Semi-nonparametric Spline Modifications to the Cornwell-Schmidt-Sickles Estimator: An Analysis of U.S. Banking Productivity

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This paper modifies the Cornwell, Schmidt and Sickles (CSS) (1990) time-varying specification of technical efficiency to allow for switching patterns in temporal changes, which may occur more than once during the sample period. For this purpose, we move from the (second-order) polynomial specification of the standard CSS to a spline function set up, while keeping CSS's flexibility regarding the cross section dimension. The spline function specification of the temporal pattern of technical efficiency can accommodate more than one turning point and thus can be non-monotonic. This allows the modeler to account for firm or individual efficiency gains that can occur relatively quickly, for example, changes related to regulation or policy changes, as well as those related to ownership/organization changes (e.g., merger or acquisitions).

1. Introduction

One of the interesting aspects of performance evaluation analysis is dynamic benchmarking, namely identifying and estimating the temporal patterns of efficiency per se and its role as an inherent component of productivity growth. In many cases, the effect of efficiency changes has been found as important as technical change in determining the evolution of productivity growth. One such example is the case of Japan during the period 1979-1988 as illustrated in Fare *et al*. (1994), where changes in technical efficiency and the catching-up process that resulted was found to be the most important source of growth for aggregate labor productivity.

Some will argue, however, that the accuracy of such empirical findings may depend on how time-varying technical efficiency has been modeled. Usually, a linear time trend is used to capture the time pattern of efficiency changes (e.g. Kumbhakar, 1990; Battese and Coelli, 1992; Cuesta, 2000). This is a rather restrictive formulation of time-varying efficiency as its changes over time are given by a constant rate. It is also common to assume that the time pattern of technical efficiency is uniform for all producing units in the sample or the firms in the industry. Even though the assumption of a common temporal pattern is restrictive, it may not be an unreasonable approximation for putty-clay type industries.

On the other hand, several flexible specifications have been proposed in the literature for modeling time-varying efficiency, such as those of Cornwell, Schmidt and Sickles (1990), Kumbhakar (1990), Lee and Schmidt (1993), Lee (2006), Karagiannis and Tzouvelekas (2007), Ahn, Lee and Schmidt (2007, 2013), and Kneip, Sickles, and Song (2012). Among them the Cornwell, Schmidt and Sickles (1990) (CSS) is still one of the more flexible specifications of temporal variation in efficiency, as it is relatively easy to implement and can accommodate settings in which technical inefficiency and the inputs or other regressors (i.e., environmental variables) can be correlated.[2] Although the specification used in the CSS analysis allows for firm-specific patterns of time-varying efficiency by means of a quadratic

[2] Much more general mixed types of so called "environmental" variables can also be controlled for with the CSS model, such as additive effects that impact the slope coefficients of the "environmental" variables. In these specifications of the CSS estimator the "environmental" variables impact the frontier as well as the level of efficiency, unlike most two-step models wherein there is separability between the frontier and efficiencies of the cross-section units (Wang, 2002; Wang and Schmidt, 2002; Simar and Wilson, 2007).

function of time, such a specification was chosen in their analysis for purposes of illustration and due to the erratic behavior of higher order terms with their airline data.

The aim of this paper is to generalize the CSS time-varying specification to allow for more general patterns of temporal changes, which in turn will allow for multiple turning points. For this purpose, we specify a spline function setup, while retaining CSS's flexibility in the cross-section dimension. This generalization puts more emphasis on firm heterogeneity in terms of growth rates rather than level differences in efficiency. The spline function specification of the temporal pattern of technical efficiency can also be non-monotonic due to periods in which firms may face radical regulation or policy changes, as well as shocks related to changes in ownership/organization (e.g., merger or acquisitions).

In the next section we briefly discuss the general productivity model we employ and decompositions into technology and efficiency change that can be made with it. We then briefly outline the CSS estimator that has been used in many applications to measure such important aspects of economic growth and point out its generality in addressing environmental effects, an attribute of the estimator that has apparently been missed by many researchers. In Section 3 we present a spline modification of the CSS estimator and discus a semi-parametric method for its estimation. Section 4 discusses the banking data and the empirical results from the illustration we use to introduce the new estimator, while section 5 concludes.

2. Econometric Specification of the Productivity Model and the CSS Estimator

Regression-based approaches to decompose productivity growth into technical change and efficiency change components can be based on a rather straightforward generic model of production using a multiple output / multiple input technology specified by means of the input or the output distance function (Caves, Christensen and Diewert, 1982; Coelli and Perelman, 1999; Sickles, Good, and Getachew, 2002). Distance functions that are linear in parameters, such as the linear in logs Cobb-Douglas or translog or linear in levels generalized-Leontief or quadratic, constitute the predominant functional forms used in productivity studies. Treatments for unobserved technical efficiency (heterogeneity) can be motivated with the following classical model for a single output technology estimated with panel data assuming unobserved firm effects:

$$y_{it} = x_{it}\beta + \eta_i(t) + v_{it}$$

where $\eta_i(t)$ represents the unit specific fixed effect that may be time varying, x_{it} is a vector of (log) regressors, some of which may be endogenous and correlated with the error v_{it} and/or the effects $\eta_i(t)$. This is the basic regression model for single output technologies used in many empirical studies.

We start with a relatively simple representation of the input distance function as an *n*- input, *m*-output deterministic distance function $D_I(Y, X)$ given by the Young index, described in Balk (2008):

$$D_I(Y, X) = \frac{\prod_{j=1}^{m} Y_{jit}^{\gamma_j}}{\prod_{k=1}^{n} X_{kit}^{\delta_k}},$$
$$with\ 0 < D_I(Y, X) \leq 1.$$

The input distance function is non-decreasing, linearly homogeneous, and concave in *X*, as well as non-increasing and quasi-concave in *Y*. After taking logs, adding a disturbance term v_{it} to account for nonsystematic error in observations, functional form misspecifications, etc., and a technical efficiency term $\eta_i(t)$ to reflect the nonnegative difference between the upper bound of unity for the distance function and the observed value of the distance function for firm *i* at time *t*, we can write the simple Cobb-Douglas input distance function as:

$$x_{it} = \sum_{j=1}^{m} \gamma_j y_{jit} + \sum_{k=2}^{n} \delta_k x_{kit}^* + \eta_{it} + v_{it}$$

where $x_{kit,k=2,..,n}^* = \ln(X_{kit} / X_{1it})$. After redefining a few variables, the distance function can be written in the canonical form as:

$$x_{it}^* = z_{it}\beta + \eta_i(t) + v_{it}.$$

where $z_{it} = [x_{it}^* \ y_{it}]$.

Despite the fact that it has been criticized for its rather restrictive assumption of separability of outputs and inputs, as pointed out by Coelli (2000), among many others, the Cobb-Douglas specification remains a reasonable and parsimonious first-order local approximation to the true function. The translog input distance function, wherein the second-order terms allow for greater flexibility and proper local curvature, lifts the assumed separability of outputs and inputs. The translog specification can be framed in this canonical model representation of a linear panel model with unit-specific and time-varying heterogeneity. For the input distance function the translog takes the following form:

$$x_{it} = \sum_{j=1}^{m} \gamma_j y_{jit} + \frac{1}{2}\sum_{j=1}^{m}\sum_{l=1}^{m} \gamma_{jl} y_{jit} y_{lit} + \sum_{k=2}^{n} \delta_k x_{kit}^{*} + \frac{1}{2}\sum_{k=2}^{n}\sum_{p=2}^{n} \delta_{kp} x_{kit}^{*} x_{pit}^{*}$$
$$+ \sum_{j=1}^{n}\sum_{k=2}^{n} \theta_{jk} y_{jit} x_{kit}^{*} + \eta_{it} + v_{it}$$

Returns to scale, which can be derived from the parameter estimates of the distance function and data, are defined as the degree to which a firm's total output increases as it inputs increase proportionally. It is derived as the inverse of the sum of the partial derivatives of the input distance function. That is,

$$RTS_{it} = \left[\sum_{j=1}^{n} \frac{\partial D_I(X,Y)}{\partial Y_{jkit}} \right]^{-1}$$

If this measure exceeds one then there is a presence of economies of scale indicating that a firm is operating below the optimal scale level and can increase its total output by employing additional inputs. On the other hand, if it is smaller than one then a firm is experiencing diseconomies of scale and should reduce its input levels to achieve optimal input/output combination.

The Allen output elasticities of substitution can also be directly derived from the parameter estimates of the distance function and data, which for output j and l are given by:

$$E_{y_j y_l} = \frac{\left[D_I(X,Y) + \dfrac{\partial^2 D_I(X,Y)}{\partial y_j \partial y_l} \right]}{\dfrac{\partial D_I(X,Y)}{\partial y_j} \times \dfrac{\partial D_I(X,Y)}{\partial y_l}}$$

A negative value of this measure indicates that the outputs are substitutes, while a positive value indicates that these are complements. The magnitude of this measure indicates the strength of the substitute/complementarity relationship.

The CSS model is the vehicle we use for estimating efficiency change using the frontier methods we specify below. If we assume that innovations are available to all firms and that firm-specific idiosyncratic errors are due to relative inefficiencies then we can decompose sources of *TFP* growth in a variety of ways. The overall level of innovation (innovation is assumed to be equally appropriable by all firms) can be proxied by past R&D expenditures, patent activity, the time index approach of Baltagi and Griffin (1988), exogenous time trends, or stochastic time trends (Bai, Kao and Ng, 2009). Various specifications of time varying inefficiency have been introduced into the productivity literature in addition to Cornwell, Schmidt, and Sickles, for example by Kumbhakar (1990), Battese and Coelli (1992), and Lee and Schmidt (1993), among many others. The CSS panel stochastic frontier model itself extends the basic panel data model of Pitt and Lee (1981) and Schmidt and Sickles (1984) to allow for heterogeneity in slopes as well as intercepts. Thus, in the model $y_{it} = x_{it}\beta + \eta_i(t) + v_{it}$ the effects are specified as $\eta_i(t) = W_{it}\delta_i$. The number of coefficients of *W*, the terms in the vector δ_i, depend on different factors representing heterogeneity in slopes. In their application to the U.S. commercial airline industry, CSS specified these factors as $W_{it} = (1, t, t^2)$, although this was intended by the authors to be a parsimonious parameterization useful for their application. The CSS estimator is not limited to a specification wherein the effects are quadratic in time.

A common construction can relate this model to the standard panel data model. Let $\delta_i = \delta_0 + u_i$ and $\delta_0 = E[\delta_i]$. Then the model can be written as:

$$x^*_{it} = Z_{it}\beta + W_{it}\delta_0 + \varepsilon_{it},$$
$$\varepsilon_{it} = W'_{it}u_i + v_{it}.$$

Here u_i are assumed to be identically and independently distributed (*i.i.d*) zero mean random variables with covariance matrix Δ. The disturbances v_{it} are also taken to be *i.i.d.* random variable with a zero mean and constant variance σ^2, and uncorrelated with the regressors and u_i. In matrix form the model is:

$$x^* = Z\beta + W\delta_0 + \varepsilon,$$
$$\varepsilon = Qu + v$$

where $Q = diag(W_i)$, $i = 1, \cdots, N$ is a $NT \times NL$ matrix, and *u* is the associated $NL \times 1$ error vector.

Three different estimators can be derived based on differing assumptions regarding the correlation of the efficiency effects and the regressors, specifically, the correlation between the error term *u*, and regressors Z and *W*. These are the *within* and GLS, which we employ in this paper, and *the efficient IV* estimator.[3] We briefly discuss below the *within* and GLS estimators, which we will modify using the spline extensions in the following section.

The *within* estimator allows for correlation between all of the regressors and

[3] Details on the efficient IV estimator can be found in the CSS paper.

the effects. Let $P_Q = Q(Q'Q)^{-1}Q'$, $M_Q = I - P_Q$. Then the CSS *within* (CSSW) estimator of β is given by:

$$\hat{\beta}_w = (Z'M_Q Z)^{-1} Z'M_Q x^*.$$

The GLS estimator is consistent when no correlation exists between technical efficiency and the regressors, as in Pitt and Lee (1981), Schmidt and Sickles (1984) and many others that utilize this standard random effects assumption. The variance of the composed error is given by

$$\mathrm{cov}(\varepsilon) = \Omega = \sigma^2 I_{NT} + Q'(I_N \otimes \Delta)Q.$$

CSS show that

$$\Omega^{-1/2} = \frac{1}{\sigma} M_Q + F$$

where $F = Q(Q^{'-1/2}[\sigma^2 I_{NL} + (Q^{'1/2}(I_N \otimes \Delta)(Q^{'1/2}]^{-1/2}(Q^{'-1/2}Q')$. The transformed model is thus $\Omega^{-1/2}y = \Omega^{-1/2}X\beta + \Omega^{-1/2}W\delta_0 + \Omega^{-1/2}\varepsilon$. CSS provide formulae for feasible consistent estimates of $\Omega^{-1/2}$.

For either the *within* or GLS estimators the δ_0 are estimated by regressing the residuals for firm *i* on $W_{it} = (1, t, t^2)$ and the fitted values from this regression provide consistent $(T \to \infty)$ estimates of $E[\eta_i(t)]$. This is analogous to the approach in Schmidt and Sickles (1984) when there is no temporal variation in the unit specific technical efficiencies. Relative efficiencies, normalized by the consistent estimate of the order statistics identifying the most efficient unit, are then calculated as $\widehat{\eta}(t) = \max_j[\widehat{\eta}_j(t)]$. Efficiency of firm *i* relative to the most efficient firm is then given by $RE_i(t) = \widehat{\eta}(t) - \widehat{\eta}_i(t)$.[4]

3. Spline Model Specification

Another flexibility aspect in the specification of technical efficiency involves

[4] Firm-specific relative efficiencies can be identified along with the overall growth in innovation that diffuses to all firms for the GLS estimator. Under appropriate orthogonality assumptions, a similar term can be identified for the Hausman-Taylor type efficient IV estimator. Thus for these two estimators total factor productivity can be decomposed into technical change and efficiency change. Such a decomposition is not possible for the CSS *within* estimator as the overall technical change term specified as quadratic in time is not identified after the within transformation if the effects are also specified as quadratic in time.

monotonicity over time. In many previous specifications (e.g., Battese and Coelli, 1992; Cuesta, 2000), the effect of the passage of time on technical inefficiency is necessarily monotonic and thus may be either efficiency-enhancing or efficiency-impeding, but not both. Others, such as Lee and Schmidt (1993) and Lee (2006), allow for more general patterns. We consider below the quadratic spline as a special case of restricted least squares and thus, once the number of knots is estimated (and tested for sequentially), a general spline estimator can be specified (Buse and Lim, 1977). Depending on the number of break (turning) points, which may be determined by either prior information regarding the sector under consideration (Bottasso and Conti, 2009) or by the process suggested by Fox (1998), the time pattern can be rather flexible, curved or monotonic. The latter two options can be tested statistically as nested model specifications.

The Diewert-Wales (1992) quadratic spline function can be incorporated into the CSS specification in order to obtain a flexible and parsimonious specification of the temporal pattern of technical efficiency, allowing for more than one turning point. This specification allows for firm-specific patterns of temporal variation of technical efficiency and captures effects not visible in those models that assume a common pattern of technical efficiency. In addition, we can test (i) for the existence of a common temporal pattern for all firms in the sample, as well as (ii) the hypothesis of time-varying technical efficiency for all or some of the firms in the sample.

The spline extensions of the CSS model can be introduced by assuming a single time break $t_1 \in [0,T]$ in the inefficiency level of firm *i*. An extension to a model with multiple time breaks is straightforward and also is briefly discussed. Following Diewert and Wales (1992), the inefficiency function of the CSS model can be represented by a quadratic spline function as follows:

$$u_{it} = \begin{cases} \delta_{i0} + \delta_{i1} t + \delta_{i2}^{(1)} t^2 & \text{if } t \le t_1 \\ \delta_{i0} + \delta_{i1} t_1 + \delta_{i2}^{(1)} t_1^{\ 2} + (t - t_1)(\delta_{i1} + 2\delta_{i2}^{(1)} t_1) + (t - t_1)^2 \delta_{i2}^{(2)} & \text{if } t > t_1 \end{cases}$$

Here the superscripts on the quadric term parameters relate to the periods before and after the time break point t_1.

The above can be rewritten as

$$u_{it} = \begin{cases} \delta_{i0} + \delta_{i1} t + \delta_{i2}^{(1)} t^2 & \text{if } t \le t_1 \\ \tilde{\delta}_{i0} + \tilde{\delta}_{i1} t + \tilde{\delta}_{i2} t^2 & \text{if } t > t_1 \end{cases}$$

where

$$\tilde{\delta}_{i0} = \delta_{i0} + (\delta_{i2}^{(2)} - \delta_{i2}^{(1)})t_1^2$$
$$\tilde{\delta}_{i1} = [\delta_{i1} + 2(\delta_{i2}^{(1)} - \delta_{i2}^{(2)})t_1^2]$$
$$\tilde{\delta}_{i2} = \delta_{i2}^{(2)}$$

Clearly, the above model specification allows for the levels and slopes of inefficiencies to differ across these two time periods. Notice that, if $\delta_{i2}^{(1)} = \delta_{i2}^{(2)} = \delta_{i2}$ then the above model collapses to the standard CSS model with no breaks. Also, it is worthwhile to note that, both the inefficiency function and its first derivative with respect to time are continuous at t_1. The continuity feature is crucial here as it allows for a smooth transition from one state to another (no jumps).

In order to proceed with the estimation of the CSS model with the quadratic spline specification, let first the matrix of time regressors to be denoted as follows:

$$W_i = \begin{bmatrix} W_{1i} & 0 \\ 0 & W_{2i} \end{bmatrix}$$

where $W_{1i} = [1, t, t^2] I(t \le t_1)$, $W_{2i} = [1, t, t^2] I(t > t_1)$ and $I(\cdot)$ is an indicator function. Subsequently, define $Q = diag(W_i)$ and $M_Q = I - Q(Q'Q)^{-1}Q'$ as a projection matrix onto column space of Q. Then while fixing t_1, the *within* estimator of the structural parameters is given by:

$$\hat{\beta}_W = (X'M_Q X)^{-1} X'M_Q y$$

and the concentrated sum of squared errors, which is a function of the observed data and time, is given by:

$$S(t) = \hat{e}'\hat{e} = (y - X(X'M_Q X)^{-1} X'M_Q y)'(y - X(X'M_Q X)^{-1} X'M_Q y)$$

where $\hat{e} = y - X\hat{\beta}_W$ represents the *within* residuals. The time break t_1 is estimated by minimizing $S(t)$ using grid search techniques. That is,

$$\hat{t}_1 = \arg\min_t S(t)$$

Once $\hat{t}_1$ is estimated, which represents the least square estimator of t_1, the slope coefficient is obtained and the residual variance is estimated by dividing $S(\hat{t})$ by the degrees of freedom, $N(T-1)$.

The model with a single time break can be extended to accommodate multiple time breaks. A model with k time breaks can be represented as follows:

$$u_{it} = \begin{cases} \delta_{i0} + \delta_{i1}t + \delta_{i2}^{(1)}t^2 & \text{if } t \le t_1 \\ \delta_{i0} + \delta_{i1}t_1 + \delta_{i2}^{(1)}t_1^{\ 2} + (t_2 - t_1)(\delta_{i1} + 2\delta_{i2}^{(1)}t_1) + (t_2 - t_1)^2\delta_{i2}^{(2)} & \text{if } t_1 < t \le t_2 \\ .... & ... \\ \delta_{i0} + \delta_{i1}t_{k-1} + \delta_{i2}^{(k-1)}t_{k-1}^{\ 2} + (t_k - t_{k-1})(\delta_{i1} + 2\delta_{i2}^{(k-1)}t_k) + (t_k - t_{k-1})^2\delta_{i2}^{(k)} & \text{if } t_{k-1} < t \le t_k \\ \delta_{i0} + \delta_{i1}t_k + \delta_{i2}^{(k)}t_k^{\ 2} + (t - t_k)(\delta_{i1} + 2\delta_{i2}^{(k)}t_k) + (t - t_k)^2\delta_{i2}^{(k+1)} & \text{if } t > t_k \end{cases}$$

Similar to the single time break model, the multiple time breaks are estimated via a grid search algorithm. Joint estimation of the time breaks requires a grid search over a large number of time break combinations. Therefore, the time breaks are sequentially estimated as suggested in Hansen (1999). The sequential estimation of the threshold parameters is consistent for large *t*. A drawback of the sequential estimation method is that it yields asymptotically efficient estimates only for the last time break in the estimation process. The previous estimates are contaminated by the presence of the neglected time breaks. We follow Bai (1997) and utilize a refinement estimation of the time break parameters, which amounts to re-estimating the time break parameters backwards, each time holding the estimates of the previous time breaks fixed.

It is important to test whether the time breaks are statistically significant or not. However, the distribution of t_1 is nonstandard which complicates inference. In particular, testing for the presence of a time break becomes problematic since t_1 is not identified under the null hypothesis of no threshold and conventional tests would have distributions that are also nonstandard (this is known as the Davies' Problem (1977) in the econometrics literature). Following Hansen (1999), we utilize a bootstrap method detailed in Yu (2012) to simulate the asymptotic distribution of the classical LR test in developing our inferences on the time break estimates.

The bootstrap testing is carried out in the following steps:

1. Estimate the model under the null and alternative hypotheses and calculate the LR statistic as $LR = (S_0 - S_1)/\sigma^2$
2. Construct a sample of residuals estimated under the null hypothesis (i.e., $\hat{e}$) and treat this sample as an empirical distribution to draw samples from in the bootstrap replications

3. Fix the data and draw (with replacement) a sample of size N from the empirical distribution above and use these errors to create a bootstrap sample
4. Using the bootstrap sample, estimate the model under the null and alternative hypotheses and calculate the bootstrap value of the likelihood ratio statistic LR^b
5. Repeat this procedure a large number of times and calculate the percentage of draws for which LR^b exceeds LR
6. Reject the null hypothesis if the percentage above exceeds the desired confidence level.

4. Data and Empirical Results

In this section we provide empirical evidence on a comparison between the two specifications of the CSS model based on the second-order polynomial and based on the spline function using a rather homogenous and balanced sample of large (too-big-to-fail) US banks. The data are from the quarterly consolidated reports on condition and income (Call Reports) for U.S. commercial banks collected and administrated by the Federal Reserve Bank of Chicago (FED Chicago) and the Federal Deposit Insurance Corporation (FDIC). The particular sample used in this study covers the period from the first quarter of 1984 to the second quarter of 2010 (i.e., 106 quarters) and includes a total of 45 banks, whose total asset size was at least US$ 10 billion as of the second quarter of the 2010 fiscal year. The total number of observation is therefore 4,770. Table 2 in the appendix of the paper lists the names of these 45 banks along with their book value of total assets.

We employ the intermediation approach of Sealey and Lindley (1977), according to which banks are viewed as financial intermediates that collect deposits and other funds to transform them into loanable funds by using capital and labor. In this approach, deposits are viewed as inputs as opposed to outputs as in the production approach. We consider five outputs: (i) real estate loans (Y_1); (ii) commercial and industrial loans (Y_2); (iii) loans to individuals (Y_3); (iv) securities (Y_4); and (v) off-balance sheet items (Y_5). On the input side we have: (i) demand deposits (X_1); (ii) time and savings deposits (X_2); (iii) labor (X_3); (iv) capital (X_4); and (v) purchased funds (X_5). Descriptive statistics of these data are provided in Table 3.

We estimate the proposed spline specification of the CSS model using both the *within* and the GLS estimator, as with the latter we can separate the effect of technical change from that of changes in technical efficiency even though both are modeled by time trends; this is an important aspect in the productivity decomposition analysis. We have also tried the Hausman-Taylor estimator in the case where some of the explanatory variables are not orthogonal with the "effects" capturing unobserved heterogeneity. Based on the estimated Hausman-Wu test statistic of 26.9 (p-value = 1), we have no evidence of simultaneity bias and, moreover, the GLS estimator fits better with the data at hand. In addition and for comparison purposes we also estimate the conventional version (i.e., second-order polynomial) of the CSS model.

Estimation of the proposed spline specification of the CSS model involves both the estimation of the values of the structural model parameters, as well as of that of the unknown time breaks. For this purpose, the proposed model is estimated by first minimizing the concentrated sum of squared residuals using a grid search over possible time periods to determine the time breaks, as in Almanidis (2013), and then by estimating the values of the structural parameters. In the estimation algorithm it is assumed that the timing of the breaks is the same for all banks but this does not necessarily imply that each bank will experience the break. This is apparent from the fact that efficiency is firm-specific in the CSS model.

We estimate twelve time-break points based on the bootstrap test with 10,000 replications at each stage of the time-break search. These time-break points occur in the third quarter of 1986, the second quarter of 1988, the third quarter of 1990, the fourth quarter of 1991 and 1994, the first quarter of 1997, the third quarter of 1999, and 2001, the second quarter of 2004, the third quarter of 2006, and the second quarters of 2007 and 2008. We are able to relate ten out of the twelve estimated break points to the dates of significant developments of the U.S. banking industry, such as deregulatory reforms, technological and financial breakthroughs, and economic and financial turmoil periods. Table 1 below reports the estimated break points, along with the p-values of the null hypothesis of the model with k time-breaks versus the model with k+1 time-breaks. The last column of Table 1 describes the corresponding significant developments of the U.S. banking industry.

Table 1: Break Point Estimates

#	Break Point	P-value	Significant Developments of the U.S. Banking Industry
1	1986.Q3	0.0141	The Federal Reserve granted commercial bank holding companies with the power to underwrite corporate securities in 1987
2	1990.Q3	0.0024	The Federal Reserve allowed the operating commercial banks to underwrite corporate securities in 1989
3	1991.Q2	0.0007	The beginning of the savings and loan crisis
4	1994.Q1	0.0012	Enactment of the Reigle-Neal Interstate Banking and Branching Efficiency Act of 1994, which allowed the interstate banking and branching
5	1997.Q1	0.0008	-
6	1999.Q3	0.0001	Enactment of the Gramm-Leach-Bliley Act Financial Services Modernization Act of 1999, which granted broad-based securities, investment, and insurance power to commercial banks
7	2000.Q4	0.0026	The introduction of the internet banking and check clearing through imaging technology
8	2001.Q4	0.0000	The financial crisis of early 2000s
9	2004.Q2	0.0011	-
10	2006.Q3	0.0001	Collapse of the U.S. housing market bubble in mid-2006 and the subsequent dramatic increase in delinquencies and default rates on subprime residential-mortgage-backed securities (RMBS)
11	2007.Q3	0.0001	The beginning of the 2007-2010 financial crisis
12	2008.Q2	0.0007	The failure of Washington Mutual Bank on September 25, 2008[5]

The results under the spline and second-order polynomial specifications for the translog case are reported in Tables 4 and 5, respectively. Overall, most parameter estimates are statistically significant at the 5% confidence level, as indicated by their respective t-statistics. There are notable differences between the estimated parameters in Tables 4 and 5. These differences are mainly attributed to the differences in the efficiency pattern specification. Table 6 reports the estimated spline specification parameters under the Cobb-Douglas case, which show significant differences from the estimates obtained under the translog case, when comparing their respective marginal effects of the inputs on outputs for a particular entity and time.

The estimated average efficiency patterns under the translog spline specification for both the *within* and GLS model are depicted in Figure 1. Figure 2 displays the respective average efficiencies under the translog polynomial specification. A comparison of these two figures shows that, on average, the estimated average efficiencies are smaller with the spline specification than with the conventional specification of the CSS model. In particular, the average technical efficiency from the spline specification is around 64% for both *within* and the GLS

[5] Washington Mutual Bank's failure is considered the largest failure in the history of the U.S. with $307 billion in failed assets.

models while the corresponding figures for the conventional specifications are at 69-71%. Overall, there are no material differences in the cross-sectional distribution of efficiency scores. There are however significant differences in the temporal pattern of efficiency scores, as depicted in Figure 1 and Figure 2.

Comparing with Figure 2, where the average efficiencies of the conventional CSS model are presented, it is evident that there is a non-monotonic time pattern from the spline specification around the aforementioned turning points. In contrast, the average efficiencies predicted by the conventional specification reached a maximum around the first half of 2006 and decreased thereafter.

Table 7 reports the return to scale (RTS) estimates of the average bank during the sample period. Most of our results indicate that large commercial banks in the U.S. have been operating at constant returns to scale since the mid-1990s, although the *within* model fails to reject the hypothesis of constant return to scale throughout the entire sample period.[6] These findings are consistent with those of McAllister and McManus (1993), Wheelock and Wilson (2001), and Almanidis (2013), among others.

The Allen output elasticity estimates are reported in Table 8 for both the *within* and GLS models. It is shown that real estate loans are substitutes to the other four outputs and that there is a strong complementary relationship between the commercial & industrial loans and securities.

5. Concluding Remarks

In this paper we present a specification of the CSS (1990) model that allows for more flexible patterns of temporal changes in technical efficiency. The model is based on a second-order spline function, which can accommodate more than one turning point over time. This non-monotonic temporal pattern depicts in a much more flexible way firm heterogeneity in terms of growth rates and it is particularly suitable for analyzing efficiency changes during periods of regulation or policy changes. We estimate such a model using a translog input distance function for a sample of large (too-big-to-fail) U.S. commercial banks using a semi-parametric approach and a grid search algorithm. Our empirical results revealed the presence of twelve time break points during the period 1984-2009 indicating a highly non-monotonic time pattern of

[6] We conjecture that this is due to the large standard errors estimated under the *within* model.

average technical efficiency. A bootstrap method was utilized to formally test for the presence of the time breaks.

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Table 2: The Sample of Large U.S. Commercial Banks

#	Name	Total Assets as of 2010.Q2 (in thousands of US$)
1	ASSOCIATED BK NA	22,464,346
2	BANCO POPULAR DE PR	31,946,078
3	BANCORPSOUTH BK	13,419,373
4	BANK OF AMER NA	1,518,957,376
5	BANK OF HAWAII	12,833,506
6	BANK OF NY MELLON	175,993,919
7	BANK OF OK NA	16,621,041
8	BANK OF THE WEST	61,142,025
9	BNY MELLON NA	11,232,029
10	BRANCH BKG&TC	149,200,262
11	CAPITAL ONE NA	123,523,256
12	CITIBANK NA	1,157,872,377
13	CITIZENS BK	10,577,080
14	CITY NB	20,894,782
15	COMERICA BK	55,778,553
16	COMMERCE BK NA	18,206,514
17	COMPASS BK	65,076,236
18	DEUTSCHE BK TC AMERICAS	42,306,022
19	DISCOVER BK	59,501,376
20	FIFTH THIRD BK	110,026,302
21	FIRST HAWAIIAN BK	14,747,881
22	FIRST TENNESSEE BK NA MMPHS	26,043,011
23	FIRST-CITIZENS B&TC	18,157,241
24	FIRSTMERIT BK NA	14,504,652
25	FROST NB	17,109,253
26	HARRIS NA	44,942,749
27	HSBC BK USA NA	183,594,485
28	HUNTINGTON NB	51,211,773
29	JPMORGAN CHASE BK NA	1,568,099,165
30	KEYBANK NA	90,662,103
31	MANUFACTURERS & TRADERS TC	67,250,835
32	MB FNCL BK NA	10,601,648
33	NORTHERN TC	66,624,304
34	NORTHERN TR NA	12,180,904
35	PNC BK NA	251,074,433
36	REGIONS BK	131,010,758
37	STATE STREET B&TC	157,473,855
38	SUNTRUST BK	160,508,908
39	SUSQUEHANNA BK	13,757,160
40	SYNOVUS BK	31,990,515
41	U S BK NA	278,464,182
42	UNION BK NA	83,842,450
43	VALLEY NB	14,092,454
44	WELLS FARGO BK NA	1,073,285,133
45	ZIONS FIRST NB	16,989,397

Table 3: Summary Statistics (mean and standard deviation) of Model Variables

	1984:Q1	1993:Q1	2000:Q1	2009:Q3
Real estate loans (Y_1)	867,896	3,300,626	11,900,000	44,700,000
	(2,027,284)	(6,149,823)	(26,400,000)	(89,100,000)
Commercial & industrial	2,431,577	3,260,927	10,500,000	18,400,000
loans (Y_2)	(6,560,456)	(6,707,246)	(22,100,000)	(36,600,000)
Loans to individuals (Y_3)	593,755	1,625,214	3,902,677	11,600,000
	(1,145,667)	(3,330,108)	(7,491,437)	(23,400,000)
Securities (Y_4)	1,868,698	4,149,224	3,485,955	44,000,000
	(3,627,331)	(5,668,735)	(7,679,057)	(96,200,000)
Off-balance sheet items (Y_5)	2.31005	1.62116	1.36456	1.03391
	(5.75334)	(1.68702)	(1.3120)	(1.05039)
Total demand deposits (X_1)	987,153	2025,806	4,332,390	8,138,886
	(1,509,827)	(2,885,831)	(8,887,760)	(17,300,000)
Total time & savings	4,991,220	9,629,871	30,300,000	104,000,000
deposits (X_2)	(11,900,000)	(18,800,000)	(59,800,000)	(214,000,000)
Labor (X_3)	4,164	6,014	13,563	23,781
	(7,620)	(9,644)	(25,614)	(45,427)
Capital (X_4)	104,298	253,391	611,758	1,231,292
	(232,928)	(476,984)	(1,150,663)	(2,170,180)
Purchased funds (X_5)	2,027,599	3,943,412	13,000,000	13,000,000
	(3,920,692)	(6,500,932)	(23,600,000)	(24,000,000)

y_1=log(real estate loans), y_2= log(commercial & industrial loans), y_3=log(loans to individuals), y_4=log(securities), y_5=log(off balance sheet items), x_1=log(total time & savings deposits/ total demand deposits), x_2=log(labor/total demand deposits), x_3=log(capital/ total demand deposits), x_4=log(purchased funds/total demand deposits) and t=time trend.

Table 4: Estimated Model Parameters: Translog Spline Specification

	Within		GLS	
	Estimated Parameter	Standard Error	Estimated Parameter	Standard Error
y_1	0.2438	0.0824	0.2268	0.0110
y_2	0.5996	0.0627	0.5880	0.0122
y_3	-0.6124	0.0590	-0.6496	0.0095
y_4	0.0054	0.0170	0.0093	0.0045
y_5	0.0403	0.0340	0.0152	0.0069
x_1	0.4935	0.1193	0.4460	0.0170
x_2	-0.2328	0.1106	-0.1338	0.0182
x_3	0.2179	0.0968	0.1598	0.0198
x_4	-0.1011	0.0372	-0.1035	0.0095
y_1y_1	-0.1256	0.0059	-0.1207	0.0010
y_2y_1	0.0029	0.0031	0.0020	0.0008
y_2y_2	-0.0315	0.0034	-0.0332	0.0009
y_3y_1	0.0549	0.0040	0.0540	0.0006
y_3y_2	-0.0060	0.0031	-0.0042	0.0006
y_3y_3	-0.0295	0.0043	-0.0310	0.0007
y_4y_1	0.0002	0.0010	0.0006	0.0003
y_4y_2	-0.0005	0.0009	-0.0006	0.0002
y_4y_3	0.0059	0.0009	0.0056	0.0002
y_4y_4	-0.0037	0.0004	-0.0032	0.0001
y_5y_1	0.0036	0.0018	0.0034	0.0004
y_5y_2	0.0016	0.0017	0.0025	0.0005
y_5y_3	-0.0004	0.0016	0.0006	0.0004
y_5y_4	-0.0009	0.0006	-0.0013	0.0002
y_5y_5	0.0070	0.0014	0.0052	0.0005
x_2y_1	0.0322	0.0089	0.0315	0.0017
x_2y_2	-0.0267	0.0078	-0.0269	0.0018
x_2y_3	0.0122	0.0073	0.0133	0.0013
x_2y_4	-0.0079	0.0023	-0.0073	0.0006
x_2x_1	-0.0151	0.0055	-0.0112	0.0011
x_2x_2	0.0592	0.0219	0.0536	0.0045
x_3y_1	-0.0842	0.0108	-0.0754	0.0017
x_3y_2	0.0629	0.0092	0.0606	0.0022
x_3y_3	-0.0011	0.0083	-0.0063	0.0015
x_3y_4	0.0049	0.0028	0.0065	0.0008
x_3x_1	0.0375	0.0056	0.0331	0.0013
x_3x_2	-0.0270	0.0194	-0.0246	0.0037
x_3x_3	-0.0732	0.0262	-0.0484	0.0054
x_4y_1	0.0578	0.0072	0.0525	0.0015
x_4y_2	-0.0332	0.0082	-0.0295	0.0017
x_4y_3	-0.0189	0.0064	-0.0165	0.0010
x_4y_4	0.0006	0.0022	-0.0014	0.0006
x_4x_1	-0.0051	0.0040	-0.0060	0.0011
x_4x_2	0.0607	0.0138	0.0601	0.0028
x_4x_3	0.0823	0.0179	0.0620	0.0035
x_4x_4	-0.1114	0.0153	-0.0956	0.0031
x_5y_1	-0.0154	0.0040	-0.0157	0.0009
x_5y_2	0.0086	0.0040	0.0061	0.0010
x_5y_3	0.0141	0.0034	0.0148	0.0008
x_5y_4	0.0010	0.0010	0.0009	0.0003
x_5x_1	-0.0068	0.0027	-0.0057	0.0008
x_5x_2	-0.0316	0.0081	-0.0276	0.0020
x_5x_3	-0.0184	0.0070	-0.0257	0.0018
x_5x_4	-0.0276	0.0075	-0.0215	0.0017
x_5x_5	0.0561	0.0050	0.0544	0.0014
y_1t	-0.0003	0.0003	-0.0001	0.0000
y_2t	-0.0003	0.0002	-0.0003	0.0001
y_3t	0.0007	0.0002	0.0006	0.0000
y_4t	0.0001	0.0001	0.0001	0.0000
x_1t	0.0004	0.0001	0.0004	0.0000
x_2t	-0.0030	0.0005	-0.0028	0.0001
x_3t	0.0031	0.0005	0.0032	0.0001
x_4t	-0.0005	0.0003	-0.0004	0.0001
x_4t	0.0003	0.0002	0.0001	0.0001
t	-	-	0.0001	0.0003
t^2	-	-	0.0000	0.0000

Table 5: Estimated Model Parameters: Translog Polynomial Specification

	Within		GLS	
	Estimated Parameter	Standard Error	Estimated Parameter	Standard Error
y_1	0.4983	0.0693	0.4631	0.0411
y_2	0.5564	0.0734	0.5340	0.0456
y_3	0.0428	0.0692	0.0002	0.0354
y_4	-0.0219	0.0211	-0.0285	0.0166
y_5	-0.3928	0.0490	-0.4229	0.0257
x_1	1.0991	0.1566	0.8951	0.0636
x_2	-0.4570	0.1444	-0.2680	0.0680
x_3	-0.0017	0.1235	-0.0108	0.0741
x_4	-0.2234	0.0518	-0.1799	0.0354
y_1y_1	-0.1128	0.0055	-0.1124	0.0037
y_2y_1	0.0212	0.0040	0.0217	0.0031
y_2y_2	-0.0581	0.0044	-0.0585	0.0033
y_3y_1	0.0241	0.0037	0.0234	0.0023
y_3y_2	0.0172	0.0038	0.0181	0.0022
y_3y_3	-0.0641	0.0044	-0.0631	0.0025
y_4y_1	0.0110	0.0012	0.0109	0.0010
y_4y_2	-0.0028	0.0010	-0.0024	0.0008
y_4y_3	0.0042	0.0008	0.0044	0.0007
y_4y_4	-0.0051	0.0005	-0.0053	0.0004
y_5y_1	0.0157	0.0023	0.0151	0.0017
y_5y_2	-0.0078	0.0023	-0.0064	0.0018
y_5y_3	-0.0032	0.0025	-0.0032	0.0016
y_5y_4	0.0011	0.0008	0.0012	0.0007
y_5y_5	0.0114	0.0024	0.0117	0.0018
x_2y_1	0.0434	0.0108	0.0460	0.0064
x_2y_2	-0.0610	0.0098	-0.0628	0.0066
x_2y_3	-0.0114	0.0095	-0.0069	0.0048
x_2y_4	-0.0158	0.0025	-0.0153	0.0021
x_2x_1	0.0359	0.0073	0.0381	0.0040
x_2x_2	-0.1748	0.0263	-0.1562	0.0166
x_3y_1	0.0529	0.0108	0.0457	0.0063
x_3y_2	0.0589	0.0118	0.0621	0.0081
x_3y_3	0.0025	0.0090	-0.0004	0.0057
x_3y_4	0.0195	0.0034	0.0188	0.0028
x_3x_1	-0.0149	0.0079	-0.0162	0.0048
x_3x_2	0.0652	0.0221	0.0490	0.0138
x_3x_3	0.2767	0.0319	0.2843	0.0200
x_4y_1	-0.0181	0.0075	-0.0156	0.0054
x_4y_2	-0.0319	0.0087	-0.0327	0.0063
x_4y_3	-0.0167	0.0065	-0.0168	0.0039
x_4y_4	-0.0115	0.0026	-0.0114	0.0021
x_4x_1	0.0031	0.0060	0.0016	0.0041
x_4x_2	0.0641	0.0166	0.0671	0.0105
x_4x_3	-0.2449	0.0218	-0.2433	0.0131
x_4x_4	0.1603	0.0182	0.1571	0.0117
x_5y_1	-0.0416	0.0043	-0.0402	0.0033
x_5y_2	0.0216	0.0049	0.0210	0.0037
x_5y_3	0.0134	0.0041	0.0126	0.0028
x_5y_4	0.0013	0.0013	0.0012	0.0011
x_5x_1	-0.0158	0.0037	-0.0156	0.0028
x_5x_2	0.0747	0.0104	0.0690	0.0073
x_5x_3	-0.1197	0.0088	-0.1124	0.0065
x_5x_4	0.0359	0.0083	0.0359	0.0062
x_5x_5	-0.0037	0.0065	-0.0048	0.0051
y_1t	0.0010	0.0003	0.0010	0.0001
y_2t	-0.0002	0.0002	-0.0002	0.0001
y_3t	0.0016	0.0002	0.0016	0.0001
y_4t	-0.0001	0.0001	-0.0001	0.0001
x_1t	-0.0004	0.0002	-0.0004	0.0001
x_2t	0.0041	0.0006	0.0037	0.0002
x_3t	-0.0003	0.0006	-0.0003	0.0003
x_4t	-0.0012	0.0004	-0.0011	0.0002
x_4t	-0.0014	0.0002	-0.0014	0.0002
t	-	-	-0.0022	0.0012
t^2	-	-	0.0000	0.0000

Table 6: Estimated Model Parameters: Cobb-Douglas Spline Specification

	Within		GLS	
	Estimated Parameter	Standard Error	Estimated Parameter	Standard Error
y_1	-0.3768	0.0084	-0.3769	0.0006
y_2	-0.1296	0.0067	-0.1359	0.0006
y_3	-0.1834	0.0063	-0.1832	0.0005
y_4	-0.0070	0.0009	-0.0068	0.0002
y_5	-0.0180	0.0023	-0.0177	0.0004
x_1	0.4918	0.0101	0.4910	0.0010
x_2	0.2923	0.0099	0.2946	0.0010
x_3	0.1171	0.0071	0.1139	0.0011
x_4	0.0365	0.0036	0.0389	0.0007
t	-	-	0.0002	0.0000

Table 7: Average Return to Scale (RTS) Estimates: Translog Spline Specification

	Within			GLS		
	RTS	Standard Error[7]	95% Confidence Interval	RTS	Standard Error	95% Confidence Interval
1984.Q1	1.2282	0.1176	(0.9978, 1.4587)	1.1559	0.0216	(1.1136,1.1982)
1984.Q2	1.2245	0.1188	(0.9917, 1.4574)	1.1528	0.0218	(1.1100,1.1955)
1984.Q3	1.2251	0.1182	(0.9934, 1.4569)	1.1534	0.0217	(1.1109,1.1959)
1984.Q4	1.2184	0.1208	(0.9816, 1.4552)	1.1476	0.0222	(1.1040,1.1911)
1985.Q1	1.2212	0.1190	(0.9878, 1.4545)	1.1504	0.0218	(1.1076,1.1932)
1985.Q2	1.2176	0.1203	(0.9817, 1.4534)	1.1473	0.0221	(1.1040,1.1906)
1985.Q3	1.2094	0.1227	(0.9689, 1.4500)	1.1402	0.0226	(1.0960,1.1844)
1985.Q4	1.2000	0.1255	(0.9541, 1.4459)	1.1323	0.0231	(1.0871, 1.1776)
1986.Q1	1.1969	0.1254	(0.9511, 1.4427)	1.1299	0.0231	(1.0847, 1.1750)
1986.Q2	1.1870	0.1283	(0.9355, 1.4385)	1.1214	0.0236	(1.0751, 1.1676)
1986.Q3	1.1875	0.1274	(0.9377, 1.4372)	1.1222	0.0234	(1.0763, 1.1681)
1986.Q4	1.1728	0.1326	(0.9129, 1.4327)	1.1096	0.0244	(1.0617, 1.1575)
1987.Q1	1.1733	0.1311	(0.9162, 1.4303)	1.1104	0.0241	(1.0631, 1.1577)
1987.Q2	1.1701	0.1321	(0.9112, 1.4289)	1.1078	0.0243	(1.0602, 1.1554)
1987.Q3	1.1675	0.1326	(0.9075, 1.4274)	1.1058	0.0244	(1.0580, 1.1536)
1987.Q4	1.1630	0.1338	(0.9007, 1.4253)	1.1021	0.0246	(1.0539, 1.1504)
1988.Q1	1.1613	0.1336	(0.8994, 1.4232)	1.1007	0.0245	(1.0526, 1.1489)
1988.Q2	1.1581	0.1350	(0.8935, 1.4227)	1.0980	0.0248	(1.0494, 1.1467)
1988.Q3	1.1591	0.1343	(0.8958, 1.4223)	1.0992	0.0247	(1.0508, 1.1476)
1988.Q4	1.1541	0.1366	(0.8864, 1.4217)	1.0949	0.0251	(1.0456, 1.1441)
1989.Q1	1.1515	0.1364	(0.8842, 1.4187)	1.0929	0.0251	(1.0438, 1.1420)
1989.Q2	1.1487	0.1374	(0.8794, 1.4179)	1.0905	0.0253	(1.0410, 1.1400)
1989.Q3	1.1451	0.1380	(0.8745, 1.4156)	1.0876	0.0254	(1.0379, 1.1374)
1989.Q4	1.1402	0.1402	(0.8654, 1.4150)	1.0832	0.0258	(1.0326, 1.1338)
1990.Q1	1.1406	0.1392	(0.8678, 1.4134)	1.0839	0.0256	(1.0337, 1.1340)
1990.Q2	1.1405	0.1392	(0.8677, 1.4134)	1.0838	0.0256	(1.0337, 1.1340)
1990.Q3	1.1405	0.1392	(0.8677, 1.4132)	1.0840	0.0256	(1.0339, 1.1341)
1990.Q4	1.1392	0.1403	(0.8642, 1.4143)	1.0827	0.0258	(1.0321, 1.1333)
1991.Q1	1.1431	0.1377	(0.8732, 1.4131)	1.0867	0.0253	(1.0371, 1.1362)
1991.Q2	1.1407	0.1391	(0.8680, 1.4134)	1.0847	0.0256	(1.0346, 1.1348)
1991.Q3	1.1399	0.1399	(0.8657, 1.4141)	1.0841	0.0257	(1.0337, 1.1345)
1991.Q4	1.1434	0.1391	(0.8708, 1.4160)	1.0875	0.0256	(1.0374, 1.1376)
1992.Q1	1.1457	0.1384	(0.8745, 1.4170)	1.0897	0.0254	(1.0398, 1.1395)
1992.Q2	1.1417	0.1402	(0.8668, 1.4166)	1.0859	0.0258	(1.0354, 1.1365)
1992.Q3	1.1414	0.1401	(0.8669, 1.4159)	1.0857	0.0258	(1.0352, 1.1362)
1992.Q4	1.1415	0.1410	(0.8651, 1.4180)	1.0860	0.0260	(1.0351, 1.1369)
1993.Q1	1.1430	0.1400	(0.8686, 1.4174)	1.0874	0.0258	(1.0369, 1.1379)
1993.Q2	1.1419	0.1408	(0.8659, 1.4178)	1.0865	0.0259	(1.0357, 1.1373)
1993.Q3	1.1421	0.1408	(0.8662, 1.4181)	1.0868	0.0259	(1.0360, 1.1376)
1993.Q4	1.1392	0.1422	(0.8605, 1.4179)	1.0843	0.0262	(1.0330, 1.1357)
1994.Q1	1.1394	0.1419	(0.8612, 1.4175)	1.0843	0.0261	(1.0331, 1.1355)
1994.Q2	1.1384	0.1422	(0.8597, 1.4171)	1.0835	0.0262	(1.0321, 1.1348)
1994.Q3	1.1390	0.1417	(0.8613, 1.4167)	1.0842	0.0261	(1.0331, 1.1353)
1994.Q4	1.1374	0.1425	(0.8580, 1.4168)	1.0831	0.0263	(1.0316, 1.1345)
1995.Q1	1.1352	0.1425	(0.8560, 1.4145)	1.0813	0.0262	(1.0299, 1.1326)
1995.Q2	1.1309	0.1442	(0.8482 ,1.4135)	1.0774	0.0265	(1.0253, 1.1294)
1995.Q3	1.1278	0.1449	(0.8439, 1.4118)	1.0749	0.0267	(1.0226, 1.1271)
1995.Q4	1.1264	0.1462	(0.8399, 1.4128)	1.0738	0.0269	(1.0209, 1.1266)
1996.Q1	1.1271	0.1451	(0.8426, 1.4116)	1.0745	0.0267	(1.0221, 1.1269)
1996.Q2	1.1173	0.1488	(0.8256, 1.4090)	1.0660	0.0274	(1.0122, 1.1198)
1996.Q3	1.1094	0.1514	(0.8126, 1.4063)	1.0590	0.0279	(1.0042, 1.1137)
1996.Q4	1.1034	0.1534	(0.8028, 1.4040)	1.0539	0.0283	(0.9984, 1.1094)
1997.Q1	1.1020	0.1532	(0.8017, 1.4024)	1.0529	0.0283	(0.9975, 1.1083)
1997.Q2	1.0898	0.1574	(0.7814, 1.3983)	1.0422	0.0290	(0.9853, 1.0992)
1997.Q3	1.0887	0.1569	(0.7812, 1.3961)	1.0414	0.0289	(0.9847, 1.0981)
1997.Q4	1.0851	0.1587	(0.7740, 1.3962)	1.0383	0.0293	(0.9809, 1.0957)
1998.Q1	1.0826	0.1589	(0.7712, 1.3940)	1.0363	0.0293	(0.9789, 1.0938)
1998.Q2	1.0802	0.1599	(0.7667, 1.3936)	1.0341	0.0295	(0.9763, 1.0920)
1998.Q3	1.0820	0.1586	(0.7712 ,1.3927)	1.0358	0.0292	(0.9785, 1.0931)
1998.Q4	1.0756	0.1610	(0.7599 ,1.3912)	1.0302	0.0297	(0.9720, 1.0885)
1999.Q1	1.0762	0.1602	(0.7621, 1.3902)	1.0309	0.0295	(0.9730, 1.0888)
1999.Q2	1.0756	0.1605	(0.7610, 1.3902)	1.0306	0.0296	(0.9726, 1.0886)
1999.Q3	1.0627	0.1649	(0.7394, 1.3859)	1.0193	0.0304	(0.9597, 1.0789)

[7] The standard errors are estimated using the Delta method.

1999.Q4	1.0544	0.1678	(0.7255, 1.3834)	1.0123	0.0310	(0.9516, 1.0730)
2000.Q1	1.0489	0.1697	(0.7163, 1.3814)	1.0073	0.0313	(0.9460, 1.0687)
2000.Q2	1.0440	0.1715	(0.7078, 1.3802)	1.0031	0.0316	(0.9411, 1.0652)
2000.Q3	1.0459	0.1702	(0.7123, 1.3795)	1.0049	0.0314	(0.9435, 1.0664)
2000.Q4	1.0450	0.1717	(0.7085, 1.3815)	1.0044	0.0317	(0.9422, 1.0665)
2001.Q1	1.0523	0.1686	(0.7218, 1.3828)	1.0113	0.0311	(0.9504, 1.0722)
2001.Q2	1.0502	0.1697	(0.7176, 1.3829)	1.0096	0.0313	(0.9483, 1.0710)
2001.Q3	1.0493	0.1703	(0.7155, 1.3830)	1.0090	0.0314	(0.9474, 1.0706)
2001.Q4	1.0473	0.1722	(0.7098, 1.3848)	1.0072	0.0318	(0.9448, 1.0696)
2002.Q1	1.0632	0.1650	(0.7397, 1.3867)	1.0220	0.0304	(0.9624, 1.0816)
2002.Q2	1.0613	0.1658	(0.7363, 1.3863)	1.0206	0.0305	(0.9607, 1.0804)
2002.Q3	1.0567	0.1676	(0.7282, 1.3852)	1.0166	0.0309	(0.9561. 1.0772)
2002.Q4	1.0525	0.1695	(0.7204, 1.3847)	1.0133	0.0312	(0.9520, 1.0745)
2003.Q1	1.0501	0.1701	(0.7167, 1.3834)	1.0113	0.0313	(0.9499, 1.0727)
2003.Q2	1.0449	0.1727	(0.7064, 1.3834)	1.0070	0.0319	(0.9445, 1.0694)
2003.Q3	1.0455	0.1717	(0.7091, 1.3820)	1.0077	0.0316	(0.9457, 1.0697)
2003.Q4	1.0488	0.1711	(0.7134, 1.3842)	1.0110	0.0315	(0.9492, 1.0728)
2004.Q1	1.0470	0.1715	(0.7110, 1.3831)	1.0099	0.0316	(0.9480, 1.0718)
2004.Q2	1.0444	0.1725	(0.7064, 1.3824)	1.0077	0.0318	(0.9454, 1.0700)
2004.Q3	1.0437	0.1731	(0.7044, 1.3830)	1.0076	0.0319	(0.9451, 1.0702)
2004.Q4	1.0342	0.1774	(0.6865, 1.3819)	0.9991	0.0327	(0.9349, 1.0633)
2005.Q1	1.0342	0.1770	(0.6873, 1.3810)	0.9992	0.0327	(0.9352, 1.0632)
2005.Q2	1.0247	0.1811	(0.6698, 1.3796)	0.9907	0.0334	(0.9251, 1.0562)
2005.Q3	1.0235	0.1808	(0.6690, 1.3779)	0.9898	0.0334	(0.9244, 1.0552)
2005.Q4	1.0202	0.1826	(0.6623, 1.3781)	0.9869	0.0337	(0.9208, 1.0530)
2006.Q1	1.0200	0.1825	(0.6623, 1.3776)	0.9868	0.0337	(0.9207, 1.0528)
2006.Q2	1.0163	0.1840	(0.6557, 1.3769)	0.9836	0.0340	(0.9170, 1.0502)
2006.Q3	1.0175	0.1831	(0.6587, 1.3763)	0.9849	0.0338	(0.9187, 1.0512)
2006.Q4	1.0080	0.1873	(0.6408, 1.3752)	0.9766	0.0346	(0.9088, 1.0444)
2007.Q1	1.0098	0.1862	(0.6448, 1.3747)	0.9784	0.0344	(0.9110, 1.0458)
2007.Q2	1.0090	0.1869	(0.6427, 1.3752)	0.9778	0.0345	(0.9102, 1.0454)
2007.Q3	1.0079	0.1870	(0.6414, 1.3744)	0.9769	0.0345	(0.9092, 1.0445)
2007.Q4	1.0042	0.1897	(0.6323, 1.3761)	0.9735	0.0351	(0.9048, 1.0423)
2008.Q1	1.0032	0.1904	(0.6301, 1.3764)	0.9728	0.0352	(0.9038, 1.0418)
2008.Q2	1.0071	0.1886	(0.6375, 1.3767)	0.9764	0.0349	(0.9081, 1.0448)
2008.Q3	0.9951	0.1949	(0.6131, 1.3772)	0.9658	0.0361	(0.8950, 1.0365)
2008.Q4	0.9920	0.1982	(0.6035, 1.3805)	0.9630	0.0368	(0.8909, 1.0351)
2009.Q1	0.9960	0.1954	(0.6130, 1.3789)	0.9669	0.0362	(0.8960, 1.0379)
2009.Q2	0.9946	0.1964	(0.6097, 1.3795)	0.9660	0.0364	(0.8946, 1.0373)
2009.Q3	0.9977	0.1952	(0.6151, 1.3803)	0.9691	0.0362	(0.8982, 1.0400)
2009.Q4	0.9955	0.1971	(0.6092, 1.3817)	0.9673	0.0365	(0.8957, 1.0389)
2010.Q1	0.9969	0.1962	(0.6123, 1.3815)	0.9688	0.0364	(0.8975, 1.0401)
2010.Q2	0.9979	0.1960	(0.6138, 1.3821)	0.9698	0.0363	(0.8986, 1.0410)

Table 8: Allen Elasticities of Substitution: Translog Spline Specification

CSSWSP	y1	y2	y3	y4	y5
y1	-				
y2	-291.52	-			
y3	-37.24	78.72	-		
y4	-517.75	1160.59	0.007	-	
y5	-77.47	0.0058	0.048	0.0032	-
CSSGSP	y1	y2	y3	y4	y5
y1	-				
y2	-269.06	-			
y3	-45.25	53.22	-		
y4	-815.55	1014.14	0.006	-	
y5	-116.23	0.0070	0.043	0.0023	-

Figure 1: Average Efficiencies over Time: Translog Spline Specification

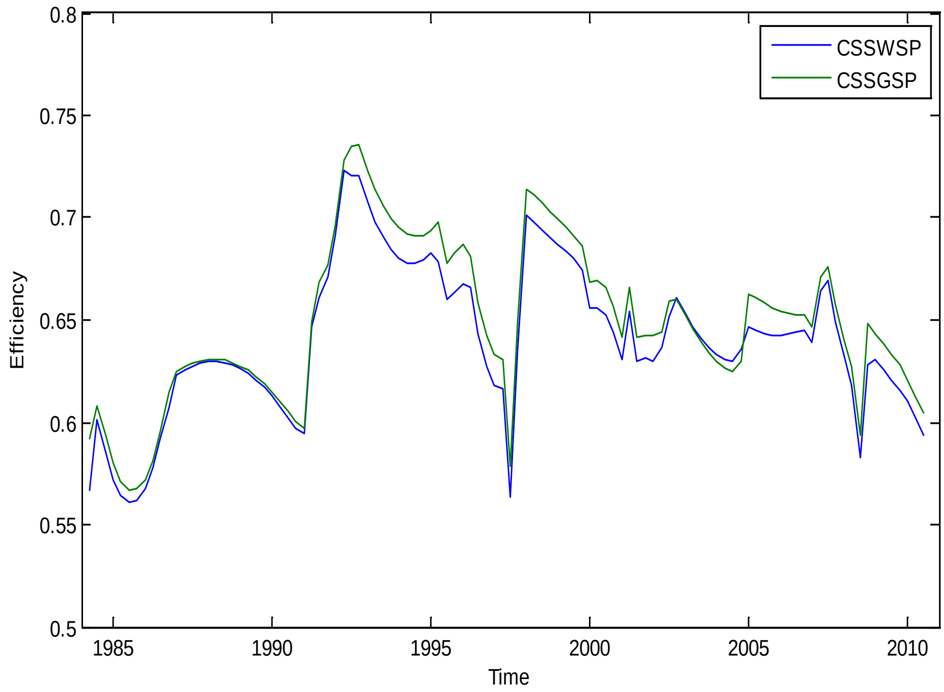

Figure 2: Average Efficiencies over Time: Translog Polynomial Specification

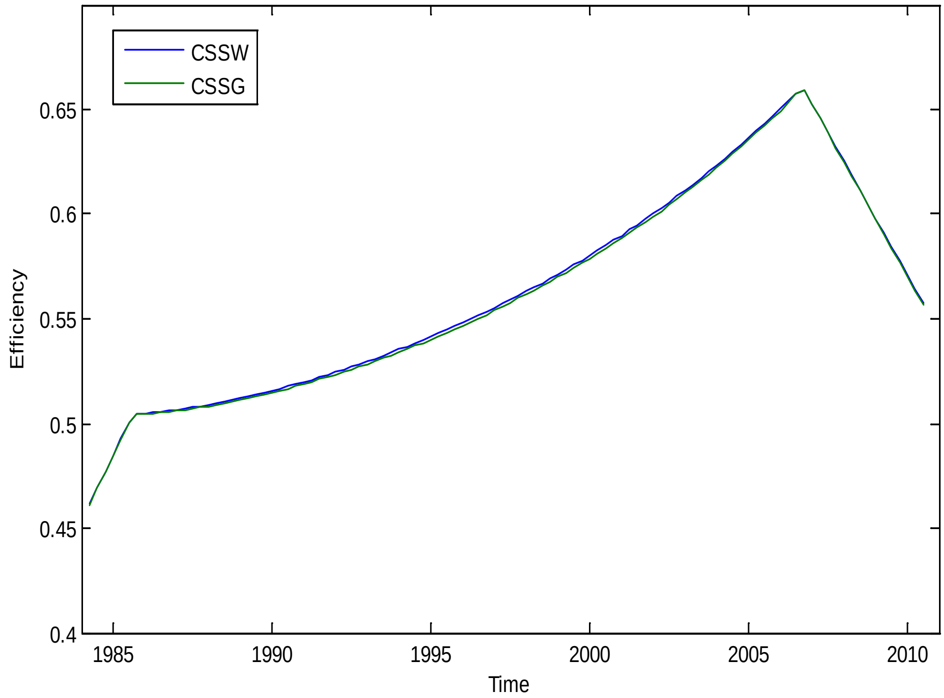

Figure 3: Average Efficiencies over Time: Cobb-Douglas Spline Specification