RICE UNIVERSITY

Nonlinear Aeroelastic Analysis of UAVs: Deterministic and Stochastic Approaches

By

Thomas Woodrow Sukut, 2d Lt USAF

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

Master of Science

APPROVED, THESIS COMMITTEE

Dr. Pol D. Spanos, Chair
Lewis B. Ryan, Professor of Mechanical Engineering and Materials Science

Dr. Andrew J. Meade
Professor, Mechanical Engineering
Department Chair

Dr. Andrew J. Dick
Assistant Professor

HOUSTON, TEXAS
April 2012
ABSTRACT

Nonlinear Aeroelastic Analysis of UAVs: Deterministic and Stochastic Approaches

By

Thomas Woodrow Sukut, 2d Lt USAF

Aeroelastic aspects of unmanned aerial vehicles (UAVs) is analyzed by treatment of a typical section containing geometrical nonlinearities. Equations of motion are derived and numerical integration of these equations subject to quasi-steady aerodynamic forcing is performed. Model properties are tailored to a high-altitude long-endurance unmanned aircraft. Harmonic balance approximation is employed based on the steady-state oscillatory response of the aerodynamic forcing. Comparisons are made between time integration results and harmonic balance approximation. Close agreement between forcing and displacement oscillatory frequencies is found. Amplitude agreement is off by a considerable margin. Additionally, stochastic forcing effects are examined. Turbulent flow velocities generated from the von Karman spectrum are applied to the same nonlinear structural model. Similar qualitative behavior is found between quasi-steady and stochastic forcing models illustrating the importance of considering the non-steady nature of atmospheric turbulence when operating near critical flutter velocity.
I would like to thank several individuals for contributions to my work here at Rice University. First and foremost I would like to thank Dr. Pol Spanos for giving me the opportunity to work under his advisement. His robust mentorship in class and in various stages of development of this thesis was invaluable.

I would also like to thank my fellow Air Force colleagues for their varied support, especially Matt Guertin and Tony Puntel. Thanks for checking my calculations, helping me debug my code, and tearing me away from the computer every now and then to relax and live a little. Lots of luck in your Air Force careers.

Finally I would like to thank my family and friends for their support up to this point in my personal and professional life. I could not have done it without them. Their love and support has helped me achieve a long list of achievements. I'm proud to now add this master's thesis to that list.

The views expressed in this article are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U.S.
# Contents

Acknowledgments ................................................................................................. iii  
List of Figures ........................................................................................................ vi  
List of Tables .......................................................................................................... viii  
Nomenclature ......................................................................................................... ix  
Introduction ........................................................................................................... 1  
  1.1. Motivation ........................................................................................................ 1  
  1.2. Aeroelasticity Explained ................................................................................. 3  
  1.3. Past and Present Research ............................................................................. 7  
  1.4. Application to UAVs. ..................................................................................... 11  
  1.5. Scope ............................................................................................................... 15  
Structural Model .................................................................................................... 16  
  2.1. Equations of Motion ...................................................................................... 17  
  2.2. UAV-Specific Parameters ............................................................................. 23  
Aerodynamic Modeling ........................................................................................... 34  
Numerical Integration ............................................................................................. 40  
  4.1. Application ..................................................................................................... 41  
Harmonic Balance ................................................................................................... 45  
  5.1. Application ..................................................................................................... 46  
Stochastic Modeling ............................................................................................... 52  
  6.1. Turbulence Modeling ..................................................................................... 53  
  6.2. Application ..................................................................................................... 55  
Results and Discussion ......................................................................................... 57  
  7.1. Quasi-steady Aerodynamics with Constant Velocity ................................... 58  
  7.2. Harmonic Balance ......................................................................................... 68  
  7.3. Quasi-steady Aerodynamics with Stochastic Velocity ............................... 74  
Concluding Remarks ............................................................................................. 77
8.1. Future Work ......................................................................................... 79

References .................................................................................................. 81
List of Figures

Figure 1-1: Coalescence of Vibration Modes [4].................................................. 5
Figure 1-2: Components of Aeroelastic Analysis [6]............................................10
Figure 2-1: Structural Model [18]........................................................................17
Figure 2-2: Representative HALE UAV Designs. (a) Predator, (b) Reaper, (c) Global Hawk. Note: Not to scale [23].................................................................25
Figure 2-3: NACA 632 – 415 airfoil [25]..............................................................28
Figure 3-1: Lift Curve for the NACA 632 – 415 Airfoil [25]............................39
Figure 4-1: Comparison of Numerical Results with Exact Steady-State Solution ..................................................................................................................44
Figure 4-2: Coupled Aerodynamic Model and Numerical Integration Algorithm ......................................................................................................................44
Figure 5-1: Harmonic Balance Solver Algorithm ..................................................51
Figure 6-1: Simulink Block Diagram of Turbulence Generation .......................55
Figure 7-1: Pitch and Plunge vs. Time, V = 50 m/s ...........................................59
Figure 7-2: Pitch and Plunge vs. Time, V = 87 m/s ...........................................59
Figure 7-3: Pitch and Plunge vs. Time, V = 89 m/s ...........................................60
Figure 7-4: Pitch and Plunge vs. Time, V = 90 m/s ...........................................61
Figure 7-5: Pitch and Plunge vs. Time, V = 95 m/s ...........................................61
Figure 7-6: Aerodynamic Force and Displacement Amplitude as a Function of the Velocity........................................................................................................63
Figure 7-7: Frequency of Oscillation as a Function of Velocity .........................64
Figure 7-8: Pitch and Plunge vs. Time, V = 10 m/s ...........................................65
Figure 7-9: Pitch and Plunge vs. Time, V = 50 m/s ...........................................65
Figure 7-10: Pitch and Plunge vs. Time, $V = 80 \text{ m/s}$ .......................................................... 66
Figure 7-11: Pitch and Plunge vs. Time, $V = 90 \text{ m/s}$ .......................................................... 67
Figure 7-12: Aerodynamic Forces Fit to 1$^{\text{st}}$ Order Fourier Series ..................................... 69
Figure 7-13: Harmonic Balance Model, $V = 90 \text{ m/s}$ .......................................................... 70
Figure 7-14: Harmonic Balance Model, $V = 90 \text{ m/s}$ (Zoomed in) ...................................... 71
Figure 7-15: Harmonic Balance Model, $V = 95 \text{ m/s}$ .......................................................... 72
Figure 7-16: Harmonic Balance Model, $V = 100 \text{ m/s}$ .......................................................... 72
Figure 7-17: Harmonic Balance Approximation Error as a Function of Velocity
................................................................................................................................................. 73
Figure 7-18: Pitch and Plunge vs. Time with Stochastic Velocity Input,
Mean Velocity = 85 m/s .............................................................................................................. 75
Figure 7-19: Pitch and Plunge vs. Time with Stochastic Velocity Input,
Mean Velocity = 89.2 m/s ......................................................................................................... 75
Figure 7-20: Pitch and Plunge vs. Time with Stochastic Velocity Input,
Mean Velocity = 95 m/s .............................................................................................................. 76
List of Tables

Table 1-1: General UAV Classifications and Characteristics [16].......................... 12
Table 2-1: Typical UAV parameters in standard units. Metric units given in parenthesis.................................................................................................................. 24
Table 2-2: Structural Model Parameters........................................................................ 32
Table 4-1: Numerical Integration Parameters................................................................... 42
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>HALE</td>
<td>High-Altitude, Long-Endurance</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>LCO</td>
<td>Limit Cycle Oscillation</td>
</tr>
<tr>
<td>$h$</td>
<td>Plunge displacement</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Pitch displacement</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$EA$</td>
<td>Elastic Axis</td>
</tr>
<tr>
<td>$AC$</td>
<td>Aerodynamic Center</td>
</tr>
<tr>
<td>$CG$</td>
<td>Center of Gravity</td>
</tr>
<tr>
<td>$I_{EA}$</td>
<td>Moment of inertia about the elastic axis</td>
</tr>
<tr>
<td>$x_\alpha$</td>
<td>Non-dimensional distance between EA and CG</td>
</tr>
<tr>
<td>$x_\beta$</td>
<td>Distance between EA and AC</td>
</tr>
<tr>
<td>$c$</td>
<td>Chord length</td>
</tr>
<tr>
<td>$b$</td>
<td>Semi-chord length</td>
</tr>
<tr>
<td>$l$</td>
<td>Span length</td>
</tr>
<tr>
<td>$a$</td>
<td>Non-dimensional location of EA</td>
</tr>
<tr>
<td>$EI, GJ$</td>
<td>Flexural and Torsional Rigidity</td>
</tr>
<tr>
<td>$K_h, K_\alpha$</td>
<td>Stiffness coefficients (plunge and pitch)</td>
</tr>
<tr>
<td>$C_h, C_\alpha$</td>
<td>Damping coefficients (plunge and pitch)</td>
</tr>
<tr>
<td>$L$</td>
<td>Aerodynamic lift force</td>
</tr>
<tr>
<td>$M$</td>
<td>Aerodynamic moment</td>
</tr>
<tr>
<td>$C_{L\alpha}$</td>
<td>Lift curve slope</td>
</tr>
<tr>
<td>$C_L, C_M$</td>
<td>Force coefficients (lift and moment)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
</tr>
<tr>
<td>$V$</td>
<td>Air velocity</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1. Motivation

Aircraft offer seemingly endless technological possibilities, yet in order to fully take advantage of them it is necessary to have a complete understanding of every process involved. In the century since man's first flight, understanding the mechanisms responsible for such a feat has been a major priority. Basic aerodynamics, structural considerations, navigation techniques, and control systems have all evolved around the goal of taking advantage of our atmosphere. As technology advances more questions have been raised which have continued to fuel man's quest for answers. Modern research is concerned less with how to get an aircraft to fly than with how to get it to fly better; how can the boundaries of possibility be expanded?
One of the major challenges facing aircraft designers today is that of aeroelasticity. Complex interactions between dynamics, solid mechanics, and aerodynamic forces can create problems if not well understood and analyzed. Aircraft structural fatigue, passenger discomfort, decreased performance, and even catastrophic failure can result. Early in the history of the airplane this problem was minimal due to low flight speeds, large factors of safety, and moderate performance requirements. Today’s aircraft, however, are expected to push the physical limits in terms of speed, altitude, maneuverability, endurance, range, and cost. Designers are turning to lightweight materials for use with high-powered engines to reduce weight in order to carry more fuel and payload. These lightweight materials exhibit more flexibility than conventional aircraft materials which when used at higher speeds and altitudes pose possible aeroelastic concerns. With current computational resources it is becoming more feasible to model and correct for aeroelastic deficiencies, yet advancements in the theory and practice of aeroelastic analysis fail to match the pace of performance requirements. Thus further innovation is inhibited until the phenomenon of aeroelasticity can be fully understood.

In the preceding direction, this work aims to break down aeroelastic phenomenon into its basic components, apply current techniques to generate a detailed structural model, and then use various nonlinear methods to analyze said model in the aeroelastic framework. The end goal of this work is to weigh the feasibility, effectiveness, and cost of the various analysis methods. Though generally applicable to most aircraft designs, for the purposes of this thesis the analysis will
be carried out in the context of a typical high-altitude, long-endurance (HALE) unmanned aerial vehicle (UAV).

1.2. Aeroelasticity Explained

The term aeroelasticity is used to refer to any phenomena in which inertial, aerodynamic, and elastic forces interact [1]. These interactions can be as inconsequential as a slight vibration, and as catastrophic as a complete structural failure. Aeroelastic analysis encompasses the fields of dynamics, structural mechanics, and aerodynamics, and they all hold equal importance. Aeroelasticity can refer to numerous structural/aerodynamic interactions such as wind interactions with power lines, suspension bridges, and buildings. For the purposes of this thesis, however, aeroelastic analysis will be focused on the behavior of an aircraft wing in an air stream. In aircraft aeroelasticity, aerodynamic forces on flexible bodies (wings, control surfaces, etc.) produce displacements that cause the body to interact with the air around it. Inertial forces are encountered, as well as changes in the aerodynamic forces based on the changing incidence of the body with the airflow. These additional forces produce additional displacements which, in turn, produce additional changes in the forces. As no structure is completely rigid, certain manifestations of this phenomenon is expected in all aircraft. However, the rigidity and structural damping of the aircraft wing should be such that this phenomenon is minimal or nonexistent during normal operation. For instance, if an aircraft wing is disturbed, perhaps through a wind gust, the desired behavior is that the wing stiffness and damping along with the aerodynamic and inertial interactions
should work to quickly return the wing to a stable equilibrium. If the disturbance is instead maintained or amplified through the structural, aerodynamic, and inertial interactions, the behavior can possibly result in material fatigue and ultimately structural failure. The main design tradeoff which affects aeroelasticity is weight versus wing stiffness. Strong stiff wings weigh more, require more fuel, and do not permit as much payload, but they are less likely to encounter aeroelastic interactions. To prevent the reduced efficiency from heavier wings, stiffness is usually sacrificed for lighter weight components which tend to be more flexible and therefore more prone to aeroelastic interactions.

Aeroelasticity can be broken into two main categories: static and dynamic. Cases in which the inertial forces play a negligible role are referred to as static. In static aeroelasticity, the aerodynamic forces are simply greater than the elastic restoring forces [2]. The resulting instability is called divergence and if not detected and corrected quickly, can be catastrophic. Dynamic aeroelasticity involves influential inertial forces and the associated instabilities are referred to as flutter [2]. As Reference [3] describes, when a cantilever wing is disturbed, the wing oscillates about its equilibrium until eventually the oscillations damp out. If the same wing is subjected to an airflow the damping effect increases as the flow velocity increases to a certain point. After this point, however, further increases in the flow velocity result in a rapid decrease in the damping. The point at which the damping reaches zero and sustained oscillations are possible is called the critical flutter velocity. By further increasing the flow velocity the wing enters a phase of
negative damping in which even minute disturbances can result in large violent oscillations. These oscillations can have both bending and torsional components.

Flutter is often thought of as a resonance problem. In reality, flutter is a self-excited phenomenon. Reference [4] describes the mechanism responsible for flutter as a coalescence of the natural frequencies of the structure. When the frequencies approach each other the airfoil extracts energy from the airstream which results in oscillations. This behavior is shown in Figure 1-1. At point A the frequencies are distinct, no energy extraction occurs, and the system damping causes the amplitude to decrease. At point B the frequencies have approached each other such that disturbances result in harmonic oscillations with constant amplitude. Point C demonstrates the coalescence of the frequencies and the introduction of flutter [4]. In the linear case this behavior will continue to grow in amplitude until wing failure. In the nonlinear case various mechanisms can result in limit cycle oscillations (LCO).

**Figure 1-1: Coalescence of Vibration Modes [4]**
From this description one can see that aircraft flutter is generally experienced as a high frequency oscillation of the wing. Adequate control systems can be developed to correct for this behavior. However, to precisely control the aeroelastic behavior for a particular aircraft design the behavior must first be modeled accurately for the control system to be effective. The high frequency of the oscillations does not permit proper human compensation in most cases [1].

Divergence and flutter can both be encountered suddenly and without warning. The role of aeroelastic analysis is to predict the operational conditions which might permit such behavior, so that these conditions can be avoided entirely either through design accommodations or operational limitations.

Another aspect of aeroelasticity is that of aileron reversal. This phenomenon occurs at certain flight conditions where by increasing the velocity the aileron effectiveness decreases to zero and then becomes negative [2]. Though not generally catastrophic in itself, consequences of this behavior can lead to failure and ineffective aircraft handling. For example, if one attempts to reduce loading on a wing, the result may actually be an increased loading on the wing if the ailerons have reversed. Overloading the wing can then lead to fatigue and failure. Though this thesis does not explicitly address aileron reversal, it is mentioned herein as a possible source of excitation that can lead to static and dynamic aeroelastic behavior and one that any aircraft operator should be aware of.
1.3. Past and Present Research

Aeroelasticity issues have been encountered since the birth of the airplane. Widely known as the “first in flight,” the Wright brothers’ successful airplane was actually almost beaten by a competitor. Professor Samuel P. Langley’s first flight was only a few days before the Wright brothers’. Langley’s flight, however, was not successful and its failure is widely attributed to wing torsional divergence [1]. Langley’s monoplane design lacked the necessary torsional rigidity to accommodate the applied aerodynamic moment and the plane crashed into the Potomac River after the wing failed. The Wright Flyer, on the other hand, used a biplane design that employed “wing warping” for lateral control. This enabled the pilot to control the craft without drastically altering the wing characteristics [2]. The biplane design featured twin wings reinforced with cables between them. The result was much higher torsional rigidity than a comparable monoplane before successful “stressed skin” designs were later realized in the 1930s [2]. Because of the success of the Wright Flyer and failure of Langley’s plane, designers opted for the proven biplane design. Had the Wright Flyer not been successful, perhaps early aircraft designers might have sooner understood the necessary torsional requirements necessary for monoplanes.

As most early aircraft designs were based on the successful biplane, the first major aeroelastic issues encountered involved tail flutter [1]. At the beginning of World War I pilots of the Handley Page 0/400 bomber experienced violent oscillations of the fuselage and tail. In what is often cited as the first documented
investigation into the flutter problem, the investigators found that the aircraft had two principal modes of vibration [1]. One mode involved torsional oscillations of the fuselage, while the other mode caused the left and right horizontal tail surfaces to oscillate 180° out of phase. Coupling between these two modes was possible which resulted in the violent oscillations that were experienced. The solution to this was to connect the left and right horizontal tail sections to the same torque tube to prevent the second mode from occurring [1].

The importance of aeroelastic considerations with respect to wings was not readily apparent until later during the war with the development of the Fokker D-8 high performance monoplane. [1]. Static tests showed the wing able to withstand adequate design load factors, but it was found that as the aerodynamic load increased, the angle of attack at the wing tips increased more than at the wing root meaning the wings were experiencing excess tip loading which led to structural failure [1]. Other aircraft designs of the time were also beginning to push the limits of technology and understanding. Increased flight speeds exerted increased aerodynamic loads which the structures were not designed to handle properly in dynamic situations. Numerous aircraft losses and the prospect of losing a competitive military advantage resulted in the first serious research efforts to understand and fix the root causes of aeroelasticity. Unfortunately, designers were unable to understand or simply did not trust many of the theories put forth in the 1920s and 1930s [1].
In analyzing aeroelastic problems, linear techniques can be quite insightful. However, their applicability to real-world nonlinear problems is limited. Linear analysis techniques can help predict the speed at which flutter or divergence might occur, but may not be useful for determining the speeds at which a real-world nonlinear system might develop self-excited oscillations [2]. Linear analysis techniques also cannot predict the effects of small perturbations which might result in an oscillatory response. These self-excited or perturbed oscillations can lead to what is called limit cycle oscillations (LCO). LCO are steady-state oscillations which pose long-term fatigue problems and can affect passenger comfort and pilot endurance [2]. Cyclical loading and unloading can cause material deterioration and the formation of small cracks. While the loading may not exceed the failure limits of the material, repeated cyclic loading can eventually lead to failure due to fatigue [5]. With modern computational capabilities, nonlinear analysis techniques are able to address these problems, though they are still quite expensive in terms of both computational resources and time.

In order to understand the complex interactions, researchers have broken aeroelasticity down into several subsets. Figure 1-2 shows the three major subsets as well as their interactions to form intermediate fields of study which are also pertinent to the subject at hand. The aeroelastician must be well versed in the fields of dynamics, solid mechanics, and unsteady aerodynamics. Several well known texts are available detailing the progress in aeroelastic analysis. For additional information the reader is referred to References [1], [2], [3], and [6], among others.
The research community has made great strides in modeling and understanding aeroelastic phenomena, yet no complete grasp of the complexities involved has been achieved. A great deal of aeroelastic analysis is reactive in that it focuses on analyzing preliminary aircraft designs for any potential problems. To this end many researchers are focused on accurately modeling the aerodynamic environment and structural components of the design while maintaining computational efficiency. Reference [7] presents a detailed computational aeroelastic analysis of an aircraft using a finite element structural model coupled to an aeroelastic analysis code. Computational analysis models are very common in
recent literature. References [8], [9], [10], [11], and [12] are just a sample of all the various papers available. From these references, it is apparent that there are many ways to model and analyze aeroelastic phenomena. Models range from simple linear single degree of freedom structural models excited by simple linear aerodynamic models, to fully nonlinear finite element models coupled to complete unsteady aerodynamic codes. Each approach has advantages and disadvantages, and each offers insights into the mechanism of aeroelasticity. The broader perspective is that although aeroelasticity is not completely understood. The problem can be modeled and analyzed for use in aircraft design and operations by making various assumptions. One ultimate goal is to make aeroelastic considerations proactive, where certain performance characteristics can be incorporated that take advantage of the aeroelastic interactions.

1.4. Application to UAVs

UAVs are aerial vehicles designed to perform tasks which traditionally require a manned vehicle. Typical UAV configurations are either fixed wing or rotary wing and can range from several inches to hundreds of feet long [13]. Their missions are as varied as their configurations as well. UAVs have been designed for weather and atmospheric research, reconnaissance and surveillance, conventional combat, remote sensing, mapping, traffic monitoring, and search and rescue to name a few [13], [14]. Though no universal UAV classification exists, by convention UAVs are grouped according to size, weight, and capabilities. The vast majority of UAVs fall into one of three different categories. The first category covers most UAVs which
consist of conventional aircraft designs used for low altitudes and relatively short durations. The second category covers high altitude and long duration missions. The third category includes unconventional designs and those with special launch and recovery modes [15]. Table 1-1 shows some general UAV classifications and characteristics.

**Table 1-1: General UAV Classifications and Characteristics [16]**

<table>
<thead>
<tr>
<th>Category</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tactical UAV</td>
<td>Ubiquitous 50 to 1000 lb deployable air vehicle</td>
</tr>
<tr>
<td>Endurance UAV</td>
<td>Capable of extended missions, typically 24 hours or longer</td>
</tr>
<tr>
<td>Vertical Takeoff &amp; Landing (VTOL)</td>
<td>Typically rotary wing UAV</td>
</tr>
<tr>
<td>Man Portable UAV</td>
<td>Being light enough to be back-packed by an individual and launched by hand-throwing or sling-shot mechanism</td>
</tr>
<tr>
<td>Optionally Piloted Vehicle (OPV)</td>
<td>Capable of both manned or unmanned flight operations, typically an adaptation of a general aviation aircraft</td>
</tr>
<tr>
<td>Micro Air Vehicle (MAV)</td>
<td>Limited in size to dimension no larger than 15 cm (6 in)</td>
</tr>
<tr>
<td>Research UAV</td>
<td>Developed for specific research investigations, typically with no production intent</td>
</tr>
</tbody>
</table>

Low-altitude tactical systems are highly desirable because of their ability to perform dangerous tasks without the risk of losing the pilot. Because the design does not require a pilot, the aircraft can be smaller and more robust than conventional manned platforms with similar performance [15]. Additionally, because of the operating limits imposed on the aircraft by the physical limitations of the human body, pilotless aircraft can have higher performance and larger flight envelopes. Removing the pilot can also result in larger payload capacity or even longer range due to increased fuel capacity. However, the increased capabilities, such as larger flight envelopes, are predicated on adequate aeroelastic performance which may be difficult to achieve.
High-altitude long-duration missions are well-suited to UAVs because of the complex challenges of manning such missions. Special equipment and crew rotations necessary for this mission result in large and heavy aircraft. Again, by eliminating the operators the crafts are much smaller, less-detectable, and able to carry more payload and fuel to perform the same mission as their traditionally manned counterparts [15]. High-altitude missions are also desirable for UAVs from a safety perspective. Few aircraft operate at higher altitudes so issues of traffic management and airspace impingement are reduced. Furthermore, challenges of communication and wind uncertainty are also reduced at higher altitudes. HALE UAVs are also of note for their increased range due to further line-of-sight capability.

Traditionally, robots have been designed to complete “dull, dirty, or dangerous” tasks. In the UAV world—at least thus far—the same holds true. Modern UAVs are designed to collect intelligence, monitor weather, track targets, etc. [14]. The rationale here is that it is less-expensive and less manpower-intensive to operate UAVs in “dull and dirty” situations, and less damaging to a unit to lose a UAV than to lose an aircraft and its pilot in “dangerous” situations. The combination of decreased risk to the pilot and fewer aircraft restrictions leads to the development of faster, lighter, and less-expensive aircraft able to perform traditional manned missions [14]. However, with this comes the fact that smaller militaries and combatants could be able to acquire and develop fleets with similar capabilities to traditionally larger manned fleets. The advantage here lies in the superior UAV
design and performance, part of which includes the aircraft’s aeroelastic performance.

The calculation and modeling of aerodynamic vehicle properties is essential to ensure proper flight characteristics. With the proliferation of computers and improvements in processing power, developers are turning more toward computational methods for determining aerodynamic properties [17]. Software based on numerical methods gives designers the ability to reduce the time required to analyze different designs. Similarly, analysis techniques that have not previously been applied to the aeroelastic problem provide new insight and opportunities for more robust designs.

Effective modeling of dynamic systems is one of the most important parts of understanding how they operate. However, dynamic models are often quite complex and difficult to create and implement. Dynamic systems with flexible components further complicate this process and, as with most aircraft, UAVs employ lightweight flexible materials in their construction. While it may be possible to model the UAV as if it were a rigid structure, the real world flexing structure can have drastic effects on the dynamics. This can lead to large discrepancies between the modeled performance and the actual performance. Researchers are thus faced with making very complex models which can take very long and be very expensive. However, by invoking certain assumptions various aspects of the performance can be predicted within an accepted level of accuracy.
1.5. Scope

This thesis aims to apply numerical integration and harmonic balance analysis methods to a typical binary aeroelastic model to gauge their effectiveness and costs. Two models are developed: one with deterministic forcing and one with stochastic forcing. Both models are analyzed via numerical integration while only the deterministic model is also analyzed with a harmonic balance approach. A basic background in aerodynamic and structural analysis is assumed, while relevant sources are included for reference. An applicable structural model is developed in Chapter 2 after which the relevant parameters are defined in the context of a HALE UAV. Chapter 3 provides background information on the aerodynamic model and simplifying assumptions used. Background information on the numerical integration scheme employed for the nonlinear system at hand is given in Chapter 4. The method of harmonic balance is developed in Chapter 5. Chapter 6 presents some stochastic analysis methodology including the basis of the stochastic parameters used. A discussion of the results is included in Chapter 7, and Chapter 8 incorporates some concluding remarks and potential future research areas.
Chapter 2

Structural Model

The model used herein is a simple two-degree-of-freedom (2DOF) airfoil model that accommodates motion in pitch and plunge. The model is referred to in the literature as the typical section and shown in Figure 2-1. The airfoil can pitch about an elastic axis which is defined as being perpendicular to the shear center of the airfoil. The structural stiffness in pitch and plunge is modeled with linear springs. Noteworthy points on the airfoil include the center of gravity, elastic axis, and aerodynamic center. The force model used herein will be discussed in depth in a later chapter.
2.1. Equations of Motion

Using the above system model from Reference [18] with slight modification, the two-degree-of-freedom equations of motion for pitch ($\alpha$) and plunge ($h$) can be derived through the use of Lagrange’s Equations by calculating the potential and kinetic energies of the system. The sign convention for both $\alpha$ and $h$ is shown in Figure 2-1 with the arrow pointing in the positive sense. The $\hat{i}$ frame is fixed inertially, while the $\hat{b}$ frame is fixed to the airfoil with its origin at the center of gravity and oriented as shown above with the $\hat{b}_1$ axis towards the leading edge. An energy formulation is used to create the equations of motion [2]. The potential

![Figure 2-1: Structural Model [18]](image)

---

**Figure 2-1: Structural Model [18]**
energy consists entirely of the energy stored in the springs and can be modeled with the equation

\[ U = \frac{1}{2} K_h h^2 + \frac{1}{2} K_\alpha \alpha^2 \]  

(1)

where \( K_h \) and \( K_\alpha \) are the linear plunging and pitching stiffness coefficients, respectively.

To calculate the kinetic energy, the velocity of the center of mass—point c.g. in the figure above—is required.

\[ V_{c.g} = V_{EA} + \dot{\alpha} \hat{b}_3 \times x_\alpha \hat{b}_1 \]  

(2)

Here, \( V_{EA} \) is the velocity of the elastic axis, and \( x_\alpha \) is the distance between the elastic axis and the center of gravity which has been non-dimensionalized by the semi-chord. The variable \( x_\alpha \) is taken to be positive if the center of gravity lies further towards the trailing edge than the elastic axis does. The symbol \( (\cdot) \) denotes the first time derivative whereas the symbol \( (\cdot') \) denotes the second time derivative. Based on the definitions introduced and the coordinate systems chosen,

\[ V_{EA} = -\dot{\theta} \hat{i}_2 \]  

(3)

and thus

\[ V_{c.g} = -\dot{\theta} \hat{i}_2 + \dot{\alpha} x_\alpha \hat{b}_2. \]  

(4)
The kinetic energy is then given by

$$T = \frac{1}{2} m \mathbf{v}_{CG} \cdot \mathbf{v}_{CG} + \frac{1}{2} I_{CG} \dot{a}^2$$

(5)

where $I_{CG}$ is the moment of inertia about the center of mass. Using the relationship between the inertial and body frames, the kinetic energy can be rewritten as

$$T = \frac{1}{2} m (\dot{h}^2 + x_a \dot{\alpha}^2 \cos (\alpha) + 2 x_a \dot{h} \dot{\alpha} \cos (\alpha)) + \frac{1}{2} I_{CG} \dot{a}^2$$

(6)

$$T = \frac{1}{2} m (\dot{h}^2 + x_a \dot{\alpha} \cos (\alpha)) + \frac{1}{2} I_{EA} \dot{\alpha}^2,$$

(7)

where $I_{EA}$ is the moment of inertia about the elastic axis. It is given by the equation

$$I_{EA} = I_{CG} + m x_a^2 \cos^2 (\alpha).$$

(8)

The next step is to find the generalized forces acting on the system. This is done by using the method of virtual work where the work done by a virtual displacement due to the external forces is calculated [2]. The external forces in this case are the aerodynamic lift and moment. It is assumed that the aerodynamic forces act at the center of pressure. To calculate the virtual displacement due to the lift, the velocity of the aerodynamic center is required. That is,

$$\mathbf{v}_{AC} = -\dot{h} \mathbf{i}_2 + \dot{\alpha} x_{\alpha} \mathbf{b}_2.$$  

(9)

Then, the virtual displacement due to the lift is

$$\delta \mathbf{P}_{AC} = -\delta h \mathbf{i}_2 + \delta \alpha x_{\alpha} \mathbf{b}_2.$$  

(10)
The angular velocity of the wing is $\dot{\alpha} \hat{b}_3$ which gives the virtual rotation due to the moment as

$$\delta R_{Ac} = -\delta \alpha \hat{b}_3.$$  

(11)

The total virtual work done by the aerodynamic forces is thus,

$$\delta W = L[\delta h + \delta \alpha x_\beta] + M\delta \alpha.$$  

(12)

From this, the generalized forces become

$$Q_h = -L$$

and

$$Q_\alpha = M + x_\beta L,$$

(13)

where $L$ and $M$ are the aerodynamic lift force and moment, respectively, and $x_\beta$ is the distance between the elastic axis and the aerodynamic center. The variable $x_\beta$ is taken to be positive if the elastic axis lies further towards the trailing edge than the aerodynamic center does. From thin airfoil theory, the aerodynamic center is located at the quarter chord $\left(\frac{c}{4}\right)$ of a symmetric airfoil. Now the equations of motion can be derived by combining all the pieces through the use of Lagrange’s Equations as follows [2]. Specifically,

$$\frac{d}{dt} \left( \frac{\partial (T-U)}{\partial h} \right) - \frac{\partial (T-U)}{\partial h} = Q_h$$

and

$$\frac{d}{dt} \left( \frac{\partial (T-U)}{\partial \alpha} \right) - \frac{\partial (T-U)}{\partial \alpha} = Q_\alpha,$$

(14)
Substituting Equations (1), (7), and (13), into Equations (14) the resulting nonlinear undamped, coupled, two degree of freedom aeroelastic equations of motion can be cast in the form

\[ \begin{align*}
    m\ddot{h} + mx_a \cos(\alpha) \ddot{\alpha} - mx_a \sin(\alpha) \dot{\alpha}^2 + K_h h &= -L \\
    I_{EA} \ddot{\alpha} + mx_a \cos(\alpha) \ddot{h} + K_a \alpha &= M + x_p L.
\end{align*} \tag{15} \]

From this system one can see the coupling is the result of the inertial forces and the pitch angle. The offset distance \( x_a \) between the center of mass and the elastic axis results in a mass imbalance that causes the bending and torsion modes to become coupled. One can see that if this term were to become zero, as in the case where the center of mass was coincident with the elastic axis, and the pitch displacements were taken to be small such that \( \sin(\alpha) \approx 0 \) and \( \cos(\alpha) \approx 1 \), the bending and torsion modes would be completely uncoupled and the inertia and stiffness matrices would be diagonal.

To model the effects of viscous damping, the Rayleigh Dissipation Function is calculated and included in the Lagrange equations. Specifically,

\[ D = \frac{1}{2} C_h \dot{h}^2 + \frac{1}{2} C_\alpha \dot{\alpha}^2. \tag{16} \]

Here, \( C_h \) and \( C_\alpha \) are the linear plunge and pitch damping coefficients, respectively.
This relationship is incorporated into Lagrange’s Equations as

\[
\begin{align*}
\frac{d}{dt} \left( \frac{\partial (T-U)}{\partial h} \right) + \frac{\partial D}{\partial h} - \frac{\partial (T-U)}{\partial h} &= Q_h \\
\frac{d}{dt} \left( \frac{\partial (T-U)}{\partial \alpha} \right) + \frac{\partial D}{\partial \alpha} - \frac{\partial (T-U)}{\partial \alpha} &= Q_\alpha.
\end{align*}
\]  

(17)

Substituting Equations (1), (7), (13) and (16), into Equations (17), the nonlinear damped, coupled, two degree of freedom aeroelastic equations of motion for pitch and plunge become

\[
\begin{align*}
m\ddot{h} + m x_\alpha \cos(\alpha) \ddot{\alpha} - m x_\alpha \sin(\alpha) \dot{\alpha}^2 + C_h \dot{h} + K_h h &= -L = L_{\text{tot}} \\
I_{EA} \ddot{\alpha} + m x_\alpha \cos(\alpha) \dot{h} + C_\alpha \dot{\alpha} + K_\alpha \alpha &= M + x_\beta L = M_{\text{tot}}.
\end{align*}
\]  

(18)

It is important to note here the assumptions inherent in this model as they may be the source of future accuracy and/or cost considerations. Specifically, linear springs have been assumed to model the stiffness in both the pitch and plunge degrees of freedom. This assumption is warranted because most materials exhibit a linear elastic region for small deflections. However, if simulations require larger deflections that exceed this linear region, then the accuracy of the computations will be reduced. Furthermore, viscous damping has been assumed. This assumes that the energy dissipated within the system is proportional to the velocity. This is also assumed to be a linear relationship, which tends to hold fairly accurately for small deflections but will introduce inaccuracies at larger ones.
2.2. UAV-Specific Parameters

Once the general structural model has been derived, the necessary parameters must be defined. Parameters such as the airfoil dimensions, mass properties, and structural stiffness and damping must be tailored to the particular aircraft and structural model. All of these properties can vary greatly among different aircraft designed for different missions; parameters of HALE UAVs are likely to be much different than those of a supersonic fighter jet. As the aircraft of interest is a HALE UAV, it is useful to examine the properties of existing HALE aircraft.

The first prominent American military HALE UAV was the Predator which was followed by the Reaper and the Global Hawk [19]. Characteristics of these three UAVs are contained within Table 2-1. One can see a large variance in the primary missions and operational capabilities. However, these three UAVs all have similar structural designs to enable high-altitude long-endurance operations as shown in Figure 2-2. By examining the similarities a representative airfoil model can be created for a generic HALE UAV.
Table 2-1: Typical UAV parameters in standard units. Metric units given in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Mission</strong></td>
<td>Armed reconnaissance, airborne surveillance, and target acquisition</td>
<td>Remotely piloted hunter/killer weapon system</td>
<td>High-altitude, long-endurance intelligence, surveillance, and reconnaissance</td>
</tr>
<tr>
<td><strong>Thrust</strong></td>
<td>115 hp</td>
<td>900 hp</td>
<td>7600 lbs</td>
</tr>
<tr>
<td><strong>Wingspan [ft] (m)</strong></td>
<td>55 (16.75)</td>
<td>66 (20.12)</td>
<td>116 (35.36)</td>
</tr>
<tr>
<td><strong>Length [ft] (m)</strong></td>
<td>27 (8.23)</td>
<td>36 (10.97)</td>
<td>44 (13.41)</td>
</tr>
<tr>
<td><strong>Height [ft] (m)</strong></td>
<td>6.9 (2.1)</td>
<td>12.5 (3.81)</td>
<td>15.2 (4.63)</td>
</tr>
<tr>
<td><strong>Weight/mass [lbs] (kg)</strong></td>
<td>1130 (512.56)</td>
<td>4900 (2222.6)</td>
<td>11,350 (5148.3)</td>
</tr>
<tr>
<td><strong>Max Takeoff Weight/mass [lbs] (kg)</strong></td>
<td>2250 (1020.6)</td>
<td>10,500 (4762.7)</td>
<td>26,750 (12,133.6)</td>
</tr>
<tr>
<td><strong>Speed [mph] (m/s)</strong></td>
<td>135 (60.35)</td>
<td>230 (102.82)</td>
<td>391 (174.80)</td>
</tr>
<tr>
<td><strong>Ceiling [ft] (m)</strong></td>
<td>25,000 (7620)</td>
<td>50,000 (15,240)</td>
<td>60,000 (18,288)</td>
</tr>
</tbody>
</table>
Figure 2-2: Representative HALE UAV Designs. (a) Predator, (b) Reaper, (c) Global Hawk. Note: Not to scale [23]
From Figure 2-2 one can see all three designs contain high aspect ratio wings, the aspect ratio is defined as the ratio of the square of the span to the planform area. Long skinny wings have a high aspect ratio whereas short fat wings have a low aspect ratio. The importance of this design feature is that it reduces the induced drag [5]. This allows the aircraft longer endurance as it requires less fuel to maintain its velocity as a similar aircraft with a lower aspect ratio. One of the drawbacks of high aspect ratio wings is reduced stiffness. Long, lightweight, skinny wings tend to be more flexible than shorter fatter wings. As Reference [13] explains, large span, high aspect ratio wings pose difficulties with respect to aeroelastic phenomena. These difficulties are rooted in the low structural weight fraction necessary for successful HALE designs. Reference [13] continues on to explain the limited understanding of the behavior of HALE designs in the unsteady aerodynamics inherent in aeroelasticity; most work in this area aims to avoid aeroelastic interactions rather than incorporate design features that take advantage of them.

Another similarity among the HALE aircraft designs is the inclusion of cambered wings. Cambered wings produce lift at zero angle of attack. This feature also results in less induced drag than a symmetric wing producing the same amount of lift [5]. The benefit is the same as before: increased endurance. Some other common wing features are sweep, dihedral, and taper. Wing sweep is often used to increase the aircraft’s critical Mach number so as to avoid the detrimental effects of supersonic flow and the associated increase in drag when traveling at high subsonic speeds. However, with HALE aircraft, speed is typically not a major design
consideration. Accordingly, the wing sweep used in the three HALE UAVs considered is very small if included at all. Dihedral is mainly used in increasing an aircraft’s stability. As dihedral can also increase drag and decrease lift, the HALE designs being considered do not incorporate this feature. Wing taper is often employed to reduce the effects of downwash from tip vortices [5]. Downwash increases induced drag which can decrease endurance. However, in the three designs considered, the taper ratio is very small and as likely the result of weight reduction as the result of induced drag reduction. Taper also contributes to the aspect ratio by reducing the planform area.

Taking into account the various design parameters discussed, a representative model would have a high aspect ratio, a cambered airfoil, and a flexible wing with little to no taper or sweep and no dihedral. Due to the two-dimensional nature of the structural model being studied the flexibility resulting from the high aspect ratio and taper can be evaluated in the stiffness of the springs. Airfoil shape can be chosen so as to incorporate camber. As there is no dihedral and only minimum sweep, the binary nature of the model should capture the full scale dynamics to an acceptable level of accuracy.

The airfoil chosen is the NACA 632 — 415. The NACA 6-series airfoils were developed to maintain laminar flow over a large part of the chord which results in a decreased minimum drag value [24]. The numeric designation of the 6-series contains several important pieces of information about the airfoil. The first number is always 6; this tells the series designation. Chord wise position of maximum
pressure in tenths is represented by the second digit. The third digit denotes the ideal lift coefficient in tenths. Finally, thickness-to-chord ratio is represented by the final two digits. If the designation contains a subscript, it indicates the lift coefficient range in tenths above and below the value of ideal lift coefficient in which favorable pressure gradient and low drag exist [24]. Knowing this information and the shape of the NACA 632 – 415 airfoil depicted in Figure 2-3 one can say it meets the desired camber characteristics with an adequate lift coefficient for a HALE mission. All of the relevant aerodynamic properties necessary for the modeling herein were obtained from the lift and drag curves in Reference [25]. This airfoil has also been used in several published works in various applications and in several production aircraft [26], [27], [28], [29].

![Figure 2-3: NACA 632 – 415 airfoil [25]](image)

Although the bending and torsional stiffness in a three-dimensional wing are dependent on the three-dimensional wing shape and the material of which it is comprised, the model at hand is a rigid two-dimensional airfoil section. Therefore, the bending and torsional stiffness represented by the linear springs in Equations (18) should be chosen to be representative of the three-dimensional wing bending and torsional stiffness. One way to do this is by applying the assumed modes
method [30]. The assumed modes method extends the principle of virtual displacements to produce a generalized parameter model of a continuous system that approximates the flexible behavior of the system [30]. Also referred to as the Ritz method, the advantage is the ability to reduce an infinite degree of freedom problem to one with $n$ manageable degrees of freedom while maintaining reasonable accuracy. The $n$ degrees of freedom result from the $n$ assumed modes. By increasing the number of assumed modes, the reduced order approximation converges to the original infinite degree of freedom system [2]. Following the derivations in either Reference [2], [30], [31], or [32], if the wing is approximated as a beam, the potential bending strain energy can be calculated as follows:

$$ U = \frac{1}{2} \int_{0}^{s} EI \left( \frac{\partial^2 z}{\partial y^2} \right)^2 dy $$

(19)

Here, $z$ is taken to be the transverse deflection based on an assumed bending shape, $y$ is the span-wise coordinate along the wing, $s$ is the wing span, and $EI$ is the flexural rigidity. The bending deformation can be described by the expression

$$ z(y,t) = \psi(y) q(t) = \left( \frac{y}{s} \right)^2 q(t) $$

(20)

containing an assumed deformation shape, $\psi$. Here $q$ is taken to be the coordinate of interest, and $s$ is the span of the finite wing. Note the chosen quadratic assumed shape. Substituting Equation (20) into Equation (19) and solving yields the potential bending strain energy of the finite wing. That is,
As done before in Equation (1), the potential energy of a spring is found using the equation

\[ U = \frac{1}{2} \int_0^s EI \left( \frac{2}{s^2} q \right)^2 \, dy = \frac{2EI}{s^3} q^2. \]  

(21)

Combining Equation (21) and Equation (22), the expression for the equivalent bending stiffness

\[ K_h = \frac{4EI}{s^3}. \]  

(23)

can be expressed in terms of the flexural rigidity.

An analogous method can be applied to the torsional component resulting in the following expression for the equivalent torsional stiffness in terms of the torsional rigidity, \( GJ \). Here a linear displacement shape is assumed yielding

\[ K_\alpha = \frac{GJ}{s}. \]  

(24)

The only other remaining step for obtaining \( K_\alpha \) and \( K_h \) involves choosing torsional and flexural rigidity values representative of a HALE UAV. While this method is not exact, it is a very good engineering approximation to capture the finite wing structural parameters with only a 2DOF airfoil model. Increasing the number of assumed mode shapes will result in a better approximation. To base the selection
of the flexural and torsional rigidities on common engineering practice, the values used in the Goland wing model of Reference [33] are used. However, the Goland wing has an aspect ratio less than 5. In order to match the HALE UAV aspect ratios of approximately 15-20, the span used in equating the finite wing stiffness to the equivalent spring stiffness in Equations (23) and (24) is scaled up. The effect of this scaling is to sufficiently model the stiffness of the high aspect ratio wing as modeled by the chosen 2DOF model. The final flexural and torsional rigidity values are provided in Table 2-2 below.

In the final model the chord length is chosen based on the three HALE UAVs studied earlier. Reference [23] gives the root chord dimensions as being between roughly 3.5 ft and 6 ft. The value of 6 feet is chosen to be representative of the three HALE UAVs and also applicable to the Goland wing structural values. Wing density is taken from the Goland model. The remaining parameters are calculated based on the chord length, span length, density, and flexural and torsional rigidity values. Small viscous damping terms are included based on the assertions made in Reference [3]. The damping is taken to be proportional to the stiffness. The resulting values for the various design parameters are given in Table 2-2.
### Table 2-2: Structural Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>35.72 kg</td>
</tr>
<tr>
<td>Moment of Inertia about Elastic Axis</td>
<td>$I_{EA}$</td>
<td>8.60 kg m$^2$</td>
</tr>
<tr>
<td>Chord length</td>
<td>$c$</td>
<td>1.83 m</td>
</tr>
<tr>
<td>Finite wing span length</td>
<td>$s$</td>
<td>30.48 m</td>
</tr>
<tr>
<td>2DOF airfoil span length</td>
<td>$l$</td>
<td>1 m</td>
</tr>
<tr>
<td>Distance between elastic axis and center of gravity non-dimensionalized by the semi-chord</td>
<td>$x_\alpha$</td>
<td>0.2</td>
</tr>
<tr>
<td>Distance between the elastic axis and the aerodynamic center</td>
<td>$x_\beta$</td>
<td>0.1463 m</td>
</tr>
<tr>
<td>Flexural rigidity</td>
<td>$EI$</td>
<td>$9.75 \times 10^6$ N m$^2$</td>
</tr>
<tr>
<td>Torsional rigidity</td>
<td>$GJ$</td>
<td>$9.83 \times 10^3$ N/m</td>
</tr>
<tr>
<td>Linear plunge stiffness coefficient</td>
<td>$K_h$</td>
<td>$3.23 \times 10^4$ N/m \ rad</td>
</tr>
<tr>
<td>Proportional damping value</td>
<td>$\epsilon$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Linear plunge damping coefficient</td>
<td>$C_h$</td>
<td>$\epsilon K_h \frac{N}{m}$</td>
</tr>
<tr>
<td>Linear pitch stiffness coefficient</td>
<td>$K_\alpha$</td>
<td>$3.23 \times 10^4$ N/m \ rad</td>
</tr>
<tr>
<td>Proportional damping value</td>
<td>$C_\alpha$</td>
<td>$\epsilon K_\alpha \frac{Nm}{rad}$</td>
</tr>
</tbody>
</table>
In the simulations, the aircraft is assumed to be traveling at 30,000 ft altitude which is representative of HALE UAV operations as shown in Table 2-1. The airfoil model is given a prescribed initial deflection in both the pitch and plunge degrees of freedom of $h(0) = h_0 = -0.1 \, m$, $\alpha(0) = \alpha_0 = 6^\circ$. The initial pitch and plunge rates are taken to be zero. These initial conditions represent the aircraft flying at a small angle of attack and experiencing small static wing bending under the applied lift force or perhaps the initial deflections produced by a disturbance such as a wind gust. A similar scenario would be seen in a real world flight of a similar aircraft with flexible wings. Additional model details specific to the various analysis methods will be included in their respective ensuing chapters.
In order to accurately model the forces experienced by an airfoil, one must understand the basics of aerodynamic theory. The continuity equation and Bernoulli’s equation provide a basis for understanding the generation of aerodynamic lift. As airflow encounters an airfoil it splits into two different flows: one above the airfoil and one below it. With a cambered airfoil or a symmetric airfoil at an angle of attack, the flow over the top of the airfoil is constricted more than the flow underneath the airfoil. This causes the flow over the top to increase in velocity to satisfy the continuity equation. Any increase in the fluid velocity must also be met with a decrease in the fluid pressure to satisfy Bernoulli’s equation. Lift results from the difference in the low pressure above the airfoil and the higher pressure below the airfoil [5].
From aerodynamic theory one knows that the lift force, $L$, exerted on an airfoil by the airflow around it is given by the equation

$$L = \frac{1}{2} \rho V^2 C_{la} \alpha S,$$

where $\rho$ is the air density, $V$ is the free stream air velocity, $C_{la}$ is the lift curve slope, $\alpha$ is the angle of attack, and $S$ is a characteristic area. The resulting force acts at the center of pressure. The expression of the aerodynamic moment is similar but includes a necessary moment arm

$$M = \frac{1}{2} \rho V^2 C_M c,$$

where $C_M$ is the moment coefficient and $c$ is the chord length. The resulting moment is centered at the aerodynamic center of the airfoil and remains relatively constant for varying angles of attack [34]. According to thin airfoil theory, the aerodynamic center is located at the quarter-chord for symmetric airfoils [35]. As Figure 2-3 shows, the chosen airfoil is not symmetric. However, even with cambered airfoils, the aerodynamic center is close to the quarter-chord for subsonic speeds [34]. The expressions in Equations (25) and (27) are valid for steady airflow where the density and velocity are constant and the airfoil is steady.

Another important aspect of aerodynamic modeling is compressibility. The expressions in Equations (25) and (27) assume incompressible flow. This assumption is warranted for free stream velocities under approximately 100 m/s. When the free stream velocity exceeds this speed the large pressure changes
experienced by the airflow can result in large density changes. Density fluctuations affect the magnitudes of the lift and drag produced by the airflow over the airfoil [5]. To account for compressibility effects the Prandtl-Glauert correction is used. This correction is based on the linearized velocity potential equation and is applicable for Mach numbers approximately between 0.3 and 0.7. Mach numbers higher than 0.7 produce transonic effects which must be addressed through other means. Compressibility is neglected for flows below Mach 0.3.

The interdependency of the airfoil motion and aerodynamic forces poses a challenge to aeroelasticians. Most classical aerodynamic theory is based on steady flow or steady motion, but aeroelastic problems often involve time-dependent fluid motion [6]. Airfoils undergoing aeroelastic interactions oscillate in the flow which alters the circulation about the airfoil. The study of this time-dependent fluid motion is called unsteady aerodynamics. Aerodynamic forces acting on a body oscillating in an unsteady flow are generated by vorticity (or circulation) and apparent mass (or inertial) contributions [3]. These two categories are referred to as circulatory and non-circulatory, respectively. Due to the Helmholtz theorem, the total circulation must be zero. When the circulation about the airfoil changes vortices are shed into the wake to balance the overall circulation [2]. Shed vortices affect the flow field by imparting unsteady flow back to the airfoil. This influence decreases as the vortex travels away from the airfoil. Based on this complex interaction, completely modeling unsteady aerodynamics is very involved and not well-understood. Theodorsen [36] and Wagner [37] have both put forth unsteady aerodynamic models, and complex computational fluid dynamics (CFD) software can model these
effects, but all of them must be formulated in such a way as to interface with the necessary structural model. The computational resources required for these models are also very costly.

As stated in Reference [38], a complete understanding of unsteady airfoil behavior has not been attained. However, by studying experimental data some qualitative models have been produced which encompass the essential physics involved [38]. One of the biggest challenges of unsteady aerodynamic modeling is dynamic stall. An oscillating airfoil may experience large angle of attack variations which result in flow separation, stall, and reattachment. This sequence generates large variations in the lift as compared to those predicted by simpler quasi-steady models. However, the quasi-steady models can still produce insightful results [38].

The quasi-steady assumption states that the aerodynamic characteristics of a moving airfoil can be equated to the aerodynamic characteristics of the same airfoil when sampled instantaneously at discrete points in time [3]. In other words, the aerodynamic characteristics of a moving airfoil are taken to be a function of only the instantaneous properties of the airfoil. Rather than the current aerodynamic forces being influenced by shed vortices and other unsteady effects, the instantaneous lift and moment are calculated using only the instantaneous angle of attack, free stream velocity, and pitch and plunge rates. It follows that when calculating the forces using such a model, increasingly smaller time steps will better approximate the forcing of the complete system. Even so, it is important to note that the unsteady effects may
have important influence on the results. Equations for calculating the quasi-steady lift and moment coefficients are as [3]:

\[
C_{L_{qs}} = C_{L_{st}} \left[ \alpha + \frac{\dot{h}}{V} + b \left( \frac{1}{2} - a \right) \frac{\dot{\alpha}}{V} \right] \\
C_{M_{qs}} = -\frac{c \pi}{8V} \dot{\alpha},
\]

where \( b \) is the semi-chord and \( a \) is a nondimensional parameter denoting the location of the elastic axis as shown in Figure 2-1. These equations take into account motion of the airfoil through the pitch and plunge rates. The advantage of the quasi-steady assumption over full unsteady aerodynamics is that computational requirements are reduced while still achieving reasonably accurate qualitative behavior. Further, depending on how the model is set up, the effects of structural nonlinearities can be distinguished from those of the unsteady aerodynamics. As mentioned above, both circulatory and non-circulatory forces are encountered in aeroelasticity. However, as Reference [3] points out, the non-circulatory terms are neglected in the quasi-steady assumption as their contributions to the simple bending-torsion airfoil flutter of a cantilever wing are negligible. For the purposes of gaining initial insight into the aeroelastic problem the quasi-steady assumption is used in the aerodynamic modeling of this paper.

Using the quasi-steady assumption along with the linear lift curve slope are likely sources of error. For the airfoil used, the linear region of the lift curve slope is valid for angles of attack between approximately ±12° as shown in Figure 3-1. As
dynamic stall behavior has not been included in this model, angles of attack larger than $\pm 12^\circ$ are likely to give results that are incorrect.

Figure 3-1: Lift Curve for the NACA 632 - 415 Airfoil [25]
Chapter 4

Numerical Integration

Once the nonlinear equations of motion have been determined and an applicable aerodynamic model has been chosen, the next step is to adopt an appropriate method of analysis. The complex nature of nonlinear equations has resulted in many different analytical techniques including numerical integration, equivalent linearization, perturbation methods, and harmonic balance [39]. One of the methods chosen for comparison herein is direct numerical integration. Several algorithms exist which provide this capability, although as with other methods, there are advantages and disadvantages. The advantages include limited initial effort and some algorithms display unconditional stability. Most algorithms require only the equations of motion, initial conditions, time step, and desired tolerance. One of the biggest drawbacks is cost; most numerical integration algorithms require very small step sizes for precise convergence which results in lengthy computation
time. If the goal of solving the equations is to determine the steady state response, this may result in quite lengthy time spans that, when combined with very small step sizes, can be computationally prohibitive. However, with the use of today's computing power most simple equations can be numerically integrated to reasonable accuracy within an acceptable amount of time.

4.1. Application

The numerical integration scheme used herein is contained within the Matlab command ODE45. This algorithm is based on the explicit Runge-Kutta (4, 5) formula defined by the Dormand-Prince pair [40]. This pair is a 4\textsuperscript{th} and 5\textsuperscript{th} order approximation which has the benefits of extended regions of absolute stability and small principle truncation terms. ODE45 is a one-step solver in that it requires only the solution of the previous time step to compute the solution at the subsequent time step. This numerical integration scheme is chosen for its accurate results and simple implementation. Implementation of this scheme requires that the 2\textsuperscript{nd} order nonlinear equations given by Equations (18) be recast as a 1\textsuperscript{st} order system

\[
\begin{align*}
\frac{dy_1}{dt} &= y_3, \\
\frac{dy_2}{dt} &= y_4, \\
\frac{dy_3}{dt} &= -x_{\alpha} \cos(y_2) \frac{dy_4}{dt} + x_{\alpha} y_2^2 \sin(y_2) - \frac{c_h}{m} y_3 - \frac{K_h}{m} y_1 - \frac{L}{m}, \\
\end{align*}
\]

and

\[
\frac{dy_4}{dt} = -\frac{mx_{\alpha} \cos(y_2)}{l_{EA}} \frac{dy_3}{dt} - \frac{c_h}{l_{EA}} y_4 - \frac{K_h}{l_{EA}} y_2 + \frac{M}{l_{EA}} + \frac{x_{\beta L}}{l_{EA}}
\]

where \(y_1 = h, y_2 = \alpha, y_3 = \dot{h}, \) and \(y_4 = \dot{\alpha}.\)
The ODE45 command also requires initial conditions as well as the desired error tolerances and time span of integration. The error tolerances specify the desired accuracy of each iteration in order to reduce any error propagation within the calculations. The time step need not be specified as the algorithm chooses its own step size at each iteration to achieve the desired accuracy [41]. However, to increase accuracy—at the cost of computation time—one can specify an initial time step as well as a maximum time step. The values used in the computations herein are given in Table 4-1 below along with the initial conditions presented earlier in section 2.2.

<table>
<thead>
<tr>
<th>Table 4-1: Numerical Integration Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute Tolerance</td>
</tr>
<tr>
<td>Relative Tolerance</td>
</tr>
<tr>
<td>Initial Step Size</td>
</tr>
<tr>
<td>Maximum Step Size</td>
</tr>
</tbody>
</table>

As a basis for the accuracy of the numerical integration method used—and due to the lack of experimental data for the nonlinear system model being analyzed—a generic linear problem is solved both numerically and analytically. The example system had the following form:

$$M \ddot{x} + C \dot{x} + K x = F,$$

(30)

where $M$, $C$, and $K$ are 2x2 mass, damping, and stiffness matrices, respectively; $x$ is a
2x1 state vector; and \( F \) is a 2x1 forcing vector. The values and theory used in this problem are from Reference [42]. The pertinent numerical values are

\[
M = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.3 & -0.2 \\ -0.6 & 0.3 \end{bmatrix}, \quad K = \begin{bmatrix} 10 & -6 \\ -6 & 22 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad F = \begin{bmatrix} \cos(3t) \\ -\cos(3t) \end{bmatrix}
\]

As shown in Figure 4-1, the system was numerically integrated and compared to the exact analytical steady-state solution from classical vibration theory. The error between the two methods using the numerical algorithm described above with the parameters in Table 4-1 is negligible. Based on this data one can reasonably say the numerical integration procedure gives accurate results.

Once the numerical integration scheme is validated, it is coupled with the quasi-steady aerodynamic model. As shown in Figure 4-2, the process is rather simple. The initial set up for this algorithm is quite straight forward and consists of casting the equations of motion in the form given in Equation (29) and defining all the relevant parameters for the simulation (mass, chord, stiffness, time step, etc.).
Figure 4-1: Comparison of Numerical Results with Exact Steady-State Solution

Figure 4-2: Coupled Aerodynamic Model and Numerical Integration Algorithm
Chapter 5

Harmonic Balance

Another nonlinear analytical method applied to the problem at hand is harmonic balance. Herein, harmonic balance is chosen as one way to predict the steady-state behavior of the system subjected to harmonic forcing. This method, though limited to cases of harmonic excitation, is deemed applicable to aeroelastic analysis as the lift and moment forces present during aeroelastic interactions can often excite steady state oscillatory responses (LCO) in both pitch and plunge. LCO are more common in transonic flight conditions than subsonic conditions. For this reason, some researchers have attributed LCO mostly to the nonlinear aerodynamic forcing [43]. However, the understanding of nonlinear systems and LCO—especially in the context of aeroelasticity—is incomplete and thus no definitive correlations can be made. Reference [43] examines the effects of nonlinear transonic
aerodynamics on the production of LCO using a harmonic balance approach building on Reference [44].

The method of harmonic balancing is employed through the use of a Fourier series expansion. The forcing terms are modeled as an $n^{th}$-order Fourier series expansion of which all the necessary coefficients can be computed from the known forcing function. The steady-state response of the system is also assumed to be an $n^{th}$-order Fourier series expansion. The excitation and the assumed response are then substituted into the equations of motion and harmonic terms of the same frequency are grouped together. This results in a system of $2n$ linear equations for each degree of freedom which are solved for the $2n$ unknown coefficients of the assumed Fourier series expansion of the steady-state response [45]. Often it is found that only a single fundamental frequency is required for accurate results, although additional higher-order components can be added for better accuracy at the cost of increased computation time [39]. The author of Reference [45] has found this method applicable even for problems in which the nonlinear terms are large.

### 5.1. Application

To apply the method to the problem at hand, the lift and moment terms are assumed to be $n^{th}$-order purely harmonic terms of the form:

\[
L_{tot} = L_1 \sin(\tau) + L_2 \cos(\tau) + \cdots + L_{2n-1} \sin(n\tau) + L_{2n} \cos(n\tau),
\]
\[
M_{tot} = M_1 \sin(\tau) + M_2 \cos(\tau) + \cdots + M_{2n-1} \sin(n\tau) + M_{2n} \cos(n\tau)
\]
where $\tau = \omega t$. Note here that the Fourier series expansions are taken to have zero-mean. This has been done to reflect the zero-mean behavior of the response of the actual system. The plunge and pitch responses are also assumed to be zero-mean $n^{th}$-order harmonic terms of the form

\[
h = h_1 \sin(\tau) + h_2 \cos(\tau) + \cdots + h_{2n-1} \sin(n\tau) + h_{2n} \cos(n\tau),
\]
and
\[
\alpha = \alpha_1 \sin(\tau) + \alpha_2 \cos(\tau) + \cdots + \alpha_{2n-1} \sin(n\tau) + \alpha_{2n} \cos(n\tau)
\]

If $n$ is taken to equal 1, Equations (32) and (33) reduce to

\[
L_{tot} = L_1 \sin(\tau) + L_2 \cos(\tau),
\]
\[
M_{tot} = M_1 \sin(\tau) + M_2 \cos(\tau),
\]
\[
h = h_1 \sin(\tau) + h_2 \cos(\tau),
\]
and
\[
\alpha = \alpha_1 \sin(\tau) + \alpha_2 \cos(\tau).
\]

Plugging Equations (34) into Equations (18) yields

\[
-m_\omega \dot{h}_1 \sin(\tau) - m_\omega \dot{h}_2 \cos(\tau) - mx_\alpha \omega^2 \alpha_1 \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin(\tau)
\]
\[
- mx_\alpha \omega^2 \alpha_2 \sin(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos^2(\tau)
\]
\[
- mx_\alpha \omega^2 \alpha_2 \sin(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin^2(\tau)
\]
\[
+ C_h \omega \dot{h}_1 \cos(\tau) - C_h \omega \dot{h}_2 \sin(\tau) + K_h h_1 \sin(\tau) + K_h h_2 \cos(\tau)
\]
\[
= -L_1 \sin(\tau) - L_2 \cos(\tau)
\]
and

\[-l_{EA} w^2 \alpha_1 \sin(\tau) - l_{EA} w^2 \alpha_2 \cos(\tau)\]

\[-m x_d w^2 h_1 \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin(\tau)\]

\[-m x_d w^2 h_2 \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos(\tau) + C_d w \alpha_1 \cos(\tau)\]

\[-C_d w \alpha_2 \sin(\tau) + K_d \alpha_1 \sin(\tau) + K_d \alpha_2 \cos(\tau)\]

\[= M_1 \sin(\tau) + M_2 \cos(\tau).\]

The complex nonlinear nature of these equations makes generating and solving the system of $2n$ linear equations for the unknown coefficients computationally prohibitive. To solve this problem, the original equations of motion—Equations (18)—are linearized and analyzed with the harmonic balance method to derive an initial estimate of the coefficients of the nonlinear problem. To solve the nonlinear equations—Equations (35) and (36)—orthogonality is enforced and the equations are averaged over an interval as in Reference [46] resulting in $2n$ equations for each degree of freedom. Specifically,
\[-\frac{1}{2}m\omega^2 h_1 - \frac{1}{2}C_h\omega h_2 + \frac{1}{2}K_h h_1
+ \omega^2 m x_\alpha \left[-\alpha_1 \int_0^{2\pi/\omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin^2(\tau) \, dt \right.
\left. - \alpha_2 \int_0^{2\pi/\omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos^2(\tau) \sin(\tau) \, dt \right.
\left. - \alpha_2^2 \int_0^{2\pi/\omega} \sin(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin^3(\tau) \, dt \right.
\left. + \alpha_1 \alpha_2 \int_0^{2\pi/\omega} \sin(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos(\tau) \sin^2(\tau) \, dt \right] = -\frac{1}{2} L_1 \]

\[-\frac{1}{2}m\omega^2 h_2 + \frac{1}{2}C_h\omega h_1 + \frac{1}{2}K_h h_2
+ \omega^2 m x_\alpha \left[-\alpha_1 \int_0^{2\pi/\omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin(\tau) \cos(\tau) \, dt \right.
\left. - \alpha_2 \int_0^{2\pi/\omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos^2(\tau) \sin(\tau) \, dt \right.
\left. + \alpha_1 \alpha_2 \int_0^{2\pi/\omega} \sin(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos(\tau) \sin^2(\tau) \, dt \right] = -\frac{1}{2} L_2 \]

\[-\frac{1}{2} l_{EA} \omega^2 \alpha_1 - \omega^2 h_1 m x_\alpha \int_0^{2\pi/\omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \sin^2(\tau) \, dt
\left. - \omega^2 h_2 m x_\alpha \int_0^{2\pi/\omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos(\tau) \sin(\tau) \, dt \right]\]
\left[ -\frac{1}{2} C_h \omega \alpha_1 + \frac{1}{2} K_h \alpha_1 = \frac{1}{2} M_1 \right]
The resulting expressions—Equations (37)-(40)—contain nonlinear integral terms. Using $\alpha_1$ and $\alpha_2$ from the linear approximation as an initial estimate, the nonlinear integrals are determined numerically and the result is then substituted back into Equations (37)-(40) resulting in a system of $2n$ linear equations of the unknown coefficients for each degree of freedom. The system is solved for new values of the coefficients which are used to re-evaluate the nonlinear integrals and generate new values of the unknown coefficients. This iterative process is continued and coefficient values at each iteration are compared to the values at the previous iteration. Once a specified tolerance between successive iterations is reached, the final coefficients are then taken to be the correct ones as assumed in Equations (34). A flow chart describing the computational process is included blow in Figure 5-1.

\[
-\frac{1}{2} l_{EA} \omega^2 \alpha_2 - \omega^2 h_1 m x_a \int_0^{2\pi / \omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos(\tau) \sin(\tau) \, d\tau \\
- \omega^2 h_2 m x_a \int_0^{2\pi / \omega} \cos(\alpha_1 \sin(\tau) + \alpha_2 \cos(\tau)) \cos^2(\tau) \, d\tau + \frac{1}{2} C_n \omega \alpha_1 \\
+ \frac{1}{2} K_a \alpha_2 = \frac{1}{2} M_2. \tag{40}
\]

In mechanizing this method, the initial set up is rather involved. The equations—Equations (37)-(40)—must be derived in addition to the linear harmonic balance equations. The complexity of these equations is dependent on the order of the Fourier series used. Once the method is implemented, however, the computations only require several seconds to run depending on the tolerance used. Using a tolerance of $10^{-4}$ requires only 2-3 iterations and less than five seconds.
Figure 5-1 details the process of the harmonic balance solver. Once the coefficients are solved for using this algorithm the assumed responses given in Equations (34) can be plotted and compared against the time integration results.

![Figure 5-1: Harmonic Balance Solver Algorithm](image)
Once the structural equations of motion, aerodynamic modeling, and time integration pieces have been completed, another relevant behavior to investigate is that due to stochastic excitation. The use of stochastic excitation within the construct of aeroelasticity is of use based on the source of the excitations. Aerodynamic lift and moment forces are generated through the interaction of airflow over an airfoil. Often, the airflow is laminar and smooth resulting in a constant force. However, anyone who has ridden in an airplane knows that aircraft often experience turbulence. Turbulence can cause a number of changes within the production of aerodynamic forces. Varying air velocities can cause the resulting force to vary as the relationship between the force and the air velocity is of the order of velocity squared (see Equation (25)). Further, turbulent airflow over an airfoil can result in separation which reduces the magnitude of lift and moment.
while producing additional drag. These issues are apparent for any airfoil traveling through the atmosphere, but become especially important to consider for airfoils experiencing aeroelastic interactions which can result in motion of the airfoil as well as phenomena such as vortex shedding and stall. The model herein does not explicitly account for separation effects, but does account for changes in lift and moment based on a stochastic velocity input.

6.1. Turbulence Modeling

Wind turbulence has been studied for many decades. Interactions between wind and structures are important to understand as history has shown through catastrophes like the Tacoma-Narrows bridge collapse. Over the years researchers have developed models of wind turbulence based on theory and empirical data. One of the most widely used models is the von Karman spectrum attributed to its developer, Theodore von Karman. It was later adapted for use in wind engineering [47]. The von Karman spectrum represents the gust velocity power spectral density and has different forms for the vertical/lateral components and the longitudinal component. Herein the longitudinal component of the gust is considered. The pertinent expression is

\[ S_u(\omega) = \frac{2\sigma_u^2 L_u}{\pi V} \frac{1}{\left[ 1 + \left( 1.339 L_u \frac{\omega}{V} \right)^2 \right]^{5/6}} \]  

(41)
where $\sigma_u$ represents the standard deviation of the gust velocity, $L_u$ is the scale height, $V$ is the mean wind velocity, and $\omega$ is the circular frequency in rad/s [48].

From the spectrum in Equation (41), time histories can be generated that encompass the necessary statistical characteristics. To generate time histories, the von Karman Continuous Wind Turbulence Model within the Simulink Aerospace Blockset was used. This block passes white noise through a filter designed to produce velocity spectra consistent with the von Karman spectrum [49]. The white noise used is band-limited with a unit variance. Several input parameters are required, including low altitude intensity defined by the wind speed at 20 ft, wind direction at 20 ft, turbulence intensity, scale height, time step, wingspan, altitude, and seed numbers for the white noise generation [49]. As described in Military Specification MIL-F-8785C [50] and Military Handbook MIL-HDBK-1797 [51], the turbulence scale height and intensity has been divided into two distinct regions, each a function of the altitude. For low altitudes—below 1000 ft—the intensity is a function of the wind velocity at 20 ft ($W_{20}$). Typically, for light, moderate, and severe turbulence $W_{20}$ is taken to be 15, 30, and 45 knots, respectively. The equation for the intensity, $\sigma_w$, then is as [49]

$$\frac{\sigma_u}{\sigma_w} = \frac{1}{(0.177 + 0.000823h)^{0.4}},$$

$$\sigma_w = 0.1W_{20},$$
where \( h \) is the altitude in feet. The scale height for low altitude turbulence is similar. Specifically,

\[
L_u = \frac{h}{(0.177 + 0.000823h)^{1.2}}.
\] (43)

Altitudes above 2000 ft have a constant scale length \( L_u = 2500 \text{ ft} \). Turbulence intensity for this region is generated from a lookup table based on the altitude and probability of the turbulence intensity being exceeded. For altitudes between the 1000 and 2000 ft, the values are linearly interpolated [49]. The Simulink block diagram of the turbulence model is included below in Figure 6-1.

6.2. Application

Using the numerical integration framework from Chapter 4, a similar model was developed. The process is exactly the same as in Figure 4-2, except the velocity

![Figure 6-1: Simulink Block Diagram of Turbulence Generation](Image)
used in the calculation of the aerodynamic forces is sampled from a stochastic time history generated by the Simulink turbulence model previously described.
By coupling the quasi-steady aerodynamic model and structural model derived above, numerical analysis was performed within Matlab. The computations were run on an Intel Core2 2.40GHz processor with 3GB RAM. The time step chosen for updating the aerodynamic forces was chosen to be $\Delta t = \frac{c}{5000}$. This was done based on the step size chosen in Reference [27]. In that paper the authors chose a step size of $\Delta t = \frac{c}{100V}$. No qualifications were given for this expression, but based on the results presented in their paper was found to be adequate. The removal of the velocity dependency was done to remove any potential variations in the data due to different sampling times at different velocities. The structural equations of motion were integrated in time while the aerodynamic forces were updated at each iteration using this time step. Using this time step the simulations for the constant velocity case took roughly 30 minutes to compute 60 seconds of data. With the
added complexity of generating the stochastic velocity time history the stochastic simulations averaged roughly four hours and 20 minutes to compute 60 seconds of data. The large difference between the two methods results from having to compute the stochastic velocity time history, save it to a file, and then read in values at every iteration. Accessing data in other files is very time consuming.

7.1. Quasi-steady Aerodynamics with Constant Velocity

A range of flow velocities was used to determine the velocity at which the oscillations might become detrimental to the craft. Figure 7-1 and Figure 7-2, below, show the pitch and plunge displacement as a function of time. Note the different time scales on the two figures. As the velocity is increased, the aerodynamic effects cause the damping to decrease resulting in a longer settling time. In Figure 7-1 the displacements have both settled after only a little more than 4 seconds whereas in Figure 7-2 it takes nearly a minute.

As the velocity increases past 87 m/s the behavior becomes very lightly damped until at 89 m/s the behavior depicted in Figure 7-3 is seen. It was found that air velocities below approximately 89.2 m/s either damped out or were sustained at constant amplitude.
Figure 7-1: Pitch and Plunge vs. Time, $V = 50 \text{ m/s}$

Figure 7-2: Pitch and Plunge vs. Time, $V = 87 \text{ m/s}$
For velocities larger than 89.2 m/s, the oscillations increased to a bounded steady state limit cycle oscillation. Again the settling time was dependent on the velocity however in the opposite sense it had before; higher velocities reached steady-state oscillations faster than those closer to the critical velocity. This behavior can be seen in Figure 7-4 and Figure 7-5. Both figures have the same time scale. Figure 7-4 does not approach steady-state amplitude until approximately 60 seconds whereas Figure 7-5 takes just over 10 seconds.
Figure 7-4: Pitch and Plunge vs. Time, $V = 90 \text{ m/s}$

Figure 7-5: Pitch and Plunge vs. Time, $V = 95 \text{ m/s}$
Note that the amplitude of the angle of attack oscillations is beyond the linear region for which the analysis scheme used applies. However, this behavior cannot be corrected for without an appropriate dynamic stall model.

From Table 2-1, the cruise velocities for the HALE UAVs ranged from approximately 60-175 m/s. While the representative model analyzed herein was not meant to match any of the HALE UAVs in Table 2-1, the model did incorporate various aspects common among them. The critical velocity of approx. 89.2 m/s is encompassed by the range given in Table 2-1. From this one might argue that the analysis holds some realism even given the numerous simplifying assumptions.

As the velocity increases past the onset of LCO, the amplitude of the LCO also increases. Figure 7-6 shows how the amplitude changes with the velocity. A trend between the lift force amplitude and plunge amplitude is evident, and the same can be said for the moment and the angle of attack. The data also appear to have an asymptotic relationship with the critical velocity which is noted by the red dashed line. Further, the amplitudes appear to change linearly as the velocity increases past approximately 95 m/s.
Figure 7-6: Aerodynamic Force and Displacement Amplitude as a Function of the Velocity

Figure 7-7 presents the frequency of oscillation as a function of the velocity. A red dashed line represents the critical velocity found earlier. Although the frequency variation in the figure is quite small (less than one rad/s variation) there is a distinct corner located at the critical velocity. Further, note how similar the frequencies of the aerodynamic forces and structural displacements are. This is not too surprising as the aerodynamic forces are directly dependent on the angle of attack, and the angle of attack is also dependent on the aerodynamic forces. It is pointed out that the data used in Figure 7-7 are not calculated to reach steady-state; only 60 seconds of data was simulated. For the post-critical velocity simulations, the steady-state behavior was reached in the form of LCO. However, with the
computational resources used and the necessary length of data required to reach steady state for velocities just prior to the critical velocity, the requisite computation time was prohibitive for this study. Additionally, no effort was made to optimize the time step so as to achieve an acceptable accuracy with minimal computation time.

Figure 7-7: Frequency of Oscillation as a Function of Velocity

Figure 7-1 through Figure 7-7 show the aeroelastic behavior of the system, but the cause of such behavior is also relevant. Using the same model, behavior at velocities much smaller than the critical velocity was simulated. Figure 7-8 shows the behavior at 10 m/s. The frequencies of the pitch and plunge oscillations are distinctly different. An interesting phenomenon is encountered as the velocity is increased.
Figure 7-8: Pitch and Plunge vs. Time, $V = 10$ m/s

Figure 7-9: Pitch and Plunge vs. Time, $V = 50$ m/s
As Figure 7-9 shows, at 50 m/s the pitch amplitude begins at approximately the same amplitude, yet is not damped out as quickly. This is consistent with the previously mentioned displacement figures; the damping decreases with increasing velocity. However, note the drastic change in the plunge displacement. The initial amplitude peak has reduced in amplitude and the damping has actually increased. Further, a secondary frequency has become apparent. Further increases in the velocity make this secondary frequency more prominent. Figure 7-10 shows the displacements at 80 m/s. The secondary frequency in the plunge displacement appears to have become dominant. Similarity to the pitch frequency is also noted. An additional increase in the velocity to 90 m/s shows a continuance of this behavior.

![Pitch and Plunge vs. Time, V = 80 m/s](image)

**Figure 7-10: Pitch and Plunge vs. Time, V = 80 m/s**
However, as Figure 7-11 shows, 90 m/s is past the critical velocity and LCO behavior has started to occur. One explanation for the emergence of LCO is the coalescence of the pitch and plunge frequencies. When the velocity is very low, the pitch and plunge modes are fairly independent of each other. As the velocity increases the plunge frequency becomes more and more dependent on the pitch frequency. This is expected as the lift and moment forces are a function of the pitch angle. The behavior shown here is indicative of the flutter mechanism described by Figure 1-1.

![Graph of Pitch Displacement vs. Time and Plunge Displacement vs. Time at 90 m/s](image)

**Figure 7-11: Pitch and Plunge vs. Time, V = 90 m/s**

Furthermore, classical flutter analysis has found that if dynamic stall were taken into account, one might expect the high frequency of the pitch velocity to result in
separated flow (stall) and thus decreased aerodynamic forces. The resulting behavior would likely appear differently than shown here.

7.2. Harmonic Balance

From the harmonic balance formulation described previously, a computer model was developed within the Matlab environment. Forcing data from the quasi-steady numerical integration model was fit to a single harmonic of the form:

\[ F = A \sin(\omega t) + B \cos(\omega t) \]  \hspace{1cm} (44)

Using the \textit{fit} command in Matlab, the parameters \(A, B\), and \(\omega\) were calculated for both the aerodynamic lift and moment based on the forcing data obtained from the numerical integration. The \textit{fit} command in Matlab uses a nonlinear least squares method to fit the data to a first order Fourier series expansion. The coefficient of determination is of the order 0.99999. Figure 7-12 below shows the superposition of the forcing data and the resulting first order Fourier series. This case was done for a velocity of 90 m/s. The time scale was modified to accurately show the agreement. Parameters, \(A, B\), and \(\omega\) for both the lift and moment were then input into the nonlinear harmonic balance solver. The goal of this analysis is to attempt to predict the steady-state oscillation amplitude and frequency if the amplitude and frequency of the forcing is known.
Using the harmonic balance solver developed in section 5.1, the harmonic forcing data for each velocity was input in terms of the amplitudes and frequency of oscillation. From this, the solver computed the coefficients of the assumed response used from Equation (34). For a velocity of 90 m/s, the results are shown in Figure 7-13.

**Figure 7-12: Aerodynamic Forces Fit to 1st Order Fourier Series**
Figure 7-13: Harmonic Balance Model, $V = 90 \text{ m/s}$

The behavior is not as expected. For the pitch displacement the harmonic balance approximation is very good initially, but then the amplitude diverges to approach the LCO amplitude. For the plunge displacement the behavior is almost the opposite; initially the approximation is poor but approaches the LCO amplitude much better than the pitch degree of freedom. There appears to be some mechanisms at work that prevent the harmonic balance from properly predicting the steady-state oscillatory behavior. Presently these mechanisms are unknown and require additional investigation to determine the cause of the disagreement. Some areas for further examination include the implementation of the equations of motion within the numerical integration model, the coding of the harmonic balance.
model, or possibly the addition of the aerodynamic forcing on the structure. With respect to frequency agreement, Figure 7-14 shows a rather close match. Frequency agreement was seen at all velocities for both degrees of freedom. This is not surprising, however, as the displacement frequency is directly related to the forcing frequency.

![Figure 7-14: Harmonic Balance Model, V = 90 m/s (Zoomed in)](image)

For larger velocities the initial agreement in the pitch displacement is not present as seen in Figure 7-15 and Figure 7-16. However, in the plunge degree of freedom the harmonic balance approximation approaches the time integration results as velocity is increased. A similar trend is seen in the pitch degree of freedom but is not as apparent.
Figure 7-15: Harmonic Balance Model, $V = 95 \text{ m/s}$

Figure 7-16: Harmonic Balance Model, $V = 100 \text{ m/s}$
For a small range of post-critical velocities Figure 7-17 shows how the error changes with respect to velocity. The error is calculated by the equation

\[
\% \text{Error} = \frac{|HB - TI|}{TI} \times 100, \tag{45}
\]

where \( HB \) is the magnitude of the harmonic balance approximation and \( TI \) is the magnitude of the time integration data. A linear relationship is seen that has the same rate of change for both the pitch and plunge degrees of freedom. As the figure shows, however, the rate of change is very gradual. The pitch degree of freedom would require very high velocities that are unreasonable for such an aircraft as modeled herein to reach agreement using this trend.

![Harmonic Balance Approximation Error as a Function of Velocity](image)

Figure 7-17: Harmonic Balance Approximation Error as a Function of Velocity
7.3. Quasi-steady Aerodynamics with Stochastic Velocity

Using a stochastic velocity input based on the von Karman spectrum, the results are similar, yet display the sensitivity of the system to variations in the velocity. Within the von Karman turbulence model the turbulence intensity was taken to be severe. Figure 7-18, Figure 7-19, and Figure 7-20 show the pitch and plunge generated by stochastic velocity inputs. The stochastic velocity time histories are shown in the center plots in each figure. A red dashed line in the center plot also represents the critical velocity found previously. In Figure 7-18 the mean velocity is below the critical velocity and thus based on the results presented above one would expect the displacements to damp out to zero eventually. But, when the variability of the wind turbulence is taken into account, the velocity contains periods both above and below the critical velocity. When the critical velocity is exceeded, the displacements show behavior similar to LCO. When the velocity later decreases below the critical velocity the displacements converge to zero as one would expect. The same behavior is seen in Figure 7-19, where the mean velocity is right at the critical velocity. In Figure 7-20 where the mean velocity is above the critical velocity, the displacements vary as in the previous two figures. However, as the velocity is almost exclusively above the critical velocity, the displacements merely vary between LCO of different amplitude.
Figure 7-18: Pitch and Plunge vs. Time with Stochastic Velocity Input, Mean Velocity = 85 m/s

Figure 7-19: Pitch and Plunge vs. Time with Stochastic Velocity Input, Mean Velocity = 89.2 m/s
Figure 7-20: Pitch and Plunge vs. Time with Stochastic Velocity Input, Mean Velocity = 95 m/s
Chapter 8

Concluding Remarks

The relative inexpensive nature of UAVs lends itself towards inexpensive methods of analysis—both monetarily and computationally—as well as inexpensive test vehicles. If such cost savings could be realized, the utilization of UAVs would grow in leaps and bounds. Improved analysis techniques would allow designers and engineers to spend more time and effort on other challenges which would open the door to new and exciting technologies.

To that end, the goal of this thesis has been to determine the applicability of modeling post-critical flutter behavior via the harmonic balance method as well as to examine the effects of stochastic forcing. As aircraft design and analysis is an expensive endeavor, any means by which costs can be reduced are beneficial. The harmonic balance has been an attempt to reduce the costly process of numerical integration of the equations of motion. Numerical integration is a great tool for
simple systems, however as complexity is added in the structure or force model the feasibility of accurate numerical results becomes much more costly in terms of time and computational resources. To avoid this cost, it was hoped that the LCO could be accurately modeled by knowing the applied harmonic forcing. Based on the findings herein, the applied forcing was found to be harmonic once LCO was achieved. Using this forcing the frequency of oscillation for both pitch and plunge displacement has been modeled quite closely. Amplitude, on the other hand, was off by as much as almost 63%. Some possible areas of discrepancy here might include the choice to neglect the unsteady effects or perhaps lack of a dynamic stall model. The only conclusion that can be made at this time concerning the applicability of the harmonic balance method is that more investigation is needed.

In examining the effects of stochastic forcing on aeroelastic behavior the results were rather expected. Velocities below the critical value are damped out at different rates depending on the magnitude of the velocity. Exceeding the critical velocity, on the other hand, results in LCO. One major finding from this study, however, is that there exists a range of notable velocities around the critical velocity. The width of this range depends on the turbulence intensity. In essence this creates a buffer zone around the critical velocity which should be avoided when operating in turbulent conditions.

Although this analysis is considerably simplified and contains many areas for improvement, the insights garnered here are quite useful. Baseline characteristics
have been observed that can be improved through incorporating additional model components that better represent the physics of the real world problem.

8.1. Future Work

Based on the findings of this work, there are several opportunities for further study. Aerodynamic modeling stands as the greatest opportunity to improve the results gained from this analysis. Incorporating a fully unsteady aerodynamic model would increase the realism of the analysis by including wake and inertial effects. Several models have been proposed based on the work of Theodorsen [36] and Wagner [37]. Beddoes [52] has also developed a computationally efficient model for use in rotor analysis based on an indicial formulation. Including these more advanced models would likely increase the requisite computation time. However, perhaps the additional aerodynamic considerations would lead to better agreement between the harmonic balance method and the time integration results. Extending the harmonic balance formulation to more than one harmonic excitation might also result in better agreement.

The angle of attack results presented herein exceeded the region where the linear lift curve slope applies. Accordingly, worthwhile model improvements should also contain some type of dynamic stall model. Perhaps this entails simply fitting a polynomial to the lift curve to capture the dynamics at higher angles of attack, or perhaps requires building on previous work by Leishman and Beddoes [53] or McAlister, et al [54].
Further investigation into the stochastic effects is also worthwhile. Employing a stochastic linearization approach [55] might reveal additional important characteristics of the aeroelastic problem within the framework of turbulent aerodynamics.

Another major area of possible improvement is to look at the aeroelastic behavior of an entire finite wing. As mentioned before, many researchers have spent considerable effort in developing complex finite element models of entire aircraft which are then coupled to unsteady aerodynamics code. By refining the number of elements, higher order vibration modes can be modeled which can also contribute to the frequency coalescence shown previously. Further, better accuracy in the displacements can be achieved with higher order models than with the simple 2\textsuperscript{nd} order model analyzed herein. Taking into account three-dimensional effects is also an avenue for improvement.

In terms of the simulation presented, computation time improvements can also be made by finding an optimal time step which gives accurate results and minimal computation time. Further, the code can be perhaps modified to run in parallel with a multi-node system. Stochastic velocity time histories could also be generated via auto-regressive (AR) [56] or auto-regressive moving-average (ARMA) [57] algorithms. Perhaps this would speed up the stochastic modeling.
References


