Classification Techniques for Undersampled Electromyography and Electrocardiography

by

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ABSTRACT

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Electrophysiological signals including electrocardiography (ECG) and electromyography (EMG) are widely used in clinical environments for monitoring of patients and for diagnosis of conditions including cardiac and neuromuscular disease. Due to the wealth of information contained in these signals, many additional applications would be facilitated by full-time acquisition combined with automated analysis. Recent performance gains in portable computing devices and large scale computing platforms provide the necessary computational resources to process and store this data; however, challenges at the sensor level have prevented monitoring systems from reaching the practicality and convenience necessary for widespread, continuous use. In this thesis, we examine the feasibility of applying techniques from the compressive sensing field to the acquisition and analysis of electrophysiological signals. These techniques allow signals to be acquired in compressed form, thereby providing a means to reduce power consumption of monitoring devices. We demonstrate the effects of several methods of compressive sampling and reconstruction on standard compression and reconstruction error metrics. Additionally, we investigate the effects of compressive sensing on the accuracy of automated signal analysis techniques for extracting useful information from ECG and EMG signals.
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Chapter 1

Introduction

Electrophysiology is the study of electrical activity and properties of biological systems. Many important physiological phenomena in humans occur through electrical means, including the conduction of sensory and control signals, the processing of information by the nervous system, and the contraction of muscles [1]. Due to the wide variety of electrical signals generated by physiological processes and the integral role played by electrophysiology, acquisition of these signals has found numerous applications in medicine, science, and human-computer interfacing [2]. In the medical field these signals are widely used for monitoring the vital signs of patients during invasive medical procedures; for diagnosis of conditions such as cardiac disease [3], epilepsy [4], and neuromuscular disease [5]; and for control of computer user interfaces and prosthetic limbs by amputees [6].

The information content of electrophysiological signals has stimulated great interest in techniques for automated interpretation. In particular, full time monitoring of electrophysiological signals would require automated processing due to the large quantity of data generated. Pattern recognition algorithms have gained recent attention for their excellent performance in these applications due to robustness to noise and artifacts encountered in non-ideal acquisition environments [7]. These algorithms are able to discover complex relationships in data given only a set of training examples: input vectors and desired output labels for which a rule to associate inputs with outputs is sought. We review one of the most successful automated pattern classifi-
cation techniques, the Support Vector Machine (SVM), which is notable for its firm theoretical foundation [8]. The process of training the SVM is formulated as a convex optimization problem, avoiding many of the difficulties with convergence to local minima faced by other classification methods such as neural networks [9]. Since the original development of the SVM, improvements have given it the ability to construct nonlinear decision surfaces, to handle errors in the training data, and to make classification decisions to distinguish more than two classes [10]. Performance of the SVM in classifying ECG arrhythmia is reviewed before continuing on to discuss improvements in acquisition efficiency gained through the application of newly-developed sampling techniques.

Despite the wide availability of electrophysiological acquisition systems for clinical applications, the technology has not found widespread use in full-time monitoring. Monitoring devices are often cumbersome to wear and require periodic reapplication of electrodes, and external electrodes and wires are susceptible to noise and motion artifacts as the subject goes about daily activities [11]. Minimally invasive implanted acquisition devices can solve some of these issues but present their own challenges. Power consumption is a particularly difficult challenge in the development of these devices due to the limitations of implantable energy storage and power generation techniques. The severe power constraints imposed on implantable devices present a need for aggressive power optimization in the development of practical implanted electrophysiological signal acquisition systems [12].

Recent developments in the field of Compressive Sensing (CS) have provided new signal sampling techniques for improving efficiency of these acquisition devices. Conventional Nyquist-Shannon sampling theory allows for the digital acquisition of band-limited signals by sampling at a rate of at least twice their highest frequency compo-
nent; however, this sampling method often leads to the acquisition of redundant data that is later discarded by compression algorithms. Results in the field of Compressive Sensing have provided more efficient methods of acquiring some of these types of signals by reducing the quantity of data needed to be acquired. CS allows an improvement over Nyquist-rate sampling for the class of sparse signals, ones which can be represented in some basis using a small number of nonzero coefficients [13]. These coefficients are measured by calculating projections of the signal to be acquired onto sampling vectors, and the measurement process is linear and nonadaptive [14], facilitating efficient hardware implementation. The gains in sampling efficiency afforded by CS come at the cost of a more difficult, nonlinear reconstruction process. Nevertheless, the trade-off may be advantageous for the acquisition of sparse signals when computational resources are relatively expensive at the sensor and cheap at the point of reconstruction. Theoretical results in CS provide some sufficient conditions for reconstruction of sparse signals from CS projections [15], and numerous algorithms have been developed to perform this reconstruction. In addition, extensions of CS have been developed to make use of additional knowledge of the signal beyond sparsity [16].

In this thesis, we study the feasibility of applying CS to electrophysiological measurements. We begin with a review of electrophysiology and acquisition of electrophysiological signals in Chapter 2. We then review the SVM and its use in automated analysis of electrophysiological signals in Chapter 3. We begin Chapter 4 with a review of compressive sensing theory and algorithms and continue by examining the effects of compression at varying levels and subsequent reconstruction via several algorithms on common error metrics such as signal-to-noise ratio for EMG and ECG signals. Due to the difficulty in extrapolating these performance metrics to auto-
mated analysis performance, we evaluate the effects of CS reconstruction error on
the accuracy of pattern recognition techniques applied to two specific applications of
automated electrophysiological signal analysis: cardiac arrhythmia classification from
ECG signals and muscle contraction force classification from EMG signals.
Chapter 2

Electrophysiology

Electrophysiology is the study of electrical activity and properties of biological systems. Electrophysiological signals are generated by the conduction of ions between and within several types of cells, including neurons and muscle fibers [17]. Neurons provide organisms with a means to communicate and process sensory and control information necessary for survival. In humans, neurons relay sensory information to the central nervous system for processing and provide control signals to muscles for reaction to this information, and they also carry control stimuli to glands to influence chemical processes and to translate electrical signals into chemical signals in the form of hormones [18].

A neuron functions by generating electrical potential spikes across its cell membrane in response to external stimuli. Ion pumps in the neuron cell membrane continuously transport positively-charged sodium ions out of the cell while transporting positively charged potassium ions into the cell, maintaining a potential of roughly $-65 \text{ mV}$ across the cell membrane at rest [19]. Specialized proteins embedded in the cell membrane known as a voltage-gated ion channels remain closed, holding back the sodium and potassium concentration gradients produced by the ion pumps, until an external stimulus raises the neuron’s membrane potential [20]. For small increases in membrane potential, a few sodium ion channels open, but the ion pumps are able to return the cell membrane to its resting potential. However, once the membrane potential is raised above a particular threshold, sodium ion channels open in great
enough numbers that the ion pumps can no longer lower the membrane potential. The rising voltage opens yet more sodium ion channels and raises the membrane potential further in a positive feedback loop. The membrane potential rises rapidly until the sodium ion channels become deactivated and potassium ion channels open, allowing a rapid outflux of potassium ions to quickly lower the cell membrane back toward its resting potential; the entire process occurs in approximately one millisecond and is known as an action potential [21].

After producing an action potential the voltage-gated ion channels require some refractory period before they can be reactivated; this gives the ion pumps time to reset the ion concentrations to the resting state [22]. Successive action potentials have largely the same amplitude and shape, so information is encoded in their timing [23]. Neurons signal other cells through connections called synapses [24]. Electrical synapses allow bidirectional conduction of current between neurons so that an action potential in one neuron may trigger action potentials in its neighbors, while chemical synapses allow one way communication through the excretion of signaling molecules known as neurotransmitters during an action potential. These neurotransmitters diffuse across the synapse to a receiving cell and bind to receptors, which may open or inhibit ion channels or affect the production of other chemicals in the cell [19].

Due to the wide variety of sources of electrophysiological activity, measurement of it can yield vast amounts of information concerning the function and state of biological systems. This information has a wide range of uses in for medical, scientific, and consumer applications. Table 2.1 lists several types of electrophysiological signal by source.
2.1 Electromyography

Electromyography (EMG) is the study of electrical activity generated by muscles, excluding the heart. EMG is used in medical settings for studying and diagnosing neuromuscular disease—a broad category of disorders which impair the function of muscles [25]. The causes of these disorders can be neurological, as in the case of amyotrophic lateral sclerosis [26], multiple sclerosis [27], Parkinson’s disease [28], and Huntington’s disease [29]; other types such as muscular dystrophy arise from pathology of muscle tissue [30]. EMG monitoring additionally provides a means to monitor progress of rehabilitation after injury [31] and is a promising method for developing new interfaces through which humans can control computers [32].

The EMG signal is generated during muscle contraction, which begins with the production of an action potential by a motor neuron in the brain; this action potential is relayed to a lower motor neuron in the spinal cord [19]. The lower motor neuron transmits the action potential to specialized synapses connecting it to muscle fibers, known as neuromuscular junctions [33]. During an action potential a neurotransmitter called acetylcholine is released by the neuron into the neuromuscular junction. It diffuses across and binds to receptors on the muscle fiber. This binding opens ion
channels on the muscle cell membrane, causing an action potential to propagate across it. The propagating charge density activates calcium release channels within the muscle fiber, and the subsequent increase in calcium concentration activates the production of contraction force by protein filaments within the muscle fiber [34].

When one motor neuron produces an action potential, all of the muscle fibers innervated by that neuron contract. These fibers and the single controlling neuron are known as a motor unit; large skeletal muscles have more fibers in a motor unit than small, precise muscles [35]. When a motor unit contracts, the combination of all the resulting electrical activity is called a motor unit action potential (MUAP) [36]. The EMG signal consists of a sequence of MUAPs from several motor units within one muscle and potentially some in nearby muscles. The individual MUAPs last only a few milliseconds, so individual motor units contract repeatedly in rapid succession to produce steady muscle contraction force. Contraction force of a given muscle may be modulated via changing the rate of production of MUAPs by its constituent motor units [37].

2.1.1 EMG Acquisition

EMG may be acquired using two methods. Surface EMG is measured using electrodes attached to the skin, while intramuscular EMG is acquired from electrodes embedded directly in muscle tissue. Surface EMG is typically measured using bipolar electrodes, pairs of conductive plates attached to the skin. Propagation of EMG signals through fat and skin tissue has the effect of low-pass filtering and spreading the signals, so surface EMG has relatively low spatial resolution and little information about individual motor units [35]. In contrast, intramuscular EMG is measured closer to its source giving greater spatial resolution. Less filtering occurs between the motor units
and electrodes, so this method yields higher signal-to-noise ratio than surface EMG. Because of its fine spatial resolution, intramuscular EMG is capable of isolating the activity of individual motor units [38].

In addition to the electrical potentials measured in an EMG study, several characteristics of the contraction are recorded. Contraction effort is usually specified as a percentage of maximal voluntary contraction (MVC), the strongest contraction that the subject can voluntarily produce. Three example EMG recordings at different contraction efforts are shown in Figure 2.1. Contractions are further described as concentric when the muscle shortens, eccentric when the muscle lengthens, or isometric when the muscle remains a constant length.

![Figure 2.1](image.png)
2.2 Electrocardiography

The recording of electrical potentials produced by the heart is known as electrocardiography (ECG or EKG) [39]. ECG signals can be measured by attaching electrodes to a subject’s skin near the heart. The electrical potentials at each electrode are amplified, filtered, and sampled, and potential differences between electrode pairs are interpreted for medical diagnosis. The number of electrodes used for recording ECG may vary depending on the application: Two electrodes are needed as a minimum, but more are often used in clinical ECG monitoring to gain spatial sensitivity [40].

The heart is responsible for circulating blood throughout the body for the purpose of delivering oxygen and nutrients to cells and removing waste products of cellular metabolism [41]. The heart is a rhythmic pump; each pumping cycle is called a heartbeat. The heart has two sides: The left side receives oxygenated blood from the lungs and pumps it to the rest of the body, while the right side receives this blood after oxygen has been removed and pumps it to the lungs. On each side, blood enters the heart into an upper chamber called the atrium. At the start of each heartbeat, the atria contract, moving blood into lower chambers known as ventricles. Between the atria and ventricles are one-way valves that allow the ventricles to contract strongly without forcing blood back to the atria; a powerful contraction is needed to develop sufficient pressure to supply the entire body with blood.

Since the heart is a type of muscle, ECG is generated in a similar manner to EMG. However, the heartbeat is initiated by an electrical impulse generated by a group of cells located atop the right atrium, called the sinoatrial (SA) node, which functions as the body’s pacemaker by adjusting the delay between successive impulses [42]. An impulse originating from the SA node travels across the heart, first reaching the atria and causing them to contract. Subsequently, the impulse reaches the atrioventricu-
lar (AV) node which delays it before propagating it to the ventricles. The impulse is distributed over the ventricle walls, producing a powerful contraction followed by relaxation while the atria refill in preparation for the next pulse. Both the rate of impulse production by the SA node and the velocity of conduction through the AV node are modulated by the central nervous system [43]: Stimulation from the sympathetic nervous system increases heart rate during stressful situations, while stimulation from the parasympathetic nervous system, responsible for general maintenance of the body at rest, decreases heart rate.

2.2.1 ECG Acquisition

The electrical activity generated during a heartbeat can be observed to determine whether the heart is functioning properly. The major structures of the ECG waveform, illustrated in Figure 2.2, are known as the P-wave, QRS-complex, T-wave,
and U-wave. The P-wave indicates contraction of the atria at the beginning of a heartbeat, the QRS complex corresponds to the contraction of the ventricles, and the T-wave denotes the repolarization of the ventricles as they recover [44]. The U-wave is not always present and its origin is uncertain [45].

ECG is often used to monitor the status of medical patients. Any interruption in the patient’s heart function can be detected from the ECG signal, so medical personnel can be notified immediately to take ameliorative action. In addition to simply checking whether or not the heart is functioning, ECG may be used to monitor heart rate. An abnormally fast heart rate, for example, may indicate low blood pressure; to ensure adequate blood supply to the body, the heart reacts by increasing speed [46].

Other medical conditions may be indicated by irregularity in the heart rhythm, known as cardiac dysrhythmia or arrhythmia. Because the various stages of heart contraction are visible in the ECG waveform, the waveform may indicate the specific region of the heart affected by an abnormal structural or chemical condition. Figure 2.3 shows an example type of arrhythmia known as premature ventricular contraction (PVC), which occurs when the ventricles contract before or during the atrial contraction. When this occurs the ventricles typically contain less blood than normal, so contraction volume is reduced [47]. Isolated instances of PVC are common and usually harmless, but prolonged episodes can indicate medical problems [48].

A chaotic heart rhythm can indicate ventricular fibrillation, a condition where the ventricular muscle of the heart begins contracting in an uncoordinated manner, rendering it unable to pump blood effectively [49]. Immediate defibrillation, administration of an electric shock, is required to restore the proper functioning of the heart. In patients at high risk for ventricular fibrillation, an implantable cardioverter-
defibrillator (ICD) may be connected to the heart to perform defibrillation when necessary [50].

Some arrhythmia types may have no immediate symptoms but raise long-term risk for other medical conditions. One of the most common types of arrhythmia, atrial fibrillation, may cause palpitations or chest pain for some people but produce no noticeable symptoms in others; however, the presence of atrial fibrillation raises risk for stroke by a factor of five [51]. Atrial fibrillation is caused when heart’s natural pacemaker impulse is overwhelmed by spurious activity generated in the atrium or pulmonary veins and can be observed in the ECG signal as diminished P-wave along with persistent low amplitude oscillation added to the normal ECG shape [52]. Other arrhythmia types have no harmful effect on their own but may trigger more dangerous
arrhythmias when structural heart problems develop due to cardiovascular disease. For example, premature atrial contraction occurs when the atrium contracts before receiving an impulse from the sinoatrial node; it is not harmful on its own but has been observed to trigger episodes of atrial fibrillation [53]. Heart rhythm can also indicate several acute medical conditions including electrolyte imbalance [54] and adverse drug interactions [55].

Some types of arrhythmia occur only while the heart is stressed, so a cardiac stress test may be performed via strenuous exercise or medication to provoke the condition during ECG monitoring [56]. If a stress test is unable to reproduce the arrhythmia, *ambulatory ECG* may be used to monitor a patient as they go about their normal activities [57]. Ambulatory ECG is the use of a portable ECG device, called a Holter monitor, that is worn by the patient for acquiring long-term ECG recordings. The Holter monitor consists of a recording device with several electrodes attached to the chest area by adhesive gel. After a monitoring period of a few days, the device is returned to medical staff for analysis of the recording. For more infrequent arrhythmia, a Holter monitor may be cumbersome due to the discomfort caused by attachment of electrodes to the skin and the frequent need to reapply them with adhesive gel. In this situation the patient may carry a cardiac event monitor [58]. When the patient notices an unusual heart rhythm, conductive plates on the device are pressed against the skin to acquire a recording.
Chapter 3

Analysis of Electrophysiological Signals

Due to the wealth of useful information contained in electrophysiological signals, numerous techniques have been developed to extract that information for applications such as medical diagnosis and human-computer interfaces. Much of the analysis performed in clinical use of electrophysiology is done by human expert interpretation; for example, many heart arrhythmias may be diagnosed by observing the shape, timing, or absence of structures in the ECG waveform and comparing those with sets of rules that indicate various conditions [59]. Neurologists use EMG to measure physiological properties such as conduction velocity of muscles and nerves to diagnose neuropathic and myopathic conditions [60].

In other applications, pattern recognition techniques have largely superseded these rule-based models. For example, when analyzing multiple EMG recordings of the forearm to detect specific hand gestures, hundreds of motor units contribute to the measured signal [61]. The contribution of each fiber is filtered by tissue between itself and the electrodes, and the geometry of this tissue changes as the arm is moved [62,63]. Due to these complexities it is often infeasible to develop a microscale physical model for interpreting these signals. This difficulty has driven interest in the use of automated methods to find relationships between electrophysiological signals and the underlying physiological phenomena by which they are produced. These automated pattern recognition methods comprise the field of machine learning [64]. Techniques from the machine learning field have been demonstrated on EMG for prosthetic limb
control [65–68] and on ECG for beat classification [69–74] among other applications.

In supervised machine learning, these relationships are discovered by analyzing a set of training examples, which consists of inputs known as features and desired outputs called labels. The goal of supervised learning is to analyze these training data to produce a mapping from features to labels that generalizes to features outside the training set. The Support Vector Machine (SVM) is one of the most successful approaches to supervised learning; it functions by finding the maximum margin hyperplane separating two classes of training data represented as vectors in some vector space [75]. Once trained, the SVM classifies newly-observed vectors by determining to which side of the hyperplane the observation lies.

3.1 Support Vector Machine Classification

A hyperplane may be parametrized as the set of all vectors $\mathbf{x}$ such that $\mathbf{w} \cdot \mathbf{x} - b = 0$, where $\mathbf{w}$ is a vector perpendicular to the hyperplane, and $\frac{b}{||\mathbf{w}||}$ is the perpendicular distance from the hyperplane to the origin. In order to train an SVM, $\mathbf{w}$ and $b$ are chosen such that they maximize the margin of the training examples [8]; the margin is illustrated in Figure 3.1. If the margin boundaries are parametrized $\mathbf{w} \cdot \mathbf{x} - b = 1$ for one class and $\mathbf{w} \cdot \mathbf{x} - b = -1$ for the other, then the margin size is $\frac{2}{||\mathbf{w}||}$. In order to train the SVM the margin size is then maximized, or equivalently $\frac{1}{2}||\mathbf{w}||^2$ is minimized, subject to the constraint that the training examples of each class lie on the correct side of the margin:

$$y_i(\mathbf{w} \cdot \mathbf{x}_i - b) \geq 1$$
Newly observed vectors $x_i$ may then be classified by evaluating the following decision function to assign the output class:

$$\text{sgn}(w \cdot x_i - b)$$

When training data are not linearly separable due to noise or mislabeling, the optimization problem may be modified by introducing slack variables $\xi_i$ and the regularization parameter $C$ [76]. The slack variables allow training vectors to violate the margin but penalize the objective function when this occurs, and the parameter $C$ is a scaling factor for this penalty. This form—known as the Soft Margin SVM—is expressed:

$$\min_{w, b, \xi} \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{\ell} \xi_i$$

subject to $y_i(w \cdot x_i - b) \geq 1 - \xi_i$

$\xi_i \geq 0$
In practice the SVM is usually solved in its dual form by constructing the Lagrangian
and deriving the KKT conditions, yielding:

$$\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{\ell} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{\ell} \alpha_i \alpha_j y_i y_j (x_i \cdot x_j) \\
\text{subject to} & \quad \sum_{i=1}^{\ell} \alpha_i y_i = 0 \\
& \quad 0 \leq \alpha_i \leq C
\end{align*}$$

where the $\alpha_i$ are Lagrange multipliers. The advantage of solving in this form is that
the problem can be reformulated to allow the use of nonlinear decision surfaces [10].
This is made possible due to the fact that the optimization depends only on inner
products between the training vectors. The inner products may be replaced by a
kernel function, which can allow the SVM to find nonlinear decision boundaries by
nonlinearly mapping the data to a higher-dimensional space in which a linear decision
boundary is found. The kernel function $K(u, v)$ is constructed such that $K(u, v) = \phi(u) \cdot \phi(v)$, where $\phi$ is the nonlinear mapping to a space of higher dimension. For
example, given vectors $u = [u_1, u_2]$ and $v = [v_1, v_2]$, and the kernel $K(u, v) = (u \cdot v)^2$,
we have:

$$K(u, v) = (u_1 v_1 + u_2 v_2)^2$$
$$= u_1^2 v_1^2 + 2u_1 u_2 v_1 v_2 + u_2^2 v_2^2$$
$$= [u_1^2, \sqrt{2} u_1 u_2, u_2^2] \cdot [v_1^2, \sqrt{2} v_1 v_2, v_2^2]$$
$$= \phi([u_1, u_2]) \cdot \phi([v_1, v_2])$$
$$= \phi(u) \cdot \phi(v)$$

where $\phi : \mathbb{R}^2 \to \mathbb{R}^3$ is defined $\phi([u_1, u_2]) := [u_1^2, \sqrt{2} u_1 v_2, u_2^2]$. Some common choices
of kernel function include the following [77]:

\[
\begin{align*}
\text{polynomial:} & \quad K(u, v) = (u \cdot v + c)^d \\
\text{radial basis function:} & \quad K(u, v) = e^{||u - v||^2/(2\sigma^2)} \\
\text{sigmoidal:} & \quad K(u, v) = \tanh(\kappa u \cdot v - \delta)
\end{align*}
\]

SVMs may also be used for distinguishing multiple classes [78]. Using a *one-against-one* approach, one SVM classifier is trained for each combination of class pairs. The classification output is selected as the class with the most votes. SVMs may be trained in a *one-against-all* approach as well by training a single SVM for each class to distinguish it from all the others. In this arrangement the SVM with highest output makes the class selection.

### 3.1.1 Preprocessing for SVM Classification

As SVMs operate on vectors of input data, a number of transformations may be applied to input vectors before introducing them to the SVM. This process, known as *feature extraction*, is used for several purposes: It can reduce the size of SVM input vectors, improving computational performance; it can improve robustness to noise [79]; and it can remove the effects of noninformative signal properties such as scaling or shifts [80], when appropriate. A number of ad-hoc techniques have been successfully applied to electrophysiological signal feature extraction. In the case of EMG analysis, these features are calculated over successive windows of signal samples. Some of the successfully-used features include statistical properties such as signal variance or higher order moments; autoregressive model coefficients; integral of absolute values; waveform length or total variation; number of zero crossings; number of slope sign changes; median frequency; and amplitude histograms [81–84]. Signals
may also be preprocessed by some linear transform such as the Fourier, wavelet, or Hadamard transforms before calculating these features in the resulting domain [85].

If large feature vectors are extracted from the signal, dimensionality reduction methods can be applied to reduce the size of the classifier inputs in order to improve the computational performance. **Principal component analysis (PCA)** is one of the most common dimensionality reduction techniques. PCA transforms the input vectors into an orthogonal set of principal components, where the largest principal component is a vector that lies in the direction of greatest variance of the input vectors [86]. Dimensionality reduction is accomplished by retaining only a subset of the principal components consisting of the ones of greatest magnitude. Like the SVM, PCA may be augmented with kernels to perform the decomposition in a higher dimensional nonlinear mapping of the input space [87]. Figure 3.2 shows features extracted from record 106 of the MIT-BIH arrhythmia database decomposed with both linear PCA and kernel PCA onto the two largest principal components, corresponding to the axes. Features are chosen as the coefficients from the four lowest subbands of the Haar wavelet decomposition, along with the sum of absolute values of the signal coefficients. The top figure shows a linear decomposition, while the bottom uses a third degree polynomial kernel. The data are nearly separable in the linear PCA decomposition, but still there remain a substantial number of premature ventricular contractions that are not separable from normal beats. Kernel PCA succeeds in producing a better separation with two components; in addition, the classes are linearly separable.

**3.1.2 ECG Classification with SVM**

Since the majority of information contained in the ECG waveform is related to its shape, QRS detection is one of the most useful automatic analysis techniques. The
QRS complex indicates contraction of the ventricles, the most crucial component of a heartbeat; thus its absence indicates a medical emergency such as ventricular fibrillation or cardiac arrest. QRS detection is also used to find heart rate by determining the duration between successive R-waves. QRS detection algorithms are typically
compared by their sensitivity:

\[ Se = \frac{TP}{TP + FN}, \]

and precision, or positive predictive accuracy:

\[ +P = \frac{TP}{TP + FP}, \]

where TP is the number of successfully detected beats, FP the number of falsely detected beats, and FN the number of missed beats. Some of the successful approaches to QRS detection (achieving greater than 99% sensitivity and positive predictive accuracy) include time-domain waveform shape [88–91], wavelet transforms [92–95], and phase space portraits [96,97].

QRS detection is also the first step in analyzing arrhythmia in ECG signals: Other components of the heartbeat are localized relative to the QRS complex. The shape of the QRS itself also provides useful diagnostic information. For example, a prolonged QRS complex can indicate damage to the ventricles caused by a heart attack [98]. This damaged tissue may cause the cardiac action potential to propagate less quickly, noticeably changing the ECG signal. Once the QRS complex has been located, signal features are extracted with reference to the QRS locations, and automated pattern recognition may be applied to determine the type of beat.

The standard benchmark for evaluating QRS detection and arrhythmia classification methods is the MIT-BIH Arrhythmia Database, which consists of 48 half-hour recordings of ambulatory ECG sampled at 360 Hz [99]. The signals are annotated with the temporal location of each QRS complex and with the type of beat: normal or arrhythmic, including several types of premature beats; escape beats; bundle branch block beats; and ectopic beats.
ECG arrhythmia classification was performed using the system illustrated in Figure 3.3. QRS detection was implemented using the algorithm described in [100]: A high frequency, low amplitude oscillation is added to a band-pass filtered version of the ECG recording and the number of zero crossings is counted. During the QRS complex peak, the added oscillation no longer produces zero crossings, thus the rate of zero crossings may be used to find peaks. The amplitude of the oscillation is adaptively adjusted to exclude longer duration peaks, such as the T-wave. This method achieves sensitivity 99.72% and precision 99.62% over the entire MIT-BIH database.

![Figure 3.3: ECG Beat Classification System](image)

ECG features were extracted from windows of 128 samples (356 ms at the sampling rate of 360 Hz) centered on each detected QRS location. Features were calculated by performing a decomposition of the window into the Haar wavelet basis and retaining only the four lower subbands of coefficients. The mean of the absolute values of signal amplitudes over the window was concatenated with the wavelet coefficients to form each feature vector. Each record in the MIT-BIH database contains a 30 minute ECG recording. The training feature set was drawn from the first 15 minutes, and the test set was drawn from the remaining 15 minutes. Classification was performed using a soft margin SVM with polynomial kernel \( K(\mathbf{u}, \mathbf{v}) := (\mathbf{u} \cdot \mathbf{v} + 1)^2 \) and with misclassification penalty factor \( C = 1 \).

Classifier accuracy was evaluated on several records from the MIT-BIH Arrhythmia Database. The confusion matrix of classification results is shown in Table 3.1.
Table 3.1: Confusion matrix for classification of arrhythmia in MIT-BIH record 106.

<table>
<thead>
<tr>
<th></th>
<th>Detected</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>V</td>
</tr>
<tr>
<td>Actual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>668</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>2</td>
<td>332</td>
</tr>
<tr>
<td>∅</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| Precision | 99.7% | 100.0% |

Table 3.1: Confusion matrix for classification of arrhythmia in MIT-BIH record 106.

for record 106, which consists of normal QRS beats (N) and premature ventricular contractions (V). Overall accuracy is 99.70%, sensitivity is 99.70%, and precision is 99.80%. Accuracy is calculated as

\[
\text{accuracy} = \frac{TC}{TC + FC + FP + FN}
\]

where TC is the number of successful classifications, FC is the number of failed classifications on correctly-detected beats, FP is the number of false positive QRS beats detected, and FN is the number of beats missed by the QRS detector. The row labeled ∅ shows the false positive beats from the QRS detector and which class the SVM assigned to these spurious beats. The column labeled with ∅ shows the number of beats from each class that were missed by the QRS detector. Sensitivity and precision of the classifier within each beat class are shown in the last column and last row of the table, respectively.

The confusion matrix for record 233 is shown in Table 3.2. This record contains beats of four classes; in addition to the normal QRS beats and premature ventricular contractions contained in record 106 there are atrial premature beats (A) and fusions of ventricular and normal beats (F). Examples of these beats are shown in Figure 3.4 taken from a segment of the recording. The classification was performed using a
one-against-one multiple SVM approach and voting strategy. For this record, overall accuracy is 97.59%, sensitivity is 97.59%, and precision is 97.79%

Classifier performance suffers on the atrial premature beats and fusion beats, partly due to the lack of training examples for these beat types; the first 15 minutes of the recording, used as the training set, contain only three examples of atrial premature contraction and eight examples of fusion beats. Additionally, the fusion beats are difficult to discern from normal beats. Still, the overall classification accuracy remains high.
Table 3.2: Confusion matrix for classification of arrhythmia in MIT-BIH record 233.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>V</th>
<th>A</th>
<th>F</th>
<th>∅</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>1104</td>
<td>0</td>
<td>12</td>
<td>1</td>
<td>0</td>
<td>98.84%</td>
</tr>
<tr>
<td>V</td>
<td>3</td>
<td>393</td>
<td>5</td>
<td>11</td>
<td>3</td>
<td>94.7%</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>75.0%</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>50.0%</td>
</tr>
<tr>
<td>∅</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

3.2 Applications of Continuous Monitoring and Automated Analysis of Electrophysiology

According to the American Heart Association, Cardiovascular Disease (CVD) accounts for one third of all deaths in the United States, and CVD cost an estimated $286 billion in medical expenses and lost productivity during the year 2007 alone [101]. Many CVD cases can be attributed to lifestyle factors including diet, lack of physical activity, smoking, and stress. Making ECG monitoring more accessible to the general public could allow earlier diagnosis of CVD and greater awareness of CVD risk factors. These benefits could translate into greater lifespans for those willing to make the necessary lifestyle changes and lower medical costs due to earlier detection of disease and less diagnosis performed in clinical environments. Continuous monitoring of ECG can also improve response time during medical emergencies such as cardiac arrest, ventricular fibrillation, heart attack, or trauma by automatically notifying emergency personnel [102].

Full-time monitoring of EMG has numerous applications as well. EMG is a suc-
cessful and widely-used method for controlling limited degree-of-freedom prosthetic hands; some types of commercially-available prosthetic arms use surface EMG to estimate the contraction force of remaining muscles in the arm of amputees [103]. The user modulates contraction force of the particular measured muscle to modulate the contraction force of a gripping device on the prosthesis. This scheme is sufficient for prostheses with one or two degrees of freedom and has been used to produce preprogrammed gestures, but surface EMG has not successfully scaled to the high degree-of-freedom prosthetic hands needed to approach the functionality of an intact human hand. For example, the Modular Prosthetic Limb, shown in Figure 3.5 [104], has 22 degrees of freedom, far more than have been successfully controlled using surface EMG.

Currently the best performing method of controlling high degree-of-freedom prosthetic limbs is via intracortical neural implants, which are electrode arrays implanted directly in the motor cortex of the brain [105]. However, several medical challenges prevent widespread use of these implants. Implantation requires invasive brain surgery and has been demonstrated only in severe cases of paralysis. These implants cause damage to brain tissue both during insertion and over time as scar tissue develops around the electrodes; furthermore, this scar tissue sheath insulates the electrodes from brain activity resulting in diminished signal quality [106].

One of the promising methods of providing prosthetic control is through implanted intramuscular EMG sensors. The Implantable Myoelectric Sensor (IMES), shown in Figure 3.6 [107], is one such device currently in development. This device strikes a balance between invasive cortical implants and noninvasive skin surface signal acquisition; the IMES sensor may be injected directly into the muscle of interest using a hypodermic needle [108].
Continuous electrophysiological signal monitoring also has applications in electroencephalography (EEG) for detecting and predicting seizures [109], which could allow those suffering from epilepsy to operate motor vehicles provided that a monitoring system could give enough warning ahead of a seizure. EEG has also been shown effective in predicting awareness of drivers [110]; thus EEG monitors could warn
drivers or other operators of heavy machinery when at risk of falling asleep [111]. Brain-computer interfaces have been implemented via EEG monitoring as well to provide severely paralyzed individuals the ability to enter text into computers [112].

3.3 Challenges in Acquisition of Electrophysiology for Continuous Analysis

Recent performance gains in portable computing devices such as smartphones, together with the availability of large-scale cloud computing platforms, provide the necessary computational resources to process and store continuous electrophysiological recordings [113]. However, there remain several technical challenges in developing these monitoring systems, particularly at the sensor level. One of the barriers to widespread use of electrophysiological recording is the difficulty of maintaining electrode attachment. In a clinical or laboratory setting, electrodes are attached to the skin with conductive, adhesive gel, which is often reported to be uncomfortable for extended use and tends to dry after several hours, affecting its electrical properties [114]. Dry electrodes which couple either resistively or capacitively with the skin have been developed [115], but these electrodes present other challenges. They are much more susceptible to motion artifacts from friction with the skin due to less stable attachment. They also have higher impedance in general than wet electrodes; this property predisposes them to interference from other electronics as well as input saturation caused by the slow draining of accumulated static charge [116].

Some of the problems of maintaining electrode connection can be solved by using implantable acquisition devices; however, this option presents its own difficulties. Power consumption is one of the main barriers to use of implanted acquisition de-
vices. Batteries of the type now used in pacemakers have a capacity of roughly 5 mWh with an expected 10 year lifespan [117], yielding roughly 60 nW average power supply. Analog-to-digital converters have successfully been designed to operate within this power budget [118, 119]; however, this level is insufficient for transmitting the acquired data outside the body. One of the practical options for power delivery is the use of near-field coupling between inductive coils on the implanted device and an external power supply system [120]. This wireless power supply technique has been demonstrated at power levels as low as 100 µW on the implant side and 2.5 mW on the external supply side [121]. However, these techniques are constrained to a maximum safe power dissipation in the tissue surrounding the implant, both from the radiation absorbed by the tissue and by the power consumption of the implanted device [122]. Other power source options include the harvesting of kinetic [123], thermal [124], or chemical [125] energy from the body itself; however, a recent review of these techniques found them to produce less than 100 µW [126].

Considering these difficulties in supplying power to implanted signal acquisition devices, optimization of power consumption is crucial to the feasibility of implanted acquisition devices. We will now discuss new sampling techniques from the field of compressive sensing which can facilitate the development of implanted continuous signal acquisition systems able to meet these stringent power requirements.
Chapter 4

Classification of Electrophysiological Signals using Compressive Sensing

Conventional Nyquist-Shannon sampling theory allows for the digital acquisition of band-limited signals by sampling at a rate of at least twice their highest frequency component. However, many signals of interest have large bandwidth but relatively small information content. Recent results in the field of Compressive Sensing have demonstrated that there are more efficient methods of acquiring some signals of this class. Compressive Sensing (CS) allows for the sampling and recovery of sparse signals—those which can be represented in some basis using a small number of nonzero basis coefficients—from a small number of linear projections [127]. Theoretical results in the CS area provide sufficient conditions for reconstruction of sparse signals from these projections [15], and a wide variety of algorithms have been developed to perform this reconstruction.

Conventional signal acquisition systems do not exploit signal sparsity during sampling; they sample at the Nyquist rate, which depends only upon the signal bandwidth [128]. Sparsity is sometimes exploited after sampling by applying transform coding to the measurements, as is done by a variety of image, audio, and video codecs. For example, the JPEG-2000 image codec transforms images to the wavelet domain before performing quantization and entropy coding [129]. This compression process reduces space required to store the signal and saves power and time needed to transmit the signal. Still, much of the sampling effort is redundant so large numbers of bits
are discarded during the compression process. Compressive sensing allows a reduction in the number of measurements and avoids the separate computation necessary to apply transform coding. The acquisition and compression can be combined into a single step that is both linear and nonadaptive [14]. This advantage comes at the cost of greater difficulty in recovering signals from their measurements. Nevertheless, this trade-off may be advantageous where sampling is difficult or expensive.

4.1 Sparsity

A discrete signal $x$ of length $N$ can be considered a vector in $\mathbb{R}^N$. As such, it may be represented as a linear combination of basis vectors $\psi_i$:

$$x = \sum_{i=1}^{N} \alpha_i \psi_i = \Psi \alpha,$$

where $\Psi \in \mathbb{R}^{N \times N}$, $\alpha \in \mathbb{R}^N$ where the basis vectors $\psi_i$ form the columns of $\Psi$, and the coefficients $\alpha_i = \langle x, \psi_i \rangle$. The signal $x$ is considered to be $K$-sparse if $K$ or fewer of the coefficients are nonzero [13]. If a sparse signal is corrupted by additive noise, as are many real-world signals, it may occur that none of the coefficients are exactly zero. In the context of compressive sensing, these signals are considered compressible or approximately sparse if the magnitude of their coefficients decays as a power law or faster [130]. More precisely, if their coefficients $\alpha$ are computed and sorted in order of decreasing magnitude:

$$|\alpha|_{(1)} \geq |\alpha|_{(2)} \geq \cdots \geq |\alpha|_{(N)},$$

where $|\alpha|_{(n)}$ is the $n$-th largest coefficient (in magnitude), then for some constant factor $c$ and decay rate $d$

$$|\alpha|_{(n)} \leq c \cdot n^{-d}, \quad n = 1, 2, \ldots, N$$
Many of the results and techniques of compressive sensing have been extended to this class of signals. Figure 4.1 and Figure 4.2 show the magnitude of ECG and EMG coefficients, respectively, sorted in decreasing order on a log-log scale, demonstrating that both are approximately sparse in the wavelet domain.

### 4.2 Compressed Sampling

Compressed samples are obtained by taking linear projections $y = \Phi x$, where $\Phi \in \mathbb{R}^{M \times N}$ and $M < N$. Since the number of measurements $M$ is less than the dimensionality of the signal $N$, the signal has been compressed by a factor of $\rho = \frac{M}{N}$, known as the compression ratio. The sampling operator $\Phi$ is a linear mapping from the *data domain* $\mathbb{R}^N$ to the *measurement domain* $\mathbb{R}^M$. The rows of $\Phi$ are sampling vectors onto
which the signal is projected to form the compressed measurements. If $x$ is sparse in some basis $\Psi$, then it may be represented as $x = \Psi \alpha$ where $\alpha$ has few significant entries. The operator $\Psi$ is an invertible mapping from the sparsity domain to the data domain. The transformations between domains involved in the compressive sampling and reconstruction process are illustrated in Figure 4.3.

One of the advantages of the linear measurement process is that CS can be applied to signals that are sparse in any basis. The sparsity basis is not needed during signal acquisition—only the reconstruction algorithm needs knowledge of it. Sampling involves only applying $\Phi$ to the signal $x$, while recovery of the signal coefficients $\alpha$ requires solving the inverse problem $y = \Phi \Psi \alpha$. For notational convenience, from now on we will assume $x$ is sparse in the standard basis ($\Psi = I^N$, and $x = \alpha$) but due to linearity of the sampling process the results can be directly extended to any sparsity
Figure 4.3: The compressive sampling and reconstruction process. $x$ is the signal being acquired, $y$ is the vector of compressed samples, $\hat{\alpha}$ is the vector of recovered coefficients, and $\hat{x}$ is the recovered signal.

basis. We will now refer to $\Phi$ as the *sampling operator*.

The sampling operator must satisfy certain conditions to allow recovery of a signal from compressed samples. One of these sufficient conditions is known as the *restricted isometry property (RIP)*, which requires that the sampling operator act as an orthonormal system on sparse signals [131]. More precisely, $\Phi$ should satisfy

$$(1 - \delta_K)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_K)\|x\|_2^2$$

where $\delta_K$ is known as the $K$-restricted isometry constant, which determines how closely the sampling operator approximates an orthonormal system when acting on $K$-sparse signals. This constant is fundamental to the construction of performance bounds for many compressive sensing reconstruction methods [132].

Verifying the restricted isometry property for a given sampling matrix $\Phi$ is infeasible in general, as it is thought to require testing of every combination of $K$ columns.
However, several classes of random matrices have been determined to have small restricted isometry constants. It has been shown that matrices with independent and identically distributed (i.i.d.) Gaussian or Bernoulli random variables as elements satisfy the RIP with high probability [127]. This result has further been generalized to all matrices of i.i.d. subgaussian random variables [133]. Other constructions for the sampling operator include the partial Fourier ensemble, where $M$ rows are chosen from the Fourier matrix at random and the columns of the resulting matrix are rescaled to have unit 2-norm [134]. The partial Hadamard ensemble is constructed from the Hadamard matrix similarly. The advantage of these constructions is that $O(N \log N)$ operation transforms, such as the fast Fourier transform and fast Walsh-Hadamard transform may be used to implement the sampling process more efficiently than the $O(MN)$ operations required in general for matrix multiplication.

The partial Fourier and Hadamard ensembles suffer the drawback of higher restricted isometry constants than the random matrices for most classes of naturally occurring signals. A hybrid approach, called structurally random matrices, augments the partial ensembles with a prerandomizer to improve the restricted isometry constant in these applications [135]. The prerandomizer is implemented either as multiplication of the input samples with a random Bernoulli sequence or as a random permutation of the input samples, thus the $O(N \log N)$ performance of the partial ensemble constructions can be retained.

4.2.1 Physical Implementation of the Sampling Operator

Since the sampling process used in compressive sensing is linear and non-adaptive, it is feasible in some applications to implement the sampling operator in the analog domain. Several novel sensing architectures have been developed to exploit this ability.
For example, the Single-Pixel Camera consists of a single photodetector element and a digital micromirror device used to form projections of incoming light with sampling vectors. The micromirror implements these projections by reflecting particular patterns of incoming light onto the photodetector while deflecting the remainder [136]. This architecture allows the use of expensive detectors which would be prohibitive to scale into an array. Another example is the Analog-to-Information converter, which can acquire high bandwidth analog signals with a lower-rate analog-to-digital converter [137]. The system modulates an input signal with a Nyquist-rate pseudorandom sequence and integrates over several periods of the sequence. An ADC then samples the output of the integrator, which is subsequently reset to begin measuring the next sample. The benefit of this architecture is that generating pseudorandom bits at high speed and modulating the input signal with them is much easier than sampling at high speed.

### 4.3 Reconstruction from Compressed Samples

Recovering a signal $x$ from compressed samples $y = \Phi x$ is an underdetermined linear inverse problem with an infinite number of solutions. To successfully recover the signal, prior information must be exploited to focus the search toward the desired solution. When the signal is known to be sparse, one reasonable approach to recovery is to look among all the possible solutions for the one of greatest sparsity. With high probability, the sparest solution is the original signal [15]. To be explicit, the problem of reconstructing a sparse signal from compressed samples can be expressed as follows:

$$\minimize_{x} \|x\|_0$$

subject to $y = \Phi x$. 
where $y$ is a vector of compressed samples, $\Phi$ is the sampling operator, $x$ is a sparse signal, and $\|\cdot\|_0$ is the $\ell^0$ pseudonorm—the number of nonzero elements in a vector. Unfortunately, exact solution in this form is intractable; at this time there are no known methods of solving it faster than exhaustive, combinatorial search. However, practical methods have been developed for approximating the solution. In addition, some theoretical results in compressive sensing allow the problem to be reformulated into one that is easier to solve.

### 4.3.1 $\ell^1$ Minimization

The $\ell^0$ minimization problem can be reformulated as an $\ell^1$ minimization, as shown by Candés and Tao [131]:

$$
\begin{align*}
\text{minimize} \quad & \|x\|_1 \\
\text{subject to} \quad & y = \Phi x.
\end{align*}
$$

This restatement of the recovery problem is known as Basis Pursuit (BP), and it may be solved by Linear Programming. Polynomial-time algorithms able to solve this problem include the simplex method and interior point method [138]. If the measurements are corrupted by noise, the BP problem may be modified by changing the hard equality constraint to a squared error penalty on the objective function:

$$
\begin{align*}
\text{minimize} \quad & \frac{1}{2}\|y - \Phi x\|^2_2 + \lambda \|x\|_1 \\
\text{subject to} \quad & \|x\|_1 < t
\end{align*}
$$

This formulation, called Basis Pursuit Denoising (BPDN) [139], is a quadratic program which may also be solved in polynomial time. Another similar variation, the least absolute shrinkage and selection operator (LASSO) [140], is defined:

$$
\begin{align*}
\text{minimize} \quad & \frac{1}{2}\|y - \Phi x\|^2_2 \\
\text{subject to} \quad & \|x\|_1 < t
\end{align*}
$$
4.3.2 Sparse Approximation

Sparse approximation consists of a family of techniques for approximately solving the \( \ell^0 \) minimization problem. Some of these techniques fall in the class of greedy sparse approximation, in which a dictionary for the compressed measurements \( y \) is formed from a subset of the columns of \( \Phi \). At each iteration, more atoms are added to the dictionary by selecting via some optimality criteria. For example, Orthogonal Matching Pursuit (OMP) begins by selecting the column of \( \Phi \) that has the highest correlation with the measurement \( y \). It then projects out the contribution from the chosen column to form a residual. The algorithm repeats, starting from the residual, and terminates after selecting a desired number of atoms. OMP is capable of recovering \( K \)-sparse signals from \( O(K \log N) \) measurements [141].

Several variants of greedy sparse approximation have been studied to improve on the performance of OMP. Stagewise OMP (StOMP) proceeds similarly to OMP, except multiple atoms are added to the dictionary during each iteration [142]. Compressive Sensing Matching Pursuit (CoSaMP) [143] and the closely-related Subspace Pursuit (SP) [144] select a fixed-size set of dictionary atoms from the columns of \( \Phi \). The signal is then recovered as the least-squares solution in the subspace generated by these atoms. A new set of atoms is chosen based on the residual, and the algorithm repeats in the new subspace. At the end of each iteration, the subspace is pruned by limiting it to a fixed size \( K \), keeping only the greatest magnitude coefficients.

Iterative thresholding is another successful family of techniques for sparse approximation. These algorithms repeatedly apply the following iteration

\[
x^{(n+1)} = T(x^n + \Phi^H(y - \Phi x^n))
\]

where \( T \) is some thresholding operation. Iterative Hard Thresholding (IHT) uses a
thresholding operator which sets coefficients below some threshold to zero [145]. Iterative Soft Thresholding (IST) reduces the magnitude of all coefficients by a constant. These algorithms are robust to noise and efficient: They require two matrix-vector multiplications per iteration and converge linearly [146], [147]. Other variations include Two-Stage Thresholding (TST) which adds a second thresholding stage to each iteration of IHT. This algorithm has been shown to generalize CoSaMP and Subspace Pursuit [148].

Another approach to sparse approximation is to relax the \( \ell^0 \) norm minimization to a smooth function that may be minimized using gradient descent. This approach is taken by the Smoothed \( \ell^0 \) (SL0) algorithm [149], which approximates the \( \ell^0 \) norm as

\[
\|x\|_0 \approx \sum_{i=1}^{N} (1 - f_{\sigma}(x_i)), \quad f_{\sigma}(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right)
\]

The \( f \) are Gaussians with zero mean and standard deviation \( \sigma \). For large \( \sigma \) the approximant is a sum of broad Gaussians, which is a smooth function that can be minimized using gradient descent. In the limit as \( \sigma \) decreases to 0, the Gaussians approach delta functions. Thus if \( x_i \) is zero then \( 1 - f_{\sigma}(x_i) = 0 \), and if \( x_i \) is nonzero then \( 1 - f_{\sigma}(x_i) = 1 \). By minimizing the sum of these quantities centered on each signal coefficient, coefficients are rewarded for being zero. SL0 starts with large \( \sigma \) and minimizes the approximant. Afterwards, it projects the solution \( x \) back onto the feasible set \( \{x : y = \Phi x\} \). It then reduces sigma and repeats, starting from the previous solution. SL0, like the greedy sparse approximation methods, tends to run much faster than \( \ell^1 \) minimization algorithms.
4.4 Extensions to Compressive Sensing

Compressive sensing theory was originally developed for a linear definition of sparsity while assuming no additional information about the signal. However, a large class of interesting signals have additional structure or statistical properties that may be exploited to improve the recovery process. One approach has focused on reconstructing signals that lie in low-dimensional nonlinear manifolds rather than linear subspaces. The random projections used in compressive sensing have been shown to preserve this manifold structure [150]. Reconstruction is more difficult than in regular compressive sensing, though some techniques have been tried [151]. Another approach, known as model-based compressive sensing, enforces some additional secondary structure on the signal coefficients in the sparsity basis during reconstruction [16]. For example, the signal coefficients may be constrained to form a connected wavelet tree, which has been determined an optimally sparse representation for a wide class of signals [152]. The wavelet tree model can be implemented in several of the greedy sparse approximation algorithms; this model requires that a given node’s parent in the tree must be present in the approximation before that node can be added. This restriction disallows high frequency wavelet coefficients from being added without adding all of their ancestor coefficients at lower frequency levels in the tree. A version of the subspace pursuit algorithm adapted for enforcing the connected wavelet tree model is used in later simulations.

4.5 Reconstruction of EMG from Compressed Samples

The performance of compressive sensing applied to acquisition of intramuscular EMG signals was evaluated by simulating the sensing and reconstruction process on several
recordings obtained from the EMGLAB project, dataset R007 [153]. These recordings were acquired using a pair of fine-wire electrodes separated 10 to 15mm during a 10 second isometric contraction of the brachial biceps muscle. The EMG recordings are segmented into blocks $x$ of length $N$, and $M < N$ compressive measurements $y$ are simulated by computing $y = \Phi x$. The vector $x$ is then recovered from $y$ using the SL0 and SP reconstruction algorithms. These reconstruction algorithms and the simulation code were implemented in the Python programming language using the NumPy, SciPy [154], scikits.learn [155], and LIBSVM [156] libraries. The measurement operator $\Phi$ was implemented using the Structurally Random Matrix construction, consisting of a $\pm 1$ Bernoulli sequence prerandomizer ($N$ additions or subtractions), a Walsh-Hadamard transform ($N \log N$ additions or subtractions), and a random downsampling ($M$ additions), yielding $O(N \log N)$ addition or subtraction operations for efficient implementation.

Figure 4.4 shows an example segment of EMG recording and its reconstruction at several compression ratios $\rho$. The reconstructions are performed on blocks of $N = 4096$ samples using $M = \rho N$ measurements. The SP algorithm requires the approximation sparsity $K$ to be given as a parameter; the rule $K = \frac{M}{6}$ was chosen after comparing the performance of several heuristics. The symlet-4 wavelet was chosen as the sparsity basis by comparing the $\ell_1$ norms of EMG decompositions in several bases and considering the complexity of these bases. Symlet-4 was chosen for a good balance of $\ell_1$ norm efficiency and simplicity, and its wavelet function is shaped much like the motor unit action potentials that compose the EMG signal. Since the EMG signal is not strictly sparse but belongs to the class of compressible signals, the SP sparse approximation has the effect of denoising the signal. In contrast, SL0 allows all coefficients to be nonzero, though most are similar in magnitude to the noise.
level. As compression ratio decreases, artifacts in the form of spurious, large wavelet coefficients begin to appear in the SP reconstructions. The SL0 reconstructions gain noise but artifacts are smaller than in SP reconstructions.

The SL0 and SP algorithms are among the fastest compressive sensing reconstruction algorithms available. Both were implemented using iterative least squares methods in their approximation steps and fast operators rather than dense matrix
multiplication. For the EMGLAB recordings, both algorithms run faster than real-time on one core of a 1.3 GHz Intel Core 2 Duo processor.

Figure 4.5 shows the reconstruction signal-to-noise ratio (SNR), for several compression ratios. SNR is defined using the ratio of signal energy to reconstruction error energy:

$$\text{SNR} = 10 \log_{10}(\frac{\sum x^2}{\sum (\hat{x} - x)^2}),$$

where $x$ is the original signal and $\hat{x}$ is the reconstruction. SP yields slightly lower SNR in reconstructions, but some of the difference can be attributed to its effect of denoising the signal by forcing signal coefficients to zero. Additionally, the larger artifacts produced by SP are penalized heavily by mean-squared error and therefore
For prosthesis control applications, the ability of compressive sensing and reconstruction to preserve signal features used in classifying muscle contraction force is our main concern. Classification accuracy was evaluated for distinguishing three different contraction levels, 10%, 20%, and 30% of the maximum voluntary contraction (MVC)—the greatest contraction force that can be generated by a given muscle.

Figure 4.6: Feature classification accuracy for distinguishing between three contraction efforts (10%, 20%, and 30% MVC) at various compression ratios for segments of 1024 samples.

Compressive sampling and reconstruction were performed as in the simulations described earlier but on smaller blocks of $N = 1024$ samples, or 128 ms at the sampling rate of 8 kHz. Better accuracy in both compressive sensing reconstruction and in contraction level classification may be attained using longer segments, but
these advantages come at the cost of higher latency in the control inputs. After reconstruction each block was decomposed into the symlet-4 wavelet basis, and a feature vector consisting of the energy in each wavelet subband was formed. These wavelet features were then labeled with the corresponding contraction level and used to train and evaluate a linear kernel SVM classifier. Classification accuracy was calculated as the fraction of correctly classified signal blocks and was tested using 5-fold cross-validation over the labeled EMG feature set. The experiment was repeated 10 times for each compression ratio, and average classification accuracies over these trials are shown in Figure 4.6.

These results demonstrate that successful contraction level classification is achieved at compression ratios on the order of $\rho = 0.1$. Although Subspace Pursuit appears to preserve the EMG signal morphology better than SL0, classification accuracy is somewhat better for SL0, especially at low compression ratios. This difference may be partially due to the choice of energy-based classification features that are preserved better by SL0, and the performance of SP might be improved by developing a more sophisticated method to estimate the approximation sparsity.

### 4.6 Reconstruction of ECG from Compressed Samples

Compressive sensing performance in ECG classification was evaluated on recordings from the MIT-BIH arrhythmia database. Reconstructions of a sample segment of ECG are shown in Figure 4.7. These reconstructions were performed with SL0, Subspace Pursuit, and wavelet tree model Subspace Pursuit at compression ratios of $\frac{1}{4}$ and $\frac{1}{8}$. Subspace Pursuit, especially with the wavelet tree model, performs better than SL0 in preserving signal morphology.

Reconstruction SNR for compressively sampled and reconstructed ECG is dis-
Figure 4.7: Compressed sampling recovery examples on ECG using three different algorithms. The top row shows the original recording before compression. Underneath, the left column was compressed at a ratio $\frac{1}{4}$, while the right column was compressed at a $\frac{1}{8}$ ratio.

played in Figure 4.8. SNR is averaged over all records of the MIT-BIH arrhythmia database at compression ratios from $\frac{1}{2}$ down to $\frac{1}{12}$. Above 10 dB, ECG shape is fairly well preserved, but below 10 dB, artifacts begin to overwhelm the shape. Figure 4.7
Figure 4.8: SNR of compressively sampled and recovered ECG over the entire MIT-BIH arrhythmia database.

illustrates that the three reconstruction algorithms tested produce distinct types of artifacts and degrade in somewhat different manners.

Due to the fact that SL0 does not strictly enforce sparsity, it produces a progressively noisier reconstruction as compression ratio decreases. SP begins to produce spike artifacts due to large wavelet coefficients in the higher frequency wavelet subbands; however, the overall signal complexity is not allowed to grow beyond the reconstruction sparsity, which is given as a parameter to the algorithm. Wavelet tree model SP outperforms SL0 and regular SP, both in reconstruction SNR and
in the subjective appearance of ECG, by enforcing greater sparsity than SL0 while suppressing the high frequency spike artifacts produced by regular SP.

Figure 4.9: QRS detection performance at a range of compression ratios $\rho$. 
SNR and subjective appearance are difficult metrics by which to predict the effect of CS reconstruction error on the performance of electrophysiological analysis in specific applications. Hence, the methods of Chapter 3 were applied to reconstructions of ECG signals from compressed samples in order to evaluate these effects. First, QRS detection was tested over all records of the MIT-BIH arrhythmia database. These results are displayed in Figure 4.9. QRS detection maintains high sensitivity and positive predictive value at compression ratios above $\frac{1}{4}$ but begins to break down at lower ratios. As in the SNR and subjective reconstruction quality comparisons, wavelet tree model SP significantly outperforms SL0 and regular SP at aggressive compression ratios, especially in positive predictive value.

Finally, classification accuracy was evaluated on selected records from the MIT-BIH arrhythmia database after compressive sampling and reconstruction with compression ratios from $\frac{1}{2}$ to $\frac{1}{12}$ followed by QRS detection before classification. Figure 4.10 shows the full system performance, taking into account both the QRS detection step and the beat classification step, on record 106 from the MIT-BIH arrhythmia database. Again, compression ratios above $\frac{1}{4}$ produce little effect on classification accuracy.

Results for MIT-BIH record 233 are displayed in Table 4.1. This record contains four beat types: normal QRS, atrial premature beats, premature ventricular contractions, and fusions of normal and ventricular beats. Classification is performed as described in Chapter 3, except a cubic SVM kernel function $K(u, v) = (u \cdot v + 1)^3$ is used. The cubic kernel function attains better accuracy under compression than the quadratic one used previously. At compression ratios down to $\frac{1}{3}$ there is no significant effect on classification performance. Below ratios of $\frac{1}{4}$, accuracy is significantly diminished for SL0 and SP, though wavelet tree model SP maintains high performance.
down to ratios of $\frac{1}{6}$. These results indicate that ECG analysis accuracy is virtually unaffected by CS reconstruction error at compression ratios down to $\frac{1}{3}$. In some tasks, compression down to ratios of $\frac{1}{6}$ is allowable using model-based SP without significant effect on classification performance.
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Table 4.1: Accuracy, sensitivity, and precision of SVM arrhythmia classification applied to compressively sampled and reconstructed ECG (MIT-BIH record 233) at various compression ratios with three CS reconstruction algorithms.
Chapter 5

Conclusion

The results of these experiments demonstrate the feasibility of compressive sampling and reconstruction in the acquisition of electrophysiological signals for automated analysis. Specifically, two promising applications of automated analysis were investigated: classification of muscle contraction force from intramuscular EMG recordings and classification of cardiac arrhythmia from ECG recordings. Existing methods for performing these classification tasks have received great attention in the literature due to the benefits of these applications; however, the lack of practical acquisition devices for continuous monitoring of electrophysiology has prevented these techniques from achieving large-scale use. The experiments conducted in this thesis demonstrate that classification accuracy can be maintained under signal distortion created by compressive sampling and reconstruction. ECG QRS detection and classification accuracy were not significantly affected at compression ratios of $\frac{1}{4}$ or higher, and intramuscular EMG contraction force classification maintained good performance at compression ratios as low as $\frac{1}{10}$.

Of the three reconstruction algorithms tested, wavelet tree model-based compressive sensing performed best: It exceeded the performance of regular SP and SL0 in all comparisons. Although the compressive sensing process trades sensor complexity for reconstruction complexity, the CS reconstruction and classification algorithms implemented for these simulations all run faster than real-time on laptop hardware. Due to the severe power constraints imposed on continuous electrophysiological mon-
itoring systems, this tradeoff is likely worthwhile. The gains in acquisition efficiency afforded by CS on the sensor side provide the designer of electrophysiological acquisition systems a compromise: With a modification to the sampling architecture, power consumption may be balanced with signal quality. These gains can help ease the restrictive power consumption limitations of implantable signal acquisition devices and bring the many benefits and applications of full-time electrophysiological signal monitoring closer to realization.
Bibliography


[33] Z. W. Hall and J. R. Sanes, “Synaptic structure and development: the neuro-

[34] R. Cooke, “The sliding filament model,” The Journal of General Physiology,

[35] R. Merletti and P. Parker, Electromyography (Physiology, Engineering, and

[36] D. Dumitru and J. C. King, “Motor unit action potential duration and muscle

[37] A. M. Taylor, E. A. Christou, and R. M. Enoka, “Multiple features of motor-
unit activity influence force fluctuations during isometric contractions,” Journal

[38] S. Nawab, R. Wotiz, and C. De Luca, “Multi-receiver precision decomposition
of intramuscular emg signals,” in Engineering in Medicine and Biology Society,
2006. EMBS ’06. 28th Annual International Conference of the IEEE, pp. 1252


[40] R. Schilling, D. Davies, and N. Peters, “Clinical developments in cardiac activ-

[41] A. M. Katz, Physiology of the heart / Arnold M. Katz. Raven Press, New York :
, 1977.


[56] M. V. Jelinek and B. Lown, “Exercise stress testing for exposure of cardiac arrhythmia,” *Progress in Cardiovascular Diseases*, vol. 16, no. 5, pp. 497 – 522,


C. Papadelis, Z. Chen, C. Kourtidou-Papadeli, P. D. Bamidis, I. Chouvarda, E. Bekiaris, and N. Maglaveras, “Monitoring sleepiness with on-board electro-


[125] F. Stetten, S. Kerzenmacher, A. Lorenz, V. Chokkalingam, N. Miyakawa, R. Zengerle, and J. Ducree, “A one-compartment, direct glucose fuel cell for powering long-term medical implants,” in Micro Electro Mechanical Systems,


1998.


75

10.1007/s00041-008-9041-1.


[155] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos,