RICE UNIVERSITY

Three Essays on Sovereign Default and Robust Policy Design

by

Xin Li

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE Doctor of Philosophy

APPROVED, THESIS COMMITTEE:

Ted Loch-Temzelides, Chair
Professor of Economics

Borghan Narajabad, co-Chair
Economist at Board of Governors of the Federal Reserve System

Mahmoud El-Gamal
Professor of Economics

Songying Fang
Assistant Professor of Political Science

Houston, Texas
April, 2014
Chapter 1 discusses the optimal fiscal response of a small open economy to business cycle fluctuations at the presence of sovereign default risks. The most recent sovereign debt crisis in Europe has demonstrated that the risk of sovereign default is not a problem in developing economies only. However, empirical studies show that fiscal policy tends to be countercyclical or acyclical in developed small open economies and procyclical in developing countries. This chapter presents a general equilibrium model with endogenous government spending, external debt financing, and sovereign default decisions for a small open economy. The model shows that developed countries’ acyclical fiscal response to productivity fluctuations can be motivated by their larger size of public sectors, lower demand elasticity of public goods, and lower volatilities of domestic investments relative to foreign investments, compared to their developing counterparts. Along this line, the recently observed fiscal policy graduation in some Latin American countries can be rationalized by the shifts in the characteristics of their public sectors towards developed countries. The model also implies that fiscal austerity is always optimal for countries with sufficiently high debt-to-output ratio, and the optimal consolidation consists of tax hikes, cuts in public consumption but not in public investment.

Based on Chapman, Fang, Li and Stone (2013), Chapter 2 studies the effect of new official bailouts on capital markets when borrowing countries economic state is private information. We first analyze a game-theoretical model of crisis lending that
incorporates bargaining, compliance and enforcement. The presence of asymmetric information yields two interesting scenarios. There are conditions under which lending reduces the risk of a deepening crisis and reduces the risk premium demanded by market actors. On the other hand, the political interests that make lenders willing to lend weaken the credibility of commitments to reform, and the act of accepting an agreement reveals unfavorable information about the state of the borrower’s economy. The net “catalytic” effect on the price of private borrowing depends on whether these effects dominate the beneficial effects of the liquidity the loan provides. Decomposing the contradictory effects of crisis lending provides an explanation for the discrepant empirical findings about market reactions, especially with regard to IMF programs. We test the implications of our theory by examining how sovereign bond yields are affected by IMF program announcements, loan size, the scope of conditions attached to loans, and measures of the geopolitical interests of the United States, a key IMF principal.

Based on Li, Narajabad, and Temzelides (2013), Chapter 3 turns to the study of robust policy design when decision makers are concerned about model uncertainty. We study a dynamic stochastic general equilibrium model where agents are concerned about model uncertainty regarding climate change. An externality from greenhouse gas emissions adversely affects the economy’s capital stock. We assume that the mapping from climate change to damages is subject to uncertainty, and we adapt and use techniques from robust control theory in order to study efficiency and optimal policy. We obtain a sharp analytical solution for the implied environmental externality, and we characterize dynamic optimal taxation. A small increase in the concern about model uncertainty can cause a significant drop in optimal energy extraction. The optimal tax which restores the social optimal allocation is Pigouvian. Under more general assumptions, we develop a recursive method and solve the model computationally. We find that the introduction of uncertainty matters qualitatively and quantitatively. We study optimal output growth in the presence and in the absence of concerns about uncertainty and find that these can lead to substantially different conclusions.
ACKNOWLEDGEMENT

I would like to express my deepest gratitude to my advisor and thesis committee chair, Dr. Ted Loch-Temzelides, and my advisor and co-chair, Dr. Borghan Nara-jabad, for their continuous support to my research and for their patience, motivation, enthusiasm, and immense knowledge.

I would also like to express my appreciation to my committee members, Dr. Mahmoud El-Gamal and Dr. Songying Fang, for their generous help and inspiration throughout my graduate years.

I am very grateful to my co-authors, Dr. Randall Stone and Dr. Terry Chapman, and my former supervisor at the IMF, Dr. Yanliang Miao. The knowledge and professional experience they have shared with me are valuable assets in my life.

Last but not least, I want to thank my dear wife, Ronghua Guo, whose solicitude has been a great consolation to me when we are not living in the same city.
Contents

Abstract ii
List of Figures vii
List of Tables ix

1 Optimal Fiscal Policy in A General Equilibrium Model of Sovereign Default 1
1.1 Introduction ................................ 1
   1.1.1 Literature Review .......................... 4
1.2 Stylized Facts and Empirical Findings .................. 8
   1.2.1 How to measure Fiscal Policy .......................... 8
   1.2.2 Stylized Differences in the Public Sectors .............. 10
   1.2.3 Fiscal Policy Graduation .................. 14
1.3 Model ................................... 15
   1.3.1 Agents ................................... 15
   1.3.2 Recursive Equilibrium .................. 20
   1.3.3 Endogenous Output Loss of Default ......... 21
1.4 Quantitative Analysis .......................... 23
   1.4.1 Calibration ................................ 23
   1.4.2 Standard Results of Sovereign Default Models ........ 26
   1.4.3 Cyclical Co-movement of Fiscal Policy ............. 29
   1.4.4 Optimal Composition of Fiscal Consolidation .... 33
1.5 Conclusion ................................ 34

2 Mixed Signals: Crisis Lending and Capital Markets 36
2.1 Introduction ................................ 36
2.2 Market Reactions to Crisis Lending ........................ 39
2.3 Model .................................................. 43
2.4 Equilibrium Results ..................................... 46
2.5 Empirical Implications .................................... 55
2.6 Research Design .......................................... 57
2.7 Conclusion ............................................... 74

3 Robust Dynamic Optimal Taxation and Environmental Externalities

3.1 Introduction .............................................. 77
3.2 The Model ............................................... 81
3.3 The Analytical Solution .................................. 84
3.4 The Computational Solution and Calibration ............... 94
    3.4.1 Varying the Approximating Distribution ............... 103
    3.4.2 Varying the Resource Feasibility Constraint ............. 106
3.5 Conclusion .............................................. 109

Bibliography ................................................. 116

Appendix I .................................................. 127

Appendix II .................................................. 135
Figures

1.1 Cyclicality of Different Measures of Fiscal Policy ................................................. 9
1.2 Stylized Facts in Both Groups of Countries .................................................. 12
1.3 The Relationship between Government Consumption and GDP per Capita .................................................. 13
1.4 Relative Volatilities of Government Investment and Consumption .................. 14
1.5 Public Consumption Cross Country Comparison .................................................. 15
1.6 Output Loss of Default ..................................................................................... 22
1.7 Debt Limits and steady State Debt Levels, Model (II) .......................................... 27
1.8 Debt Limits and Optimal Next-Period Debt Levels, Model (II) .......................... 28
1.9 Optimal Composition of Fiscal Consolidation, Model (II) ..................................... 34

2.1 The Effect of Political Bias on the Equilibrium Conditionality $x_b$ and $x_p$ .......................................................... 49
2.2 The Effect of Political Bias on the Probability of Accepting a Loan Agreement .................................................................................. 50
2.3 The Effect of Political Bias on the Probability of Defection Conditional on Accepting a Loan Agreement .................................................................................. 51
2.4 The Expected Duration of Punishment for Defection ........................................... 51
2.5 M’s Posterior Beliefs about $\theta$ Conditional on $G$ Accepting A Loan ........... 52
2.6 The Total Effect of $\beta$ on Equilibrium Interest Rates ......................................... 53
2.7 The Effect of the Uncertainty of $\theta$ on the Equilibrium Interest Rates ......... 54

3.1 The Effect of Penalty Parameter $\alpha$ on Optimal Carbon Emissions, $E$ ................. 91
3.2 The Effect of $\alpha^{-1}$ on $E$ ................................................................................ 91
3.3 The Effect of Model Deviation as Measured by Entropy, $\delta$, on $E$ .................... 92
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>Optimal Use of Energy</td>
<td>99</td>
</tr>
<tr>
<td>3.5</td>
<td>Increase in Global Temperatures</td>
<td>101</td>
</tr>
<tr>
<td>3.6</td>
<td>Capital Stock and Output</td>
<td>102</td>
</tr>
<tr>
<td>3.7</td>
<td>Optimal Use of Energy when $R_0 = 253.8$</td>
<td>106</td>
</tr>
<tr>
<td>3.8</td>
<td>Increase in Global Temperatures when $R_0 = 253.8$</td>
<td>107</td>
</tr>
<tr>
<td>3.9</td>
<td>Capital Stock and Output when $R_0 = 253.8$</td>
<td>108</td>
</tr>
<tr>
<td>3.10</td>
<td>Optimal Use of Energy when $R_0 = 8000$</td>
<td>109</td>
</tr>
<tr>
<td>3.11</td>
<td>Increase in Global Temperatures when $R_0 = 8000$</td>
<td>110</td>
</tr>
<tr>
<td>3.12</td>
<td>Optimal Use of Energy when $R_0 = \infty$</td>
<td>111</td>
</tr>
<tr>
<td>3.13</td>
<td>Optimal Use of Energy when $R_{coal} = 666$</td>
<td>112</td>
</tr>
<tr>
<td>3.14</td>
<td>Increase in Global Temperature when $R_{coal} = 666$</td>
<td>113</td>
</tr>
<tr>
<td>3.15</td>
<td>Capital Stock and Output when $R_{coal} = 666$</td>
<td>114</td>
</tr>
</tbody>
</table>
Tables

1.1 Instrument Variable Panel Regression with Country Fixed Effect . . . 11
1.2 Benchmark Parameter Values .............................................. 25
1.3 Statistical Moments in the Models and in the Data ..................... 30

2.1 Effect of IMF Program Initiation and U.S. Influence on Bond Yields . 64
2.2 Conditional Effects of New IMF Program Announcements ......... 69
2.3 Fixed Effects IV Regression ................................................. 73

3.1 Parameter Values ............................................................... 98
3.2 Robust Externalities ........................................................... 106
Chapter 1

Optimal Fiscal Policy in A General Equilibrium Model of Sovereign Default

1.1 Introduction

The most recent sovereign debt crisis in Europe has demonstrated that the risk of sovereign default is not a problem in developing economies only. To battle the soaring risk premium on sovereign bonds, governments in south European countries were advised by the IMF and the ECB to implement fiscal austerity to harness their growing external debts, and to restore market confidence. The reason for this advice lies in the post-crisis consensus that fiscal policy is a more efficient tool against economic recessions than conventional monetary policy. This is especially true for countries that do not have independent monetary policy (e.g., countries in a monetary union) and for countries in which the nominal interest rate is very close to its lower bound of zero. Putting monetary policy aside, this paper constructs a sovereign default model to simultaneously study the interactions between default decision, fiscal policy, and debt dynamics.

A bulk of empirical studies has shown that stylized business cycle features are quite different between developing and developed countries. In particular, the fiscal policy tends to be procyclical in developing countries and countercyclical (or acyclical) in developed countries.\(^1\) In this paper, I develop a consolidated model of sovereign default with a comprehensive fiscal authority to simultaneously rationalize the countercyclical (or acyclical) fiscal policy of developed countries and the

\(^1\)The next subsection gives a brief review on related literatures.
procyclical fiscal policy of developing countries.

The benchmark model features an infinite-horizon economy with domestic households, domestic firms, a central government (fiscal authority), and foreign lenders. Households value private consumption, government’s public consumption, and leisure. Government’s public consumption interacts with private consumption goods, directly affecting households’ period utility. Domestic firms produce private consumption goods using labor, public investment and foreign investment, which are imperfect substitutes. The government collects lump-sum taxes from households, issue sovereign bonds on international credit market, and then allocates the available resource between public consumption and public investment. The payment of sovereign debt is not enforceable, and the government is allowed to dishonor its debt every period. I assume that when a country chooses to default, it loses access to international credit markets temporarily, and domestic firms lose access to a fraction of foreign investment. Foreign lenders are risk-neutral and offer loans in a complete market. As a result, the risk premium of the sovereign bond is determined to compensate the sovereign default risk.

Below, I proceed by discussing three important implications of the benchmark model. First, the imperfect substitutability between public investment and foreign investment results in an efficiency loss in the final good production sector when the firms are partially excluded from international investment market. Additionally, the efficiency loss could be amplified by TFP shocks, leading to the so-called financial amplification mechanism as illustrated in Mendoza and Yue (2012). Under this

---

2Following the sovereign default literature, I assume that the centralized government is the only agent that makes borrowing and investment decisions, or in other words, the government makes borrowing and investment decisions on behalf of households and firms. Therefore, throughout this paper, I do not distinguish between domestic public investment and domestic investment and between private borrowing and public borrowing.

3See Fuentes and Saravia (2010), and Albuquerque (2003).
mechanism, the percentage output cost of default to be an increasing and convex function of TFP shocks, implying that the country is more likely to default when TFP shocks are negative. Therefore, given the level of debt, the country is facing a high (low) interest rate premium in bad (good) times. This is a key determinant to understand debt dynamics implied by sovereign default models.

Second, a key underlying premise is that developed countries’ production technology is more biased towards home technology and that public consumption are valued more in developed countries than in their developing counterparts. It has two consequences. The resulting larger share and the lower volatility of public consumption in developed economies make the reduction of public good more costly in bad times. Additionally, the stronger bias towards domestic investment raises the relative volatility of FDI, making advanced economies more reliant on domestic capital inputs, particularly in bad times. These considerations jointly encourage advanced economies to implement more conservative fiscal policies. Consequently, developed countries are more likely to maintain a larger fiscal room in normal times in order to borrow at a low interest premium in bad times. In contrast, developing countries are more likely to face prohibitively high borrowing cost in bad times and have to shift from debt financing to the more costly tax financing.

Third, my model predicts that the optimal fiscal consolidation should consist of tax hikes, cuts in governments’ public consumption but not in public investment. To improve the country’s fiscal condition through consolidation, the government can raise its revenue by imposing higher taxes, and/or cut its expenditure by reducing public consumption and public investment. Tax hikes and cuts in public consumption should be implemented simultaneously in an optimal consolidation plan because the optimal allocation of resource must equalize the marginal utilities of private consumption and public consumption. Moreover, reallocating the additional tax revenue to public investment is beneficial when the marginal productivity of capital inputs
(measured in private consumption goods) exceeds one, the marginal cost of imposing one additional unit of tax.  

1.1.1 Literature Review

In the past decade, a large number of empirical studies have documented that developed countries and developing countries follow quite different fiscal policies. Kaminsky et al. (2004) show that low and middle-income countries are in favor of procyclical fiscal policies while OECD countries tend to implement either countercyclical or acyclical fiscal policies. Talvi and Vegh (2005), among others, find similar results that the correlation between government spending and GDP in G7 countries is zero (acyclical). In addition, Ethan Ilzetzki and Carlos A. Vegh (2008) test a quarterly dataset for 49 countries with several econometric models, and find “overwhelming evidence to support the idea that pro-cyclical fiscal policy in developing countries is in fact truth and not fiction.”

There are at least two competing classes of models that can be used to address the different cyclical behaviors of fiscal policies of small developing or developed economies, namely, the small open real business cycle model and the sovereign default model for a small open economy. These two models differ from whether countries borrowing cost, the interest rate premium, is endogenously determined or not.

1) Small Open Real Business Cycle Model

The small open real business cycle model assumes that countries can borrow at a risk-free interest rate or with an exogenously determined risk premium. Grohe and Uribe (2003) conduct a thorough comparison between alternative small open
economy models and demonstrate that these models uniformly predict acyclical fiscal police for a typical small developed economy, Canada. This is driven by the government’s incentive to smooth out consumption in bad times through external debt financing.

Since the government in bad times can borrow at a cost as cheap as in good times and is not contained in its borrowing ability, standard small real business cycle models have difficulties to account for the procyclical fiscal policy of developing countries. To address this issue, the first attempt focuses on imperfections in asset markets. This includes, among others, Gavin and Perotti (1997), Riascos and Vegh (2003), Guerson (2003), Caballero and Krishnamurthy (2004), and Mendoza and Oviedo (2006). The imperfection in international and/or domestic asset markets limits the domestic government’s capabilities for risk sharing and consumption smoothing in bad times. This could possibly result in a procyclical fiscal policy, as is the case in developing countries.

The second attempt emphasizes the different laws governing the underlying TFP processes in developing and developed economies. Aguiar and Gopinath (2007) demonstrate that fluctuations of output in developing countries can be explained by persistent shocks to trend growth, rather than transitory shocks around a stable

---

5 In fact, Grohe and Uribe (2003) implies an acyclical trade balance for Canada. However, if a country can have a trade deficit only by the government borrowing abroad, the negative correlation between output and trade balance implies a procyclical fiscal policy and vice versa.

6 However, there is a problem if this approach is used it to rationalize the fiscal policy of developed countries because developed countries will be assumed to issue state-contingent debt in a complete and perfect asset market, which seems counterfactual. When state-contingent debt is not available, Angeletos (2002) and Buera and Nicolini (2004) have argued that the government can replicate state-contingent debt by varying the maturities of non-contingent debt. However, as is shown in Angeletos (2002), the above argument is true only if default is not an option for the government — this has also proved challenging for some peripheral European countries struggling with the ongoing financial crisis.
trend, as is true for their developed counterparts. They show that the shock-to-trend model generates the so-called current account reversals (i.e., countercyclical current accounts) while the standard business cycle model with transitory shocks yields a procyclical or acyclical current account. 7 However, it is worth to note that the qualitative implication of developed countries rests on the assumption that the country can borrow at the risk-free interest rate regardless its economic states. As will be shown below, this relationship can be reversed if the country’s borrowing cost is adjusted according to its economic states to compensate the risk of sovereign default.

(2) Sovereign Default Model for a Small Open Economy

During the past decade, there has been a growing literature on sovereign defaults. The main feature of this class of models is to allow sovereigns to default strategically. Consequently, country’s borrowing costs are endogenously determined to compensate the risk of sovereign default. The existing work on sovereign default has shed light on understanding business cycle facts in developing countries.

Alfaro and Kanczuk (2005), Aguiar and Gopinath (2006), and Arellano (2008) rest their analysis on a stochastic endowment economy, proven successful in explaining the procyclical fiscal policy of developing countries. The main logic is as follows. Since borrowing during economic recessions is expensive, corresponding to a higher risk of default, countries have strong incentives to reduce their borrowings or to borrow less than in economic booms, leading to a procyclical fiscal policy. However, these models produce a negative correlation between output and trade balance only when the persistence of TFP shocks is sufficiently large and the volatility of the shocks is sufficiently small, as is the case for developing countries. Thus, in order to rationalize the countercyclical fiscal policy in developed countries, it requires either

---

7This is in line with the first attempt in that developed countries are assumed to have access to a perfect asset market and their government can borrow at a favorable (constant) rate.
a relatively low persistence or a relatively large volatility of TFP shocks. However, such evidence has not been found yet.

To address this issue, this paper emphasizes the difference in governments’ capabilities of affecting economic activity and social welfare via fiscal measures. Empirical data shows that economic development is accompanied with an increasing demand for public good and service. In addition, production in developed countries are less (more) dependent on foreign investment (public investment) on average than in their developing counterparts. This evidence implies that the degree of the interaction between fiscal policy and economic activity may have played an important role in determining the different stylized business cycle facts in developed and developing countries.

Cuadra, Sanchez and Sapriza (2010) develop a sovereign default model, in which public expenditure directly enter households’ utility function. However, their paper assumes an exogenous output cost of default, thus cannot model government’s investment as an important policy tool to fight against business cycle fluctuations. Mendoza and Yue (2012) propose a production function that uses both domestic intermediate goods and imported inputs, a fraction of which requires working capital financing. It allows them to endogenize the output loss given default, which is increasing disproportionally with TFP through a financial amplification mechanism, by assuming both firms and the government are excluded from international credit markets after default.

The rest of the paper is organized as follows. Section 2 discusses empirical results on the cyclicality of fiscal policy and the stylized difference in public sectors between developed and developing countries. Section 3 describes the benchmark model and characterizes the equilibrium. Section 4 presents the quantitative implications of the benchmark model. Section 5 concludes.
1.2 Stylized Facts and Empirical Findings

1.2.1 How to measure Fiscal Policy

Conceptually and practically, it is imperative to choose an appropriate measure of fiscal policy. A bulk of literature measures fiscal policy using government spending and government final good consumption (for example, Ilzetzki and Vegh (2008)); alternatively, many studies use fiscal deficit (for example, Alesina, Campante, and Tabellini (2008)). This paper follows the second approach for two reasons. First, using fiscal deficit allows us to study the dynamics between fiscal policy, sovereign debt, and sovereign default. For small open economies, fiscal deficit serves a good proxy for the increment of government external debt, which in turn determines the risk of sovereign default. Consequently, a feedback loop will take place: the increasing risk of sovereign default drives up the interest spread; with other conditions fixed, the increasing borrowing cost further deteriorates government fiscal balance. Second, since a country can run current account deficit only through borrowing from abroad and the majority of external debts are borne by the government, using fiscal deficit to measure fiscal policy helps rationalize the opposite cyclicality of fiscal policy and current account balance in a certain country. Following the literature of sovereign default, my model in the next section considers a simplified case where only the government has access to international credit market. In this scenario, fiscal deficit is equivalent to net government external borrowing and current account deficit.

Figure 1.1 calculates the country-level correlations between alternative measures of fiscal policy and GDP growth for 23 developed countries and 26 developing countries for 1980-2012, using annual data drawn from World Economic Outlook (WEO) database and World Development Indicator (WDI) database. All variables are in real terms and in logs, except for current account deficit which is expressed as a percentage of GDP. Fiscal policy is measured by the first order difference in (log) government investment, government final consumption good expenditure, and current
account deficit, respectively. Simple statistics show that government investment and/or government final consumption good expenditure are procyclical or acyclical in both developed and developing countries, with a few exceptions such as Belgium, Canada, and Denmark. On the other hand, current account deficit is on average weakly countercyclical (or acyclical) in developed countries and significantly procyclical in developing countries. The results support my choice of current account deficit, or equivalently fiscal deficit, as a proxy for fiscal policy.

Moreover, I run two separate panel data regressions using two-stage-least-square methods with country fixed effect to capture the cyclicality of alternative fiscal policy

---

8Alternatively, one could also filter the data using Hodrick-Prescott filter; however, the main results are insensitive to the choice of filtering methodology as shown in Aguiar and Gopinath (2007).

9The countercyclical government consumption in Canada is well documented in the literature.
measures in developed and developing countries, respectively.

\[ \Delta g^{(k)}_{i,t} = \alpha^{(k)}_i + \Delta y_{i,t} \beta^{(k)} + (x^{(k)}_{i,t})' \gamma^{(k)} + \epsilon^{(k)}_{i,t}, \quad k = 1, 2, 3 \]

where \( \Delta y_{i,t} \) is the change in (log) real GDP, which is instrumented in the first stage regression by its one-period lag and the change in six-month treasury bills rate. In addition, \( k = 1, 2, 3 \) is a model index, indicating the candidate of fiscal policy.

In Model 1 (i.e., \( k = 1 \)), \( \Delta g^{(1)}_{i,t} \) is the change in (log) real government investment; in Model 2, \( \Delta g^{(2)}_{i,t} \) is the change in (log) real government final consumption good expenditure; and in Model 3, \( \Delta g^{(3)}_{i,t} \) takes the current account deficit as percentage of GDP. In Model 1 and 2, \( x^{(k)}_{i,t} \) contains a set of exogenous explanatory variables, such as the lags of the dependent variable on the left hand side, capturing the lag effect of government spending. In Model 3, \( x^{(3)}_{i,t} \) is an empty-set.

The results are shown in Table 1.1. Real government investment is procyclical in developing countries while real government consumption are procyclical in both groups of countries. A sound result is that current account deficit is acyclical in developed countries and significantly procyclical developing countries, consistent with the sample statistical results shown in Figure 1.1.

1.2.2 Stylized Differences in the Public Sectors

The previous subsection has shown that neither government investment nor government consumption can be used to distinguish the different fiscal behaviors between advanced and emerging economies. Instead, this paper uses fiscal deficit, or equivalently current account deficit, because it identifies the change in government’s borrowing/lending position, which directly affects the probability of sovereign default and interest risk premium. Nevertheless, government investment and government consumption are two important determinants of fiscal deficit. I will show in the next section that the observed differences in the sizes and the volatilities of government consumption and government investment are key factors driving the opposite fiscal
Table 1.1: Instrument Variable Panel Regression with Country Fixed Effect

<table>
<thead>
<tr>
<th>Model</th>
<th>Developed Country</th>
<th>Developing Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Government Investment</td>
<td>1.313</td>
<td>3.775**</td>
</tr>
<tr>
<td></td>
<td>(0.744)</td>
<td>(0.484)</td>
</tr>
<tr>
<td>2. Government Consumption</td>
<td>0.796**</td>
<td>2.000**</td>
</tr>
<tr>
<td></td>
<td>(0.088)</td>
<td>(0.327)</td>
</tr>
<tr>
<td>3. Current Account Deficit</td>
<td>0.166</td>
<td>0.679**</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>Observations</td>
<td>713</td>
<td>714</td>
</tr>
<tr>
<td>F-statistics in the first stage</td>
<td>254.271</td>
<td>111.503</td>
</tr>
</tbody>
</table>

**Significant at the .01 level. *Significant at the .05 level. Standard errors in parentheses.

policies in developed and developing countries. This subsection explores the stylized differences and the formal model will be established in the next section.

Figure 1.2 compares the average size of the public sector and its components between developed and developing countries. It revisits the well-known results that the share of the public sector of developed countries (as a percentage of GDP) is significantly larger than that of developing countries. While government spending on average accounts for more than 40 percent of GDP in developed countries this figure drops below 30 percent of GDP in developing countries. Moreover, both the shares of public consumption and investment are greater than their developing counterparts. On other hand, Figure 1.2 shows that the share of foreign direct investment (FDI) and the debt-to-output ratio is not statistically different in the two groups of countries.

Figure 1.3 further explore the quantitative relationship between government spending and GDP per capita, based on a linear regression with GDP per capita
Figure 1.2: Stylized Facts in Both Groups of Countries
and its square terms as explanatory variables. It follows that government spending is increasing in GDP per capita until it reaches about $55,000. Similar regressions of government consumption revisit the same result.

Figure 1.4 displays the (relative) volatilities of GDP, government consumption, investment, and FDI. The first panel of Figure 1.4 revisits the well-known result that the real GDP process in developing countries is twice as volatile as in developed economies. The second panel shows that in both groups of countries, real government investment is about three times as volatile as real GDP, consistent with Aguiar and Gopinath (2007). The last two panels of Figure 1.4 demonstrate that real government consumption in developing countries is approximately twice as volatile as that in developed countries while FDI inflows are about 45 percent more volatile in developed economies.
1.2.3 Fiscal Policy Graduation

Frankel, Vegh, and Vuletin (2013) find that a few developing countries, such as Brazil and Chile, have shifted from procyclical fiscal response to countercyclical response since 1998. This finding blurs the conventional distinction of fiscal policy responses between developing and developed countries. The authors argue that “the quality of institutions appears to be a key determinant of a country’s ability to graduate.” Despite the solid evidence provided, such an institutional explanation is always difficult to model in orthodox economic models.

My model provides an alternative explanation. According to the rationale of my paper, policy graduation can be explained by some fundamental shifts of Brazil and Chile’s public sectors towards developed countries. Brazil serves as a good example: Figure 1.5 shows that the share of public good consumption in Brazil has reached 20 percent of GDP on average since 1998, very close to the level of Canada and much higher than the average level of 13 percent of GDP in other Latin American
countries. It suggests that the logic explaining the fiscal behavior of developed countries apply to Brazil.

![Figure 1.5: Public Consumption Cross Country Comparison](image)

1.3 Model

I model a small open economy that borrows externally to help smooth consumption over output fluctuations. It is charged at the world interest rate plus a risk premium that is endogenously determined by the country’s debt level and capacity to repay (output). Debt obligation is neither state-contingent nor enforceable, allowing the government to choose to default strategically. If a country chooses to default, its government will be excluded from external loan market and firms will lose access to a fraction of foreign investment for some periods. Specifically, there are four groups of agents in the model, namely, domestic households, firms, government, and foreign investors.

1.3.1 Agents

(1) Households
Households choose consumption and labor supply to maximize a standard time-separable utility function \( E \sum_{t=0}^{\infty} \beta^t u(c_t, G^1_t, l_t) \) where \( 0 < \beta < 1 \), \( c_t \) and \( l_t \) denote time discount factor, private consumption and labor supply. I assume a continuum of identical households of mass one. Households receive wages at a given rate \( w_t \), profit \( \pi_t \) from firms, and public consumption \( G^1_t \) while paying a lump-sum tax \( T_t \). As standard in the literature of sovereign default model, I assume that households do not borrow directly from international market. Consequently, households’ optimization problem is:

\[
\max_{\{c_t, l_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t u(c_t, G^1_t, l_t)
\]

s.t. \( c_t = w_t l_t + \pi_t - T_t \)

To solve the model numerically, I choose \( u(c_t, G^1_t, l_t) = c_t^{1-\sigma} + b(G^1_t)^{1-\gamma} - \frac{t^1}{1+\psi} \) with \( \sigma, \gamma, \psi > 0 \). The first order conditions of private consumption and public consumption imply that \( \frac{(G^1_t)^{\gamma}}{c_t^{\gamma}} = b \).

(2) Firms

Firms produce consumption goods using labor \( l_t \), foreign investment \( m_t \), public investment \( G^2_t \) and a fixed level of capital \( k \). The production function is Cobb-Douglas with a Markov TFP shock \( A_t \):  

\[
f(G^2_t, m_t, l_t; A_t) = A_t M_t^{\alpha_M} l_t^{\alpha_L} k^{\alpha_K}, \quad \alpha_M + \alpha_L + \alpha_K = 1
\]

where the logarithm of \( A_t \) satisfies an AR(1) process, \( \log(A_{t+1}) = \rho \log(A_t) + \epsilon_t \). The intermediate good \( M_t \) is produced by \( G^2_t \) and \( m_t \) according to a CES production function

\[
M_t = [(1 - \lambda)(G^2_t)^\mu + \lambda (\tilde{m}_t)^\mu]^{\frac{1}{\mu}}
\]

where \( \tilde{m}_t = m_t^a \) with \( 1 < a < \min\{\mu^{-1}, \alpha_M^{-1}\} \) represents the localization process of the composite foreign investment \( m_t \). As we will see later, the parameter \( a \) are used to capture the average share and the volatility of foreign investment relative
to domestic investment. In addition, the foreign investment $m_t$ is represented by a Dixit-Stiglitz aggregator combining a continuum of differentiated investments $m_{jt}$ from country $j$ for $j \in [0, 1]$. That is,

$$m_t = \left[ \int_{j \in [0,1]} (m_{jt})^\nu dj \right]^{1\over \nu}. \quad (1.4)$$

The Armington curvature parameter $\mu$ satisfies $0 < \mu < 1$ to ensure imperfect substitutability between the government public investment $G^2_t$ and the foreign investment $m_t$. The ratio $1 - \lambda$ measures the bias of production technology towards public investment. Similarly, the Dixit-Stiglitz curvature parameter $\nu$ is assumed to be between 0 and 1.

The differentiated foreign investments $m_{jt}$ are rented at constant prices $p_j$ for $j \in [0, 1]$ expressed in terms of the price of final goods. Define a subset $\Omega = \{ j | j \in [0, \theta] \}$ of foreign countries that are the creditors of the home country. We assume that the home country will lose access to the foreign investment $m_{jt}$ from country $j$ if $j$ is in $\Omega$. Accordingly, define $P$ and $P_{aut}$ as the relative prices of the composite foreign investment $m_t$ conditional on “default” and “not default”, respectively.

It follows that when a country does not default,

$$P = \left[ \int_{j \in [0,1]} (p_j)^{\nu \over \mu} dj \right]^{\mu-1 \over \nu}, \quad (1.5)$$

$$m_{jt} = \left( \frac{p_j}{P} \right)^{1\over 1-\nu} m_t \quad \text{for } j \in [0, 1]. \quad (1.6)$$

And when a country defaults and is in financial autarky,

$$P_{aut} = \left[ \int_{j \in [\theta,1]} (p_j)^{\nu \over \mu} dj \right]^{\mu-1 \over \nu}, \quad (1.7)$$

$$m_{jt} = 0 \quad \text{for } j \in [0, \theta], \quad (1.8)$$

$$m_{jt} = \left( \frac{p_j}{P_{aut}} \right)^{1\over 1-\nu} m_t \quad \text{for } j \in [\theta, 1]. \quad (1.9)$$

Furthermore, I make the assumption that $p_j = \bar{p}$ for all countries, which yields

$$P = \bar{p}, \quad (1.10)$$

$$P_{aut} = (1 - \theta)^{\nu-1 \over \nu} \bar{p}. \quad (1.11)$$
Taking public investment, real wage, and the price index of composite foreign investment as given, domestic firms choose factor demands \( l_t \) and \( m_t \) to maximize current profits. The period-\( t \) profit is given by

\[
\pi_t = f(G_t^2, m_t, l_t; A_t) - P_m t - w_t l_t \quad \text{(when not default),}
\]

and

\[
\pi_{t, aut} = f(G_t^2, m_t, l_t; A_t) - P_{aut} m_t - w_t l_t \quad \text{(when default).}
\]

The F.O.Cs of firm’s problem satisfy that

\[
\frac{\partial f}{\partial m_t} = P, \quad \text{(when not default),}
\]

and

\[
\frac{\partial f}{\partial m_t} = P_{aut}, \quad \text{(when default).}
\]

Since \( \frac{P_{aut}}{P} = (1 - \theta)^{\nu + 1} > 1 \) for \( 0 < \nu < 1 \), it follows that the exclusion of the home country from a fraction of foreign investments creates a wedge between the price of foreign investment and its marginal productivity.

To complete the description of firm’s problem, it is straighthforward to show that a default event leads to a disproportionate loss of output due to the imperfect substitutability between \( G_t^2 \) and \( m_t \) and the nonlinear transformation function.\(^{10}\)

(3) Government

The government acts as a benevolent social planner that maximizes the representative household’s welfare. Each period, it chooses whether to default or not on

\(^{10}\)Arellano (2008) argues that the cost of default needs to be disproportionally increasing in output. Along this line, Mendoza and Yue (2012) shows that, under assumptions of the imperfect substitutability between the inputs of intermediate good, namely public investment and foreign investment, and the (partial) exclusion from foreign investment when a country defaults, the percentage output cost of default is convexly increasing in TFP.
its current debt $Z_t$, how much to borrow $Z_{t+1}$ if not default, and how to allocate the available resource among public consumption $G_1^t$, public investment $G_2^t$, and tax $T_t$. When the country purchases bonds $Z_{t+1} < 0$, and when it borrows $Z_{t+1} > 0$. The level of borrowing $Z_{t+1}$ is constrained to the set $[Z_{\text{min}}, Z_{\text{max}}] \subset R$. The payoff of the government is given by:

$$V(Z_t, A_t) = \max \{ V^{nd}(Z_t, A_t), V^{d}(A_t) \} ,$$

where $V^{nd}(Z_t, A_t)$ is the value when government chooses not to default, $V^{d}(Z_t, A_t)$ when defaults. When government does not default, it remains in a good standing and has access to international credit market. Therefore, $V^{nd}(Z_t, A_t)$ is defined by the Bellman equation:

$$V^{nd}(Z_t, A_t) = \max \{ u(c_t, G_1^t, l_t) + \beta E_t[V(Z_{t+1}, A_{t+1})] \}$$

s.t. $Z_t + c_t + G_1^t + G_2^t + P m_t \leq q(Z_{t+1}, A_t) Z_{t+1} + f(G_2^t, m_t, l_t; A_t)$.  

Define the net market financing from abroad $\Delta_t = q(Z_{t+1}, A_t) Z_{t+1} - Z_t$ and the trade balance $TB_t = f(G_2^t, m_t, l_t; A_t) - c_t - G_1^t - G_2^t - P m_t$. The feasibility constraint can be written as $\Delta_t + TB_t \geq 0$. As a byproduct, the F.O.Cs also imply that

$$\frac{(G_2^t)^{1-\mu}}{(m_t)^{1-a\mu}} = \frac{(1-\lambda)P}{a\lambda}$$

$$\log(G_2^t) = \left( \frac{1-a\mu}{1-\mu} \right) \log(m_t) + \text{constant}$$

and thus

$$\frac{\text{stdev}(\log(m_t))}{\text{stdev}(\log(G_2^t))} = \frac{1-\mu}{1-a\mu}$$

which will be employed in the next section to calibrate the ratio of $m_t$ to $G_2^t$ and the relative volatility of $m_t$ w.r.t. the volatility of $G_2^t$.

Similarly, the country loses access to the international credit market when it fails to honor its debt obligations $Z_t$, leading to a financial autarky (i.e., $Z_{t+1} = 0$). However, in each of the following period, this country has a probability $0 < \phi < 1$
to re-enter the world credit market with a zero liability. Therefore, the payoff from default $V^d(A_t)$ is given by the Bellman equation:

$$V^d(A_t) = \max_{\{c_t, G^1_t, G^2_t, l_t, m_t\}\in o} u(c_t, G^1_t) - v(l_t)$$

$$+ \beta \{(1 - \phi)E_t V^d(A_{t+1}) + \phi E_t V(0, A_{t+1})\} \tag{1.22}$$

s.t. \quad c_t + G^1_t + G^2_t + P_{aut} m_t \leq f(G^2_t, m_t, l_t). \tag{1.23}

Following the standard approach to Eaton-Gersovitz (1981) models, let $D(A_t) = \{A_t | V^d(A_t) \geq V^{nd}(Z_t, A_t)\}$ denote the space of default (i.e. repudiating debt is the optimal choice) and $p(Z_{t+1}, A_t) = \int_{A_{t+1} \in D(Z_{t+1})} dP(A_{t+1} | A_t)$ the probability of default.

(4) International Investors

Risk-neutral international investors offer short-term loans to the country in a competitive market. Therefore, the equilibrium price of sovereign debt $q(Z_{t+1}, A_t)$ is the one that leads to an expected return rate equaling the world risk-free rate $r^*$. That is,

$$q(Z_{t+1}, A_t) = \frac{1 - p(Z_{t+1}, A_t)}{1 + r^*}. \tag{1.24}$$

It follows immediately that the risk-adjusted interest rate of the sovereign bond $r_{t+1} = r(Z_{t+1}, A_t) = \frac{1}{q(Z_{t+1}, A_t)} - 1 = \frac{r^* + p(Z_{t+1}, A_t)}{1 - p(Z_{t+1}, A_t)}$.

1.3.2 Recursive Equilibrium

At the beginning of each period, the states of the world, $Z_t$ and $A_t$, are revealed. The government then decides whether to default on its debt obligations $Z_t$. If the government chooses to default, it loses access to international loan market, and firms lose access to a variety of foreign investment. If it chooses not to default, the government allocates available resources through net market financing $\Delta_t = q(Z_{t+1}, A_t)Z_{t+1} - Z_t$ (zero when it defaults) among public consumption, public investment and taxes.
Finally, firms produce to maximize profits while consumers choose the level of labor supply and consumption subject to their budget constraints. A country that defaults will regain access to international markets in the next period with an exogenous probability $\phi$.

A recursive general equilibrium consists of (i) a default rule $d(Z_t, A_t)$ and a debt financing rule $Z_{t+1}(Z_t, A_t)$ chosen by the government; (ii) government spending $G^1_t(Z_t, A_t)$ and $G^2_t(Z_t, A_t)$, tax $T_t(Z_t, A_t)$ and private consumption $c_t(Z_t, A_t)$; (iii) default probability $p(Z_{t+1}, A_t)$ and an equilibrium price scheme for sovereign debt $q(Z_{t+1}, A_t)$ such that:

- Given $q(Z_{t+1}, A_t)$, the default rule $d(Z_t, A_t)$ and the debt financing rule $Z_{t+1}(Z_t, A_t)$ solve the government’s problem.

- Given $q(Z_{t+1}, A_t)$ and $Z_{t+1}(Z_t, A_t)$, government spending $G^1_t(Z_t, A_t)$, $G^2_t(Z_t, A_t)$, and private consumption $c_t(Z_t, A_t)$ maximize representative agent’s one-period utility subject to the feasibility constraints.

- Tax $T_t(Z_t, A_t)$ is given by $T_t(Z_t, A_t) = q(Z_{t+1}, A_t)Z_{t+1} - Z_t - G^1_t(Z_t, A_t) - G^2_t(Z_t, A_t)$.

- Default probability $p(Z_{t+1}, A_t)$ and equilibrium price scheme $q(Z_{t+1}, A_t)$ are consistent.

### 1.3.3 Endogenous Output Loss of Default

Figure 1.6 shows the relationship between TFP shocks and percentage output loss for $\lambda = 0.39$ and $0.41$, respectively. $^{11}$ It illustrates two important properties of the output cost of default. First, the percentage output cost of default is increasing and convex in shocks $\log A_t$. The logic behind it differs from Mendoza and Yue (2012).

$^{11}$Other relevant parameters include $\mu$ and $a$, which are set to their benchmark values that generate $\sigma(m_t)/\sigma(G^2_t) = 2.51$, the case of Mexico.
Due to the convexity of the transformation function $\tilde{m}_t = m_t^\rho$ and the imperfect substitutability between $G_t^2$ and $m_t$ in the final good production function, more resource will be allocated away from $G_t^2$ to $m_t$ when $A_t$ increases. As a result, $G_t^2/m_t$ is decreasing in $A_t$. Since the country loses access to a fixed fraction of $m_t$ after defaulting on its debt, the percentage loss of output rises as $A_t$ increases. Second, given productivity $A_t$, the cost of default increases as $\lambda$ increases. It is intuitive: a larger $\lambda$ means the reliance of production technology on foreign investment is higher, thus implying a larger output loss when defaulting.

![Output Loss of Default](image)

**Figure 1.6 : Output Loss of Default**

Similar to Mendoza and Yue (2012), the convex function of output loss makes default a more attractive option when the economy experiences bad TFP shocks and thus low output. As a result, default events are more likely to be observed when the debt-to-output ratio is high.
1.4 Quantitative Analysis

I calibrate the theoretical model in Section 2 based on data for Canada and data for Mexico and denote the calibrated models as Model (I) and Model (II), respectively. The data come from three sources: World Bank national accounts data, OECD National Accounts data files, and Aguiar and Gopinath (2007). World Bank national accounts data are annually nominal series, including final consumption expenditure, government final consumption expenditure, gross fixed capital formation, and foreign direct investment net inflows. All of these variables are presented as a percentage of GDP. The data from OECD National Accounts data files and Aguiar and Gopinath (2007) are seasonally adjusted quarterly or annually real series and filtered with the HodrickPrescott filter. All series are truncated from 1980 to 2006.

1.4.1 Calibration

Each model contains 18 parameters, which I classify into four groups. The first group consists of 5 parameters $\beta$, $\sigma$, $\gamma$, $b$ and $\psi$ that describe the households’ preference; the second group contains 7 parameters $\alpha_M$, $\alpha_L$, $\alpha_K$, $\alpha$, $\mu$, $\lambda$ and $\nu$ that define the production functions of final goods and intermediate goods; the third group includes the exogenous prices $r^*$, $\bar{p}$, and the share of foreign creditors $\theta$; additionally, the last group contains 3 parameters $\phi$, $\rho$ and $\sigma_e$, which characterize the properties of TFP shocks and the reentry probability. All parameters are detailed in Table 1.2.

The main focus of this paper is to contrast the different characteristics of public sectors across emerging and developed economies. Towards this end, I fix all other parameters to be constant across Model (I) and Model (II) and set them to the same values used in the baseline model of Mendoza and Yue (2012) whenever appropriate. There are nine of them including $\beta$, $\sigma$, $\psi$, $\alpha_M$, $\alpha_L$, $\alpha_K$, $r^*$, $\bar{p}$, and $\phi$. In addition, $(\rho, \sigma_e)$ are set to $(0.85, 0.017)$ in both Model (I) and (II).\textsuperscript{12} $\theta$ and $\nu$ together creates

\textsuperscript{12}It needs to be pointed out that $(\rho, \sigma_e) = (0.85, 0.017)$ fails to capture the extremely high output
a wedge between the price of foreign investment and its marginal productivity only when a country defaults, which in turn affects the output loss given default. The values of \( \nu \) and \( \theta \) are jointly set to (0.5, 0.7), generating a wedge equal to 2.33\( \bar{p} \).

On the other hand, I calibrate two groups of parameters \((\mu, \lambda, a)\) and \((\gamma, b)\) against Canada in Model (I) and Mexico in Model (II), respectively, to capture the difference in the quality of public consumption and service, the preference of final good production technology. The parameter \( \mu \) is set to 0.65 for both models, the same value as in Mendoza and Yue (2012). Given \( \mu \), the parameter \( a \) is calibrated according to eq (1.21) to target \( \frac{\sigma(\mu)}{\sigma(G)} = 4.74 \) and 2.51 for Canada and Mexico, respectively.\(^{13}\) Finally, given \( \mu \) and \( a \), the parameter \( \lambda \) is chosen to approximately match \( \frac{m}{G} = 0.09 \) for Canada and 0.13 for Mexico at the constant price \( \bar{p} = 1 \).

\((\gamma, b)\) are jointly calibrated to (1.37, 0.45) in Model (I) to approximately match the average ratio of private consumption to government final good expenditure \( \frac{c}{G} = 2.69 \) and the relative volatility of government final good expenditure \( \frac{\sigma(G)}{\sigma(y)} = 1.21 \) in Canada.\(^{14}\) For model (II), \((\gamma, b)\) are set to (0.56, 0.79) to match the average ratios of \( \frac{c}{G} = 6.68 \) and \( \frac{\sigma(G)}{\sigma(y)} = 2.74 \) in Mexico.

To approximate the value functions and optimal policy functions numerically, I apply Chebyshev polynomial methods with 15 grid points along each dimension.\(^{15}\) The simulation method follows Mendoza and Yue (2012). I draw 1000 independent TFP processes of 500 periods and feed them to Model (I) and (II) to get 1000 persistence of 0.91 in Canada, which is 16 percent higher that the average of developed countries, as reported in Aguiar and Gopinath (2007). However, I believe it is still in the reasonable range for a representative developed small open economy.

\(^{13}\) Canada’s FDI inflows are unprecedentedly high and volatile between 1998 and 2002, which are excluded when calculating the volatility of FDI.

\(^{14}\) The ratios are computed using the original data while the volatilities are derived using the cyclical components of the data series filtered the HodrickPrescott filter.

\(^{15}\) Hatchondo, Martinez, and Sapriza (2010) show that Chebyshev polynomials or cubic spline interpolation is more efficient than discrete state space techniques with evenly spaced grid points.
simulated samples. Then, I delete the first 100 periods in each sample to calculate
the moments from the stochastic stationary states of each model.

Table 1.2: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model (I)</th>
<th>Model (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$  Time preference rate</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>$\sigma$ Coefficient of relative risk aversion ($c_t$)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\psi$ Exponential parameter of labor supply</td>
<td>0.455</td>
<td>0.455</td>
</tr>
<tr>
<td>$\gamma$ Coefficient of relative risk aversion ($G_t^1$)</td>
<td>1.37</td>
<td>0.56</td>
</tr>
<tr>
<td>$b$ Weight coefficient of public consumption</td>
<td>0.45</td>
<td>0.79</td>
</tr>
<tr>
<td>$\alpha_M$ Share of intermediate good in final good production</td>
<td>0.43</td>
<td>0.43</td>
</tr>
<tr>
<td>$\alpha_L$ Share of labor in final good production</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>$\alpha_K$ Share of capital in final good production</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>$a$ Parameter in capital good transformation function</td>
<td>1.42</td>
<td>1.32</td>
</tr>
<tr>
<td>$\mu$ Armington curvature parameter</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$\lambda$ Armington weight of foreign investment</td>
<td>0.41</td>
<td>0.39</td>
</tr>
<tr>
<td>$\nu$ Dixit-Stiglitz curvature parameter</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$r^*$ World risk-free interest rate</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\theta$ Share of foreign creditors</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$\bar{p}$ Relative price of differentiated foreign investments</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\phi$ Reentry probability</td>
<td>0.083</td>
<td>0.083</td>
</tr>
<tr>
<td>$\rho$ AR(1) coefficient of TFP shocks</td>
<td>0.085</td>
<td>0.085</td>
</tr>
<tr>
<td>$\sigma_\epsilon$ Standard deviation of TFP shocks</td>
<td>0.017</td>
<td>0.017</td>
</tr>
</tbody>
</table>
1.4.2 Standard Results of Sovereign Default Models

In this subsection, I summarize the qualitative relationships between price function, debt dynamics and TFP shocks based on the results from Model (II).\textsuperscript{16} It is not surprising that these results should be in line with the conventional wisdoms in sovereign default models, such as in Arellano (2008), Bi (2012), and Hatchondo, Martinez, and Roch (2012), and Mendoza and Yue (2012). Nevertheless, I exploit the concepts of the safety level of debt, debt limit and fiscal space, which altogether help better understand the different business cycle facts in developed and developing countries.

Figure 1.7 plots the equilibrium bond price as a function of the current productivity $A_t$ and the beginning of the next period debt $Z_{t+1}$.\textsuperscript{17} It shows that, with other things equal, the sovereign bond price monotonically decreases as debt level increases or TFP shock decreases. In addition, there exists a narrow band in the space of $A_t$ and $Z_{t+1}$, in which the market value of sovereign bonds $q(Z_{t+1}, A_t)$ starts to drop very quickly. It follows from $r_{t+1} = 1/q(Z_{t+1}, A_t) - 1$ that the risk-adjusted interest rate on sovereign bonds increases sharply from the world risk-free rate $r^*$ to infinity. This band can be depicted in Figure 1.8 by the upper and lower contours of price levels $\{(Z_{t+1}, A_t)|q(Z_{t+1}, A_t) = 0.99\}$ and $\{(Z_{t+1}, A_t)|q(Z_{t+1}, A_t) = 0.005\}$, respectively. With these counters, I can define the safety level of debt, debt limit and fiscal space:

1. The safety level of debt. The highest contour $q(Z_{t+1}, A_t) = 0.99$, which approximately equals to $1/(1 + r^*)$, can be used to indicate the safety level of debt, such that a positive risk premium exists for any combination of $(Z_{t+1}, A_t)$ in the northwest side of this contour. The safety level of debt is an increasing function

\textsuperscript{16}Model (I) produces similar qualitative results.

\textsuperscript{17}For illustrative purpose, debt is normalized by the model’s stochastic steady state output in all figures.
of the current productivity $A_t$, implying that a debt level considered being safe in normal times might not be safe in a crisis featured by low productivity.

(2) Debt limit. For any given productivity, debt limit is defined as the debt level at which the price of sovereign debt approaches zero, e.g., $q(Z_{t+1}, A_t) = 0.01$. Debt limit is used to measure the government’s ability to service its debt. As standard in the literature, the debt limit is an increasing function of $A_t$.

(3) Fiscal space. Following Ostry and et al. (2010) and Ghosh and et al. (2013), fiscal space is defined as the distance between the current debt ratio and the debt limit.

Figure 1.7 : Debt Limits and steady State Debt Levels, Model (II)
Figure 1.8: Debt Limits and Optimal Next-Period Debt Levels, Model (II)

Figure 1.8 displays the optimal level of debt $Z_{t+1}$ as a function of $A_t$, given $Z_t = 0$ (low debt) and $Z_t = 0.1$ (high debt). When $A_t$ is sufficiently large and $Z_t$ is far below the corresponding safety level of debt, $Z_{t+1}$ increases as $A_t$ decreases, which is driven by the motivation to smooth out consumption in economic downturns. However, this trend is curved by the concern of sovereign default as a country is approaching the “dangerous band” when $A_t$ is sufficiently low. In this situation, the risk premium of sovereign bonds increases rapidly, making it more difficult for the government to smooth out consumption using debt financing. As a result, it is optimal for the government to reduce its contemporaneous borrowing and households’ consumption in exchange of a large fiscal space for future borrowings.
1.4.3 Cyclical Co-movement of Fiscal Policy

Table 1.3 compares several business cycle moments in Canada and Mexico with their simulated counterparts using Model (I) and Model (II), respectively. The model does well in matching the overall pattern of business cycles in both economies. Model (I) produces an insignificant correlation between fiscal policy and output for Canada while Model (II) a negative correlation between the two variables for Mexico.\textsuperscript{18} It is consistent with the stylized facts that fiscal policy in developed countries tends to be countercyclical or acyclical while it tends to be procyclical in emerging economies.

The acyclical fiscal response of developed economies can be explained as follows. The larger share and the lower volatility of $G_1^t$ in developed economies make it costly from them to reduce $G_1^t$ in bad times. Additionally, the stronger bias towards the domestic technology $G_2^t$ leads to a large volatility of FDI relative to domestic investment, makes advanced economies more reliant on $G_2^t$ in bad times. The above characteristics of the public sectors in developed countries motivate these countries to implement more conservative fiscal policies. Consequently, developing countries are more likely to have little fiscal space, face prohibitively high borrowing cost, and have to shift from debt financing to tax financing, which is more costly. Meanwhile, due to more inelastic demand of public consumption and investment, developed countries are inclined to leave a larger fiscal room in normal times in order to borrow at a relative low cost in bad times.

\textsuperscript{18}Since countries can have a trade deficit only through governments’ external debt financing, $TB_t = -\Delta_t$. As a result, $\rho(TB_t/y_t, y_t) = -\rho(\Delta_t/y_t, y_t)$. 
Table 1.3 : Statistical Moments in the Models and in the Data

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Model (I)</th>
<th>Data (Canada)</th>
<th>Model (II)</th>
<th>Data (Mexico)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output Process</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(y_t)$</td>
<td>4.82</td>
<td>1.64</td>
<td>4.91</td>
<td>2.48</td>
</tr>
<tr>
<td>$\rho(y_{t-1}, y_t)$</td>
<td>0.862</td>
<td>0.91</td>
<td>0.871</td>
<td>0.82</td>
</tr>
<tr>
<td><strong>Correlation with Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(G^1_t, y_t)$</td>
<td>0.99</td>
<td>-0.17</td>
<td>0.99</td>
<td>0.63</td>
</tr>
<tr>
<td>$\rho(G^2_t, y_t)$</td>
<td>0.99</td>
<td>0.77</td>
<td>0.99</td>
<td>0.91</td>
</tr>
<tr>
<td>$\rho(T_t, y_t)$</td>
<td>0.99</td>
<td>0.73</td>
<td>0.98</td>
<td>0.77</td>
</tr>
<tr>
<td>$\rho(TB_t/y_t, y_t)$</td>
<td>-0.075</td>
<td>-0.20 (0.21)</td>
<td>-0.247</td>
<td>-0.74 (0.14)</td>
</tr>
<tr>
<td>$\rho(c_t, y_t)$</td>
<td>0.99</td>
<td>0.88</td>
<td>0.98</td>
<td>0.92</td>
</tr>
<tr>
<td><strong>Other Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(c_t)/\sigma(y_t)$</td>
<td>0.78</td>
<td>0.77</td>
<td>0.72</td>
<td>1.24</td>
</tr>
<tr>
<td>$E(Z_{t+1}/y_t)$</td>
<td>0.07</td>
<td>0.22</td>
<td>0.10</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 1.3 shows that both models are consistent with the actual data that public investment and output are positively correlated in both countries. It is because the marginal productivity of capital is higher in economic booms. Model (I) fails to generate the negative correlation between government’s public consumption and output in Canada. It is because of that the government in the model always equalizes the marginal utilities between private consumption and public consumption and that private consumption is lower in bad times due to the constrained ability to borrow. Additionally, Model (I) and (II) replicate the basic fact that tax revenue is positively correlated with output, but with a larger magnitude than the actual data. This is due to the assumed incompleteness of credit markets, in which the government cannot fully insure against negative TFP shocks. Consequently, the government has
to impose higher taxes and spending more on public consumption in goods times.

Model (I) implies that the volatility of private consumption does not exceed the volatility of output, which is consistent with Canada’s data. However, Model (II) yields a similar result that private consumption is less volatile than output, which contradicts the data of Mexico. That consumption process is more volatile than output is a common feature in developing countries, as shown in Aguiar and Gopinath (2007). Previous sovereign default models tend to explain this phenomenon by the countercyclical trade balance in good times, or equivalently, the procyclical fiscal stance.\footnote{For instance, Arellano (2008) stresses the persistence of TFP shocks. As TFP shocks are persistent, a positive shock today will probably lead to a positive shock tomorrow. Therefore, in good times, the government is enticed to borrow more and consume more at a low interest rate, leading to a procyclical policy.} However, in my model, the simulated $\rho(\Delta_t, y_t)$ in good times is countercyclical in both developed and developing countries. It implies that the procyclical policy has been weakened or even reserved if the government can actively affect production and utility via proper fiscal instruments.

The models yield a slightly smaller volatility of output of Canada than that of Mexico, which contradicts the data that the output volatility of Canada is two third of Mexico. This is because this paper does not intend to match the exact persistence and volatility of output in both countries by calibrating the TFP process for each country. Instead, it stresses the difference in governments’ ability to affect economy activity. Towards this end, I isolate the effect of TFP shocks on the choice of fiscal policy by using the same TFP process for both developed and developing countries. I argue that my choice of $(\rho, \sigma_e) = (0.85, 0.017)$ falls in the range of values used in previous works.\footnote{For example, Mendoza and Yue (2012) choose $(\rho, \sigma_e) = (0.945, 0.017)$ while Cuadra, Sanchez, and Sapriza (2010) take $(\rho, \sigma_e) = (0.85, 0.006)$.} Meanwhile, as is mentioned in the introduction, the previous work on sovereign default failed to explain the countercyclical fiscal policy by the
difference in TFP shocks due to the absence of a comprehensive fiscal authority.

Finally, model (I) and (II) predicts a debt-to-output ratio of 7 percent of GDP in Canada, 3 percent lower than in Mexico. It may seem counterfactual given that the gross external-debt-to-output ratio is about 70 percent in Canada in recent years. Note that some developed countries, such as Canada, also have a considerable amount of foreign assets, which significantly lowers the net external debt ratio. Due to data limitation, I use the net International Investment Position (IIP) to represent the net external debt ratio in Canada. Direct calculation shows that the average IIP in Canada from 1997 to 2011 is -21.8 percent of GDP, less than Mexico’s actual debt-to-output ratio of 29 percent.  

Nevertheless, it is necessary to point out that the model falls short in capturing the record high debt ratios in peripheral European countries over the past decades. There are a few ways to further increase the debt ratio. For example, Arellano (2008) points out that reducing $\beta$ or $\phi$ tends to increase the average debt ratio. In addition, I have explored the consequence of varying the persistence $\rho$ and the volatility $\sigma_e$ of TFP shocks and found that increasing $\rho$ or reducing $\sigma_e$ tends to lower the average debt ratio. The intuition is the following: when negative TFP shocks are more persistent, the country has to reduce its debt holdings in normal times and good times in order to leave adequate fiscal space for borrowings in bad times. This result, however, implies that Model (I) will fail to capture the high debt ratios in Southern European counties even if I apply a more persistent TFP process to these countries, unless I alter other key parameters simultaneously. While this is an interesting

---

21 The International Investment Position data from 1997 to 2011 are collected from the website of Statistics Canada at http://www.statcan.gc.ca. Data show that the IIP of Canada is steadily declining from 1997 to 2009 with a rebound in 2010 and 2011.

22 The debt external debt data are collected from Eurostat at http://epp.eurostat.ec.europa.eu. Calculation shows that the average net external debt ratios of Greece, Italy, Portugal, and Spain from 1997 to 2011 are 0.73, 0.31, 0.53, and 0.61, respectively.
question that requires further discussion and exploration, I think possible reasons include the easy access of European countries to lenders of last resort (e.g. the ECB and the IMF) for low-interest loans, prevailing nominal wage rigidities (e.g. in Portugal), and decentralized borrowing mechanism (e.g. in Spain).

1.4.4 Optimal Composition of Fiscal Consolidation

Given productivity $A_t$, Figure 1.9 presents the relationships between the amount of net external financing $\Delta_t$ and the optimal allocation of public goods $G^1_t$, domestic investment $G^2_t$ and Tax $T_t$. Recall $\Delta_t = q(Z_{t+1}, A_t)Z_{t+1} - Z_t$ denotes the amount of financing available to the government; therefore, a positive (negative) $\Delta_t$ stands for fiscal expansion (consolidation). It shows that the optimal tax $T_t$ and public investment $G^2_t$ increase with the scale of consolidation while public consumption $G^1_t$ decreases with consolidation efforts. In other words, an optimal consolidation path consists of tax hikes, cuts in public consumption but not in public investment. The reason is as follows. A rise in taxes results in a drop in private consumption, leading to a decrease in public consumption to equalize the marginal utilities of $c_t$ and $G^1_t$. Moreover, the increase in public investment $G^2_t$ is driven by the fact that the marginal productivity of $G^2_t$ is increasing in the equilibrium labor supply $l_t$ and the latter is increasing with the scale of consolidation due to the wealth effect. Therefore, reallocating the additional tax revenue to public investment becomes more attractive when the marginal productivity of $G^2_t$ is greater than one, the marginal cost of lump-sum tax.

Although my model studies fiscal policy in a very simple way, it leads to policy implications that are in line with a wide range of empirical studies, as well as IMF policy advice for countries suffering high risks of sovereign default. First, revenue-based consolidations could be more favorable than previously thought. Second,

$^{23}$See Abbas et. al (2011), and Mauro and Villafuerte (forthcoming).
Perotti (2011) reexamines the four largest multi-year fiscal consolidations (Denmark, 1982-86; Ireland 1987-90; Finland 1992-98; and Sweden 1993-98) and finds that cuts in government spending are less effective than was widely believed. In addition, post-crisis empirical studies tend to agree that spending multipliers are growing faster than revenue multipliers under adverse economic conditions, leading to a greater gap between the two in economic recessions. This implies that reducing government expenditure, particularly public investment, will cause a great loss in output, raise the debt-to-output ratio and thereby the borrowing cost for the country.

1.5 Conclusion

This paper simultaneously rationalizes the countercyclical fiscal policy of developed countries and the procyclical fiscal policy of developing countries using a general

---

24See Baum, Poplawski, Ribeiro, and Weber (2012), and Batini, Callegari and Melina (2012).
equilibrium model of sovereign default. The model highlights differences in the biases of production technology towards public investment, as well as in the shares and volatilities of public good consumptions, between the two types of countries. It turns out that the difference in governments’ ability to ameliorate business cycle fluctuations through fiscal instruments is key to understand its choice of fiscal policy over business cycles. The larger share and the lower volatility of public goods in developed economies raise the welfare loss of reducing them in bad times. Additionally, the stronger bias towards the domestic technology leads to a large volatility of FDI relative to domestic investment, making advanced economies more reliant on domestic investment in bad times. These common characteristics of the public sectors in developed countries provide incentives for them to implement more conservative fiscal policies than their developing counterparts.

The model also explains the difference in some key business cycle moments between the two types of economies. In addition, the model implies that the optimal fiscal consolidation should consist of tax hikes and cuts in public consumption but not in public investment. I acknowledge that the interaction between fiscal policy, sovereign default and business cycles should be an interesting subject for further research. For instance, models that feature longer debt maturities, decentralized borrowing or frictions in the labor market may also explain the countercyclical fiscal behaviors in developed small open economies.
Chapter 2

Mixed Signals: Crisis Lending and Capital Markets

2.1 Introduction

Crisis lending is intended to restore confidence in capital markets. Current efforts to shore up the euro zone focus on conditional lending to reassure investors in sovereign bonds. Similarly, the International Monetary Fund has long claimed that its lending acts as a “seal of approval” on national economic policies, which catalyzes private capital flows. Yet the empirical evidence is mixed. A likely explanation for this mixed evidence is that the political incentives to engage in crisis lending lead to countervailing effects, and quantitative studies of market reactions to crisis lending have not modeled these political dynamics. International lending is a political decision, which results from bargaining between the borrower and the lender. The terms of crisis lending depend on their relative bargaining power and the quality of their relationship. Furthermore, the inferences that private investors draw from observing crisis lending depend on what new information it reveals, which in turn depends on this bargaining process.

The recent debt crisis in the euro zone illustrates the countervailing effects of crisis lending on the calculations of private actors. When multilateral actors lend to Greece, for example, the infusion of liquidity reduces the short-term risk of involuntary default, which should reassure bond holders, and any new commitments that Greece makes to undertake fiscal reforms as a condition for receiving the loan should

---

1E.g. Rodrik (1995) argues that multilateral lending can provide a seal of “good health” to markets.
improve its prospects for long-term solvency. On the other hand, the Greek decision to accept a bailout reveals information about the severity of the crisis. The fact that the government is willing to accept a particular deal resolves some of the investors’ uncertainty about the government’s private information, because governments that are more confident about the future hold out for more generous terms. Finally, capital markets must make an assessment of how likely the promised reforms are to be implemented.

Lender motivations play an important role in these calculations. EU lending to Greece reflects the priority of maintaining political and economic union in Europe. Greece is critical because the fiscal vulnerabilities in Spain, Italy and other countries raise the risk that a default in one euro country could lead to rapid contagion. Moreover, Greece is important because German and French banks are heavily exposed to Greek debt, so a Greek default might lead to a banking crisis in the core countries. Consequently, creditors are anxious to make a deal. This should be reflected in bargaining: Greece should receive more generous loans than less pivotal countries facing similar circumstances, but the conditions attached to the loans should be less rigorous. For the same reasons, future enforcement of those conditions is expected to be lax, which undermines the reassurance provided by conditionality. Thus, strategic importance influences the terms of loan packages through several channels that can have countervailing effects on the reactions of capital market participants.

We build on these insights by developing a game-theoretic model of interactions between a “lender of last resort,” a borrowing country government, and a private market actor. Our model introduces three theoretical innovations. First, it models (1) bargaining over the terms of a loan program, (2) implementation and enforcement of conditionality, and (3) the reactions of market actors, allowing these three processes that are generally treated in isolation to affect one another. Second, it

\(^2\)We assume that market actors are informed about the conditions attached to loans. Data on
introduces private information on the part of the government of the borrowing country, which makes possible bargaining failure and adverse selection. The market and the lender are not fully informed about the underlying state of the borrowing country’s economy. Third, we model the probability of a financial crisis as a random variable that is a function of market interest rates, which in turn depend upon the probability of a crisis, allowing for the possibility of self-fulfilling expectations. The interest rate is determined competitively, so market actors price their capital to offset risk, and the price of capital is itself a component of a country risk profile. This setup allows us to examine how market actors make inferences about the likelihood of a crisis and factor these considerations into their investment decisions, which in turn determine interest rates.

Our theoretical analysis indicates that the effect of crisis lending on market expectations depends on bargaining, so we are unlikely to observe one, straightforward “catalytic” effect. When drawing inferences about investment environments, market actors consider the observable characteristics of loan programs, what loan announcements reveal about the underlying state of the economy, and how future bargaining will be affected by the political bias of the lender. Because of adverse selection, we expect the effect of new lending announcements to be harmful to the investment climate, once we control for the salutary effects of liquidity and conditionality. Furthermore, because of moral hazard, we expect lending to have less beneficial effects in countries that are especially favored by lenders of last resort. On the other hand, liquidity and conditionality should improve the investment climate and lower in-

---

IMF conditionality were not publicly released until recently, but a determined observer has always been able to obtain this information.

---

As Blustein (2001) and reports by the Independent Evaluation Office of the IMF document, even multilateral financial institutions like the IMF have often been as much “in the dark” as market actors regarding detailed information about a country’s Central Bank reserves and liabilities and about the solvency of the banking sector.
terest rates—and countries that are the favorites of lenders should receive more of the former and less of the latter. The net effects of crisis lending should depend on the various elasticities involved, but the mechanisms predicted by the model are straightforward and amenable to quantitative testing. We test these hypotheses using data from the IMF’s Monitoring of Agreements (MONA) database for the period 1992-2002, and we find support for the predictions of the model.

Our central empirical finding is that geopolitically important countries are offered larger loans, on softer terms, and with less rigorous enforcement of conditionality, and the perverse effect is that crisis lending is least effective, in terms of lowering bond yields, in the countries of greatest importance to the lender. The net effect of lending can reduce or increase bond yields, depending on the relative weights of the countervailing influences of the liquidity, adverse selection, and moral hazard effects. Crisis lending causes market confidence to deteriorate when there is substantial ex ante uncertainty and the borrowing country is politically or economically important. This suggests that design features of IOs that may be necessary to secure the “buy in” of major powers, such as the IMF governance structure that allows key shareholders to exert informal influence over lending decisions, can have unintended consequences that undermine their effectiveness (Stone 2011).

2.2 Market Reactions to Crisis Lending

Sovereign crisis lending initiatives aim to provide immediate liquidity and reassure market actors. The lender of last resort may be an individual government, a “pivotal” or critical member of an international financial institution, the median member of a coalition of lending countries, or even a large financial institution with incen-

---

4Borrowers and lenders are willing to engage in transactions that increase bond yields, because under some circumstances loans that reduce the probability of a crisis nevertheless lead the market agent to revise her estimate of the probability of a crisis upwards.
tives for defensive lending in the face of a crisis. Although we focus our empirical analysis on IMF programs, our theoretical analysis holds insights for a range of crisis-lending relationships. Crisis lenders typically impose conditionality. For instance, in the interwar period the banker J.P. Morgan offered a crisis loan to Great Britain, but Britain rejected the conditionality attached to the loan and was subsequently compelled to go off of the Gold Standard. The Bank of International Settlements extended credit to Austria and Germany in 1931 in an effort to stem a spreading banking crisis, and demanded far-reaching policy changes in return. In more recent times, the U.S. government provided extraordinary financing to Mexico in 1982 and again in 1995. The Paris Club and London Club have engaged in debt rescheduling to alleviate financial crises and restore investor confidence, and, as noted above, the European Central Bank and European Commission have taken on a crisis lending role in the current European financial crisis. In each of these cases, a lender with significant economic or political interest in an country in crisis extended a loan, typically with reforms attached, in an effort to forestall a deepening economic crisis and restore investor confidence.

The effectiveness of IMF programs in catalyzing private capital flows has received considerable attention in the economics literature (Bird and Rowlands 2002; Brune et al. 2004; Edwards 2005; Mody and Saravia 2003; Eichengreen et al. 2007). Restoring investor confidence is a key element of the IMF mission and a critical component in evaluating whether IMF programs are beneficial for participating countries. Yet empirical research on the catalytic effect displays mixed findings. For instance, correcting for selection, Edwards (2006) finds that IMF programs generate net outflows of portfolio investment, and Jensen (2004) finds a similar effect for foreign direct investment. Mody and Saravia (2003) find a positive effect of IMF programs only in cases of intermediate financial risk, which the authors characterize as instances

---

5See Steinwand and Stone 2008 for a review.
when IMF programs are viewed as joint commitments between a government and the IMF. Eichengreen and Mody (2000) find evidence that IMF lending decreases bond spreads, while Cottarelli and Giannini (2002) find little evidence that IMF interventions catalyze investment. It is clear that catalytic effects vary considerably across types of countries, but there is little consensus about the systematic sources of this variation.\footnote{Another possible reason for discrepant findings is that different studies have different types of catalysis in mind. Some focus on FDI while others focus on portfolio investment, and some focus on indicators of country risk, like bond yields, while others focus on investment flows.} To date, the question of how IMF programs influence international markets has not been studied with sensitivity to fundamental political factors.

In this paper we shift from a focus on average catalytic effects to an effort to theoretically identify the multiple countervailing forces that influence private market actors. As we discussed above, there are several factors that influence market actors’ investment decisions. An agreement with a crisis lender indicates that additional resources are available to stave off involuntary default, which should reassure capital markets. On the other hand, the fact that a country needs a rescue package reveals troubling information about the country’s economic fundamentals, which can make private capital market actors balk at lending to the government. These two opposing effects of crisis lending may explain the mixed findings, including the finding that IMF programs may actually lead to capital flight. We go one step further, noting that the fact that crisis lending decisions are also subject to the political biases of the major principals further complicates the inferences that market actors can draw from observing an agreement.

Recent cross-national empirical evidence confirms that geopolitical interests influence multilateral lending decisions. For instance, the interests of the United States have been shown to exert a broad influence over IMF lending, including the likelihood of receiving an IMF program (Thacker 1999), loan size and conditions (Stone
2008, 2011, Copelovitch 2010), and the credibility of IMF conditions (Stone 2002, 2004, 2011, Dreher et al. 2009b). Similarly, temporary membership in the UN Security Council or significance to U.S. foreign policy can affect the disbursement of World Bank Loans (Dreher et al. 2009a, Kilby 2009). Extant work has not examined whether this influence affects the responses of private capital markets to lending, however, so it is not surprising that previous studies have not reached consensus on a general pattern of catalytic effects. Our theoretical analysis provides an explanation for this lack of consensus: market actors are sophisticated about the incentives of the crisis lenders, and they take them into account when they draw inferences from new loan announcements.

The model that we present in the next section is an infinitely repeated game between a lender of last resort, a government managing an economy under stress, and a representative market actor. In each period, the lender may offer a loan and a package of associated policy conditions to be implemented by the borrower, and the government may accept or reject the package. The government is free to renege on its policy commitments, so the model captures the borrower’s time-consistency problem. A financial crisis may occur stochastically in any period, with a probability that is a function of the market interest rate, in addition to crisis financing, the state of the economy, and the government’s implementation of reforms. In the equilibrium that we study, the lender punishes defection by denying the government access to its funds in the future; however, it subsequently restores the country to good standing if the government implements the required reforms. The market actor invests only if the bond yield is high enough to compensate for sovereign risk, so in equilibrium market expectations set the interest rate. We allow the lender to place different weights on outcomes in different countries, and we examine the implications of this bias for the reactions of the capital market to a loan agreement.

Our analysis of the model shows that, first, crisis lending programs are hetero-
geneous treatments. Our bargaining-theoretic framework generates the expectation that different countries will receive programs that contain more or less stringent conditions for policy reform, on one hand, and will receive more or less generous financing, on the other. If this is the case, empirical studies that estimate a uniform effect of such diverse treatments will be misspecified; in principle, the effect of crisis lending programs should be conditional on their terms. Consequently, in the empirical analysis that follows, we control for conditionality and financing when we investigate the effect of program participation. Second, our model suggests that the effects of lending programs depend on the political biases of the lenders. Influential borrowers are less likely to fully implement the contracted conditions because they anticipate that rigorous enforcement is not credible. Our empirical analysis therefore models this influence.

2.3 Model

In this section we develop an infinitely repeated game to analyze the strategic interactions between a government, a lender of last resort, and the international capital market. Let $G$ denote a government, $L$ the lender, and $M$ a representative market actor. The state of the economy in period $t$ is a random variable, $\theta \in \{\theta_1, \theta_2\}$, where $\theta$ measures $G$’s financing gap, which must be financed in order to overcome a potential crisis. Let $\theta_1 < \theta_2$. For convenience, we will sometimes refer to $\theta_1$ as a good economy, and $\theta_2$ as a bad economy. Furthermore, $\theta$ is independently and identically drawn in all subsequent periods.

At the beginning of a period (or the stage game), nature draws the state of the economy, $\theta = \theta_i$ for $i \in \{1, 2\}$, from the time invariant distribution $\pi = (\pi_1, \pi_2)$, where $\pi_1 + \pi_2 = 1$. $G$ learns the true value, but neither $L$ nor $M$ does. Then, $L$ moves first and offers a new loan agreement to $G$. The offer is a take-it-or-leave-it offer, $(s, x)$, where $s \in [0, \bar{s}]$ is the size of the loan with a budget constraint at $\bar{s}$, and
$x \in [0, \bar{x}]$ is the level of reforms, or conditionality, required in exchange for the loan.

After receiving a proposal $(s, x)$ from $L$, $G$ decides whether to accept or reject the offer. For notational convenience, let $(d_x, d_s) \in \{0, 1\} \times \{0, 1\}$ denote the decisions of $G$ on $x$ and $s$, where 1 represents acceptance, and 0 rejection. There are four possible combinations of $G$’s choice: it can reject both, accept both, accept only the loan but refuse to implement the reforms, or implement the reforms without accepting the loans.\(^7\) Let $x_G$ and $s_G$ denote the realized level of reforms and loan size based on $G$’s choice. It is easy to see that $s_G = d_s s$, and $x_G = d_x x$. Moreover, to capture the idea that $L$ cannot fully predict $G$’s decision in each period, we introduce a vector of i.i.d choice-specific shocks, $\epsilon = [\epsilon(0, 0), \epsilon(0, 1), \epsilon(1, 0), \epsilon(1, 1)]$, to $G$’s utility function. $G$ observes the realization of $\epsilon$ and chooses its optimal response $(d_x, d_s)$ by taking into account $\epsilon(d_x, d_s)$. $L$ only knows the distribution of $\epsilon$, so from $L$’s perspective, $G$’s choice of $(d_x, d_s)$ is probabilistic.

Next, $M$ observes $(s, x)$ and $d_s$, i.e., whether $G$ accepted the loan or not, updates its belief about $G$’s economic conditions, and forms beliefs about $G$’s choice of reforms using Bayes’ rule, and then sets a competitive interest rate, $r$. That is, the equilibrium yield makes a representative agent indifferent in expectation between buying and selling bonds, given the available information. At the end of the period, nature moves again and reveals whether an economic crisis takes place as a function of the state of the economy, $\theta$, and the choices that the actors made. The stage game is repeated infinitely, with $G$ and $L$ discounting their future payoffs by $\delta_G$ and $\delta_L$, respectively.

Now we elaborate on the probability of crisis at the end of each period. The economy will experience a crisis if $G$ cannot meet its obligations, which occurs when: $r\bar{I}(\theta-(1+x_G)s_G)-\phi_\theta \geq 0$, where $\theta-(1+x_G)s_G$ is the difference between the country’s financing gap and the size of the loan augmented by the reforms implemented by $G$.

\(^7\)The last scenario is considered for completeness.
The term \((1 + x_G)s_G\) implies that reforms \((x_G)\) allow a country to utilize external financing \((s_G)\) more effectively. Thus, \(r\mathbb{I}(\theta - (1 + x_G)s_G)\) is the interest payment that \(G\) owes the private market for the remaining financing gap, where the truncation function, \(\mathbb{I}(\theta - (1 + x_G)s_G)\), ensures that the amount is well-behaved.\(^8\) Finally, \(\phi_\theta\) is a stochastic shock to \(G\)’s revenue that will be realized at the end of the period. Let \(F_{\phi_\theta}(\cdot)\) be the cumulative distribution function of \(\phi_\theta\). Then we can derive the probability of crisis as \(p(s_G, x_G, r; \theta) = F_{\phi_\theta}[r\mathbb{I}(\theta - (1 + x_G)s_G)].\(^9\)

We now proceed to specify the actors’ payoffs. \(G\)’s utility function for each period is a function of its observables, \(-bx_G - p(s_G, x_G, r; \theta)\), and unobservables, \(\epsilon(d_x, d_s)\):

\[
u_G(\theta, \epsilon; d_x, d_s; s, x, r) = -bx_G - p(s_G, x_G, r; \theta) + \epsilon(d_x, d_s), \tag{2.1}
\]

where \(b\) is a constant that scales the cost to \(G\) of implementing reforms. The choice-specific random shock, \(\epsilon(d_x, d_s)\), captures short-term fluctuations in political constraints known only to the government. Note that \(\theta\) and \(\epsilon\) are state variables, \((d_x, d_s)\) are \(G\)’s choice variables that determine \(s_G\) and \(x_G\), and \(s, x\) and \(r\) are choice variables of \(L\) and \(M\) that enter \(G\)’s utility function as parameters.

We allow \(L\)’s lending decision to be influenced by its geopolitical interests in the country. Let \(\beta \in [0, \bar{\beta}]\) capture this bias. Then, \(L\)’s utility function for each period is:

\[
u_L(s, x; d_x, d_s; r; \beta, \theta) = -c(1 - \beta)s_G - (1 + \beta)p(s_G, x_G, r; \theta), \tag{2.2}
\]

where \(c\) is a constant that measures the unit opportunity cost of crisis lending, \(s_G\), for \(L\). The geopolitical importance of \(G\) influences \(L\)’s lending decision in two ways: higher levels of importance give \(L\) a direct incentive to increase the size of a loan,

---

\(^8\)That is, \(\mathbb{I}(\theta - (1 + x_G)s_G) = \theta - (1 + x_G)s_G\) if \((1 + x_G)s_G \leq \theta\), and \(\mathbb{I}(\theta - (1 + x_G)s_G) = 0\) otherwise.

\(^9\)We discuss some of the properties of \(p(s_G, x_G, r; \theta)\) in Appendix I.
and also amplify the negative effect if a crisis occurs, providing a reinforcing indirect incentive.

Lastly, let $M$ be a representative agent in a competitive market, so that the price of debt exactly offsets the risk of default.\(^{10}\) $M$ is risk neutral so that its utility function is the same as its expected net profit per unit of investment in $G$:\(^{11}\)

$$u_M(r; s_G, x_G, \theta, \bar{v}) = [1 - \bar{p}(s, x, d_s, r)]r - \bar{p}(s, x, d_s, r) - \bar{v},$$

(2.3)

where $\bar{v}$ is the transaction cost in percentage, and $\bar{p}(s, x, d_s, r) = \mathbb{E}\{p(s, x, d_s, r, \theta) | s, x, d_s, r\}$ is the expected crisis probability w.r.t. $d_s$ and $\theta$, given the observables $(s, x)$ and $d_s$ and the competitive interest rate, $r$.

This is a game of incomplete information, and we solve for a perfect Bayesian equilibrium (PBE).\(^{12}\)

### 2.4 Equilibrium Results

To facilitate the explanation of the equilibrium results, we define a punishment phase and a bargaining phase. A punishment phase begins if $G$ did not comply with its agreement with $L$ in the previous period; otherwise, $G$ is in good standing, and the period takes place in a bargaining phase. Each phase may include multiple periods. As is generally true for infinitely repeated games, our model has multiple equilibria. Informed by our empirical observations, we focus on an equilibrium where $L$ adopts a limited punishment strategy for defection.

On the equilibrium path, $G$ chooses to defect if the cost of implementing conditionality is high. $L$ prefers that $G$ comply with its agreement, so $L$ adopts a punishment strategy designed to minimize $G$'s incentive to defect. We consider the following punishment strategy for $L$: if $G$ defects, the game enters a punishment

---

\(^{10}\)We assume that $G$ will default if, and only if, a crisis occurs.

\(^{11}\)Without loss of generality, we use per unit profit to represent $M$’s utility.

\(^{12}\)For technical details, see Appendix I.
phase in which $L$ offers $s = 0$ until $G$ has implemented a threshold level of reform, $x_p$. This threshold is endogenous. A high threshold is more effective at deterring $G$ from defecting, but implies that punishment periods last longer in expectation. Withholding credit for a long period of time is costly to $L$, and this cost is magnified when $L$ faces a high $\beta$ country, so the optimal threshold is a function of the geopolitical importance of the borrower, $\beta$. The following proposition characterizes an equilibrium in which $L$ adopts such a limited punishment strategy, offering two levels of conditionality contingent on $G$’s compliance behavior.\footnote{The proof of the proposition is in Appendix I. There exist multiple equilibria that are similar to the one we analyze, in which $L$ plays a punishment strategy that allows the borrower to return to good standing by implementing some level of conditionality in the future. However, Proposition 1 focuses on the equilibrium in which $s_b^*(\beta)$, $x_b^*(\beta)$ and $x_p^*(\beta)$ are chosen to be optimal for $L$. Therefore, it is the optimal equilibrium from $L$’s point of view.}

**Proposition 1.** If the marginal crisis-reducing effect of $s$ decreases as $s$ increases, and the unit cost of loan, $c$, is sufficiently large, then for any $\beta$, $b$, $c$ and $p(s, x; \theta)$, there exists a perfect Bayesian equilibrium of the game with the following equilibrium strategies:

(1) The game begins in a bargaining phase. If the game is in a bargaining phase, $L$ offers $G$ a bundle, $(s_b^*(\beta), x_b^*(\beta))$; if $G$ defects in any period by choosing $(d_x, d_s) = (0, 1)$, the game enters a punishment phase where $L$ offers $G$ a bundle, $(0, x_p^*(\beta))$. The punishment phase lasts until $G$ chooses to implement $x_p^*(\beta)$ in some period, in which case $L$ treats the next period as beginning a new bargaining phase. $L$’s choice of $s_b^*(\beta)$, $x_b^*(\beta)$ and $x_p^*(\beta)$ maximizes $L$’s expected lifetime utility on the equilibrium path.

(2) In any period, for all histories in which $L$ has not deviated from its strategy in (1), $G$ chooses $(d_x, d_s)$ to optimize its expected lifetime utility given $\theta$ and $\epsilon$. If $L$ has deviated, $G$ chooses $(d_x, d_s)$ to optimize its one-period expected utility, and $L$ offers $(s_o, x_o) = (0, 0)$ in all future periods.
(3) In any period, $M$ chooses $r^*(\beta)$ to clear the current period loan market after observing the outcome of the interaction between $L$ and $G$.

Because the number of choice variables makes the model computationally intensive, we present below the numerical results of the equilibrium. The parameters of the model are calibrated to match some aspects of the empirical data; however, our model produces significant new predictions. To examine the robustness of the results, we have perturbed the parameters around their benchmark values, and we find that the qualitative relationships do not change. Below we show several findings that center around the relationship between our key parameter $\beta$ and the choices made by $G$, $L$ and $M$ in equilibrium.

**Lender’s equilibrium strategy**

The lender is willing to provide a loan whose size is increasing as $\beta$ rises, until it reaches the budget constraint. To simplify the analysis, we focus our discussion on the case where the budget constraint is binding.\footnote{In the numerical results presented, we choose the quota $\bar{s} = 0.2$, which is sufficiently small compared with $\theta_2 = 1.5$, so that $L$ lends the maximum amount to a country with $\beta > 0.12$.}

Figure 2.1 shows how $\beta$ affects $x^*_b$, the level of conditionality when $G$ is in good standing, and $x^*_p$, the level of conditionality required to return to good standing when $G$ is in a punishment phase. It shows that when $s$ reaches $\bar{s}$, both $x^*_b$ and $x^*_p$ are decreasing in $\beta$. Moreover, $x^*_b$ steadily decreases while $x^*_p$ remains constant till about $\beta = 0.4$ and then decreases sharply. In other words, the lender’s incentives to provide support erode the substantive policy commitments that the government is expected to meet and make the lender more forgiving when the borrower has reneged. For a range of $\beta$, the lender holds the borrower to a higher standard after defecting ($x^*_p > x^*_b$), but for very important borrowers the standards to return to good standing are lower than the initial level of conditionality ($x^*_p < x^*_b$).

To foreshadow the results that we present in the next section, note that these two
variables have countervailing effects on the probability that a government reneges on its policy commitments after accepting a loan, $Pr(d_x = 0|d_s = 1)$. On the one hand, $x^*_b$ measures the cost of implementing the conditions tied to $L$’s loan, so less conditionality, or smaller $x^*_b$, makes it less attractive to defect. On the other hand, a smaller $x^*_p$ reduces the future cost of returning to good standing after reneging, which makes defection a more attractive option in the present for $G$.

![Figure 2.1: The Effect of Political Bias on the Equilibrium Conditionality $x_b$ and $x_p$](image)

The government’s equilibrium strategy

Figures 2.2 and 2.3 show the equilibrium choice probabilities of $G$ in the bargaining phase. Figure 2.2 shows the relationship between $\beta$ and $G$’s probability of accepting a loan, $Pr(d_x = 1)$. First, given any state of the economy, $\theta$, countries with larger $\beta$ are more likely to accept $L$’s offer. Second, given a value of $\beta$, $G$ is more likely to accept an offer if the state of the economy is unfavorable ($\theta_2$), because it has greater need for funds in order to stave off involuntary default. This logic drives the adverse selection effect: countries that accept IMF offers, on average, are worse candidates for investment than countries that reject them. We will elaborate
this point as we discuss how market actors update their beliefs on $\theta$.

Figure 2.3 plots the probability of reneging on promises to implement economic reforms after accepting a crisis loan, $Pr(d_x = 0|d_s = 1)$, against $\beta$. We can see that both probabilities first decrease and then increase in the region where $s = \bar{s}$. The decrease results from the fact that the equilibrium conditionality in the bargaining phase, $x^*_b$, decreases steadily, while the conditionality in the punishment phase, $x^*_p$, remains high initially (see Figure 2.2). Once $x^*_p$ begins to decrease, the incentive to defect increases for more influential countries because they incur smaller costs from defecting. This reflects a moral hazard problem for this set of countries because they know that they can get back to good standing with relative ease. Countries with worse economic conditions are less likely to defect: the higher probability of involuntary default reinforces their commitment.

Figure 2.2 : The Effect of Political Bias on the Probability of Accepting a Loan Agreement
Figure 2.3: The Effect of Political Bias on the Probability of Defection Conditional on Accepting a Loan Agreement

Figure 2.4 shows that the expected duration of a punishment phase, $ET(\beta)$, decreases in $\beta$. That is, more important countries spend less time in the punishment phase. This is because the equilibrium conditionality for the punishment phase, $x_p^*$, decreases as $\beta$ increases, so it is easier for higher $\beta$ countries to implement the reforms necessary to return to good standing.
Market Actors’ Equilibrium Response

Figure 2.5 shows how the market actor updates its belief about $G$’s initial economic conditions after observing that $G$ accepts a loan agreement ($d_s = 1$). Specifically, $M$ updates its beliefs to put more weight on the possibility that economic conditions are unfavorable when it observes that $G$ has accepted a loan, i.e., $Pr(\theta = \theta_2|d_s = 1) > Pr(\theta = \theta_1|d_s = 1)$, for all $\beta$. This is due to adverse selection: countries with more favorable economic fundamentals are more likely to reject any given offer. Note that when $s$ is already at $\bar{s}$, the difference in the beliefs becomes smaller as $\beta$ increases, because as a country becomes more important, the terms of $L$’s offer become more favorable; as a result, more and more good economies will accept the offer and thus $M$ can infer less about the characteristics of countries that accept an offer.

![Figure 2.5: M’s Posterior Beliefs about $\theta$ Conditional on $G$ Accepting A Loan](image)

Figure 2.6 illustrates the total effect of $\beta$ on the market equilibrium interest rates when a country accepts a loan, $r_{acc}$, which represents the net effect of the consequences of bias that operate through liquidity, conditionality, and implementation.
There are two regimes corresponding to those in Figure 2.1, separated by the vertical red line. The increase in liquidity, $s^*_b$, as a function of political bias, drives the downward slope in the first regime, while the declining level of conditionality, $x^*_b$ and $x^*_p$, drive the upward slope in the second regime. In other words, countries that are important to crisis lenders receive larger loans, and this effect reduces the interest rates that they pay to private lenders. Once this effect ceases to operate, however, the fact that important countries are asked to implement less conditionality and are less likely carry out their commitments drives up interest rates for important countries.

Figure 2.6: The Total Effect of $\beta$ on Equilibrium Interest Rates

The Effect of Crisis Lending

Our model does not provide a straightforward answer to the question of whether crisis lending increases or decreases equilibrium interest rates on average, because there are several mechanisms operating that have countervailing effects. The net effect depends on the parameters of the model. To illustrate this, Figure 2.7 plots the equilibrium interest rate for countries that accept and reject crisis lending packages as a function of the ex ante uncertainty about the state of the economy, $\text{Var}(\theta)$, while
holding our other parameters constant. Intuitively, lending can only be harmful in our model because of the adverse selection effect; liquidity is unambiguously beneficial, and conditionality is only helpful to the extent that it is implemented, but cannot increase interest rates. The magnitude of the adverse selection effect depends on how much difference there is between the favorable and unfavorable economic states of the world. If there is not much uncertainty, or these states are fairly similar, the net effect of lending is to reduce interest rates. As uncertainty increases, the fact that market agents draw unfavorable inferences when they observe crisis lending becomes increasingly important, and the net effect of lending becomes to increase interest rates.

![Figure 2.7: The Effect of the Uncertainty of $\vartheta$ on the Equilibrium Interest Rates](image)

Given these theoretical findings, our empirical analysis focuses on disaggregating and testing the several mechanisms that we have identified by which crisis lending effects interest rates, rather than seeking to identify an average effect of lending per se.
2.5 Empirical Implications

In this section we investigate the empirical implications of our theoretical results. Our theoretical model focuses on the effects of crisis lending on short-term borrowing costs, so we focus empirically on sovereign bond yields. Our model leads to hypotheses about the political determinants of loan size and conditions, and the effects of loan size, conditionality and lender bias on equilibrium interest rates. We distinguish between three categories of effects: bargaining, adverse selection, and moral hazard.

Our first set of hypotheses deals with the effect of bargaining on loan terms. The lender’s political bias on behalf of the borrower has a monotonically increasing effect on equilibrium loan size, and for most of the parameter space has a decreasing effect on loan conditionality.

$H1$ (Bargaining and Loan Terms): Political bias is associated with lower conditionality and higher liquidity.

Loan terms, in turn, influence the market interest rate, because they have direct effects on the probability of a financial crisis. Increased liquidity—an expanded crisis loan—decreases the probability of a crisis, all else equal, so it decreases the interest premium due to political risk. Similarly, increased conditionality reduces the probability of a crisis, because implementing economic reforms reduces the size of the financing gap that must be filled by the private sector. Consequently, conditionality reduces the equilibrium market interest rate. Our model predicts that more conditionality and larger loans will depress interest rates.

$H2$ (Loan size and interest rates): Larger crisis loans are associated with lower interest rates.

$H3$ (Conditionality and interest rates): Higher conditionality is associated with lower interest rates.

Crisis lending is not randomly distributed, so the class of countries that negotiate
crisis lending deals differs systematically from the population of non-participants (Vreeland 2003). In our model, governments have private information about the health of their economies, which influences the return to participation in a crisis bailout. Specifically, governments that face weaker economic fundamentals have stronger incentives to accept a loan, so countries that accept lending packages reveal themselves to be in worse circumstances than the average country without a loan, and thus, on average, more prone to subsequent crisis. Therefore, after controlling for the effects of loan size and conditionality, we expect the onset of a loan to generate increased interest rates.

\textbf{H4 (Adverse selection): Crisis lending announcements are associated with higher interest rates, controlling for conditionality and amount of financing.}

Our model assumes that it is more costly for lenders to deny support to politically influential borrowers, which reduces the sanctions that influential borrowers face when they renege on their commitments to implement reforms. This can be expressed in the expected duration of the punishment phase, which is lower for influential countries, as depicted in Figure 2.4. This occurs because, as Figure 2.1 illustrates, the policy reforms that are required to bring their programs back on track are less arduous. As a result, the expected cost of punishment is lower for influential countries, which implies that their incentives to comply with conditionality are weaker, as illustrated by Figure 2.3.\textsuperscript{15} Influential countries implement a lower proportion of their policy commitments, so they are more subject to financial crises, and consequently they pay higher risk premia. In our model, this effect is present only for countries participating in conditional lending programs, because we model capital markets as fully liquid. However, if the model were extended to allow for

\textsuperscript{15}Figure 2.3 shows a non-monotonic effect, because more important countries receive larger loans on less onerous terms, but also face less enforcement. Controlling for liquidity and conditionality, however, the model predicts less compliance as countries become more important.
long-term lending, this effect would be present for influential countries regardless
of whether they were currently participating in a conditional lending program. In
either case, however, the effect should be strongest for countries that are currently
participating, because future participation is discounted and uncertain.

*H5 (Moral hazard): Political bias exerts an upward pressure on equilibrium in-
terest rates, and this effect is strongest for countries that are crisis borrowers.*

### 2.6 Research Design

To capitalize on data availability and a comparable set of cases of lending, we focus
on crisis lending by the International Monetary Fund, using data drawn from the
IMF’s Monitoring of Agreements Database (MONA). The data span the period from
1992 to 2002 and cover the 66 countries that were not members of the OECD and
for which data on bond yields were available from IFS. Our dependent variable is
the nominal yield of short-term sovereign bonds issued in home country currency
and measured at the end of each month.¹⁶ Our quantities of interest are the effect
of new program announcements, conditional on measures of U.S. influence over the
IMF, the effects of those measures of influence when a new program is announced,
and the effects of conditionality and loan size. We treat conditionality and loan size
as endogenous.¹⁷ We use a dummy variable for a month in which a new program is

---

¹⁶Our empirical analysis thus speaks to the short-term (within one month) reaction of financial
market actors, as opposed to longer term flows like FDI, or lagged effects of lending on investment
inflows. While we recognize that these are also important measures of the market catalysis, we
focus here on short term perceptions that might influence future crisis dynamics, as the change in
interest rates for a country can either substantially ease or exacerbate economic crisis.

¹⁷To our knowledge, this is the first empirical analysis to control for both loan size and number
of conditions and treat loan terms as endogenous in a study of the effects of loan programs on
endogeneity and conditionality, but both study economic growth rather than bond yields.
announced to capture the short-term effects of new program announcements. Any information contained in the lending decision should be reflected in this short-term effect. To capture conditional effects, we regress interest rates on the new program dummy, influence variables $x$ new program, influence variables $x$ no new program, conditionality and loan size, and controls.¹⁸

We do not correct for selection bias, for several reasons. Corrections for selection bias, as for example those produced by Heckman-type selection models, attempt to estimate the hypothetical effect that a treatment might have if it were applied randomly. This is not the question that we want to ask, because we are interested in estimating the selection effect. Instead, after controlling for the liquidity and conditionality channels through which IMF programs exercise their treatment effects, we interpret the estimated coefficient on IMF program initiation as due to selection bias. To be precise, our theoretical model predicts adverse selection—worse candidates for programs are those that choose to participate—so we anticipate that program initiation will be associated with increased interest rates, after controlling for financing and policy conditionality. The coefficient, although biased from the point of view of a treatment effects model, is exactly the theoretical quantity that we want to estimate.

The reader might be concerned, however, that failing to control for selection leaves the error term correlated with our regressors, which biases the estimates on our other coefficients. This is a question about whether the assumptions of our empirical model correspond to the assumptions of our theoretical model. The theoretical model presented above assumes that the only private information relevant to the game belongs to the government. The short-term interest rate is determined

¹⁸This is mathematically equivalent to the usual practice of including the dummy variable, the interaction term and the uninteracted covariate in the regression, but allows us to report our quantities of interest and the hypothesis tests appropriate for them in Table 2.1.
by market actors after the government decides to accept a loan and before it reveals whether it will implement conditionality, so it can only depend on public information and any private information held by market actors. The terms of the loan offer are set by the lender, so they are not correlated with the government’s private information. The lender’s bias is assumed to be public knowledge, and is operationalized below in terms of publicly observable variables, so it cannot be correlated with the government’s private information. Similarly, our control variables measure publicly observable information. Consequently, none of our estimates will be biased as long as the market actors’ private information is uncorrelated with the government’s private information, and our theoretical model assumes that this is the case.\footnote{In Heckman’s model, there is a single player with private information, so that both her decision and the error term are correlated with her private information. In contrast, in our model, the private information determining program participation and the second stage error term belong to two different players, so the independence assumption is much weaker. Our theoretical model could be enriched to allow for insider trading, which would violate the assumption of uncorrelated private information, but in order for the effect to be substantively important it would have to affect the equilibrium interest rate, which implies that the information is no longer private. In any case, we think the assumption of uncorrelated private information is relatively innocuous in our application because the effects of any bias would have to be quite large to overturn the results we report below, and our results rely on models with country fixed effects that capture most of the likely sources of endogeneity. Furthermore, Heckman-type models are inefficient, and our theoretical model rules out all of the candidates for instruments for program participation.}

Measures of U.S. Influence

Our theory does not provide guidance about what particular interests motivate the United States to interfere in IMF program design, so we take an eclectic approach and allow for a range of variables to exert effects that reflect alternative interests. Following extant studies (e.g. Thacker 1999, Stone 2004, Oakley and Yackee 2006), we operationalize U.S. interests in terms of similarity of the borrowing country with
the United States in alliance portfolios and UN General Assembly voting patterns, and U.S. bank exposure in recipient countries (Copelovitch 2010, Stone 2011). We also tested for effects of U.S. foreign aid and U.S. exports, but those variables did not yield significant results, so they are not included in the specifications that we report below.

**Instrumental Variables**

The model indicates that IMF conditionality, loan size, and market responses are endogenous to U.S. interests, so we adopt an instrumental variables approach. The validity of instrumental-variables analysis depends on the strength and exogeneity of the instruments. We use the following instrumental variables, which are correlated with loan size and conditionality, but are not strongly correlated with bond yields. We have argued that conditionality and loan size are endogenous to our measures of U.S. influence; our theoretical model predicts that countries with a high ”β” are more likely to receive loans, and those loans are, on average, larger and have fewer conditions. Therefore, the exogeneity of instruments is important with respect to our measures of U.S. influence. These instruments can be thought of as non-political determinants of IMF program terms, theoretically and empirically separate from the geopolitical concerns in which we are interested. The instruments collectively pass the Sargan test of overidentifying restrictions.²⁰

²⁰Empirically, the highest correlation between our instrumental variables and treasury bill rates is $\rho = .17$ for the case of total outstanding commitments, followed by $\rho = .14$ for countries with extended IMF program commitments. These instruments are not highly correlated with our measures of U.S. influence, which theoretically drive loan size and conditionality. The highest correlation between affinity scores and any instrument is $\rho = .15$ for number of countries participating, which is perhaps the least likely of our instruments to have a causal association with a particular country’s affinity score with the U.S. U.S. commercial bank exposure is also not strongly correlated with any of our instruments ($\rho < .02$), with the exception of its moderate correlation of $\rho = .15$ with the
Number of countries participating: Przeworski and Vreeland (2000, 2001) and Vreeland (2003) argue that the IMF becomes reluctant to lend when its resources are stretched thin because of the need to hold something in reserve for future crises. This might lead the Fund to make smaller loans or extract more extensive conditionality in return for scarce funds. Alternatively, the number of countries participating in IMF programs might be an index for systemic vulnerabilities that magnify the risks of contagion. This could lead the IMF to offer more generous lending terms, including larger loans and more limited conditionality. We find that the number of participants increases IMF willingness to extend credit and reduces conditionality.

Ratio of Prior commitments of IMF financing to IMF Quota: The IMF has formal rules about access to credit measured in terms of multiples of a country’s contributed quota. These rules can be waived, but the Executive Board is reluctant to extend credit substantially beyond previous precedents, and such decisions lead to extended discussions. To the extent that quotas represent constraints on IMF lending, previous commitments reduce the amount of credit available, and should reduce the size of new lending arrangements. Alternatively, the defensive lending hypothesis holds that countries that owe substantial amounts to the IMF may more easily qualify for additional credits because the Fund seeks to prevent any of its debtors from going into default. We find support for the hypothesis that prior commitments constrain new credits, and not for defensive lending.

Extended program: This is a dummy variable that codes arrangements that are designed to be disbursed over more than one year, including the EFF, ESAF, ratio of prior IMF commitments to IMF quota. Alliance similarity with the U.S. is not strongly correlated with any instrument. The results that follow are robust to the one-by-one exclusion of each instrument (the results are in the on-line appendix).
and PRGF. Such programs are typically intended to follow successful Stand-By arrangements and deepen structural reforms, so they typically involve more extensive conditionality and larger commitments of financing.

The inclusion of these instruments in our instrumental variables regressions below consistently yields first-stage F statistics of over 119 and 611 for our equations predicting loan size and conditionality, which is well over the of 10 suggested by Staiger and Stock (1997), and these variables are statistically significant predictors of loan size and number of conditions.\textsuperscript{21}

**Control Variables**

We use a common set of controls, including our three measures of U.S. influence interacted with a dummy for months in which there is no new program announcement (alliance patterns, UN voting patterns, and U.S. bank exposure). We control for economic variables that are correlated with interest rates and the terms of crisis lending (foreign debt, GDP per capita, reserves as a share of GDP, population). In addition, we control for missing data, which is a measure derived from a principal components analysis of the missingness of 19 time series reported by member countries to the IMF. Countries that fail to report these data are likely to have low administrative capacity, and this is associated with higher conditionality and higher interest rates. IMF standing is a measure of past non-performance of conditionality, which is derived from a 12-month moving average of a dummy variable that measures whether a country has an IMF program that is suspended for non-performance. Past non-performance is associated with additional conditionality and higher interest rates.\textsuperscript{21}

\textsuperscript{21}Neither number of countries currently under IMF programs nor extended commitments is a strong predictor of loan size, though both are strong predictors of number of conditions.
Results

The results of three models are presented in Table 2.1 below. The first model uses OLS to provide a baseline for comparison, and the second and third use instrumental variables (2SLS) to model the endogeneity of conditionality and the size of IMF lending facilities predicted by our model. The second model allows for cross-sectional and time series variation, and the third uses country fixed effects to focus on over-time variation within countries. It is important to control for fixed effects for several reasons in this particular analysis, but perhaps most importantly this prevents country heterogeneity in the size of bond markets across countries from biasing the results.\textsuperscript{22} The results are broadly consistent across the three models, but there are important differences that we highlight below. The coefficient of IMF program initiation is statistically insignificant in the first two models when the three U.S. influence variables take a value of zero, but is significantly associated with higher interest rates in the fixed-effects specification. As we will see below when we interpret the conditional effects, however, IMF program initiation is statistically significant in all three models across most of the range of the U.S. influence variables. Note that this variable measures the short-term effect of initiating a new IMF program, which is our theoretical quantity of interest, not the steady-state effect of having an IMF program.

In the 2SLS estimates we focus on the second stage estimates that predict in-

\textsuperscript{22}A related argument is that perhaps bond supply rationing drives bond yields (e.g. Stiglitz and Blinder 1983). We are confident that our results are not driven by supply rationing for several reasons. First, cross-national heterogeneity in supply strategies will largely be controlled for by country fixed effects. Second, within country bond rationing is unlikely to occur during economic crisis when governments want to prevent skyrocketing interest rates. Third, there is not a compelling reason to think that supply rationing would be systematically linked to our right hand side variables, and as such the omission of a bond supply control is unlikely to generate bias in coefficient estimates (although it may reduce the model’s overall explanatory power).
Table 2.1: Effect of IMF Program Initiation and U.S. Influence on Bond Yields

<table>
<thead>
<tr>
<th></th>
<th>OLS Coefficient (std. error)</th>
<th>p-value</th>
<th>2SLS Coefficient (std. error)</th>
<th>p-value</th>
<th>2SLS (Fixed Effects) Coefficient (std. error)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMF Program</td>
<td>0.94 (5.80)</td>
<td>0.87</td>
<td>15.82 (6.10)</td>
<td>0.00</td>
<td>14.97 (6.06)</td>
<td>0.00</td>
</tr>
<tr>
<td>IMF Credit</td>
<td>1.06 (0.72)</td>
<td>0.14</td>
<td>-0.03 (0.01)</td>
<td>0.00</td>
<td>-0.03 (0.01)</td>
<td>0.00</td>
</tr>
<tr>
<td>Conditions</td>
<td>0.68 (0.07)</td>
<td>0.00</td>
<td>-1.02 (0.13)</td>
<td>0.01</td>
<td>-1.20 (0.13)</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>New IMF Program</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alliance portfolio</td>
<td>5.17 (12.73)</td>
<td>0.69</td>
<td>9.25 (13.48)</td>
<td>0.493</td>
<td>43.88 (14.11)</td>
<td>0.00</td>
</tr>
<tr>
<td>UN Voting</td>
<td>35.22 (6.01)</td>
<td>0.00</td>
<td>16.85 (6.16)</td>
<td>0.01</td>
<td>15.40 (6.14)</td>
<td>0.01</td>
</tr>
<tr>
<td>U.S. Bank Exposure</td>
<td>-97.51 (21.43)</td>
<td>0.35</td>
<td>1907.328</td>
<td>0.00</td>
<td>1931.70</td>
<td>0.00</td>
</tr>
<tr>
<td>Exposure</td>
<td>(104.86)</td>
<td>(473.32)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No New Program</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alliance portfolio</td>
<td>-15.86 (1.15)</td>
<td>0.00</td>
<td>9.73 (4.16)</td>
<td>0.02</td>
<td>42.65 (5.90)</td>
<td>0.00</td>
</tr>
<tr>
<td>UN Voting</td>
<td>5.27 (0.67)</td>
<td>0.00</td>
<td>6.30 (0.96)</td>
<td>0.00</td>
<td>5.25 (0.98)</td>
<td>0.00</td>
</tr>
<tr>
<td>U.S. Bank Exposure</td>
<td>7.71 (14.09)</td>
<td>0.59</td>
<td>38.94 (24.43)</td>
<td>0.11</td>
<td>85.47 (28.34)</td>
<td>0.00</td>
</tr>
<tr>
<td>Control Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population</td>
<td>-0.13 (0.01)</td>
<td>0.00</td>
<td>-0.004 (0.04)</td>
<td>0.918</td>
<td>0.36 (0.10)</td>
<td>0.00</td>
</tr>
<tr>
<td>Foreign Debt</td>
<td>0.33 (0.02)</td>
<td>0.00</td>
<td>0.179 (0.065)</td>
<td>0.01</td>
<td>0.36 (0.09)</td>
<td>0.00</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>-0.61 (0.06)</td>
<td>0.00</td>
<td>-0.10 (0.02)</td>
<td>0.00</td>
<td>-.113 (0.03)</td>
<td>0.00</td>
</tr>
<tr>
<td>Reserves/GDP</td>
<td>-24.32 (1.89)</td>
<td>0.00</td>
<td>-40.0 (3.70)</td>
<td>0.00</td>
<td>-47.39 (3.92)</td>
<td>0.00</td>
</tr>
<tr>
<td>Missing Data</td>
<td>8.27 (1.57)</td>
<td>0.00</td>
<td>9.07 (1.84)</td>
<td>0.00</td>
<td>8.43 (1.86)</td>
<td>0.00</td>
</tr>
<tr>
<td>IMF Standing</td>
<td>2.63 (0.90)</td>
<td>0.00</td>
<td>-2.92 (1.24)</td>
<td>0.02</td>
<td>-3.39 (1.24)</td>
<td>0.01</td>
</tr>
<tr>
<td>Constant</td>
<td>26.93 (0.74)</td>
<td>0.00</td>
<td>22.10 (2.41)</td>
<td>0.00</td>
<td>-1.64 (3.50)</td>
<td>0.638</td>
</tr>
<tr>
<td>Fixed Effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F test</td>
<td></td>
<td></td>
<td>42.53</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of obs</td>
<td>8,373</td>
<td></td>
<td>8,373</td>
<td>8,373</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rho</td>
<td></td>
<td></td>
<td>0.66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
terest rates, though the first-stage estimates (Presented in Table 2.3) confirm our theoretical expectation that political importance increases loan size but depresses the number of conditions attached to a loan. IMF credit is measured as the monthly change in aggregate IMF commitments in the month in which a new program is introduced, so it represents a short-term effect. The effect is substantively and statistically insignificant in the baseline OLS model. In the second model, which treats the loan amount as endogenous, however, IMF credit is highly significant, and is estimated to reduce interest rates on average by 16 percentage points. Comparing models 2 and 3 makes it clear that most of this effect is due to cross-sectional variation across countries rather than to within-country variation over time. The coefficient remains highly significant in the fixed-effects specification, but the size of the estimated effect drops to an average of 6.5 percentage points. Nevertheless, the results indicate that even in this conservative specification, particular countries experience greater gains in investor confidence, all else equal, when they receive larger infusions of IMF credit, and the effects can be substantial.

The estimated effect of conditionality on bond yields differs across the three models, but does so in a way that makes us confident in our interpretation of the results. The OLS estimate indicates that conditionality, contrary to theory, increases bond yields. However, when we model the endogeneity of conditionality—countries that have failed to implement conditionality in the past are subject to more conditionality in the present, for example—this effect is reduced by one-third. When we further control for fixed effects that capture a wide range of country-level variables that affect both conditionality and credit-worthiness, the result is reversed. Focusing on the 2SLS results with fixed effects, it is clear that when a particular country is subject to more conditionality, its interest rates are lower, as predicted. The results indicate that conditionality has a substantial depressing effect on bond yields. Conditionality is measured as a count of types of conditions contained in a particular
program review, ranging from 0 to 19 and averaging almost 6, so conditionality is estimated to depress bond yields under IMF programs by 4.6 percentage points on average. A one standard-deviation increase in conditionality, or 3.6 more conditions, is sufficient to depress interest rates by another 2.7 percentage points.

The results for our three measures of U.S. influence generally support the model’s prediction that bias increases bond yields. The results strengthen when we control for endogeneity and become uniformly significant across measures of influence when we also control for country fixed effects. The similarity in alliance portfolios has a consistently positive coefficient, but is only significant when we control for fixed effects. This suggests that the variation in alliance commitments that is important is taking place within countries over time, for example, as East European countries dropped out of the Warsaw Pact and joined NATO. In the fixed-effects specification, increasing alliance similarity with the United States by one standard deviation is estimated to increase interest rates by 8.4 percent in the month of a new IMF program announcement. To put this result in context, the alliance similarity between the United States and Poland increased by 65 percent of one standard deviation in this sample between 1990 and 2000. UN voting similarity also has consistently positive coefficients, which are significant in the OLS, 2SLS, and 2SLS with fixed effects specifications. The estimated marginal effect of increasing voting similarity with the United States by one standard deviation is to increase interest rates by 11.2 percent in the month of a program announcement. Comparing the results with and without fixed effects, it appears that almost 60 percent of this effect is due to over-time variation in voting similarity within countries.

The exposure of U.S. banks to particular countries tells a similar story. The OLS coefficient is negative. However, modeling the endogeneity of conditionality and IMF credit reverses the effect, and shows that countries that are important to U.S. banks pay much higher interest rates when they receive new IMF programs.
Examining the results of the reduced-form equations makes clear why endogeneity plays an important role in the interpretation of these effects. (See Table 2.3.) The exposure of U.S. banks plays a major role in explaining the size of IMF loans to particular countries, and IMF credit in turn reduces interest rates. When we take into account this important indirect effect of bank exposure that operates through IMF credit, we find that the direct effect of U.S. bank exposure (which our model attributes to the moral hazard effect) is to substantially increase interest rates. On average, this effect increases interest rates by 12.9 percent, which negates about 80 percent of the average effect of increasing IMF credit. Increasing the exposure of U.S. banks by one standard deviation increases interest rates by an estimated 26.4 percent. Comparing the results with and without fixed effects indicates that 56 percent of this effect is attributable to over-time variation in U.S. bank exposure to particular countries rather than to cross-national variation.

Our theory does not make predictions about the effect of U.S. influence variables when no new program has been announced, because the model includes the simplifying assumption that interest rates are recontracted every period. We include these variables as controls, in order to isolate the short-term effects of the same variables during new program initiation months. However, we conjecture that a richer model that allowed bonds to have longer maturities would generate expectations for these variables that are similar to the ones our model generates during announcement months, but that the effects should be smaller. Our intuition is that variations in IMF credibility should affect borrowers with on-going programs and non-borrowers as well as new borrowers, because resorting to IMF financing is always part of the game tree for developing countries and emerging markets. The effects should be smaller, however, because future program participation would be uncertain and discounted for current non-participants. Four of the six hypothesis tests that we perform with models that account for endogeneity support this hypothesis. Simi-
larity of alliance portfolios with the United States has essentially the same effect when there is no new program as when there is a new program announcement in the two-stage least squares specification with fixed effects. Similarity of UN voting records has significant effects that raise bond yields, although the effects when a country does not have a new program announcement are only 20 to 30 percent as large as when a new program is announced. Bank exposure has statistically significant effects that are five percent as large when there is no new program as when there is a program announcement. These results broadly support our conjecture.

Our control variables have the expected effects. Foreign debt increases bond yields, richer countries pay lower interest rates, central bank reserves lower interest rates, missing data increases interest rates, and poor standing with the IMF increases interest rates.

Because the interpretation of interaction effects is not straightforward, Table 2.2 presents the conditional effects of announcing a new IMF program with U.S. influence measures fixed at their means and at one standard deviation above their means. The effect of initiating a new IMF program is highly significant in the 2SLS equations when all three U.S. influence measures are fixed at their mean, and extracts a risk premium of 26.65 percentage points (the 95% confidence interval of the effect runs from 17.23 to 34.08 percentage points). The effects are even stronger in the fixed effects specification. The effects become stronger when the U.S. influence measures are increased. Increasing alliance similarity with the United States is associated with only a slight change in interest rates, but the effect of a new program is approximately 14 percent greater in countries that vote in alignment with the United States in the UN to a degree that puts them one standard deviation above the mean.

The most dramatic estimated effects of U.S. influence occur in countries that

---

23The difference in coefficients is not statistically significant.
Table 2.2: Conditional Effects of New IMF Program Announcements

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>2SLS</th>
<th>2SLS (Fixed Effects)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient (std. error)</td>
<td>Coefficient (std. error)</td>
<td>Coefficient (std. error)</td>
</tr>
<tr>
<td>All variables at their means</td>
<td>-0.02 (1.87)</td>
<td>25.65 (4.30)</td>
<td>41.05 (4.72)</td>
</tr>
<tr>
<td>Alliance S-score 1 std. dev. above mean</td>
<td>0.96 (3.25)</td>
<td>27.41 (5.37)</td>
<td>49.39 (6.05)</td>
</tr>
<tr>
<td>UN Voting S-score 1 std. dev. above mean</td>
<td>12.98 (2.94)</td>
<td>31.88 (4.62)</td>
<td>46.73 (5.00)</td>
</tr>
<tr>
<td>US Bank exposure 1 std. dev. above mean</td>
<td>-1.41 (2.37)</td>
<td>52.83 (10.40)</td>
<td>68.57 (10.54)</td>
</tr>
</tbody>
</table>

are important to U.S. banks. Controlling for endogeneity and allowing for cross-sectional effects, equation 2 finds that the effect of a new program announcement is to increase bond yields by nearly 53 percentage points in countries that are one-standard-deviation more important to U.S. banks than average. One standard deviation is a bit under 2 percent of total U.S. foreign bank assets, so it is not near the high water mark set by Mexico in 1995 of 18 percent. This is approximately the level reached by Colombia in the early 1990s, and Greece, the Philippines, South Korea, South Africa and Venezuela in the late 1990s. This effect is weaker in the model with fixed effects, indicating that a substantial portion of the effect is due to cross-country comparisons. However, 55 percent of the effect is attributable to over-time variation within particular countries.

What is the total effect of political influence on bond yields for IMF program participants? In other words, what is the cumulative effect of our measures of U.S. influence on bond yields, both operating directly and indirectly through IMF credit and conditionality? Table 2.3 displays the results of model 3, but now with first stage
estimates reported. By adding the coefficients of U.S. influence across the stages, we can estimate the aggregate, or net effect as it operates through increasing loan size, decreasing conditionality, and the direct moral hazard and adverse selection effects.

Consider the loan the IMF extended to Russia to counter a crisis of confidence in the sovereign bond market in July 1998. At the time, Russia’s alliance profile and UN voting profiles vis-a-vis the United States were close to their average levels, so they are estimated to have had no substantial effects on the terms of the loan, but Russia’s share of U.S. bank lending had risen over the previous two years to almost five percent of total foreign assets (approximately 3% above the mean for Russia in this sample). Under U.S. pressure, the IMF scrambled to assemble its largest loan to Russia, activating its General Arrangements to Borrow in order to secure the necessary resources. This in turn required U.S. Congressional action, prompting Treasury Secretary Robert Rubin to write to House Speaker Newt Gingrich, "Our interest in successful political and economic reform in Russia is compelling. A collapse of the ruble would undoubtedly strengthen Russian opponents of reform, who include ultra-nationalists and Communists."  

According to model 3, the scale of U.S. bank exposure is estimated to have boosted the size of the IMF loan to Russia by 1.65 billion SDRs, or approximately 26 percent of the 6.3 billion SDRs that the IMF committed. The large size portion of the loan attributed to U.S. bank exposure, in turn, is estimated to have depressed bond yields by 54 points. On the other hand, the large scale of U.S. bank exposure is estimated to have had a direct effect of raising Russian bond yields by 58 percentage points, which is attributable to moral hazard. In addition, program initiation is estimated to have raised the premium on Russian bonds by another 15 percentage points, which is attributable to adverse selection. The net effect of political influence, as measured through U.S.

---


25 The 17.1 billion dollar headline figure announced at the time included loans from the World Bank and Japan.
bank exposure, is thus estimated to be 19 percentage points.

Capital markets initially reacted to the loan announcement with some optimism, and Russian bond yields declined in anticipation of the loan package announcement. Yet shortly after the announcement, bond yields began to rise to crisis levels, reaching 75 percent by early August—25 points above the Russian average treasury bill rate for the sample—and soared to 150 percent by the middle of August as it became clear that the Russian government was considering default (Sturzenegger and Zettelmeyer 2006, 98). Amid increasing market panic, Russia defaulted on some obligations, suspended inter-bank payments and devalued the ruble in late August. The dynamics driving investor expectations during the crisis were complex, but our theoretical model suggests that the terms of the bailout may have signaled that first, the extent of the Russian crisis was larger than anticipated, and second, the importance of the Russian economy to IMF principals was such that it could acquire bailout funding without implementing the longer-term structural reforms necessary to return to fiscal solvency. Indeed, although the July program included a far-reaching set of reforms intended to restore fiscal solvency, signals began to leak out within days of signing the accord that the Russian government did not seriously intend to implement them. As Blustein puts it, “during the 1990s, the Russians had usually heard ‘yes’ when it came to seeking aid from the IMF, to the point that the mantra ‘too big and too nuclear to fail’ pervaded attitudes of many market participants about the country” (2001, 238). Russia’s geopolitical and economic importance created a perception that it would continue to receive IMF funding, making the ultimate decision of the IMF to allow default a surprise for many. At the same time, however, perceptions of geopolitical importance created concerns about the underlying state of the Russian economy and fears about future crises. These concerns created a self-fulfilling prophecy as the combination of rising bond yields, capital flight, and bank runs drove the economy into collapse. Blustein
concludes that “it is reasonable to wonder whether Russia was set up for the colossal letdown of 1998 because it had been told ‘yes’ too many times in the past” (2001, 239).
In summary, we find several pieces of evidence that support our model. We find that conditionality decreases and the scale of financing increases with some of
our measures of IMF influence, as hypothesized. We also find that conditionality and liquidity exert strong depressing effects on bond yields. We find robust direct effects of measures of U.S. influence—alliances, UN voting patterns, and U.S. bank exposure—on the yields of sovereign bonds, which are consistent with the moral hazard hypothesis that countries that enjoy privileged access to U.S. decision makers pay additional risk premia. We find that the initiation of new IMF programs is associated with an increase in the risk premium, controlling for conditionality and loan size, and that the risk premium increases more sharply in the presence of U.S. influence. These results hold in models that treat conditionality and loan size as endogenous variables, as the theory specifies is appropriate, and are robust in a model with fixed effects.

2.7 Conclusion

We have investigated the theoretical and empirical linkages between IMF programs, loan size, conditionality, informal political influence and the responses of international financial markets. Our formal model extends the model in Stone (2002) in three important ways. It incorporates bargaining over the terms of IMF programs, it allows the government to have an information advantage about the state of its finances, and it treats the probability of a crisis and the equilibrium interest rate as mutually endogenous. This richer model allows us to make predictions about the relationship between IMF bias—which we interpret as pressure to lend to countries that are important to the IMF’s leading shareholder, the United States—and the scale of financing, the associated conditionality, the enforcement of conditionality, the probability of a financial crisis, and equilibrium exchange rates. The model’s results suggest that IMF lending can catalyze private capital flows under some circumstances, but that catalytic effects are highly contextual. This may account for the fact that empirical studies of catalytic effects have reached every possible con-
clusion. The results indicate that the informal influence that biases IMF lending has similarly complex and contradictory effects on capital markets: influence decreases conditionality but increases liquidity, exacerbates moral hazard problems but ameliorates adverse selection. The net effect on capital flows depends on the various elasticities involved, so it is theoretically ambiguous. However, the mechanisms posited in the model are amenable to empirical testing, and the empirical findings support them unambiguously. Our empirical results indicate that the net effect of IMF lending depends on the borrower’s access to U.S. influence, and that the greater the access, the worse the consequences.

Our empirical results can be read as qualified support for the practice of conditional lending, since we find that increasing the scope of conditionality reduces the yield on government bonds. This indicates that market actors believe that the reforms promoted by the IMF improve the probability that they will be repaid. Since the success of the Fund at managing financial crises and limiting international contagion depends upon the perception that its programs are successful, this suggests that rather than implementing plans to streamline conditionality, it might better serve its purposes by expanding it. In addition, we find that larger IMF loans are more effective at stemming capital flight than smaller ones, all else equal. On the other hand, we find evidence that the net effect of announcing a new program, controlling for the effects of liquidity and conditionality, is to raise the cost of borrowing. This indicates that program announcements do not serve as seals of approval, but rather reveal that the government’s financial situation is insecure. Furthermore, we find that the negative effect of announcing a program on market confidence increases when the borrowing country is important to U.S. foreign policy. This is consistent with the finding of our model that enforcement of conditionality is less rigorous for influential borrowers, which consequently are less likely to implement conditionality, and more likely to suffer a financial crisis. This interpretation, furthermore, is
consistent with the finding that measures of U.S. interest in potential borrowing
countries are directly associated with higher bond yields, and that these effects are
greatest when a new program is announced. Borrowing from the IMF is always a
potential future strategy, so the IMF’s lack of credibility with influential countries
affects their policies when they do not have active IMF programs, as well as when
they do. The effect is strongest, however, when a new program is announced.

In combination, the theoretical model and the empirical estimates provide a
clear picture of the effects of informal influence on capital markets. When bor-
rowing countries are able to draw on U.S. influence, conditionality is reduced but
liquidity is increased. When informal influence is at its peak and uncertainty about
economic fundamentals is substantial, the announcement of a new IMF program
leads to capital flight. This analysis therefore provides an example of the broader
trade-off involved in governance arrangements that allow powerful countries to ex-
ert informal influence in exchange for “buy-in” to multilateral institutions. Such
arrangements exacerbate the time consistency problems that powerful states face,
frequently leading to unintended policy outcomes. In this case, it is precisely the
countries that the United States most wants to help to avoid financial crises that
are able to derive the least benefit from IMF involvement.
Chapter 3

Robust Dynamic Optimal Taxation and Environmental Externalities

3.1 Introduction

We study optimal taxation in a dynamic stochastic general equilibrium model where agents are concerned about model uncertainty. We assume that an externality through global temperature changes resulting from greenhouse gas emissions (GHG) adversely affects the economy’s capital stock and, thus, output. Its precise effects, however, are subject to uncertainty. In order to model the effect of the emissions created by economic activity on the environment, we employ the framework in Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2013). 1 While they assume that the mapping from climate change to damages is subject to risk, in our model this mapping is subject to Knightian uncertainty. We study the implications of this assumption using a robust control approach. We believe that this is an appropriate application of uncertainty in economic modeling. After all, man-made climate change is unprecedented, and there is an ongoing heated debate about its potential effects. Our approach can perhaps be thought of as a first step towards addressing the critique that economic models consistently under-assess risk (Stern, 2013). While our model does not include the risks of large-scale human migration or conflict resulting from climate change, it proposes a robust control approach as an alternative to standard probability distribution-based modeling. More specifically, concerned about

1 Acemoglu, Aghion, Bursztyn, and Hemous (2012) study related issues. See Nordhaus and Boyer (2000) and Stern (2007) for earlier work that also points to the importance of uncertainty.
model uncertainty, a social planner in our model maximizes social welfare under a “worst-case scenario.”

In addition to taking into consideration model-uncertainty, there are two other differences between our assumptions and those in GHKT. First, we find it convenient to assume that the environmental externality affects output indirectly, through the capital stock. As a result, the theoretical analysis in our model brings different results, although the two assumptions lead to identical results if we assume 100% capital depreciation (as we do in the computational part). A second difference is that we use estimates about total fossil fuel supplies that are significantly larger than theirs. This is partly due to adding the supply of unconventional oil and gas, but mainly due to considering estimated methane hydrate resources.\footnote{See Boswell and Collett (2011), Hartley, Medlock, Temzelides, and Zhang (2012), and references therein for a more detailed discussion on total estimated fossil fuel resources.}

Under plausible assumptions, we obtain a sharp analytical solution for the implied pollution externality, and we characterize dynamic optimal taxation. A small increase in the concern about model uncertainty can cause a significant drop in optimal energy extraction. The optimal tax, which restores the social optimal allocation, is Pigouvian. Under more general assumptions, we develop a simple recursive method that allows us to solve the model computationally. We find that the introduction of uncertainty matters in the sense that our model produces results that are qualitatively different, for example, in terms of oil consumption, from GHKT. At the same time, we find that concerns about uncertainty do not affect renewable energy adoption. The reason is that the margin that determines short-term decisions regarding energy sources is driven by two factors: the trade-off between higher versus lower total energy consumption, and the choice of coal versus gas/oil, rather than by renewable energy use. We find that oil-use in our model can be flat for some parametrizations. We study optimal output growth in the presence and in the
absence of concerns about uncertainty and find that the results can be very different.
In the worst case scenario, optimality implies that a small sacrifice in yearly output
can prevent a large future welfare loss.

Since the green energy sector does not create emissions in our model, we find
that the optimal path for the use of green energy does not directly depend on the
level of concern about model uncertainty. However, since green energy, coal, and
oil are substitutes, model uncertainty does affect the use of green energy indirectly,
through its impact on coal and oil. We also find that an increase in the concern
about model uncertainty causes a significant decline in the use of coal, while the
use of oil is slightly delayed. Holding other parameters fixed, the optimal path of
oil consumption is determined jointly by the resource scarcity effect and by the
model uncertainty effect. Naturally, we do not find a significant difference in oil
consumption when the scarcity effect dominates. However, when we consider a
higher level of initial resources of fossil fuel, the concern about model uncertainty
substantially discourages the use of oil.

As we mentioned, our work builds on the model in GHKT. In addition, we rely
on existing work in robust control theory from both economics and engineering. In
the traditional stochastic control literature, uncertainties in the system are modeled
using probability distributions. The goal there is to derive a policy that works best
"on average." In contrast, given a bound on uncertainty, robust control is concerned
with optimizing performance under a so-called worst-case scenario. Hansen and
Sargent (2001) introduced techniques from robust control theory to dynamic eco-

---

3See also Barrage (2013). Previous related work includes Hotelling (1931), Dasgupta and Heal
ski (2009), Krusell and Smith (2009), and Ploeg and Withagen (2012, 2012). GHKT (2013) provide
an excellent review of this literature.

4See, for example, Lewis (1986) and Chandrasekharan (1996).
onomic decision making problems.\(^5\) They pointed out the connection between the max-min expected utility theory of Gilboa and Schmeidler (1989) and the applications of robust control theory proposed by Anderson et al. (2000) and Dupuis et al. (1998). Hansen, Sargent, Turmuhambetova and Williams (2005) give a thorough introduction to the robust control approach, and develop a variety of tools required to make it useful in an economics context. They discuss applications to a wide range of problems within the Linear-Quadratic-Gaussian world.\(^6\)

As is standard in the robust control literature, our paper postulates the problem of optimal fossil fuel extraction as a two-person zero-sum dynamic game: in each stage, a social planner (a representative household in the decentralized version) maximizes social welfare (lifetime utility) by choosing the level of energy extraction, consumption, labor and capital investment. Then, a malevolent player chooses alternative distributions in order to minimize the respective payoff. Our work contributes to the existing literature of applications of robust control in economics in two ways. First, it explores a class of models under a non-quadratic objective and non-linear constraints. In that regard, we demonstrate that models of the type in GHKT (2013) can be restated in a robust control framework. We then derive some sharp analytical results, and compute the resulting model numerically. Second, we employ the exponential distribution as the approximating distribution. While existing studies usually employ the Linear-Quadratic model combined with Gaussian distributions in order to produce analytical solutions, our work shows that the approximating distribution for models with log-utility and full depreciation of capital

\(^5\text{See Knight (1921), Savage (1954), Ellsberg (1961), Gilboa and Schmeidler (1989), Hansen and Sargent (2001 and 2010) for related research.}\)

\(^6\text{Related work includes Hansen, Sargent and Tallarini (1999), Hansen and Sargent (2003), Col- gey, Colacito, Hansen and Sargent (2008). See Williams (2008) for a review. In a recent paper, Bidder and Smith (2012) use robust control theory to study the implications of model uncertainty for business cycles generated through “animal spirits.”}\)
can be drawn from either the normal or the exponential family.

The paper proceeds as follows. Section 2 presents the basic model. Section 3 studies the model analytically, while Section 4 presents our numerical and quantitative findings. A brief conclusion follows. Appendix II contains some technical material.

3.2 The Model

In order to characterize the optimal policy for the case where there is a concern about climate change and model uncertainty, we first formulate a general framework for the ”robust planner’s problem,” a benchmark that we will subsequently compare to decentralized market solutions.

Time, $t$, is discrete and the horizon is infinite. The world economy is populated by a $[0,1]$-continuum of infinite-lived representative agents with utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t).$$

The function $u$ is a standard concave period utility function, $C_t$ represents final-good consumption in period $t$, and $\beta \in (0,1)$ is the discount factor. The final goods sector uses energy, $E$, capital, $K$, and labor, $N$, to produce output. Labor supply is inelastic. The economy’s capital stock depreciates at rate $\delta \in (0,1)$. Henceforth, $\tilde{K}$ represents the end-of-period capital (before interacting with the climate factor through the process described below). The feasibility constraint in the final goods sector is given by

$$C_t + \tilde{K}_{t+1} = Y_t + (1 - \delta)K_t.$$ 

There are four production sectors. The final-goods sector, indexed by $i = 0$, produces the consumption good. The corresponding production function is given by $Y = F(K, N_0, E)$. Thus, in addition to capital and labor, production of the final good
requires the use of energy, $E$. The three energy-producing sectors for oil, coal, and green energy (labelled by $i = 1, 2, 3$, respectively) produce energy amounts $E_1$, $E_2$ and $E_3$ (measured in carbon equivalents). The oil sector is assumed to produce oil at zero cost. We denote by $R$ the total oil energy stock, and we impose the resource constraint, $R_t \geq 0$, for all $t$. Both the coal and the green energy sectors use linear technologies

$$E_i = A_iN_i, \ i = 2, 3$$

(3.3)

We follow GHKT in modeling a simplified carbon cycle as follows. The variable $S$ (measured in units of carbon content) represents the GHG concentration in the atmosphere in excess of the pre-industrial level. We denote by $P$ and $T$ the permanent and temporary components of $S$, respectively. These evolve according to the following.

$$P' = P + \phi_L(E_1 + E_2)$$

(3.4)

$$T' = (1 - \phi)T + (1 - \phi_L)\phi_0(E_1 + E_2)$$

(3.5)

$$S' = P' + T'$$

(3.6)

We introduce model uncertainty regarding climate change through a stochastic variable, $\gamma$, which reduces the end-of-period capital stock $\tilde{K}'$ by a factor of $h(S', \gamma)$ to $K'$. That is, $K' = h(S', \gamma)\tilde{K}'$. We use $\pi(\gamma)$ to denote the approximating distribution of $\gamma$, while $\hat{\pi}(\gamma)$ denotes the welfare-minimizing distribution, and $m(\gamma) = \frac{\pi(\gamma)}{\hat{\pi}(\gamma)}$ is the likelihood ratio. The distance, $\rho$, between $\hat{\pi}(\gamma)$ and $\pi(\gamma)$ is measured by relative entropy:

\[ \text{\footnote{In GHKT, } \gamma \text{ directly affects output. For technical reasons, we find it convenient to assume that } \gamma \text{ adversely affects the economy’s capital stock. The two assumptions lead to identical results when there is 100% capital depreciation (as we assume for our numerical results).} } \]
\[
\rho(\hat{\pi}(\gamma), \pi(\gamma)) = E[m(\gamma) \log m(\gamma)] = E[\log m(\gamma)] = \int [m(\gamma) \log m(\gamma)]\pi(\gamma) d\gamma \quad (3.7)
\]

As is standard in robust control, the concern about model uncertainty is represented by a two-person zero-sum dynamic game in which, after observing the choice of a social planner, a malevolent player chooses the worst specification of the model in each period. This game proceeds as follows.\(^8\) At the beginning of a period, the state; i.e., the value of \((K, N, P, T, R)\) is revealed. Then, the planner chooses \((C, E_i, N_i, \tilde{K}', P', T', S', R')\) in order to maximize social welfare. After observing the planner’s choice, nature (the ”malevolent player”) chooses an alternative distribution \(\hat{\pi}(\gamma)\) or, equivalently, \(m(\gamma)\), to minimize welfare. Note that any deviation from the approximating distribution will be penalized by adding \(\alpha \rho(\hat{\pi}(\gamma), \pi(\gamma))\) to the objective function. Here, \(\alpha\) represents the magnitude of the ”punishment.” A greater \(\alpha\) means a greater penalty associated with the deviation of \(\gamma\) from its approximating distribution, thus, a lower concern about robustness.

---

\(^8\)Our attention will be restricted to a particular type of equilibrium, the so-called Markov perfect (or feedback) equilibrium. This equilibrium is strongly time-consistent.
This leads to the following social planner’s problem:

\[
V(K, N, P, T, R) = \max_{\{C,E_i,N_i,K',P',T',S',R'\}} \min_{m(\gamma)} \{u(C) + \beta \int [m(\gamma)V(K', N', P', T', R') + \alpha m(\gamma) \log m(\gamma)] \pi(\gamma) d\gamma \}
\]

s.t.

\begin{align*}
E_i &= A_i N_i; \ i = 2, 3 & (3.9) \\
E &= (\kappa_1 E_1^p + \kappa_2 E_2^p + \kappa_3 E_3^p)^{1/\rho} & (3.10) \\
N &= N_0 + N_2 + N_3 & (3.11) \\
\tilde{K}' &= F(K, N_0, E) + (1 - \delta) K - C & (3.12) \\
K' &= h(S', \gamma) \tilde{K}' & (3.13) \\
R' &= R - E_1 \geq 0 & (3.14) \\
N' &= A_N N & (3.15) \\
P' &= P + \phi_L(E_1 + E_2) & (3.16) \\
T' &= (1 - \phi) T + (1 - \phi L) \phi_0(E_1 + E_2) & (3.17) \\
S' &= P' + T' & (3.18) \\
1 &= \int m(\gamma) \pi(\gamma) d\gamma & (3.19)
\end{align*}

Under a set of additional assumptions, the social planner’s problem can be solved analytically, and we will focus on the analytical solution first. We will discuss the decentralized problem and show that the socially optimal allocation can be restored by imposing appropriate fossil fuel taxes on the energy-producing sector.

### 3.3 The Analytical Solution

For the remainder of this section, we will make the following additional assumptions. While these assumptions are admittedly strong, they allow us to fully solve the model analytically. As we shall see, certain aspects of the solution remain instructive in the next Section, when the restrictive assumptions are dropped and the model is
solved numerically.

(A1) The period utility function is given by $u(C) = \log(C)$.

(A2) Capital depreciates fully; i.e., $\delta = 1$.

(A3) The production function is given by $F(K, N_0, E) = A_0 K^\theta N_0^{1-\theta - \nu} E^\nu$.

(A4) The damage function is given by $h(S', \gamma) = e^{-S^\gamma}$.

(A5) The approximating distribution for $\gamma$ is exponential with mean $\lambda^{-1}$ and variance $\lambda^{-2}$; i.e., $\pi(\gamma) = \lambda e^{-\lambda \gamma}$.

(A6.1) $\phi_L = 0$.

(A6.2) $\phi = 0$.

(A7) There is a single fossil energy sector producing oil at zero cost. Production is subject to a resource feasibility constraint: $R' \geq 0$. As a result, $N_1 = 0$ and $N_0 = N$.

(A8) There is no population growth, and the aggregate labor supply is normalized to 1. That is, $A_N = 1$ and $N = 1$ in all periods.

(A9) There is no technology improvement. That is, $A_0$ is constant over time. We normalize $A_0 = 1$.

---

9 There exists a constant, $\Delta$, such that if the GHG concentration, $S$, is greater than $\frac{1}{\Delta}$, the system cannot be “robustified,” in the sense that the value of the game goes to negative infinity. However, if the economy starts with an initial $S_0 < \frac{1}{\Delta}$, then $S_t$ will converge to $\frac{1}{\Delta}$ as $t \to +\infty$.

10 The exponential distribution with mean $\lambda^{-1}$ is the maximum-entropy distribution among all continuous distributions supported in $[0, \infty]$ that have mean $\lambda^{-1}$. The worst case distribution for $\gamma$ is also exponential with mean $(\lambda^*)^{-1}$ and variance $(\lambda^*)^{-2}$, where $\lambda^* = \lambda(1 - \Delta S^*) = \lambda(1 - \Delta \phi_0 CE)(1 - \Delta S)$. That is, $\pi^*(\gamma) = \lambda^* e^{-\lambda^* \gamma}$. Since $\lambda^* = \lambda(1 - \Delta S^*) < \lambda$, the worst case mean of $\gamma$, $(\lambda^*)^{-1}$, is strictly greater than the approximating mean, $\lambda^{-1}$.

11 If $\phi_L > 0$, we need to depict the dynamics of $P$ and $T$ separately before we sum them in order to obtain the dynamics of $S$. Assuming that $\phi_L = 0$ allows us to express the dynamics of $S$ without the need to consider $P$ and $T$ separately. That is, $S' = (1 - \phi)S + \phi_0 E$. Moreover, (A6.1) and (A6.2) imply that $S' = S + \phi_0 E$, which is necessary for an analytical solution.
The resource feasibility constraint is not binding.\textsuperscript{12}

We will first solve the social planner’s problem. We will then discuss the decentralized problem and show that the socially optimal allocation can be restored by implementing fossil fuel taxes on the energy-producing sector.

Under A1-A10, the social planner’s problem can be rewritten as:

\[
V(K, S) = \max_{\{C,E,K',S\}} \min_{m(\gamma)} \{u(C) + \beta \int [m(\gamma)V(K', S') + \alpha m(\gamma) \log m(\gamma)] \pi(\gamma) \} \tag{3.20}
\]

s.t.

\[
\tilde{K}' = F(K, E) - C \tag{3.21}
\]

\[
K' = h(S', \gamma) \tilde{K}' \tag{3.22}
\]

\[
S' = S + \phi_0 E \tag{3.23}
\]

\[
1 = \int m(\gamma) \pi(\gamma) d\gamma \tag{3.24}
\]

where \( h(S', \gamma) = e^{-S'\gamma} \) and \( F(K, E) = K^\theta E' \). To solve this problem, we first guess that \( V(\cdot) \) takes the form

\[
V(K', S') = f(S') + \bar{A} \log(K') + \bar{D} = f(S') + \bar{A} \log(h(S', \gamma) \tilde{K}') + \bar{D} \tag{3.25}
\]

where \( \bar{A} \) and \( \bar{D} \) are undetermined coefficients. The functional form for \( f(\cdot) \) will be derived when we solve the minimizing player’s problem.

First, we define the robustness problem (the inner minimization problem) by

\[
\mathcal{R}(V)(\tilde{K}', S') = \min_{m(\gamma)} \int [m(\gamma)V(K', S') + \alpha m(\gamma) \log m(\gamma)] \pi(\gamma) d\gamma \tag{3.26}
\]

s.t.

\[
K' = e^{-S'\gamma} \tilde{K}' \tag{3.27}
\]

\[
1 = \int m(\gamma) \pi(\gamma) d\gamma \tag{3.28}
\]

The F.O.N.C. for \( m(\gamma) \) implies that

\textsuperscript{12}Later we provide a sufficient condition for (A10).
\[ m^*(\gamma) = \frac{\exp\left(-\frac{V(K',S')}{\alpha}\right)}{\int \exp\left(-\frac{V(K',S')}{\alpha}\right) \pi(\gamma) d\gamma} = (1 - \Delta S') e^{\Delta S' \lambda} \] (3.29)

or, equivalently,

\[ \tilde{\pi}^*(\gamma) = m^*(\gamma) \pi(\gamma) = \lambda^* e^{-\lambda^* \gamma} \] (3.30)

where we define \( \Delta = \frac{\lambda}{\alpha} \) and \( \lambda^* = \lambda(1 - \Delta S') \).\(^{13}\) Thereby,

\[ R(V)(\tilde{K}', S') = \int [m^*(\gamma)V(K', S') + \alpha m^*(\gamma) \log m^*(\gamma)] \pi(\gamma) d\gamma \]

\[ = -\alpha \log\left[ \int \exp\left(-\frac{V(K',S')}{\alpha}\right)\pi(\gamma) d\gamma \right] \] (3.31)

Substituting equation (3.25) into equation (3.31), we obtain

\[ R(V)(\tilde{K}', S') = f(S') + \bar{\alpha} \log(\tilde{K}') + \bar{D} + H(S'; \alpha, \bar{\alpha}) \] (3.32)

where \( H(S'; \alpha, \bar{\alpha}) \), the robust version of the externality from carbon emissions, is given by

\[ H(S'; \alpha, \bar{\alpha}) = -\alpha \log\left[ \int h^{-\frac{\bar{\alpha}}{\alpha}}(S'; \gamma) \pi(\gamma) d\gamma \right] \] (3.33)

It follows from (A4)-(A5) that

\[ H(S'; \alpha, \bar{\alpha}) = \alpha \log(1 - \Delta S') \] (3.34)

Next, we define the optimal choice problem (the outer maximization problem). Using the analysis above, this problem can be written as

\[ V(K, S) = \max_{\{C, E, K', S'\}} \{ \log(C) + \beta R(V)(\tilde{K}', S') \} \] (3.35)

\(^{13}\)The worst case distribution of \( \gamma \) remains exponential with a distorted mean \( (\lambda^*)^{-1} \) and variance \( (\lambda^*)^{-2} \).
or equivalently,

\[ f(S) + \bar{A}\log(K) + \bar{D} \]

\[ = \max_{C,E} \left\{ \log(C) + \beta [f(S') + \bar{A}\log(\bar{K}') + \bar{D} + H(S'; \alpha, \bar{A})] \right\} \] (3.36)

s.t.

\[ \bar{K}' = F(K, E) - C \] (3.37)

\[ S' = S + \phi_0 E \] (3.38)

\[ H(S'; \alpha, \bar{A}) = \alpha \log(1 - \Delta S') \] (3.39)

The F.O.N.C. imply

\[ C = \frac{F(K, E)}{1 + \beta \bar{A}} \] (3.40)

\[ -\phi_0 \left[ \frac{\partial f(S')}{\partial S'} + \frac{\partial H(S'; \alpha, \bar{A})}{\partial S'} \right] = \frac{1 + \beta \bar{A} \frac{\partial F(K,E)}{\partial E}}{\beta \frac{F(K,E)}{E}} \] (3.41)

Noting that \( H(S; \alpha, \bar{A}) \) is a logarithmic function of \( S \), we guess that \( f(S) = \bar{B}\log(1 - \Delta S) \), where \( \bar{B} \) is an undetermined coefficient. As a result, the above F.O.N.C. can be simplified to

\[ C = \frac{K^\theta E^\nu}{1 + \beta \bar{A}} \] (3.42)

\[ \frac{\beta \phi_0 \Delta(\alpha + \bar{B})}{1 - \Delta S'} = \frac{\nu(\beta \bar{A} + 1)}{E} \] (3.43)

After some tedious derivations, we obtain

\[ \bar{A} = \frac{\theta}{1 - \beta \theta} \] (3.44)

\[ \bar{B} = \frac{1}{1 - \beta} \left[ \alpha \beta + \frac{\nu}{1 - \beta \theta} \right] \] (3.45)

The expression for \( \bar{D} \) is more complicated and less intuitive. Substituting \( \bar{A} = \frac{\theta}{1 - \beta \theta} \) into the F.O.N.C., we obtain the optimal allocation. We summarize the above discussion in the following.

**Proposition 1.** Assume that (A1)-(A10) hold. The two-person zero-sum dynamic game described by eq(3.20)-eq(3.24) admits a feedback (Markov perfect) equilibrium.
The equilibrium strategies are given by:

\[ C^\star = (1 - \beta \theta)K^\theta E^\nu = (1 - \beta \theta)K^\theta[c_E(1 - \Delta S)]^\nu \]  
(3.46)

\[ E^\star = c_E(1 - \Delta S) \]  
(3.47)

\[ S'^\star = S + \phi_0 c_E(1 - \Delta S) \]  
(3.48)

\[ \hat{\pi}^\star(\gamma) = \lambda^* e^{-\lambda^* \gamma} \]  
(3.49)

where \( c_E = \frac{\nu(1-\beta)}{\beta \alpha(1-\beta \theta) + \nu \phi_0 \Delta} \) and \( \lambda^* = \lambda(1 - \Delta S'^\star) \).

A few technical remarks are in order. First, the function \( V(K, S) \) is increasing in \( K \), decreasing in \( S \), and jointly concave in \( K \) and \( S \). The value of \( \bar{A} \) is the same as in the model without concern about model uncertainty. Both \( E^\star \) and \( S'^\star \) are affine functions of \( S \). In addition, it can be shown that, given \( S \), both \( E^\star \) and \( S'^\star \) are increasing functions of \( \alpha \). This is intuitive since a greater \( \alpha \) implies a larger resulting penalty from a deviation of \( \gamma \) from its approximating distribution, thus, a lower concern about model-uncertainty. Note that \( C^\star \) is affected by \( S \) only through \( E^\star \). This is due to logarithmic utility. As a result, a greater concern about model-uncertainty will lower both \( E^\star \) and \( C^\star \). The value of the externality from one unit of emissions evaluated at \( E^\star \) is given by

\[ \lambda^* = -\beta \frac{\partial V(K', S')}{\partial E} \bigg|_{K'^\star, S'^\star} = \beta \phi_0 \Delta (\bar{B} + \alpha) \]  
\[ \frac{\nu}{c_E(1 - \beta \theta)(1 - \Delta S)} = \frac{\nu}{(1 - \beta \theta)E^\star} \]  
(3.50)

Our model so far is similar to the oil regime in GHKT, except that we assume that the resource constraint is not binding. Since \( S_{t+1} = S_t + \phi_0 E_t \), we arrive at the following expression for the aggregate oil extraction

\[ \sum_{t=0}^{+\infty} E_t = \lim_{t \to +\infty} \phi_0^{-1}(S_t - S_0) = \phi_0^{-1}(\frac{1}{\Delta} - S_0) \]  
(3.51)

Thus, the resource constraint is not binding if and only if the aggregate oil reserves are greater than \( \phi_0^{-1}(\frac{1}{\Delta} - S_0) \). Figures 3.1, 3.2, and 3.3 below illustrate how \( E^\star \)
responds to a concern about model-uncertainty. Figures 3.1 and 3.2 show how $E^*$ reacts to a change in the penalty parameter, $\alpha$, in the multiplier version of the game.

We can also study the effect of change in concern about uncertainty on energy use. Figure 3.3 refers to the equivalent constraint game, in which $\hat{\pi}(\gamma)$ is constrained in a closed ball of radius $\delta$ centered at $\pi(\gamma)$, denoted by $B_\delta(\pi(\gamma))$. Direct calculation shows that the distance between $\hat{\pi}^*(\gamma)$ and $\pi(\gamma)$, as measured by entropy is given by

$$\rho(\hat{\pi}^*(\gamma), \pi(\gamma)) = \log(1 - \Delta S^{*\pi}) + \frac{\Delta S^{*\pi}}{1 - \Delta S^{*\pi}}$$

(3.52)

Since $\hat{\pi}^*(\gamma)$, which is chosen by the minimizing player, must be on the boundary of $B_\delta(\pi(\gamma))$, we have that $\rho(\hat{\pi}^*(\gamma), \pi(\gamma)) = \delta$. Recall that $\rho$ measures the relative entropy of $\pi$ and $\hat{\pi}$. Figure 3.3 shows how $E^*$ changes as we relax $\delta$, allowing for more uncertainty about the approximating model. In Appendix II, we show that $\frac{\partial E^*}{\partial \delta}|_{\delta=0} = -\infty$. That is, even an infinitesimal concern about model uncertainty can cause a significant drop in the optimal energy extraction.

Robust control modeling can be introduced in different ways. So far we used a closed-loop zero-sum dynamic game in which the social planner moves first in each period. Alternatively, we can construct a game with the same information structure by interchanging the order of max and min in eq(3.20). The two games differ only in terms of the timing protocol. However, both lead to the same (unique) feedback saddle-point equilibrium if certain conditions are satisfied. More precisely, if (A1)-(A10) hold, then the objective in (3.20) is strictly concave in $C$ and $E$, and strictly convex in $m(\gamma)$. Consequently, the two closed-loop zero-sum dynamic games admit the same unique pure strategy saddle-point Nash equilibrium, which is the one described in Proposition 1.

Let us now turn to the decentralized problem. Suppose a percentage tax, $\tau_t$, is imposed on emissions, $E_t$. Since the extraction cost of energy (the cost of creating emissions) is zero, it must be true that
Figure 3.1: The Effect of Penalty Parameter $\alpha$ on Optimal Carbon Emissions, $E$

Figure 3.2: The Effect of $\alpha^{-1}$ on $E$
The above equation captures the one-to-one relationship between $E_t$ and $\tau_t$. Therefore, to achieve the optimal emissions level, $E_t = c_E(1 - \Delta S)$ in eq(3.47), we must impose $\tau_t = \nu c_E^{\nu-1}(1 - \Delta S)^{\nu-1}K_t^\theta$. It is straightforward to show that $\tau_t = \frac{\lambda^*}{w(C_t^*)}$, where $C_t^*$ is the optimal consumption, given by eq(3.46). That is, the optimal tax on emissions is equal to the corresponding GHG externality measured in units of the consumption good. It remains to show that $C_t^*$ can be recovered under the optimal tax. This can be shown using the representative household’s problem as follows.

Since we have established a one-to-one relationship between $E_t$ and $\tau_t$, we may assume without loss of generality that the planner chooses $E_t$. Further, assume that $E_t$
is chosen as a function of $S_t$ only. Given $E = E(S_t)$, $k$, $K$, and $S$, a representative household solves:

$$V(k, K, S) = \max_{c, k'} \min_{\hat{\pi}(\gamma)} \left\{ u(c) + \beta \hat{E}_\gamma \left[ V(k', K', S') + \alpha \log \left( \frac{\hat{\pi}(\gamma)}{\pi(\gamma)} \right) \right] \right\}$$  \hspace{1cm} (3.54)

subject to

$$c + \tilde{k}' = r(K, S)k + \tau(K, S)E(S) + \pi^{profit}$$  \hspace{1cm} (3.55)

$$\hat{K}' = G(K, S)$$  \hspace{1cm} (3.56)

$$k' = e^{-\gamma S'} \tilde{k}'$$  \hspace{1cm} (3.57)

$$K' = e^{-\gamma S'} \hat{K}'$$  \hspace{1cm} (3.58)

$$S' = S + \phi_0 E(S)$$  \hspace{1cm} (3.59)

where $u(c) = \log(c)$, $r(K, S) = \theta K^{q-1}[E(S)]^v$, $\tau(K, S) = \nu K^{q}[E(S)]^{v-1}$, $\pi^{profit}$ is the firm’s profit, and $\hat{K}' = G(K, S)$ is the equilibrium transition law for the aggregate capital stock. Here, $(k, K, S)$ stands for the beginning-of-period and $(\tilde{k}', \hat{K}', S')$ for the end-of-period state, respectively. Notice that $(\tilde{k}', \hat{K}')$ is not equal to the beginning-of-next-period state, $(k', K')$, due to capital deterioration by a factor $e^{-\gamma S'}$. In addition, $\hat{E}_\gamma$ is calculated with respect to the worst case distribution for $\gamma$, $\hat{\pi}(\gamma)$, as chosen by the minimizing player. Since the minimizing player moves after the maximizing player, the worst distribution is, in general, conditional on the end-of-period state, $(\tilde{k}', \hat{K}', S')$. It can be shown that the optimal consumption sequence satisfies the following Euler equation:

$$u'(c^*) = \beta \int e^{-\gamma S'} r(K', S') u'(c^{**}) e^{-\frac{\nu(K', S')}{\alpha}} \pi(\gamma) d\gamma \int e^{-\frac{\nu(K', S')}{\alpha}} \pi(\gamma) d\gamma$$  \hspace{1cm} (3.60)

This yields the following Proposition.

**Proposition 2.** Assume that (A1) - (A10) hold. The optimal energy consumption is $E = c_E(1 - \Delta S)$. The optimal tax is $\tau_t = \frac{\lambda^*}{\mu(c^*)}$, with tax proceeds rebated lump-

---

14This is without loss of generality, since our goal is to recover the optimal emissions in eq(3.47), which only depends on $S_t$. 

sum to the representative consumer. The resulting competitive equilibrium allocation coincides with the solution to the planner’s problem. That is, $c^* = C^* = (1 - \beta \theta)K^{\theta}[c_E(1 - \Delta S)]^\nu$.

### 3.4 The Computational Solution and Calibration

In this Section we first extend the analytical model by relaxing assumptions (A6.1) and (A6.2). For our baseline model, we will assume that $\pi(\gamma)$, the approximating distribution of $\gamma$, is exponential. As we now allow for $\phi_L > 0$, we need to introduce two additional state variables ($P$ and $T$), since keeping track of the sum $S = P + T$ will no longer suffice. We will also relax (A7) by incorporating a "coal" and a "green" sector into the model. Furthermore, we will relax (A8) and (A9) by allowing $A_2N_2$ and $A_3N_3$ to grow at a rate of two percent per year. Last, we will drop (A10).
The social planner’s problem becomes:

\[
V(K, N, P, T, R) = \max_{\{C, E_1, E_2, E_3, K', P', T', R'\}} \min_{m(\gamma)} \{u(C) + \beta \int [m(\gamma)V(K', N', P', T', R') + \alpha m(\gamma) \log m(\gamma)] \pi(\gamma) d\gamma \}
\]  

s.t.

\[
E = (\kappa_1 E_1^p + \kappa_2 E_2^p + \kappa_3 E_3^p)^{1/\rho}
\]  

\[
\tilde{K}' = F \left( K, N \left( 1 - \frac{E_2}{A_2 N} - \frac{E_3}{A_3 N} \right), E \right) - C
\]  

\[
K' = h(S', \gamma) \tilde{K}'
\]  

\[
A_2' N' = (1 + g) A_2 N
\]  

\[
A_3' N' = (1 + g) A_3 N
\]  

\[
R' = R - E_1 \geq 0
\]  

\[
P' = P + \phi_L (E_1 + E_2)
\]  

\[
T' = (1 - \phi) T + (1 - \phi_L) \phi_0 (E_1 + E_2)
\]  

\[
S' = P' + T'
\]  

\[
1 = \int m(\gamma) \pi(\gamma) d\gamma
\]

To solve this problem we first argue that most of the analysis conducted in Section 3 carries over. The only difference is that the function \( f(\cdot) \) no longer has a closed form expression. We will again apply the outer-inner loop method used in Section 3. The inner loop minimization problem is unchanged, while the outer loop maximization problem will be solved in parts. In that regard, it is important to note that solving the optimization problem for \( E_i, P', T', \) and \( R' \) can be carried out separately from solving for \( C \) and \( \tilde{K}' \). Furthermore, the solution to the second optimization problem remains the same as in Section 3; i.e., \( C^* = (1 - \beta \theta) Y^* \) and \( \tilde{K'}^* = \beta \theta Y^* \), where \( Y^* \) denotes the optimal output level. After substituting for \( C^* \), the optimization problem for \( E_i, P', T', \) and \( R' \) can be simplified, leading to the dynamic programming
problem below:

\[
\begin{align*}
   f(N, P, T, R) &= \max_{E_1, E_2, E_3, E', T', R'} \\
   &\left\{ \frac{1}{1-\theta} \log\left(1 - \frac{E_2}{A_2 N} - \frac{E_3}{A_3 N}\right)^{1-\theta} + \beta \left[f(N', P', T', R') + \alpha \log(1 - \Delta S')\right]\right\}
\end{align*}
\]

s.t.

\[
\begin{align*}
   E &= (\kappa_1 E_1^p + \kappa_2 E_2^p + \kappa_3 E_3^p)^{1/p} \\
   N' &= (1 + g)N \\
   R' &= R - E_1 \geq 0 \\
   P' &= P + \phi_L(E_1 + E_2) \\
   T' &= (1 - \phi)T + (1 - \phi_L)\phi_0(E_1 + E_2) \\
   S' &= P' + T'
\end{align*}
\]

Next, we characterize the optimality conditions for \( E_3, E_2, \) and \( E_1 \), respectively.

The first-order condition for \( E_3 \) implies

\[
\frac{\nu \kappa_3}{E_3^{1-p} E^p} = \frac{1 - \theta - \nu}{A_3 N_0}
\]

The first-order condition for \( E_2 \) gives

\[
\frac{1 - \theta - \nu}{A_2 N_0} = \frac{\nu \kappa_2}{E_2^{1-p} E^p} + (1 - \beta \theta) \beta \left[ \phi_L \left( \frac{\partial f}{\partial P'} - \frac{\alpha \Delta}{1 - \Delta S'} \right) + (1 - \phi_L) \phi_0 \left( \frac{\partial f}{\partial T'} - \frac{\alpha \Delta}{1 - \Delta S'} \right) \right]
\]

Applying the envelope theorem to \( P \) and \( T \) gives

\[
\frac{\partial f}{\partial P} = \beta \left( \frac{\partial f}{\partial P'} - \frac{\alpha \Delta}{1 - \Delta S'} \right) \quad \text{(3.81)}
\]
\[
\frac{\partial f}{\partial T} = \beta (1 - \phi) \left( \frac{\partial f}{\partial T'} - \frac{\alpha \Delta}{1 - \Delta S'} \right) \quad \text{(3.82)}
\]

Defining \( \hat{\lambda}_P = -(1 - \beta \theta) \frac{\partial f}{\partial P} \) and \( \hat{\lambda}_T = -(1 - \beta \theta) \frac{\partial f}{\partial T} \) to be the marginal values of the externality caused by \( P \) and \( T \), respectively, the first-order condition for \( E_2 \) becomes

\[
\frac{1 - \theta - \nu}{A_2 N_0} = \frac{\nu \kappa_2}{E_2^{1-p} E^p} - \left[ \phi_L \hat{\lambda}_P + \frac{(1 - \phi_L) \phi_0 \hat{\lambda}_T}{1 - \phi} \right]
\]
It is easy to see that the marginal externality caused by $E_2$ (or $E_1$) is given by

$$\Delta^S = \phi_L \Delta^P + \frac{(1 - \phi_L)\phi_0}{1 - \phi} \Delta^T$$  \hspace{1cm} (3.84)

Thus, we obtain

$$\frac{\nu\kappa_2}{E_2^{-p}E^p} - \Delta^S = \frac{1 - \theta - \nu}{A_2 N_0}$$  \hspace{1cm} (3.85)

This has the same form as the corresponding equation in GHKT, but under a different interpretation for $\hat{\lambda}_t^S$. To see the difference, it is convenient to restore the time index, $t$. From eq(3.81) and eq(3.82) we have

$$\Delta^P_t = \sum_{j=1}^{+\infty} \frac{\beta_j}{1 - \Delta S_{t+j'}} = \sum_{j=1}^{+\infty} \frac{\beta^j}{1 - \Delta S_{t+j'}}$$  \hspace{1cm} (3.86)

$$\Delta^T_t = \sum_{j=1}^{+\infty} [\beta(1 - \phi)]^j = \sum_{j=1}^{+\infty} [\beta(1 - \phi)]^j = \sum_{j=1}^{+\infty} \left[\frac{\beta^j}{1 - \Delta S_{t+j'}} - \frac{(1 - \phi_L)\phi_0}{1 - \phi} [\beta(1 - \phi)]^j\right]$$  \hspace{1cm} (3.87)

The second equality in either equation is obtained by using $(1 - \beta\theta)\alpha\Delta = (1 - \beta\theta)\alpha \bar{\lambda} = \theta\lambda^{-1} = \theta\bar{\gamma}$, where $\lambda^{-1} = \bar{\gamma}$ is the mean of $\gamma$ under the approximating model. It follows immediately that $\hat{\lambda}_t^S$ can be expressed as

$$\Delta^S_t = \sum_{j=1}^{+\infty} \left[\phi_L \frac{\beta^j}{1 - \Delta S_{t+j'}} + \frac{(1 - \phi_L)\phi_0}{1 - \phi} [\beta(1 - \phi)]^j\right]$$  \hspace{1cm} (3.88)

It is instructive to consider the case when $\alpha \to +\infty$; i.e., when there is no concern about model uncertainty. Observe that $\Delta \to 0$ as $\alpha \to +\infty$. Therefore,\textsuperscript{15}

$$\lim_{\alpha \to +\infty} \Delta^S_t = \theta\bar{\gamma} \sum_{j=1}^{+\infty} \left[\frac{\phi_L \beta^j}{1 - \Delta S_{t+j'}} + \frac{(1 - \phi_L)\phi_0}{1 - \phi} [\beta(1 - \phi)]^j\right] = \theta\bar{\gamma} \left[\frac{\phi_L \beta}{1 - \beta} + \frac{(1 - \phi_L)\phi_0 \beta}{1 - (1 - \phi)\beta}\right]$$  \hspace{1cm} (3.89)

\textsuperscript{15}Contrasting this with the corresponding equation in GHKT $\left(\hat{\lambda}_t^S = \bar{\gamma} \left[\frac{\phi_L \beta}{1 - \beta} + \frac{(1 - \phi_L)\phi_0 \beta}{1 - (1 - \phi)\beta}\right]\right)$, we identify two differences. First, eq(3.89) contains an additional term ($\theta$). This is because GHG directly affect aggregate capital instead of output in our model. Second, the externality related to $P$ and $T$ is weighted by $\beta$ in eq(3.89). This is because GHG in our model affect next period’s capital rather than the current one.
Finally, the first-order condition for $E_1$ yields

$$
\frac{\nu \kappa_1}{E_1^{1-\rho} E^\rho} - \tilde{\Lambda}^S = \beta \left[ \frac{\nu \kappa_1}{(E'_1)^{1-\rho}(E')^\rho} - (\tilde{\Lambda}^S)' \right]
$$

(3.90)

Note that the operator $E_t$ does not appear on the right-hand-side, as the planner optimizes under the worst case scenario, rather than averaging over all cases. As the planner’s problem has a similar structure as in the analytical model, it can be shown that analogues of Propositions 1 and 2 hold in this environment. We numerically solve the above problem for the cases where $\alpha = 0.01$ and $\alpha = 100$. We use the same parameter values as in GHKT, except for $R_0$, which is set to 800 as in Rogner (1997). Figures 3.4 through Figure 3.6 plot the computed optimal paths.

**Table 3.1 : Parameter Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi$</th>
<th>$\phi_L$</th>
<th>$\phi_0$</th>
<th>$\theta$</th>
<th>$\nu$</th>
<th>$\beta$</th>
<th>$\rho$</th>
<th>$1 + g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.0228</td>
<td>0.2</td>
<td>0.393</td>
<td>0.3</td>
<td>0.04</td>
<td>0.985</td>
<td>-0.058</td>
<td>1.02</td>
</tr>
<tr>
<td>Parameter</td>
<td>$P_0$</td>
<td>$T_0$</td>
<td>$R_0$</td>
<td>$\kappa_1$</td>
<td>$\kappa_2$</td>
<td>$A_{2,0}$</td>
<td>$A_{3,0}$</td>
<td>$\lambda^{-1}$</td>
</tr>
<tr>
<td>Value</td>
<td>103</td>
<td>699</td>
<td>800</td>
<td>0.5008</td>
<td>0.08916</td>
<td>7.693</td>
<td>1.311</td>
<td>2.379 $\times$ 10$^{-5}$</td>
</tr>
</tbody>
</table>

Figure 3.4 describes the optimal paths for the use of green energy, coal, and oil, as well as the resulting carbon concentration in the atmosphere, conditional on different levels of concern about model uncertainty. For simplicity, we refer to the optimal path under $\alpha = 100$ as the ”non-robust optimal path,” and to the path under $\alpha = 0.01$ as the ”robust optimal path.” Since the green energy sector does not inject carbon into the atmosphere, the optimal path for the use of green energy does not directly depend on the level of concern about model uncertainty regarding the externality from carbon emissions. However, since green energy, coal, and oil are substitutes, model uncertainty considerations do affect the use of green energy indirectly, through its impact on the ”dirty” energy sectors — coal and oil.
Figure 3.4: Optimal Use of Energy
We find that an increase in the concern about model uncertainty causes a significant decline in the use of coal. In contrast, the use of oil is delayed, but only slightly. As the supply of oil is finite, the decline rate of oil-use depends not only on model uncertainty, but also on resource scarcity. As we will show in the next Section, an initial stock of oil equaling $R_0 = 800GtC$ is low enough so that the resource scarcity effect overwhelms the model uncertainty effect in determining the optimal use of oil in the economy. This explains why we do not observe a sharp decrease in the optimal use of oil when the concern about model uncertainty increases. Finally, straightforward calculation shows that the difference in energy use in the two optimal paths leads to a significant difference in the associated carbon accumulation. Our model predicts that if there is a "small" concern about model uncertainty ($\alpha = 100$), or if model uncertainty is not incorporated into the model ($\alpha = 0.01$), atmospheric carbon concentrations will reach a level as high as $1350GtC$ (net of preindustrial levels) after 180 years. However, this number is reduced by 40% to about $800GtC$ if concerns about model uncertainty are incorporated and addressed through the corresponding optimal tax, restoring the optimal energy path under $\alpha = 0.01$.

Figure 3.5 demonstrates a direct consequence of the above analysis: based on the mapping from carbon concentrations to global temperatures used in the RICE model, 
\[ T(S_t) = 3 \ln\left(\frac{S_t}{\bar{S}}\right)/\ln 2, \]
the global average temperature will rise by 3.8 degree Celsius 180 years from now if the concern about model uncertainty is addressed, and by 5.3 degrees Celsius otherwise.

The graphs in the first (second) column in Figure 3.6 describe the paths of total damages as a percentage of the capital stock, and as a function of the capital stock, and of output, respectively, assuming that the approximating model (worst case model) for $\gamma$ is the true model.\(^{16}\) In each graph, the green-dashed line (blue-
solid line) represents the outcome when energy is extracted based on the non-robust (robust) optimal path. The main findings can be summarized as follows. If the approximating model for $\gamma$ is the true model, pursuing the robust optimal path for energy consumption would further reduce total damages by an additional 1 percent 180 years from now. However, due to a more conservative use of oil and coal in the final good sector, such a policy will also reduce both capital stock and output in the long run. Since utility depends only on consumption (which is proportional to output), this implies that the welfare loss from over-estimating the concern about uncertainty would be rather small. In contrast, if the true distribution of $\gamma$ evolves according to the worst case model in each period (second column of Figure 3.6), the cost of implementing the non-robust optimal policy is rather large. In fact, the non-robust policy, which overlooks concerns about model uncertainty, will dramatically
Figure 3.6: Capital Stock and Output
reduce the entire capital stock in 120 years, resulting in a large reduction in output and welfare.\footnote{The dramatic effects on capital, output, and social welfare are partly due to the assumption that the approximating distribution of $\gamma$ is exponential. As we discuss next, the losses are somewhat reduced, though still large, if the approximating distribution of $\gamma$ is assumed to be normal. The exponential distribution is one way to capture the extreme effects in Stern (2013) in the context of our model.}

### 3.4.1 Varying the Approximating Distribution

Here we further explore the implications of assumption (A5). To this end, we now assume that the approximating distribution of $\gamma$ is normal with mean $\bar{\gamma}$ and variance $\sigma^2$; i.e., $\pi(\gamma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\gamma-\bar{\gamma})^2}{2\sigma^2}}$. This creates two key differences. First, the normal distribution provides us with two degrees of freedom: the mean, $\bar{\gamma}$, reflecting the planner’s prior expectation regarding damages, and the variance, $\sigma^2$, indicating the prior regarding model uncertainty. In comparison, recall that the exponential distribution only used one parameter, $\lambda$, which determined both the mean and the variance of $\gamma$.\footnote{As we shall see below, assuming that $\gamma$ is normally distributed can also eliminate the ”breaking point” for $S$, which is always present when $\gamma$ follows an exponential. This is because the exponential distribution has a ”fat” tail, thus, allowing more room for nature to create a worst-case-scenario given a level of penalty, $\alpha$.}

We have:

$$H(S'; \alpha, A) = -(\bar{\gamma} + \frac{\bar{\gamma}^2}{2\alpha} S') A S'$$

$$\hat{\pi}^*(\gamma) \sim \mathcal{N}(\bar{\gamma} + \frac{\bar{\gamma}^2}{\alpha} S^2, \sigma^2)$$

It is straightforward to show that $H(\cdot)$ is strictly negative, strictly increasing in $\alpha$, and strictly decreasing in both $\bar{\gamma}$ and $\sigma^2$. In addition, the worst case distribution for $\gamma$ also follows a normal distribution, and $\hat{\pi}^*(\gamma)$ and $\pi(\gamma)$ differ only in their means. That is, when choosing the worst case model, nature only alters the mean of $\gamma$.
rather than its variance. As a by-product, the relative entropy of $\hat{\pi}^*(\gamma)$ with respect to $\pi^*(\gamma)$ is given by

$$\rho(\hat{\pi}^*(\gamma), \pi^*(\gamma)) = \frac{\hat{A}^2 \sigma^2 S'^2}{2\alpha^2} \quad (3.93)$$

To complete the model, we need to replace the term $\alpha \log(1 - \Delta S')$ in eq(3.72) with $-(\gamma + \frac{4\sigma^2}{2\alpha}) \bar{A} S'$. Accordingly, the optimality conditions for $E_1$, $E_2$, and $E_3$ remain intact, except that the values of the externality associated with $P$, $T$, and $E_2$ (or $E_1$), respectively, are now as follows:

$$\hat{\Lambda}_t^P = \frac{\beta \theta \gamma}{1 - \beta} + \frac{\theta \hat{A} \sigma^2}{\alpha} \sum_{j=1}^{+\infty} \beta^j S_{t+j} \quad (3.94)$$

$$\hat{\Lambda}_t^T = \frac{\beta (1 - \phi) \theta \gamma}{1 - \beta (1 - \phi)} + \frac{\theta \hat{A} \sigma^2}{\alpha} \sum_{j=1}^{+\infty} [\beta (1 - \phi)]^j S_{t+j} \quad (3.95)$$

$$\hat{\Lambda}_t^S = \phi_L \hat{\Lambda}_t^P + \frac{(1 - \phi_L) \phi_0}{1 - \phi} \hat{\Lambda}_t^T \quad (3.96)$$

Note that $\hat{\Lambda}_t^S$ reduces to the previous expression as $\alpha \to +\infty$, or as $\sigma^2 \to 0$. That is,

$$\hat{\Lambda}_t^S = \theta \gamma \left[ \frac{\phi_L \beta}{1 - \beta} + \frac{(1 - \phi_L) \phi_0 \beta}{1 - (1 - \phi) \beta} \right] \text{, as } \alpha \to +\infty, \text{ or } \sigma^2 \to 0 \quad (3.97)$$

We will consider three cases regarding the initial stock of fossil fuel: $R_0 = 253.8 \text{GtC}$, $R_0 = 8000 \text{GtC}$, and $R_0 = \infty$. While the $R_0 = \infty$ case is for expository purposes only, the other two cases are of interest. Indeed, the total stock of oil and gas is estimated to exceed 8000 GtC if methane hydrates are included.\(^{19}\) For each case, we numerically solve the above problem for $\alpha = 0.01$ and for $\alpha = +\infty$.\(^{20}\)

\(^{19}\)Estimated resources of methane hydrates vary, but they alone can amount to as much as $2.1 \times 10^4 \text{GtC}$. Of course, only a small fraction of these resources is recoverable using today’s technologies. See Boswell and Collett (2011). See also Hartley, Medlock, Temzelides, and Zhang (2012) and references therein.

\(^{20}\)To draw an even closer comparison with GHKT, we have re-scaled $\gamma$ by a factor of $1/\theta$, where $\theta$ is the share of capital. The reason is that, given a Cobb-Douglas specification in final goods production, and given 100% depreciation of capital, a proportional damage of $e^{-\gamma S'}$ on capital is equivalent to a proportional damage of $e^{-\theta \gamma S'}$ on output. Accordingly, the mean and variance of $\gamma$ in the approximating model are set to $\bar{\gamma} = 7.93 \times 10^{-5}$ and $\sigma^2 = 2.65 \times 10^{-8}$, respectively.
Below we plot the same quantities as those shown in Figure 3.4 through Figure 3.6, but under the assumption that the approximating distribution of $\gamma$ is normal. Our focus here is to compare the effects of model uncertainty on optimal oil-use under different values of $R_0$. As we have discussed earlier, holding other parameters fixed, the optimal path of oil consumption is determined jointly by the resource scarcity effect and the model uncertainty effect. First, note that we can hardly identify a difference between the robust and the non-robust optimal path for oil-consumption when the scarcity effect dominates, that is, when $R_0$ is sufficiently small. Figure 3.7 shows that when $R_0 = 253.8 GtC$, the non-robust optimal paths replicate their counterparts in GHKT. In this case, model uncertainty delays the optimal use of oil only slightly. However, Figure 3.10 displays an altogether different pattern. When $R_0$ is set to $8000 GtC$, although both paths are still decreasing over time, model uncertainty discourages the use of oil substantively. Finally, as $R_0$ goes to infinity, as shown in Figure 3.12, we observe a qualitative difference between the two paths. On the one hand, the non-robust optimal path allows the use of oil to grow unboundedly, partially due to the technological progress in the coal and green sectors. On the other hand, the increasing trend in oil consumption is curbed due to the externality caused by carbon emissions.

We now turn to a comparative analysis of the damages resulting from fossil fuel consumption. GHKT assume $R_0 = 253.8 GtC$ and estimate damages of $56.9/ton$ of carbon using an annual discount rate of 1.5% and $496/ton$ under a rate of 0.1%. When $\beta = 0.985^{10}$, and if there is no concern about model uncertainty ($\alpha = \infty$), the welfare loss implied by our model equals $0.985^{10} \times 56.4 = 48.5/ton$. This number is independent of the approximating distribution for $\gamma$, the initial stock of oil, and of the future path of the GHG concentration. When $\alpha = 0.01$, however, these factors can matter substantially, as seen below. If the approximating distribution is normal,
the losses are given in the following Table.

<table>
<thead>
<tr>
<th>$R_0$ (GtC)</th>
<th>$\alpha = 0.01$</th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 1$</th>
<th>$\alpha = 100$</th>
<th>$\alpha = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>253.8</td>
<td>239.60</td>
<td>70.65</td>
<td>50.85</td>
<td>48.52</td>
<td>48.49</td>
</tr>
<tr>
<td>8000</td>
<td>276.60</td>
<td>90.60</td>
<td>55.08</td>
<td>48.57</td>
<td>48.49</td>
</tr>
<tr>
<td>$\infty$</td>
<td>318.70</td>
<td>103.06</td>
<td>63.42</td>
<td>56.49</td>
<td>48.49</td>
</tr>
</tbody>
</table>

### 3.4.2 Varying the Resource Feasibility Constraint

In order to further explore the model’s implications, we now report the results for the case where oil is in infinite supply, while coal is constrained under an initial stock $R_{coal} = 666 GtC$. This case demonstrates that the optimal use of oil mimics...
Figure 3.8: Increase in Global Temperatures when $R_0 = 253.8$
Figure 3.9: Capital Stock and Output when $R_0 = 253.8$
that in the case when both oil and coal are in infinite supply. In addition, the use of coal increases steadily at the beginning and then starts to drop.

3.5 Conclusion

We studied optimal taxation in a dynamic stochastic general equilibrium model where agents are concerned about model uncertainty regarding climate change. Our model builds on (GHKT, 2013). We used robust control theory in order to model the uncertainty associated with climate change. In addition, we used an estimate of fossil fuel that includes methane hydrates as part of the supply of unconventional natural gas. While this huge resource is not readily available with today’s technology, we believe that it is appropriate to include it given the long-term modeling that we follow throughout this exercise. Finally, we assumed a fat-tailed distribution of
damages as a way to capture the extreme effects discussed in Stern (2013).

We obtained a sharp analytical solution for the implied externality, and we characterized the optimal tax. We found that a small increase in the concern about model uncertainty can cause a significant drop in optimal energy extraction. The optimal tax which restores the social optimal allocation was shown to be Pigouvian. Under more general assumptions, we developed a recursive method that allowed us to solve the model computationally. We showed that the introduction of uncertainty matters in a number of ways, both qualitatively and quantitatively. This dependence relies heavily on specific assumptions about the magnitude of fossil fuel reserves.

Our model can be extended in many ways. In the current version, the growth rate of renewables is assumed to be independent from the concern about model uncertainty. It would be interesting to endogenize growth in renewable energy pro-
Figure 3.12: Optimal Use of Energy when $R_0 = \infty$
Figure 3.13: Optimal Use of Energy when $R_{\text{coal}} = 666$
Figure 3.14: Increase in Global Temperature when $R_{\text{coal}} = 666$
Figure 3.15: Capital Stock and Output when $R_{\text{coal}} = 666$
ductivity. A related extension could involve using a distortionary tax on labor to subsidize R&D in renewables in order to study the effects on energy composition and growth. Additionally, we could study a benchmark case where coal supply is constrained, while assuming infinite supply of gas and oil.
Bibliography


[38] M. Fuentes and D. Saravia, “Sovereign defaulters: Do international capital markets punish them?” *Journal of Development Economics*, vol. 91, no. 2,


Appendix I

The appendix is organized as follows. **A1** provides a partial equilibrium analysis. Specifically, we first show that M’s optimization problem can be reduced to a simple substitution of the equilibrium interest rate into the crisis probability. We then show that G’s optimization problem can be seen as solving an infinite horizon Markov Decision Process (MDP) on the equilibrium path, and a static MDP off the equilibrium path. **A2** proves Proposition 1.

A1. Partial Equilibrium Analysis

In our model, the three players play an infinitely repeated game, where L, G and M move sequentially in each period. Because we assume a competitive international loan market, M’s problem is to choose an interest rate that satisfies the following condition:

\[ [1 - \bar{p}(s, x, d_s, r)]r - \bar{p}(s, x, d_s, r) = 0. \]  

The expected crisis probability \( \bar{p}(s, x, d_s, r) \), conditional on the observables \((s, x)\) and \(d_s\), is updated according to the Bayes Rule:

\[
\bar{p}(s, x, d_s, r) = \mathbb{E}\{p(s|d_s, x|d_x, r, \theta)|s, x, d_s, r\} \\
= \sum_{\theta \in \{\theta_1, \theta_2\}} \sum_{d_x \in \{0, 1\}} p(s|d_s, x|d_x, r, \theta) Pr(d_x|\theta, s, x, r) Pr(\theta) \\
= \frac{\sum_{\theta \in \{\theta_1, \theta_2\}} \sum_{d_x \in \{0, 1\}} p(s|d_s, x|d_x, r, \theta) Pr(d_x|\theta, s, x, r) Pr(\theta)}{\sum_{\theta \in \{\theta_1, \theta_2\}} \sum_{d_x \in \{0, 1\}} Pr(d_x|\theta, s, x, r) Pr(\theta)},
\]

where \( Pr(d_x, d_s|\theta, s, x, r) \), which will be explored in greater details later, is the probability of G to choose \((d_x, d_s)\) given L’s offer \((s, x)\) and M’s choice of \(r\).

M’s problem implies that \( r = r(d_s, s, x) \). That is, given L’s offer \((s, x)\), the equilibrium interest rate is an implicit function of G’s choice, \(d_s\). Computationally, then, we can solve M’s problem first and substitute \( r = r(d_s, s, x) \) into \( p(s_G, x_G, r; \theta) \),
which enters into $L$’s and $G$’s utility functions. For simplicity, we rewrite the crisis probability as $p(s_G, x_G; \theta)$.

In contrast, $L$ and $G$ face dynamic problems. Their actions today affect their value functions (or continuation values). We now show that $G$ solves an infinite horizon Markov Decision Process (MDP) on the equilibrium path, and a static MDP off the equilibrium path. In doing so, we also derive $L$’s calculation of $G$’s choice probabilities both on and off the equilibrium path, which will be useful for the proofs of Lemma 1 and Proposition 1.

(1) $G$’s strategy on the equilibrium path

On the equilibrium path, $L$ offers a time invariant plan $(s_b, x_b)$ and $(0, x_p)$, and $G$ maximizes its expected value function by choosing $(d_x, d_s)$ in each period. Thus, $G$ solves a recursive dynamic programming problem on the equilibrium path. Moreover, since $G$’s current choice determines the phase (state) of the next period game, which in turn defines its next period action space, $G$’s problem is one of Markov Decision Process (Rust 1994).

Because $G$ has complete information about the shock to its economy, $\epsilon$, it makes a deterministic decision about $(d_x, d_s)$ in equilibrium. On the other hand, $L$ does not know $\epsilon$, therefore, $L$ derives a system of conditional choice probabilities regarding $G$’s decision. We now show how $G$ makes the optimal choice given $\epsilon$, and how $L$ calculates $G$’s choice probabilities without knowing the realization of $\epsilon$.

Let $I_p \in \{0, 1\}$ be an indicator of the phase of the current phase of the game, where 0 represents the bargaining phase, and 1 the punishment phase. Define $A_G(I_p)$ as the set of available actions of $G$ in phase $I_p$, where $A_G(0) = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$ and $A_G(1) = \{(0, 0), (1, 0)\}$. In addition, given $\epsilon$, define $G$’s value function, $V_G(\cdot)$,
and choice-specific value function, \( v_G(\cdot) \), as:

\[
V_G(I_p, \theta, \epsilon) = \max_{(d_x, d_s) \in A_G(I_p)} [v_G(I_p, \theta; d_x, d_s) + \epsilon(d_x, d_s)],
\]

and

\[
v_G(I_p, \theta; d_x, d_s) = \tilde{u}_G(I_p, \theta; d_x, d_s) + \delta \sum_{\theta'} \sum_{I_p'} \int V_G(I_p', \theta', \epsilon') \phi(\epsilon') d\epsilon' Pr(I_p'|I_p; d_x, d_s) Pr(\theta').
\]

Here, \( \tilde{u}_G(\theta; d_x, d_s) \) is an abbreviation for \( \tilde{u}_G(\theta; d_x, d_s; s_b \times I_p, x_b \times (1 - I_p) + x_p \times I_p, r) \) and the superscript on the right hand side (RHS) denotes the next period.

Equations (3) and (4) define a contraction mapping operator (Rust 1994); therefore, the choice-specific value function, \( v_G(I_p, \theta; d_x, d_s) \), can be solved by value function iteration. This is means \( G \)'s optimal choice on the equilibrium path can be expressed as:

\[
(d_{x}^{on}, d_{s}^{on}) = \arg \max_{(d_x, d_s) \in A_G(I_p)} [v_G(I_p, \theta; d_x, d_s) + \epsilon(d_x, d_s)].
\]

To calculate \( L \)'s conditional choice probabilities regarding \( G \)'s decision, we assume that \( \epsilon \) (a vector) has a multivariate extreme-value distribution, which leads to a conditional multinomial logit representation of the choice probabilities:

\[
Pr(d_x, d_s|I_p, \theta) = \Pr \{ v_G(I_p, \theta; d_x, d_s) + \epsilon(d_x, d_s) \geq v_G(I_p, \theta; \tilde{d}_x, \tilde{d}_s) + \epsilon(\tilde{d}_x, \tilde{d}_s), \forall (\tilde{d}_x, \tilde{d}_s) \in A_G(I_p)|I_p, \theta \}
\]

\[
= \frac{\exp[v_G(I_p, \theta; d_x, d_s)]}{\sum_{(d_x, d_s) \in A_G(I_p)} \exp[v_G(I_p, \theta; d_x, d_s)]}
\]

That is, from \( L \)'s perspective, given state \( (I_p, \theta) \), \( G \) will choose the action \((d_x, d_s)\) with probability \( Pr(d_x, d_s|I_p, \theta) \). Furthermore, \( L \) can derive \( G \)'s choice probabilities conditional only on \( I_p \), \( Pr(d_x, d_s|I_p) \), by integrating out \( \theta \) in \( Pr(d_x, d_s|I_p, \theta) \). Using the probabilities \( L \) can calculate its expected utility from offering an arbitrary pair.

---

21 Note that although \((s_b, x_b, x_p)\) are the state variables in \( G \)'s problem, we suppress the notations in the equations because they are parameters for the analysis of \( G \)'s equilibrium strategy.
of \((s, x)\) to \(G\), and therefore be able to choose the pair that gives it the highest expected utility in equilibrium.

(2) Off the equilibrium path

Off the equilibrium path, \(L\) makes an offer, \((s_o, x_o)\), which is independent of history, and \(G\) chooses \((d_x, d_s)\) to maximize its one-period utility. Therefore, \(G\)’s optimal decision off the equilibrium path is reduced to a static MDP, for which there exists an analytical solution. Given an arbitrary \((s_o, x_o)\) of the equilibrium path, \(G\)’s optimal choice off the equilibrium path is reduced to a static MDP, for which there exists an analytical solution. Given an arbitrary \((s_o, x_o)\) of the equilibrium path, \(G\)’s optimal choice off the equilibrium path is reduced to a static MDP, for which there exists an analytical solution.

\[
(d_x^{off}, d_s^{off}) = \arg \max_{(d_x, d_s) \in \mathcal{A}(I_p)} [\bar{u}_G(\theta; d_x, d_s) + \epsilon(d_x, d_s)] ,
\]

and

\[
Pr(d_x, d_s|\theta) = \frac{\exp[\bar{u}_G(\theta; d_x, d_s)]}{\sum_{(d_x, d_s) \in \mathcal{A}(I_p)} \exp[\bar{u}_G(\theta; d_x, d_s)]}
\]

A2. Proof of Proposition 1

We first prove Lemma 1, which shows that given \(G\)’s equilibrium strategy \(L\) has no incentive to deviate from the equilibrium path. In addition, it shows how \(L\) will behave off the equilibrium path, given \(G\)’s equilibrium strategy.

Assumption 1: \(-\frac{\partial^2 p(s, x, \theta)}{\partial s^2} \leq 0\) for all \(x \in [0, \bar{x}]\), \(s \in [0, \bar{s}]\), and \(\theta \in \{\theta_1, \theta_2\}\). That is, the marginal crisis reducing effect of \(s\) decreases as \(s\) increases.

Assumption 2: \(c \geq \frac{1+\beta}{1-\beta}(1 + \frac{\bar{x}}{1+e^{\bar{x}}}) \left(-\frac{\partial p(s, \theta)}{\partial s} \right)_{s=0}\) for all \(\beta \in [0, \bar{\beta}]\) and \(\theta \in \{\theta_1, \theta_2\}\), where \(\bar{x} = \arg \max \{\frac{s}{e^{sx}+1}\}\). That is, the unit cost of loan, \(c\), is sufficiently large.

Lemma 1. Suppose assumptions 1 and 2 are satisfied, and \(b\) is sufficiently large. Then, given \(G\)’s off-the-equilibrium strategy, offering \((s_o, x_o) = (0, 0)\) weakly dominates all other offers off the equilibrium path for \(L\).
Proof. Let $Eu_L(s, x; \theta) = \sum_{(d^*_x,d^*_s)} Pr(d^*_x, d^*_s | s, x; \theta) u_L(d^*_x, d^*_s; \theta)$ be $L$’s expected one-period utility conditional on $\theta$, and $Eu_L(s, x) = \sum_{\theta \in \{\theta_1, \theta_2\}} \pi(\theta) Eu_L(s, x; \theta)$ be $L$’s unconditional utility. To show $L$’s optimal strategy is to offer $(s, x) = (0, 0)$ off the equilibrium path, it suffices to verify that, under the two assumptions $Eu_L(s, x; \theta)$ is decreasing in $s$ for any $x \in [0, \bar{x}]$ and $\theta \in \{\theta_1, \theta_2\}$.

First, fix $x$ and $\theta$, and evaluate the expectation with respect to $G$’s choice probabilities conditional on $\theta$. Consider a change from $(0, x)$ to $(s, x)$, where $s > 0$, we want to show $Eu_L(0, x; \theta) - Eu_L(s, x; \theta) \geq 0$.

\[
Eu_L(0, x; \theta) - Eu_L(s, x; \theta) = [Pr(0, 0 | 0, x, \theta)u_L(0, 0; \theta) - Pr(0, 0 | s, x, \theta)u_L(0, 0; \theta)] \\
+ [Pr(1, 0 | 0, x, \theta)u_L(0, 0; \theta) - Pr(1, 0 | s, x, \theta)u_L(0, 0; \theta)] \\
+ [Pr(0, 1 | 0, x, \theta)u_L(0, 0; \theta) - Pr(0, 1 | s, x, \theta)u_L(s, 0; \theta)] \\
+ [Pr(1, 1 | 0, x, \theta)u_L(0, 0; \theta) - Pr(1, 1 | s, x, \theta)u_L(s, x; \theta)]
\]

Using $u_L(0, x; \theta) = u_L(0, 0; \theta)$ and eq(8) to simplify the above equation, we have

\[
Eu_L(0, x; \theta) - Eu_L(s, x; \theta) = Pr(0, 1 | s, x, \theta)[u_L(0, 0; \theta) - u_L(s, 0; \theta)] - Pr(1, 1 | s, x, \theta)[u_L(s, x; \theta) - u_L(0, 0; \theta)] \\
= \beta e^{-p(s,0;\theta)}[-p(0,0,\theta) + \bar{c}s + p(s,0;\theta)] - e^{-bx-p(s,x;\theta)}[-\bar{c}s - p(s,x;\theta) + p(0,0;\theta)] \\
= \beta e^{-p(s,0;\theta)} e^{-p(0,0;\theta)} + e^{-p(s,0;\theta)} + e^{-bx-p(0,x;\theta)} + e^{-bx-p(s,x;\theta)}
\]

where $\bar{c} = \frac{(1-\beta)c}{1+\beta}$.

Since the denominator in the above equality is strictly positive, to show $Eu_L(0, x; \theta) - Eu_L(s, x; \theta) \geq 0$, we only need to show

\[
e^{-p(s,0;\theta)}[-p(0,0,\theta) + \bar{c} + p(s,0;\theta)] - e^{-bx-p(s,x;\theta)}[-\bar{c}s - p(s,x;\theta) + p(0,0;\theta)] \geq 0,
\]

which is equivalent to

\[
\bar{c} \geq \frac{e^{-bx-p(s,x;\theta)}[p(0,0,\theta) - p(s,x;\theta)] + e^{-p(s,0;\theta)}[p(0,0,\theta) - p(s,0,\theta)]}{se^{-p(s,0;\theta)} + e^{-bx-p(s,x;\theta)}} \tag{9}
\]
If Assumption 1 is satisfied and the unit cost of implementation the reforms, $b$, is sufficiently large, then we can show that the supremum of the RHS of eq(9) is attained as $s$ converges to 0. That is,

$$
\sup_{s \in [0, s]} \{ \text{RHS of eq(9)} \} = \lim_{s \to 0} \frac{e^{-bx-p(s,x;\theta)}[p(0,0,\theta) - p(s,x;\theta)] + e^{-p(s,0;\theta)}[p(0,0,\theta) - p(s,0,\theta)]}{s[e^{-p(s,0;\theta)} + e^{-bx-p(s,x;\theta)}]}
= \frac{e^{-bx}}{1 + e^{-bx}} \left( - \frac{\partial p(s,x;\theta)}{s|_{s=0}} \right) + \frac{1}{1 + e^{-bx}} \left( - \frac{\partial p(s,0;\theta)}{s|_{s=0}} \right)
= (1 + \frac{x}{1 + e^{bx}}) \left( - \frac{\partial p(s,0;\theta)}{s|_{s=0}} \right),
$$

(10)

where the second equality is obtained by applying Hopital’s rule, and the last equality results from the fact that $\frac{\partial p(s,x;\theta)}{s|_{s=0}} = (1 + x) \frac{\partial p(s,0;\theta)}{s|_{s=0}}$.

Now, relax $x$. Then a sufficient condition for $EU_L(0,x;\theta) - EU_L(s,x;\theta) \geq 0$ is

$$
c \geq \frac{1 + \beta}{1 - \beta} (1 + \frac{x}{1 + e^{bx}}) \left( - \frac{\partial p(s,0;\theta)}{s|_{s=0}} \right)
$$

(11)

for all $\beta \in [0, \bar{\beta}]$ and $\theta \in \{\theta_1, \theta_2\}$, where $\bar{x} = \arg \max \{\frac{x}{e^{bx}+1}\}$.

Now we turn to the proof of Proposition 1.

**Proof.** $G$’s equilibrium strategy

On the equilibrium path: Because $L$ cannot observe $\epsilon$, it cannot detect whether $G$ deviated from its equilibrium strategy. This in turn means that $G$ cannot influence $L$’s strategy by deviating from the equilibrium path. Consequently, $G$’s can at best optimize its expected value function given $L$’s optimal offer on the equilibrium path.

Off the equilibrium path: Since off the equilibrium path $L$ makes an offer, $(s_\alpha, x_\alpha)$, which is independent of history, it follows that $G$’s choice does not have any inter-temporal effect, thus it is optimal for $G$ to maximize its one-period utility.

$L$’s equilibrium strategy

For this part we first show that $L$’s off-the-equilibrium strategy is the best response to $G$’s off-the-equilibrium strategy. We then show that $L$ has no incentive to
deviate from the equilibrium path.\textsuperscript{22}

Off the equilibrium path: Given that \( G \) switches to a myopia strategy off the equilibrium path by maximizing its one-period utility, \( L \) could at best choose an offer \((s, x)\) that maximizes its one-period payoff. Lemma 1 shows that if the unit cost of loan, \( c \), is sufficiently large and the marginal crisis reducing effect of \( s \) decreases as \( s \) increases, then \( L \)'s optimal offer in this case is \((s_o, x_o) = (0, 0)\).

Note that when \( L \) offers \((0, 0)\), it will receive an expected one-period payoff of

\[
u_L = \sum_{i=1}^{2} \pi(\theta_i) u_L(s = 0, x = 0; \theta_i),
\]

which is independent of \( G \)'s strategy and whether the game is on or off the equilibrium path. We will used the fact in the next part of the proof.

On the equilibrium path: since the option of offering \((0, 0)\) is still available, \( L \) can guarantee a payoff of \( u_L \) defined above in each period by offering \((0, 0)\). It follows that \( L \) has no incentive to deviate from the equilibrium path.

Now we show how \( L \) determines the optimal offer \((s_b^*(\beta), x_b^*(\beta))\) for the bargaining phase, and \( x_p^*(\beta) \) for the punishment phase. Using the conditional choice probabilities of \( G \)'s decision, \( \Pr(d_x, d_s|I_p, \theta) \), in eq (6), \( L \) maximizes its value function w.r.t. \( s_b, x_b \) and \( x_p \). Since \( G \)'s choice probabilities cannot be expressed as analytical functions of \( s_b, x_b \) and \( x_p \), \( L \)'s expected value function is not a closed form function, either. We therefore use numerical methods to calculate \( s_b^*(\beta), x_b^*(\beta), \) and \( x_p^*(\beta) \).

Let \( V^L(I_p, \theta) \) denote \( L \)'s value function conditional on the observable state \( I_p \) and the unobservable state \( \theta \). Since both \( I_p \) and \( \theta \) are binary variables, \( V^L \) can be represented by a 4 \( \times \) 1 column vector, \( V^L = (V_{01}^L, V_{02}^L, V_{11}^L, V_{12}^L)' \), where \( V_{ij}^L \) is \( L \)'s value function when \( I_p = i \) and \( \theta = \theta_j \) for \( i \in \{0, 1\}, j \in \{1, 2\} \). Then, by definition,

\textsuperscript{22}Note that in this game there is no Bayesian updating for \( L \) since \( \theta \) and \( \epsilon \) are independently drawn in each period. \( L \) calculates its expected utility using the stationary distributions of \( \theta \) and \( \epsilon \).
$V^L$ solves the following linear system of equations:

$$V^L = Eu^L + \delta_L P^L V^L \quad (12)$$

Here, $P^L$ is a $4 \times 4$ transition probability matrix, where $P^L_{2i+m,2j+n}$ denotes the probability for $G$ to transit from $I_p = i$ and $\theta = \theta_m$ to $I_p' = j$ and $\theta' = \theta_n$ for $m, n \in \{1, 2\}$. In addition, $Eu^L = (Eu^L_{01}, Eu^L_{02}, Eu^L_{11}, Eu^L_{12})'$ is a column vector of $L$’s current expected utility, where $Eu^L_{ij}$ is $L$’s current period expected utility when $I_p = i$ and $\theta = \theta_j$ for $i \in \{0, 1\}, j \in \{1, 2\}$. Accordingly, $Eu^L_{ij}$ is given by:

$$Eu^L_{ij} = \sum_{(d_x,d_s) \in A_G(I_p=i)} Pr(d_x,d_s|I_p = i, \theta = \theta_j) \times u_L(s, x; d_x, d_s, r; \beta, \theta = \theta_j) \quad (13)$$

Since $L$ cannot observe $\theta$ when making an offer, it follows from eq (12) that $L$’s objective function is:

$$\max_{(s_b,x_b,x_p)} \sum_{j \in \{1, 2\}} \pi_j V^L_{0j}, \quad (14)$$

where $\pi = (\pi_1, \pi_2)$ is the stationary distribution of $\theta$.

In the second step, we employ numerical methods to find the globally optimal $s_b^*(\beta)$, $x_b^*(\beta)$ and $x_p^*(\beta)$ of eq (14). We then obtain the equilibrium interest rates given by eq (1), and the equilibrium choice probabilities given by eq (6). \qed
Appendix II

Here we demonstrate that the optimal level of GHG, $E^*$, has the following properties: $\frac{\partial E^*}{\partial \delta} < 0$ and $\frac{\partial E^*}{\partial \delta}|_{\delta=0} = -\infty$, where $\delta$ is the upper bound for entropy allowed in the constraint game.

Proof. Recall that $E^* = c_E(1 - \Delta S)$ and $\delta = \log(1 - \Delta S^*) + \frac{\Delta S^*}{1 - \Delta S^*}$, where $S^* = S + \phi_0 c_E(1 - \Delta S)$. Define $a = \alpha^{-1}$ and $b = 1 - \Delta S^* = (1 - \Delta \phi_0 c_E)(1 - \Delta S)$. It follows immediately that $E^*$ is decreasing in $a$. In addition, since both $\Delta$ and $c_E$ are functions of $a$, it follows that $b$ is a function of $a$:

$$b(a) = [1 - \Delta(a)\phi_0 c_E(a)](1 - \Delta(a)S)$$

(1)

It is easy to see that $b$ is decreasing in $a$. Thus, it defines $a$ as an implicit function of $b$, with a negative slope. Moreover, we can rewrite $\delta$ as:

$$\delta = \log b + \frac{1 - b}{b}$$

(2)

which defines $b$ as an implicit function of $\delta$. Direct calculation shows that $\frac{\partial b}{\partial \delta} = -\frac{a^2}{1 - b} < 0$, as $b \in (0, 1)$. Thus,

$$\frac{\partial E^*}{\partial \delta} = \frac{\partial E^*}{\partial a} \frac{\partial a}{\partial b} \frac{\partial b}{\partial \delta} < 0$$

(3)

Evaluating this at $\delta = 0$, we obtain

$$\frac{\partial E^*}{\partial \delta}|_{\delta=0} = \left(\left.\frac{\partial E^*}{\partial a}\right|_{a=0}\right) \left(\left.\frac{\partial a}{\partial b}\right|_{b=1}\right) \left(\left.\frac{\partial b}{\partial \delta}\right|_{\delta=0}\right)$$

(4)

It is straightforward to show that the first two terms on the right hand side in the above expression are strictly negative and finite, and the last term goes to $-\infty$. Therefore, $\frac{\partial E^*}{\partial \delta}|_{\delta=0} = -\infty$.  

\[\square\]