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VAT Reform in a Developing Country. A CGE Model with an Informal Sector

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ABSTRACT

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I develop a computable general equilibrium model that quantifies the redistributive effects and the efficiency gains of broadening the base of the Value Added Tax (VAT) by removing the zero-rating of food. I incorporate an informal retail sector in the supply of goods with two distinctive features: an indirect tax on the informal retail sector and a productivity gap between the formal and informal retail sectors. In the model the efficiency effect of the reform not only depends on the correction of the price distortion between food and the rest of the goods, induced by the zero-rate on food, but also on the changes of the average productivity of the retail sector. The size of the effect is determined by the indirect tax on the informal sector and by household’s elasticity of substitution between purchases in the formal and informal sector. I calibrate the model for Mexico and I simulate a revenue-neutral harmonization of the VAT. The results show that the efficiency gains from uniform taxation are partially offset by a reallocation of resources to the less productive informal sector. The reform has a regressive effect despite the incorporation of the informal sector as low income households can not fully avoid the tax by buying food in the informal sector.
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Chapter 1

Dissertation overview

The Value Added Tax (VAT) is one of the most important topics in the fiscal policy literature as this tax has been introduced in more than 150 countries around the world. Important efforts have been done assessing an optimal rate structure that enhances economic efficiency. So far, the common answer given by academic experts is a VAT with a single rate and no exemptions. However, this regime is rarely favored by most of policy makers as they follow the principle of reducing the tax on "basic goods" that are consumed proportionally more by low income households. For this reason, the rule in most of the countries is to have a VAT with multiple rates: reduced rates, and in some cases exemptions, for basic goods and a higher general rate for the rest of the goods.

I analyze a revenue neutral VAT reform, or harmonization of the VAT rate, consisting on eliminating the exemption of the VAT on food while introducing a single rate to all consumption. This implies to increase the tax on food and to reduce the tax on the rest of the goods. So far, the main conclusion of the existing literature is that the potential welfare gain, or loss for some studies, due to a change of the economic efficiency of the tax comes at the expense of a regressive effect on the distribution of the tax burden by income deciles. This dissertation shows that the efficiency and income redistribution effects of the reform depends on the indirect
tax on the informal sector and on household’s elasticity of substitution between purchases in the formal and informal sector.

I develop an model that despite of been closer to the economic environment faced in the emerging economies, its specification has not been sufficiently explored by the existing literature. First, my model considers the harmonization VAT as an stand alone reform and not in the context of a substitution of trade taxes for a VAT with a single rate. Although this policy is suitable for economies heavily dependent on the production of exportable raw materials, that is not the case for middle income developing nations with stronger domestic markets and a VAT regime already put in place.

Second, my model considers the effect of the informal sector on determining the welfare changes of the VAT reform. Despite that the informal sector represents around forty percent of total consumption in most of the developing world; this sector had been neglected in the analysis of the VAT reform. However, for most of these existing studies, the economy is represented by two sectors: a formal sector with tradable goods and a informal sector with non tradable goods, as home production. This view is not accurate as the informal sector is not an isolated part of the economy that operates parallel to the formal sector. Instead, in the informal sector is fully integrated with the formal sector in the supply chain of goods and services. Therefore it cannot be assumed that the informal sector does not share the burden of the taxes levied on the formal sector, as in most of the existing literature. On the demand size, the informal sector is an alternative retail option that is considered by households shopping decisions. For that reason in my model I split the retail of
goods between formal and informal retail sectors. I incorporate an informal retail sector in the supply of goods with two distinctive features: an indirect tax on the informal retail sector and a productivity gap between the formal and informal retail sectors.

The theoretical contributions of my dissertation can be grouped in two categories. On efficiency grounds, the standard literature states that a harmonizing the VAT rate removes the price distortion between untaxed food and taxed goods and therefore enhances the efficiency of the tax system. However, given that the informal sector has a lower productivity, as suggested in the literature, as households shift consumption from the formal to the informal retail sector, there is an impact on the average productivity of the retail sector. Within this context, in my model the efficiency effect of the reform not only depends on the correction of the price distortion between food and the rest of the goods, induced by the zero-rate on food, but also on the changes of the average productivity of the retail sector. The size of the effect is determined by the indirect tax on the informal sector and by household’s elasticity of substitution between purchases in the formal and informal sector.

On equity grounds the incorporation of the informal sector modifies the redistributive effects of VAT harmonization analyzed in the standard literature. Food is usually exempted to favor low income households who spend a larger fraction of their income on this good. Therefore with a single VAT rate, low income households are taxed proportionally more than high income households. However, this argument neglects that low income households can shift their consumption from the informal to the informal sector and therefore avoid, at least partially, the regressive effect of
the tax increase. My model analyzes how regressive is the reform once the partial
tax avoidance of the informal sector is incorporated.

The empirical contribution of the dissertation consists on the econometric esti-
mation of the demand system including informal sector purchases using Mexican
households’ survey data and the calibration of the revenue neutral VAT reform for
Mexico. The informal sector, although it represents a significant share of Mexican
households’ consumption, has not been included in the estimation of households’
demand. This is an important contribution as such estimations are nonexistent and
the calibration of the model requires some parameters that have not been estimated
previously. In order to calibrate the theoretical model, I estimate a demand system
for Mexican households incorporating the informal sector in order to determine key
parameters in the model. The estimation of the elasticity of substitution between
formal and informal sector is important, not only for VAT reform but for any policy
that modifies, via taxes or subsidies, any of these factors. For this reason the em-
pirical contribution of these estimations is not limited to the scope of this dissertation.

The calibration of the model shows the magnitude of VAT harmonization in
Mexico, where the federal government is considering to reform the VAT in order
to strengthen its tax income heavily dependent on oil revenue. Currently, there is
no study that analyzes the reform in the context of a general equilibrium for this
country. Therefore the empirical calibration contributes substantially to give some
reference for the tax design of the VAT reform that will be sent to the Mexican
Congress later this year.
The dissertation is divided into four chapters. The first two chapters are focused on the theoretical analysis of the reform. In the first chapter I review the existing theoretical and empirical contributions in the literature of the VAT reform. The second chapter is the General Equilibrium model that describes the environment where households make consumption decisions. In the first appendix I include an analytical solution of a simplified version of the model to indicate the theoretical effect of the tax reform.

The third and the fourth chapters are focused on the empirical analysis applied for the Mexican Economy. In the third chapter, I show the methodology and the econometric estimations using extensive survey data on households’ consumption. In the fourth chapter I calibrate the model for the Mexican economy, defining the value of the main parameters and computing the steady state values used as a benchmark in the simulation of the reform effects. At the end of that chapter I include the general conclusions of this dissertation.
Chapter 2

Literature review

2.1 Introduction

The Value Added Tax (VAT) with a single rate and no exemptions has been prescribed as the optimal rate structure that maximizes social welfare. However, this regime is rarely favored by policy makers. Instead most actual VAT systems have multiple rates: low rate or exemptions for basic good (i.e. food or medicines) and higher rates for luxury goods. For that reason, several authors have prescribed a VAT rate harmonization of the VAT rate, that is migrating the tax system from a multi-rate system to a single rate.\(^1\)

Although the harmonization of VAT rates is the most common reform proposed recently (i.e. Colombia in December 2012 or Mexico in 2013) with the exception of Piggott and Whalley (2001) most of the existing literature has analyzed the VAT harmonization within the context of a broader reform process, but not as an isolated reform. For instance Emran and Stiglitz (2005) analyze the substitution of trade taxes for the introduction of VAT regime or Bye et al. (2012) analyzed a non-uniform VAT reform. This has motivated me to focus my research toward the analysis of the VAT harmonization as an stand-alone reform.

\(^1\)See Crawford et al. (2008) for a broad summary.
Additionally, by including an informal sector in a general equilibrium model, I explicitly model some efficiency tradeoffs that have not been fully explored in the literature. I explicitly analyze the efficiency gains from removing the price distortion between taxed and untaxed goods and how these gains are increased or reduced by the existence of a retail sector in the informal sector. On the one hand, the introduction of the VAT contributes to the expansion of the informal sector by firms who want to avoid the new tax (Piggott and Whalley, 2001), but on the other hand it also has an effect on existing informal sector firms as the new tax acts as tax on inputs purchased in the formal sector (Keen, 2008). I also consider effects on income redistribution that have not been fully analyzed. Piggott and Whalley (2001) argue that the expansion of the informal sector mitigates the regressive effects of removing the exemptions of basic goods. I show, however, that low-income individuals in the informal sector can only partially avoid the tax as they bear the indirect input tax mentioned previously in the form of higher prices on purchases from informal sector retailers.

In the following pages I explain with more details each of the contributions enumerated previously and I provide an introduction to the main issues that have been considered in the formulation of the model presented in the next chapter.
2.2 The paradigm of a single VAT rate

Differentiated consumption tax rates are the rule not the exception among national VAT systems. With the notable exceptions of New Zealand, Denmark, the Dominican Republic and a few other countries, VAT exemptions are present in almost all the VAT regimes in the world. The efficiency and equity effects of these exemptions have been analyzed in the literature, which has concluded that a single VAT rate is desirable on efficiency grounds, although it might imply some regressivity of the tax.2

Efficiency considerations

The main argument in favor of a single VAT rate is that VAT exemptions represent price distortions that distort consumption and production decisions, moving the economy away from the optimal allocation (Agha and Haughton, 1996). On the production side, the Production Efficiency Theorem (Diamond and Mirrlees, 1971) states that optimal taxes should be zero on all intermediate goods leaving the relative input prices unchanged and setting the economy on its production frontier. On the consumption side, Atkinson and Stiglitz (1976) argue that under certain conditions the optimal taxation of final goods is uniform.3 Together, the results of both Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1976) imply that the optimal consumption tax is a single and flat rate on all consumption goods with no exemptions.

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2 Bird and Gendron (2007)

3 The authors assume that the utility function is weakly separable in leisure and consumption, that preferences for goods do not depend on individual skills and the government has a non-linear tax on income that is set optimally.
Equity considerations

VAT systems in the world usually have at least two rates: the general consumption rate and zero-rated or exempted goods (Bird, 2007). The rationale for this is the achievement of equity goals. VAT exemptions commonly target goods heavily consumed by the poor, so that these goods do not become prohibitively expensive (Deaton and Stern, 1986). Usually, a uniform VAT rate on consumption is expected to be regressive as low-income households spend a higher proportion of their income on consumption goods than do high-income households. For these reasons basic items such as passenger transport, medical services, charitable work, education and cooking fuel are commonly tax exempted.

Higher tax rates on "luxury" goods and lower rates on "basic" goods not only represent price distortions, and therefore efficiency losses, but are also an ineffective means of increasing progressivity as these reduced rates are generally poorly targeted and ineffective (Ebrill et al., 2001). Although high-income households spend relatively less of their income on 'basic food', they are likely to spend more in absolute terms and hence receive more benefit than the poor from such concessions. Additionally, the existing studies on incidence of the tax systems have not reached a clear conclusion whether tax differentiation has successfully achieved tax progressivity. Overall, the literature has concluded that the elimination of tax rate differentiation leads to efficiency gains but also regressive effects. In the

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4 See Bird and Gendron (2007) for excellent tables about the VAT structure in the world.
5 Some examples of this literature are Colombia: Rutherford, Light, and Barrera (2005); Ethiopia: Munoz and Cho (2003); Dominican Republic: Jenkins, Jenkins, and Kuo (2006); Peru: Haughton (2005); Russia: Decoster and Verbina (2003); Munoz and Cho (2003); South Africa: Botes (2001) or Go et al. (2005).
following section, I analyze recent literature incorporating the informal sector to this analysis and explore how these results are modified.

2.3 The VAT and the informal sector

The informal sector represents between thirty to forty percent of the economic activities in the developing world (Schneider, 2002) and generally the larger the informal economy, the more the countries rely on indirect taxes (Alm, Martinez-Vazquez, and Schneider, 2004). The informal sector is relevant for tax policy as it represents an opportunity to avoid taxes. In this context, an increase of the VAT rate would imply an expansion of the informal sector and a reduction of both formal sector output and aggregate output (Keen, 2008).

The incorporation of the informal sector in a VAT reform analysis is far from simple, as it is difficult to accurately specify the relationship between the formal and informal sector. Traditionally, the informal sector was modeled as the result of a binding restriction on entry in the formal sector: a minimum wage, start-up cost, etc. (Harris and Todaro, 1970). Under this view, the informal sector is a "residual" sector for those households and firms that cannot enter the formal sector. Most of the existing literature examining VAT reforms consider the informal sector in this way (Ab-Iorweth and Whalley, 2002; Keen, 2008; Piggott and Whalley, 2001). In these models, there is a dichotomy between the formal and informal sector, commonly modeled as home production, as both sectors

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6 The informal sector is part of the economy that escapes tax coverage. The OECD estimates that the average size of the informal sector over 1989-1993 as a percentage of GDP is 39% for developing countries and is 12% for OECD countries.
operate independently from the other. Households and firms cannot belong to both sectors simultaneously. This is one of the main weaknesses, as households’ purchases are usually divided between both sectors. By assuming that households can purchase in both sectors simultaneously, my model corrects this flaw. The main idea is that household consumption combines purchases in the formal and informal sector. Additionally, on the production side I assume that both sectors are integrated (i.e., informal sector producers use inputs from the formal sector). This approach seems to be more accurate with respect to what is observed in practice.

As I will describe in the next chapter, in my model, the informal sector consists of small firms, commonly self-employed workers or family businesses, that avoid the regulations and taxes of the formal sector. Similar to Maloney (2004) and Heckman and Pages (2004), I assume free mobility between both sectors but the interaction between them is different than analyzed in the existing literature. On the production side, formal and informal sector firms are fully integrated in the production and retail of the final consumption goods, as suggested by Keen (2008). On the demand side, survey data shows that households seem to supply labor to both sectors simultaneously (Levy, 2008); thus, in the model, households split their time between formal and informal firms. Similarly, households can purchase goods in both sectors, formal and informal sector as suggested by the survey data shown in the third chapter.

**Informal sector and efficiency effects**

The existing literature has assumed that the informal sector is less productive than the formal sector. The main reason is that firms in the informal sector have limited access to capital and low productive technologies (Loayza, 1996). This lack of
capital also leads to higher labor intensity of production (Hines, 2004) and to the impossibility of fully exploiting economies of scale (Gordon and Li, 2005; De-Paula and Scheinkman, 2006). Overall, all these factors result in lower productivity in the informal sector relative to the formal sector.

Removing VAT exemptions induces a shift of production and consumption of the newly taxed goods from the efficient taxed formal sector to the inefficient untaxed informal sector. Emran and Stiglitz (2005) find that the introduction of the VAT leads to an increase in the size of the informal sector. The authors find that expanding consumption taxes to include more goods requires a higher rate on all goods due an increase of tax evasion through the informal sector.

The existing literature has not reached a consensus as to whether a rate harmonization reform results in a net efficiency gain or loss when the informal sector is considered. On one hand, Piggott and Whalley (2001) analyze the distortion of production across the formal and informal sectors in their simulation of broadening the consumption tax base in Canada from only manufactured goods to all goods and services. The authors find that efficiency gains due to the elimination of the price distortion are diminished by the shift of consumption of newly taxed goods to the untaxed informal sector.

However, some authors claim that removing the existing price distortions under a VAT harmonization reform does not result in an expansion of the informal

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7Specifically, the reform replaced a 13.25% tax on manufactured goods by a 7% tax on all goods and services.
sector. Instead, there is an efficiency gain from removing these exemptions as they represent input distortions that benefit the informal sector. For instance, Ab-Iorweth and Whalley (2002) conclude that eliminating the VAT exemption on food would improve the efficiency of the VAT system. The authors analyze this tax in the context of meals production in the formal sector (restaurants) and informal sector (home production). The idea is that restaurants in the formal sector can deduct the new VAT on food, while the household production cannot, given that it is not a market activity. Overall, since the existing exemption represents an advantage for informal sector firms, the authors conclude that the exemption of food is socially costly in terms of welfare. Using a general equilibrium model, Ferri et al. (2009) arrive at the same conclusion when analyzing the exemption of VAT on food in Spain.

My model extends Piggott and Whalley (2001), Ab-Iorweth and Whalley (2002), Emran and Stiglitz (2005) and Keen (2008) taking some features of these models. With the exception of Keen (2008) in all these models formal and informal sector producers are completely separated. In contrast, my model assumes that both sectors are integrated in a two-stage production process. In the first stage, an intermediate good is produced in the formal sector and in the second stage, there are two retail sectors, formal and informal sectors, that buy the intermediate good and sells it to final consumers. This structure has two main advantages. First, it captures not only the input distortions of tax differentiation from the retailer perspective but also those at the final demand. The models mentioned consider the efficiency effects exclusively from the producers’ perspective only capturing the deviations from the optimal input price ratio. However, these models do not directly consider the efficiency gains from removing the distortion in final prices faced by the consumer (Atkinson and Stiglitz,
1976). This is important as the relevant efficiency effects are subject not only to the elasticity of substitution between inputs, but also to the elasticity of substitution between consumption goods. Including both effects more accurately captures the general equilibrium effects of a VAT rate harmonization reform.

The second advantage is that my model structure allows to capture the effect of taxation suggested by Ab-Iorweth and Whalley (2002) and clearly identified by Keen (2008). Despite the increase in tax induces a rise in informal sector, this expansion might not be as large as suggested by Emran and Stiglitz (2005). Keen (2008) argues that the drastic fall of tax revenue due tax avoidance underestimates the extent to which the VAT in developing countries succeeds in extracting tax revenues from the informal sector. Keen’s point is that most of the models neglect the fact that the invoice-and-credit VAT indirectly taxes production in the informal sector by denying input credits to non-registered firms (i.e., the VAT paid on formal sector production used as inputs by the informal sector). Thus, when the VAT credit chain is broken, the VAT becomes an input tax. If this is the case, the VAT might be one of the least costly means of indirectly taxing the informal sector.\textsuperscript{8} This indirect mechanism is one of the main gaps in the existing literature and it is one of the contribution of my model. Despite Keen (2008) identifies this features, he does not analyze the effect of this indirect tax in the context of general equilibrium.

\textit{The informal sector and VAT income redistribution effects}

\textsuperscript{8}The magnitude of this indirect tax depends on several factors, such as the extent to which informal sector producers purchase inputs from the formal sector. Similarly, the indirect tax is also affected by whether the VAT is shifted forward from registered firms to informal sector consumers.
The informal sector also has important implications in terms of the equity of the tax. As discussed earlier there is a common belief, mostly among policy makers, that broadening the consumption tax base is regressive because low-income households spend a larger fraction of their income on basic goods like food and medicines. Therefore, taxing these goods equally to other goods is likely to increase their price, which would make such goods, many of which are necessities, unaffordable for low-income households Ahmad and Stern (1987). For that reason, exempting basic goods like food has been popular, not only in the political arena but also in academia among some authors who argue that food, should not only be exempt from VAT but also zero-rated Kreklewetz (2004).

My model contributes to the discussion by assessing the redistributive effects of VAT base broadening in the case of the informal sector. Piggott and Whalley (2001) illustrate this suggesting that VAT base broadening including all goods is welfare worsening for the rich, as they do not have access to the informal economy to avoid the tax. In the case of the poor, they can shift demand to the informal sector, whereby they can avoid the tax and increase their own welfare. Therefore, a VAT reform in the context of the informal sector might not be as regressive as it is commonly believed. However, Piggott and Whalley (2001) assume that low-income people can avoid the entire tax. This is not necessarily true if we consider the indirect tax on informal sector retailers mentioned previously and incorporated in my model. In general, the more the informal sector is indirectly taxed the less it contributes to offset the regressive effects of the reform as suggested by Piggott and Whalley (2001).
2.4 Conclusions

The existing literature has included the formal and informal sector as two parallel sectors with limited interaction between them. There is an existing gap in the literature of one model that incorporates both sectors in a closer interaction between them. Overall, the literature has neglected the fact that informal sector retailers also buy inputs from the formal sector. This not only affects the relative input prices faced by retailers at the end of the supply chain but also this ultimately affects consumers on the demand side. Thus welfare gains are the result of a basic tradeoff between reducing the distortion in the relative price of food and the rest of the goods versus introducing a distortion in the newly taxed good between formal and the less productive informal sector.

The model presented in the next chapter incorporates this tradeoff. On efficiency grounds, I account for the expansion of the informal sector due to the introduction of a tax on food and the effect on average productivity. At the same time, I consider the indirect tax on the informal sector. In terms of equity, my model incorporates the fact that low-income households can avoid the tax Piggott and Whalley (2001) but they can only do it partially as they also pay the indirect input tax that is shifted to them from informal sector retailers Keen (2008).
Chapter 3

A CGE Model with an Informal Sector

3.1 A CGE Model with an Informal Sector

The goal of the model is to analyze the VAT harmonization eliminating existing VAT on food. The model is focused on the efficiency and redistributive effects of this reform considering the features of the informal sector and the VAT not included in the literature presented in the previous chapter.

In the model the formal and informal sector are integrated not in production but in retail sales of of goods and services. This is a different approach than the existing literature where the informal sector usually is modeled as home production or as a supply of non-tradable goods (e.g. Ab-Iorweth and Whalley (2002)).

Additionally my model incorporates the effect of the credit of VAT paid on inputs, one of the main features of the VAT. For formal sector retailers, the VAT paid for inputs is just deducted from the VAT collected from final consumers. However, informal sector retailers do not get credited for inputs purchased from formal sector retailers therefore, the informal sector does not fully evade taxation (Keen, 2008). In this case when the exemptions are removed, households buying in the informal sector can not fully evade the tax. This has important implications in the general equilibrium effects that are captured by my model and extends the results of the
existing literature.

Regarding the efficiency effects of the reform, Emran and Stiglitz (2005) suggest that the efficiency gains from removing price distortions are diminished by the expansion of a less productive informal sector. However, when the formal and informal sector are integrated, this trade off is affected by the extent that the VAT reaches the informal sector as an indirect input tax, as the introduction of the VAT might not expand the informal sector as much as the authors suggest.

Regarding equity aspects, Piggott and Whalley (2001) have suggested that removing VAT exemptions on goods largely consumed by low income households might not be as regressive as it is commonly believed. In my model I extend this result showing that the progressivity of the reform depends on the fraction of consumption allocated in the informal sector, the differences in productivity between the formal and informal sector, the indirect tax mentioned previously and households’ elasticities of substitution between formal and informal sector purchases.

Overall, this CGE model is the first one in analyze the effect of the indirect tax on the informal sector and the changes in the aggregate productivity due to the expansion of the informal sector. Before introducing the general equilibrium model, in the following section I use a simplified example to introduce these reform effects capture by my model stressing on what has being said in the existing literature and the extensions to these contributions. In the second section of this chapter, I introduce the general equilibrium model with all its components.
3.1.1 A Cobb-Douglas example

To illustrate the reform effects captured by my model, I introduce a simplified example that shows some of the main features captured by the general equilibrium model shown in the second section. This simplified version consists in an economy with households, firms and government. Households consume two goods: a VAT taxed good $M$ and VAT exempted good $F$. Households allocate all their initial endowment $X$ of the consumption of these goods paying the after tax prices $P_j(1 + \tau_j)$ solving the following problem:

$$\text{Max } U(M, F) = M^{\alpha_M} F^{\alpha_F} \quad \text{s.t.} \quad P_M(1 + \tau_M)M + P_F(1 + \tau_F)F = X \quad (3.1)$$

and getting the demand functions:

$$M^* = \frac{\alpha_M}{P_M(1 + \tau_M)} X \quad F^* = \frac{\alpha_F}{P_F(1 + \tau_F)} X \quad (3.2)$$

Firms produce each good using a constant returns to scale technology: $M = A_M X$ and $F = A_F X$ maximizing the profit functions:

$$\Pi_M = P_M A_M X - X \quad \Pi_F = P_M A_F X - X$$

such that the supply functions are given by:

$$P_M = \frac{1}{A_M} \quad P_F = \frac{1}{A_F}$$

Plugging (3.2) into (3.1) we obtain household’s indirect utility function (house-
hold welfare) defined as:

\[
\nu(p, A, X) = \left[ \frac{\alpha_M A_M M}{(1 + \tau_M)} \right]^{\alpha_M} \left[ \frac{1 - \alpha_M A_F F}{(1 + \tau_F)} \right]^{\alpha_F} X
\]  

(3.3)

The welfare effects of the tax reform are determined by changes of this indirect utility function expressed as the Equivalence Variation (EV) and defined as:

\[
EV = \nu(p^1, A, X) = \nu(p^0, A, (1 - \psi)X)
\]

where \(p^0 = (p^0_M, p^0_F)\) and \(p^1 = (p^1_M, p^1_F)\) are the vector of prices before (time = 0) and after (time = 1) the tax reform, \(A = (A_M, A_F)\) is the productivity vector and \(\psi\) is the EV expressed as the fraction of households’ income that must be given in order to compensate any change in their utility function or in other words how much money households are willing to pay in order to avert a price increase. In the case where there is a single VAT rate, such that \(\tau_M = \tau_F = \tau\), the EV \(\psi^1\) is defined as:

\[
\psi^1 = \frac{\tau}{1 + \tau}
\]  

(3.4)

3.1.2 Efficiency effects

As mentioned in the previous chapter, optimal taxation theory prescribes the existence of no distortions in household decisions. Indeed the optimal tax that minimizes the tax burden \(\psi^1\) should be zero (see Proposition 1 in Appendix 1). However, if this is not possible, the second best is to set a single flat rate for all goods such that the EV, the welfare loss of the tax, is defined by (3.4). However, most of the VAT systems have differentiated rates where one of the goods has a lower rate. For instance suppose that good F is VAT exempt or taxed at a reduced rate such that \(\tau_F < \tau_M\).
In this case the equivalence variation represented by $\psi^2$, is defined as:

$$\psi^2 = 1 - \frac{1}{(1 + \tau_M)^{\alpha_M}(1 + \tau_F)^{\alpha_F}}$$ (3.5)

Independently of how the tax rates $\tau_M$ and $\tau_F$ are set, the welfare loss of having differentiated rates is larger than in the case where there is a single tax rate that is $\psi^2 \geq \psi^1$. Thus order to minimize the DWL the policy implication is to set $\tau_M = \tau_F = \tau$ where $\psi^2 = \psi^1$ (see Proposition 2 in Appendix 1). This is the main motivation in the literature to argue that the harmonization of the VAT rate is efficiency enhancing.

Emran and Stiglitz (2005) incorporate the informal sector in the analysis of this policy recommendation. Their main finding is that the efficiency gains of removing price distortions are netted by the efficiency losses of expanding a less productive informal sector. To illustrate this point, suppose that households can consume the VAT exempted good F in either the formal sector ($FF$) or in the informal sector ($FI$), where they partially evade the VAT. In this case households solve the following problem:

$$\max U(M_{FF}, FI) = M^{\alpha_M}FF^{\alpha_F}FI^{\alpha_I} \quad \text{s.t.} \quad P_M(1 + \tau_M)M + P_{FF}(1 + \tau_F)FF + P_{FI}FI(1 + \xi\tau_F) = X$$ (3.6)

where $\alpha_M + \alpha_F + \alpha_I = 1$. Notice that informal sector purchases pay only a fraction $\xi$ of the consumption tax. Additionally, suppose that the production technology $FI = A_I X$, similar to $M$ and $FF$ technologies defined previously. In this case at the
equilibrium, households indirect utility function is defined as:

\[ \nu_i(M^* F F^*, FI^*) = \left[ \frac{\alpha_M A_M}{(1 + \tau_M)} \right]^{\alpha_M} \left[ \frac{\alpha_F A_F}{(1 + \tau_F)} \right]^{\alpha_F} \left[ \frac{\alpha_I A_I}{(1 + \xi \tau_F)} \right]^{\alpha_I} X \]  

(3.7)

and the Equivalence Variation is defined as:

\[ \psi^{2a} = 1 - \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_F)^{\alpha_F}(1 + \xi \tau_F)^{\alpha_I}} \]  

(3.8)

Similarly to the previous example, the optimal tax rate is zero (see Proposition 1a). In the case when the informal sector fully evades the tax, that is \( \xi = 0 \), and if it is not possible to set the tax rate equal to zero, the second best is to set a flat VAT tax to all goods (Proposition 2b).

This setup is more general as we can incorporate two main features incorporated in the existing literature: First, the assumption of differences in productivity: the formal sector is productive than the informal sector and, second, the informal sector purchases are indirectly taxed by the consumption tax.

Regarding the differences in productivity between formal and informal sector, Emran and Stiglitz (2005) conclude that the expansion of a less productive informal sector reduces the efficiency gains of harmonizing the consumption tax rate. To illustrate this point assume that \( A_M = A_F = A \) and \( A_I < A \). Under these assumptions, we get that the E.V. can be expressed as: ¹

¹See proposition 3 for details.
\[
\psi^3 = 1 - \left( \frac{A_I}{A} \right)^{\alpha_I} \left( \frac{1}{(1 + \tau)^{\alpha_M} (1 + \tau_F)^{\alpha_F} (1 + \xi \tau_F)^{\alpha_I}} \right)
\]  
(3.9)

In this case the difference in productivity increases the DWL as \( \psi^3 > \psi^{2a} \) (Proposition 3 in Appendix 1). Thus this efficiency loss partially offsets the efficiency gains of eliminating the price distortions.

Regarding the VAT as an indirect tax, the existing literature of the VAT reform has neglected this feature as several authors have assumed that the informal sector fully evades the tax (e.g. Emran and Stiglitz (2005)). In this case \( \xi = 0 \) and the EV can be expressed as:

\[
\psi^4 = 1 - \left( \frac{A_I}{A} \right)^{\alpha_I} \frac{1}{(1 + \tau)^{\alpha_M} (1 + \tau_F)^{\alpha_F}}
\]  
(3.10)

However, this case neglects the fact that the informal sector is indirectly taxed when the credit chain of VAT is broken (Keen, 2008). Emran and Stiglitz (2005) efficiency loss overestimates the expansion of the informal sector as it is clear that \( \psi^4 > \psi^3 \). My model incorporates the indirect taxation of the informal sector, represented in this example by the term \( \xi \). The indirect taxation of the informal sector partially offsets the negative effect of the productivity gap between the formal and informal sectors as:

\[
\frac{\partial \psi^3}{\partial \xi} = - \left[ \left( \frac{A_I}{A} \right)^{\alpha_I} \frac{\alpha_I \tau_F}{(1 + \tau)^{\alpha_M} (1 + \tau_F)^{\alpha_F} (1 + \xi \tau_F)^{\alpha_I+1}} \right] < 0
\]  
(3.11)

given that all the terms in straight brackets are positive. The main result of
my model is that the extend that the informal sector is taxed is critical to assess the efficiency losses of the VAT reform. The existing analysis of the VAT reform assuming that the informal sector fully evades the case clearly underestimates the efficiency gains of harmonizing the VAT rates.

### 3.1.3 Distributive effects

The distributive effect of the VAT reform has been controversial. There is the common perception that removing the existing VAT exemptions affects low-income households when they spend a larger fraction of their income on basic goods. To illustrate this point assume that the VAT burden can be expressed as the fraction of total tax paid divided by household’s income:

\[
\Phi_i = \frac{\tau_M M_i + \tau_F F_i + \xi \tau_F I_i}{X_i} \quad (3.12)
\]

Using the demand functions, defined in equation (3.2) and the equilibrium prices; the tax burden is defined as:

\[
\Phi_i = \left[ \frac{\alpha^i_M A_M}{(1 + \tau_M)} + \frac{\alpha^i_F A_F}{(1 + \tau_F)} + \xi \tau_F \frac{\alpha^i_F A_F}{(1 + \xi \tau_F)} \right]
\]

To assess the progressivity of the tax system I assume two types of households: high income households, the rich \((i = R)\) and low income households, the poor \((i = P)\). The main difference between them are the parameters values in their utility functions. First, I assume that the VAT exempted good is basic good that is consumed proportionally more by low income households, thus \(\alpha^P_F > \alpha^R_F\). Second, low income households consume a larger fraction of their income in the informal
sector, thus $\alpha_I^P > \alpha_I^R$. Notice that these two assumptions imply that $\alpha_M^R > \alpha_M^P$.

The progressivity of the tax system $\Theta$ is expressed as the difference between the tax burden paid by rich and poor households defined as:

$$
\Theta = \Phi^R - \Phi^P
$$

$$
\Theta = \left[ \tau_M \frac{\alpha_M^R A_M}{(1 + \tau_M)} + \tau_F \frac{\alpha_F^R A_F}{(1 + \tau_F)} + \xi \tau_F \frac{\alpha_I^R A_I}{(1 + \xi \tau_F)} \right] - \\
- \left[ \tau_M \frac{\alpha_M^P A_M}{(1 + \tau_M)} + \tau_F \frac{\alpha_F^P A_F}{(1 + \tau_F)} + \xi \tau_F \frac{\alpha_I^P A_I}{(1 + \xi \tau_F)} \right]
$$

$$
\Theta = \left[ \tau_M \frac{A_M (\alpha_M^R - \alpha_M^P)}{(1 + \tau_M)} + \tau_F \frac{A_F (\alpha_F^R - \alpha_F^P)}{(1 + \tau_F)} + \xi \tau_F \frac{A_I (\alpha_I^R - \alpha_I^P)}{(1 + \xi \tau_F)} \right] \quad (3.13)
$$

In the absence of the informal sector ($\alpha_I^P = \alpha_I^R = 0$) and when the good FF is VAT exempt ($\tau_F = 0$) the progressivity of the tax system $\Theta^0$ is:

$$
\Theta^0 = \tau_M \frac{A_M (\alpha_M^R - \alpha_M^P)}{(1 + \tau_M)} > 0\quad \text{given that } \alpha_M^R > \alpha_M^P
$$

Thus under these assumptions, the exemption of the basic good makes the tax schedule is progressive. For this reason, it is commonly believed that the harmonization of the VAT is regressive as it removes these exemptions. For instance if we set $\tau_F > 0$ the progressivity of the tax system $\Theta^1$ is defined as:
\[ \Theta^1 = \left[ \tau_M \frac{A_M(\alpha^R_M - \alpha^P_M)}{(1 + \tau_M)} + \tau_F \frac{A_F(\alpha^R_F - \alpha^P_F)}{(1 + \tau_F)} \right] \]

that is clearly larger than \( \Theta^0 \). This increase in the regressivity of the tax is the main argument against harmonization of the tax rates. Indeed the regressive effect of removing the exemptions is shown by the partial derivative of \( \Theta^1 \) respect to \( \tau_F \) defined as:

\[
\frac{\partial \Theta^1}{\partial \tau_F} = \frac{A_F(\alpha^R_F - \alpha^P_F)(1 + \tau_F) - \tau_F A_F(\alpha^R_F - \alpha^P_F)}{(1 + \tau_F)^2} = \frac{A_F(\alpha^R_F - \alpha^P_F)}{(1 + \tau_F)^2} < 0
\]

as

\[ \alpha^R_F < \alpha^P_F \]

The regressive effect of the reform is determined by the fraction of the income allocated by low income households on the purchases of the basic good \( FF \). The larger this fraction the more regressive is the reform given that \( \frac{\partial \Theta^1}{\partial \alpha^P_F} \) always takes negative values as:

\[
\frac{\partial \Theta^1}{\partial \alpha^P_F} = - \frac{A_F \tau_F}{(1 + \tau_F)} < 0
\]

However, the reform can be progressive if the informal sector is incorporated in the analysis. Piggott and Whalley (2001) argue that low income households avoid the tax while increasing their purchases in the the informal sector. Evidently this reduce
the regressivity of the tax reform that primarily affects high income households. In our example this can be shown assuming that the basic good, originally VAT exempt, can be purchased in the informal sector. Particularly by low income households who allocate a larger fraction of their consumption than high income households. That is \( \alpha^P_I > \alpha^R_I > 0 \). For simplicity assume no differences in technology: \( A_M = A_F = A_I = A \) and that VAT rates are harmonized to a single rate: \( \tau_M = \tau_F = \tau \). In this case I rewrite equation (3.13) as follows:

\[
\Theta^2 = \frac{\tau A (\alpha^R_M - \alpha^P_M)}{(1 + \tau)} + \frac{\tau A (\alpha^R_F - \alpha^P_F)}{(1 + \tau)} + \frac{\xi \tau A (\alpha^R_I - \alpha^P_I)}{(1 + \xi \tau)}
\]

given that \( \alpha^i_M + \alpha^i_F = 1 - \alpha^i_I \) then:

\[
\Theta^2 = A \left[ \frac{\tau}{1 + \tau} \left( (1 - \alpha^R_I) - (1 - \alpha^P_I) \right) + \frac{\xi \tau}{1 + \xi \tau} (\alpha^R_I - \alpha^P_I) \right] = A \tau \left[ \frac{1}{1 + \tau} - \frac{\xi}{1 + \xi \tau} \right] (\alpha^P_I - \alpha^R_I)
\]

When low income households can fully evade the tax through informal sector purchases, that is \( \xi = 0 \), the reform is progressive as:

\[
\Theta^2 = A \left[ \frac{\tau}{1 + \tau} \right] (\alpha^P_I - \alpha^R_I) > 0 \text{ given that } \alpha^P_I > \alpha^R_I.
\]

In other words, low income households can offset the regressive effects of the reform evading the tax purchasing in the informal sector. This is basically Piggot
My dissertation extends this result in two ways. First, I incorporate the VAT as an indirect tax on the informal sector, as suggested by Keen (2008), assuming that $\xi > 0$. Second, I incorporate the redistributive effect of the differences in productivity assuming that the productivity of the informal sector is just a fraction of the productivity in the formal sector such that $A_I = \nu_A A$ where $\nu_A \in (0, 1)$.

Under these assumptions, the regressivity of the tax system after introducing a single rate can be expressed as:

$$
\Theta^3 = \left[ \frac{\tau A (\alpha^R_M - \alpha^P_M)}{(1 + \tau)} + \frac{\tau A (\alpha^R_F - \alpha^P_F)}{(1 + \tau)} + \xi \tau \nu_A A (\alpha^R_I - \alpha^P_I) \right] \\
= \left[ \frac{1}{(1 + \tau)} (\alpha^P_I - \alpha^R_I) + \frac{\xi \nu_A}{(1 + \xi \tau)} (\alpha^R_I - \alpha^P_I) \right] A \tau \\
= \left[ \frac{1}{(1 + \tau)} - \frac{\xi \nu_A}{(1 + \xi \tau)} \right] \tau A (\alpha^P_I - \alpha^R_I) 
$$

(3.14)

In this case the regressivity of the VAT reform depends on the relationship between the formal and informal sector determined in this example by three factors:

- The difference in the share of total consumption spent in the informal sector between poor and rich households: $\alpha^P_I - \alpha^R_I$.

- The difference in productivity between both sectors: $\nu_A$.

- The indirect tax in the informal sector: $\xi$.

Regarding the differences in the consumption in the informal sector, in this example we get that any increase differences in the share of consumption in the informal
sector affects the progressivity of the tax as follows:

\[
\frac{\partial \Theta^3}{\partial (\alpha^P_I - \alpha^R_I)} = \left[ \frac{1}{1 + \tau} - \frac{\xi \nu_A}{(1 + \xi \tau)} \right] \tau A
\]

The effect on the tax incidence depends on the sign of the first term in brackets. Piggot and Whalley (2002) find that if low income households consume a larger fraction of the good in the informal sector the reform is not regressive. This would imply that this happens when:

\[
\frac{\partial \Theta^3}{\partial (\alpha^P_I - \alpha^R_I)} < 0 \quad \text{but this is true if and only if} \quad \frac{\xi \nu_A}{(1 + \xi \tau)} > \frac{1}{(1 + \tau)} \quad \text{given that} \quad \xi \nu_A < 1 \quad \text{we must have that} \quad 1 + \tau > 1 + \xi \tau.
\]

Piggot and Whaley’s result does not hold in this case. The fact that the low income households consume a larger fraction of their income in the informal sector is not a sufficient condition to assess that the reform is progressive. Evidently the differences in productivity and the indirect tax are important determinants of this result. The incorporation of these two factors is one of the contributions of my dissertation to the existing literature.

Regarding the productivity between formal and informal sector we have that the effect of \( \nu_A \) on the progressivity of the tax is defined by:

\[
\frac{\partial \Theta^3}{\partial \nu_A} = -\frac{\xi}{(1 + \xi \tau)} \tau A (\alpha^P_I - \alpha^R_I) < 0
\]

In this case the lower the productivity gap between the formal and informal (a
high value of $v_A$) the less regressive is the reform. The intuition behind this result is that $v_A$ is the inverse of the substitution rate between consumption in the formal and informal sector. Notice that the consumption prices in this model are the inverse of the productivity factor therefore the equilibrium relative price of FI relative to FF is given by:

$$\frac{P_I}{P_F} = \frac{A}{A_I} = \frac{1}{v_A}$$

This relationship implies that a reduction of the productivity gap increases the cost of shifting consumption from the formal to the informal sector. In other words, for low income households is more expensive the shift their consumption to effectively avoid the consumption tax on the basic good and therefore offset the regressive effect of the reform. This result has not been fully explored in the existing literature.

Regarding the indirect tax on the informal sector, represented by $\xi$, we have that its effect on the progressivity of the tax is given by:

$$\frac{\partial \Theta^1}{\partial \xi} = \frac{v_A A \tau (\alpha_P^I - \alpha_I^R)}{1 + \xi \tau} + \frac{\xi \tau^2 v_A A (\alpha_P^I - \alpha_I^R)}{(1 + \xi \tau)^2}$$

$$= \left[ \frac{v_A A \tau}{1 + \xi \tau} \right] \left[ 1 + \frac{\xi \tau}{1 + \xi \tau} \right] (\alpha_P^I - \alpha_I^R) > 0 \quad \text{given that} \quad \alpha_P^I > \alpha_I^R \quad (3.15)$$

This result implies that the more the informal sector is indirectly taxed by the VAT the more regressive is the reform. In an extreme case when the informal sector can not fully avoid the tax. $\xi = 1$, low income households can not evade the tax and therefore the reform is regressive. Notice that the regressivity of the reform also
depends on the difference in the share of consumption in the informal sector and in the productivity gap.

The main conclusion is that the existence of the informal sector has not a direct and clear effect on the progressivity of the reform. Instead, the integration between the formal and informal is a central elements to determine if the reform is regressive or not.

In the next section I show with more detail the general equilibrium model that in essence shows the efficiency an redistributive effects shown in this section.

### 3.2 Model Overview

The General Equilibrium evaluates the effects of harmonizing the VAT rates to a single rate to all consumption eliminating of existing VAT exemptions. The model captures the effects of this reform in terms of efficiency, income redistribution tax revenue and compensatory mechanisms intended to offset potential regressive effects of the single rate regime.

The model consists in a closed economy with a representative households, firms and government. In this economy there are two types of goods: VAT taxed goods, labeled as $M$ goods, and VAT exempted goods, labeled as $F$ goods. However, this exemption is removed after the reform is introduced.

The supply and demand of these goods is represented in Figure 1. On the right side of this figure, the production of goods occurs in two phases: production of
Figure 3.1: The general equilibrium model

\[ \text{Max } U(C, \ell) \]
\[ \{MF, MI, FF, FL, \ell\} \]

Subject to

Budget constraint:
\[ P_{MF} MF + P_{MI} MI + P_{FF} FF + P_{FL} FL = a_{avg}(H - \ell) + \ell K + T^{\ell} \]

Time constraint
\[ H_i = \ell_i + L_i + L_i' \]

Leisure time

Government budget constraint (tax collection)
\[ T^{\ell} = \tau^{\ell}_{MF} (P_{MF} M_{MF} + P_{MI} M_{MI}) + \tau^{\ell}_{FF} (P_{FF} M_{FF} + P_{MF} M_{MF}) + \tau^{\ell}_{avg} \ell L^{\ell} \]

\[ K_i = K_{MF} i + L_{MF} + M_{MF} + K_{MI} i + L_{MI} + M_{MI} + K_{FF} i + L_{FF} + M_{FF} \] (Capital market)

\[ \theta_i (H_i - \ell_i) = L_i + L_i' = L_{MF} + L_{MI} + L_{FF} \] and \[ (1 - \theta_i) (H_i - \ell_i) = L_i' = L_{MF} + L_{FI} \] (Formal and informal labor markets)
intermediate goods and retail of these intermediary goods that are transformed into
final consumption goods. All firms are assumed to behave competitively. Production
firms produce the intermediary VAT taxed and VAT exempted goods: \( M_{M^{\text{int}}} \) and
\( F_{F^{\text{int}}} \). The production of these goods is assumed to be entirely in the formal sector.
Production firms use capital and labor to produce the intermediate goods. Retailers
use this production as an input, in addition to capital and labor, and resale in either
formal or informal sector. Retailing of VAT-taxed good is divided between the formal
\( M_S^F \) and informal sectors \( M_S^I \). Similarly, the VAT exempted good retailing is also
split between formal \( F_S^F \) and informal \( F_S^I \) sectors. As we showed in the preceding
section, the relationship between the formal and informal sector is essential in the
results derived by this model.

The informal sector has two main characteristic features. First, informal sec-
tor retailers do not collect taxes (VAT or income tax) whereas retailers in the formal
sector do. In the case of the VAT, formal sector retailers charge the tax on \( M_S^F \) and
\( F_S^F \) (once the VAT exemption is removed). In contrast, informal sector retailers of
\( F_S^I \) and \( M_S^I \) do not charge any VAT at all. Similarly, informal sector firms do not
pay income tax on wages for labor inputs of each good: \( L_{M_I} \) and \( L_{F_I} \). I assume this
combined structure between formal and informal sector in order to incorporate the
idea of Keen (2008) where the VAT works as an input tax, as in my model informal
sector retailers can not deduct the VAT paid for the intermediate good purchases
from the formal sector.

Second, I am assuming differences in technology of retailing. Retailers in the
formal sector have a higher productivity than retailers in the informal sector. This
enables the model to capture the production distortion mentioned by Piggott and Whalley (2001). In this model, the difference in technology is defined assuming that retailers in the informal sector has a constant returns to scale technology whereas formal sector retailers have an increasing returns to scale technology following Piggott and Whalley (2001).

The demand of these goods is described on the left hand side of Figure 1. Households maximize an utility function that depends on total consumption $C_i$ and leisure $\ell_i$. Consumption is composite good defined by a nested structure of goods following Diamond and Zodrow (2008). The consumption good $C_i$ is a composite good that includes the VAT exempted good $F_i$ and the VAT taxed good $M_i$. $F_i$ is also a composite good that includes purchases of the untaxed good in the formal sector and informal sector: $FF^d_i$ and $FI^d_i$ respectively. Similarly, $M_i$ is a composite good that includes purchases of the originally untaxed good in both sectors: $MF^d_i$ and $MI^d_i$.

Households spend their total disposable wealth for consumption determined by:

1) labor income determined by household’s labor supply to formal and informal sector firms: $L^F$ and $L^I$, and the wage earned in formal and informal sectors: $w_f$ and $w_i$.

2) capital income determined by supplying their capital endowment $K_i$ to firms who pays a rate of return $r_k$ of capital,

3) government lump sum transfer $T^G_i$ funded by the VAT and income taxes proceeds.
Finally, at equilibrium input prices \( \{r_k, w_f, w_i\} \), households’ optimal demand \( \{MF_i^d, MI_i^d, FF_i^d, FI_i^d\} \) and firms’ optimal supply of consumption goods \( \{M_F^S, M_I^S, F_F^S, F_I^S\} \) clear implying that the input markets: intermediate goods, capital and labor market also clear (equalities shown at the bottom of Figure 1).

In the following section, I explain with more detail the functional form of each of the components of this model and I provide the specific definition of equilibrium employed.

### 3.2.1 The representative household

**Household’s utility function**

As shown in the left side of Figure 1, households allocate optimally their time and disposable wealth between consumption and leisure \( \ell_i \). Households maximize a Constant Elasticity of Substitution (CES) utility function defined as:

\[
Max U_i(C_i, \ell_i) = \left[ \alpha_i^{\frac{1}{\sigma_u}} C_i^{\frac{\sigma_u-1}{\sigma_u}} + (1 - \alpha_i^{\frac{1}{\sigma_u}}) \ell_i^{\frac{\sigma_u-1}{\sigma_u}} \right]^{\frac{\sigma_u}{\sigma_u-1}} = [U_i^*]^{\frac{\sigma_u}{\sigma_u-1}} \tag{3.16}
\]

where:

- \( C_i \) is composite consumption good,
- \( \ell_i \) is leisure time,
- \( \alpha_i^{\frac{1}{\sigma_u}} \) is the individual preference parameter of the composite consumption good,
\( \sigma_u \) is the elasticity of substitution between the composite consumption good and leisure.

The composite consumption good \( C_i \) groups taxed good and untaxed good consumption with a CES function defined as:

\[
C_i(M_i, F_i) = \left[ \alpha_i^{c} \frac{1}{\sigma_c} M_i^{\frac{1}{\sigma_c-1}} + (1 - \alpha_i^{c}) \frac{1}{\sigma_c} F_i^{\frac{1}{\sigma_c-1}} \right]^{\frac{\sigma_c}{\sigma_c-1}} = \left[ \tilde{C}_i \right]^{\frac{\sigma_c}{\sigma_c-1}} \tag{3.17}
\]

where:

\( M_i \) is the composite VAT taxed good consumption,

\( F_i \) is the composite VAT exempted good consumption,

\( \sigma_c \) is the elasticity of substitution between taxed good and untaxed good consumption,

\( \alpha_i^{c} \) is the individual preference parameter for taxed good consumption.

The composite VAT taxed good \( M_i \) is determined by household purchases in formal and informal sector defined by the following CES function:

\[
M_i(MF_i, MI_i) = \left[ \alpha_i^{m} \frac{1}{\sigma_m} MF_i^{\frac{1}{\sigma_m-1}} + (1 - \alpha_i^{m}) \frac{1}{\sigma_m} MI_i^{\frac{1}{\sigma_m-1}} \right]^{\frac{\sigma_m}{\sigma_m-1}} = \left[ \tilde{M}_i \right]^{\frac{\sigma_m}{\sigma_m-1}} \tag{3.18}
\]

where:

\( MF_i \) is VAT taxed good purchases from formal sector retailers,
$MI_i$ is VAT taxed good purchases from informal sector retailers (consumers do not pay the statutory taxes for these transactions),

$\alpha^M_i$ is the utility preference weights of taxed good purchases in the formal sector,

$\sigma_m$ is the elasticity of substitution between the purchases in the formal and informal sectors.

Likewise, the composite originally untaxed good consumption $F_i$ is divided between purchases in formal and the informal sector.

$$F_i(FF_i, FI_i) = \left[ \alpha^F_i \frac{\sigma_f^{-1}}{\sigma_f} FF_i^{\frac{\sigma_f^{-1}}{\sigma_f}} + (1 - \alpha^F_i) \frac{1}{\sigma_f} FI_i^{\frac{\sigma_f^{-1}}{\sigma_f}} \right]^{\frac{\sigma_f}{\sigma_f-1}} = \bar{F}_i^{\frac{\sigma_f}{\sigma_f-1}} \tag{3.19}$$

where:

$FF_i$ is the consumption of VAT exempt good in the formal sector. Note that after the reform, these purchases are subject to the VAT.

$FI_i$ is the consumption of VAT exempt in the informal sector. These purchases remain untaxed after the reform.

$\alpha^F_i$ is the utility preference weights of untaxed good consumption in the formal sector,

$\sigma_m$ is the elasticity of substitution between the VAT exempt consumption in formal and informal sectors.
Households time allocation problem and labor supply

As shown on the left hand side of Figure 1, households are endowed with $H_i$ hours of time to be allocated between leisure ($\ell_i$) and labor supply $L_i$

$$H_i = L_i + \ell_i$$ (3.20)

Household labor supply is split between labor supply to formal ($L^F_i$) and informal sector sector ($L^I_i$) such that:

$$L_i = L^I_i + L^F_i$$ (3.21)

The labor supply to both sectors is determined by: 1) the after-tax wage in the formal sector $\hat{w}_f$, 2) the wage in the informal sector $w_I$ and 3) household’s endowment of labor efficiency units in the formal sector $\theta_F$ and informal sector $\theta_I$. Based on this, households determines the fraction of their disposable time $H_i - \ell_i^*$ (e.g. labor supply) devoted to the formal sector $\theta_i$ and consequently to the informal sector $1 - \theta_i$. Thus $\theta_i$ can be also defined as:

$$\theta_i = \frac{L^F_i}{L^I_i + L^F_i}$$ (3.22)

Households set $\theta^*_i$ such that it maximizes household’s average wage received in both sectors $w_{avg}$ defined as:

$$Max_{\theta_i} \ w_{avg} = \theta_i \hat{w}_f \theta_F + (1 - \theta_i) w_f \theta_I$$ (3.23)

and determines, labor supply in the formal and informal sector:
\[
L^F_i = \theta_i^* \theta_F (H_i - \ell_i^*) \\
L^I_i = (1 - \theta_i^*) \theta_I (H_i - \ell_i^*)
\]  

(3.24)

At the equilibrium, from the market clearing conditions, household’s labor supply is divided among firms’ labor demands in both formal and informal sector:

\[
L^F_i = L_{Mint} + L^{MF}_i + L^{FF}_i \\
L^I_i = L^{MI}_i + L^{FI}_i
\]  

(3.25)

\(L_{Mint}\) is the labor demand for \(M_{int}\) production,

\(L^{MF}_i\) is the labor demand for \(MF\) production,

\(L^{FI}_i\) is the labor demand for \(FI\) production.

\(L^{FF}_i\) is the labor demand for \(FF\) production,

\(L^{MI}_i\) is the labor demand for \(MI\) production.

\(L_{Mint}\) is the labor demand for \(M_{int}\) production,

\(L^{MF}_i\) is the labor demand for \(MF\) production,

\(L^{FI}_i\) is the labor demand for \(FI\) production.

\(L^{FF}_i\) is the labor demand for \(FF\) production,

\(L^{MI}_i\) is the labor demand for \(MI\) production.

\(L^{FI}_i\) is the labor demand for \(FI\) production.

**Household’s Total Disposable Wealth and budget constraint**

Households budget constraint states that total consumption must be equal to their Total Disposable Wealth (TDW) such that:

\[
\hat{P}_{MF} MF_i + \hat{P}_{MI} MI_i + \hat{P}_{FF} FF_i + \hat{P}_{FI} FI_i = TDW
\]  

(3.26)
where:

\[ \hat{P}_{MF} \] is the after-tax price of \( MF \) in the formal sector,

\[ P_{MI} \] is the price of \( MI \) in the informal sector,

\[ \hat{P}_{FF} \] is the after-tax price of \( FF \) in the formal sector,

\[ P_{FI} \] is the price of consumption \( FI \) in the informal sector

and the after tax prices are defined as:

\[
\hat{P}_{MF} = (1 + \tau_{MF}^V)P_{MF} \\
\hat{P}_{FF} = (1 + \tau_{FF}^V)P_{MF} \\
\hat{w}_f = (1 - \tau^w)w_f
\]

where \( \tau^V \) is the Value Added Tax on consumption goods and \( \tau^w \) is the personal income tax in the formal sector. Note that before the reform \( \tau_{MF}^V = 0 \). After the reform, the VAT rate harmonization implies an increase of this rate such that \( \tau_{FF}^V = \tau_{MF}^V \).

Total Disposable Wealth is determined by three sources:

1) labor income (based on the labor supply \( (H_i - \ell_i) \) and the optimal \( w^*_{avg} \) determined as in (3.23)),

2) capital income (based on the fixed capital endowment \( K_i \) and return of capital \( r_k \)), 
3) and the government’s lump-sum transfer $T^G$.

defined as:

$$TDW = w^*_{avg}(H_i - \ell_i) + r_kK_i + T^G_i$$  \hspace{1cm} (3.27)

### 3.2.2 Firms

The supply of VAT taxed and VAT exempt goods consists in two stages. In the first stage, producers produce an intermediate good. In the second stage, retailers buy the intermediate good and sell it to consumers. Whereas all the production firms are assumed to be in the formal sector, retailing firms are in both formal and informal sector. Thus it is in the retailing sector where VAT taxation can be avoided by firms and consumers.

Regarding the VAT burden, I assume forward shifting. Producers shift the VAT to retailers and who shift the tax forward to consumers. Similarly, I assume that the burden of the income tax on labor in the formal sector is fully paid by wage earners. All firms are assumed to be in a perfect competition environment.

**Taxed good supply**

*Taxed good* firms produce an intermediate good using the following Cobb-Douglas technology:
\[ M_{int} = A_{Mint} K_{Mint}^\beta M_{int} L_{Mint}^\beta \]  

(3.28)

\( M_{int} \) is the taxed good intermediate good sold to retailers,

\( K_{Mint} \) is the capital input in taxed good production,

\( L_{Mint} \) is the labor input in taxed good production.

Taxed good producers maximize the profit function:

\[
\Pi_{Mint} = \hat{P}_{Mint} M_{int} - r_k K_{Mint} - w_f L_{Mint} - \tau_{MF} \} P_{Mint} M_{int} \\
= P_{Mint} M_{int} - r_k K_{Mint} - w_f L_{Mint} 
\]  

(3.29)

where \( \hat{P}_{Mint} = (1 + \tau_{MF}) \). Note that the tax \( \tau_{MF} \) does not affect profits as it is fully shifted to retailers. Producers just collect the tax and pay it to government. The retailing of this good is divided between formal and informal sector retailers. Retailers in the formal sector use the Cobb-Douglas technology:

\[
M_F = A_{MF} K_{MF}^\beta M_{MF} L_{MF}^\beta M_{int,F}^{1-\beta_{MF}-\beta_{L}}. 
\]  

(3.30)

\( M_F \) is taxed good retailing in the formal sector,

\( A_{MF} \) is total productivity factor of retailing in the formal sector,

\( K_{MF} \) is capital used by taxed good retailers in the formal sector,

\( L_{MF} \) is labor used by taxed good retailers in the formal sector,

\( M_{int,F} \) is the intermediate good purchases from producers.
and maximize the profit function:

$$\Pi_{MF} = \hat{P}_{MF}M_F - \tau_k K_{MF} - w_j L_{MF} - \hat{P}_{Mint}M_{int} + \tau_{MF}^V (P_{Mint}M_{int} - P_{MF}M_F)$$

$$= P_{MF}M_F - \tau_k K_{MF} - w_j L_{MF} - P_{Mint}M_{int}$$

(3.31)

Note that the VAT on this good, $\tau_{MF}^V$, does not affect the profit function. As the VAT collected from consumers is fully paid to government AND the VAT paid on intermediate good purchases is fully credited from retailer’s tax liabilities (e.g. tax credit).

Retailers in the informal sector use the following technology:

$$M_I = A_{MI} L_{MI}^{\beta_{MI}} K_{MI}^{\beta_{MI}} M_{int,I}^{1-\beta_{MI}}$$

(3.32)

$M_I$ is taxed good retailing in the informal sector,

$A_{MI}$ is total productivity factor of taxed good retailing in the informal sector,

$L_{MI}$ is labor used by taxed good informal sector retailers,

$K_{MI}$ is capital used by taxed good informal sector retailers,

$M_{int,I}$ is intermediate good purchases by taxed good informal sector retailers,

Following Piggot and Whalley (2001,) I am assuming that the informal sector is less productive hence the total productivity factor in $A_{MF} > A_{MI}$. This is one the most important features of the model as technology seems to affect the efficiency
effects of the reform as suggested by Ab Iorweth, and Whalley (2002) who evaluates constant and increasing returns to scale but assuming the same technologies for both sector.

Retailers in the informal sector maximize:

\[ \Pi_{MI} = P_{MI} M_I - w_I L_{MI} - r_k K_{MI} - (1 + \tau_{MF}^V) P_{int,I} M_{int,I} \]  

(3.33)

Note that firms in the informal sector cannot deduct the VAT paid for the intermediary good: \( \tau_{MF}^V P_{int,I} M_{int,I} \). Thus this represents an input tax that affects the profits as suggested by Keen (2008).

**VAT exempt good supply**

Similar to the VAT taxed good, firms produce an intermediate good using the Cobb-Douglas technology:

\[ F_{int} = A_{Fint} K_{Fint}^{\beta_{Fint}} L_{Fint}^{1-\beta_{Fint}} \]

(3.34)

\( F_{int} \) is the untaxed intermediate good production,

\( K_{Fint} \) is the capital input in untaxed good production,

\( L_{Fint} \) is the labor input in untaxed good production.

Producers maximize the profit function:
\[ \Pi_{F_{int}} = \hat{P}_{F_{int}}F_{int} - r_kK_{F_{int}} - w_fL_{F_{int}} - \tau^V_{FF}P_{F_{int}}F_{int} \]

\[ = P_{F_{int}}F_{int} - r_kK_{F_{int}} - w_fL_{F_{int}} \]

(3.35)

where \( \hat{P}_{F_{int}} = (1 + \tau^V_{FF}) \). Given that this good is exempt before the reform, initially \( \tau^V_{MF} = 0 \). After the reform, the VAT rate harmonization implies an increase of this rate, however it does not affect producers’ profits as the tax is fully shifted forward to retailers.

Retailing firms of this good are also divided between formal and informal sector firms. Retailers in the formal sector have the following technology:

\[ F_F = \frac{A_{FF}K_{FF}^{\beta^F_K}L_{FF}^{\beta^F_L}F_{int,F}^{1-\beta^F_K-\beta^F_L}}{F_{int,F}} \]

(3.36)

\( F_F \) is untaxed good retailing in the formal sector,

\( A_{FF} \) is total productivity factor of retailing in the formal sector,

\( K_{FF} \) is capital input of retailers in the formal sector,

\( L_{FF} \) is labor input of retailers in the formal sector,

\( F_{int,F} \) is the intermediate purchases from producers.

and they maximize the profit function:

\[ \Pi_{FF} = \hat{P}_{FF}F_F - r_kK_{FF} - w_fL_{FF} - P_{F_{int}M_{int}} + \tau^V_{FF} (P_{F_{int}}F_{int} - P_{FF}F_F) \]

\[ = P_{FF}F_F - r_kK_{FF} - w_fL_{FF} - P_{F_{int}M_{int}} \]

(3.37)
Similarly to the producers case, after the harmonization of the VAT rates the increase of the tax $\tau_{FF}$ does not affect retailers profits in the formal sector.

Retailers in the informal sector use the following technology:

$$F_I = A_{FI} L_{FI}^{\beta_{FI}} K_{FI}^{\beta_{FI}} F_{int,I}^{1-\beta_{FI}}$$  \hspace{1cm} (3.38)

$F_I$ is untaxed good retailing in the informal sector,

$A_{FI}$ is total productivity factor of untaxed good retailing in the informal sector,

$L_{FI}$ is labor input in the untaxed good retailing informal sector,

$K_{FI}$ is capital input in the untaxed good retailing informal sector,

$F_{int,I}$ is intermediate purchases from untaxed good producers.

Informal sector retailers maximize:

$$\Pi_{FI} = P_{FI} F_I - w_I L_{FI} - r_k K_{FI} - (1 + \tau_{FF}) P_{F_{int}} F_{int,I}$$  \hspace{1cm} (3.39)

Similarly to the VAT taxed good, retailers in the informal sector do not recover the VAT paid for the intermediary good for that reason, the introduction of the tax on this good represents an input tax.

### 3.2.3 Government

Government is assumed to have zero cost and a balanced budget. All the proceeds of taxation are used to fund the lump sum transfer given to households mentioned earlier. Government budget constraint is defined as:
\[ T^G = \tau_{MF}^V (P_{M\text{int}} M_{\text{int}} + P_{MF} M_{F}) + \tau_{FF}^V (P_{F\text{int}} F_{\text{int}} + P_{FF} F_{F}) + \tau^w w_i L_i^F \]  

(3.40)

### 3.2.4 Demand and supply functions

The optimal allocation of households (maximizing utility) and firms (maximizing profits) determine the following supply and demand functions:

**Consumption demand**

Households optimal consumption \( \{MF_i^*, MI_i^*, FF_i^*, FI_i^*\} \), are given by:

\[
MF_i^* = \alpha_i^m \alpha_i^c \alpha_i^u \left[ w_i^* H_i + r K_i + T_G^i \right] \hat{P}_{MF}^{-\sigma_m} \times \\
\times \left[ \alpha_i^m \hat{P}_{MF}^{1-\sigma_m} + (1 - \alpha_i^m) \hat{P}_{MI}^{1-\sigma_m} \right]^{\sigma_m-\sigma_c \over 1-\sigma_m} \hat{P}^{-1} 
\]  

(3.41)

\[
MI_i^* = (1 - \alpha_i^m) \alpha_i^c \alpha_i^u \left[ w_i^* H_i + r K_i + T_G^i \right] \hat{P}_{MI}^{-\sigma_m} \times \\
\times \left[ \alpha_i^m \hat{P}_{MF}^{1-\sigma_m} + (1 - \alpha_i^m) \hat{P}_{MI}^{1-\sigma_m} \right]^{\sigma_m-\sigma_c \over 1-\sigma_m} \hat{P}^{-1} 
\]  

(3.42)

\[
FF_i^* = \alpha_i^F (1 - \alpha_i^c) \alpha_i^u \left[ w_i^* H_i + r K_i + T_G^i \right] \hat{P}_{FF}^{-\sigma_f} \times \\
\times \left[ \alpha_i^F \hat{P}_{FF}^{1-\sigma_f} + (1 - \alpha_i^F) \hat{P}_{FI}^{1-\sigma_f} \right]^{\sigma_f-\sigma_c \over 1-\sigma_f} \hat{P}^{-1} 
\]  

(3.43)

\[
FI_i^* = (1 - \alpha_i^F) (1 - \alpha_i^c) \alpha_i^u \left[ w_i^* H_i + r K_i + T_G^i \right] \hat{P}_{FI}^{-\sigma_f} \times \\
\times \left[ \alpha_i^F \hat{P}_{FF}^{1-\sigma_f} + (1 - \alpha_i^F) \hat{P}_{FI}^{1-\sigma_f} \right]^{\sigma_f-\sigma_c \over 1-\sigma_f} \hat{P}^{-1} 
\]  

(3.44)
where:

\[ \bar{P} = \alpha_i^u \left[ \alpha_i^c \left[ \alpha_i^m \bar{P}_{MF}^{1-\sigma_m} + (1 - \alpha_i^m) \bar{P}_{MI}^{1-\sigma_m} \right]^{1-\sigma_c} \right]^{1-\sigma_u} + (1 - \alpha_i^c) \left[ \alpha_i^F \bar{P}_{FF}^{1-\sigma_f} + (1 - \alpha_i^F) \bar{P}_{FI}^{1-\sigma_f} \right]^{1-\sigma_f} + (1 - \alpha_i^u) w^* \text{avg}^{1-\sigma_u} \]

\[ \times \left[ \alpha_i^c \left[ \alpha_i^m \bar{P}_{MF}^{1-\sigma_m} + (1 - \alpha_i^m) \bar{P}_{MI}^{1-\sigma_m} \right]^{1-\sigma_c} \right]^{1-\sigma_c} \]

(3.45)

and

\[ w^*_{\text{avg}} = \theta_i^* \left( \hat{w}_F \theta_F^* \bar{L}_{i*}^F \right) + (1 - \theta_i^*) w_1 \theta_I \bar{L}_{i*}^I \]

(3.46)

thus following the definition of \( \theta_i \) in equation (3.22)

\[ \theta_i^* = \frac{\bar{L}_{i*}^F}{(\bar{L}_{i*}^F + \bar{L}_{i*}^I)} \]

(3.47)

**Labor supply**

\( \bar{L}_{i*}^F, \bar{L}_{i*}^I \) depend on the optimal leisure consumption defined as:

\[ \ell_i^* = \frac{1 - \alpha_i^u}{w^*_{\text{avg}}^{1-\sigma_u}} \left[ \frac{w^*_{\text{avg}} H_i + r_k K_i + T_i^G}{\alpha_i^u P_{C}^{1-\sigma_u} + (1 - \alpha_i^u) w^*_{\text{avg}}^{1-\sigma_u}} \right] \]

(3.48)

Therefore the optimal labor supplies in the labor market are defined as:

\[ \bar{L}_{i*}^F = \theta_i^* \theta_F \left( H_i - \frac{1 - \alpha_i^u}{w^*_{\text{avg}}^{1-\sigma_u}} \left[ \frac{w^*_{\text{avg}} H_i + r_k K_i + T_i^G}{\alpha_i^u P_{C}^{1-\sigma_u} + (1 - \alpha_i^u) w^*_{\text{avg}}^{1-\sigma_u}} \right] \right) \]

(3.49)
\[ L_i^* = (1 - \theta^*_u) \theta_l \left( H_i - \frac{1 - \alpha_i^u}{u_{avg}^{\sigma_u}} \left[ \frac{u_{avg}^* H_i + r_k K_i + T_i^G}{\alpha_i^u P_{C}^{1 - \sigma_u} + (1 - \alpha_i^u) w_{avg}^{1 - \sigma_u}} \right] \right) \] (3.50)

On the supply side, firms supply functions (producers and retailers) satisfy the **Pricing condition** (price equals marginal cost). Therefore the supply functions \( P_{MF}, P_{MI}, P_{FM} \) and \( P_{FF} \), are defined as follows:

\[
P^*_{MF} = \frac{w_l \beta_{MK}^M \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) A_{MK}}{\beta_{MK}^M \beta_{MK}^M \beta_{MK}^M \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) A_{MK}}
\] (3.51)

\[
P^*_{MI} = \frac{w_l \beta_{MK}^M \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) A_{MK}}{\beta_{MK}^M \beta_{MK}^M \beta_{MK}^M \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) A_{MK}}
\] (3.52)

\[
P^*_{FF} = \frac{w_l \beta_{MK}^M \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) A_{MK}}{\beta_{MK}^M \beta_{MK}^M \beta_{MK}^M \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) \beta_{MK}^M (1 - \beta_{MK}^M - \beta_{MK}^L) A_{MK}}
\] (3.53)
\[ P_{FI} = \frac{w_I^{\beta_{FI}} + (1 - \beta_{Fi}^{\text{int}})(1 - \beta_{K}^{\beta_{FI}} - \beta_{L}^{\beta_{FI}})r_k^{\beta_{FI}} + \beta_{Fi}^{\text{int}}(1 - \beta_{K}^{\beta_{FI}} - \beta_{L}^{\beta_{FI}}) (1 + \tau_{FI})^{1 - \beta_{K}^{\beta_{FI}} - \beta_{L}^{\beta_{FI}}}}{\beta_{FI}^{\beta_{FI}} \beta_{K}^{\beta_{FI}} (1 - \beta_{K}^{\beta_{FI}} - \beta_{L}^{\beta_{FI}}) (1 - \beta_{Fi}^{\beta_{Ki}}) (1 - \beta_{Fi}^{\beta_{Li}}) (1 + \tau_{FI})^{1 - \beta_{K}^{\beta_{FI}} - \beta_{L}^{\beta_{FI}}}} \]

(3.54)

In the case of producers of the intermediate goods, the supply functions are defined by

\[ P_{M_{\text{int}}} = \frac{r_k^{\beta_{M_{\text{int}}}} w_f^{1 - \beta_{M_{\text{int}}}}}{\beta_{K}^{\beta_{M_{\text{int}}}} (1 - \beta_{M_{\text{int}}}) (1 - \beta_{K}^{\beta_{M_{\text{int}}}}) A_{M_{\text{int}}}} \]  

(3.55)

\[ P_{F_{\text{int}}} = \frac{r_k^{\beta_{F_{\text{int}}}} w_f^{1 - \beta_{F_{\text{int}}}}}{\beta_{K}^{\beta_{F_{\text{int}}}} (1 - \beta_{F_{\text{int}}}) (1 - \beta_{K}^{\beta_{F_{\text{int}}}}) A_{F_{\text{int}}}} \]  

(3.56)

These supply function determines the input demand for each of these firms.

In labor markets firms’ demand are given by:

**Producers’ labor demand**

\[ L_{F_{\text{int}}}^{*} = \left( \frac{1 - \beta_{K}^{\beta_{F_{\text{int}}}}}{\beta_{K}^{\beta_{F_{\text{int}}}}} \right) r_k w_f^{\beta_{F_{\text{int}}}} F_{F_{\text{int}}}^{\beta_{F_{\text{int}}}} A_{F_{\text{int}}} \]  

(3.57)

\[ L_{M_{\text{int}}}^{*} = \left( \frac{1 - \beta_{M_{\text{int}}}}{\beta_{M_{\text{int}}}} \right) r_k w_f^{\beta_{M_{\text{int}}}} M_{\text{int}}^{\beta_{M_{\text{int}}}} A_{M_{\text{int}}} \]  

(3.58)

**Formal sector retailers’ labor demand**

\[ L_{MF}^{*} = \left( \frac{P_{M_{\text{int}}}}{1 - \beta_{K}^{\beta_{MF}} - \beta_{L}^{\beta_{MF}}} \right)^{1 - \beta_{L}^{\beta_{MF}} - \beta_{K}^{\beta_{MF}}} \left( \frac{w_f}{\beta_{L}^{\beta_{MF}}} \right)^{\beta_{L}^{\beta_{MF}} - 1} \left( \frac{r_k}{\beta_{K}^{\beta_{MF}}} \right)^{\beta_{K}^{\beta_{MF}}} M_{F}^{\beta_{K}^{\beta_{MF}}} A_{M_{F}} \]  

(3.59)
\[ L^*_F = \left( \frac{P_{F\text{int}}}{1 - \beta F_F^* - \beta F_L^*} \right)^{1 - \beta F_F^* - \beta F_L^*} \left( \frac{w_F}{\beta F_L^*} \right)^{\beta F_L^* - 1} \left( \frac{r_k}{\beta F_K^*} \right)^{\beta F_K^*} \frac{F_F}{A_{F_F}} \] (3.60)

Informal sector retailers’ labor demand:

\[ L^*_I = \left( \frac{\hat{P}_{F\text{int}}}{1 - \beta F_I^* - \beta F_L^*} \right)^{1 - \beta F_I^* - \beta F_L^*} \left( \frac{w_I}{\beta F_L^*} \right)^{\beta F_L^* - 1} \left( \frac{r_k}{\beta F_K^*} \right)^{\beta F_K^*} \frac{F_I}{A_{F_I}} \] (3.61)

\[ L^*_M = \left( \frac{\hat{P}_{M\text{int}}}{1 - \beta M^* - \beta M^*} \right)^{1 - \beta M^* - \beta M^*} \left( \frac{w_I}{\beta M^*} \right)^{\beta M^* - 1} \left( \frac{r_k}{\beta M_K^*} \right)^{\beta M_K^*} \frac{M_I}{A_{M_I}} \] (3.62)

In capital markets, firms’ capital demands are given by:

Producers’ capital demand

\[ K^*_{M\text{int}} = \left( \frac{\beta M\text{int}}{1 - \beta M_{K}} \right)^{1 - \beta M_{K}} \frac{M_{\text{int}}}{A_{M\text{int}}} \] (3.63)

\[ K^*_{F\text{int}} = \left( \frac{\beta F\text{int}}{1 - \beta F_{K}} \right)^{1 - \beta F_{K}} \frac{F_{\text{int}}}{A_{F\text{int}}} \] (3.64)

Formal sector retailers’ capital demand

\[ K^*_{M_F} = \left( \frac{P_{M\text{int}}}{1 - \beta M_{K} - \beta M_{L}} \right)^{1 - \beta M_{K} - \beta M_{L}} \left( \frac{r_k}{\beta M_{K}} \right)^{\beta M_{K} - 1} \left( \frac{w_I}{\beta M_{L}} \right)^{\beta M_{L}} \frac{M_F}{A_{M_F}} \] (3.65)
\[ K_{FF}^* = \left( \frac{P_{F int}}{1 - \beta_K^{FF} - \beta_L^{FF}} \right)^{1-\beta_K^{FF}-\beta_L^{FF}} \left( \frac{r_k}{\beta_K^{FF}} \right)^{\beta_K^{FF}-1} \left( \frac{w_f}{\beta_L^{FF}} \right)^{\beta_L^{FF}} \frac{F_F}{A_{FF}} \] (3.66)

**Informal sector retailers’ capital demand**

\[ K_{MI}^* = \left( \frac{\hat{P}_{M int}}{1 - \beta_K^{MI} - \beta_L^{MI}} \right)^{1-\beta_K^{MI}-\beta_L^{MI}} \left( \frac{r_k}{\beta_K^{MI}} \right)^{\beta_K^{MI}-1} \left( \frac{w_I}{\beta_L^{MI}} \right)^{\beta_L^{MI}} \frac{M_I}{A_{MI}} \] (3.67)

\[ K_{FI}^* = \left( \frac{\hat{P}_{F int}}{1 - \beta_K^{FI} - \beta_L^{FI}} \right)^{1-\beta_K^{FI}-\beta_L^{FI}} \left( \frac{r_k}{\beta_K^{FI}} \right)^{\beta_K^{FI}-1} \left( \frac{w_I}{\beta_L^{FI}} \right)^{\beta_L^{FI}} \frac{F_I}{A_{FI}} \] (3.68)

Finally, retailers intermediate good demands are given by:

**Formal sector retailers’ intermediate good demands**

\[ M_{int,F}^* = \left( \frac{P_{M int}}{1 - \beta_K^{MF} - \beta_L^{MF}} \right)^{-\beta_K^{MF}-\beta_L^{MF}} \left( \frac{r_k}{\beta_K^{MF}} \right)^{\beta_K^{MF}} \left( \frac{w_f}{\beta_L^{MF}} \right)^{\beta_L^{MF}} \frac{M_F}{A_{MF}} \] (3.69)

\[ F_{int,F}^* = \left( \frac{P_{F int}}{1 - \beta_K^{FF} - \beta_L^{FF}} \right)^{-\beta_K^{FF}-\beta_L^{FF}} \left( \frac{r_k}{\beta_K^{FF}} \right)^{\beta_K^{FF}} \left( \frac{w_f}{\beta_L^{FF}} \right)^{\beta_L^{FF}} \frac{F_F}{A_{FF}} \] (3.70)

**Informal sector retailers’ intermediate good demands**

\[ M_{int,I}^* = \left( \frac{\hat{P}_{M int}}{1 - \beta_K^{MI} - \beta_L^{MI}} \right)^{-\beta_K^{MI}-\beta_L^{MI}} \left( \frac{r_k}{\beta_K^{MI}} \right)^{\beta_K^{MI}} \left( \frac{w_I}{\beta_L^{MI}} \right)^{\beta_L^{MI}} \frac{M_I}{A_{MI}} \] (3.71)
\[ F_{\text{Fint,} I}^* = \left( \frac{\hat{P}_{\text{Fint}}}{1 - \beta_{K}^{F I} - \beta_{L}^{F I}} \right)^{-\beta_{K}^{F I} - \beta_{L}^{F I}} \left( \frac{r_{k}}{\beta_{K}^{F I}} \right)^{\beta_{K}^{F I}} \left( \frac{w_{I}}{\beta_{L}^{F I}} \right)^{\beta_{L}^{F I}} \left( \frac{F_{I}}{A_{FI}} \right) \] (3.72)

3.2.5 General Equilibrium and Market Clearing Conditions

The equilibrium in this model is defined by the set the feasible allocation of \( \{MF^*, MI^*, FF^*, FI^*\} \) and the set of input prices \( \{r_k^*, w_f^*, w_i^*\} \) such that

1) Given prices the allocation household maximize the utility function in equation (3.16) subject to the time restriction, equation (3.20), and the budget constraint, equation (C9).

2) Producers and retailers maximize their profit functions defined by equations (3.29), (3.31), (3.33), (3.35), (3.37) and (3.39).

3) Government has a balanced budget satisfying the budget constrain in equation (3.40).

4) All the final good and input markets: labor, capital and intermediate good, clear satisfying the following restrictions:

\[*\] **Consumption goods market clearing conditions**
Taxed good formal sector  \[ M_F = \sum_i M_{Fi}^* \] (3.73)

Untaxed good informal sector  \[ M_I = \sum_i M_{Ii}^* \] (3.74)

Taxed good formal sector  \[ F_F = \sum_i F_{Fi}^* \] (3.75)

Untaxed good informal sector  \[ F_I = \sum_i F_{Ii}^* \] (3.76)

[⋆] **Inputs market clearing conditions**

Labor market formal sector  \[ \sum_i L_{Fi}^* = L_{MF}^* + L_{Mint}^* + L_{FF}^* + L_{Fin}^* \] (3.77)

Labor market in informal sector  \[ \sum_i L_{Ii}^* = L_{MI}^* + L_{FI}^* \] (3.78)

Capital market  \[ \sum_i K_i^* = K_{MF}^* + K_{MI}^* + K_{Mint}^* + K_{FF}^* + K_{FI}^* + K_{Fin}^* \] (3.79)

Intermediate taxed good good  \[ M_{Mint} = M_{Mint,F}^* + M_{Mint,I}^* \] (3.80)

Intermediate untaxed good good  \[ F_{Fin} = F_{Fin,F}^* + F_{Fin,I}^* \] (3.81)
3.3 Conclusions

The model creates a theoretical environment closer to what it is observed in the developing countries. The informal sector is included as a fully integrated part of the supply of goods and services. The interaction between formal and informal sector is achieved through the two-step supply process where informal sector retailers purchase the intermediate good from formal sector producers. This is the key link that represents the VAT chain and how the informal sector is indirectly taxed by the VAT. Additionally, there are differences in productivity between the formal and informal retail sectors. This is important as any change in the size of the formal sector will have an effect on the average productivity of the retail sector and therefore welfare effects.

On the demand size, the informal sector is assumed to be an essential component of households shopping decisions. In the model, households decide not only in terms of what to buy: the untaxed or the taxed good, but also where to buy: formal or informal retail sector. In this way the model captures the behavioral response among goods and between sector when the relative prices change after the reform is implemented. Additionally, the model can be computed by income decile. This allows to measure the effect of the reform on the income distribution.

In the model the efficiency effect of the reform not only depends on the correction of the price distortion between food and the rest of the goods, induced by the zero-rate on food, but also on the changes of the average productivity of the retail sector. The size of the effect is determined by the indirect tax on the informal
sector and by household’s elasticity of substitution between purchases in the formal and informal sector.

The model, also gives the option of simulate different scenarios by modifying the technology gap between retail sectors or by changing the indirect tax of the informal sector. This will enable an empirical assessment of the different cases that have been analyzed in the existing literature and how the changes introduced in my model modify these results. In this way, the model enables the estimation of the combined effect of the welfare gains from removing the distortion on the relative price of food and the rest of the goods and the productivity effect of the expansion of the informal sector. The following two chapters are devoted to the estimation of the relevant parameters and the calibrations for the Mexican economy.
Chapter 4

Elasticities of substitution for Mexican household between formal and informal sectors

4.1 Introduction

In this chapter, I estimate a demand system for the Mexican economy. The analysis focuses on two issues. First, I provide a new classification of goods that is based on their tax treatment and allows to capture the effects of potential changes in households demand in response to the enactment of tax reforms. This is a contribution to the existing literature of household demand studies that omit relevant substitutions between goods that are classified within the same group. Particularly, because goods with different tax treatment are included in the same consumption group. (i.e. Urzua (2004)).

Second, I estimate a demand system that considers substitutability in two dimensions. The first one is among consumption goods, as in all the existing estimations of the demand system. The second is substitutability among retail sectors, as changes in relative prices lead consumers to not only evaluate their consumption bundles but also their shopping options. Some goods like clothing can be purchased from both formal and the informal sector retailers, while others goods, like education or medical services, can be purchased in the private or public sector. So far, no study has considered substitution into purchases in the informal sector.
even though it accounts for 45 percent of households’ expenditures.

Additionally, in the case of Mexico, there are several estimates of demand functions using household data, however the most recent dates back to 2001 (Urzua, 2001). Therefore the existing literature requires an update of the cross elasticities of substitution.

In the first section of this chapter, I review the existing literature. In the second section, I introduce the base model and its extension to incorporate the substitution between the formal and informal sectors. In the third section, I summarize the data set and all the changes and assumptions made in order to make the estimation possible. In the fourth section, I summarize the main results, particularly the estimation of the elasticity of substitution for the CES function between the formal and informal sectors, a key parameter in the calibration shown in the next chapter. In the last section I provide the conclusions.

### 4.2 Contributions to the existing literature

Since the seminal work of Timmer and Alderman (1979), the estimation of demand functions using survey data has dealt with two main issues: microeconomic consistency and econometric feasibility. Regarding the microeconomic foundations, the Almost Ideal Demand System (AIDS) proposed by Deaton and Muellbauer (1980) has been the most prominent approach, given its theoretical advantages over the competing Rotterdam model and the translog models.\(^1\) In the case of Mexico, the

\(^1\)The Rotterdam model derives Marshallian demand functions and is based on the absolute price of each good, the trans log model specification of the indirect utility function.
AIDS model has a long tradition, given the availability of high quality survey data. Jarque (1987), Campos (2002) and more recently Urzua (2004) estimate household demand systems based on the AIDS model. However, these estimates require an update these results, as to my knowledge there is no existing estimation using the most recent household data from 2010.

The second contribution is to estimate the demand system following a consumption structure suitable for the analysis of a VAT reform. Traditionally, all these estimations include in the same group some goods with different tax treatment and even different types of goods. For instance, Urzua (2001) groups in the same category: processed food, clothing and appliances. Hence, in this estimation I employ a different classification of goods that considers two key factors: the type of good and tax treatment. Instead of using 5 or 6 group of goods, as in the papers mentioned, I introduced a more detailed classification with 13 different groups. The advantage is that I group goods with more similar characteristics. For instance, I separate the expenses on gasoline, due to its completely different tax treatment. Similarly, I separate expenses related to entertainment from other expenses related to housekeeping, as we hardly believe that the elasticity of both goods is similar. In the same way, I separate health services from medicines, instead of broadly defined them as health or book purchases from the payment of tuition and fees, instead of broadly denominate them as education.

The third and most important contribution is that none of these existing de-

\footnote{Golan et al. (2001) estimate a AIDS system for the demand of meat in Mexico, Valero-Gil (2006) includes some variation of the quality of production.}
mand systems in the literature has ever included alternative shopping options where households can buy the same good. Households’ choices are not only among consumption goods (what to buy) but also among retailers or service providers (where to buy). This is relevant as a particular good, say, clothing, is priced differently depending where is purchased (a department store or a street vendor). Similarly, it might not be appropriate to group in the same category medical expenses incurred in a private hospital with the highly subsidized fees paid in the public hospitals. This is important as after a change in relative prices households can adjust their demand across shopping options without changing the total quantity consumed of a good. This situation is overlooked when the demand system only considers substitution among goods.

The main contribution of this chapter is to estimate a demand system that considers consumption substitutability in two dimensions: among consumption goods and between retailing sectors. Attaining this goal requires an extension of the traditional demand systems to a multi-dimension budgeting process. Based on the relative prices in one dimension, households substitute among goods (i.e., food vs clothing), whereas in a second dimension, households pick the retailer or service provider from different sectors. For some goods the only alternative is the private-formal sector. That is the retailer is a private firm that I assume fully complies with the pertinent tax (i.e., large grocery stores or department stores). This is the case of gasoline and transportation. For a second group of goods, the formal sector competes with a complementary informal retail sector. For some goods, like clothing or food, the alternative is the informal sector (mom and pop stores or street vendors), who at most partially comply with the
VAT. For other goods like education or medical services the alternative is the public sector, where there are no taxes at all and in some circumstances the service is free.

The incorporation of a shopping choice among retail sectors is important for three reasons. The first one is empirical. In the developing world, and in some cases even in developed economies, the informal and public sector account for a substantial fraction of households’ consumption, particularly for those with low income. In the case of Mexico, survey data suggest that 45.26 percent of total consumption is purchased in either the informal sector or in the public sector. For that reason, I include the three sectors: formal, informal and public sector in the analysis. Second, the incorporation of this second dimension affects the substitutability among goods, especially for those goods that exhibit low cross-price elasticities. In principle, it might be the case a good has small cross-price elasticities respect other goods, but a high substitutability between the shopping options.

**Microeconomic considerations**

In recent years, there have been many extensions to the AID model to ensure econometric feasibility while increasing the number of consumption goods.\(^3\) The main issue arises when the number of goods included in the model has to be reduced in order to preserve the degrees of freedom. The most common solution has been to assume a nested consumption structure in a multistage budgeting. In the first stage, the consumer allocates the budget among fewer broader categories of groups (e.g. food, clothing, etc.) so that the econometric estimation is possible. In the second stage, each group of expenditures is broken up in sub-categories (e.g., food is divided

\(^3\)See Ozcelik and Sahinili (2009)
between meat, vegetables, etc.).

The introduction of this multistage method is possible for certain types of consumer’ preferences. One common extension is a multistage AIDS or the Florida PI models (Seale et al. (1991); Theil (1996)). Both models estimate the marginal shares of each consumption category, regressing the average budget share of all groups on relative prices and a real income term, as proposed by Working (1943). The underlying assumption is that in the first stage, both models assume strongly separable preferences, commonly referred to as block independence (Theil, 1996). In essence, this assumption implies that the consumer’s utility from consuming, say food, is unaffected by the consumption share of good in another category such as transportation.

However, in the second stage, some authors argue that block dependence has to be maintained (Seale and Regmi, 2006). The argument is that in the latter stage of the multistage budgeting process, the shares of the subcategories depend on the levels of the broader categories at the first stage (e.g., the share of vegetables and meat depends on how much is spent on food). The problems connected with the first-stage allocation arise from the two necessary conditions required if the prices of all goods in a group are to be replaced with a single price index: First, preferences must be homothetic for all commodities in the same group, which implies that all the conditional income elasticities must be one. Second, preferences must be strongly

---

4 Gao et al. (1996) is good examples of this method

5 The Florida-PI use the differential approach to consumer demand describing the budget share as a function of a linear-real-income term, a quadratic-pure-price term, a cubic-substitution term, and an error term. For more description see Theil et al. (1989)
separable. In this case neither the AIDS nor the Florida PI can be estimated as they assume block independence. In order to solve this issue Edgerton (1997) shows that two stage budgeting will lead to an approximately correct allocation if preferences are weakly separable and the group price indices used do not vary too greatly with the expenditure level. Based on these assumptions, Seale and Regmi (2006) use an extension of the Florida model referring it as the Florida-Slutsky (FS) model.

For this reason, I use the Florida-Slutsky model to estimate my demand functions.\(^6\) One common critique of this approach is that weak separability implies that commodities can be partitioned into a number of "separate" groups, where a change in the price of a commodity in one group affects the demand for all commodities in another group in the same manner. This precludes any heterogeneous responses within a particular group. Some empirical evidence suggests that the effect of an increase in expenditures on food is not the same for all items included in this group (Eales and Unnevehr, 1988). It could be expected that this is also true for purchases of a good between formal and informal sectors. In other words, the effect of expenditure on one particular good must have a differentiated effect on the purchases from different retailers.

I correct for this group effect using Thompson (2004) solution for the multi stage AIDS model. Thompson argues that the calculation of the elasticities of substitution computed at the second stage have to include a differentiated effect from changes in the expenditure of the group at the first stage. For that reason Thompson

\[^6\] The Florida-Slutsky model may be written as a conditional system, that is, the demand for good \(i\) contained in group \(S_g\) is conditional on total group expenditure.
proposed a different formula for the computation of the elasticities of substitution, a
formula that is used in the estimation of the demand system. In the next section I
provide more detail on the different components of the demand system.

### 4.3 Household’s demand model

The econometric specification of the model is far from trivial. The main issue is that
the AIDS model uses expenditures weights instead of quantities in order to make the
model consistent with Deaton and Muellbauer (1980) model. From an econometric
point of view, this creates an endogeneity problem leading to biased parameters.\(^7\)
Thompson (2004) addresses this issue by estimating not only prices, but also income
in separate equations. In the case of substitutability among goods, the idea is to
get an estimate of the income elasticity for each good instead of assuming that the
group elasticity is one. This estimation allows one to estimate differentiated group
effects in the second phase of the estimation. Overall, the estimation consists of
two phases: income and consumption by group of goods. The estimation of the
substitutability between retail sector is done in a similar way. In the first stage I
estimate an expenditure function and in the second phase I estimate the demand
functions between sectors.

#### 4.3.1 Substitutability among consumption goods

The estimation of the cross-price elasticity of substitution is done in three phases. In
the first phase, I correct for the endogeneity of the price elasticity by estimating a
separate equation for income. The estimation of the AIDS model implicitly assumes

---

\(^7\)For instance, Dhar et al. (2003) find evidence of price and expenditure endogeneity in empirical
demand analysis of soft drinks.
that the elasticity of group expenditure with respect to income is 1 for all goods. This implies that the share-weighted average of the supposed income elasticities will also be one. This implication affects the estimates obtained in the subsequent stages. In order to relax this assumption, I follow Thompson (2004) who argues that the endogeneity of the weights can be solved by estimating the income parameter in an independent equation:

\[
\log y_i = D_0 + d_A \log (CPI_i) + d_cX_i + \varepsilon_i
\] (4.1)

where \(y_i\) is household income and \(CPI\) is a general consumer price deflator and \(X_i\) is a vector of socio-demographic variables, defined in Appendix 2 and described in the next section. The important parameter is the term \(d_A\) that captures the effect of households purchases on income. In the second phase, I estimate the expenditure of broader categories and I use the linear approximation of the AIDS model (LAID). The LAID model gives the share equations in an n-good system as:

\[
w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \log p_j + \beta_i \log \left( \frac{y_i}{P_i} \right) + \varepsilon_i
\] (4.2)

where \(w_i\) is the expenditure share of income of the \(i^{th}\) good, \(\alpha_i\) is the constant coefficient in the \(i^{th}\) share equation, \(\gamma_{ij}\) is the slope coefficient associated with the \(j^{th}\) good in the \(i^{th}\) share equation, \(p_j\) is the price on the \(j^{th}\) good. Commonly, the weight is estimated using the total expenditure of goods given; however, this could be an additional source of endogeneity. For this reason, the total expenditure is replaced by aggregate income \(y_i\) and the price index \(P_i\) defined by:

---

\(^8\) Although several models use a quadratic function, this functional form rises to additional difficulties (Buse (1994); Green and Alston (1990)).
\[
\log P_i = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j
\] (4.3)

The model is estimated using Minimum Square in three stages.\(^9\) In the first stage, as in Blanciforti and Green (1983) I estimate equation (4.2) using an initial guess of the nonlinear \(P\) in the AIDS model by specifying a linear Stone index defined by:

\[
\log P = \sum_{j=1}^{n} w_j \log p_j
\] (4.4)

In the second stage, I estimate equation (4.3) using the estimates \(\hat{\gamma}_{ij}\) to approximate the adjust the aggregate price index as follows:

\[
\log P_i = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \hat{\gamma}_{ij} \log p_i \log p_j
\] (4.5)

Then, I use the estimates to re-estimate equation (4.2) as the linear approximate AIDS (LA-AIDS) model. The model implies the following restrictions on the parameters in the nonlinear AIDS model:

\[
\sum_{i=1}^{n} \alpha_i = 1; \quad \sum_{i=1}^{n} \beta_i = 0; \quad \sum_{j=1}^{n} \gamma_{ij} = 0; \quad \gamma_{ij} = \gamma_{ji}
\] (4.6)

The third condition determines the homogeneity of the system while the last condition is usually referred as the symmetry condition. One advantage of the AIDS model is that the homogeneity and symmetry restrictions are satisfied while using consumption weights instead of quantities. Although at the theoretical level, using weights impose symmetry in the system, at the empirical level this feature does not

\(^9\)See Urzua (2001) for further discussion of the estimation techniques.
necessarily lead to a symmetric demand system.

With the econometric estimates I compute the cross-price elasticities. Traditionally, the price elasticities of substitution are obtained by taking the derivative of these equations respect to income. The Marshallian income elasticity is defined as:

$$\eta_i = 1 + \frac{\beta_i}{E[w_i]}$$  \hspace{1cm} (4.7)

Similarly, deriving the expression respect the logarithm of the prices, the cross-price elasticity is defined as:

$$\eta_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{E[w_i]} - \frac{\beta_i}{E[w_i]} \left[ \alpha_i + \sum_{k=1}^{n} \gamma_{kj} \log P_k \right]$$  \hspace{1cm} (4.8)

The estimation of the first phase modifies the Marshallian elasticities definitions using elasticities formulas derived by Thompson (2004) defined as:

$$\hat{\eta}_i = dy \left( 1 + \frac{\beta_i}{E[w_i]} \right)$$  \hspace{1cm} (4.9)

Similarly, deriving the expression in terms of the logarithm of the prices, the cross-price elasticity is defined as:

$$\hat{\eta}_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{E[w_i]} - \left[ \frac{\beta_i}{E[w_i]} - d_A \left( 1 + \frac{\beta_i}{E[w_i]} \right) \right] \left[ \alpha_i + \sum_{k=1}^{n} \hat{\gamma}_{kj} \log P_k \right]$$  \hspace{1cm} (4.10)

Notice that the income elasticity here is the same as the AI-only expenditure elasticity if \( dY = 1 \) and the price elasticity is the same as the AI-only price elasticity if \( dA = 0 \). That is, if the income elasticity of the group expenditure is unity (\( dY = 1 \)), then the traditional AI expenditure elasticities equal the income elasticities. Similarly,
if the group expenditure is invariant with respect to own-price \((dA = 0)\), then the estimated cross-price elasticity is the same as in the traditional AI model.

### 4.3.2 Substitutability between retail sectors

The second dimension of substitutability of consumption considers the goods that are purchased either in the formal, informal or in the public sector. In a similar way I estimate the cross-price elasticities in two phases. In the first phase, I estimate the expenditure of each category with an equation similar to equation (4.1). However in this case instead of estimating income, I estimate aggregate expenditure defined as:

\[
\log e_k = D_0 + d_A \log p_k + d_e \log (P_i) + d_c X_k + \varepsilon_k
\] (4.11)

I use an extension of the AIDS model to define the fraction of the expenditure in certain category that is spent in a specific retail sector defined by the equation:

\[
w_{ik} = \alpha_k + \sum_{l=1}^{n} \gamma_{kl} \log p_l + \beta_k \log \left(\frac{e_k}{P_k}\right) + \varepsilon_k
\] (4.12)

where \(w_{ik}\) is the share associated with the \(k\) sector of category \(i\). In this case \(\gamma_{lk}\) the slope coefficient associated with complementary subsector, \(P_k\) is the adjusted category price by the subsector correlations and is defined as:

\[
\log P_k = \alpha_k + \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j
\] (4.13)

in the nonlinear AIDS model. In this case the income elasticity, the Marshallian price elasticity and the Hicksean cross-price elasticities are defined in a similar way as in equations (4.9) and (4.10).
4.3.3 Elasticities of substitution

While the price elasticity measures the percentage response of quantity demanded to a percentage change in own price, the *elasticity of substitution* (ES) between two goods measures the percentage response in the ratio of their quantities relative to the ratio of their marginal utilities, usually expressed as prices. The elasticity of substitution measures the curvature of an isoquant and the substitutability between goods. In this case, the utility is assumed to remain constant. For that reason, the computation of ES requires the computation of the Hicksian compensated cross-price elasticity, that is defined as:

\[
\hat{\epsilon}_{ij} = \frac{\gamma_{ij}}{E[w_i]} - \left[ \frac{\beta_i}{E[w_i]} - d_A \left( 1 + \frac{\beta_i}{E[w_i]} \right) \right] \times \left[ \alpha_i + \sum_{j=1}^{n} \hat{\gamma}_{kj} \log P_k \right] + E[w_i]d_y \left[ 1 + \frac{\beta_i}{E[w_i]} \right]
\]

(4.14)

Since Hicks (1932) introduced the elasticity of substitution, many variations and generalizations of this pivotal concept have been developed, but there is still much confusion about what is measured by the various forms of the ES. Traditionally, in the empirical literature, the estimation can be derived from the demand cross-price elasticities using the Allen-Uzawa ES defined as:

\[
\sigma_{ij}^A = \frac{\hat{\eta}_i}{E[w_i]}
\]

(4.15)

Allen (1938) and Uzawa (1962) classify a pair of inputs as direct substitutes \( \sigma_{ij}^A > 0 \) (complements, \( \sigma_{ij}^A < 0 \)) if an increase in the price of one causes an increase (decrease) in the quantity demanded of the other. However, this definition relies
on absolute changes neglecting the fact that the substitution between inputs is also affected by the change of the quantity of the good whose price is changing. Taking into account this effect requires one to consider the relative changes of the price and quantity ratios. Morishima (1967) proposes that the elasticity of substitution be defined as:

\[ \sigma_{ij} = \frac{\partial \ln \left( \frac{q_i}{q_j} \right)}{\partial \ln \left( \frac{p_i}{p_j} \right)} = \hat{\eta}_{ij} - \hat{\eta}_{jj} \]  

(4.16)

The Morishima elasticity classifies a pair of inputs as direct substitutes \( \sigma_{ij}^M > 0 \) (complements, \( \sigma_{ij}^M < 0 \)) if an increase in the price ratio \( \left( \frac{p_i}{p_j} \right) \) causes the quantity of the other to increase (decrease) relative to the quantity of the input whose price has changed \( \left( \frac{q_i}{q_j} \right) \). Thus, considering an increase of \( p_j \), the Morishima elasticity leans more toward substitutability, since a substitution of \( q_j \) for \( q_i \) will lead to a higher \( \sigma_{ij} \).

The implication of this elasticity is that, if two goods are direct substitutes according to the Allen-Uzawa criterion, theoretically they must be direct substitutes according to the Morishima criterion, but if two inputs are direct complements according to the Allen-Uzawa criterion, they can be either direct complements or direct substitutes according to the Morishima criterion, depending on how the quantities and price ratios change. In the next section I summarize the data used for the estimations of the cross-prices elasticities and elasticities of substitution shown in the fourth section.
4.4 Household consumption classification

The Encuesta Nacional de Ingreso y Gasto de los Hogares (ENIGH) is the most important consumption survey at household level. The survey covers the consumption of 27,086 households. The price information was obtained from Federal Consumers Protection Office (PROFECO). Based on this information the variables were created as described in this section.

4.4.1 Household’s consumption goods classification

The ENIGH identifies 731 categories of expenditure.\textsuperscript{10} For the demand system I group them in 13 different groups as follows:

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>Cereals, meat, dairy products, vegetables, etc</td>
</tr>
<tr>
<td>Beverages</td>
<td>Non alcoholic beverages</td>
</tr>
<tr>
<td>Alcohol and Tobacco</td>
<td>Alcoholic beverage and tobacco</td>
</tr>
<tr>
<td>Clothing</td>
<td>Cloths and shoes</td>
</tr>
<tr>
<td>Books</td>
<td>Textbooks, newspapers, etc</td>
</tr>
<tr>
<td>Housing</td>
<td>Personal and house services, utilities</td>
</tr>
<tr>
<td>Medicines</td>
<td>Medicines</td>
</tr>
<tr>
<td>Medical Services</td>
<td>Hospitals, doctors, medical insurance</td>
</tr>
<tr>
<td>Education</td>
<td>Registration and tuition.</td>
</tr>
<tr>
<td>Transportation</td>
<td>Public transportation</td>
</tr>
<tr>
<td>Gasoline</td>
<td>Diesel, high and low octane gasoline</td>
</tr>
<tr>
<td>Entertainment</td>
<td>Entrainment</td>
</tr>
<tr>
<td>Auto</td>
<td>Automobile, trucks, boats, motorcycles, etc</td>
</tr>
</tbody>
</table>

Table 4.1 : Household consumption categories

For the expenditure classification, I differentiate services from goods as the substitutability between them is quite different. For instance, previous studies have

\textsuperscript{10}See de Ingresos y Gastos de los Hogares ENIGH (2010) handbook for a complete description of each of them.
combined transportation services with communication expenses or education services with education articles (e.g. Seale and Regmi (2006)). Similarly, goods like appliances and footwear have been grouped together without any reasonable justification. Additionally, I carefully group products with a similar tax treatment. For instance, gasoline should not be included with other goods like transportation or autos due to its special tax treatment.

4.4.2 Retail sectors

The survey identifies 18 categories of place of purchase. Using additional survey information, I created five additional categories in order to have a more accurate description of households’ purchases. Regarding medical services and medicines, the survey identifies the institution that provides the medical services and the medicines.\(^{11}\) I use this information to assess the place of purchase of those services. Similarly, for the expenditure on education the survey identifies whether a child attends a private or public school and if the school is public or private. Thus I can identify whether those expenses were done in public or private school. Finally, regarding non-monetary expenses, the survey identifies the institution that provides the non-monetary aid.\(^{12}\)

\(^{11}\)The survey identifies the medical expenses paid to private physicians, social security institutions and state organism health institutions (IMSS, ISSSTE, PEMEX, Navy, Army), federal government health institutions (SSA, SSA and Seguro Popular, DIF, public universities) and other philanthropic institutions (Cruzt Roja, Cruz Verde, etc.) and social programs (PROGRESA, OPORTUNIDADES).

\(^{12}\)As sources of aid, the survey identifies local government (municipal or state level), federal government and government programs (Diconsa, Liconza, Oportunidades and Sedesol) and private sources.
4.4.3 Sub-categories at the second stage of consumption

I reclassified the 23 categories in three different sectors: formal sector, informal sector, and public sector according to the distribution reported in the survey.\textsuperscript{13} In the case of expenditures of transportation, gasoline and auto I assume that there is no substitution as the consumption is clearly concentrated in only one sector. Overall, the first five categories are divided between formal and semi-informal sectors. Medicines, medical services and education are divided between private and public sectors as shown in the following table:

4.5 Price information

Prices are one of the main important sources of information. Its accuracy is crucial for the assessment of the elasticities values. The survey contains more than 730 items that can be reported by each of the 27,086 households.\textsuperscript{14} In total, the demand system required 19,772,780 prices to be estimated. The survey does not report directly the prices. For some products, the expenditure and the quantity consumed of a particular item are reported making it possible for me to estimate the price, as expenditure divided by quantity. Unfortunately quantity is not reported for all non-food items and for all households.\textsuperscript{15} In total only 40 percent of prices are directly estimated from household information. The remain-

\textsuperscript{13}In the cases whew some retailer represent a low fraction of consumption, I reclassified the consumption in other category. For instance, Government sector represented 1.28\% in food, 6.95\% for beverages, 0.05 for alcohol and tobacco, 0.77 for clothing, 0.22 for books. In all these cases its has been reclassified as semi-formal sector consumption. The informal sector represented.

\textsuperscript{14}For a detail description of each of this category see the methodological handbook of the de Ingresos y Gastos de los Hogares ENIGH (2010).

\textsuperscript{15}The survey reports quantity for food categories. In the case of non-food categories quantities are not directly reported. However for some households the quantity is reported when the non-food category is a non-monetary transfer or if the item was purchased with credit or through installments.
### GOODS PURCHASED IN TWO RETAIL SECTORS

<table>
<thead>
<tr>
<th>Formal sector</th>
<th>Informal sector</th>
<th>Private hospitals</th>
<th>Private schools</th>
<th>Pharmacies</th>
<th>Liconsa and Liconsa (government stores)</th>
<th>Public hospitals and clinics</th>
<th>Social security</th>
<th>OPORTUNIDADES</th>
<th>Non monetary contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>Super market and grocery stores</td>
<td>Municipal markets (Mercado Popular)</td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
</tr>
<tr>
<td>Beverages</td>
<td>Departamental stores</td>
<td>Nom and pops (tienda de abarrotes)</td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
</tr>
<tr>
<td>Alcohol and Tobacco</td>
<td>Overseas purchases</td>
<td>Poultry shops, butcher, bakeries, etc.</td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
</tr>
<tr>
<td>Clothing</td>
<td>Convenience stores</td>
<td>Restaurants and cafeterias</td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
</tr>
<tr>
<td>Books</td>
<td></td>
<td>Tanguis, farmers markets, street vendors</td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
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<tr>
<td>Housing</td>
<td></td>
<td></td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
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<tr>
<td>Entertainment</td>
<td></td>
<td></td>
<td>Private hospitals</td>
<td>Liconsa and Liconsa (government stores)</td>
<td>Public hospitals and clinics</td>
<td>Social security</td>
<td>OPORTUNIDADES</td>
<td>Non monetary contributions</td>
<td></td>
</tr>
</tbody>
</table>

### GOODS PURCHASED IN ONE RETAIL SECTOR

<table>
<thead>
<tr>
<th>Transportation</th>
<th>Formal sector</th>
<th>Buses and Public transit systems</th>
<th>Gas stations</th>
<th>Auto car dealers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline</td>
<td>Gas stations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auto</td>
<td>Auto car dealers</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ing 60 percent of the information had to be completed using four different approaches.

The first one is to use the average price information estimated from other households with similar characteristics. The average is computed based on the place of purchase, as described in Table 4.2, size of the city and household geographic location.\textsuperscript{16} For instance if one household did not report the price of a particular item shopped in a departmental store in a city with more than one million of inhabitants in north Mexico, I use the average price computed from other households who did in the same place and in a city of similar size. Overall, I estimated 45.6 percent of the prices in this way.\textsuperscript{17}

The second approach is used for categories like housekeeping services and fees, professional services, where we cannot impute a certain quantity. In these cases, the price equals the monthly expenditure. In the case of durable goods, like furniture or appliances, I divide the expenditure by 12 months. Similarly, in the case of vehicles I divide by 36 months. Using this method I completed 7.73 percent of all prices.

The third approach is to use the expenditure reported, but adjust the expenditure by the number of key consumers identified in the household. For instance, I divided the expenses on automobile maintenance services by the number of cars owned by the household, also reported in the survey. Among all the consumption categories using this method, education and medical services are important as the

\textsuperscript{16}See Appendix 2 for definitions.

\textsuperscript{17}For 10.59\% of prices it was possible to use the three categories combined. For 8.43 I can only use place of purchase and city size and for 4.77\% place of purchase and geographic zone. Finally for 5.7 I used place of purchase and for 16.09 I use the national average.
literature has neglected the distinction between public and private consumption. In the case of education, the survey allows me to identify if the school is private or public. In this case, I identified the number of students per household in public or private school. Additionally, I separate the expenditure made for each of the students for registration fees (paid by both public and private schools) and tuition (paid only by private schools) and I estimate the expenditure per student setting this as the price. The survey also contains detailed information of the level of education, thus each level of education elementary, secondary, high school, etc., had its own imputed price. Similarly, in the case of medical services, I divide the expenses in a public or private hospital by the number of people reporting health issues, also reported in the survey. Overall I imputed 2.21 percent of the prices using this method.

The fourth approach consists of using the average price of the items from an external source. In the case of transportation prices, I used the official prices of subway, buses, and urban commuters for representative cities for each of the regions.\textsuperscript{18} Gasoline prices are considered as the average prices observed in 2010, when the survey was conducted.\textsuperscript{19} Overall I estimated the remaining 3.47 percent of prices in this way. I verify the accuracy of the price estimations for the top 50 categories that account for 70 percent of all household consumption. I compare the average price calculated to the averages released by the Consumer Division of the General Attorney Office: (PROFECO).

\textsuperscript{18}In the case of taxi rides, the fares are base in two different ways. In some cases the city has a flat rate depending on the zone. In these cases it is assumed the middle fare. In the cases where the taxi ride cost depends on a combined rate based on an initial fix fee plus a per kilometer fee, it is assumed a ride of 10 km.

\textsuperscript{19}For low octane gasoline (Magna) it is assumed a price of 8.76 pesos per liter, for high octane gasoline (Premium) 10.1 pesos per liter and for diesel 9.12 pesos per liter.
One common flaw in the literature is the estimation of the demand system including only the items consumed by households. This leads to an endogeneity problem as the estimated system does not include all the relative prices. I correct this, estimating the system with all goods regardless if they were consumed by the household or not. For that purpose, I compute a price index that is imputed to the items with zero demand. In this way I effectively capture all of the relative prices in the demand system. For each item category, I create 160 indexed based on regional areas (4 regions), size of the city (4 sizes) and income deciles (10). Then for the items not consumed by the household I impute the price index accordingly. In this way, I include all the goods in the computation of the price index.

4.6 Socio-demographic variables

The ENIGH provide specifies household socio-demographic characteristics that are useful to identify consumption patterns. In Appendix 2, I provide more details about the categories and definitions listed below.

Demographic variables

Regarding demographic variables, I include location variables like country region and city size and income decile. Information of household head as sex, age and education level. Regarding household composition, I am classifying each member depending on their age group, sex and school attendance situation. Additionally, I am including information about how many members perceived income and how many are unemployed.
*Households’ wealth*

Household assets are important for certain categories of expenditure. For instance, in the case of housing expenses, I identified households type of house: houses, apartments or just a room, as this defines the expenditure in housing services. Similarly, I include household ownership of the property (own property, mortgaged, rented or borrowed). Similarly, I quantify the automobiles own by the household and it condition (brand new or old). These variables are determinants for categories like gasoline, autos or transportation.

*Food variables*

The estimation of the food weights also considers the fraction of expenditures devoted to meat, vegetables, dairy products, cereals (including tortilla) and other products. Additionally I created two indexes, one that measures the shopping frequency and the second based on their own assessment if they are properly nourished or not.

*Education and Health variables*

Regarding public goods, I have a detailed description of households characteristics relevant for the estimation of education, medical services and expenses. A more detailed description of these variables can be found in Appendix 2. In the case of education and book expenses I breakup each of the households’ member category by school attendance and education level. Additionally, I introduce dummy variables identifying households recipients of scholarships (cash or tuition waivers) from government or private institutions.
Health variables are divided in three main categories: health condition, medical services and institutional coverage. Regarding health condition, several variables expressed as the fraction of people with chronic health condition, receive medical attention, household members with disabilities. I developed an index of medical attention. The models also include other variables regarding some meaning reason whenever the household does not get medical attention like no money, no clinic, bad service, personal reason or no medicines. Finally, household affiliation to any health institution like social security or any public institution is also considered in the estimation.

Finally, I also include information about whether the households is enrolled in any social program like PROGRESA as government transfers represent a relevant fraction of low income households’ consumption.

4.7 Econometric Estimates of Uncompensated Price and Cross-price Elasticities

Using the information described previously, I estimate regressions 1 to 5 to compute two separate cross-price elasticities representing each of the phases described previously. Similarly, I derive the elasticities of substitution between food and other goods and between the formal and informal sectors that will be used in the calibration of the model in the next chapter.
4.7.1 Own and cross-price elasticities at aggregate level

The first cross-price elasticity matrix shows the substitutability among the consumption goods shown in Appendix 2). The own-price elasticities, the main diagonal of the matrix, are shown in the Table 4.1. Alcohol & tobacco and beverages are elastic goods while food, entertainment and medical services are inelastic, with values ranging around -0.7 followed by gasoline, auto, education and clothing whose elasticity is around -0.5. Finally, transportation, medicines and housing are the most inelastic goods. The price elasticity of food is 0.95.

![Uncompensated price elasticities](image)

Figure 4.1 : Uncompensated price elasticities

Regarding the cross-price elasticities, I provide a graphical representation of the cross-price elasticity matrix with shaded areas according to the value of the elasticity. There are two elasticities represented in the diagram. The first elasticity can be read is following along one row. In this case, the color represents the effect of a change
in the price of the good labeled in the row on the quantity demanded of other goods (represented by columns). The elasticity is read following a specific column. In this case the color represents the effect that the price of the other goods (represented by rows) have on the quantity demanded of the good labeled in the column. For easy visualization, the darker the color, the more negative the elasticity of substitution, meaning that both goods tend to be complements. Similarly, the lighter the color, the more positive the elasticity of substitution meaning that both goods are substitutes.

Figure 4.2 : Uncompensated cross-price elasticities
Overall, **housing, transportation and clothing** not only have a low own-price elasticity, but also low uncompensated price elasticity as a change in the price of these goods have an effect below 0.5 in most cases (note the yellow cells). However, changes in the price of other goods do have an impact on the demand for these items (note dark cells in the columns of each of these goods). **Entertainment, books and medical services**, have a high substitutability with respect to other goods, as a one percent increase in the price of any of these goods have an effect above 0.5 on most of the other goods (see the lighter rows). Following these goods by column, we can see that the impact of a price increase of the other goods on these items is more symmetric. A pairwise comparison shows that medical services and medicines are strong substitutes (see the light squares at their respective intersection) suggesting a propensity to self-medication when the price of physician consultation increases. **Food, beverages, medicines and education** show a high complementarity with respect other goods, with elasticities ranging between -0.6 and -1.5, (see dark colored rows). In the case of food and beverage, this complementarity is symmetric (see dark colored columns) columns with mostly relatively dark cells. In particular, the pairs beverage-alcohol & tobacco, auto-transportation and medicines-food show that they are strong complements.

### 4.7.2 Income and cross-price elasticities between retail sectors at the aggregate level

The second set of cross-price elasticity of substitution is defined with respect to substitution between the retail sectors within each consumption group. The following figures show the uncompensated income elasticities and cross-price elasticities of substitution between formal (private) and informal (public) sectors by type of good.
The matrix is shown in Appendix 2.

Regarding income elasticity (left graph) for **beverages, alcohol & tobacco, clothing, medicines and housing** the income elasticity in the formal sector is larger than in the informal sector. For these goods, more income leads to spend a larger fraction of consumption in the formal sector. In the case of medicines, an increase in income has a minimal effect on purchases in the formal sector, but significantly increases it does in the consumption in the public sector. **Food and education** have similar income elasticities in both retail sectors. For **books, medicines and medical services**, the income elasticity in the informal (public) is larger than in the formal (private) sector, however in the case of books the elasticities are small thus this difference might not be relevant. In the case of medical services, an increase in income has a considerable effect on reducing the consumption in the public sector and increasing it in the private sector.

![Figure 4.3: Income elasticity by retail sector](image)

Regarding the own-price elasticities by sector (right graph), for food, alcohol and
tobacco and medicines the demand in the informal sector is more elastic than in the formal sector. Traditionally, food and medicines are considered inelastic goods. However, for both goods demand in the informal (public) sector is elastic; in particular, in the case of food where the own-price elasticity in the informal sector is twice as large the one in the formal sector. For the rest of the goods the demand in the formal sector is more elastic than the demand in the formal sector. Noteworthy are the cases of books and education and to a lesser extent medical services where the demand in the formal sector is quite elastic.

![Figure 4.4: Own-price elasticities of expenditure](image)

The interaction between the formal (private) sector versus the informal (public) sector is captured by the cross-price elasticities. The following graph shows that the cross-price elasticities between the formal and informal sectors are far from being symmetric. Only for beverages, medical services and alcohol the cross-price elasticities between formal-informal and informal-formal do have the same sign. In the case of beverages both sectors are substitutes in both directions (see upper right quadrant of the figure below) whereas in the case of alcohol & tobacco and medical services
both sectors are complements (low left quadrant). However, in the case of medical services, consumption in the public sector is very inelastic to changes in the price in the formal sector.

Figure 4.5: Mapping the cross-price elasticities between formal and informal sector

For the rest of the goods there are two situations. The first one is the case of food and medicines the consumption in the formal sector is a substitute of the informal sector, while the consumption in the informal sector is a complement of the formal sector (see upper left quadrant). The second one is the case of housing, clothing and books. In this case the formal sector is a complement of the informal sector whereas the informal sector is substitute of the consumption in the formal sector (low right quadrant).
For both goods, an increase in the relative price of the formal sector with respect to the informal sector leads to a shift of consumption from one sector to as households substitute consumption between sectors before reducing overall consumption. However the mechanism is different. In the first group, the complementarity between informal and formal sector are the key factor that leads the shifting while for the second group the substitutability is the driving force.

4.7.3 Own and cross-price elasticities by income decile

The cross-price elasticities are shown in Appendix 2. Base on the trends observed among income deciles, I classify the consumption goods in four main groups.

The first group consists in goods that become more elastic as income increases. That is the case of medicines, automobiles, food and alcohol. In the case of the first two the increase of the elasticity is exponential for the top income deciles. The second group, are goods whose elasticity decreases with income. That is the case of clothing, entertainment and books.

The third group are goods whose elasticities peaks for a particular income level. For instance, for transportation and beverages, the demand is more elastic for the V and VI deciles. In the case of housing services the demand is practically inelastic for low income households, but becomes elastic for the upper half of the income distribution. Finally, the fourth group are those goods that do not show a clear trend as the elasticity increases and decreases throughout the income distribution showing multiple peaks. For education, the peaks is at deciles III and X; for medical services the peak is at the deciles III and VIII; whereas in the case of gasoline the elasticity increases at both extremes of the income distribution.
Figure 4.6: Increasing and decreasing own-price elasticities by income decile

Regarding the cross-price elasticities Figure 4.8 shows the changes of the cross-price elasticities in the formal and informal sector (the values are shown in Appendix 2). The series labeled as $Q_{informal}$, and represented by a dashed blue line, refer to the effect on the consumption in the informal sector (public sector) due to a one percent increase in the price in the formal sector (private sector). The inverse relationship is shown by the series labeled as $Q_{formal}$, represented by a dashed red line.

Regarding food, books and education, there elasticities are roughly constant throughout the income distribution. In the case of food, the absolute value of both elasticities increases for the top half of the income distribution. This implies that high income deciles are more susceptible to shift more food consumption between both sectors due to a change in relative prices. In the case of clothing, housing and medicines the demand at bottom half of the distribution is more elastic than
the upper half. In the case of housing, the demand of the top three deciles is quite inelastic.

Finally, for alcohol & tobacco, medical services and beverages, demand is more elastic in the middle part of the income distribution. In the case of medical services, for the bottom and the top of the income distribution public and private medical services are complements. To a lesser extent we can observe a similar pattern in the case of medicines. This might be explained by the fact that low income people do not have the means to pay for private medical services, and thus only get medical coverage from public institutions, whereas households in the high income deciles
primarily rely on private services.

4.8 Estimation of elasticities of substitution for a CES function

The estimation of the demand system has a particular purpose: the estimation of the elasticities of substitution that will be used in the nested functions. In this last section I show that the estimates were obtained based on the demand system.

For the computation of the elasticities of substitution, I calculate the Hicksian own and cross-price elasticities (see Appendix 2) using equations (4.14) and (4.16). The elasticity of substitution computed using these estimates are shown in Appendix 2.

Elasticities of substitution between food and other goods

The computation of the elasticity of substitution between food and all other goods, denoted as \( \sigma_c \) in the model, requires to the weighted average of the estimated of the elasticity of substitution between food and each of the other goods. As shown in Figure 4.9, the elasticity of substitution decreases as income increases. Low income households tend to substitute more food purchases than high income households when the price of food increases, In the aggregate the elasticity of substitution between food and all other goods is \( \sigma_c = 0.75 \).

Elasticities of substitution between the formal and informal sectors \( \sigma_F \) and \( \sigma_M \)

The elasticity of substitution between formal and informal sectors there is not only one is shown in Appendix 2 and graphed in the following figure. The goods have
Figure 4.8: Cross-price elasticity of substitution
been sorted in an increasing order of the value of the elasticity of substitution. Beverages, clothing and housing show the lowest elasticities of substitution, while alcohol, education and medical services have the largest elasticities of substitution.

These estimates are used to calibrate the elasticities of substitution between food and all other goods in both the formal and the informal sectors in the CES functions in the CGE model analyzed in the next chapter. At the aggregate level, the elasticity of substitution between formal and informal sector food consumption is \( \sigma_{F}^i = 0.36 \). In the case of other goods, the weighted average of all other goods is \( \sigma_{M}^i = 0.45 \).

**Elasticities of substitution between formal and informal sector**

The estimation of the elasticities of substitution by income deciles is shown in Figure 4.11. Overall, the consumption categories can be classified in three different groups.
1. For books, alcohol, and education, we observe a sharp increase in the substitutability between the formal and informal sector, particularly for deciles VI to IX where the substitutability between both retail sectors is quite large. These deciles show the biggest increase in their consumption in the informal (public) sector after a price increase in the relative price of formal (private) sector.

1. For food, beverages, and housing, the elasticities of substitution tend to be small. For beverages and housing, the elasticity slightly decreases with income, suggesting that low income deciles will have a relatively large increase in their consumption of goods in the informal sector after an increase in the price of the formal sector. For housing, we observe that there is a small increase with income.
1. For clothing, medicines and medical services, there is a sharp decrease in the elasticity of substitution as income increases. This implies that the bottom third of the distribution will show the largest increase after a change in the relative price in the formal sector.

4.9 Conclusions

The estimation of the demand system for Mexican households described in this chapter reveals some interesting facts. First, the traditional grouping of goods loses key information about the substitutability between consumption goods, particularly for those goods that currently benefit from tax exemptions such as food, medicines, medical services, education and books.
The incorporation of substitutability between the formal and informal sectors adds additional information that cannot be omitted in the analysis of tax reform. The first fact, not directly addressed in this dissertation, is that the relationship between both retail sectors is not symmetric. An increase in the price in formal sector does not have the same impact on consumption in the informal sector as does the effect of changes in the price of the informal sector on the consumption in the formal sector. This challenges the standard literature as the empirical results contradict the theoretical assumption that the Slutsky equation is symmetric.

The second is regarding some goods that apparently are inelastic. The apparent low elasticity of these goods is due to substitution between formal and informal sectors, such as housing expenditures and food. In the context of the harmonization of the VAT this finding is relevant as both categories account for a substantial fraction of households consumption.
5.1 Introduction

I estimate the efficiency effects of a revenue neutral VAT reform and its incidence across the income distribution. I simulate a reform under which an increase in the VAT rate on food is accompanied by an offsetting reduction of the VAT rate on other goods. The goal is to set a single VAT rate to all goods. In the initial equilibrium the VAT rate on food (good $F$ in the model) is zero, the standard VAT rate on other goods (good $M$) is 16 percent. The average VAT rates in the two formal and informal sectors are 10.43 and 4.2 percent respectively.\footnote{The current statutory VAT rate is 16\%. However, among good M I includes the consumption of VAT exempted goods as education or medical expenses. For this reason, the effective rate is 10.40\%. For the purposes of this chapter, this is the effective rate on other goods as no tax change is assumed beside the VAT on food.} Given this initial equilibrium, I simulate the five scenarios summarized in Table 5.1.

In the first case, which serves as a benchmark, I assume no informal retail sector therefore there is no tax evasion and no productivity gap between the retail sectors. The goal is to measure the efficiency gains that would be obtained from eliminating the price wedge between food and other goods induced by the zero rate on food. I neglect the effects of tax evasion in the informal sector and the differences in productivity between the formal and informal retail sectors in order to compare the
<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Effective VAT rate</th>
<th>Productivity gap</th>
<th>Indirect VAT on informal sector</th>
<th>Effect measured</th>
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<tbody>
<tr>
<td></td>
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<td>60 % of VAT</td>
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*Price wedge reduction*

*Effect of productivity gap*

*Effect of tax evasion*

*Combined effect: productivity gap + tax evasion*

*Reform effects on the Mexican Economy*
results obtained to those of Emran and Stiglitz (2005), Piggott and Whalley (2001) and Bye et al. (2012).

In the second case, I introduce an informal retail sector assuming a productivity gap between the formal and informal sectors. The goal is to measure the effect of the change in the average productivity of the retail sector induced by the VAT reform. In this case I keep the assumption that the informal sector is fully taxed in order to capture only the effect of the productivity change on households’ welfare.

In the third case, I assume no productivity gap between formal and informal retail sectors, instead I assume that the informal sector fully evades the tax. In this case, I analyze the efficiency effects when the VAT can be avoided and therefore the distortion of the relative price of food and other goods is not fully corrected by removing the zero rate on food. This scenario is similar to those analyzed by Ab-Iorweth and Whalley (2002) and Ferri et al. (2009). In the fourth case I assume full evasion of the tax in the informal sector and productivity differences between formal and informal retail sectors. In this case, I capture the combined effect of all the elements mentioned previously. This case is similar to the one analyzed by Emran and Stiglitz (2005).

In the last case, I incorporate Keen’s critique of Emran and Stiglitz (2005) results by assuming some indirect taxation in the informal sector. In this case, the VAT paid on inputs is never credited or refunded by firms in the informal sector since they evade the tax. As a consequence there is some indirect taxation on the informal sector, issue that has not been fully explored in the literature. Although Munk
(2009) incorporates some indirect tax on the informal sector, the author examines this case in a theoretical model that does not include any productivity difference between the formal and informal sectors. In this last scenario I provide my preferred estimates of the combined effects of VAT harmonization in Mexico: the effect of the price wedge, the effect of the productivity gap between the formal and informal retail sectors, and the effect of partial indirect taxation of the informal sector due to taxation under the VAT of some of its inputs.

In the first section of this chapter I show the national accounts data used in the calibration. In the second section, I review the key literature regarding the parameter values used in the calibration of the model. In the third section, I show the calibration of the initial equilibrium for each of the five cases mentioned previously. In the fourth section I evaluate the efficiency effects of the harmonization reform and finally, in the last section I analyze the incidence of the tax reform on households at different points in the income distribution.

5.2 The National Accounts data and calibration

The calibration of the model uses the national accounts (NA) data obtained from Sistema Nacional de Cuentas Nacionales published by Instituto Nacional de Estadística Geografía e Informática (SNCN-INEGI). The estimation of the tax base is shown in (Table 5.2).² Currently, the Mexican government only collects 49 percent of all the

²The VAT base uses the adjusted aggregate demand by the taxable consumption. I subtract gross fixed capital formation, inventory variation, exports and government consumption and VAT exempt private consumption such as imputed rent, net acquisitions in foreign markets and construction. The estimated VAT base equals 28.5 percent of GDP (row F).
potential tax collection. The non-collected VAT is around two percent of the GDP. This is explained by the evasion of the tax, the reduced VAT on the border regions and the zero rate on food, that accounts for almost one percent of the GDP.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Billions of pesos</th>
<th>% of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Aggregate demand in 2010</td>
<td>17,153</td>
</tr>
<tr>
<td></td>
<td>Minus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gross fixed capital formation</td>
<td>2,584</td>
</tr>
<tr>
<td></td>
<td>Inventory variation</td>
<td>499</td>
</tr>
<tr>
<td></td>
<td>Exports (FOB)</td>
<td>3,965</td>
</tr>
<tr>
<td></td>
<td>Government consumption</td>
<td>1,542</td>
</tr>
<tr>
<td>B</td>
<td>Private Consumption</td>
<td>8,563</td>
</tr>
<tr>
<td>C</td>
<td>VAT-Exempt Private Consumption</td>
<td>1,667</td>
</tr>
<tr>
<td>C.1</td>
<td>Imputed rent</td>
<td>1,111</td>
</tr>
<tr>
<td>C.2</td>
<td>Net acquisitions in foreign markets</td>
<td>-60</td>
</tr>
<tr>
<td>C.3</td>
<td>Construction</td>
<td>616</td>
</tr>
<tr>
<td>D = B-C</td>
<td>Aggregate demand subject to VAT at market prices</td>
<td>6,896</td>
</tr>
<tr>
<td>E</td>
<td>VAT revenue (National Accounts-Minister of Finance)</td>
<td>505</td>
</tr>
<tr>
<td>F = D-E</td>
<td>VAT Base</td>
<td>6,392</td>
</tr>
<tr>
<td>G</td>
<td>Potential VAT collection</td>
<td>1,023</td>
</tr>
</tbody>
</table>

Table 5.2: VAT base estimation (Billions of pesos and as GDP percentage)

The distribution of the VAT base across the goods and the retail sectors, referred as $P_{MF} \cdot MF + P_{MI} \cdot MI + P_{FF} \cdot FF + P_{FI} \cdot FI$ in the model, is computed based on the information provided by the ENIGH(2010). Regarding the distribution between food and other goods, the data show that 27 percent of households
Figure 5.1: Food expenditure distribution by retail sectors and by income deciles

consumption is food with the rest going to the purchase of other goods.\footnote{INEGI-SNC. Regarding the informal sector, from 1998 to 2003 INEGHI published the Satellite account of the informal sector; however the information has not been published since then. For that reason, I used the distribution reported in de Ingresos y Gastos de los Hogares ENIGH (2010).} This gives an idea of how significant is the impact of the VAT reform on Mexican household’s consumption. Worth to mention is that the fraction of consumption shows significant differences across the income distribution. As Figure 5.1 shows, for low income households, food consumption represents around thirty percent of total consumption whereas for high income deciles it does represent less than ten percent. This is the main reason why the tax on food has had strong opposition in Mexico, as in principle low income households would pay proportionally more than the tax increase.

However, as mentioned in the introduction, consumption in the informal sector is a significant factor that is neglected in the estimation of the effects of the VAT. At the aggregate level, purchases in the formal sector accounts for 54.9 percent of all consumption leaving the remaining 45.1 of purchases in the informal sector.
Table 5.3: Distribution of the VAT base

<table>
<thead>
<tr>
<th>Concept</th>
<th>Value (% VAT base)</th>
<th>Model symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FINAL GOOD PRODUCTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAT Base</td>
<td>100</td>
<td>$P_{MF} \times MF + P_{MI} \times MI + P_{FF} \times FF + P_{FI} \times FI$</td>
</tr>
<tr>
<td>By good</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food consumption</td>
<td>27.1</td>
<td>$P_{FF} \times FF + P_{FI} \times FI$</td>
</tr>
<tr>
<td>Other goods consumption</td>
<td>72.9</td>
<td>$P_{MF} \times MF + P_{MI} \times MI$</td>
</tr>
<tr>
<td>By sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formal Sector</td>
<td>54.9</td>
<td>$P_{MF} \times MF + P_{FF} \times FF$</td>
</tr>
<tr>
<td>Informal Sector</td>
<td>45.1</td>
<td>$P_{FF} \times FF + P_{MI} \times MI$</td>
</tr>
<tr>
<td><strong>INTERMEDIATE PRODUCTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>65.4</td>
<td>$P_{Mint} \times Mint + P_{Fint} \times Fint$</td>
</tr>
<tr>
<td>By good</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>17.7</td>
<td>$P_{Fint} \times Fint$</td>
</tr>
<tr>
<td>Other goods</td>
<td>47.7</td>
<td>$P_{Mint} \times Mint$</td>
</tr>
<tr>
<td>By sectors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formal Sector</td>
<td>37.4</td>
<td>$P_{Mint} \times Mint, F + P_{Fint} \times Fint, F$</td>
</tr>
<tr>
<td>Informal Sector</td>
<td>28.1</td>
<td>$P_{Mint} \times Mint, I + P_{Fint} \times Fint, I$</td>
</tr>
</tbody>
</table>

Source: Own calculations using SNCN-ENIGH (2010)

This leaves a considerable fraction of purchases without full taxation affecting the incidence of the VAT harmonization. As it is shown in Figure 5.1, the informal sector represents a larger fraction of food purchases for low income households. Thus if we consider the fraction of food purchases in the informal sector, we observe an even distribution across income deciles. Therefore, the incidence of the reform will depend on the extend that the informal sector purchases are indirectly taxed.

Another determinant factor of the reform effects is the Value Added created at the second stage of production. Regarding this, the aggregate data shows 34.6
percent of the VAT is created by the retail sectors. The Value Added created at the second stage of the supply of goods determined the size of the tax base that is subject to an indirect tax in the informal sector.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Value (% VAT base)</th>
<th>Model symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HOUSEHOLD INCOME</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Disposable Income</td>
<td>100.0</td>
<td>TDW</td>
</tr>
<tr>
<td>Labor income</td>
<td>64.7</td>
<td>w_a*(Li)</td>
</tr>
<tr>
<td>Capital income</td>
<td>14.6</td>
<td>r_k*K_i</td>
</tr>
<tr>
<td>Government transfers</td>
<td>2.0</td>
<td>T_G</td>
</tr>
<tr>
<td>External Transfers</td>
<td>18.7</td>
<td>T_e</td>
</tr>
<tr>
<td><strong>GOVERNMENT INCOME</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Government Income</td>
<td>17.2</td>
<td></td>
</tr>
<tr>
<td>Consumption taxes</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>Income taxes</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>Corporate Taxes</td>
<td>5.5</td>
<td></td>
</tr>
</tbody>
</table>

Source: Own calculations using SNCN-ENIGH (2010)

Table 5.4: Distribution of households and government income

Regarding households’ income structure, ENIGH reports each of its components. For consistency, I use monetary income to estimate the size of total disposable income (TDW). Table 5.4 shows that labor income is the principal component, accounting for almost half of TDW, followed by capital income. The amount of government

---

4 The estimation of the intermediate demand is based on the in NA intermediate consumption minus the intermediate consumption by government, financial, mining and construction sectors. Additionally, I subtract the imputed intermediate demand due to gross capital formation and exports.

5 Labor income is defined as all labor compensation: wages, commissions and fringe benefits. Capital income is the income reported as business profits and investment return proceeds. Government transfers are defined as all the monetary transfers received plus the monetary value of transfers in kind distributed through social programs reported by the Minister of Finance. Finally, external transfers consist in remittances, pensions, annuities, etc.
For the capital stock, I consider the gross capital formation as a proxy of the capital stock owned by households. INEGI reports capital formation by production sector enabling me to identify the fraction of capital employed for food production and other goods. Regarding, the labor markets distribution between formal and informal sector, I assume that 48 percent of the labor force is in the informal sector, based on the estimates of Briseno (2008).

Table 5.5: Distribution of capital and labor

<table>
<thead>
<tr>
<th>Concept</th>
<th>Value (% VAT base)</th>
<th>Model symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CAPITAL MARKETS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital stock</td>
<td>8.1</td>
<td>$K_i$</td>
</tr>
<tr>
<td><strong>By use</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food production</td>
<td>0.8</td>
<td>$K_{f1}$</td>
</tr>
<tr>
<td>Non Food production</td>
<td>4.5</td>
<td>$K_{m1}$</td>
</tr>
<tr>
<td>Food Retailing</td>
<td>0.8</td>
<td>$K_{f2}+K_{m2}$</td>
</tr>
<tr>
<td>Non food Retailing</td>
<td>2.0</td>
<td>$K_{m2}+K_{m1}$</td>
</tr>
<tr>
<td><strong>By sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formal Sector</td>
<td>7.2</td>
<td>$K_{pf}+K_{mf}+K_{mf1}+K_{f1}$</td>
</tr>
<tr>
<td>Informal Sector</td>
<td>0.9</td>
<td>$K_{pf}+K_{m1}$</td>
</tr>
<tr>
<td><strong>LABOR MARKETS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of labor force informal sector</td>
<td>0.48</td>
<td></td>
</tr>
</tbody>
</table>

Source: Own calculations using SNCN-ENIGH (2010)
5.3 Parameter values

The specification of the production and utility functions use parameters estimated in the existing literature. In those cases where it was not possible to find any estimate available I estimated them directly. That is the case if the elasticity of substitution estimated in the previous chapter.

Parameters in the utility function

Regarding households utility function, due to the lack of empirical results in the literature, I estimated the elasticities of substitution between consumption goods and between formal and informal sector (chapter 3). The elasticity of substitution between food and other goods $\sigma_c$, shown in Table 5.6, decreases as income increases. This implies that low income households will respond more than high income households to changes in the relative price between food and the rest of the goods. Regarding the substitutability between formal and informal retail sectors, the elasticity of substitution of food purchases between formal and informal sector $\sigma_F$ also increases with income. However the opposite occurs for the elasticity of substitution of other goods between formal and informal sector $\sigma_M$.

Finally, the values of $\alpha_c$, $\alpha_F$, $\alpha_M$ are estimated using the values of sigma, the observed relative prices and the budget shares obtained from survey data. Given that the demand functions for the CES utility function are determined by:

$$M(p_M, p_F, TDW) = \left( \frac{\alpha_c}{p_M} \right)^{\sigma_c} \frac{TDW}{(\alpha_c^{\sigma_c} p_M^{1-\sigma_c} + (1 - \alpha_c^{\sigma_c})^{1-\sigma_u} p_F^{1-\sigma_u})}$$
\[ F(p_M, p_F, TDW) = \left( \frac{1 - \alpha_c}{p_F} \right)^{\sigma_c} \frac{TDW}{(\alpha_c^{\sigma_c} p_M^{1-\sigma_c} + (1 - \alpha_c^{\sigma_c})^{1-\sigma_u} p_F^{1-\sigma_u})} \]

and given that the budget share is defined as:

\[ \nu_c = \frac{M(p_M, p_F, TDW)}{F(p_M, p_F, TDW)} \]

therefore

\[ \nu_c = \left( \frac{\alpha_c p_F}{1 - \alpha_c p_M} \right)^{\sigma_c} \]

thus

\[ \alpha_c = \frac{\nu_c^{\sigma_c} \left( \frac{p_M}{p_F} \right)}{1 + \nu_c^{\sigma_c} \left( \frac{p_M}{p_F} \right)} \tag{5.1} \]

The values of \( \alpha_M \) and \( \alpha_F \) are estimated using a similar formula and the results are summarized in Table 5.6.

*Parameters in the production function*

On the production side, I assume Cobb Douglas production functions. The implicit assumption of constant returns to scale is consistent with the weighted average of the estimated elasticities of substitution for Mexico that is 0.96. Regarding the factor shares, there is an extensive literature estimating for the manufacturing sector assuming a Cobb Douglas function. Garcia-Verdu (2005) estimates factor shares for different sectors in the economy, whereas Salgado and Bernal (2011) estimate factor shares for the food industry.
Table 5.6: CES components by income decile

<table>
<thead>
<tr>
<th>Decile</th>
<th>CES consumption Food and Other goods</th>
<th>CES other goods Formal and Informal</th>
<th>CES food Formal and Informal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_i^f$</td>
<td>$\frac{p_M}{p_f}$</td>
<td>$\alpha_i^f$</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.75</td>
<td>106.20</td>
<td>0.25</td>
</tr>
<tr>
<td>I</td>
<td>1.29</td>
<td>40.09</td>
<td>0.18</td>
</tr>
<tr>
<td>II</td>
<td>1.45</td>
<td>27.19</td>
<td>0.21</td>
</tr>
<tr>
<td>III</td>
<td>0.98</td>
<td>40.15</td>
<td>0.26</td>
</tr>
<tr>
<td>IV</td>
<td>1.08</td>
<td>33.13</td>
<td>0.24</td>
</tr>
<tr>
<td>V</td>
<td>1.01</td>
<td>51.93</td>
<td>0.28</td>
</tr>
<tr>
<td>VI</td>
<td>1.00</td>
<td>27.96</td>
<td>0.30</td>
</tr>
<tr>
<td>VII</td>
<td>0.82</td>
<td>25.82</td>
<td>0.31</td>
</tr>
<tr>
<td>VIII</td>
<td>0.93</td>
<td>18.03</td>
<td>0.56</td>
</tr>
<tr>
<td>IX</td>
<td>0.63</td>
<td>26.65</td>
<td>0.70</td>
</tr>
<tr>
<td>X</td>
<td>0.36</td>
<td>90.69</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Table 5.7: Elasticity of substitution between capital and labor

<table>
<thead>
<tr>
<th>Good</th>
<th>Weights</th>
<th>Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>23%</td>
<td>0.92</td>
</tr>
<tr>
<td>Beverage</td>
<td>2%</td>
<td>0.98</td>
</tr>
<tr>
<td>Tobacco</td>
<td>0%</td>
<td>1.39</td>
</tr>
<tr>
<td>Clothing</td>
<td>6%</td>
<td>0.92</td>
</tr>
<tr>
<td>Electricity and Fuels</td>
<td>4%</td>
<td>0.93</td>
</tr>
<tr>
<td>Transportation</td>
<td>19%</td>
<td>0.88</td>
</tr>
<tr>
<td>Education</td>
<td>9%</td>
<td>0.82</td>
</tr>
<tr>
<td>Appliance</td>
<td>6%</td>
<td>0.85</td>
</tr>
<tr>
<td>Personal Care</td>
<td>8%</td>
<td>1.30</td>
</tr>
<tr>
<td>Medicines</td>
<td>1%</td>
<td>0.95</td>
</tr>
<tr>
<td>Other</td>
<td>21%</td>
<td>1.295</td>
</tr>
<tr>
<td>Average</td>
<td>1</td>
<td>1.012</td>
</tr>
</tbody>
</table>


One critical factor in the estimated reform effects is the productivity gap between formal and informal sector retailers. Lagakos (2008) estimates that the total productivity factor of retailing in the formal sector is three times bigger than in the informal sector. One important issue is that the author assumes that goods sold by formal and informal sector retailers are exactly the same, and therefore they are perfect substitutes. However, this assumption is not completely accurate as this estimate does not exclusively reflect the difference in technology but also some other characteristics of the products sold by each sector (i.e. differences in quality). Although it is hard to get enough information that accounts for that, this is an issue that might affect the results of my model as I am assuming that both goods are imperfect substitutes. Lagakos (2008) also estimates differences in labor intensity for each sector. In the case of the formal sector, the author estimates 0.77 for the informal retail sector and 0.15 for the formal retail sector.
The tax rates

In each case calibrated, the initial equilibrium reflects the existing taxes in the economy. Regarding the income tax, I estimate the marginal tax rate for each income decile based on the tax schedule in the Income Tax Act (Ley del Impuesto sobre la Renta).\(^6\) From the ENIGH (2010) I took wages and other compensation of labor reported in the formal and informal sector. For this calculation, employment was assumed to be in the formal sector when the household reported contributions to any Social Security institution: IMSS, ISSSTE, etc. Additionally all the income from tips and gratuities was added to the informal sector revenue. Households average monthly salary was determined by diving income by the number of household members. Once the wage was computed I was able to identify the corresponding income bracket and the statutory marginal that applied to each case. Corporate taxes are defined as income reported from household-owned business. In this case, the model does not consider two parallel markets for capital: one in the formal and another in

\(^6\)The income tax is determined by two components: a fixed fee in pesos, plus a marginal rate, expressed as a percentage, depending on the income bracket.
the informal sector. Therefore I estimate an average evasion for all capital. I estimate the evasion of the corporate tax of 44 percent.

<table>
<thead>
<tr>
<th>Decile</th>
<th>Income tax rate</th>
<th>Value Added Tax Distribution of tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOTAL</td>
<td>18% 7%</td>
<td>100%</td>
</tr>
<tr>
<td>I</td>
<td>2% 0%</td>
<td>2%</td>
</tr>
<tr>
<td>II</td>
<td>2% 0%</td>
<td>3%</td>
</tr>
<tr>
<td>III</td>
<td>6% 1%</td>
<td>4%</td>
</tr>
<tr>
<td>IV</td>
<td>6% 3%</td>
<td>5%</td>
</tr>
<tr>
<td>V</td>
<td>6% 3%</td>
<td>7%</td>
</tr>
<tr>
<td>VI</td>
<td>6% 4%</td>
<td>8%</td>
</tr>
<tr>
<td>VII</td>
<td>18% 5%</td>
<td>10%</td>
</tr>
<tr>
<td>VIII</td>
<td>18% 7%</td>
<td>12%</td>
</tr>
<tr>
<td>IX</td>
<td>18% 8%</td>
<td>17%</td>
</tr>
<tr>
<td>X</td>
<td>28% 9%</td>
<td>33%</td>
</tr>
</tbody>
</table>

Table 5.9: Estimation of tax rates by Income Decile

Government expenditure and external transfers

Government transfers are estimated as the fraction of government revenue that is allocated to monetary transfers. The distribution of these transfers is available in the annual study of tax and expenditure incidence published by the Minister of Finance. Regarding external transfers, I take the distribution of the reported exogenous sources of income reported in ENIGH (2010).

5.4 Calibration of Initial Equilibrium and Stability analysis

The model solution is characterized by a system of thirteen equations. Recall that given the analytic solution defined in section 2.5 in the second chapter, any computed equilibrium automatically satisfies the market clearing conditions for the intermediate and final consumption goods defined in equations (3.74) to (3.76) and
The solution to the model requires solving six microeconomic restrictions and seven macroeconomic restrictions. The \textbf{microeconomic restrictions} are defined as:

- labor supply constraint (equation (3.20)):
  \[ r_1 = \frac{L_i - (H_i - l_i)}{L_i} - 1 \]  
  \hspace{1cm} (5.2)

- household’s budget constraint (equation (??)):
  \[ r_2 = \frac{\hat{P}_{MF}MF_i + \hat{P}_{MI}MI_i + \hat{P}_{FF}FF_i + \hat{P}_{FI}FI_i}{TDW_i} - 1 \]  
  \hspace{1cm} (5.3)

- market clearing conditions for the labor markets in the formal and informal sectors (equations (3.78) and (3.79)):
  \[ r_3 = \frac{L_{Fi}^F - (L_{MF} + L_{Mint} + L_{FF} + L_{Fint})}{L_{MF} + L_{Mint} + L_{FF} + L_{Fint}} \]  
  \hspace{1cm} (5.4)

and

\[ r_4 = \frac{L_{FI}^I - (L_{MI} + L_{FI})}{L_{MI} + L_{FI}} \]  
  \hspace{1cm} (5.5)

- capital market clearing condition (equation (3.80)):
  \[ r_5 = \frac{K_i - (K_{MF} + K_{MI} + K_{Mint} + K_{FF} + K_{FI} + K_{Fint})}{K_{MF} + K_{MI} + K_{Mint} + K_{FF} + K_{FI} + K_{Fint}} \]  
  \hspace{1cm} (5.6)

- and the government budget constraint (equation (3.40))
Additionally, the calibrated equilibrium satisfies seven macroeconomic restrictions defined by the national account data.

<table>
<thead>
<tr>
<th>Restriction</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_7 ) Food market size</td>
<td>0.27</td>
</tr>
<tr>
<td>( r_8 ) Informal sector</td>
<td>0.45</td>
</tr>
<tr>
<td>( r_9 ) Intermediate consumption</td>
<td>0.65</td>
</tr>
<tr>
<td>( r_{10} ) Total tax revenue</td>
<td>0.17</td>
</tr>
<tr>
<td>( r_{11} ) VAT revenue</td>
<td>0.07</td>
</tr>
<tr>
<td>( r_{12} ) Income tax revenue</td>
<td>0.04</td>
</tr>
<tr>
<td>( r_{13} ) Corporate taxes</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.10: Macroeconomic calibration targets

The solution of these equations is given by the equilibrium prices: the return of capital \( r_k \), the wage in the formal and informal sectors \( w_F \) and \( w_I \), the tax evasion parameter of VAT \( \theta_m \), the income tax \( \theta_w \) and the corporate tax \( \theta_k \). The last two variables are calibrated parameters of the elasticity of substitution between labor and consumption \( \sigma_u \) and the total productivity factor in the formal retail sector \( A_{Mint} \) as of the rest of the factors are defined in terms of this parameter.

Regarding the calibration of the income distribution, I use the equilibrium prices \( \hat{P}_{MF}, \hat{P}_{MI}, \hat{P}_{FF} \) and \( \hat{P}_{FI} \) for each decile \( i \in [1, 10] \) to compute the final demand for each of the goods: \( MF_i, MI_i, FF_i \) and \( FI_i \) for each income decile. The estimated demand for each decile satisfies the consumption distribution between goods and sector of \( \hat{P}_{MF}MF_i, \hat{P}_{MI}MI_i, \hat{P}_{FF}FF_i \) and \( \hat{P}_{FI}FI_i \) according to the survey data as follows:
Table 5.11: Calibration targets for income distribution

<table>
<thead>
<tr>
<th>Decile</th>
<th>Total Consumption A + B</th>
<th>Food A</th>
<th>Other goods B</th>
<th>Formal Sector A</th>
<th>Informal Sector B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOTAL</td>
<td>100.00</td>
<td>72.92</td>
<td>27.08</td>
<td>54.94</td>
<td>45.06</td>
</tr>
<tr>
<td>I</td>
<td>2.83</td>
<td>1.59</td>
<td>1.24</td>
<td>1.23</td>
<td>1.59</td>
</tr>
<tr>
<td>II</td>
<td>3.42</td>
<td>1.93</td>
<td>1.49</td>
<td>1.53</td>
<td>1.89</td>
</tr>
<tr>
<td>III</td>
<td>5.49</td>
<td>3.48</td>
<td>2.06</td>
<td>2.58</td>
<td>2.91</td>
</tr>
<tr>
<td>IV</td>
<td>5.53</td>
<td>3.48</td>
<td>2.05</td>
<td>2.61</td>
<td>2.92</td>
</tr>
<tr>
<td>V</td>
<td>7.35</td>
<td>4.91</td>
<td>2.45</td>
<td>3.66</td>
<td>3.70</td>
</tr>
<tr>
<td>VI</td>
<td>8.35</td>
<td>5.50</td>
<td>2.85</td>
<td>4.21</td>
<td>4.14</td>
</tr>
<tr>
<td>VII</td>
<td>9.44</td>
<td>6.53</td>
<td>2.92</td>
<td>4.85</td>
<td>4.60</td>
</tr>
<tr>
<td>VIII</td>
<td>11.09</td>
<td>7.78</td>
<td>3.32</td>
<td>6.16</td>
<td>4.93</td>
</tr>
<tr>
<td>IX</td>
<td>17.53</td>
<td>13.44</td>
<td>4.09</td>
<td>10.02</td>
<td>7.51</td>
</tr>
<tr>
<td>X</td>
<td>28.96</td>
<td>24.34</td>
<td>4.62</td>
<td>18.08</td>
<td>10.88</td>
</tr>
</tbody>
</table>

The calibration of the model was done by minimizing a norm of a vector including the 13 restrictions. The goal is to bring the norm to zero. I used simulate annealing (SA) from MATLAB optimization toolbox. 

Calibrated Equilibrium I compute five different initial equilibria, one for each specification defined in Table 5.1. Each of the first four simulations focus on some of the specific features that have been identified in the existing literature. Although I will be referencing to each of them, note that a direct comparison of the results is not possible, as I am only simulating the effects of the VAT harmonization, whereas in the literature the harmonization of the tax rate is combined with other tax changes.

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7Originally proposed by Kirkpatrick et al. (1983), simulated annealing (SA) is a probabilistic method to find a good approximation to the global optimum of a given function in a large search space. It is often used when the search space is discrete. For certain problems, simulated annealing may be more efficient than exhaustive enumeration as the goal is to rule out the case of converging in a local optimum. The simulated Annealing algorithm is based on changes in the probability of finding another optimum looking for slow decrease in the probability of accepting worse solutions as it explores the solution space.
or trade tax reforms. However, the VAT harmonization has effects that are roughly similar to these case.  

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model specification</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity gap</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Productivity ratio</td>
<td>( \frac{A_{MF}}{A_{MI}} )</td>
<td>2.90</td>
<td>1.00</td>
<td>2.90</td>
<td>2.90</td>
</tr>
<tr>
<td>Tax avoided by informal sector</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Partial</td>
<td></td>
</tr>
<tr>
<td>Evasion parameter</td>
<td>( \theta_m )</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>Equilibrium prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate</td>
<td>( r_i )</td>
<td>0.57</td>
<td>0.60</td>
<td>0.66</td>
<td>0.69</td>
</tr>
<tr>
<td>Wage formal sector</td>
<td>( w_f )</td>
<td>69.48</td>
<td>88.18</td>
<td>92.80</td>
<td>109.36</td>
</tr>
<tr>
<td>Wage informal sector</td>
<td>( w_i )</td>
<td>43.65</td>
<td>61.70</td>
<td>61.01</td>
<td>45.00</td>
</tr>
</tbody>
</table>

\( \theta_m \) Represents the fraction of the VAT that is avoided.

Table 5.12: Benchmark calibration results

Case 1. **No informal sector, no technological gap and no tax evasion.** In this case, on the demand side this implies that the CES functions \( M_i \) and \( F_i \) entirely depend on the consumption in the formal sector thus \( \alpha_i^m = \alpha_i^f = 1 \). On the supply side there is no productivity gap between formal and informal sector retailers, thus \( \frac{A_{MF}}{A_{MI}} = 1 \). In this case households can only supply labor to one sector (\( \theta_i = 1 \)) and there is no tax evasion (\( \theta_m = 0 \)). Regarding corporate and personal income taxes, I assume the average tax rate in each case as no evasion is assumed. In this case I estimate the welfare gains of the reform identified by Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1976).  

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8In the particular case of the trade tax reform, one exportable good is taxed with a tariff while the rest of the taxes are not. When a VAT is introduced, the tariff on that good is eliminated and a general VAT on all the goods is introduced.
Case 2. **Technological gap without tax evasion.** In this case I assume a productivity gap between the formal and informal retail sectors for both goods as in Emran and Stiglitz (2005). Therefore $\frac{A_{MF}}{A_{MI}} = \frac{A_{FF}}{A_{FI}} = 2.9$. I am isolating the technological gap effect from the tax evasion effect. For that reason I assume that the informal sector is fully taxed, thus the fraction of the tax avoided by the informal sector is set $\theta_m = 0$. This is an extreme situation of the indirect tax on the informal sector referred by Keen (2008). In this case the harmonization is an instrument to extract resources from the informal sector and therefore it provides a tax advantage for the formal sector. The productivity gap can be noticed in the equilibrium wages as the formal sector wage is substantially higher than the wage in the informal sector.

Case 3. **No technological gap with tax evasion.** In this case I measure the welfare effects of tax evasion without considering any technological gap, thus $\frac{A_{MF}}{A_{MI}} = \frac{A_{FF}}{A_{FI}} = 1$. I assume that the informal sector fully evades the tax, therefore $\theta_m = 1$. This case is important as tax avoidance of the new tax on food diminishes the corrective effect of the reform on the distorted relative price between food and other goods. This case is similar to Ab-Iorweth and Whalley (2002) analysis of VAT reform in Canada.

Case 4. **Technological gap with tax evasion.** In this case I want to consider the combined welfare effect of the productivity gap and tax evasion once the reform is enacted; therefore $\frac{A_{MF}}{A_{MI}} = \frac{A_{FF}}{A_{FI}} = 2.9$ and $\theta_m = 1$, a similar case to Piggott and Whalley (2001). Similarly to the previous case, the efficiency gains of correcting the tax distortion on consumption prices due the food exemption are diminished by the tax avoidance. Additionally, the welfare gains are also affected by the changes
in the average productivity in the economy due to the existence of the technological
gap between formal and informal sectors.

Case 5. Technological gap with partial tax evasion. This case adds the
efficiency effects of the indirect taxation of the informal sector through the taxation
of business inputs that are not subsequently credited, stressed by Keen (2008). This
case provides my preferred estimates for the effects of the VAT harmonization reform
in Mexico.

5.5 Efficiency analysis at the aggregate level

To examine the efficiency properties of the harmonization reforms, I calculate the
Equivalent Variations (EV), defined as how much income a consumer would pay to
avoid the price changes associated with the reform. Traditionally, the equivalent
variation is computed in terms of an indirect utility function.

The equivalent variation can be defined in terms of the value function as follows:

\[ V(p_c^1, \omega, TDW) = V(p_c^1, \omega, (1 - \varphi)TDW) = (1 - \varphi)V(p_c^1, \omega, TDW) \] (5.7)

Given that the calibrated version gives a value of \( \sigma_u = 1 \) the household
utility function is approximated as a Cobb-Douglas function. In this case the EV,
can computed as:

\[ \varphi = 1 - \left( \frac{p_c^0}{p_c^1} \right)^{\alpha_u} \left( \frac{\omega_a^0}{\omega_a^1} \right)^{1-\alpha_u} \] (5.8)
For presentation purposes, in Table 5.13 I express the welfare gains as a fraction of total consumption, as a fraction of the GDP, and as a fraction of current VAT collections in order to provide an indication of the relative magnitudes of the simulated efficiency changes.

**Case 1. No informal sector and no technological gap** Using a theoretical model, Emran and Stiglitz (2005) show that in the absence of an informal sector, when the goods are weak substitutes in consumption and production and the good whose tax rate is reduced bears the highest tax burden, then the reform is welfare improving. The simulation in Case 1 satisfies these conditions as the tax on M was positive whereas the tax on F was zero and the elasticity of substitution between F and M is less than one. In this case eliminating the price wedge between food and other goods results in a welfare gain of 0.36 percent of consumption. Although Piggott and Whalley (2001) do not estimate the equivalent variation for this case, they estimate a 1 percent increase in household utility. Similarly, Bye, Strom, and Avitsland (2012) estimate a welfare gain of 0.40 percent of consumption for Norway.

The increase of the VAT on food eliminates the price distortion between F and M and therefore eliminates the efficiency losses identified by Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1976). The new tax on food leads households to reduce the consumption of F more than the consumption of M (by 1.19 percent and 0.15 percent respectively). As a consequence, the ratio $Q_F/Q_M$, that measures the consumption of food relative to the consumption of other goods, drops 1.04 percent.

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9The authors simulate a VAT reform that expand the covers goods and services.
Case 2. Technological gap without tax evasion. In this case I measure the efficiency effect of adding the informal retail sector. I assume no tax evasion in order to isolate the welfare change of the productivity gap between the formal and informal sectors. In this case, total consumption falls more than previously. However, the welfare gains of the reform are 0.64 percent of total consumption, more than in the previous case (0.36). In this case, good M is more elastic than F, therefore the expansion of the formal sector in M is proportionally larger than the drop in F.
Note that the change in the ratio $MF/MI$ is larger than the drop of $FF/FI$. As a result, there is a net productivity gains. Finally, note that in this case there is less substitution between F and M as the ratio $F/M$ only changes 0.49 percent, half of what is observed in the absence of the informal sector. Therefore, most of the welfare change comes from the substitution between the formal and informal sectors within each good. As a conclusion for this particular case, when the informal sector is introduced in the model, the changes in the existing price wedge between the two goods are less important than the change in aggregate productivity.

Case 3. No technological gap with tax evasion. When the informal sector fully evades the tax, $\theta_m = 1$, the reform generates a welfare loss of 0.29 percent of total consumption. Households evade the tax on food by purchasing in the informal sector and therefore the distortion between $P_F$ and $P_M$ is not fully eliminated as in case 1. This result is comparable to Emran and Stiglitz (2005), who find that an alternative of evading the tax results in a welfare loss of 0.8 percent of total consumption. However, given that the informal sector fully evades the tax, there is more incentive to shift consumption away from the formal sector when the tax on food is higher. This result is similar to that obtained by Piggott and Whalley (2001) who suggest that the reform is welfare worsening when households have the alternative of evading the tax. Although the authors do not provide any estimation of the Equivalent Variation, in their calibrations they show a utility loss of 0.82 percent.

Case 4. Technological gap with tax evasion. In this case I analyze the combined effect of tax evasion and the change in productivity, thus $A_{MF}/A_{MI} = A_{FF}/A_{FI} = 2.9$ and $\theta_m = 1$. The reform has a minimal efficiency loss of 0.04 percent of total consumption, as the efficiency losses of tax evasion are almost offset by the efficiency gains of reducing the price distortion between F and M. This result is similar to that obtained by Piggott and Whalley (2001) who suggest that the reform is welfare worsening when households have the alternative of evading the tax. Although the authors do not provide any estimation of the Equivalent Variation, in their calibrations they show a utility loss of 0.82 percent.
As in the previous case, the informal sector has a tax advantage over the formal sector and therefore, households have more incentives to shift from formal to informal sector when their relative price changes. Note that when the tax on food is increased, the shift from the formal to the informal sector is bigger and consequently the ratio $FF/FI$ falls more than in the previous cases $\Delta(FF/FI) = -2.76$. At the same time, when the tax on M is reduced the growth of the formal sector, $\Delta(MF/MI) = 0.86$, is less than in case 2, when the informal sector does not evade the tax.

**Case 5. Technological gap with partial tax evasion.** Contrary to the previous cases, where the informal sector either evades or complies with the VAT, in this case I analyze an intermediate situation where the informal sector pays a fraction of the tax. In this case I incorporate the indirect tax on the informal sector through the taxation of inputs as suggested by (Keen, 2008). In his theoretical model, an indirect tax on the informal sector reduces its potential expansion once the VAT on food is introduced. In addition, I explore how this effect interacts with the change in the productivity of the informal sector, an exercise that has not been investigated in the literature. Thus, I set $A_{MF}/A_{MI} = 2.9$ and $\theta_m = 0.6$, values estimated for the Mexican economy.

The reform generates welfare gains of 0.59 percent of total consumption. Although the model can not be directly compared to any of the existing simulations in the literature, the magnitude of the welfare gains is similar to that obtained by Munk (2009) who estimates a welfare change of 0.53 using a quantitative calibration of theoretical data. However, the author does not consider any technological gap between formal and informal sectors in his estimations.
5.6 Policy Implications

The shift of consumption between formal and informal sector is the main determinant of the welfare effects of the revenue neutral reform. For each of the simulated reforms, Figure 5.2 represents the change in efficiency, consumption of food relative to other goods $\Delta(F/M)$, the consumption in the formal sector relative to the informal sector for food $\Delta(FF/FI)$ and for other goods $\Delta(MF/MI)$.

Figure 5.2 shows that when the informal sector is fully taxed (cases 1 and 2), the reform is welfare enhancing as households reduce the over-consumption of food induced by the tax exemption of food. In the case of a productivity gap between retail sectors, case 2, the net expansion of the more productive formal sector increases the welfare gains. However, when the informal sector fully avoids the tax (cases 4 and
Figure 5.3 : Price distortion and change in the relative price of $P_F/P_M$.

5), the reform has a negative impact on welfare. In these cases the drop of the consumption of F in the formal sector, whose tax is increased, is significantly larger than the increase in the formal sector of consumption of M, whose tax is reduced (compare the change $\Delta(MF/MI)$ to $\Delta(FF/FI)$). The intuition behind this result is simple: households ability to avoid the tax undermines the correction of the price ration between F and M induced by the VAT harmonization.

The changes in consumption in the formal and informal sectors are the main determinant of the efficiency effects of the reform in two ways: The first one is the impact of the reform on the price distortion between food and other goods. Figure 5.4 shows that the efficiency gains due to the reduction in this distortion is smaller when there is a tax on the informal sector. The introduction of the VAT on M,
but keeping F untaxed, creates a drop in the relative price $P_F/P_M$ of almost ten percent. For illustration purposes, the absolute value of this change is depicted by each of the gray bars in each case. In the absence of the informal sector (case 1), the increase of the relative price of food and other goods corrects more than half of the distortion. Particularly when the productivity gap between sectors is considered (case 2). However, once households have the ability to avoid the tax, the VAT has a minimum effect on changing the relative price $P_F/P_M$ (case 3). The change is even smaller if productivity differences between the retail sectors are considered (cases 4 and 5). The magnitude of these results depend on the value of the evasion parameter $\theta_m$. However, the data show a general trend: the more the informal sector is taxed indirectly, the bigger the reduction of the price distortion between food and other goods.

The second effect is the impact of the reform on average productivity. Figure 5.4 shows the change in the average productivity of the combined retail sector (formal and informal) once the reform is introduced. Given that $A_{MF}/A_{MI} = A_{FF}/A_{FI}$, the average productivity can be defined as

$$\text{Average productivity} = \frac{A_{MF}(P_{MF}MF_i + P_{FF}FF_i) + P_{MI}MI_i + P_{FI}FI_i}{P_{MF}MF_i + P_{FF}FF_i + P_{MI}MI_i + P_{FI}FI_i} \quad (5.9)$$

In the cases when a productivity gap is assumed (cases 2, 4 and 5), the relative increase of households purchases in the formal sector leads to a productivity increase of the retail sector. The magnitude of the change in average productivity depends primarily on two factors: (1) the size of the productivity gap assumed $A_{MF}/A_{MI}$ and (2) the elasticities of substitution between formal and informal sector $\sigma_m$ and $\sigma_f$ and between F and M $\sigma_c$. Given that in this particular case $\sigma_m > \sigma_f$, the positive effect
on purchases of M in the formal sector is larger than the negative effect on purchases of F in the formal sector. For this reason, the larger the productivity gap, the bigger the welfare gains obtained with the VAT harmonization reform.

5.7 Welfare analysis by income decile

Regarding the reform effects on income distribution, it is commonly argued that removing the exemption of food has a regressive impact on income distribution. Indeed, the primary motivation for the VAT exemption on food is to reduce the tax burden on low income households since they consume a larger fraction of their income on food. In the academic literature, Piggott and Whalley (2001) is the only study that analyzes the progressivity of the introduction of a VAT with a single rate in a general equilibrium context. With a two agent model: rich and poor, the
The authors simulate the incidence effect of the introduction of the VAT in Canada. The authors conclude that although the reform reduces aggregate welfare, the reform is progressive as low income households show a welfare improvement while rich households experience a welfare loss. The key feature underlying the Piggot and Whalley result is that low income households avoid the VAT increase by purchasing more goods in the informal sector. However, as mentioned previously, the authors analyze the introduction of the VAT as a substitute of a sales tax on manufactured goods thus they do not provide any further insight regarding the VAT on food.

(Keen, 2008) criticizes this result arguing that the VAT is an indirect tax on the informal sector by denying input credits to informal sector firms for the VAT paid on inputs purchased from the formal sector. Under these circumstances, it is reasonable to expect that a fraction or perhaps all of the tax on inputs is shifted forward to final consumers, implying some taxation on purchases in the informal sector. This effect is not considered by Piggot and Whalley as in their paper low income households avoid the tax. Keen’s critique is incorporated in the simulated scenarios. Particularly in cases 3, 4 and 5.

To evaluate the effect of the indirect tax on income distribution, I calculate the welfare changes and the tax burden of the reform by income decile. The left panel of Figure 5.2 shows that when the informal sector is fully taxed (case 2) the reform is regressive: the middle and high income deciles get a higher welfare gain than deciles at the bottom of the income distribution. Low income households pay the VAT on food regardless where they shop.
Figure 5.5: Change in welfare and distribution of tax burden by income decile

The situation changes when it is assumed that the informal sector avoids all the tax (case 4). In this case the simulation generates a similar welfare effect to that obtained by Piggott and Whalley (2001). Although the reform reduces welfare overall, high income deciles have the largest welfare loss as low income households evade the tax by shifting consumption towards the informal sector. Thus the reform is progressive.

However, as case 5 shows, this result changes once I introduce the indirect tax on the informal sector suggested by (Keen, 2008). Under this assumption, the reform is regressive as the welfare gains of the top two deciles are higher to those of the rest of the income distribution. However, the reform is not as regressive as in case 1 when the informal sector is fully taxed.

The results show that evasion in the informal sector is the key parameter that determines the progressivity or the regressivity of this reform. To illustrate this, in the right panel of Figure 5.5 I compute the average tax burden under different levels
of indirect tax on the informal sector, as determined by the parameter $\theta_m$ in the model. In case 2, I assume full taxation ($\theta_m = 0$); in case 5, I assume partial tax ($\theta_m = 0.6$) and in case 4 I assume full evasion ($\theta_m = 1$). Before the reform, the tax burden has a progressive profile, as high income deciles bear a larger tax burden than low income deciles. After the reform, when the informal sector is fully taxed, the tax burden is equally distributed among income deciles: low and high income deciles bear the same tax burden. Therefore the reform is regressive given the disproportionally increase of the tax burden for low income deciles compared to high income deciles.

When the informal sector fully avoids the tax (case 4), the tax burden distribution is progressive across income deciles, as the one observed before the reform. Although the reform is slightly regressive as the difference in the tax burden between high and low income households reduces. A similar result is obtained when an indirect tax is assumed (case 5) as the reform reduces the progressivity of the tax burden distribution.

The main conclusion is that VAT reform is less regressive when the informal sector can avoid the tax. Even in the case in which the informal sector is indirectly taxed, the tax burden distribution remains progressive. However, this result depends on the relationship between the elasticities of substitution of M and F $\sigma_c$ and the elasticities of substitution between formal and informal sector for goods M and F: $\sigma_m$ and $\sigma_f$.

Regarding $\sigma_c$, the estimates of the previous chapter show that higher income deciles have greater substitutability between F and M compared to low income deciles. Figure 5.2 shows the relationship between $\sigma_c$ and the welfare effects of the reform.
In the absence of the informal sector, the maximum welfare gains occur when both goods are close to $\sigma_c = 1.1$. However, when the informal sector fully evades the tax, the relationship between $\sigma_c$ and the welfare gains tend to disappear (see the flat relationship in Figure 5.2 for case 4). In there is an indirect tax on the informal sector, as there is a negative relationship between But $\sigma_c$ is relevant when there is and the welfare gains. In this case, the more food and other goods are substitutes, the smaller are the gains from the harmonization reform.

However, as mentioned previously, the substitutability between formal and informal sector is a key factor to determine the the welfare effects of the reform. Figure 5.7 shows the relationship between the welfare effect of the reform and the differences in the elasticity of substitution between formal and informal sectors $\sigma_m$ and $\sigma_f$. The empirical data presented in Chapter 3 indicate that for low income people the sub-
Figure 5.7: Welfare change and the elasticity of substitution between M and F ($\sigma_c$)

Substitutability between formal and informal sector is larger for food purchases than for other goods. The opposite relationship characterizes high income people. A way to summarize this is to compute the difference between $\sigma_m$ and $\sigma_f$: for low income households the difference is negative and for high income the difference is positive.

When the informal sector fully evades the tax, the difference between these elasticities has no effect on the welfare gains from the reform. However, once the indirect tax on food due to the taxation of inputs is considered, the welfare gains increase the larger is the differences between these elasticities. For high income households, the welfare benefit of the tax reduction on M more than offset the loss due to the increase of the tax on food declines whereas for low income households the opposite happens. This reveals that the welfare gains of the reform increase as long as the reduction of the tax rate is on the more elastic good, implying more consumption shifted from the
informal to the formal sector in good M and less consumption shifted in the opposite direction in good F.

5.8 Conclusions

The calibration results show that the welfare effects of VAT harmonization reform change once we account for the informal sector respect the case when no informal sector is assumed. On efficiency grounds, in the absence of the informal sector, the harmonization of the VAT rate corrects the distortion of the relative prices between food and other goods induced by the zero VAT on food. However, once we account for the informal sector, under the conditions assumed in the model, the change in the average productivity is the main driving factor of the welfare effects. The reform is welfare improving if the elasticity of substitution between formal and informal sector consumption is larger for the good whose tax is reduced (other goods) than for the good whose tax is increased (food). The intuition behind this is that the expansion of the informal sector on good M offsets its contraction in good F. Additionally, the welfare gains are also largely affected by the existing productivity gap between formal and informal sector retailers and how much is the indirect tax on the informal sector.

On equity grounds, when the informal sector is included in the analysis, the reform is not as regressive as the standard analysis where there is no informal sector at all. This is because low income households can avoid the tax while shifting consumption from formal to informal sector retailers. However, as shown in the last section of this chapter, this result also depends on the difference of elasticities of
substitution between formal and informal sector across the income distribution.

The findings of this chapter extend the results in the existing literature as the indirect taxation of the informal sector is a crucial determinant of the welfare effects of the reform. Overall, modeling the informal sector as fully integrated component of the supply of goods and services gives different policy prescriptions. In the case of Mexico, the harmonization of the VAT can be a reform that fosters the formal sector with a minimum effect on the existing progressivity of the VAT. Indeed, in the case of a non-revenue neutral reform, the additional revenue could be allocated to offset any potential regressive effect of the reform.
Appendix

The Cobb Douglas Example

**Proposition 1:** The optimal tax \( \tau \) is zero.

*Proof.* Define the after-tax price vectors \( p^0 = (P_M, P_F), p^1 = (P_M(1 + \tau), P_F(1 + \tau)) \), productivity vector \( A^0 = (A_M, A_F) \) and the scalar \( X \) to denote household’s income. In this case the Equivalence Variation expressed as the fraction of income \( \psi^1 \) is defined as:

\[
EV = \nu(p^1, A^0, X) = \nu(p^0, A^0, (1 - \psi^1)X)
\]

\[
= \left[ \frac{\alpha_M A}{P_M(1 + \tau)} \right]^{\alpha_M} \left[ \frac{1 - \alpha_M A}{P_F(1 + \tau)} \right]^{\alpha_F} X = \left[ \frac{\alpha_M A}{P_M} \right]^{\alpha_M} \left[ \frac{1 - \alpha_M A}{P_F} \right]^{\alpha_F} (1 - \psi^1)X
\]

\[
= \frac{1}{1 + \tau} = (1 - \psi^1)
\]

\[
\psi^1 = 1 - \frac{1}{1 + \tau} = \frac{\tau}{1 + \tau}
\]

thus \( EV = 0 \Leftrightarrow \tau = 0 \) Q.E.D.

**Proposition 2:** If the tax can not be zero, the second best is to set a single flat rate for all goods. A reduction of the consumption tax on food increases the DWL.

*Proof.* Define \( p^2 = (P_M(1 + \tau), P_F(1 + \tau^V_F)) \). Then the equivalent variation \( \psi^2 \) is defined as:
\[ EV = \nu(p^1, A^0, X) = \nu(p^0, A^0, (1 - \psi^1)X) \]

\[
\begin{align*}
\left[ \frac{\alpha_M A}{P_M} \right]^{\alpha_M} \left[ \frac{(1 - \alpha_M) A}{P_F} \right]^{\alpha_F} X &= \left[ \frac{\alpha_M A}{P_M(1 + \tau_M)} \right]^{\alpha_M} \left[ \frac{(1 - \alpha_M) A}{P_F(1 + \tau_F)} \right]^{\alpha_F} (1 - \psi^2)X \\
\frac{1}{(1 + \tau_M)^{\alpha_M}(1 + \tau_F)^{\alpha_F}} &= (1 - \psi^2)
\end{align*}
\]

\[ \psi^2 = 1 - \frac{1}{(1 + \tau_M)^{\alpha_M}(1 + \tau_F)^{\alpha_F}} \tag{C2} \]

If a flat tax \( \tau \) has a lower equivalence variation then \( \psi^2 \) in (C2) is larger than \( \psi^1 \) in (C1). Without loss of generality, assume a reduction of good F tax rate (this include the possibility that the good is exempt), that \( \tau_M^V = \tau \) and that \( \tau_F^V < \tau \) thus if \( \psi^2 > \psi^1 \) then

\[
\Rightarrow 1 - \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_F^V)^{\alpha_F}} > 1 - \frac{1}{1 + \tau} \quad \Rightarrow \quad \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_F^V)^{\alpha_F}} < \frac{1}{1 + \tau} \\
\Rightarrow (1 + \tau)^{\alpha_M}(1 + \tau_F^V)^{\alpha_F} < 1 + \tau \quad \Rightarrow \quad 1 + \tau_F^V < 1 + \tau \iff \tau_F^V < \tau
\]

Thus, in order to minimize the DWL it is optimal to set \( \tau_M^V = \tau_F^V = \tau \) in that case \( \psi^2 = \psi^1 \). Q.E.D.

\textit{Consumer demands derivation}

First we maximize the demand of each of the components of the composite consumption good. Thus we

\[ \text{Max } U_i(MF, FI, FF) = M^{\alpha_M} FI^{\alpha_I} FF^{\alpha_F} \tag{C3} \]
FOC

\[ M \quad \alpha_M M^{\alpha_M-1} F I^{\alpha_I} F F^{\alpha_F} = \lambda P_M (1 + \tau^V_M) \quad (C4) \]

\[ FI \quad \alpha_I M^{\alpha_M} F I^{-\alpha_M-\alpha_F-1} F F^{\alpha_F} = \lambda P_{FI} (1 + \tau^V_{FI}) \quad (C5) \]

\[ FF \quad \alpha_F M^{\alpha_M} F I^{\alpha_I} F F^{\alpha_F-1} = \lambda P_{FF} (1 + \xi^V_F) \quad (C6) \]

dividing \((C4)\) by \((C5)\) and \((C4)\) by \((C6)\) we get

\[ \frac{\alpha_M}{\alpha_I} \frac{FF}{M} = \frac{P_M (1 + \tau^V_M)}{P_{FF} (1 + \tau^V_F)} \quad \Rightarrow \quad FF = \frac{P_M (1 + \tau^V_M)}{P_{FF} (1 + \tau^V_F)} \frac{\alpha_I}{\alpha_M} M \quad (C7) \]

\[ \frac{\alpha_M}{\alpha_I} \frac{FI}{M} = \frac{P_M (1 + \tau^V_M)}{P_{FI} (1 + \xi^V_F)} \quad \Rightarrow \quad FI = \frac{P_M (1 + \tau^V_M)}{P_{FI} (1 + \xi^V_F)} \frac{\alpha_I}{\alpha_M} M \quad (C8) \]

Plugging into the budget constraint:

\[ P_M (1 + \tau^V_M) M + P_{FF} (1 + \tau^V_F) \left( \frac{P_M (1 + \tau^V_M)}{P_{FF} (1 + \tau^V_F)} \frac{\alpha_I}{\alpha_M} M \right) + P_{FI} (1 + \xi^V_F) \left( \frac{P_M (1 + \tau^V_M)}{P_{FI} (1 + \xi^V_F)} \frac{\alpha_I}{\alpha_M} M \right) = X \quad (C9) \]

and solving for \(M\) and plugging into \((C7)\) and \((C8)\) we get the demand functions

\[ M^* = \frac{\alpha_M}{P_M (1 + \tau^V_M)} X \quad FF^* = \frac{\alpha_F}{P_{FF} (1 + \tau^V_F)} X \quad FI^* = \frac{\alpha_I}{P_{FI} (1 + \xi^V_F)} X \quad (C10) \]

The indirect utility function is given by
\[ \nu_i(M^* FF^*, FI^*) = \left[ \frac{\alpha_M}{P_M(1 + \tau_M^V)} \right]^{\alpha_M} \left[ \frac{\alpha_F}{P_{FF}(1 + \tau_F^V)} \right]^{\alpha_F} \left[ \frac{\alpha_I}{P_{FI}(1 + \xi \tau_F^V)} \right]^{\alpha_I} X \quad (C11) \]

On the supply side I assume a constant returns to scale technology for the production of each

\[ M = A_M X \quad FF = A_{FF} X \quad FI = A_{FI} X \quad (C12) \]

The firm maximizes the profit function:

\[ \Pi_M = P_M A_M X - X \quad \Pi_M = P_M A_M X - X \quad \Pi_M = P_M A_M X - X \quad (C13) \]

The supply functions are given by

\[ P_M = \frac{1}{A_M} \quad P_{FF} = \frac{1}{A_{FF}} \quad P_{FI} = \frac{1}{A_{FI}} \quad (C14) \]

Plugging the equilibrium prices in the indirect utility function

\[ \nu_i(M^* FF^*, FI^*) = \left[ \frac{\alpha_M A_M}{P_M(1 + \tau_M^V)} \right]^{\alpha_M} \left[ \frac{\alpha_F A_{FF}}{P_{FF}(1 + \tau_F^V)} \right]^{\alpha_F} \left[ \frac{\alpha_I A_{FI}}{P_{FI}(1 + \xi \tau_F^V)} \right]^{\alpha_I} X \quad (C15) \]

**Proposition 1a:** In an economy with an informal sector, the optimal tax \( \tau \) is zero.
Proof. Define the after-tax price vectors \( p^0 = (P_M, P_{FF}, P_{FI}) \), \( p^{1a} = (P_M(1 + \tau), P_{FF}(1 + \tau), P_{FI}(1 + \xi \tau)) \), productivity vector \( A^0 = (A_M, A_{FF}, A_{FI}) = (A, A, A) \) and the scalar \( X \) to denote household’s income. Assume a single tax rate to all consumption such that \( \tau_M^V = \tau_F^V = \tau \). Thus the Equivalence Variation \( \psi^{1a} \) is:

\[
EV = \nu(p^{1a}, A^0, X) = \nu(p^0, A^0, (1 - \psi^{1a})X)
\]

\[
\left[ \frac{\alpha_M A}{P_M(1 + \tau)} \right]^{\alpha_M} \left[ \frac{\alpha_F A}{P_{FF}(1 + \tau)} \right]^{\alpha_F} \left[ \frac{\alpha_I A}{P_{FI}(1 + \xi \tau)} \right]^{\alpha_I} X = \left[ \frac{\alpha_M A}{P_M} \right]^{\alpha_M} \left[ \frac{\alpha_F A}{P_{FF}} \right]^{\alpha_F} \left[ \frac{\alpha_I A}{P_{FI}} \right]^{\alpha_I} (1 - \psi^{1a})X
\]

\[
\frac{1}{(1 + \tau)^{\alpha_M + \alpha_F (1 + \xi \tau)}^{\alpha_{FI}}} = (1 - \psi^{1a})
\]

\[
\psi^{1a} = 1 - \frac{1}{(1 + \tau)^{\alpha_M + \alpha_F (1 + \xi \tau)}^{\alpha_{FI}}}
\]

thus \( EV = 0 \iff \tau = 0 \) Q.E.D.

**Proposition 2b:** In an economy where the informal sector can fully evade the tax, \( \xi = 0 \), a reduction of the consumption tax on purchases \( FF \) and \( FI \) increases the DWL. If the tax can not be zero, the second best is to set a single flat rate for all goods.

Proof. Without loss of generality, assume that \( \tau_M^V = \tau \) and that \( \tau_F^V < \tau \) thus \( p^{2a} = (P_M(1 + \tau), P_{FF}(1 + \tau^V_F), P_{FI}(1 + \xi \tau^V_F)) \). Then the equivalent variation \( \psi^{2a} \) is defined as:
EV = \nu(p^{2a}, A^0, X) = \nu(p^0, A^0, (1 - \psi^{2a})X)

\[
\left[ \frac{\alpha_M A}{P_M(1 + \tau)} \right]^{\alpha_M} \left[ \frac{\alpha_F A}{P_{FF}(1 + \tau_{FF})} \right]^{\alpha_F} \left[ \frac{\alpha_I A}{P_I(1 + \xi\tau_{FI})} \right]^{\alpha_I} X = \left[ \frac{\alpha_M A}{P_M} \right]^{\alpha_M} \left[ \frac{\alpha_F A}{P_{FF}} \right]^{\alpha_F} \left[ \frac{\alpha_I A}{P_I} \right]^{\alpha_I} (1 - \psi^{2a}).
\]

\[
\frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_{FF})^{\alpha_F}(1 + \xi\tau_{FI})^{\alpha_I}} = (1 - \psi^{2a})
\]

\[
\psi^{2a} = 1 - \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_{FF})^{\alpha_F}(1 + \xi\tau_{FI})^{\alpha_I}}
\]

(C17)

In this case we show that having a single consumption tax rate has a lower E.V. therefore \( \psi^{2a} \) in (C17) is larger than \( \psi^{1a} \) in (C16). We compute the difference \( \psi^{2a} - \psi^{1a} > 0 \):

\[
\Rightarrow 1 - \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_{FF})^{\alpha_F}(1 + \xi\tau_{FI})^{\alpha_I}} > 1 - \frac{1}{(1 + \tau)^{\alpha_M + \alpha_F}(1 + \xi\tau)^{\alpha_I}}
\]

\[
\Rightarrow \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_{FF})^{\alpha_F}(1 + \xi\tau_{FI})^{\alpha_I}} < \frac{1}{(1 + \tau)^{\alpha_M + \alpha_F}(1 + \xi\tau)^{\alpha_I}}
\]

\[
\Rightarrow (1 + \tau)^{\alpha_M}(1 + \tau_{FF})^{\alpha_F}(1 + \xi\tau_{FI})^{\alpha_I} > (1 + \tau)^{\alpha_M + \alpha_F}(1 + \xi\tau)^{\alpha_I}
\]

\[
\Rightarrow \left( \frac{1 + \tau_{FF}}{1 + \tau} \right)^{\alpha_F} \left( \frac{1 + \xi\tau_{FI}}{1 + \xi\tau} \right)^{\alpha_I} > 1
\]

when the informal sector evades the tax, \( \xi = 0 \) thus

\[
\Rightarrow \frac{1 + \tau_{FF}}{1 + \tau} > 1 \Leftrightarrow \tau_{FF} < \tau
\]

Thus, in order to minimize the DWL it is optimal to set \( \tau_M^V = \tau_F^V = \tau \) in that case \( \psi^2 = \psi^{1a} \). Q.E.D.
**Proposition 3:** Differences in productivity increases the DWL of the tax.

*Proof.* Define \( p^3 = (P_M(1 + \tau), P_{FF}(1 + \tau_F^V), P_{FI}(1 + \xi\tau_F^V)) \) and the productivity vector \( A^1 = (A, A, A_{FI}) \). The equivalent variation \( \psi^3 \) is derived as:

\[
EV = \nu(p^3, A^1, X) = \nu(p^0, A^0, (1 - \psi^3)X)
\]

\[
\left[ \frac{\alpha_M A}{P_M(1 + \tau)} \right]^{\alpha_M} \left[ \frac{\alpha_F A}{P_{ FF}(1 + \tau_F^V)} \right]^{\alpha_F} \left[ \frac{\alpha_{FI} A_{FI}}{P_{FI}(1 + \xi\tau_F^V)} \right]^{\alpha_{FI}} X = \left[ \frac{\alpha_M A}{P_M} \right]^{\alpha_M} \left[ \frac{\alpha_F A}{P_{FF}} \right]^{\alpha_F} \left[ \frac{\alpha_{FI} A}{P_{FI}} \right]^{\alpha_{FI}} (1 - \psi^3)X
\]

\[
\psi^3 = 1 - \left( \frac{A_{FI}}{A} \right)^{\alpha_{FI}} \left( \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_F^V)^{\alpha_F}(1 + \xi\tau_F^V)^{\alpha_{FI}}} \right)
\]

(C18)

Given (C18), we have that differences in productivity increase the DWL respect the case where there is a flat tax \( \tau \) and no differences in productivity as \( \psi^3 > \psi^{2a} \) given that:

\[
\psi^3 - \psi^{2a}
\]

\[
\Rightarrow
\]

\[
1 - \left( \frac{A_{FI}}{A} \right)^{\alpha_{FI}} \left( \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_F^V)^{\alpha_F}(1 + \xi\tau_F^V)^{\alpha_{FI}}} \right) - \left( 1 - \frac{1}{(1 + \tau)^{\alpha_M}(1 + \tau_F^V)^{\alpha_F}(1 + \xi\tau_F^V)^{\alpha_{FI}}} \right)
\]

The first term is always positive as \( 1 - \left( \frac{A_{FI}}{A} \right)^{\alpha_{FI}} \) if \( 1 > \left( \frac{A_{FI}}{A} \right)^{\alpha_{FI}} > 0 \) implying that \( A > A_{FI} A \) that is always true. Given that the second term is always positive we have that \( \psi^3 - \psi^{2a} > 0 \).
Stability analysis of calibrated parameters $\sigma_u$ and $A_{Mint}$.

The equilibrium prices and consequently the calibrated equilibrium depend on the calibrated parameters. One of the conditions to assure the global optimum is that the norm of the system of equations reaches its minimum, ideally zero, at the calibrated value. For that purpose, I compute several equilibrium points assigning different values of $\sigma_u$ and $A_{Mint}$ and I graph the value of the optimized norm of each local equilibrium in order to find the global equilibrium. As the following graphs show, the equilibrium paths are clearly selecting the optimal point at the calibrated value.

Regarding the stability of the computed equilibrium, $\sigma_u$ seems to have a larger effect on the norm than $A_{Mint}$. As the right hand side graph shows, small deviation of $\sigma_u$ from the equilibrium has an exponential effect on increasing the value of the norm.
<table>
<thead>
<tr>
<th>Group</th>
<th>Variable</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographic</td>
<td>Location</td>
<td>North: BC3, BCN, BON, CH, COA, NL, TAS, DDI</td>
</tr>
<tr>
<td></td>
<td></td>
<td>West: NAX, JAI, COI, PAC, S1P, AGS, GYD, WCM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Central: GR0, HGD, MEX, TUX, VUE, MOR, YER, D1 and</td>
</tr>
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<td></td>
<td></td>
<td>South: GRO, OAX, CH3, IAR, CAMP, YUC, OIR</td>
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<tr>
<td></td>
<td>City size</td>
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<td></td>
<td></td>
<td>15,000 to 99,999 inhabitants,</td>
</tr>
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<td></td>
<td></td>
<td>2,500 to 14,999 inhabitants and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>less than 2,500 inhabitants.</td>
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<tr>
<td></td>
<td>Sex</td>
<td>Male, Female</td>
</tr>
<tr>
<td></td>
<td>Age</td>
<td>Years</td>
</tr>
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<td>Household head</td>
<td>Education level</td>
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</tr>
<tr>
<td></td>
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<td>complete highschool,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>incomplete undergraduate</td>
</tr>
<tr>
<td></td>
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<td>complete undergraduate</td>
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<td></td>
<td></td>
<td>graduate studies</td>
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<td>Household composition</td>
<td>Dummy variables</td>
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<td>female</td>
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<tr>
<td></td>
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<td>adults</td>
</tr>
<tr>
<td></td>
<td></td>
<td>elderly</td>
</tr>
<tr>
<td></td>
<td>Daily Intake perception</td>
<td>0 = without problems</td>
</tr>
<tr>
<td></td>
<td>Index</td>
<td>20 = worried about food intake</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70 = limited food intake</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100 = no food</td>
</tr>
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<td>Shopping frequency index</td>
<td>0 = monthly</td>
</tr>
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<td>30 = biweekly</td>
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<td></td>
<td>75 = weekly</td>
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<tr>
<td></td>
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<td>90 = every three days</td>
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<td>100 = daily</td>
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<td>Type of property</td>
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<td></td>
<td>Other</td>
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<td></td>
<td>Property Ownership</td>
<td>Own property</td>
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<td>Rented</td>
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<td>Vehicles</td>
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<tr>
<td></td>
<td>New Vehicles</td>
<td>Brand new vehicles (cars, trucks, etc)</td>
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Table D1: Household consumption categories
<table>
<thead>
<tr>
<th>Group of variable</th>
<th>Variable</th>
<th>Categories</th>
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<td><strong>Health condition</strong></td>
<td><strong>Health problems</strong></td>
<td>Household members with chronic health problems</td>
</tr>
<tr>
<td></td>
<td><strong>Child attention</strong></td>
<td>Medical attention to children and mothers</td>
</tr>
<tr>
<td></td>
<td><strong>Disability</strong></td>
<td>Household members with some disability</td>
</tr>
<tr>
<td></td>
<td><strong>Medical attention</strong></td>
<td>Household members who get medical attention when needed</td>
</tr>
<tr>
<td></td>
<td><strong>Automedication</strong></td>
<td>Household members self-medicating</td>
</tr>
<tr>
<td></td>
<td><strong>Frequency index</strong></td>
<td>100: Always 80: Frequent 60: Occasionally 20: Sometimes 0: Never</td>
</tr>
<tr>
<td><strong>Medical services</strong></td>
<td><strong>Reason no attention</strong></td>
<td>No money Personal reason No clinic No medication Bad service</td>
</tr>
<tr>
<td></td>
<td><strong>(Dummy variable for each category)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>segsoc</strong></td>
<td>Household member contributing to soc. segsoc</td>
</tr>
<tr>
<td></td>
<td><strong>segpop</strong></td>
<td>Seguro Popular ISSSTE</td>
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<tr>
<td><strong>Institutional coverage</strong></td>
<td><strong>Affiliation counters:</strong></td>
<td>ISSSTE states ISSSTE states</td>
</tr>
<tr>
<td></td>
<td><strong>(Households members affiliated to medical institution)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Medical service provider</strong></td>
<td><strong>Affiliation counters:</strong></td>
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<td></td>
<td>Other providers</td>
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<tr>
<td><strong>Type of School</strong></td>
<td><strong>Education expenses</strong></td>
<td>Amount received</td>
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<td><strong>Scholarships public institutions</strong></td>
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<tr>
<td></td>
<td><strong>Scholarship private institutions</strong></td>
<td>Amount received</td>
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<td><strong>Tuition waivers</strong></td>
<td>Amount waived</td>
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<td><strong>Social Programs</strong></td>
<td><strong>Oportunidades</strong></td>
<td>Household enrolled in OPORTUNIDADES</td>
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<tr>
<td></td>
<td><strong>Government transfers</strong></td>
<td>Government transfers as fraction of household income</td>
</tr>
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Table D2: Household consumption categories
### UNCOMPENSATED CROSS-PRICE ELASTICITIES OF SUBSTITUTIONS (MARSHALLIAN)

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<tbody>
<tr>
<td>Food</td>
<td>-0.77</td>
<td>-0.59</td>
<td>-0.51</td>
<td>-0.60</td>
<td>-0.64</td>
<td>-0.58</td>
<td>-0.60</td>
<td>-0.47</td>
<td>-0.60</td>
<td>0.85</td>
<td>-0.17</td>
<td>-0.73</td>
<td>-0.26</td>
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<tr>
<td>Beverages</td>
<td>-0.42</td>
<td>-1.38</td>
<td>-0.62</td>
<td>-0.42</td>
<td>-0.61</td>
<td>-0.22</td>
<td>-0.39</td>
<td>-0.24</td>
<td>-0.31</td>
<td>1.44</td>
<td>-0.31</td>
<td>-0.11</td>
<td>-0.43</td>
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<tr>
<td>Alcohol and Tobacco</td>
<td>-1.15</td>
<td>-1.50</td>
<td>-1.54</td>
<td>-0.63</td>
<td>-1.03</td>
<td>-0.68</td>
<td>-0.71</td>
<td>-0.69</td>
<td>-0.04</td>
<td>1.26</td>
<td>-0.69</td>
<td>0.11</td>
<td>-1.11</td>
</tr>
<tr>
<td>Medicines</td>
<td>-1.41</td>
<td>-0.80</td>
<td>-0.45</td>
<td>-0.40</td>
<td>-0.82</td>
<td>1.29</td>
<td>-0.66</td>
<td>-0.38</td>
<td>0.58</td>
<td>-0.00</td>
<td>-0.80</td>
<td>0.05</td>
<td>-0.36</td>
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<tr>
<td>Clothing</td>
<td>0.02</td>
<td>0.14</td>
<td>0.18</td>
<td>0.13</td>
<td>-0.36</td>
<td>0.04</td>
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Table D4: Price and cross-price elasticities between retail sectors
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Encuesta Nacional de Ingresos y Gastos de los Hogares ENIGH. Instituto Nacional de Estadística, Geografía e Informática. 2010.


C. M. Urzua. The Ahmad-Stern approach revisited: Variants and an application to Mexico. EGAP Working Papers, Tecnologico de Monterrey, Campus Ciudad de Mexico, April 2004.

