data when

![Graph 1](image1)

**FIG. 22** (color online). (top) Values of $\mu$ for the two categories. The vertical line shows the combined $\mu$ together with its associated $\pm 1\sigma$ uncertainties, shown as a green band. The horizontal bars indicate the $\pm 1\sigma$ uncertainties in $\mu$ for the different categories. The uncertainties include both statistical and systematic sources of uncertainty. (bottom) Likelihood contours on the signal-strength modifiers associated with fermions ($\mu_{ggH, t\bar{t}}$) and vector bosons ($\mu_{VBF, VH}$) shown at a 68% and 95% C.L.

![Graph 2](image2)

**FIG. 23** (color online). Distribution of $\mathcal{D}_{bkg}$ (top) and $\mathcal{D}_{dec}$ for the production-independent scenario (bottom) in data and MC expectations for the background and for a signal resonance consistent with the SM Higgs boson with $m_\gamma = 125.6$ GeV.
D. Spin and parity

To measure the spin and parity properties of the new boson, the methodology discussed in Sec. X is followed. In addition to the models tested in Ref. [31] ($0^-$ and $gg \rightarrow 2^+_m$), seven additional models are examined: $0^+_m$, $q\bar{q} \rightarrow 1^-$, $q\bar{q} \rightarrow 2^+_m$, $gg \rightarrow 2^+_m$, $gg \rightarrow 2^+_m$, $gg \rightarrow 2^+_m$. The discrimination is based on 2D probability density functions ($D_{bkg}$, $D_{jet}$), where the kinematic discriminants $D_{bkg}$ and $D_{jet}$ are defined by Eqs. (8) and (9). The $1^\pm$ and $2^+_m$ signal hypotheses are also tested by relying only on their decay information, i.e. in a production-independent way, using pairs of kinematic discriminants ($D_{bkg}$, $D_{jet}$), defined by Eqs. (10) and (11). All models and discriminants, discussed in Sec. X, are listed in Table II.

For spin and parity studies, the event categorization based on jets is not used in order to reduce the dependence on the production mechanisms. Consequently, the VBF discriminants, $P_{1d}$ and $D_{jet}$, are not used, resulting in the $Z_{2D}$ model defined in Eq. (15). Events in the mass range $106 < m_{4f} < 141$ GeV are used to perform these studies. The Higgs boson mass is assumed to be $m_{0^-} = 125.6$ GeV. The 2D probability density functions for signal and background, $P(D_{j^P}, D_{bkg})$ in Eq. (15), are obtained as 2D templates from simulation for the signal and irreducible background, and from control regions for the reducible backgrounds.

Figure 23 shows expected and observed distributions for the discriminants $D_{bkg}$ and $D_{dec}$. The distributions are very similar for the SM and all alternative signal hypotheses but differ significantly from the background. Figures 24 and 25 show distributions for the $D_{j^P}$ observables for all tested signal hypotheses. Only one alternative hypothesis is shown on each figure. The distributions show events with $D_{bkg} > 0.5$ to enhance the fraction of signal events for illustration purposes only. For the hypothesis tests, the full range of the discriminant is used.

The alternative signal models are defined by the tensor structure of couplings; however, the absolute values of couplings, and hence, the expected event yields are not uniquely defined. The cross sections for alternative signal hypotheses are left floating in the fit. The same approach is taken for the SM Higgs boson hypothesis: i.e., the overall SM Higgs boson signal strength $\mu$ is the best-fit value as it comes out from data. This way, the overall signal event yield is not a part of the discrimination between alternative hypotheses. Consequently, for pairwise tests of alternative signal hypotheses with respect to the SM Higgs boson, the

![Figure 24](color online). Distributions of $D_{j^P}$ with a requirement $D_{bkg} > 0.5$. Distributions in data (points with error bars) and expectations for background and signal are shown: six alternative $J^P$ hypotheses are shown. $J^P = 0^-$ (upper left), $0^+_m$ (upper middle), $1^-(q\bar{q})$ (upper right), $1^-$ (lower left), $1^+(q\bar{q})$ (lower middle), $1^+$ (lower right).
test statistic is defined using the ratio of signal plus background likelihoods for two signal hypotheses \( q = -2\ln(\mathcal{L}_{\text{H}}/\mathcal{L}_{\text{B}}) \). The expected distribution of \( q \) for the pseudoscalar hypothesis (blue histogram) and the SM Higgs boson (orange histogram) are shown in Fig. 26 (top). Similar distributions for the test statistic \( q \) are obtained for the other alternative hypotheses considered. The pseudoexperiments are generated using the nuisance parameters fitted in data.

To quantify the consistency of the observed test statistics \( q_{\text{obs}} \) with respect to the SM Higgs boson hypothesis (0\(^{+}\)), we assess the probability \( p = P(q \leq q_{\text{obs}} | J^{P} + \text{bkg}) \) and convert it into a number of standard deviations \( Z \) via the Gaussian one-sided tail integral:

\[
p = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x^2}{2} \right) \, dx.
\]

Similarly, the consistency of the observed data with alternative signal hypotheses (\( J^{P} \)) is assessed from \( P(q \geq q_{\text{obs}} | J^{P} + \text{bkg}) \). The \( \text{CL}_{s} \) criterion, defined as \( \text{CL}_{s} = P(q \geq q_{\text{obs}} | J^{P} + \text{bkg}) / P(q \geq q_{\text{obs}} | 0^{+} + \text{bkg}) < \alpha \), is used for the final inference of whether a particular alternative signal hypotheses is excluded or not with a given confidence level (1 - \( \alpha \)).

The expected separations between alternative signal hypotheses are quoted for two cases. In the first case, the expected SM Higgs boson signal strength and the alternative signal cross sections are equal to the ones obtained in the fit of the data. The second case assumes the nominal SM Higgs boson signal strength (\( \mu = 1 \), as indicated in parentheses for expectations quoted in Table VIII), while the cross sections for the alternative signal hypotheses are taken to be the same as for the SM Higgs boson (the 2\( e\mu \) channel is taken as a reference). Since the observed signal strength is very close to unity, the two results for the expected separations are also similar. The observed values of the test statistic in the case of the SM Higgs boson versus a pseudoscalar boson are shown with red arrows in Fig. 26 (top). Results obtained from the test statistic distributions are summarized in Table VIII and in Fig. 27.

The observed value of the test statistic is larger than the median expected for the SM Higgs boson. This happens for many distributions because of strong kinematic correlations between different signal hypotheses, most prominently seen in the \( m_{Z_{2}} \) distributions. The pseudoscalar (0\(^{+}\)) and all spin-1 hypotheses tested are excluded at the 99.9\% or higher C.L. All tested spin-2 models are excluded at the
95\% or higher C.L. The 0^h hypothesis is disfavored, with a CLs value of 4.5\%.

In addition to testing pure JP states against the SM Higgs boson hypothesis, a measurement for a possible mixture of CP-even and CP-odd states or other effects leading to anomalous couplings in the H \rightarrow ZZ \rightarrow 2e2\mu decay amplitude in Eq. (6) is performed. The D_0 discriminant is designed for the discrimination between the third and the first amplitude contributions in Eq. (6) when the phase f_{a3} between a_3 and a_1 couplings is not determined from the data [48].

For example, even when restricting the coupling ratios to be real, there remains an ambiguity where f_{a3} = 0 or \pi. The interference between the two terms (a_1 and a_3) is found to have a negligible effect on the discriminant distribution or the overall yield of events. The parameter f_{a3} is defined as

$$f_{a3} = \frac{|a_3|^2 \sigma_3}{|a_1|^2 \sigma_1 + |a_2|^2 \sigma_2 + |a_3|^2 \sigma_3},$$

where \sigma_i is the effective cross section H \rightarrow ZZ \rightarrow 2e2\mu corresponding to a_i = 1, a_{j\neq i} = 0. The 4e and 4\mu final

![Figure 26](image_url)

**Figure 26** (color online). (top) Distribution of the test statistic \( q = -2\ln(D_0 - D_0^0) \) of the pseudoscalar boson hypothesis tested against the SM Higgs boson hypothesis. Distributions for the SM Higgs boson are represented by the yellow histogram, and those for the alternative JP hypotheses are represented by the blue histogram. The arrow indicates the observed value. (bottom) Average expected and observed distribution of \(-2\Delta \ln L\) as a function of \( f_{a3}\). The horizontal lines at \(-2\Delta \ln L = 1\) and 3.84 represent the 68\% and 95\% C.L.’s, respectively.

![Figure 27](image_url)

**Figure 27** (color online). Summary of the expected and observed values for the test-statistic q distributions for the twelve alternative hypotheses tested with respect to the SM Higgs boson. The orange (blue) bands represent 1\sigma, 2\sigma, and 3\sigma around the median expected value for the SM Higgs boson hypothesis (alternative hypothesis). The black point represents the observed value.
states may lead to either constructive or destructive interference of identical leptons, and therefore to slightly different cross-section ratios. When testing the CP-odd contribution, the second term in the amplitude is assumed to be zero ($\alpha_2 = 0$). The measured value of $f_{a3}$ can be used to extract the coupling constants in any parameterization. For example, following Eq. (6), the couplings are

$$|\alpha_3| = \sqrt{\frac{f_{a3}}{(1 - f_{a3})}} \times \sqrt{\sigma_1}, \quad (19)$$

where $\sigma_1/\sigma_3 = 6.36$ for a boson with mass 125.6 GeV. The $f_{a3}$ parameter does not define the mixture of parity-even and parity-odd states, because it would also depend on the relative strength of their couplings to vector bosons.

Figure 26 (bottom) shows a likelihood scan of $-2 \ln \mathcal{L}$, where the likelihood for the event $i$, $\mathcal{L}^i = \mathcal{L}_{0a} \propto (1 - f_{a3})^{\mathcal{L}^{0}}_{2D} + f_{a3}^{\mathcal{L}^{0}}_{2D}$. The normalization due to the acceptance is accounted for in $\mathcal{L}^{i}_{2D}$, defined in Eq. (15), and the normalization of the likelihood $\mathcal{L}_{0a}$ depends on $f_{a3}$. From the likelihood scan as a function of $f_{a3}$, the fraction of a CP-odd amplitude contribution to the cross section $f_{a3} = 0.00^{+0.17}_{-0.00}$, and a limit $f_{a3} < 0.51$ at the 95% C.L., are inferred. The limit on $f_{a3}$ can be converted into a limit on amplitude constants using the convention of Eq. (6): $|\alpha_3/\alpha_1| < 2.6$ at the 95% C.L. The statistical coverage of the results obtained in the likelihood scan has also been tested, with the Feldman-Cousins approach [153] yielding a consistent result.

**XIV. SUMMARY**

The observation and the measurements of the properties of a Higgs boson candidate in the four-lepton decay channel have been presented. The four-lepton invariant mass distributions are presented for $m_{4\ell} > 70$ GeV using data samples corresponding to integrated luminosities of 5.1 fb$^{-1}$ at $\sqrt{s} = 7$ TeV and 19.7 fb$^{-1}$ at $\sqrt{s} = 8$ TeV. For the measurements, the following experimental observables are employed: the measured four-lepton mass, the mass uncertainty, kinematic discriminants, and information sensitive to the production mechanism, such as associated dijet characteristics and transverse momentum of the four-lepton system.

The observation of the new boson [20,21,31] is confirmed in the 4\ell final state, with a local significance of 6.8 standard deviations above the expected background. Upper limits at the 95% C.L. exclude the SM-like Higgs boson in the mass ranges 114.5–119.0 GeV and 129.5–832.0 GeV, for an expected exclusion range for the background-only hypothesis of 115–740 GeV. The measured mass of the new boson is $125.6 \pm 0.4$ (stat) $\pm 0.2$ (syst) GeV. The measured width of this resonance is smaller than 3.4 GeV at the 95% C.L. The production cross section of the new boson times the branching fraction to four leptons is measured to be $0.93^{+0.26}_{-0.23}$ (stat) $^{+0.13}_{-0.09}$ (syst) times that predicted by the standard model. Those associated with fermions and vector bosons are $\mu_{ggH, tH} = 0.80^{+0.36}_{-0.36}$, and $\mu_{BF, VH} = 1.7^{+2.2}_{-2.1}$, respectively, consistent with the SM expectations.

The spin parity of the boson is studied, and the observation is consistent with the pure scalar hypothesis when compared to several other spin-parity hypotheses. The fraction of a CP-odd contribution to the decay amplitude, expressed through the fraction $f_{a3}$ of the corresponding decay rate, is $f_{a3} = 0.00^{+0.17}_{-0.00}$, and thus consistent with the expectation for the SM Higgs boson. The hypotheses of a pseudoscalar and all tested spin-1 boson hypotheses are excluded at the 99% C.L. or higher. All tested spin-2 boson hypotheses are excluded at the 95% C.L. or higher.

The production and decay properties of the observed new boson in the four-lepton final state are consistent, within their uncertainties, with the expectations for the SM Higgs boson.

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[18] ALEPH, CDF, D0, DELPHI, L3, OPAL, and SLD Collaborations, the LEP Electroweak Working Group, the Tevatron Electroweak Working Group, and the SLD Electroweak and Heavy Flavour Groups, CERN Report No. PH-EP-2010-095, 2010; At this time, the most up-to-date Higgs boson mass constraints come from lepewwg.web.cern.ch/LEPEWWG/plots/winter2012, arXiv:1012.2367.

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57c CSFNSM, Catania, Italy
58a INFN Sezione di Firenze, Firenze, Italy
58b Università di Firenze, Firenze, Italy
59 INFN Laboratori Nazionali di Frascati, Frascati, Italy
60a INFN Sezione di Genova, Genova, Italy
60b Università di Genova, Genova, Italy
61a INFN Sezione di Milano-Bicocca, Milano, Italy
61b Università di Milano-Bicocca, Milano, Italy
62a INFN Sezione di Napoli, Napoli, Italy
62b Università di Napoli ‘Federico II’, Napoli, Italy
62c Università della Basilicata (Potenza), Napoli, Italy
62d Università G. Marconi (Roma), Napoli, Italy
62e INFN Sezione di Padova, Padova, Italy
62f Università di Padova, Padova, Italy
62g Università di Trento (Trento), Padova, Italy
64a INFN Sezione di Pavia, Pavia, Italy
64b Università di Pavia, Pavia, Italy
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66a INFN Sezione di Pisa, Pisa, Italy
66b Università di Pisa, Pisa, Italy
66c Scuola Normale Superiore di Pisa, Pisa, Italy
67a INFN Sezione di Roma, Roma, Italy
67b Università di Roma, Roma, Italy
68a INFN Sezione di Torino, Torino, Italy
68b Università di Torino, Torino, Italy
68c Università del Piemonte Orientale (Novara), Torino, Italy
69a INFN Sezione di Trieste, Trieste, Italy
69b Università di Trieste, Trieste, Italy
70 Kangwon National University, Chunchon, Korea
71 Kyungpook National University, Daegu, Korea
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75 Sungkyunkwan University, Suwon, Korea
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95 University of Belgrade, Faculty of Physics and Vinca Institute of Nuclear Sciences, Belgrade, Serbia
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97 Universidad Autónoma de Madrid, Madrid, Spain
98 Universidad de Oviedo, Oviedo, Spain
99 Instituto de Física de Cantabria (IFCA), CSIC-Universidad de Cantabria, Santander, Spain
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