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Novel Waveguide Structures in the Terahertz Frequency Range

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ABSTRACT

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Over the last decade, considerable research interest has peaked in realizing an efficient Terahertz (THz) waveguide for potential applications in imaging, sensing, and communications applications. Two of the promising candidates are the two-wire waveguide and the parallel-plate waveguide (PPWG). I present theoretical and experimental evidence that show that the two-wire waveguide supports low loss terahertz pulse propagation, and illustrate that the mode pattern at the end of the waveguide resembles that of a dipole. In comparison to the weakly guided Sommerfeld wave of a single wire waveguide, this two-wire structure exhibits much lower bending losses. I also observe that a commercial 300-Ohm two-wire TV-antenna cable can be used for guiding frequency components of up to 0.2 THz, although these cables are generally designed to operate only up to about 800 MHz. The parallel-plate waveguide is another promising candidate that would make an efficient THz waveguide, since it has relatively low Ohmic losses. The transverse electromagnetic mode (TEM) of this waveguide has been generally preferred since it has no cutoff frequency, and therefore no group velocity dispersion. Utilizing this
TEM mode, I study the reflection of THz radiation at the end of a PPWG, due to the impedance mismatch between the propagating transverse-electromagnetic mode and the free-space background. I find that for a PPWG with uniformly spaced plates, the reflection coefficient at the output face increases as the plate separation decreases, consistent with predictions by early low frequency ray optical theory. I observe this same trend in tapered PPWGs, when the input separation is fixed, and the output separation is varied. In another study, I investigate how to minimize diffraction losses in PPWGs by using plates with slightly concave surfaces. Using a simple “bouncing plane wave” analysis, I demonstrate how to determine an ideal radius of curvature for a waveguide operating at a given THz frequency. I perform a detailed experimental and simulation study that illustrates, for a waveguide with a plate separation of 1 cm, one can inhibit the diffraction around a frequency of 0.1 THz, when the surface has a curvature of 6.7 cm. Using much longer PPWGs (about 170 cm), I reliably measure the overall losses in a PPWG with a radius of curvature of $R=6.7$ cm, and find it to be less than 1 db/m around the design frequency (of 0.1 THz). This is very close to the lowest achieved loss to date with any terahertz waveguide.
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Chapter 1

Introduction

1.1. The Terahertz Spectral range

Over the last century, scientists have made great advances in understanding electromagnetic waves [1]. Electromagnetic waves range from radio waves and microwaves on the lower end of the electromagnetic spectrum, to ultraviolet, x-rays, and gamma rays on the higher end of the spectrum, as shown in figure 1-1. Sandwiched between the microwave and far-infrared parts of the spectrum is the Terahertz (THz) spectral range [2, 3].

The THz band encompasses a broad range of frequencies from 100 GHz to 10 THz, and bridges the gap between the microwave and optical regimes, to bring together many technologies otherwise sequestered to either the long or short ends of the spectrum. 1 THz = $10^{12}$ Hz = 0.3 mm in wavelength = 1 ps in period = 4 $meV$ in energy.
Figure 1-1 The electromagnetic spectrum. The Terahertz spectral range is sandwiched between the electronics and photonics regimes. Courtesy: The THz technology network image gallery.

THz radiation is non-ionizing and shares with microwave radiation the capability to penetrate a wide variety of non-conducting materials (wood, plastic, rubber, non-polar molecules, etc.). In addition, many chemical and biological molecules have distinctive THz spectral fingerprints. These properties and the ability to resolve picosecond dynamics make the THz spectral range one of the most versatile regions of the electromagnetic spectrum.

1.2. Terahertz Imaging and Spectroscopy

The far-infrared frequency regime has been utilized for spectroscopic purposes for several decades [4]. However, far-infrared radiation was first generated in the early 70’s with the development of ruby lasers and via optical rectification in a non-linear LiNbO$_3$ crystal [5]. In a review paper a few years later, Shen and his co-workers summarized subsequent work in the burgeoning field of far-infrared radiation generation via optical mixing in a non-linear crystal [6].
With the invention and improvement of femtosecond (fs) dye lasers [7, 8], the first broadband THz sources were designed - in the form of photoconductive switches [9-12]. The development of these switches led to the birth of terahertz time domain spectroscopy (THz TDS). The basis of THz-TDS is coherent coupling of generation and detection methods, either with another time gated switch or via electro-optic sampling [13]. This coherent coupling is achieved using ultrafast femtosecond laser pulses (with pulse widths of ~100 fs) to generate and detect single cycle electromagnetic (THz) pulses. In particular, coherent detection allows one to measure the electric field of broadband THz radiation in the time domain, which upon transformation into the frequency domain yields both the amplitude and the phase information over the entire bandwidth of the pulse.

This unique property is the main advantage of THz-TDS compared to most other spectroscopy techniques that only measure the amplitude, and thus provide an incomplete description of a system’s optical properties. Also, because the measurement THz-TDS technique is coherent, it consistently filters out incoherent radiation. Moreover, since the time measurement window is extremely narrow, the noise contribution to the measurement is extremely low. The signal-to-noise ratio (S/N) of the time-domain pulse mostly depends on experimental conditions (for instance, the averaging time), and due to the coherent detection technique, high S/N values (>60 dB) are feasible with 1 minute averaging times.

The discovery of fs-pulse generation from solid state Ti:Sapphire lasers [14] proved to be the watershed moment for the field as their high average power and
small temporal pulse widths dramatically increased the signal and bandwidth of THz spectrometers, while at the same time decreasing their size, price, and complexity. These improvements had a dramatic impact on this field of research, and it is now possible to focus on realizing many applications for THz radiation, which were previously impractical.

Early THz TDS research centered on characterizing the spectra of small molecules and gases which displayed rotational and vibrational transitions in the THz range \cite{15, 16}. Spectroscopy studies then turned to liquids \cite{17-19}, solids \cite{20-23}, biological molecules \cite{24-27}, and explosives \cite{28, 29}. Later there were unique studies on μL chemical component identification \cite{30}, and in-situ flame analysis \cite{31} which are only possible with THz spectroscopy. Thereafter, time-resolved and pump-probe techniques were developed which enabled the characterization of transient charge mobility \cite{32}, conductivity \cite{33, 34}, carrier relaxation dynamics \cite{35}, and charge transport processes \cite{36} happening at sub-ps time scales.

Besides spectroscopy, THz imaging \cite{37, 38} has also become an important field of study. Since it can provide contrasting absorption properties in a wide range of materials, THz radiation can produce sub-mm resolution images containing greater information (spectral amplitude and phase) than most conventional optical imaging techniques. THz imaging applications include quality control \cite{39}, defect analysis \cite{40}, security screening \cite{41}, and medical diagnostic imaging \cite{42}.

The future looks bright for THz spectroscopy and imaging applications, particularly with the recent development of fiber coupled plug-and-play THz
systems. Unlike gas lasers that were bulky, expensive, and complex needing expert training to operate, these new systems are cost effective, easy to operate, and can be installed to operate in nearly any condition to achieve high signal, large bandwidth, and high sample throughput – while requiring minimal computational effort to generate real-time results. These attributes together with its optical and spectroscopic properties and low energy make THz spectroscopy a powerful non-contact, non-ionizing and non-destructive analytical tool.

1.3. Terahertz Waveguides

In general, a waveguide [3] is any structure used to guide the propagation of electromagnetic waves while confining them near the propagation axis without significant loss in intensity. Other desirable properties of an ideal waveguide include broadband spectral guiding, and low dispersion. The frequency range of operation for any waveguide is dictated by the physical design and material properties of the waveguide. For instance, the most common type of waveguide for radio waves and microwaves are hollow metal tubes and coaxial cables. In the hollow metal tube, the wave is usually confined in the interior of the pipe. In the coaxial line, the propagating wave is confined in the dielectric layer between the inner conductor and the outer metal shield. In the optical region, silica based optical fibers are usually the ideal choice due to their low attenuation and low dispersion in the optical communication band (1.3-1.6 μm).
Currently, several THz applications rely heavily on the free space transport of terahertz waves via bulk optical components such as polyethylene, teflon, and silicon lenses and parabolic mirrors. However, in many real world applications, the sample or region to be measured may not be readily accessible to a line of sight beam. Commercial devices such as optical fiber based sensors and medical endoscopes exploit the guided delivery of light to the remote sensing location. For this kind of technology to be extended to the THz region, the development of an efficient THz waveguide is required.

The search is still ongoing for the “ideal waveguide” in the THz frequency range (100 GHz – 10 THz). The development of such a waveguide has been slowed down by the material properties and challenging application requirements in this spectral range. The materials most transparent to THz waves are crystalline (for instance, high resistivity Silicon), and are costly, fragile, and difficult to machine into required waveguide geometries. In spite of this, several waveguiding techniques have been investigated for the propagation of broadband THz pulses over the past decade. These techniques include coplanar transmission [43], silicon-on-insulator materials with a microstrip geometry [44], thin film microstrip lines [45], sub-mm circular metallic tubes [46], rectangular waveguides [47], dielectric fibers [48], dielectric slabs [49], parallel plate structures [50], photonic crystal fibers [51], 2-D photonic crystals [52], coaxial lines [53], single metal wires [54-57], dual metal wires [58], metallic slits [59], metal deposited hollow glass/polymer tubes [60, 61], and hollow polymer fibers [62-64]. Of all these waveguides, metal parallel-plate waveguides have shown the most promise since they exhibit low ohmic losses (0.1 cm⁻¹),
negligible dispersion, and support single mode propagation. In particular, the lowest order transverse magnetic mode (TM$_{0}$) of this waveguide can be easily coupled into and out of the waveguide by use of common lens-optical elements. Last, these metal waveguides can be machined with great ease in comparison with other THz waveguides.

### 1.4. Terahertz Generation and Detection

There are generally two methods for generating Terahertz radiation: with photoconductive antennas or via optical rectification. There are also two main ways to detect THz radiation: with photoconductive antennas (PCA) or via electro-optic sampling. Here we will focus on the most common technique for Terahertz generation and detection - using photoconductive antennas [9, 11, 12]. The PCA consists of a highly resistive direct-band gap semiconductor thin film with two electric contact pads. The film is made in most cases using a III-V compound semiconductor like GaAs. This GaAs is frequently epitaxially grown on a semi-insulating GaAs substrate (SI-GaAs), which is also a highly resistive material. The main variation between the SI-GaAs substrate and the film is the relaxation time for excited carriers. In a SI-substrate the carrier lifetime is about 500 ps, but in the film (GaAs) it is shorter than 1 ps. To shorten the carrier lifetime, one of the following techniques can be used: 1) impurity doping of the thin film [65] 2) growth of a polycrys-talline or amorphous material [66], 3) and damage by ion implantation [67]. A metalized antenna is deposited on the GaAs substrate via optical lithography.
During operation, an optical beam (with a wavelength centered around 800 nm) is focused on the gap between the electrodes. This wavelength exceeds the bandgap of the GaAs, with the optical beam photon energy \( E = h\nu \) being larger than the band gap energy \( E_g \) of GaAs, where \( E_g = 1.43 \) eV for GaAs at room temperature. When the laser pulse impinges on the GaAs substrate, each absorbed optical photon creates a free electron in the conduction band and a hole in the valence band of the film. These free carriers make the gap between the electrical contacts electrically conducting for a short time. The carriers are then accelerated by the bias field and decay with a time constant determined by the free-carrier lifetime, resulting in a pulsed photocurrent in the photoconductive antenna. Current modulation occurs in the subpicosecond regime and thus radiates a subpicosecond electromagnetic transient (THz radiation). For a Hertzian dipole antenna in free space, the terahertz electric field is proportional to the time derivative of the transient photocurrent, i.e.

\[
E_{\text{THz}} \approx \frac{\partial i(t)}{\partial t} \quad [1-1]
\]

In the case of the THz receiver, a sensitive current detector (ammeter) is attached to the electrodes, instead of the DC bias. Free carriers are generated in the same way as in the emitter, by focusing the optical beam on the gap between the electrodes. The THz electric field radiation from the emitter arrives at the same time as the free carriers being generated in the THz receiver. When the THz radiation is focused on the gap between the THz receiver electrodes, the free carriers are accelerated. The detected current is proportional to the convolution of the THz field
and the semiconductor response. For that reason, the THz pulse duration, and thus the spectrum bandwidth of THz radiation, is limited by the carrier lifetime of the detector and frequency dependence of its antenna structure. A good candidate for the receiver (at a wavelength of 800 nm) is low-temperature-grown Gallium Arsenide (LT-GaAs) as it has a short carrier lifetime, matched band-gap, and low absorption.

A typical THz-TDS setup is shown in figure 1-2. A beam of ultrafast laser pulses (with a ~ 800 nm wavelength and ~ 100 fs pulse duration) is split into a pump beam for THz generation and a probe beam for THz detection. Since the pulse width of the laser pulse is much shorter than the duration of the THz electric field, the aforementioned detection process records only an instantaneous value of the THz electric field.
Figure 1-2. A typical terahertz time-domain set-up. Light from a pulsed laser is split into two arms; one arm is used to excite the source chip made out of GaAs, while the other one is used to excite the detecting chip made out of LT-GaAs. Courtesy: The THz technology network image gallery.

In order to capture the complete THz waveform, a linear scanning stage is used to vary the travel time of the probe laser relative to the THz radiation, thereby enabling the time-domain waveform of THz radiation to be sampled by measuring the current response. Usually, a silicon hemispherical lens is attached on the backside of the substrate to increase the coupling of THz radiation into free space for the THz emitter, and to focus THz electric field onto the electrode gap for the THz receiver.
A commercial fiber coupled system from Picometrix Inc was used in two of our experiments. This system utilizes fiber-coupled photoconductive antennas for THz generation and detection. A photograph of a commonly used commercial Terahertz system is shown in figure 1-3. A typical time domain THz waveform from this system is shown in figure 1-4 (top). The single cycle electromagnetic pulse detected, has a pulse duration of several picoseconds. The corresponding spectra shown in figure 1-4(bottom) illustrate a broad bandwidth extending to 1 THz, and a linear phase within the bandwidth.

Figure 1-3. The schematic shows a typical THz time domain system. Optical fibers are used to connect the THz emitter and THz receiver with the freespace pump and probe optical beams, respectively
Figure 1-4. Top: A typical THz time-domain waveform. Bottom: Fourier transform of that time domain waveform, plotted along with its corresponding phase.
Chapter 2

A Two-wire Terahertz waveguide

2.1. Introduction and Preliminary work

Over the past decade, tremendous progress has been made in the study of terahertz (THz) waveguides. Single mode coupling and propagation of THz radiation has been demonstrated in circular and rectangular metal waveguides [68], dielectric fibers [69], plastic ribbons [70], parallel-plate waveguides [71-73], bare metal wires [54, 74-76], hollow glass waveguides [77], planar plasmonic devices [78], photonic crystal fibers [79, 80], and hollow polymer optical fibers [81]. In particular, a bare metal wire has attracted a great deal of attention for several reasons, including the simplicity of the design and its connection to plasmonics [82]. Nevertheless, since the primary mode (the Sommerfeld wave) of a single wire is radially polarized, the commonly used linearly polarized photoconductive antennas cannot be used for the efficient excitation of this mode, and thus efficient coupling to a single wire requires
the use of novel photoconductive antennas [76]. Moreover, the weakly guiding nature of the Sommerfeld wave leads to high losses when the wire is bent, which limits the practical applications of this waveguide [74, 75].

Not long ago, we demonstrated [83] through simulation and experiment that one could couple a linearly polarized surface wave from free space at a 45 degree angle to a two-wire waveguide. For the simulation work we employed a commercially available finite element modeling (FEM) software (COMSOL Multiphysics).

In the simulation geometry, each of the wires is 7 cm long and 0.6 mm in diameter. The wire separation is 1 mm. A schematic of the simulation geometry is shown in figure 2-1. The long axis of the wires is in the z-direction. Another wire (also 0.6 mm in diameter) is placed between the two wires and aligned.

![Figure 2-1. A 3D drawing of our simulation model geometry.](image-url)
perpendicular to them to act as an input coupler. This scattering mechanism for coupling to the propagating mode of the waveguide is similar to the one used in our earlier work on single wire waveguides [57]. A cylinder is placed in the model geometry such that its central axis lies at the center of the gap between the waveguide wires and the coupler wire. The cylinder is placed there in order to provide a location for the excitation source. The excitation plane is on the far circular face of the cylinder. The excitation is a linearly polarized (x-direction) wave with a frequency of 0.1 THz modeled as a plane wave with its k-vector incident on the gap between the wires and the input coupler at an incident angle of 45 degrees. This polarization is chosen such that the vector of the incident electric field is normal to the length axis of the two wires. The outer boundary of the two wires is assigned a transition boundary condition with a surface impedance defined by the conductivity of the metal and its skin depth [84].

The simulation is bounded by enclosing the waveguide geometry in a rectangular box of air (9cm x 2.5cm x 2cm), the walls of which are assigned a low-reflecting boundary condition to minimize the effects of back reflections. Before solving, the model is divided into 0.8 million tetrahedral mesh elements resulting in a model containing 1.1 million degrees of freedom. The model problem is then solved using an iterative Generalized Minimal Residual iterative Solver (GMRES) with Symmetric Successive Overrelaxion (SSOR) matrix preconditioning [85]. Figure 2-2 (a) below shows result of the simulation with the wires present, whereas figure 2-2 (b) shows the result with the wires absent.
Figure 2-2(a). A slice plot of the electric field (x-component) in the x-z plane showing the 0.1 THz wave propagating in the positive z direction with the wires present. The wires are shown in bold. (b). A slice plot of the electric field (x-component).

Figure 2-2 (a) shows a slice plot of the x-component of the electric field. The wave is shown propagating between the two wires in the x-z plane. Red indicates a large positive electric field, whereas blue indicates a large negative electric field. There is clear evidence of a guided mode propagating along the two wires, with the largest fields concentrated in the region between them. One can clearly distinguish coupling in the domain between the wires and radiation outside the domain, by noting that the waves between the wires and those outside the wires are out of
phase. Figure 2-2 (b) shows a slice plot of the electric field (x-component) in the x-z plane of the wave propagating with the wires absent. It is apparent that the guided mode is absent without the wires.

A similar geometry as in the simulation is used on our experiment. A schematic of the experimental set-up is shown in figure 2-3. It consists of a terahertz transmitter/receiver pair (fiber coupled photo-conductive antennas) and two cylindrical stainless steel metal wires. Each of the wires is 7 cm long and 0.6 mm in diameter. The wire separation is 2 mm.

![Figure 2-3. A schematic showing our experimental set-up, whose geometry resembles that of the simulation.](image)

The two-wire waveguide is supported using several pieces of Styrofoam that are nearly transparent to THz radiation. We isolate a 2 mm by 1.5 mm transverse-grid at the output end of the two wires. This grid has a total number of 63 points and the THz receiver is raster scanned along all these points in steps of 0.25 mm.
The field mapping is done for both the vertical transverse electromagnetic (TEM) polarization and for the horizontal polarization of the THz receiver, while keeping the polarization of the THz emitter undisturbed. Once the field mapping for both polarizations is done, both data sets are combined to generate the resultant electric field in magnitude and direction shown in figure 2-4 (a). Figure 2-4 (b) shows the corresponding simulation result for the mode pattern at the end of the two-waveguide with a wire separation of 1mm. Both figures indicate that the mode at the output end of the wires strongly resembles a dipole.

Figure 2-4 Left: The experimental result showing the mode pattern at the output end of a two-wire waveguide. Right: The corresponding simulation result showing the mode pattern at the output end of a two-wire waveguide
These preliminary simulation and experimental results show strong evidence of wave-guiding, and the mode pattern at the end of the waveguide strongly resembles a dipole. In spite of these promising results, the coupling of the wave, from freespace to the two-wire waveguide is relatively inefficient since the incoming wave is angled at 45° and only ~ 1% \[86\] couples to the dipole mode of the waveguide.

2.2. Experimental Results

In an effort to improve the coupling efficiency from free space to the two-wire waveguide, we try a new configuration shown in the inset of figure 2-5. Our experimental setup again consists of a THz transmitter/receiver pair (fiber coupled photoconductive antennas). Two cylindrical stainless-steel wires form the waveguide. They each have a diameter of 0.3 mm and a length of 24 cm. The center-to-center wire separation is 0.5 cm. The two-wire waveguide is supported using several pieces of Styrofoam which are nearly transparent to THz radiation. The waveguide is bent to an angle of about 45° at the transmitter-end in order to distinguish between the guided mode and the (uncoupled) wave propagating freely in space. The THz waveform represented by the dashed curve in figure 2-5 is measured when the two-wire waveguide is in place. The solid flat line is measured when the waveguide is absent, with the THz emitter and receiver undisturbed,
which indicates that there is no line-of-sight transmission of freely propagating radiation from the emitter to the receiver.

**Figure 2-5.** Dotted line: Transmitted THz pulse along a 24 cm two-wire waveguide, 0.5 cm wire separation. Solid line: measured when the two-wire waveguide is absent with the THz emitter and receiver remaining fixed in position. The inset shows the schematic of the experiment setup.

In the next experiment, we image the propagated electric field at the output end of the two-wire waveguide, in a plane perpendicular to the wire axes. The THz receiver (containing a 1.5 mm aperture for improved spatial resolution) is raster-scanned in a 8 mm by 6 mm grid, in steps of 1 mm. For the field mapping, both orthogonal electric field components of the propagating transverse-electromagnetic
(TEM) mode were obtained, by orienting the receiver parallel and perpendicular to the center-to-center line. Once the two orthogonal components were measured, these data sets were combined to generate the total electric field in magnitude and direction. This result is shown in figure 2-6.

Figure 2-6. The THz receiver is raster scanned around a 9 mm X 7 mm area at the output end of the waveguide. The field mapping is done for both the vertical (TEM) polarization and for the horizontal polarization of the THz receiver while keeping the polarization of the emitter undisturbed. The data sets for both polarizations are combined to generate the resultant electric field magnitude and direction shown in arrows.
The strongest electric field is concentrated between the wires, which decreases as one moves away from that central area. Figure 2-7(a) shows the THz waveforms for points along the center-to-center line of the two wires in figure 2-6.

![Graph showing THz waveforms for different points along the wires.](image)

**Figure 2-7(a).** THz waveforms for the points along the vertical dotted line in figure 2-6. (b) THz waveforms for the points along the solid horizontal line in figure 2-6. The waveforms have been shifted for clarity.
The electric field amplitude is seen to decrease as one moves away from the center of the wires. In the signals corresponding to the top of the upper wire (4 mm above the waveguide axis), and bottom of the lower wire (4 mm below the axis), we observe a flip in polarity (relative to the field between the wires) due to the dipolar nature of the propagating TEM mode. Figure 2-7 (b) shows the THz waveforms for the points along the solid horizontal line in figure 2-6. As anticipated, the electric field signal decreases as one moves either left or right of the two-wire center.

Next, we determine the loss characteristics of the waveguide. The attenuation is measured by comparing THz pulses that have propagated through two different waveguide lengths, as in previous experiments [68-72, 74, 75]. For both lengths, we use the same 45° bend immediately after the transmitter, which introduces a fixed coupling and bending loss. We change only the length of the straight section, and extract the intrinsic loss on a straight waveguide by normalizing out these coupling and bending effects.

The measured THz pulses from these two lengths are shown in figure 2-8 (a). The two main pulses are almost identical in shape, indicating the negligible group velocity dispersion expected for a TEM mode. The THz pulses have relatively long time scales of about 100 ps. The long time scales are due to the poor coupling efficiency of the input Gaussian beam to the two-wire waveguide. At high frequencies, the energy is more concentrated near the wire boundaries, and will be highly mismatched with the input Gaussian profile. Thus, some of the high frequency components will not be coupled into the waveguide. The relatively small
change in amplitude signifies the low-loss nature of the propagation. The corresponding amplitude spectra shown in figure 2-8 (b) are obtained by Fourier transforming the time domain pulses in figure 2-8 (a).

The amplitude attenuation coefficient derived using the measured spectra is shown in figure 2-9. For comparison, the theoretical attenuation for the two-wire waveguide (TEM mode) [87], and for a single-wire waveguide (Sommerfeld wave) are also plotted. The cylindrical wires in these calculations were modeled as having a diameter of 0.3 mm and being made of type 304 stainless steel, with a conductivity of $1.45 \times 10^6$ S/m, as in our experiments. For comparison, we show the attenuation for the TEM mode of a stainless steel parallel-plate wave guide (PPWG) with a plate separation of 0.5 cm.
Figure 2-8 (a). Transmitted THz pulses along a 60 cm and 24 cm waveguide. (b) Fourier transform spectra for the THz pulses truncated at the line shown in 2-8(a).
Figure 2-9. The amplitude attenuation coefficient for various waveguide structures. Dots: measured for two-wire from 60 cm and 24 cm stainless steel, 0.5 cm wire separation. Thick curve: theory for two-wire for 0.31 mm-diam stainless steel, 0.5 cm sep. Thin curve: Sommerfield theory for one 0.31-mm-diam stainless steel. Dotted curve: Theory for ideal parallel plate waveguide, 0.5 cm plate separation.
The results in figure 2-9 illustrate that the measured attenuation for the two-wire waveguide is relatively low, and is in reasonable agreement with its theoretical value.

In addition to measuring the attenuation of the two-wire waveguide, we also measured the bending loss. Figure 2-10(a) shows a schematic of the setup showing a 65 cm long waveguide. The straight section of the waveguide near the receiver-end is labeled L1, the bent section S, and the straight section near the emitter-end L2. The bend angle is $\theta$ and the radius of curvature is R. We keep the receiver fixed and only vary the bend angle, and the location of the emitter (which is fixed relative to the straight section L1). Since the lengths L1, S, and L2 remained the same for every bend angle, any changes in transmission with changing $\theta$ result only from variations in the loss due to bending.
Figure 2-10 (a). A schematic of the experimental set-up for the bending measurements. (b). Transmitted THz pulses along a 65 cm waveguide with various radii of curvature. Solid R = 51.6 cm (θ = 15°). Dashed line: R = 17.2 cm (θ = 45°). Thin: R = 8.6 cm (θ = 90°).

The time-domain waveforms for several values of R are shown in figure 2-10(b). As expected, the signal decreases with increasing bend angle, but the decrease is relatively slow. Even a relatively small radius (R = 8.6 cm, corresponding to a 90° bend) results in less than a factor of two change in the peak-to-peak THz field. This demonstrates the tight coupling of the propagating wave to the two
wires, in contrast to the Sommerfield wave of a single-wire where the wave is very weakly coupled. For an equivalent bend on a single wire waveguide \([75]\), the attenuation would be more than a factor of 10, considerably larger than the value measured here which is less than a factor of two. Figure 2-11 shows the peak-peak electric field, normalized to the value corresponding to no bend \((\theta = 0^\circ, R = \infty)\).

![Image of graph showing THz pk-pk electric field vs. Radius of curvature (cm)](image_url)

**Figure 2-11.** The peak-peak electric field, normalized to the value corresponding to no bend \((\theta = 0^\circ, R = \infty)\).

In addition to measuring the ohmic and bending losses, we also computed and compared the losses for various wire separations. An experiment to quantify the influence of the wire separation on the loss would be very difficult to assemble, since for each wire separation, the wire bend would have to be identical. Theoretical
curves showing the losses for various wire separations are shown in figure 2-12. It is apparent that the greater the wire separation, the lower the conductor losses.

![Graph showing the losses for various wire separations](image)

**Figure 2-12. Theoretical curves showing the influence of the wire separation on the conductor losses**

We perform one more measurement with the bare metal wires whereby we slowly separate the two wires on the straight section close to the receiver. The receiver is moved 2 mm above the center of the straight wire as shown in figure 2-13, and the electric field is sampled. The receiver is then moved 2 mm below the center of the lower wire and the measurement is repeated. We find that the radial mode (Sommerfield wave) [74, 75] of a single wire is excited at the end of the straight wire and at the end of the separated wire. Figure 2-14 shows measured
time domain waveforms for the two-wire waveguide before separating the wires, and after separating them.

Figure 2-13. A schematic of the experimental set-up showing how the wires are slowly separated close to the receiver

Once the wires are separated, the THz electric field signal is sampled 2 mm above and 2 mm below the straight wire. The flip in polarity confirms that we are indeed looking at the radially polarized mode of a single wire. The same radially polarized field is observed in the separated wire when the electric field is sampled 2 mm above and 2 mm below it.
2.1. Measurements With a Commercial 300-Ω Tv-Twin Lead Cable

In addition to the work with bare metal wires, we also tested a commercially available 300-Ω, 18 gauge standard twin-pair TV antenna cable. This is basically a two-wire waveguide, where the wires are equally spaced and completely enclosed within a dielectric medium (polyethylene), as shown in figure 3-15. Each wire

Figure 2-14. Measured time domain waveforms for the two-wire waveguide before separating the wires, and after separating them. The THz electric field signal is sampled 2 mm above and 2 mm below the straight wire. The flip in polarity is due to the radial nature of the field.
consists of seven copper strands of about 500 μm diameter woven together. The center-to-center separation is about 0.3 cm.

![Image](image.png)

**Figure 2-15. A picture of the TV-antenna twin lead cable that we used.**

The length of the cable used in our measurements is 9.5 cm. We insert this length of cable between the THz transmitter and receiver, incorporating a 90° bend so that no freely propagating radiation can be detected. After measuring the propagated signal, the antenna cable is removed, and the measurement is repeated with the emitter and receiver undisturbed. Finally, the receiver is re-positioned to face the emitter (with the distance between them kept at 9.5 cm), and the measurement is repeated for the freely propagating beam.

The THz waveforms for all three measurements are shown in figure 2-16. For the signal with the antenna cable, we observe a positive chirp in the initial signal, where the high frequency components arrive later in time. This dispersive behavior is distinct from the usual fundamental TEM mode propagating in the two-wire waveguide, and is probably caused by small air gaps between the metal wires and
the dielectric medium [88]. The long-lived oscillations following the main burst are probably due to the excitation of higher-order waveguide modes. As expected, without the cable in place, no energy is guided from emitter to receiver. When the receiver is re-positioned to detect the freely propagating wave, we observe an undistorted pulse.

Figure 2-16. Transmitted THz pulses. Solid: 9.5 cm TV-antenna cable present, with THz emitter and receiver at 90°. Thin: TV-antenna cable is absent with THz emitter and receiver at 90°. Dashed: TV-antenna cable is absent with THz emitter and receiver facing each other.
Figure 2-17 gives the spectra corresponding to the waveforms in figure 2-16. We observe greater low-frequency content in the signal with the antenna cable, compared to the free-space signal. This demonstrates that the cable has functioned as a waveguide, successfully inhibiting the diffractive losses (which are higher at low frequencies). The dramatic drop in the high-frequency content is probably due to the additional losses caused by the dielectric medium, which likely represents the dominant loss mechanism. Nonetheless, we find that this TV-antenna cable can carry frequency components up to about 0.2 THz, although these are generally designed to carry frequencies only up to about 800 MHz (Ultra-High-Frequency (UHF) TV broadcast band) [89].
The THz propagation characteristics of two-wire waveguides indicate that the mode at the end of the waveguide resembles a dipole pattern, consistent with the fundamental TEM mode of this two-conductor structure. The demonstrated low attenuation and bending loss is consistent with early results at microwave frequencies [90, 91]. It has also been demonstrated that a commercial 300 Ω twin-pair TV-antenna cable can be used to propagate radiation up to about 0.2 THz, with large bend angles.
Chapter 3

Terahertz Measurements using Electronically Controlled Optical Sampling (ECOPS)

3.1. Background

Over the past two decades, terahertz time-domain spectroscopy has become a standard technique for versatile access to the far infrared region of the spectrum. As is well known, this technique requires femtosecond optical pulses split into two beams. Typically, the path length of one beam is scanned relative to the other using a mechanical delay line. Although easy to implement, this technique has shortcomings including spot size variations during scanning, and sensitivity to misalignment. Recently, asynchronous optical sampling (ASOPS) has been employed to remedy this problem. In ASOPS [92], two lasers are used instead of one, and the
temporal delay is achieved by locking the repetition rate of one laser to that of the other, with a slight offset \( \Delta f \). Thus, the pulses in one pulse train scan past those in the other pulse train at a rate given by \( \Delta f \). This technique has been implemented by several groups in a variety of pump-probe-type measurements including terahertz time-domain spectroscopy [93, 94].

Here, we discuss an alternate method for measuring terahertz waveforms, known as electronically controlled optical sampling (ECOPS). As has been pointed out recently, ECOPS [95-97] has an advantage over ASOPS, particularly in the case of lasers with relatively low repetition rates (100 MHz or less). For a laser system with a 100 MHz rep rate, the ASOPS scheme requires a full scan of the 10-nanosecond time window between each pulse for each measurement. However, most THz time-domain measurements do not require (or desire) such a long time window (unless the measurement is focused on the study of very narrow spectral features). As a result, most of the measurement time is wasted, and moreover a very high sampling rate is required in order to measure the signals which occupy a very small fraction of the data. As a result, lasers with repetition rates in excess of 1 GHz are typically used with ASOPS, in order to obtain sufficient timing resolution. In contrast, in ECOPS the repetition rate offset between the two lasers is modulated, so that \( \Delta f \) is alternately positive and negative. In this way, one can efficiently utilize the scanning window, limiting the scan to only a small fraction of the 10-nanosecond window between pulses, thus reducing the overall measurement time. This ECOPS technique is more suitable for lasers with lower repetition rates, in the 100 MHz range or less.
3.2. Preliminary Measurements & Results

For our measurements, we use two modelocked erbium fiber lasers (Toptica) with a wavelength centered around 1550 nm. Each laser contains a ring oscillator with an erbium doped fiber amplifier. An essential part of the oscillator is a free-beam section comprising several waveplates and a polarizer, serving as artificial saturable absorber. Thus, modelocking is achieved by nonlinear polarization evolution via the optical Kerr effect.

Laser 2 (the master oscillator) runs freely with a repetition rate of 82MHz, while laser 1 (the slave oscillator) can be tuned in repetition rate by varying the length of the free-beam section. The two laser repetition frequencies are synchronized by controlling the cavity length via a stepper motor and piezoelectric transducer for coarse and fine adjustment, respectively. The phase difference between the pump and probe pulses can be controlled by an external offset voltage applied to the locking electronics for one of the lasers. Thus, the time delay between the pump and probe pulses can be repetitively scanned at the modulation frequency of the external offset voltage. The scan rate and time delay window can be adjusted by the modulation frequency and amplitude of the external offset voltage.

A feedback loop via an analog phase detector controls the relative phase difference between the pulse trains from the two lasers. The setpoint of the feedback loop is controlled externally, so that it temporarily lowers the repetition frequency of the slave laser, resulting in its pulses lagging behind the pulses of the pump laser. Once the maximum time delay ($\Delta t_{\text{max}}$) is reached, the scan is reversed.
by resetting the setpoint so that the slave laser has a slightly higher repetition rate as shown in figure 3-1.

**Figure 3-1. Schematic diagrams illustrating the ECOPS timing principle**

3.2.1. Autocorrelation and Cross-correlation Measurements

Prior to performing Terahertz experiments we perform studies on the nature of the optical pulse generated by the fiber laser system. This was particularly important because this laser system was one of the very first units manufactured by the company, and we wanted to be sure that it would operate as advertised. We begin with performing autocorrelation measurements [98] on one of the lasers.

The time duration for the pulses generated by this laser is about 100 fs. Due to the fast duration of these pulses, it isn’t possible to use a photodetector to measure the pulse intensity and phase of the pulse. Thus, one can only use a pulse to measure itself by taking its autocorrelation. This measurement involves splitting a beam into
two, and delaying one of the beams in time with respect to the other beam, then spatially overlapping both beams in a non-linear optical medium such as a second harmonic generation crystal. After propagation through the crystal, the second harmonic light travels collinearly with the two original individual beams. In the process of travel these three beams combine coherently resulting in several fringes occurring vs delay. This type of autocorrelation combines elements related to the usual autocorrelation (often referred to as the intensity correlation) and spectrum and is thus called an interferometric autocorrelation [98, 99], or the fringe-resolved autocorrelation (FRAC).

The expression for the intensity of the field envelope is given by:

\[
I_{\text{FRAC}}(\tau) = \int_{-\infty}^{\infty} \left| \left[ E(t) + E(t - \tau) \right]^2 \right|^2 dt
\]

[3-1]

\[
= \int_{-\infty}^{\infty} \left| E(t)^2 + 2E(t)E(t - \tau) + E(t - \tau)^2 \right|^2 dt
\]

[3-2]

Where \( \tau \) is the time delay, and \( E(t)^2 \) and \( E(t - \tau)^2 \) terms are due to the SHG generation of the individual beams. The interference of these two terms yield an interferogram of the second harmonic pulse. Expanding equation 3-2, we have
In words this is equivalent to:

\[ I_{FRac}(\tau) = A \text{ Constant} + \text{the Modified interferogram of } E(t) + \text{an Interferogram of the 2nd harmonic of } E(t) + \text{the Autocorrelation of } I(t) \]

Figure 3-2 illustrates this auto-correlation principle. It shows an experimental layout of the interferometric autocorrelation.

**Figure 3-2. Experimental layout for the interferometric autocorrelation**

(Courtesy of SWAMP optics)
Figure 3-3 shows interferometric fringes that were measured using one of our fiber lasers. Light from the laser is split into two pulse trains via a beam splitter (BS1). One of the pulse trains is delayed in time, prior to both beams being re-united at another beam splitter (BS2) and guided to a SHG crystal. The second harmonic light generated then travels collinearly with the two original pulse trains. These three beams combine coherently resulting in interference fringes occurring vs delay. The silicon photodiode (because of its slow response), measures the slowly varying envelope of the pulse. The measured FWHM of the autocorrelation trace is \(~70\) fs.

Figure 3-3. Experimental setup for the interferometric autocorrelation. Light from one laser is split into two pulse trains. One of the pulse trains is delayed in time, prior to both being re-united at another beam splitter and being guided to a SHG crystal. The second harmonic light generated then travels collinearly with the two original pulse trains. These three beams combine coherently resulting in interference fringes occurring vs delay.
In addition to measuring the auto-correlation, we also measured the cross-correlation of the two lasers we utilized. In general the cross-correlation of two lasers is a measure of the similarity of their optical pulses, as a function of the time-delay applied to one of them. The cross-correlation is given by the expression

\[ C(\tau) = E(t - \tau)^2 \int_{-\infty}^{\infty} I_2(t) I_1(t - \tau) \, dt \]  

[3-4]

Where \( I_1 \) is the pulse intensity for laser 1, \( I_2 \) is the pulse intensity for laser 2, and \( \tau \) is the time delay. Figure 3-4 shows a schematic of the set-up used to measure the cross-correlation. The pulse train from laser 2 is delayed in time with respect to the one from laser 1. Before arriving at the detector, the pulses are focused tightly as shown figure 3-5, in order to initiate two photon absorption at the detector. In figure 3-4, the inset showing a zoomed in view of the oscilloscope illustrates the cross correlation trace in yellow. The square wave (~10 V, 20Hz) in blue is output from a function generator and is input to the laser electronics.

As mentioned earlier, utilizing ECOPS, the phase difference between the two pulses can be controlled by an external offset voltage applied to the locking electronics for one of the lasers. Here, the time delay between these pulses was repetitively scanned at the modulation frequency (20Hz) of the external offset voltage (~10V).
Figure 3-4. Experimental setup for the Cross-correlation. In the inset, the square wave (~10 V, 20Hz) in blue is output from a function generator and is input to the laser locking electronics where it is used repetitively scan one pulse past the other, thereby mapping the cross-correlation waveform.

Figure 3-5. A schematic showing how the two pulse trains are tightly focused (in figure 3-4) prior to arriving at the detector, in order to generate the cross-correlated beam via two-photon absorption.
Next, we performed measurements to establish how much the cross-correlation drifted over time. We input a 15 Hz, 10 V waveform from the function generator to the laser electronics and used it to repetitively scan between the two pulse trains. Figure 3-6 illustrates the stability of the cross-correlation over a period of two hours. Our Terahertz measurements necessitated a stability of at least 1 ps over that period of time. The drift of our time domain pulse was measured to be about 1 ps over 2 hours at constant temperature.
Figure 3-6. Stability of the cross-correlation over time. A 15 Hz, 11 V waveform from the function generator was input to the laser electronics and used (via ECOPS) to repetitively scan between the two pulse trains.

Last, in order to utilize the measurements from our oscilloscope, we need to carefully calibrate the time scale on the oscilloscope so as to get an accurate reading on the time duration of our ultra-short pulses.
Figure 3-7. Measurement of the FWHM of the cross-correlation.

We find that by manually displacing the translation stage by 1mm, one moves the cross-correlation waveform by 600 µs on the oscilloscope. Since light travels 30 µm in 100 fs, we have:

\[
\frac{1000 \text{µm}}{30 \text{µm}} \times 100 \text{fs} \times 2 = 6667 \text{fs}
\]

Meaning that 6667 fs in real time = 600 µs in oscilloscope time

In order to measure the FWHM we need to convert the oscilloscope time (13.6 µs) into real time. Thus we have:
\[
\frac{13.6 \mu s}{600 \mu s} \times 6667 \, fs = 151 \, fs
\]

3.3. Terahertz Experiments & Results

Our terahertz experimental set-up is shown in figure 3-8. It consists of a terahertz generation and detection arm, a cross-correlation arm, and laser stabilization electronics to ensure low timing jitter and long-term stability of the waveform being measured. The pump pulse (laser #1) is used for THz generation via surface generation from p-InAs. The THz beam is guided to a low temperature Gallium arsenide (LTGaAs) photoconductive antenna using parabolic mirrors. The probe pulse (laser #2) at 1550 nm is converted to 775 nm via second harmonic generation in a Beta Barium Borate (BBO) crystal, before arriving at the LTGaAs THz detector. A small fraction of the pump and probe optical beam are picked off and sent to an amplified silicon photodiode which is used for generation of a cross-correlation signal via two-photon absorption. The cross-correlation signal is used to correct for slow drifts in the laser frequency as result of a change in room temperature or humidity. By using an oscilloscope, one can lock the cross-correlation signal to the trigger sine wave signal from a function generator which ensures long-term signal stability. A function generator provides a 20 Hz – 20 V pk-pk signal to the laser and locking electronics. Figure 3-9 shows the cross-correlation and THz time domain signal after 5000 averages, and the corresponding frequency spectrum. We obtain good signal-to-noise with low timing jitter and approximately 1 THz of spectral bandwidth.
Figure 3-8. A Schematic of the experimental arrangement
Figure 3-9. Cross-correlation signal and terahertz waveform, after 5000 Oscilloscope averages
To test the long term temporal stability of the signal waveform, we left the laser on for several hours and collected time domain measurements periodically with the oscilloscope. Figure 3-10 below indicates a stable temporal signal even after 28 hours.

Figure 3-10. Plot showing temporal stability of the THz pulse after several hours
2.1 Terahertz Generation from p-InAs:

Our ECOPS system exploited the use of a bulk (100) p-type InAs crystal for THz generation at a pump wavelength of 1550 nm. P-type InAs is classified as a narrow band semiconductor, with a band gap $E_g = 0.36$ eV, and has shown to have the highest THz emission efficiency investigated to date. THz emission from bulk InAs [100-105] is thought to be a linear process explained by the current-surge model on the semiconductor surface induced by photoexcitation. The current surge is posited to have two origins:

1. The acceleration of photoexcited carriers by the surface depletion field
2. The photo-Dember effect resulting from the difference between the diffusion velocities of the electrons and holes. The electrons usually gain much faster velocity due to their higher mobility (of about 30 000 cm$^2$/V/s), compared to holes.

In wide band gap semiconductors such as GaAs ($E_g = 1.43$ eV) or InP ($E_g = 1.34$ eV) one expects the contribution from the photo-Dember effect to be very small since the absorption length is relatively long and the depletion field is usually strong. In contrast in narrow gap semiconductors, the depletion field is commonly weaker due to the small band gap energy. When a narrow band gap semiconductor is excited with near-infrared light the absorption depth is very small (~100 nm) and the photoexcited electrons absorb kinetic energy from the considerable surplus excitation energy. This coupled with high electron mobility, enhances the photo-Dember effect in InAs.
In addition to the photo-Dember effect, and the acceleration of photoexcited carriers by the surface depletion field, optical rectification also played a role in the THz emission from p-InAs. In order to obtain the optimum signal from our crystal we performed azimuthal angle dependence measurements for the (100) p-type InAs sample. We first mounted the crystal on a high precision rotation mount, then proceeded with the measurements. Figure 3-5 shows that the peak THz amplitude was observed at an angle of 150°.

![Figure 3-11. Azimuthal angle dependence of THz radiation from our (100) p-type InAs](image)
Studying the Impedance mismatch at the output end of THz Parallel-plate waveguides

4.1. Modes in a THz Parallel Plate waveguide

The parallel plate waveguide is a well-studied structure in classical waveguide theory. The waveguide consists of two parallel plates which confine the wave in one transverse direction but extend infinitely in the other (see figure 3-1). When the input electric field of a PPWG is polarized in the direction perpendicular to the plates, only transverse magnetic (TM) modes can exist in the waveguide. On the other hand when the input electric field is polarized parallel to the plates, only transverse electric (TE) modes can exist in the waveguide. TM waves in the PPWG contain no magnetic field in the direction of propagation \((H_z = 0)\), while the TE
waves contain no electric field in the direction of propagation \((E_z = 0)\). The TE mode will be discussed in more detail in chapter 5.

![Parallel plate waveguide schematic](image)

**Figure 4-1.** A schematic of a parallel plate waveguide showing the plate spacing \(b\), and orientation of the transverse magnetic (TM) and transverse electric (TE) modes. Propagation is into/out and out of the page, in the \(z\)-direction.

Assuming a lossless wave traveling in the \(z\)-direction, the non-vanishing field components of the TM mode can be written as:

\[
\begin{align*}
H_x &= A_n \cos\left(\frac{n\pi}{b} y\right)e^{-i\beta z z} \quad [4-1] \\
E_y &= \frac{A_n \beta}{\omega_0 \varepsilon} \cos\left(\frac{n\pi}{b} y\right)e^{-i\beta z z} \quad [4-2] \\
E_z &= \frac{A_n \beta}{j\omega_0 \varepsilon} \left(\frac{n\pi}{b}\right) \sin\left(\frac{n\pi}{b} y\right)e^{-i\beta z z} \quad [4-3]
\end{align*}
\]
Where,

\[ \beta_z^2 + \left( \frac{n \pi}{b} \right)^2 = \beta_0^2 = \omega_0^2 \mu \varepsilon \]  \hspace{1cm} [4-4]

Here \( n = 0, 1, 2, 0, \ldots \) and \( 0 \leq y \leq b \). \( \omega, \mu, \beta, \text{ and } \varepsilon \) all have their usual meanings, \( b \) is the plate separation, \( A_n \) is a constant which is dependent on the excitation of the waveguide, while the subscript "0" refers to free space quantities.

### 4.1.1. The TEM mode of the parallel plate waveguide

An extraordinary characteristic of the parallel plate waveguide is that its lowest order transverse electromagnetic (TEM) mode (i.e. TM\(_0\)) is virtually dispersionless. That means, the dispersion relation, \( k = \omega / c \), is the same as that of free space, there is no cut-off frequency, and the phase and group velocities of the wave are equal to the speed of light. The TEM mode is represented in the form of a plane wave,

\[ E(z, t) = E_0 e^{i(kz - \omega t)} \] \hspace{1cm} [4-5]

inside the waveguide and vanishes everywhere else.

Since it was first observed that a parallel-plate waveguide (PPWG) could support undistorted propagation of terahertz (THz) pulses via its dominant transverse electromagnetic (TEM) mode [106], the PPWG has been exploited for a plethora of THz applications. These include spectroscopy [107, 108], imaging [109],
sensing [110], and many others [111-114]. Very recently, tapered PPWGs have been used for achieving subwavelength confinement of THz radiation (even super-focusing) over a broad spectral bandwidth [115-117] with potential applications in near-field imaging and spectroscopy of subwavelength materials. For nearly all of these applications, it is important to have a good understanding of the impedance mismatch between the propagating mode and the free space background, since this could influence the transmission throughput of the device. For example, a strong frequency dependence in this impedance mismatch may lead to significant distortions of the time-domain THz pulse, which could complicate the interpretation of near-field measurements. However, for the important case of a tapered PPWG, and even for most implementations of a conventional (un-tapered) PPWG, there is no simple analytical theory which is valid in the regime of $b \sim \lambda$ or $b<\lambda$, where $b$ is the plate separation.

4.2. Experimental studies of the Impedance mismatch at the output of a Parallel-plate waveguide

In this work, we study the impedance mismatch between the propagating TEM wave in the PPWG and the free space background, at the output face of the abruptly terminated waveguide. This mismatch results in a fraction of the propagating wave being reflected at the output facet, back into the waveguide. We quantify the impedance mismatch by measuring this back-reflected wave after it emerges from the input end of the waveguide. We study both uniformly-spaced and tapered
waveguides, and compare to the existing theoretical treatments.

A schematic of the experimental setup is shown in figure 4-2. We use two synchronized mode-locked erbium-doped fiber lasers in an ECOPS configuration [118, 119]. THz pulses are generated via surface emission from a bare p-InAs wafer, and these pulses are detected using a LT-GaAs photoconductive antenna, gated with the second harmonic of the probe laser. The THz radiation is guided using a polyethylene lens through a thin silicon beam splitter. A cylindrical teflon lens is used to focus the THz beam into the waveguide to excite the TEM mode. A metal mirror can be placed at the

Figure 4-2. Schematic of the experimental setup. THz pulses from a bare p-InAs wafer are focused into the waveguide with the right polarization to excite the TEM mode. A metal mirror can be placed at the output face of the waveguide to retro-reflect the TEM mode, or it can be removed to allow the THz pulse to emerge from the waveguide into free space. A silicon beam splitter directs the retro- reflected signal to a LT-GaAs receiver for coherent detection.
output face of the waveguide to retro-reflect the guided mode back towards the input facet, or it can be removed to allow the THz pulse to emerge from the waveguide into free space. In this way, we can compare the incident pulse propagating in the waveguide (measured by retro-reflection from the metal mirror) to the pulse reflected due to the impedance mismatch between the guided wave and free space (as measured when the retro-reflector is absent).

We compare the THz pulses measured with and without the retro-reflecting mirror, for several different plate separations, for a waveguide length of $L = 12.7$ mm. Results for $b = 1.8$ mm, 1 mm, and 0.63 mm, are plotted in figure 4-3 (a)-(c). In these figures, the reflection from the front facet of the waveguide arrives earlier in time followed by the reflection from the back of the waveguide. The feature (12 ps) immediately following the initial THz pulse and the one following the pulse reflected from the back of the waveguide are due to multiple reflections inside the silicon beam splitter. In each case, the signals arising from reflections at the front face of the waveguide are unchanged by the removal of the retro-reflecting mirror, as expected. However, the reflection originating from the back facet of the waveguide, at a later delay, changes dramatically when this mirror is removed, exhibiting a diminished amplitude and a reversed polarity. By comparing these two reflections (with and without the retro-reflecting mirror), we can estimate the impedance mismatch (or reflection coefficient) between the guided wave and free space. The amplitude spectra corresponding to the relevant reflections for $b = 1$ mm are shown in Figure 4-3 (d).
Figure 4-3. THz pulses measured with and without the retro-reflecting mirror, for a 12.7 mm long waveguide with three different plate separations: (a) $b = 1.8$ mm, (b) $b = 1$ mm, and (c) $b = 0.63$ mm. The reflection from the front end of the waveguide arrive earlier in time followed by the reflection from the back end of the waveguide. The feature (12 ps) immediately following the initial THz pulse and the one following the pulse reflected from the back end of the waveguide, are due to multiple reflections inside the silicon beam splitter. Amplitude spectra corresponding to the relevant reflections for $b = 1$ mm are shown in (d).
This reflection coefficient is expressed as:

\[
\text{reflection coefficient} = \frac{\text{THz amplitude with mirror absent}}{\text{THz amplitude with mirror present}}
\]

Figure 4-4 shows the measured amplitude reflection coefficient for a 12.7 mm long waveguide when the plate separation is varied uniformly. The width of the waveguide is chosen to be sufficiently larger (25.4 mm in size) than the input beam diameter, so that the waveguide mimics infinitely wide plates to the propagating beam. Therefore, the width has no bearing on the results.

The reflection coefficients are obtained from the Fourier transforms of the relevant truncated portions of the measured time-domain waveforms. In figure 4-4, we note that, for large plate separations, most of the propagating wave is transmitted into empty space and is not reflected back to the detector, and therefore, the reflection coefficient increases as the plate separation decreases. Additionally, for a given plate separation, the reflection coefficient increases as the wavelength increases.
Figure 4-4. Measured amplitude reflection coefficient for a 12.7 mm long PPWG, for several different plate separations. Here, the plate separation is uniform along the direction of propagation.

Figure 4-5 shows logarithmic plots of the reflection coefficient for the TEM mode versus the ratio of the plate separation to the wavelength ($b/\lambda$). The experimental data points in figure 4-5 (a) and (b) correspond to 170 GHz and 300 GHz, respectively, and are for two waveguides with $L = 12.7$ mm and 25.4 mm.
Figure 4-5. Logarithmic plots of the amplitude reflection coefficient for the TEM mode. The black squares and red circles correspond to measured values for a 12.7 mm long and a 25.4 mm long waveguide, respectively. The solid blue and green curves correspond to the calculated values using ray optical theory for thick and infinitely-thin plates, respectively. Parts (a) and (b) correspond to the frequencies of 170 GHz and 300 GHz, respectively, at which the experimental points were derived (from figure 4-4, for example). The inset in (b) shows a schematic used for the calculation.
These are plotted along with approximate results predicted by ray optical theory [120, 121]. Exact results are only available for the case where the waveguide plates are infinitely thin [122, 123].

Following the analysis in Ref. [120], for our case, where the plates have a finite thickness, the ray optical theory considers the input beam to consist of two plane waves, one traveling along and almost parallel to the top plate and the other propagating in the same manner along the bottom plate. Once the plane waves reach the end of the waveguide, they are “scattered” back after diffraction from the edges at the output end of the waveguide. The reflection coefficient is derived based on the resultant sum of these diffracted waves. The inset in figure 4-5 (b) shows the two waves $u$ and $\bar{u}$ (symbols denoting normalized amplitude) that are scattered by the lower and upper edges, respectively.

Equations (4-6) and (4-7) describe the reflection coefficient for a particular TM mode assuming only primary diffraction, where higher-order diffraction that results from multiple interactions at the output edges has been neglected.

$$\left| \Gamma_{TEM} \right| = (-1)^n u D(\alpha, \theta_n, \theta_N) + \bar{u} D(\alpha, \theta_n, \theta_N) \frac{\sqrt{2\pi k}}{2\kappa_n b} e^{i\pi/4}$$  \hspace{1cm} [4-6]
Here, \( \Gamma_{NN} \) is the reflection coefficient corresponding to the \( n \)-th mode due an incident \( N \)-th mode; \( \theta \) is the incidence angle of the decomposed waves; \( \alpha \) is the exterior wedge angle (at the output edges); and \( K_n \) is the phase constant of the \( n \)-th mode.

For our case, where propagation is purely due to the dominant TEM mode,

\[
n = 0, \quad \theta = \theta_N = 0, \quad \alpha = \frac{3}{2} \pi, \quad k = K_n = \frac{2\pi}{\lambda}, \quad \text{and} \quad u = \bar{u} = \frac{1}{2}.
\]

Combining Eqs. (4-6) and (4-7) with these substitutions, we can derive the magnitude of the reflection coefficient for the single-TEM mode as,

\[
|\Gamma_{TEM}| = 0.0613 \left( \frac{\lambda}{b} \right) \quad [4-8]
\]

Equation (4-8), although approximate, implies an (inversely) linear \( b/\lambda \) dependence. The corresponding theoretical curves {solid blue curves in figure 4-5 (a) and (b)} show reasonable agreement with the experimental results. Additionally, we have shown theoretical curves corresponding to infinitely thin plates {solid green curves in figure 4-5 (a) and (b)}, which indicate the same dependence but with a slightly higher overall magnitude. The lower magnitude in the case of thick plates is probably caused by the softening of the physical discontinuity due to the
90° edge at the output end.

In order to achieve smaller plate separations than were possible with the uniformly-spaced PPWG, next, we resort to a tapered PPWG. At smaller plate separations, the significantly reduced energy coupling into the uniformly-spaced waveguide would preclude a reliable measurement. However, using a tapered waveguide, the input plate spacing could be held constant at a sufficiently large size to couple enough energy, while the output plate spacing is varied. An axial cross-section of this tapered waveguide is shown in the inset of figure 4-6, whose output ends are formed into knife edges.

This knife-edge geometry permits us to place the retro-reflecting mirror very close to the output aperture even for the largest taper angle, which is crucial for obtaining an accurate measurement of the reference waveform. For these measurements, the plate separation of the waveguide at the input facet is kept fixed at 2.3 mm, using steel ball-bearings as spacers to provide sufficient pivoting for the surfaces of the plates. Only the output plate separation of the waveguide is varied.
Figure 4-6. THz pulses measured with and without the retro-reflecting mirror, for a 12.7 mm long waveguide with three different plate separations: (a) $b = 1.8$ mm, (b) $b = 1$ mm, and (c) $b = 0.63$ mm.
We compare the THz pulses measured with and without the retro-reflecting mirror, for several different plate separations, for a waveguide length of $L = 76.2$ mm. Results for an output plate separation of $b = 0.8$ mm, 0.5 mm, and 0.25 mm, are plotted in figure 4-6 (a)-(c). As in the un-tapered case, the amplitude of the retro-reflected signal increases as the (output) plate separation is reduced. In fact, at an output separation of 0.25 mm, the reflected pulse is a little more than half the original input signal.

As before, the reflection coefficient is derived after Fourier transforming the relevant portions of the time domain data and plotted in figure 4-7. Again, similar to the previous case (with uniformly spaced plates) the reflection coefficient is found to increase as the output plate spacing is decreased (for a given frequency) and also when the frequency is decreased (for a given output plate separation). This latter dependence has also been observed in a finite-width tapered PPWG very recently [124], using an air-photonic field imaging technique.

It should be noted that in our data analysis, we are assuming that the forward propagating wave is not affected by the placement of the mirror in deriving the incident (reference) signal. This assumption is reasonable for large plate separations, but may not be quite valid at very small plate separations, where plasmonic effects dominate [116]. This probably explains the relatively higher estimated values for the reflection coefficient at smaller separations in figure 4-7, compared to what one might expect intuitively under TEM mode propagation.
Figure 4-7. Measured amplitude reflection coefficient for a 76.2 mm long tapered PPWG with a knife-edged output end. An axial cross-section of this waveguide is shown in the inset.

Furthermore, since we are using an *adiabatic* taper, we can assume that there are no additional reflections due to the continuously varying local impedance in the tapered waveguide. The only appreciable reflection is caused by the abrupt impedance change at the output face. This implies that the reflection coefficients corresponding to both untapered and tapered waveguides having the same output plate separations, should match favorably. In fact, if we compare the curves for 0.63 mm and 0.8 mm in figures 4-4 and 4-7, we see that the differences are small and within experimental error.

We have characterized the impedance mismatch relevant for the dominant
TEM mode of THz PPWGs. In the case of a PPWG with uniformly spaced plates, the reflection coefficient at the output-facet of the PPWG increases as the plate separation decreases. This dependence is approximately linear with the ratio $b/\lambda$, as predicted by a ray optical theory. This same trend is observed in tapered PPWGs, when the input separation is fixed and the output separation is varied. The reflection coefficients that we measure, especially for the smaller plate separations, are unexpectedly high, since one would not generally associate a high impedance mismatch under TEM mode propagation. These results shed more light on the propagation of THz radiation in practical PPWGs.
Inhibiting Diffractive losses Using Curved parallel Plate waveguides

5.1. Introduction

As mentioned in Section 1.3, of all the waveguides investigated to date, metal parallel plate waveguides have shown the most promise since they exhibit low ohmic losses (0.1 cm⁻¹), negligible dispersion, and support single mode propagation. As explained in section 4.11, the transverse electromagnetic mode (TEM) of this waveguide has been the popular choice for measurements due its low loss, ease of quasi-optic coupling, and low dispersion nature due to the absence of a cutoff. In spite of its many advantages the TEM mode of this waveguide falls short of being the ideal mode for transport as it cannot offer complete 2-D energy confinement due to the one dimensional nature of the PPWG.
Towards realizing a waveguide mode that offers a complete solution (i.e. low ohmic losses, negligible dispersion, very good coupling, and 2-D energy confinement) we investigate the lowest order transverse electric mode (TE\textsubscript{1}) mode of the PPWG. This work is motivated by studies carried out in the mid-infrared region of the spectrum about three decades ago [125-127]. These studies explored the possibility of obtaining ultra-low ohmic losses (in the db/km range) with good energy confinement via the use of the TE\textsubscript{1} mode.

5.2. The Transverse Electric mode for Parallel plate waveguides

As mentioned in section 4.1, when the input electric field is polarized parallel to the plates, only transverse electric (TE) modes can exist in the waveguide (see figure 4-1).

The TE field components are given by the following equations [128]:

\[ E_x = \frac{A_n \beta_y}{\varepsilon_0} \sin(\beta_y y) e^{-\beta_z z} \quad [5-1] \]

\[ H_y = \frac{A_n \beta_y \beta_z}{\omega \mu_0 \varepsilon_0} \sin(\beta_y y) e^{-\beta_z z} \quad [5-2] \]

\[ H_z = -j \frac{A_n \beta_y^2}{\omega \mu_0 \varepsilon_0} \cos(\beta_y y) e^{-\beta_z z} \quad [5-3] \]

where \( A_n \) is the amplitude of each mode and the phase constants \( \beta \).
$\beta_y$, and $\beta_z$ are given by:

$$\beta = \frac{2\pi}{\lambda} \quad [5-4]$$

$$\beta_y = \frac{n\pi}{b} \quad [5-5]$$

$$\beta_z = \sqrt{\beta^2 - \beta_y^2} \quad [5-6]$$

where $n$ is the mode number, $\lambda$ is the wavelength, and $b$ is the spacing between the two plates. A mode can only propagate if $\beta_z$ is sufficiently high ($\beta_z > 0$). The cut-off condition is given by equation [5-5] and the situation where $\beta_z = 0$, i.e.:

$$\beta_z = \sqrt{\omega^2 \mu \varepsilon - \left(\frac{n\pi}{b}\right)^2} = 0 \quad [5-7]$$

Here $n$ is the mode number, $b$ is the plate separation, $\omega = 2\pi f$. $\mu$ and $\varepsilon$ have their usual meanings. The cut off frequency is then given by:

$$f_c = \frac{nc}{2b} \quad [5-8]$$

Exactly at cutoff, the wave would bounce between the plates without propagation in the $z$-axis. So for a wave given by $e^{-\gamma z}$, we see that there are two cases: (1) where $\gamma$ is purely imaginary, $\gamma = i\beta_z$, and (2) where $\gamma$ is a real number.
When $\gamma = \alpha$ \hspace{1cm} $\gamma = \alpha$ \hspace{1cm} Evanescent wave

$e^{-\frac{\gamma z}{c}}$

$e^{-i\beta z}$ \hspace{1cm} $\gamma = i\beta$ \hspace{1cm} Propagating wave

When $\gamma = i\beta = i \sqrt{\omega^2 \mu \varepsilon - \left(\frac{n\pi}{b}\right)^2} = j\beta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$ \hspace{1cm} $f > f_c$ [5-9]

On the other hand:

When $\gamma = \alpha = i \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon} = \beta \sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$ \hspace{1cm} $f < f_c$ [5-10]

Where $\beta = \omega \sqrt{\mu \varepsilon}$. In equation 5-10, $\alpha$ is the attenuation constant for the mode, which immediately after entering the waveguide attenuates and acts as an evanescent wave (that decays exponentially and does not propagate).

A result of this cutoff frequency is high dispersion at frequencies near the cutoff, leading to severe distortion of the THz pulse. For a plate spacing of 0.5 mm, this cutoff frequency for the TE$_1$ mode is 300 GHz, well within the bandwidth of a standard photoconductive system. For this reason, in the past, the TE$_1$ mode was not a popular choice for terahertz time-domain measurements.

Recently [129], it was shown that because the cutoff frequency is dependent on the plate separation, this dispersion can be reduced by increasing the gap between the plates from 0.5 mm to 5 mm, shifting the cutoff frequency to 30 GHz, close to the limit of the THz frequency range, thereby allowing THz propagation with negligible dispersion. The idea of moving the cut off frequency to the low
frequency end by increasing the plate separation had not been explored previously with PPWGs, but a similar concept was first applied for the transport of microwave radiation in a hollow metallic circular waveguide [125] in the 1970’s. In that case and also in the present situation, the widening of the gap between the plates has the disadvantage of permitting multimode propagation.

This is the case because the cutoff frequency of several higher order modes now lie within the input spectrum. Thus, many multiple modes could be excited leading to excessive loss, dispersion, and mode conversion. For a plate separation of \( b = 5 \text{ mm} \), and input spectrum of 1 THz, there are a total of 17 possible even-symmetry TE modes (TE\(_1\), TE\(_3\), TE\(_5\), TE\(_7\),...,TE\(_{33}\)). This multi-mode problem can be resolved by selectively exciting only the TE\(_1\) mode via mode matching. The electric field of the TE\(_1\) mode has a spatial dependence given by:

\[
\Phi_n = \sqrt{\frac{2}{b}} \sin\left(\frac{n\pi y}{b}\right)
\]

[5-11]

where \( y \) is the coordinate normal to the plates. This spatial profile is well matched to a Gaussian input beam and should, therefore enable better coupling than can be achieved with the more commonly used TEM mode, which has a flat spatial profile. Figure 5-1 shows the electric field patterns for the TE\(_1\) to TE\(_4\) mode patterns of the parallel waveguide.
Figure 5-1. Electric Field patterns for the $\text{TE}_1$ to $\text{TE}_4$ modes of the parallel plate waveguide.

Higher order modes can be eliminated by matching the diameter of the input Gaussian beam to that of the $\text{TE}_1$ mode pattern [129]. Figure 5-2 shows a calculation of the power coupling efficiency ($\eta$) from an input Gaussian beam to several lower order modes, as a function of $b/D$, where $D$ is the $1/e$ beam size of the Gaussian at the input of the waveguide. The results show that the maximum possible $\eta$ is 99%, and is achieved when $b/D = 1.42$. The figure also shows that $\eta > 90\%$ when $b/D$ is in the range $0.53 < b/D < 2.1$, which indicates that one can achieve single $\text{TE}_1$ mode excitation for a large range of input beam sizes.
Figure 5-2. The power coupling efficiency from an input Gaussian beam to the TEM mode and several even-symmetric TE modes of the PPWG versus b/D. Where b is the plate separation, and D is the 1/e input beam diameter. A maximum possible of ~99% is achieved when $b/D = 1.42$

By converting the sinusoidal TE field components $[5-1$ to $5-3]$ into exponentials, we can represent them as superpositions of upward and downward traveling plane waves of the form:

$$E_x H_y H_z \propto (e^{-i\beta_y y} \pm e^{+i\beta_y y}) e^{\pm i\beta_z z} \quad [5-12]$$
This means that the TE mode propagation in the PPWG corresponds to two plane waves traveling up and down between the two plates in the z-direction, while polarized in the plane parallel to the plates. For the rest of the discussion, we use only one plane wave to describe the propagation of the THz wave in the PPWG.

In addition to improved coupling compared to the TEM mode, the TE \(_1\) mode also has the advantage of having relatively lower ohmic loss. The equations for ohmic loss [130] for the TEM and TE \(_1\) mode are given below.

\[
\alpha_{\text{TEM}} = \frac{2nR_s}{Z_0b} \quad [5-13]
\]

\[
\alpha_{\text{TE}} = \frac{4nR_s\left(\frac{f_c}{f}\right)^2}{Z_0b\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad [5-14]
\]

where the surface resistance \(R_s = \sqrt{\frac{\pi\mu}{\sigma}}\) and \(Z_0\) is the free space impedance. These equations illustrate a dependence on frequency. In the case of the TEM mode, the attenuation increases as frequency increases due to the decreasing depth plate separation. However, for the TE modes, attenuation actually decreases with increasing frequency. See figure 5-3. When the cutoff is shifted to lower frequencies, by widening the gap between the plates, the ohmic loss for the TE modes reduces significantly compared to the TEM mode at all frequencies of the input spectrum. For instance, analytical calculations show that for aluminum plates separated by 5 mm, the ohmic loss at a frequency of 1 THz is 2.67 db/km [129].
(many orders of magnitude lower compared to the TEM mode). This loss is three orders of magnitude lower than the lowest loss experimentally demonstrated in the THz range [131] and is comparable to that of telecommunications-grade optical fibers operating at 1.55 µm.

![Figure 5-3. The attenuation constant for the TE₁ mode. The theoretical solid curves are for a plate separation, b = 0.5 mm and 5 mm. The dots are experimental. (b) Close-up of the baseline of the theoretical curve for b=5 mm.](image)
5.3. Numerical Simulation Results

These low ohmic losses could permit long distance transport of THz radiation, significantly longer than what is now feasible. However, with such long propagation distances a new concern arises which is not relevant in either fiber optics or over-moded circular waveguides: energy leakage out of the unconfined sides of the PPWG due to diffraction of the propagating wave. In fact, this would be the dominant loss mechanism in the case under consideration, where the ohmic losses are virtually negligible [129, 132]. As a possible approach to mitigating this diffraction loss, we investigate the effects of introducing a slight curvature to the inside plate surfaces based on studies carried out in the mid infrared region of the spectrum in the 1970’s [127, 133].

As mentioned before, the TE mode is equivalent to a plane wave bouncing back and forth between the two parallel plates as it propagates in the z-direction. According to the bouncing plane wave theory, introducing a slight curvature (transverse to the axis of propagation) of radius R to the inside of the metal plates imparts a lateral focusing effect to the THz beam. The focusing effect at each bounce of the plane wave can be considered as being caused by a concave mirror with a focal length of R/2. This bouncing-plane-wave picture is illustrated in figure 5-4 (a-c), which shows a particular situation where the plane wave undergoes five bounces where the bounce-to-bounce distance is 2d. It can be shown that the bounce angle is related to the frequency by
\[ \theta = \cos^{-1} \left( \frac{c}{\nu 2b} \right) \quad [5-15] \]

where the plate separation \( b \) is related to \( d \) by \( b = 2d \cos \theta \), \( \nu \) is the frequency, and \( c \) is the speed of light. After "unfolding" the propagating wave, the repetitive interactions with the curved surface can be simulated by a series of identical thin lenses having a focal length of \( R/2 \) as shown in figure 5-4 (c). This value of \( R \) corresponds to the confocal condition at this particular frequency (based on the bouncing-plane-wave argument presented above), which allows maximum power transfer through the optical system [134]. At the confocal condition of \( R = 2d \), substituting from Eq. (5-15), we obtain,

\[ R = 2d = \frac{2b^2 \nu}{c} \quad [5-16] \]

This equation can be used to deduce the required radius of curvature that would allow for maximum power transfer, for a given plate separation and preferred operating frequency. For example, when \( b = 1 \) cm and \( \nu = 0.1 \) THz, we can deduce, \( R = 6.7 \) cm.
Figure 5-4(a) The transverse cross-section of the PPWG showing the concave plate geometry. (b) Longitudinal cross-section of the PPWG showing the path of the “bouncing plane wave” for five bounces. (c) The equivalent unfolded beam profile of the bouncing plane wave, where thin lenses simulate the focusing effect caused by the curvature.

First, we present results of a numerical study into the THz propagation behavior, investigating the applicability of this curved-surface waveguide geometry.
We use a commercial finite element method (FEM) modeling software (COMSOL Multiphysics) [135] to carry out the numerical simulations, and compare the behavior of several curved-surface waveguides to the well-known behavior of a flat-surface one. In the simulation, the outer boundaries of the two plates are assigned perfect-electric-conductor boundary conditions [136]. The simulation space is bounded by enclosing the waveguide within a solid rectangular box of vacuum, the walls of which are assigned low-reflecting boundary conditions to minimize the effects of back reflections. Each of the plates is 3 cm wide and 23 cm long. For each waveguide, the plate separation is 1 cm. In the case of the curved-surface plates, the plate separation is defined to be the plate spacing along the central axis.

To initiate the simulation, a 0.1 THz wave is incident on the input gap between the plates, linearly polarized along the x axis [parallel to the (nominal) plate surfaces] to excite only the TE modes. The input spatial profile is modeled as a Gaussian with an elliptic cross-section, where the major axis is chosen to be 2 cm, which is smaller than the plate width. The minor axis is chosen to be 0.7 cm to optimize the input coupling to the single TE$_1$ mode [129]. The model is solved using an iterative generalized minimal residual solver (GMRES) with Symmetric Successive Over-Relaxation (SSOR) matrix preconditioning.

Figure 5-5 (a) shows the magnitude of the electric field component oriented in the x direction ($E_x$), along a centralized slice (between the plates) in the x-z plane, for a wave propagating inside the flat-surface PPWG. Figure 5-5 (b) shows the $E_x$ distribution for a wave propagating inside a curved-surface waveguide with a radius
of curvature of 6.7 cm. A side by side qualitative comparison of the two figures indicates that there is less lateral diffraction in the curved-surface waveguide, as the wave propagates along the waveguide. This is expected as the design radius of curvature is optimized for the (0.1 THz) frequency under consideration.

Figure 5-5 (c) shows a comparison of the fractional time-averaged power versus distance of propagation for various radii of curvature. These results were extracted from numerical simulation results such as the ones shown in Figures 5-5 (a) and (b). We note that the energy confinement improves as the curvature is increased, and that there is a significant improvement in the energy confinement at the confocal condition, when R = 6.7 cm.
Figure 5-5. Results of numerical simulations of wave propagation in two different waveguide geometries. These display the electric-field slices in the x-z plane for a 0.1 THz wave propagating inside (a) a flat-surface PPWG, and (b) a curved-surface waveguide with curvature radius $R = 6.7$ cm. The thin black vertical lines denote transverse work planes at which the power flow can be extracted. (c) Power confinement as a function of propagation length for several values of the surface curvature.
5.4. Experimental Results

Next, we present experimental results showing how a slight curvature on the inside surfaces improves the lateral confinement along the open-ended sides. For the experiment, we fabricate five waveguides using polished aluminum plates. They have the following radii of curvature: $R = 6.7$ cm, $R = 20$ cm, $R = 50$ cm, $R = 100$ cm, and $R = \infty$ (i.e. a conventional flat surface PPWG). They each have a transverse width of 3.8 cm and a center-to-center plate separation $b = 1$ cm. THz pulses are generated and detected using a conventional THz-time domain spectroscopy system based on fiber-coupled photoconductive antennas [137]. A schematic that illustrates our measurement technique is shown in Figure 5-6.

The input electric field is polarized parallel to the (nominal) plate surfaces to excite only TE modes. The THz receiver is shown scanning across the output face of one of the curved-surface waveguides. As defined in the figure, $L = 25$ cm, $W = 3.8$ cm, and $b = 1$ cm.
Figure 5-6. A curved PPWG with its input electric field polarized parallel to the PPWG to excite the TE mode. The THz receiver is shown scanning across the center axis at the output facet. In the figure, \( L = 25 \text{ cm} \), \( W = 3.8 \text{ cm} \), and the center-to-center plate separation, \( b = 1 \text{ cm} \).

The input THz beam is centered on the front face in both the \( x \) and \( y \) directions and weakly focused using two convex teflon lenses (not shown), to achieve a frequency independent \( 1/e \) input beam diameter of \( \approx 2 \text{ cm} \). This beam size was chosen to dominantly excite the TE\(_1\) mode [129]. Typical THz waveforms measured at the input and output facets of our waveguides are shown in Figure 5-7. Figure 5-7(a) shows the measured THz pulse at the input facet, as well as its amplitude spectrum (inset). Figure 5-7(b) shows THz pulses measured at the output face of a flat-surface waveguide and a waveguide with a surface curvature of \( R = 6.7 \text{ cm} \).
In both cases, during each measurement, the receiver is centered in both the x and y directions. A qualitative comparison of both output signals indicates that the dispersion introduced by the curved-surface waveguide is comparable to that introduced by the flat-surface one. Furthermore, the respective spectra (shown in the inset) indicate that there is relatively more low-frequency content in the output signal corresponding to the curved-surface waveguide. We note that this is consistent with an apparent reduction in the diffraction losses in the vicinity of 0.1 THz (design frequency) for the curved-surface waveguide, as predicted by the theoretical results.
Figure 5-7. (a) Input THz pulse, and (b) output THz pulse for a flat-surface PPWG and a curved-surface waveguide with curvature radius $R = 6.7$ cm, measured on axis. The corresponding amplitude spectra are shown as insets.
We also image the electric field at the output of the waveguides in a plane perpendicular to the axis of propagation. The THz receiver is raster-scanned in a 20 × 60 mm\(^2\) grid, in steps of 1 mm. The detected time-domain waveforms obtained from raster-scanning were Fourier transformed and their field amplitudes were used to plot two-dimensional color plots at specific frequencies. These plots shown in figure 5-8 give the electric field distribution for a flat surface waveguide, and for curved-surface waveguides with curvature radii of R = 6.7 cm, 20 cm, and 50 cm.

Panels 5-8(a), (b), and (c) correspond to the frequencies of 0.1 THz, 0.2 THz, 0.3 THz, respectively. It is evident that at each frequency, the waveguide with the radius of curvature R = 6.7 cm exhibits the most energy confinement. In figure 5-8(a), at a frequency of 0.1 THz, we observe the best improvement in energy confinement with R = 6.7 cm, as predicted by our numerical simulation results and analytical derivations. Furthermore, we note that in general, the improvement in energy confinement diminishes with increasing frequency.

In figure 5-9, we plot one-dimensional line profiles corresponding to horizontal cuts along the central axis of the two-dimensional color plots shown in figure 5-8. These profiles show the normalized electric field amplitude at the output of the waveguides, when the receiver is scanned across the x-axis centered between the plates, as shown in the inset. Again, we observe that the waveguide with the radius of curvature R = 6.7 cm displays the least amount of diffraction at each frequency shown. These results indicate that this curvature has the effect of
confining a relatively broad range of frequencies, although the confocal condition discussed above applies to only one particular frequency.
Figure 5-8. 2-D colour profiles of the THz electric field distribution measured at the output of the PPWG for a flat-surface PPWG, and for curved-surface waveguides with curvature radii of: R = 6.7 cm, R = 20 cm, and R = 50 cm.
Figure 5-9. Corresponding line profiles for the contour plots shown in figure 5-8. The line profiles show the detected electric field magnitude at the output of the waveguides, when the receiver is scanned across the x axis centered between the plates, as shown in the inset.
In figure 5-10, we plot the full-width-at-half-maximum (FWHM) of the profiles (given in figure 5-9) versus the inverse of the radius of curvature of the waveguide plates. The measured FWHM are plotted as discrete symbols, while the solid lines are least-square fits. We note that the (negative) slope of the FWHM decreases as a function of frequency, and is almost zero at 0.5 THz, indicating that the curved plates have less effect on the output beam size as the frequency increases. To quantify this effect, we show the slope of these lines versus frequency in the inset. The gradient is largest at low frequencies, which is not surprising since the radius of curvature (of R = 6.7 cm) was optimized for maximum power transfer at the lowest frequency of 0.1 THz. This result indicates the bandwidth over which the concave plates have a significant confining effect on the propagating mode. Even though the confocal condition (Eq. [5-12]) suggests that this mode confinement strategy is a narrow-band effect, it is clear from our results that the mode confinement is effective over a bandwidth of at least several hundred GHz.
Figure 5-10. Measured FWHM of the electric field profile at the central x axis of the output face of the waveguide as a function of 1/R. The discrete symbols are the measured values at several frequencies and the lines are least-square fits. The inset shows a plot of the slope of each fitted line shown in the main figure, versus frequency, and the corresponding least-square fit.

Another consideration involves the possibility that concave plates could introduce appreciable group velocity dispersion. In order to quantify and compare the dispersion behavior, we consider the fundamental equation governing the input and output relationship of the experimental system. Assuming single-mode propagation, this relationship can be expressed in the frequency domain as
\[ E_{\text{out}}(\omega) = E_{\text{in}}(\omega)TC \exp[-j\beta L] \exp(-\alpha L) \]  \[5-17\]

where \( E_{\text{out}}(\omega) \) and \( E_{\text{ref}}(\omega) \) are the complex spectral components at angular frequency \( \omega \) of the output and reference electric fields, respectively; \( T \) is the total transmission coefficient, which takes into account the reflections at the entrance at exit surfaces; and \( C \) is the amplitude coupling coefficient, which takes into account the spatial-mode mismatch at both the entrance and exit faces. \( L \) is the distance of propagation, \( \alpha \) is the attenuation constant, \( \beta \) is the phase constant.

The experimental group velocity can be estimated using [138]

\[ v_g(\exp t) = (d\beta / d\omega)^{-1} = c\beta / k_0 \], where \( \beta = \sqrt{(k_0)^2 - (\pi / b)^2} \) and \( k_0 = w / c \).

We plot \( V_{g, \exp} \) for \( R = 6.7 \) cm, 50 cm, and \( \infty \), along with, \( V_{g, \text{theory}} \) as a ratio with respect to \( c \), in figure 5-11. We find that in all three experimental cases the group velocity dispersion is negligible throughout the spectrum, except at the very low-frequency end, and comparable to the theoretical curve. This minimal dispersion behavior is due to the TE\(_1\)-mode cutoff frequency \{given by \( c/(2b) \)\} of 15 GHz being very close to the low end of the input spectrum. We also observe that within the noise level, there is no appreciable additional dispersion due to the surface curvature.
Figure 5-11. Measured values of the group velocity (with respect to c) for the waveguides with curvature R = 6.7 cm (circles), and R = 50 cm (squares), and flat, i.e., no curvature (diamonds) plotted along with the theoretical curve for a flat-surface waveguide.
In summary, we have demonstrated that it is possible to inhibit diffraction losses for the TE\textsubscript{1} mode of a PPWG operating in the THz region, by utilizing plates with slightly concave surfaces. Using a simple “bouncing plane wave” analysis, we demonstrate how to determine an ideal radius of curvature for a waveguide operating at a given THz frequency. We show both experimentally and theoretically that for a waveguide with a plate separation of 1 cm, one can inhibit the diffraction at (and around) a frequency of 0.1 THz, when the surface has a radius of curvature of 6.7 cm. These results support the possibility of realizing long range transport of THz radiation via PPWGs.

The first step to realizing long range transport in these PPWGs is by estimating their propagation losses. This loss measurement isn’t feasible with the 25 cm long PPWGs utilized here, due to the small path-lengths and the significant challenge in reliably measuring the losses there. Chapter 6 explores using long PPWGs (~170cm) to estimate these propagation losses.
Chapter 6

Measuring Propagation Losses in Curved parallel Plate waveguides

6.1. Introduction

The parallel-plate waveguide (PPWG) has drawn considerable research interest [50, 51, 107, 110, 112-114, 129, 131, 139-148], since it was first reported to support the undistorted propagation of terahertz (THz) radiation [50, 149]. From that time, the transverse-electromagnetic (TEM) mode of this waveguide has been the popular choice for measurements due its low loss, ease of quasi-optic coupling, and negligible dispersion due to the absence of a cutoff. Unlike the TEM mode, the lowest order transverse electric (TE\(_1\)) mode has a cut-off frequency, and thus this mode was largely unexplored in the THz regime until recently [129]. This work showed that the cut-off frequency could be moved to lower frequencies to reduce
the dispersion, while matching the input beam size to the plate separation to realize dominantly single-mode propagation. It also predicted the possibility of realizing ultra-low losses in the db/km range by again utilizing the TE$_1$ mode of this waveguide. These low ohmic losses could permit long distance transport of THz radiation, than what is now feasible. However, with such long propagation distances a new concern arises: energy leakage out of the unconfined sides of the PPWG due to diffraction of the propagating wave. This in fact, would be the dominant loss mechanism in the case under consideration, where the ohmic losses are virtually negligible.

We addressed this diffraction problem in chapter 5 of this thesis and in a recent article [148] where we showed that it is possible to inhibit diffractive losses for the TE$_1$ mode via slightly concave plates. Using a simple “bouncing plane wave” analysis, we demonstrated how to determine an ideal radius of curvature for a waveguide operating at a given THz frequency, via a confocal condition, which is given by: $R = \frac{2b^2\nu}{c}$, where $R$ is the radius of curvature of the inside surfaces of the waveguide, $b$ is the plate to plate separation, $\nu$ is the frequency, and $c$ is the speed of light. We showed both experimentally and theoretically that for a waveguide of length $L = 25$ cm, and with a plate separation of 1 cm, one could inhibit the diffraction at (and around) a frequency of 0.1 THz, when the inside surfaces of the PPWG have a radius of curvature of 6.7 cm.

In this current work, we extend the aforementioned idea to long PPWGs ($\approx 170$ cm). We attempt to measure the propagation losses in these long waveguides.
This loss measurement wasn’t feasible with the 25 cm long PPWGs we used previously, due to the small path-lengths and the significant challenge in reliably measuring the losses there. We select an optimal radius of curvature of 6.7 cm for our PPWGs, where the concave plates mimic a confocal mirror configuration that would allow for maximum power transfer at a frequency of 0.1 THz. We compare the overall loss in the curved waveguide with that of a waveguide with flat plates.

6.2. Experiment Techniques & Results

For the experiment, we fabricate two waveguides using polished aluminum plates. One of the waveguides has plates whose inside surfaces are slightly concave, with radius of curvature of $R = 6.7$ cm, while the other has flat plates. The two PPWGs each have a length of 167 cm, a transverse width of 3.8 cm, and a center-to-center plate separation of $b = 1$ cm. Figure 6-1 shows a schematic of our experimental setup. In the experiment, THz pulses are generated and detected using a conventional THz-time-domain spectroscopy system based on fiber-coupled photoconductive antennas [137]. The THz receiver is shown scanning across the output face of the curved-surface waveguides. The input electric field is polarized parallel to the plate surfaces to excite only TE modes. This input THz beam is centered on the front face in both the $x$ and $y$ directions and weakly focused to achieve a $1/e$ input beam diameter of $\sim 2$ cm. This beam size was chosen to dominantly excite the TE$_1$ mode.
Figure 6-1. A schematic of the curved-surface waveguide with its input electric field polarized parallel to the inner plate surfaces to excite the TE\textsubscript{1} mode. The THz receiver is shown scanning across the output face. In the figure, $W = 3.8$ cm, $L = 167$ cm, $R = 6.7$ cm, and $b = 1$ cm.

Fig. 6-2 shows a photo of the fully assembled 167 cm long flat PPWG. A closer look at the photo shows that the PPWG is comprised of several smaller waveguide pieces whose ends are carefully fit together using threaded stainless steel fasteners.
Figure 6-2. Top: A photograph of the fully assembled 167 cm long flat waveguide showing how the two parallel plates are supported. The long PPWG is in fact comprised of several smaller waveguide pieces whose ends are carefully fit together using threaded stainless steel fasteners. Bottom: A zoomed in shot at the junction of two pieces showing how the upper and lower pieces are attached.

Typical THz waveforms measured at the output facets of the 167 cm long flat-surface and curved-surface waveguides are shown in figure 6-3. During both
measurements, the THz receiver is centered in both the $x$ and $y$ directions. The respective spectra (shown in the inset) indicate that there is significantly more low-frequency content in the output signal corresponding to the curved-surface waveguide. This is consistent with an apparent reduction in the diffraction losses in the vicinity of 0.1 THz (design frequency) for the curved-surface waveguide, as predicted by the analytical and simulation results [129, 148]. This plot also clearly indicates that this curvature has the effect of confining a relatively broad range of frequencies, although the confocal condition applies to only one particular frequency. As in the previous case with short length PPWGs, a qualitative comparison of both output signals indicates that the dispersion introduced by the curved-surface waveguide is comparable to that introduced by the flat-surface PPWG.
Figure 6-3. Output THz pulses for a flat-surface and curved-surface PPWG with radius of curvature $R = 6.7$ cm measured on axis. The respective spectra are shown in the inset.

We also image the electric field at the output of the waveguides in a plane perpendicular to the axis of propagation. The THz receiver is raster-scanned in a $20 \times 60$ mm$^2$ grid, in steps of 1 mm. The detected time-domain waveforms obtained from raster-scanning were Fourier transformed and their field amplitudes were
used to plot two-dimensional color plots. Figure 6-4(a) and (b) show 2-D color plots at a frequency of 0.1 THz, all plotted on the same color scale. Figure 6-4(a) illustrates the electric field distribution at the output face of a 23.8 cm curved and flat-surface PPWG, while figure 6-4 (b) shows the distribution at the output face of 167 cm long PPWG. As with the previous case with the short-length PPWGs, we observe a significant improvement in energy confinement in the curved PPWG, whose curvature is optimized for the design frequency of 0.1 THz. The beam size at the end of the 167 cm long curved-surface PPWG also seems smaller in size laterally, compared to the one at the end of the 23.8 cm long flat PPWG. This is most likely because the beam in the short waveguide undergoes 4 bounces whereas the one in the long waveguide undergoes ~25 bounces while propagating along the entire length of the waveguide. Since the beam undergoes lateral focusing after every bounce [129], it is not surprising that the beam traveling in the long PPWG is the smaller in size in that dimension.
Figure 6-4. Two dimensional color plots at 0.1 THz showing the electric field distribution measured across the output face of (a) a 23.8 cm curved-surface and flat-surface PPWG (b) a 167 cm curved-surface and flat-surface PPWG. We observe a significant improvement in energy confinement in the curved PPWG, whose curvature is optimized for the design frequency of 0.1 THz.
To further quantify how much of the THz wave is dissipated by the time it arrives at the output end of the 167 cm long curved-surface PPWG, we perform Fourier transforms for the detected time-domain waveforms whose points are inside the 10 × 40 mm² grid shown in the inset in Figure 4. For each frequency, we integrate the electric field amplitude for all the points in that grid, and plot them versus frequency in figure 6-5. We again observe an apparent reduction in the diffraction losses in the vicinity of 0.1 THz (the design frequency) as expected, and a significant loss in energy at frequencies above 0.15 THz (outside the design frequency).

**Figure 6-5.** Fourier transform waveforms showing the integrated electric field amplitude for time domain waveforms obtained from raster scanning the receiver in a 10 × 40 mm grid at the output of the 23.8 and 167 cm curved-surface waveguide. The inset shows the location of the 10x 40 mm² whose points are integrated to obtain these fourier transforms.
Next, we quantify the THz power intensity dissipated at various PPWG lengths for the flat and also for the curved-surface waveguide. We select 5 lengths of interest: 167 cm, 143 cm, 71 cm, 47 cm, and 23.8 cm. For each of these lengths, we obtain Fourier transforms for the detected time-domain waveforms for the points inside the $10 \times 40 \text{ mm}^2$ grid shown in the inset in figure 6-5, for both the curved-surface and flat PPWG. As before for each frequency, we integrate the intensity for all points. We perform several separate measurements and plot (in figure 6-6) the normalized mean of those measurements as solid points and the standard error of the mean of the measurements as error bars. The solid curves are exponential fits to the solid points. For the curved PPWG, we observe a dramatic improvement in the energy conservation at the design frequency of 0.1 THz, and an appreciable amount of energy conservation at 0.2 THz. In comparison, close to 50% of the energy is lost in the flat PPWG at both 0.1 THz and 0.2 THz, by the time the propagating THz wave reaches the output of the 167 cm long waveguide.

In figure 6-7, we focus just on the curved PPWG and characterize how its integrated power intensity varies for five frequencies: 0.1, 0.2, 0.3, 0.4 and 0.5 THz. As expected we find that there is a significant improvement in the power confinement at the design frequency of 0.1 THz, and that it drops substantially at higher frequencies.
Figure 6-6. Normalized plots of the integrated intensity for successive waveguide lengths of the flat-surface and curved-surface waveguide. We perform several separate measurements and plot the normalized mean of those measurements as solid points and the standard error of the mean as error bars. The solid curves are exponential fits to the solid points.
Figure 6-7. Normalized plots of the integrated power intensity for successive lengths of the curved–surface waveguide showing the dependence on frequency. We perform several separate measurements and plot the normalized mean of those measurements as solid points and the standard error of the mean of the measurements as error bars. The solid curves are exponential fits to the solid points.
Lastly, we measure the propagation loss for the curved and flat PPWGs using the aforementioned integrated power intensity data taken at the end of the 5 selected lengths of interest: 167 cm, 143 cm, 71 cm, 47 cm, and 23.8 cm. We compute the logarithm of the intensity data and use the length data above to fit it using a straight line, in order to obtain the power attenuation. We plot the power attenuation in $dB/m$ versus frequency as shown in figure 6-8. As expected the loss is lowest for frequencies in the range close to the design frequency (0.1 THz). In order to better visualize the loss pattern in the curved-surface PPWG at low frequencies (0.07-0.2 THz), we calculate the average of the loss in ~ 22 GHz segments and plot them in figure 6-8. These results indicate that the loss around the design frequency is less than 1 dB/m, very close to the lowest loss measured (0.95 dB/m at 2.5 THz) for the least lossy THz waveguide to date [131].
Figure 6-8. Comparison of the propagation loss (dB/m) in the flat-surface and curved-surface PPWG. We use the integrated intensity data taken at the end of the 5 selected lengths of interest: 167 cm, 143 cm, 71 cm, 47 cm, and 23.8 cm. We compute the logarithm of the intensity data and use the length data to fit it using a straight line, in order to obtain the power attenuation.
Figure 6-9. The averaged power attenuation for the range between 0.07 and 0.2 THz for the curved-surface PPWG shown in figure 6-8. The power attenuation is averaged in successive ~ 22 GHz segments.

We have shown that it is possible to inhibit TE₁ mode diffraction losses in long PPWGs with slightly concave plates. By utilizing a confocal condition, we design a 167 cm long waveguide with a radius of curvature of R = 6.7 cm and demonstrate lateral energy confinement at a design frequency of 0.1 THz. We reliably measure propagation losses of about 1 dB/m in this curved waveguide. These losses are among the lowest measured to date for any THz waveguide.
Conclusion

We have presented theoretical and experimental evidence that show that the two-wire waveguide supports low loss terahertz pulse propagation, and illustrated that the mode pattern at the end of the waveguide resembles that of a dipole. We compared the weakly guided Sommerfeld wave of a single wire waveguide with this structure and found that the two-wire exhibits much lower bending losses. We also observed that a commercial 300-Ohm two-wire TV-antenna cable can be used for guiding frequency components of up to 0.2 THz, although these cables are generally designed to operate only up to about 800 MHz.

Next we explored THz propagation in parallel-plate waveguides. Utilizing the TEM mode of this waveguide we studied the reflection of THz radiation at the end of a PPWG, due to the impedance mismatch between the propagating transverse-electromagnetic mode and the free-space background. We found that for a PPWG with uniformly spaced plates, the reflection coefficient at the output face increases as the plate separation decreases, consistent with predictions by early low frequency ray optical theory. We also observed this same trend in tapered PPWGs, when the input separation was fixed, and the output separation varied.

In another study, we investigated how to minimize diffraction losses in PPWGs by using plates with slightly concave surfaces. Using a simple “bouncing plane wave” analysis, we demonstrated how to determine an ideal radius of
curvature for a waveguide operating at a given THz frequency. We performed a
detailed experimental and simulation study that illustrated, for a waveguide with a
plate separation of 1 cm, one could inhibit the diffraction around a frequency of 0.1
THz, when the surface has a curvature of 6.7 cm. Using much longer PPWG
(~170cm), we reliably measured the overall losses in a PPWG with a radius of
curvature of R=6.7 cm, and found them to be about 1db/m, very close to the lowest
achieved loss to date with any terahertz waveguide.
References


73. Rajind Mendis, and Daniel M. Mittleman, "Comparison of the lowest-order transverse-electric (TE1) and transverse-magnetic (TEM) modes of the parallel-plate waveguide for terahertz pulse applications," Optics Express 17, 14839 (2009).


132. R. Mendis, and D. M. Mittleman, "Comparison of the lowest-order transverse-electric (TE\textsubscript{1}) and transverse-magnetic (TEM) modes of the parallel-plate waveguide for terahertz pulse applications," Optics Express 17, 14839-14850 (2009).


135. www.comsol.com

Appendix A

Program for plotting the 2-D image files in chapter 5 & 6

%WGANALYSIS

%INPUT - ROWS/COLUMNS
cols = input('Enter number of columns [x-coord]: ');
rows = input('Enter number of rows [y-coord]: ');

% %PREALLOCATE
% for i=1:10
% f(:,:,i)=zeros(cols,rows,i);
% end;

%CALLS FUNCTION AND STORRS FFT DATA FOR EVERY POINT IN finalDatafft
for m=6:rows;
    for n=10:cols;
        a=wgFunction(m,n);
        f(n,m,:)=a(:);
    end
end

for i=1:4000;
    d(i)=sum(sum(f(:,:,i)));
end

Function for plotting the 2-D image files in chapter 5 & 6

function [a]=wgFunction(row,col)

%read in data files
fileName1=strcat('166 cm_ ',num2str(col),'_',num2str(row),'.picotd');
data=dlmread(fileName1,'	',6,0);

%CORRECT FOR DC OFFSET
data(:,2)=data(:,2)-mean(data(:,2));

%TRUNCATE: FIND 'ZERO' CROSSING
min1=500; %set to larger number so it finds the min of data in this region
for iStep1=140:160
    test1=abs(data(iStep1,2));
    if test1<min1
        min1=test1;
        timecut1=iStep1;
    end
end

min2=100;
for iStep2=3800:3900
    test2=abs(data(iStep2,2));
    if test2<min2
        min2=test2;
        timecut2=iStep2;
    end
end

cuttime=data(timecut1:timecut2,1);
cutdata=data(timecut1:timecut2,2);

nfft = 4000;

freq = (0:(nfft-1))/(nfft*(0.078125));
finalDatafft = abs(fft(cutdata(:),nfft));

%finalDatafftrev(:,1)=smooth(finalDatafft(:,1),3);
a=zeros(4000,1);

a=finalDatafft.^2; % convert from field to power