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Essays in Structural Econometrics of Auctions

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ABSTRACT

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The first chapter of this thesis gives a detailed picture of commonly used structural estimation techniques for several types of auction models. Next chapters consist of essays in which these techniques are utilized for empirical analysis of auction environments. In the second chapter we discuss the identification and estimation of the distribution of private signals in a common value auction model with an asymmetric information environment. We argue that the private information of the informed bidders are identifiable due to the asymmetric information structure. Then, we propose a two stage estimation method, which follows the identification strategy. We show, with Monte-Carlo experiments, that the estimator performs well. Third chapter studies Outer Continental Shelf drainage auctions, where oil and gas extraction leases are sold. Informational asymmetry across bidders and collusive behavior of informed firms make this environment very unique. We apply the technique proposed in the second chapter to data from the OCS drainage auctions. We estimate the parameters of a structural model and then run counterfactual simulations to see the effects

of the informational asymmetry on the government's auction revenue. We find that the probability that information symmetry brings higher revenue to the government increases with the value of the auctioned tract.

In the fourth chapter, we make use of the results in the multi-unit auction literature to study the Balancing Energy Services auctions (electricity spot market auctions) in Texas. We estimate the marginal costs of bidders implied by the Bayesian-Nash equilibrium of the multi-unit auction model of the market. We then compare the estimates to the actual marginal cost data. We find that, for the BES auction we study, the three largest bidders, Luminant, NRG and Calpine, have marked-down their bids more than the optimal amount implied by the model for the quantities where they were short of their contractual obligations, while they have put a mark-up larger than the optimal level implied by the model for quantities in excess of their contract obligations. Among the three bidders we studied, Calpine has come closest to bidding its optimal implied by the Bayesian-Nash equilibrium of the multi-unit auction model of the BES market.

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[1]This chapter is a version of the online appendix to Hasker and Sickles (2010).

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Introduction

Auctions have been used widely as a sale format since antique ages. From government procurement to used car sales they are employed prevalently across the world. With the advent of online marketplaces like eBay and Amazon consumers from all segments have come to use this sale mechanism. Their broad use and high trade volumes have made auctions an interesting research area for economists. Since the paper by Vickrey (1961) the topic has gained more and more attention, and been analyzed in many different frameworks. Their relatively isolated environment and data availability have also made auctions an attractive field to test predictions of game theoretic models. Structural econometric papers in this area have come to the scene following Paarsch (1992). Estimating unknown value distribution of bidders helps researchers to conduct several interesting policy analyses by counterfactual experiments. Several different identification results and estimation ways have been proposed for alternative environments. Athey and Haile (2007) provides a comprehensive summary of this literature. The first chapter also takes a similar role and gives a detailed picture of commonly used structural estimation techniques for various auction types and different environments. The analyses in the following chapters are conducted based on some of the methods and results listed in the first chapter.

More specifically, in the second chapter, we discuss the identification and estimation of the distribution of private values in a common value auction model with an asymmetric information structure. This study builds on the results in identification and structural estimation of first price auctions, which are summarized in section 1 of the first chapter. We argue that the private values of the informed bidder is identifiable due to the asymmetric information structure. Then, we propose a two stage estimation method, which follows the identification strategy. Third chapter studies Outer Continental Shelf auctions, where oil and gas extraction leases are sold. Informational asymmetry across bidders and collusive behavior of more informed firms make this environment very unique. We apply the technique proposed in the second chapter to data from the OCS auctions. We estimate the parameters of a structural model and then run counterfactual simulations to see the effects of the informational asymmetry on the government's auction revenue. We find that, the probability that information symmetry brings higher revenue to the government increases with the value of the auctioned tract. In the fourth chapter, we make use of the results in the multi-unit auction literature to study the Balancing Energy Services auctions (electricity spot market auctions) in Texas.We adopt the multi-unit auction model of BES studied in Hortacsu and Puller (2008). We release their assumption of additively separable supply schedules and estimate the marginal costs of bidders implied by a multi-unit auction model of the market. We then compare the estimates to the actual marginal cost data. We find that, for the BES auction we study, the three largest bidders, Luminant, NRG and Calpine, have marked-down their bids more than the optimal amount implied by the model for the quantities where they were short of their contractual obligations, while they have put a mark-up larger than the optimal level implied by the model for quantities in excess of their contract obligations. Among the three bidders we studied, Calpine has come closest to bidding its optimal implied by the Bayesian-Nash equilibrium of the multi-unit auction model of the BES market.

CHAPTER 1

A Survey on Econometrics of Auctions[1]

1.1. First Price Auctions

1.1.1. First Price Unit Demand Auctions

Most of the sales that involve state as a buyer or seller are in the form of first price auctions. The most common examples for those include highway procurement, oil and gas extraction leases, and timber sales. Having a sample of similar auctions researcher can do structural analysis to answer policy questions.

The literature in structural econometrics of auctions began in 90's with the analysis of first price auctions. Early papers in the area include Donald and Paarsch (1993) and Laffont et al. (1995). The former studies a classical problem in maximum likelihood estimation of structural auction models. In general, the distribution of bidders' valuations is estimated having a sample of bidding data from similar auctions, and the support of the distribution depends on the parameters to be estimated. This violates the application of standard asymptotics to the estimates, a major problem in maximum likelihood estimation. To tackle this problem they propose a "Piece-wise Pseudo Likelihood" estimator and derived its asymptotic properties. Their estimation method , however, is parametric and requires the computation of the nonlinear bid function which might be cumbersome for some parameterization. To solve this

[1]**This chapter is a version of the online appendix to Hasker and Sickles (2010).**

problem Guerre et al. (2000) proposes a nonparametric estimation technique where they do not need to compute the bid function, and instead they use the first order condition to get pseudo values for the bidders. In the second stage they estimate bidders' value distribution from these pseudo values nonparametrically. They then extend their method to capture auction covariates and reserve prices. All these papers assume independent private values for bidders which provide simpler bid functions than those in common value models where bidders have imperfect signals about the value of the item being auctioned prior to sale. The asymmetry in bidders' signals causes what is known as the "winner's curse" problem. The bidder knows that if she becomes the winner, she will be the one with the most optimistic signal. Therefore, the bidder updates her belief about the item accordingly. If this issue is not considered by the bidders ex ante, negative profits may come out. However, "winner's curse" is not observed in equilibrium since bidders take it into account while preparing their bids. Hendricks and Porter (2003) analyzes this issue for oil and gas lease auctions in Gulf of Mexico, and estimates the winner's curse correction of bidders nonparametrically.

One important issue in this context is the selection or participation problem. In the real auction data it is observed that not all the potential bidders participate in the auction even when reservation prices are low. Ignoring this issue is likely to cause biased estimates. Li and Zheng (2009) consider a first price auction model taking this issue into account and study entry and competition effects on bidding behavior. They follow Bayesian estimation with MCMC techniques and apply their method to timber sales. In a parallel study for timber auctions Athey et al. (2008) analyzes the issue of collusion among bidders using data from two different auction formats. Both of these papers also take the unobserved auction heterogeneity (the information observed by bidders but not by the researcher) into account.

1.1.2. Multi Unit Auctions

The papers mentioned so far are about single good auctions where in each sale only one item is auctioned. The context is different however for example for treasury or electricity auctions, in which bidders submit a price-quantity schedule instead of a single monetary amount as in single unit auctions. These are called multi unit auctions since bidders demand multiple units of the same good. Armantier and Sbai (2006) compare uniform-price and discriminatory auction formats for French treasury auctions. They find informational and risk aversion asymmetries across bidders. On the other hand, Hortacsu and Puller (2008) study bidding behavior in Texas electricity market. Using a data from Texas balancing energy market they compare actual bids with those proposed by their model, and find that basic predictions of their model are consistent with the data.

1.2. Second Price Auctions

1.2.1. e-Bay Auctions

In this section we outline standard approaches in the literature to estimating bidders' values and the entry process in eBay auctions using parametric and nonparametric methodologies. We assume throughout that the analyst is analyzing single item auctions, that the item has private values, entry is exogenous, and that the following insight from Haile and Tamer (2003) holds. This assumption is that bidders follow two intuitive rules:

(1) No bidder ever bids more than he is willing to pay.

(2) No bidder allows opponents to win at a price he is willing to pay.

The second assumption must be applied with care in eBay auctions. Even if one considers exit as long as one assumes that the steady state hypothesis holds then this insight should be correct. Why would the second highest bidder leave "money on the table" in this auction to go to another auction where the competition will be just as strict?

As explained above, bidding takes place by a proxy program. The bidder submits a reservation price and the computer raises the price until only one bidder remains. In such an auction the obvious action is to enter your reservation value (or simply "value") as your reservation price, and the assumptions above makes sure that the second highest bidder will do this. For each bidder there will be a common set of auction specific characteristics x_n (where n indicates the auction) and a private component ρ^j (where j indicates the person). If the winning price is b_n^w, r_n is the traditional open reservation price, and values are log linear then the formula for the winning bid is:

$$b_n^w = \max\left\{r_n, e^{x_n'\beta}\rho_n^{(2)} - c\right\} \tag{1.1}$$

where $\rho_n^{(2)}$ is the private component of the second highest bidder in auction n given there are I potential bidders, and c is the continuation value of bidders. We follow

Sailer (2006) by assuming that bidders can not exit an auction until the auction is finished and simplify the analysis by assuming this constant is independent of the bidder.

Note that while ρ_n^i will be from a standard distribution, $\rho_n^{(2)}$, is not since it is the second order statistic from a sample of I bids. One method to estimate (1.1) is to use a Tobit model controlling for heteroskedasticity.However while this method is consistent it is a reduced form approach, leaving the analyst with no information about the true distribution of private values or how competitive the auction is. An auction is competitive if the number of potential bidders, I, is large, and this variable can not be estimated using reduced form techniques from this equation.

A straightforward methodology at this point would be to utilize formal maximum likelihood techniques, and extend them to allow for an exogenous entry process. Let $F_n(z,\beta)$ be the cumulative distribution function of the bidders' values at z and $f_n(z,\beta)$ be the probability density function—where β may include some distribution specific coefficients. Let I_n^a be the number of active bidders in auction n—or the number who actually submitted bids, and for $i \in \{0,1\}$ $D_n^i = 1$ if $I_n^a = i$, $D_n^i = 0$ otherwise. If $I \geq I_n^a$ is the number of potential bidders in auction n the likelihood of auction n given I is:

$$
\begin{aligned}
l_n(\beta|I) &= \left(F_n(r_n,\beta)^I\right)^{D_n^0} * \\
&\quad \left(IF_n(r_n,\beta)^{I-1}(1-F_n(r_n,\beta))\right)^{D_n^1} * \\
&\quad \left(I(I-1)F_n(b_n^w,\beta)^{I-2}(1-F_n(b_n^w,\beta))f_n(b_n^w,\beta)\right)^{\left(1-D_n^0-D_n^1\right)} .
\end{aligned} \tag{1.2}
$$

Inactive bidders are somewhat problematic. Although there must have been at least I_n^a who have bid there might also be any number of bidders who thought about bidding and did not. Therefore I is often treated as a random variable that can range from I_n^a to $\overline{I}$—an arbitrary upper bound. One can view this treatment of identification and estimation of the number of bidders I as a direct treatment for what would otherwise be unobserved heterogeneity in each auction that potentially could be correlated with outcomes of the bidding process. In this section we do not explicitly correlate the number of bidders with other potential heterogeneity controls or the feed-back rating of the seller, but rather estimate this potential auction specific unobservable. For an extensive treatment of unobserved heterogeneity in first-price auctions see Bierens and Song (2008).

One could estimate the number of potential bidders in each auction as a constant (perhaps varying with the length of auction or other discrete variables) or alternatively one could estimate an entry process. The number of bidders in an auction is often modeled as a Poisson entry process. The parameter of the entry process, λ_n, is often assumed to be log-linear in a set of auction specific characteristics z_n. Notice that some auction characteristics might affect entry but not private values and vice a versa, although it is difficult to understand what these non-overlapping variables might be a priori. However based on the analysis in the literature it seems clear that feedback—for example—has a much stronger effect on entry than it does on the sales price. A convenient functional form for entry is:

$$\ln \lambda_n = z_n' \gamma \ . \tag{1.3}$$

Let T_n be the length of the auction and D_n^{sr} be an indicator set to one if there is a secret reservation price and zero otherwise. Then the total likelihood for auction n with exogenous entry is:

$$l_n(\beta, \gamma) = \frac{\Sigma_{i=I_n^a+D_n^{sr}}^{\overline{I}} \frac{(\lambda_n T_n)^i}{i!} e^{-\lambda_n T_n} l_n(\beta|i)}{\Sigma_{i=D_n^{sr}}^{\overline{I}} \frac{(\lambda_n T_n)^i}{i!} e^{-\lambda_n T_n}} \quad . \tag{1.4}$$

The lower bound on i in both the denominator and the numerator is increased by one if there is a secret reservation price, following the treatment in Bajari and Hortaçsu (2003) wherein the auctioneer is treated as another bidder if there is a secret reservation price. When analyzing eBay data sets based on spider programs that collect information on auctions that do not result in sales as well as those that do, full likelihood can be utilized, of course based on a parametric assumption for distribution of private values and the entry process.

Since one cannot be certain *a-priori* of the true distribution of bidders' values, an array of distributions can be used and results based on these different parametric distributions can be compared and analyzed, possibly with nonparametric diagnostic methods. Results can also be compared with nonparametric methods. Common one-sided distributions proposed in the literature are the folded logistic, gamma, Weibull, log-normal, and Pareto.

Another methodology proposed by Laffont, Ossard, and Vuong (1995) is to simulate the auction. Laffont et al. focused on first price auctions but the methodology is the same for eBay auctions. Imagine running S auctions with I bidders in each auction. In each simulation the second highest value is selected $(X_{sn}(\beta, I))$ and these values are averaged to form $\bar{X}_n(\beta, I) = \frac{1}{S}\Sigma_{s=1}^{S} X_{sn}(\beta, I)$. If S is large then the distance between $\bar{X}_n(\beta, I)$ and $E\left[v^{(2)}\right]$ will be small, and assuming one has the correct value for I then the distance between $\bar{X}_n(\beta, I)$ and $E\left[b_n^w\right]$ will be small. However, an unbiased methodology must take into account that in practice S is not large, and thus the objective function should compensate for the variance of the simulated estimator. This variance is $V_{Sn}(\beta, I) = \frac{1}{S(S-1)}\Sigma_{s=1}^{S}\left(X_{sn}(\beta, I) - \bar{X}_n(\beta, I)\right)^2$. Estimation of β and I are then given by:

$$\text{(1.5)} \qquad \arg\min_{\beta, I} Q_{S,N}(\beta, I) = \frac{1}{N}\Sigma_{n=1}^{N}\left[\left(b_n^w - \bar{X}_n(\beta, I)\right)^2 - V_{Sn}(\beta, I)\right]$$

where N is the number of auctions. Note that the distribution of $v^{(2)}$ will be a non-degenerate function of β and I. This allows identification of I. On an intuitive level this is because I determines the amount of "skewness" in the observed prices. Theoretically one could allow for exogenous entry as outlined above, but this could prove computationally burdensome.

Another simulation based, parametric estimation approach comes from Bajari and Hortacsu (2003). Bajari and Hortacsu (2003) specifies a structural econometric model of eBay auctions, where Bayesian approach is taken to estimate the parameters of the model. They model the eBay bidding problem in a second price sealed bid auction setting with symmetric common value assumptions and stochastic entry, where the entry of the potential bidders depends on a zero profit condition and the number of actual bidders is determined by a Poisson process with mean λ_t. At each auction, bidders only observe a private signal x_{it} about the value of the auctioned good v_t, where $x_{it} = v_t + \varepsilon_{it}$, ε_i is distributed $i.i.d$, and $v \sim N(\mu_t, \sigma_t^2)$ and $x \sim N(\mu_t, k\sigma_t^2)$ (here t is the auction, and i is the bidder subscript). Being motivated by some empirical regularities in the observed characteristics of the auction data, they estimate reduced form relations between some observable auction variables and the structural parameters. Then, they specify the likelihood function of the observed bids using the data of the bidders who have not bid at all for each auction. By this way they assign a positive likelihood to auctions with no bidders, hence make a good use of all the available data. Showing the form of the likelihood function of the bids conditional on the auction specific data and the structural parameters to be estimated, they use this likelihood function to update their prior in the Markov-chain Monte Carlo method by which they simulate the posterior distribution of the model parameters. In the paper they emphasize the advantages to using a Bayesian approach when estimating parametric auction models. First, Bayesian methods are computationally simple and easier to implement than maximum likelihood. Since in many auction models, support of the distribution of the bids depends on the parameters, it is not possible to apply standard asymptotic theory straightforwardly. Moreover, confidence intervals in a classical framework require second order asymptotic approximations, whereas the results in this paper are correct in finite samples and do not require invoking the assumptions used in second order asymptotics. Lastly, in some parametric auction models, Bayesian models are asymptotically efficient while some commonly used classical methods are not efficient.[2]

[2] see Porter and Hirano (2001).

Sailer (2006) is another interesting paper in terms of modeling and identification strategy, preferring parametric techniques for estimation. Sailer (2006) sets up an intertemporal optimization problem for an eBay bidder who faces an infinite sequence of Vickrey auctions for heterogenous products (She argues that only the bids at the last minutes of the auction, when bidders are no more able to observe the bids by their opponents, determine the sales price and the winner of the auction; that's why we can take into account only the last minutes of an eBay auction and evaluate it as a sealed bid second price auction.[3][4]) The intertemporal optimization setting is designed to reflect the tradeoff between bidding today and waiting for tomorrow, where the bidder optimally chooses a bid to maximize her expected benefit from bidding minus the cost of bidding, but still can choose to wait for the next auction without bidding in the current auction (hence not incurring the cost of bidding now) if the expected future payoff of waiting for the next auction is higher than the maximum of what she can expect to get from bidding in the current auction. The problem can be summarized as the following:

$$V_i(v_i, \mathbf{s}) = \begin{cases} \max \left\{ \max\limits_{b_i > r} E_{b_{(1)}} \left[\mathbf{1}\left\{ b_{(1)} < b_i \right\} \left(v_i - b_{(1)} \right) \right. \right. & \\ \left. \left. \quad -c_i + \mathbf{1}\left\{ b_{(1)} \geq b_i \right\} V_i^e | \mathbf{s} \right], \quad V_i^e \right\} & \text{,before win} \\ 0 & \text{,after win} \end{cases}$$

[3]Bajari and Hortacsu (2003) take a similar approach for the eBay auctions of some common value goods.

[4]Nekipelov (2007) actually shows that early bidding can benefit the bidder by entry deterrence, hence the early bids may have an effect on the result of the auction.

where $V_i(v_i, \mathbf{s})$ is the value of having the opportunity of bidding in the current auction given the realization of the valuation of bidder i for the currently auctioned good (v_i), the realizations of the characteristics of the currently auctioned good and the current auction $(\mathbf{s})$. b_i is the bid of bidder i if she decides to participate in the current auction, $b_{(1)}$ is the maximum bid of the bidder i's opponents, v_i is the realization of bidder i's valuation of the auctioned good after seeing the characteristics of the good and the auction, c_i is the cost of bidding for bidder i, and V_i^e is the continuation value of bidder i (the value she expects to get from the next auctions if she fails to win in the current auction or does not bid at all). The optimal bidding behavior of bidder i requires the following equality:

$$c_i = E_{\mathbf{s},b_i^*}\left[E_{b_{(1)}}\left[\mathbf{1}\left\{b_{(1)} < b_i^*\right\}\left(b_i^* - b_{(1)}\right)|\mathbf{s}\right]|\delta_i^* = 1\right] \tag{1.6}$$

where $\delta_i^* = 1$ denotes that it is optimal for bidder i to participate in the current auction (the entry decision is affirmative), and b_i^* is the optimal bid of her. Clearly, we could calculate the cost of bidder i if we could observe b_i^* when it is a winning bid. (We can only get biased estimates of the cost if we use the observed highest bid in place of the highest bid b_i^*). This is the point where Sailer(2006) suggests a stepwise procedure which allows her to show that both the distribution of valuations of the bidders and their costs are non-parametrically identified. Given that the model is set under the assumption of asymmetric bidders and that the available data are the bids of all loosing bidders together with the transactions prices and the identities of the bidders in each auction, the identification results from Athey and Haile (2007), Brendstrup and Paarsch (2006) and Song (2004) allow inference about the underlying parent bid distributions. Then, given these bid distributions, estimates of the unobserved winning bids can be built, which in turn are used to compute the costs from (1.6). The likelihood function of the problem is shown to be the following:

$$l = \sum_{t=1}^{T} \ln \left[\frac{f_{b_{l_t}}(b_{(2),t}|\mathbf{x}_t)(1 - F_{b_{m_t}}(b_{(2),t}|\mathbf{x}_t))}{(1 - F_{b_{l_t}}(b_{(3),t}|\mathbf{x}_t))(1 - F_{b_{m_t}}(b_{(3),t}|\mathbf{x}_t))} \right]$$

which is obtained from the summation of different auctions t due to the fact that the auctions are independent of each other. Since the parent bid distributions are bidder specific, the t dimension of the panel is not long enough to legitimize a non-parametric approach. Hence the author assumes a normal form for the bid distributions such that $f_{b_i}(b_i|x) = N\left[\mu_{b_i}, \sigma\right]$ with $\mu_{b_i} = cons + x\beta - V_i^0$, where $cons$ includes the part of the continuation value of bidder i that is common for everyone, and the individual specific part of the continuation value of i (V_i^0) is assumed to be a function of bidder i's cost c_i. Hence we can approximate the V_i^0 by a polynomial in c_i. However, the plan is to calculate the value of c_i from (1.6) after inferring the unobserved winning bids from the estimated bid distribution, for which we need to know the value of c_i. Therefore we start with a guess about the value of c_i, and this would be the beginning of an iterative process which goes on until convergence of the estimated bid distributions $\widehat{F}_{b_i|\mathbf{x}}$ and the bidding costs c_i.

A non-parametric methodology can be based on Song (2004). Much attention has been given to nonparametric alternatives to the structural models discussed above. Athey and Haile (2007) show that the parent distribution is uniquely determined if the distribution of any order statistic with a known sample size is identified. However, in eBay auctions, the number of potential bidders is generally not observable. Song (2004) addressed this issue by showing that within the symmetric independent private values model, observation of any two valuations of which ranking from the top is known non-parametrically identifies the bidders' underlying value distribution. Based on this theorem, Song argues that we can use the second and third highest bids to identify the distribution of bidders' private values.

In this technique one must assume not only that the second highest bidder bids his true value but also the third highest bidder does, thus we assume explicitly that

$$v_n^j = e^{x'_n \beta} \rho_n^j - c$$

for at least the second and third highest bidders. These bidders' values are denoted $v_n^{(2)}$ and $v_n^{(3)}$ with the corresponding error terms $\rho_n^{(2)}$, and $\rho_n^{(3)}$—note that I may vary by auction and is unknown throughout this analysis. Since we need both the second and third highest bids for estimation, all auctions with two or fewer bidders must be dropped. This methodology is not without attendant problems, however, since whether or not the third highest bids reflect the third highest bidders' true private valuations can be questioned. To deal with this issue, Song suggests that we should "use data from auctions where the first or the second highest bidder submitted a cutoff price greater than the third highest valuation late in the auction". With this in mind, she details an econometric method to decide "how late" is proper. The interested reader should see Song (2004).

In this methodology we can only use the partial likelihood of $\rho_n^{(2)}$ given $\rho_n^{(3)}$ since the full likelihood requires the unknown number of potential bidders. According to the basic theory of order statistics, the sample likelihood function can be written as:

$$L_N\left(\hat{f}\right) = \frac{1}{N}\sum_{n=1}^{N} \ln \frac{2\left[1-\hat{F}\left(\rho_n^{(2)}\right)\right]\hat{f}\left(\rho_n^{(2)}\right)}{\left[1-\hat{F}\left(\rho_n^{(3)}\right)\right]^2},$$

where

$$\hat{F}(z) = \int_{\underline{v}}^{z} \hat{f}(t)\,dt.$$

Here and below, $\underline{v}$ is the lower bound of bidders' private value. Denote $m = \min\left(v_n^{(2)}\right)$. Note that no information about $F(v)$ for $v < m$ can be observed. If a starting price set by a seller is below m with positive probability, then m is a consistent estimate of $\underline{v}$. In order to estimate the unknown distribution one can employ the method proposed by Coppejans and Gallant (2002) and use the hermite series to approximate the unknown distribution:

$$\widehat{f}(z) = \frac{\left[1 + a_1\left(\frac{z-u}{\sigma}\right) + \cdots + a_k\left(\frac{z-u}{\sigma}\right)^k\right]^2 \phi\left(z; u, \sigma, \underline{v}\right)}{\int_{\underline{v}}^{\infty}\left[1 + a_1\left(\frac{z-u}{\sigma}\right) + \cdots + a_k\left(\frac{z-u}{\sigma}\right)^k\right]^2 \phi\left(z; u, \sigma, \underline{v}\right)dz}$$

where $\phi\left(z; u, \sigma, \underline{v}\right)$ is the density of $N(u,\sigma)$ truncated at m .Then an estimator of $\widehat{f}(z)$, denoted as $\widehat{f}_N(z)$, is the maximizer of $L_N\left(\hat{f}\right)$, such that

$$\left(\widehat{a}_1, ..., \widehat{a}_k, \widehat{u}, \widehat{\sigma}\right) = \underset{a_1,\ldots,a_k,u\in R,\sigma>0}{\arg\max} L_N(\widehat{f}) = \frac{1}{N}\sum_{n=1}^{N} \ln \frac{2\left[1-\hat{F}\left(\rho_n^{(2)}\right)\right]\hat{f}\left(\rho_n^{(2)}\right)}{\left[1-\hat{F}\left(\rho_n^{(3)}\right)\right]^2}$$

Gallant and Nychka (1987), Fenton and Gallant (1996) and Coppejans and Gallant (2002) provide details of this method to approximate the unknown distribution of private values. The optimal series length varies according to the data set under consideration. One can choose the optimal series length, k^*, using the cross-validation strategy employing the Integrated Squared Error (ISE) criterion (Coppejans and Gallant, 2002).

Another work that makes use of the SNP approach developed by Gallant and Nychka (1987) is by Brendstrup and Paarsch (2006). Brendstrup and Paarsch (2006) work under the assumption of asymmetric valuations and known number of potential bidders. They develop a theoretical model of bidder behavior at single unit English auctions when valuations of the bidders are assumed to be independent draws from one of the J different classes of distributions. Then, in the light of Theorem 2 in Athey and Haile (2007), they demonstrate that the distribution of the different classes of latent valuations are nonparametrically identified when the winning bid, identity of the winner and the number of potential bidders n is observed. They propose a semi-nonparametric (SNP) estimation strategy. Finally, they extend the analysis to multi-unit auctions.

In order to show the identification of the J different classes of distributions, Brendstrup and Paarsch (2006) start from the probability density function of the second highest order statistics of n independent draws each from one of the J different types of distributions, due to Balakrishnan and Rao (1998):

$$g_{(2:n)}(y|\mathbf{F}) = \frac{1}{(n-2)!} Perm \begin{pmatrix} F_{type(1)}(y) & \cdot\ \cdot\ \cdot & F_{type(n)}(y) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ F_{type(1)}(y) & \cdot\ \cdot\ \cdot & F_{type(n)}(y) \\ f_{type(1)}(y) & \cdot\ \cdot\ \cdot & f_{type(n)}(y) \\ \left[1 - F_{type(1)}(y)\right] & \cdot\ \cdot\ \cdot & \left[1 - F_{type(n)}(y)\right] \end{pmatrix}$$

where $\mathbf{F}$ is the vector of the cumulative distribution functions of the J parent classes of distributions, $F_{type(i)}(y)$ and $f_{type(i)}(y)$ stand for the CDF and the PDF of the i'th bidder's valuation respectively, and $Perm$ is the permanent operator. Then they develop the following system of Pfaffian integral equations:

$$\mathbf{F}^0_{type}(y) = \exp\left\{\mathbf{A}^{-1} \log\left[diag\left(\int_0^y \exp\left\{-\log\left[\boldsymbol{\iota}_n - \mathbf{F}^0_{type}(u)\right]\right\} d\mathbf{G}^0(u)^T\right)\right]\right\}$$

where $\mathbf{F}^0_{type}$ and $d\mathbf{G}^0$ are $(n \times 1)$ column vectors whose ith rows equal $F^0_{type(i)}(y)$ (the true population cumulative distribution function for the class $type(i)$) and $d\mathbf{G}^0_{(2:n)}(y, i)$ (the derivative of the true population cumulative distribution function of the winning bid at an auction won by a bidder whose identity is i), respectively. From Meilijson (1981), this system of Pfaffian integral equations have a unique solution, which in turn leads the authors to conclude that the distributions of the valuations are identified from the winning bids and the identities of the winners.

Having shown the identification of the latent distributions of the valuations, the authors propose an estimator using the SNP estimator developed by Gallant and Nychka (1987) in a way to introduce covariates into the system. They define the draw of bidder i who is in class j in the tth auction as follows:

$$\log V_t^{ij} = \mathbf{x}_t \boldsymbol{\beta}_j + U_t^{ij}$$

They approximate $f_j(u)$ by

$$f_j^{p_T}(u) = \left[\sum_{k=0}^{p_T} \gamma_{jk} H_k(u) \right]^2 \exp\left(-u^2/2\right) + \varepsilon \exp\left(-u^2/2\right)$$

where $H_k(u)$ denotes an Hermite polynomial of order k. Then they implement this finite order approximation into the method of quasi-maximum likelihood , defining the estimator

$$\left\{ \widehat{f_{jT}} \right\}_{j=1}^{J} = \arg \max_{f_j \in \mathcal{F}_{jT}} \frac{1}{T} \sum_{t=1}^{T} \log g_{(2:n)}(y_t, i_t | \mathbf{F})$$

where

$$\mathcal{F}_{jT} = \left\{ f_{jT} \in \mathcal{F}_j : f_{jT}(u|\boldsymbol{\gamma}_j) = \left[\sum_{k=0}^{p_T} \gamma_{jk} H_k(u) \right]^2 \exp\left(-u^2/2\right) + \varepsilon \exp(-u^2/2), \boldsymbol{\gamma}_j \in \boldsymbol{\Theta}_{jT} \right\}$$

and

$$\boldsymbol{\Theta}_{jT} = \left\{ \boldsymbol{\gamma}_j = (\gamma_{j0}, ..., \gamma_{jp_T}) : \int_0^{\infty} f_{jT}(u|\boldsymbol{\gamma}_j) du = 1 \right\}$$

The approximation will converge to the truth by letting p_T increase at a rate that is slower than the rate at which the sample size T increases.

One of the important papers in terms of identification strategy is Adams (2007). In a setting similar to Song (2004), Adams (2007) generalizes the result that the value distribution is identified from the observed auction prices when the number of bidders is known or randomly determined (Athey and Haile, 2007). Although the number of actual bidders in an eBay auction is observable, we cannot say that it is randomly determined, because of the selection bias resulting from the format of the eBay auctions. It can be argued that there is a set of potential bidders which are randomly determined, but for which the number is not known (Song, 2004). Adams (2007) shows that the value distribution is identified when the auction prices are observed and the probability distribution over the number of potential bidders is known, under the assumption that the potential number of bidders is independent from the values of bidders in an auction. Nevertheless, it is not clear if it is possible to determine this distribution, since the existence of potential bidders in an auction can be censored. The author argues and shows in a formal proof, that some auction characteristics that affect the number of bidders but not the distribution of bidder valuations in an auction will allow us approximate this distribution arbitrarily closely under some assumptions. The auction length can be given as an example of such a characteristic: with varying auction length, the probability of having n bidders in an auction varies, but the value distribution does not. Hence, for the estimation of the structural parameters, the length of the auction is included in the function defining the probability of having n people in the auction, but not in the value function of the bidder. The author applies OLS on the price data, and uses the order statistics approach in a maximum likelihood estimation, preferring a log-normal assumption for the distribution of the value. Results of the two estimations show that OLS may substantially overestimate the average value.

Another non-parametric estimation approach comes from Haile and Tamer (2003). As mentioned at the beginning of the current section, the authors have two weak assumptions as to how to interpret the observed bids in an English auction. But it is clear that the distribution of the bidder valuations is not identified having assumed these rules which imply neither (i) a unique distribution of bids given a distribution of valuations nor (ii) a unique distribution of valuations given a distribution of bids. However, they argue that informative bounds on the distribution functions of bidder valuations are possible to identify out of this incomplete model of bidding. The identification of the upper bound for the distribution of the valuations $F(v)$ is provided by the property of the i.i.d random variables that the distribution of the ith order statistic $F_{i:n}(.)$ is related to the parent distribution $F(.)$ by

$$F(v) = \phi(F_{i:n}(.); i, n)^5 \tag{1.7}$$

together with the implication of the first assumption that $b_{i:n} \leq v_{i:n}$ for all i. Therefore

$$F_{i:n}(v) \leq G_{i:n}(v) \qquad \forall i, n, v \tag{1.8}$$

[5] see for example Arnold,Balakrishnan and Nagaraja (1992)

where i denotes the bidder, n is the total number of bids, $G_{i:n}(.)$ denotes the distribution of the ith lowest bid, and $\phi(.;i,n)$ is a strictly increasing differentiable function. Applying the monotone transformation $\phi(.;i,n)$ on both sides of (1.8) implies the identification of the upper bound $F_U(v)$ as the following:

$$F(v) \leq F_U(v) \equiv \min_{n\in\left\{2,\ldots,\overline{M}\right\},i\in\{1,\ldots,n\}} \phi(G_{i:n}(v);i,n) \tag{1.9}$$

for all v in the relevant range and higher than the reservation price. Note that there is a different bound on $F(v)$ for each pair of indices (i,n), thus the minimization operator chooses the tightest one among them. For the identification of the lower bound, the following implication of the second assumption (bidders do not allow an opponent to win at a price they are willing to beat) gains importance: The second highest valuation has to be smaller than or equal to the highest bid plus the minimum bid increment:

$$v_{n-1:n} \leq b_{n:n} + \triangle \tag{1.10}$$

Obviously this provides an upper bound on the realization of only one order statistic of the valuation at each auction, but (1.7) enables us to construct a lower bound for the latent distribution. From (1.10) we can easily write

$$F_{n-1:n(v)} \geq G^{\triangle}_{n:n}(v) \quad \forall n, v \tag{1.11}$$

where $G^{\triangle}_{n:n}(.)$ is the distribution of $B_{n:n}+\triangle$. Then, analogous to the derivation of the upper bound, we apply the monotone transformation to both sides of (1.11) and recall (1.7) to finally get the identification of the lower bound $F_L(v)$:

$$F(v) \geq F_L(v) \equiv \max_{n} \phi(G^{\triangle}_{n:n}(v); n-1, n) \tag{1.12}$$

for all v in the relevant range and greater than the reservation price. Note that whenever $v_{n-1:n} = b_{n:n}$ for some n, the lower bound implied by (1.10) is the true distribution; and whenever $b_{i:n} = v_{i:n}$ for some (i, n), the upper bound obtained from above is the true distribution. Therefore we can say that there is no cost to taking this bound approach rather than assuming the full structure of the standard model, since only when the data is inconsistent with the standard model we identify bounds on $F(.)$ rather than identify $F(.)$ itself. Consistent nonparametric estimators for the upper and lower bounds $F_U(v)$ and $F_L(v)$ are obtained by substituting the relevant empirical distribution functions for their population analogs in (1.9) and (1.12) respectively. The estimators are

$$\begin{aligned}
\widehat{F}_U(v) &\equiv \min_{n\in\left\{2,\ldots,\overline{M}\right\}, i\in\{1,\ldots,n\}} \phi(\widehat{G}_{i:n}(v); i, n), \\
\widehat{F}_L(v) &\equiv \max_{n\in\left\{2,\ldots,\overline{M}\right\}} \phi(\widehat{G}^{\triangle}_{n:n}(v); n-1, n)
\end{aligned}$$

where

$$\widehat{G}_{i:n}(v) = \frac{1}{T_n}\sum_{t=1}^{T}\mathbf{1}\left[n_t = n, b_{i:n_t} \leq v\right],$$
$$\widehat{G}^{\triangle}_{n:n}(v) = \frac{1}{T_n}\sum_{t=1}^{T}\mathbf{1}\left[n_t = n, b_{n_t:n_t} + \triangle_t \leq v\right]$$

and

$$T_n = \sum_{t=1}^{T}\mathbf{1}\left[n_t = n\right]$$

A negative property of these fairly simple non-parametric estimators is that, although they are uniformly consistent[6], these estimators can be badly biased in small samples. The authors propose a smoothing operation in order to improve the small sample performance of the estimators.

A final non-parametric methodology comes from Nekipelov (2007). In his paper, he explains two types of aggressive bidder behavior induced by the multi-auction structure of eBay in a continuous time stochastic auction model with endogenous entry, in which bidder types are differentiated by their initial information regarding the entry process. It is the only continuous time auction model that we are aware of in the literature, though there are more aspects that are specific to this paper. One is the visibility parameter θ of a given auction, which brings a stochastic component into the model: even if the bidder is completely certain about the quality of the object itself, she can be uncertain about the group of potential rivals. Bidders have prior beliefs in the form of probability distributions over the value of visibility, and they

[6]The asymptotic distribution of the estimators and the consistency of the bootstrap bands are shown in Haile and Tamer (2002).

can update their beliefs over the course of the auction. The other is that the bidder who arrives in the auction gives a best response to the entire path of price process rather than to actions of particular rival bidders. The model is composed of four key structural functions: the instantaneous demand function (this is the frequency of the poisson process that specifies the entry rate to an auction) $\lambda(t, p_t, \theta)$, the price jump size function $h(t, p_t, \eta)$ (where η is the bid increment), the distribution of valuations of the bidders $F(v)$, and the distribution of bidders' beliefs about the visibility parameter (the mean and the variance of it) $G_\mu(\mu_0)$ and $G_\sigma(\sigma_0)$. These structural functions of the model are defined to be non-parametrically identified from the data if no two different sets of characteristics of the model produces the same distribution of simulated prices. This definition requires two conditions to hold. First, the observable distribution (this is the joint distribution of timing and the sizes of price jumps for the data set used in the paper) should contain sufficient information about the structural functions of the model. Second, the structural functions of the model can be recovered by using some method of inversion of the observed joint distribution. The author lists the assumptions that allow the unique recovery from the data the set of structural functions of the model. The distribution of bidders' valuation is identified from the distribution of the number of active bidders across auctions given the price, if the bidding function is assumed to be monotone with respect to the valuation in each moment of time and that the beliefs are independent from valuations. After the distribution of the valuations is identified, it is possible to identify the mean beliefs of the bidders by the distance between their observed bidding patterns given their information and the pattern computed for an auction with given visibility and given structural functions. Lastly, sorting the bidders according to the relative number of their bids will identify the variance of the beliefs of bidders, as the model predicts that bidders with more diffuse priors bid more frequently. Finally, given the distributions of valuations and beliefs, the path of the second highest bid can be simulated for any given instantaneous demand and price jump size function, and minimizing the distance between the observed price path and the simulated price path gives us the estimate for the parameters of the Poisson process. The multi-step estimation procedure starts with non-parametric estimation of the distribution of the observed price and timing of jump data $f(p_t, N_t, t, \gamma_0)$, which is characterized by the structural parameter γ_0, using a kernel:

$$\widehat{f}(p,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{1}{h_p h_t}\sum_{i=1}^{I_k}\kappa\left(\frac{p_i^{(k)}-p}{h_p}\right)\kappa\left(\frac{t_i^{(k)}-t}{h_t}\right)$$

where N_t is the total number of bidders who have entered the auction up to time t, n is the number of observed auctions, k is the index of an auction, I_k is the number of price jumps in the auction k, $\kappa(.)$ is a kernel function and h_p and h_t are bandwidth parameters. This density estimate is consistent and asymptotically normal. In the second step the entry of the bidders N_t is simulated given the parameter vector γ, then optimal bidding problem of each bidder is solved to calculate the second highest bid in the auction at any given instant. This simulated price data is named as *the response of the structural model to the data.* Then the same non-parametric estimation procedure is applied to the simulated price data to get the estimated distribution $\widehat{f}_\gamma(p,t)$, which is also a consistent estimate. Then, Kullback-Leibler Information Criterion (KLIC) is used to compare the joint distribution of the observed and the simulated price and time of the price jump data:

$$\widehat{KLIC} = \frac{1}{n} \sum_{k=1}^{n} \sum_{i=1}^{I_k} \log \left[\frac{\widehat{f}(p_i^{(k)}, t_i^{(k)})}{\widehat{f}_\gamma(p_i^{(k)}, t_i^{(k)})} \right] \tag{1.13}$$

The idea here is to compare the empirical model with the structural model. Note that minimizing the KLIC is equivalent to maximizing

$$L_n(\gamma) = \frac{1}{n} \sum_{k=1}^{n} \sum_{i=1}^{I_k} \log \left[\widehat{f}_\gamma(p_i^{(k)}, t_i^{(k)}) \right]$$

The author shows the estimate of the parameter γ obtained by minimizing (1.13) is asymptotically normal. Markov-Chain Monte Carlo method is used to minimize (1.13) with a substantially reduced computational burden. This also gives an asymptotically normally distributed estimator, though the asymptotic variance of the estimate needs a small correction.

Hong and Nekipelov (2009) develop an efficient local instrumental variable estimation of an empirical auction model. In particular, they derive semi-parametric efficiency bounds and efficient estimators for the conditional monotone local instrumental variable model studied in Abadie, Angrist and Imbens (2002). They apply this semiparametrically efficient estimation method to analyze the relation between bid dispersion and early bidding in the dataset which is collected from a natural experiment conducted in Nekipelov (2007). One of the implications of the theoretical model developed in Nekipelov (2007) is that early bidding has competing effects on the dispersion of bids. On the one hand, early bidding deters entry and decreases the bid dispersion. On the other hand, early bidding provides more information and potentially increases both learning and bid dispersion. In an attempt to test which effect is dominant, Hong and Nekipelov (2009) apply the local instrumental variable approach trying to walk around the endogeneity problem that can arise with a simple regression of bid variation on an early bidding indicator because of the correlation of some unobserved characteristics of auction (visibility) with both early jump bidding and bid dispersion. A local instrumental variable is given by an exogenous change of supply in the natural experiment conducted in Nekipelov (2007), who auctioned off additional supply of Robbie Williams' CD on eBay. Such an exogenous increase in the supply weakly increases the incentive for early jump bidding. The set of compliers are defined as the auctions which have no early jump bid prior to the supply increase but have early jump bid after the supply increases. In the first step of the estimation, the basic result in Abadie, Angrist and Imbens (2002) is used to find the initial consistent but inefficient estimate of the parameter vector. In the second step, an estimate of the weighting matrix is formed, and efficient estimates of the parameter vector is obtained. Results of the estimation actually reveal the inefficiency of the first step estimates; although the same parameter values were obtained from the first step and at the end of second step, the standard errors from the second step are much smaller than the ones obtained from the first step estimation. Moreover, the estimates from OLS and 2SLS are compared. They find that the coefficient of the early bid indicator is positive in both, but significantly greater in 2SLS than in OLS : a result consistent with the prediction of omitted variable bias.

The field of econometrics has been providing methods for investigation of bidder behavior in eBay auctions that are well grounded in economic theory. Today, a researcher has a number of commonly used alternative structural methods like maximum likelihood (Adams 2007), non parametric methods (Song, 2004; Brendstrup and Paarsch, 2006; Nekipelov, 2007), simulation based methods (Bajari and Hortacsu, 2003) and bounds estimation of incomplete models (Haile and Tamer, 2003), as well as methods for identifying empirical relationship between various characteristics of online auctions (Lucking-Reiley et al. 2007; Hong and Nekipelov, 2009). As long as one has enough sample size for standard asymptotic results to hold, using non-parametric methods have advantages in terms of robustness to distributional misspecification, although parametric methods are more convenient with small sample sizes and high number of covariates included in the analyses, besides having a higher convergence rate. Another issue that gains importance with non-parametric estimation of structural models is the identification problem. Although we have some standard results about identification under different assumptions (Athey and Haile, 2007), development of new structural models will require new identification results (Nekipelov, 2007).

CHAPTER 2

Structural Estimation of an Asymmetric Common Value Auction Model

2.1. Introduction

Auctions have been a popular research area in the last fifty years due to their prevalence as a selling mechanism. Since the seminal paper by Vickrey (1961), researchers have shown more interest in the topic and analyzed it from different perspectives. Early papers mostly focus on the theoretical aspect of the topic[1]. Later, with the availability of the data, researchers have started to use structural models to test the theory and make counterfactual experiments for policy analysis. Most of these papers propose structural methods assuming that bidders have private values, whereas only a few study common value auctions. In this chapter, we propose a structural estimation method for a common value environment where bidders posses asymmetric information. Then we test the performance of our estimator in Monte-Carlo experiments.

Structural auction models started to garner interest in early 90's. Paarsch (1992) is one of the early papers in this area which proposes parametric estimation methods for first price auctions. An important concern in estimating a structural auction model is that the support of bids depends on the parameter to be estimated, thereby

[1]See, for example, Milgrom and Weber (1982).

violating one of the regularity conditions in maximum likelihood estimation. This causes the estimates to have a nonstandard asymptotic behavior that complicates the inference. Donald and Paarsch (1993) suggest a piecewise pseudo-likelihood estimation method to handle this problem. Another issue in estimating a structural auction model is the highly nonlinear bid function which makes the likelihood function computationally cumbersome. Guerre, Perrigne, and Vuong (2000) propose a two stage nonparametric estimation method based on the first order condition of bidders which does not require computing the nonlinear bid function. Most of the papers in the literature studied private value environments since bid function is simpler compared to that in a common value setting. Two of the exceptions are Bajari and Hortacsu (2003) and Hendricks et al (2003). The former paper focuses on the eBay environment and estimates the determinants of buyer valuations under a common value environment. Through counterfactual experiments, they quantify the winner's curse and examine the optimal selling strategies. On the other hand, Hendricks et al (2003) study OCS Wildcat leases using a common value auction model but they do not estimate the structural parameters of it. Moreover, both papers assume that bidders have symmetric information about the quality of the auctioned item.

In this paper, we study a common value auction setting where bidders have asymmetric information about the quality of the item. To the best of our knowledge, this is the first paper that proposes a structural estimation method for this environment. We assume that there is an informed bidder and a number of uninformed bidders. Informed bidder has more channels to obtain information about the item quality, and thus has more precise information than uninformed bidders have.

A classical example of this environment is the OCS drainage leases in Gulf of Mexico. Since 1954, the federal government has been holding auctions to lease the rights to extract oil and gas in Gulf of Mexico[2]. Between 1954 and 1982, each tract in the area was sold through a first price sealed bid auction. Also, for some tracts there existed some previously leased and drilled neighbor tracts which convey significant amount of information for its owners about the neighbor tracts. In general, the previous drilling information from a neighbor tract is superior to the publicly observable seismic signal. Therefore, owners of the neighbor tracts have an informational advantage over other bidders. In general, auctions in which both incumbent firms and new comers participate constitute a possible example for an asymmetric information auction environment. The incumbent firm may have an informational advantage due to its previous operations in the industry.

We use the theoretical model given in Hendricks and Porter (1988) (hereafter HP), which extends the models studied in Wilson (1967) and Engelbrecht-Wiggans et al (1983).We propose a two stage estimation for the structural model following the approach in Guerre, Perrigne, and Vuong (2000). In the first stage, we estimate the distribution of uninformed bids and then using the first order condition of the informed bidder we estimate the unobserved private information of the informed bidder. In the second stage, we estimate the distribution of the private information which then allows one to conduct counterfactual analysis. The method is computationally

[2]Indeed, tracts in Gulf of Mexico are still being leased, yet the rules of auctions changed in 1982. Since 1983, the U.S. Mineral Managemeng Service (MMS) adopted area wide leasing, which dramatically changed the bidding environment for companies.

attractive since it does not require computing the bid function of the informed bidder. Also, under certain cases it can be implemented nonparametrically. We test the performance of our estimator for both parametric and nonparametric settings.

The next section introduces the theoretical model. Section 3 explains the estimation procedure. Monte Carlo experiments to test the performance of our estimator are given in Section 4. Section 5 concludes.

2.2. Theoretical Model

The theoretical model we use is from HP. In this section we give a summary of the asymmetric common value auction model they study. Their bidding model is a version of the non-cooperative, first price, sealed bid model with asymmetric information[3], and they use it to analyze OCS drainage leases. OCS auctions have been held by the federal government since 1954 to lease oil and gas extraction rights in Gulf of Mexico. Before 1982, there were three different sales of tracts: wildcat, drainage, and development. HP studies OCS drainage leases where bidders have asymmetric information about the resource potential. They assume the following for their analysis: (i) there is no information externality between tracts sold in the same sale; (ii) each bidder is risk neutral; (iii) the bidding strategy of each bidder for a tract depends only on the state of information and competition for that tract.

The source of the information asymmetry is based on the fact that firms that already own a neighbor tract (the informed firms) have more informative signals than the signals that firms without any neighbor tract (the uninformed firms) get only

[3]This model was first introduced by Wilson (1967), and later studied by Weverbergh (1979) and Engelbrecht-Wiggan et al (1983).

from seismic surveys. This is because informed firms have private information from exploratory drilling on the neighbor tract. Since the values of tracts are expected to be spatially correlated, this drilling information is more precise than seismic survey information.

Following the notation in HP, we assume that, in each auction, there exists an informed bidder and n uninformed bidders. Focusing on an individual auction and suppressing the covariates, let V be the common value of the item being auctioned[4]. We assume that V is unknown to all bidders. All bidders have access to a public signal denoted by Z which is weakly correlated with V. Moreover, the informed bidder has additional private information about the item, X, which is more precise than Z. In other words, the private information of the informed bidder is sufficient for the public signal. The informed firm observes the realizations of X and Z prior to bidding, while uninformed firms observe only the realization of Z. The distributions, and the bidding strategies are dependent on Z, but this dependence is suppressed in the bidding strategies for the notational convenience.

The private information of the informed bidder is summarized by the real valued random variable $H = E[V \mid X, Z]$, which is assumed to have a continuous distribution $F(\cdot|z)$, with finite mean $\overline{H}$. We call H the private signal of the informed bidder. Let $\sigma(h)$ be the strategy of the informed bidder which maps the realizations of H into the nonnegative real numbers. $\sigma(h)$ is assumed to be differentiable, strictly increasing function on the range (R, ∞) ,where R is the reservation price. Also, let $\sigma^{-1}(b) = \tau(b)$ be the inverse bid function .

[4]We use capitals to denote random variables, and small letters for their realizations.

The strategy of uninformed bidder i is a distribution function $G_i(\cdot)$ over the nonnegative real numbers, and $G(b) = G_1(b) \cdots G_n(b)$ is the distribution function of the maximum of the bids submitted by n uninformed bidders.

Given the above assumptions, the problem of the informed bidder with a private signal h is,

$$\underset{\sigma}{Max} G(\sigma(h)) \cdot (h - \sigma(h))$$

First order optimality condition for the informed bidder gives the following:

$$h = \sigma + \frac{G(\sigma)}{g(\sigma)} \tag{2.1}$$

where $g(\sigma)$ is the pdf of the maximum bid of n uninformed bidders.

The problem of an uninformed bidder i, which submits a bid higher than R, is[5]

$$\underset{b}{Max} E[H - b \mid \tau(b) > h; z] \cdot F(\tau(b) \mid z) \cdot \prod_{j \neq i} G_j(b)$$

HP shows that the $(n+1)$-tuple $(\sigma^*, G_1^* \cdots G_n^*)$ is a Bayesian Nash equilibrium profile if and only if

[5]HP allows for possible differences in tract valuations or costs by defining the expected value of the drainage tract to the non-neighbor firm as $E[H \mid z] - c$, where c is a fixed, nonnegative constant. We assume this constant is zero for simplicity.

$$
(2.2) \qquad G^*(b) = \begin{cases} 1, & b > \overline{H} \\ F(\tau(b); z), & R < b < \overline{H} \\ F(\tau(R); z), & 0 \leq b < R \end{cases}
$$

$$
(2.3) \qquad \sigma^*(h) = \begin{cases} E[H \mid H \leq h; z], & h > \widehat{h} \\ R, & \widehat{h} \geq h \geq R \\ 0, & h < R \end{cases}
$$

where $\widehat{h}$ solves $E\left[H \mid H \leq \widehat{h}; z\right] = R$.

A zero bid is interpreted as no bid. It is important to point out the double advantage of the informed bidder against its competitors in order to understand the equilibrium strategies. The informed bidder not only has a more accurate estimate of the tract's value but also knows precisely what information his competitors posses. The randomization strategy of uninformed bidders is a consequence of the latter informational advantage of the informed bidder. If the uninformed bidder follows a pure strategy, since all the information known to it is also observed by the informed bidder, it would be predictable; hence it would be certain to lose on average. By randomizing, the uninformed bidder will earn positive expected profit on some tracts, and the expected profits are zero only on average. Equilibrium strategy of the informed bidder on $\left(R, \overline{H}\right]$ is uniquely determined by the condition that, in equilibrium, uninformed bidders must earn zero profits.

The equilibrium of the bidding game above gives some idea about the conditional distributions of the maximum informed and uninformed bids. HP also note that the supports of the equilibrium density functions of the informed bid $f_\sigma(\cdot)$ and maximum uninformed bid $g(\cdot)$ are identical and consist of $\{0\}$ and the interval $\left[R, \overline{H}\right]$. The two distributions differ in the probability of no bid (zero bid), and of the reservation bid R. $G^*(\cdot)$ possesses a mass point equal to $F\left(\widehat{h}\right)$ at $\{0\}$, and it is constant at this value on the interval $(0, R]$. The distribution of the neighbor bid F_σ^* also has a mass point at $\{0\}$, but it is equal to $F(R)$, which is less than $F\left(\widehat{h}\right)$. The distribution is constant at $F(R)$ on the open interval $(0, R)$, and then jumps discontinuously upward at R, the value of the mass point being $F\left(\widehat{h}\right) - F(R)$.

2.3. Structural Estimation

In this section we describe the structural estimation method for the model given in the previous section. We are interested in estimating the private signal of the informed bidder h and its distribution $F(.)$ given the bids of informed and uninformed bidders. To do this, we follow the approach in Guerre et al (2000) and propose a two step estimation method. The method is based on the first order condition of the informed bidder and does not require us to compute the nonlinear bid function nor its density. In the first step, we estimate the cdf and pdf of the maximum uninformed bid. We then plug their estimates in the first order condition of the informed bidder and obtain the estimate of the private signal of the informed bidder. Using these estimates, in the second step, we estimate the distribution of the private signal. The estimation method can be implemented both parametrically or nonparametrically. We explain in detail how to conduct the estimation for both cases.

2.3.1. Nonparametric Analysis

2.3.1.1. Identification. An important issue in any structural model is the identification of the model. We first discuss nonparametric identification of the model. Identification in this context amounts to establishing a one-to-one mapping between informed bid, σ, and private signal, h. To ensure this, we assume a sufficient condition that the cdf of the maximum uninformed bid, $G(.)$, is logconcave. This is satisfied by most of the commonly used distributions such as normal, exponential, uniform etc. The logconcavity of $G(.)$ implies that $\frac{G(\sigma)}{g(\sigma)}$is nondecreasing which in turn yields the right hand side of equation 2.1 strictly increasing. Thus, for each informed bid in the data there corresponds a unique private signal.

Nevertheless, the distribution of bids and the corresponding distribution of private signal are truncated. This is because R, the reserve price is a lower bound for bids and the corresponding private signal. To identify the distributions over their entire supports, one would need to estimate the truncation probability at R. In general, this could require additional information or structural assumptions. For example, in a private value environment with symmetrically informed bidders, one can use the total number of participating bidders to estimate the truncation probability at the reserve price, as suggested in Guerre, Perrigne, Vuong (2000). In our case, we do not need any other information nor to make other assumptions for this purpose. Knowing that there are at least one informed and one uninformed bidder in each auction suffices.

This is an advantage of our estimation method because in many applications of first price auctions with private value bidders, researchers assume a nonbinding reserve price to avoid the truncation issue. The structure of the model allows us to handle this straightforwardly.

On the other hand, an important assumption of the model necessary for the identification is that there is only one informed bidder at each auction. This makes the first order condition of the informed bidder similar to that of a bidder in a first price auction with private values. Therefore, the private signal of the informed bidder can be identified. In a symmetric common value auction environment, however, it is shown by Laffont and Vuong (1996) that private signals of bidders are not identified nonparametrically, since the first order condition of bidders changes due to winner's curse correction[6]. Because informed bidders know all the information available to uninformed bidders, winner's curse is not an issue for informed bidders. Hence, they do not account for winner's curse in their maximization problem and their private signals turn out to be identified. This is a very interesting point since, even though the symmetric common value model is not identified nonparametrically, the asymmetric common value model in this paper is identified. Should there exists more than one informed bidder at the auction, identification result holds no longer because informed bidders, in this case, should also correct for winner's curse that arises due to unobserved private information of other informed bidders.

2.3.1.2. Estimation. Regarding the estimation strategy, it also follows the steps of identification. In the first stage, we nonparametrically estimate the distribution

[6]See Hendricks et al (2003) for a symmetric common value auction environment with participation.

and density functions, $G(.)$ and $g(.)$, of the maximum uninformed bid using the uninformed bid data. Then, we plug in the informed bid, σ, and the estimated distribution and density, $\widehat{G}(.)$ and $\widehat{g}(.)$, of the maximum uninformed bid in equation 2.1 to estimate private signal of the informed bidder. Following Guerre et al (2000) we call the estimated private signals as *pseudo-signals*. In the second stage, this sample of private pseudo-signals is used to estimate nonparametrically the underlying distribution of h, $F(.)$. In this case, we assume that auctions are homogenous. However, the methodology can straightforwardly be extended to capture covariates by using conditional distributions of bids and the private signal.

We assume that there are T auctions, and for each auction $t = 1, ..., T$ there exist one informed bidder and n_t uninformed bidders. As a consequence of the equilibrium of the model, we are interested only in the maximum of the uninformed bids. Thus, we have T maximum uninformed bids, $\{b_t\}_{t=1}^{T}$, and T informed bids, $\{\sigma_t\}_{t=1}^{T}$. Also, let T_b be the number of maximum uninformed bids higher than or equal to R. In the first step, we use the uninformed bids, $\{b_t\}_{t=1}^{T}$, to estimate $G(.|b \geq R)$, $g(.|b \geq R)$, and the truncation probability $G(R)$ nonparametrically by the following empirical distribution and the kernel density estimators, respectively:

$$\widehat{G}(b|b \geq R) = \frac{1}{T_b} \sum_{t=1}^{T_b} I(b_t \leq b) \tag{2.4}$$

$$\widehat{g}(b|b \geq R) = \frac{1}{T_b h_g} \sum_{t=1}^{T_b} K_g \left(\frac{b - b_t}{h_g} \right) \tag{2.5}$$

$$\widehat{G}(R) = \frac{1}{T}\sum_{t=1}^{T} I(b_t = 0)$$

where h_g is a bandwidth and $K_g(\cdot)$ is a kernel with compact support. We then estimate the unconditional cdf and pdf of maximum nonneighbor bid by $\widehat{G}(b) = \widehat{G}(b|b \geq R).(1 - \widehat{G}(R))$ and $\widehat{g}(b) = \widehat{g}(b|b \geq R).(1 - \widehat{G}(R))$, respectively.

Moreover, it is well known that kernel estimators are biased close to the boundary of the support. So, if one works with distributions with bounded supports, one can trim the distribution of the maximum uninformed bid following Guerre et al (2000) to handle this issue. In particular, R and the maximum of the maximum uninformed bids, $b_{\max} = \max\{b_t\}_{t=1}^{T}$, can be taken as lower and upper bounds, respectively. Then, for the second stage, only signals corresponding to bids, that are away from the boundaries, are taken into consideration, as follows:

$$\widetilde{h}_t = \begin{cases} \sigma_t + \frac{\widehat{G}(\sigma_t)}{\widehat{g}(\sigma_t)} & \text{, if } R + h_g \leq \sigma_t \leq b_{\max} - h_g \\ +\infty & \text{,otherwise} \end{cases} \tag{2.6}$$

The sample of pseudo signals $\{\widetilde{h}_t\}_{t=1}^{T}$ is then used to estimate the density of the signals. A truncation issue as in the first stage of the estimation also exist in the second stage. The pseudo signals obtained from the FOC are higher than $\widehat{h}$, and in order to estimate the density of signals over its entire support we need to estimate the truncation probability at $\widehat{h}$. Since there exists at least one informed bidder in each auction, we can use the ratio of zero or reserve informed bids for this purpose.

Let T_h be the number of pseudo signals in the sample. Then we can use the following estimators:

$$\widehat{f}(h|h > \widehat{h}) = \frac{1}{T_h h_f}\sum_{t=1}^{T_h} K_f\left(\frac{h - \widetilde{h}_t}{h_f}\right) \tag{2.7}$$

$$\widehat{F}(\widehat{h}) = \frac{1}{T}\sum_{t=1}^{T_h} I(\sigma_t = 0)$$

where h_f is a bandwidth and K_f is a kernel with compact support again. We can then estimate the unconditional density of signals by $\widehat{f}(h) = \widehat{f}(h|h > \widehat{h}).(1 - \widehat{F}(\widehat{h}))$.Note that although we do not know the value of $\widehat{h}$ in advance, we can still carry out the estimation since it is sufficient to know whether a pseudo signal is greater than $\widehat{h}$.

2.3.2. Parametric Estimation

Having shown nonparametric identification and estimation, now we describe the parametric version of the estimation method. In most cases, auctions are heterogenous, and use of covariates is necessary to capture the heterogeneity. Due to the curse of the dimensionality in nonparametric estimation, parametric estimation can be attractive for heterogeneous data. Therefore, we explicitly include covariates in the parametric estimation. In the first stage, we estimate the distribution of the maximum uninformed bid conditional on the publicly available information, $G_t(.|z_t)$, by maximum likelihood. Then, we plug the informed bid as well as the estimated conditional distribution and density, $\widehat{G}_t(.|z_t)$ and $\widehat{g}_t(.|z_t)$, in equation 2.1 to estimate the signals.

In the second stage, we estimate the conditional distribution of signal by maximum likelihood again.

Suppose that $G_t(.|z_t)$ is a distribution with a parameter vector μ_t which is a function of the publicly available information , i.e. $\mu_t = \Omega(z_t)$ for all t. Because we assume a parametric form for $G_t(.|z_t)$, we can handle the truncation in the likelihood function. Denoting the maximum uninformed bids and covariates by $\mathbf{b}$ and $\mathbf{Z}$, respectively, we construct the following likelihood function:

$$\log L(\Omega|\mathbf{b}, \mathbf{Z}) = \sum_{t=1}^{T} [I(b_t > 0) \cdot \log(g_t(b_t|z_t)) + I(b_t \leq R) \cdot \log(G_t(R_t|z_t))]$$

where R_t is the announced reservation price in auction t. We get the estimates of the parameters of the distribution of uninformed bids as the following:

$$\widehat{\Omega} = \arg\max_{\Omega} \log L(\Omega|\mathbf{b}, \mathbf{Z}))$$

Now for each auction t, we can plug the estimates in equation 2.1 to find the pseudo signal of the informed bidder as follows:

$$\widetilde{h}_t = \sigma_t + \frac{\widehat{G}_t(.|z_t)}{\widehat{g}_t(.|z_t)}$$

Let T_h be the number of signals greater than $\widehat{h}_t$, as before. In the second stage of the estimation, we use these pseudo signals $\widetilde{h}_t$ for $t = 1, ..., T_h$ to estimate the distribution of H, conditional on the publicly available information. We basically follow the procedure in the first stage and assume that H has a parametric distribution with parameter β, which is a function of the public information as $\beta = \Gamma(z_t)$. Denoting the set of pseudo private signals by $\widetilde{\mathbf{h}}$, we estimate the parameters of this density by maximizing the following likelihood function:

$$\log L(\Gamma|\widetilde{\mathbf{h}}, \mathbf{Z}) = \sum_{t=1}^{T} \Big[I(\widetilde{h}_t > \widehat{h}_t) \log(f(\widetilde{h}_t|z_t)) + I(\widehat{h}_t \geq \widetilde{h}_t \geq R_t) \log(F(\widehat{h}_t|z_t) - F(R_t|z_t))$$

$$+ I(\widetilde{h}_t \leq R_t) \log(F(R_t|z_t)) \Big]$$

Note that contrary to nonparametric estimation, one needs to calculate $F(\widehat{h}_t|z_t)$ since it appears in the above likelihood function. However, $\widehat{h}_t$ is unknown before estimation. This could cause a problem depending on the data on hand. In practice, if no informed bidder submitted the reserve price[7], then one could carry out the estimation without problem. Such a situation may occur if, for example, the reserve price is too low or nonbinding. Otherwise, one would need to make additional assumptions. Depending on the data and the environment, one assumption could be that R_t and $\widehat{h}_t$ are very close so that the second term in the likelihood function is negligible. Recall that we do not have this problem in nonparametric estimation which is another advantage of it.

We get the estimates of the parameters of the distribution of private signals of informed bidders as the following:

[7]For example, this is the case in the data we use for the empirical application in the next chapter.

$$\widehat{\Gamma} = \arg\max_{\Gamma} \log L(\Gamma|\widetilde{\mathbf{h}}, \mathbf{Z}))$$

2.4. Monte Carlo Experiments

In this section we demonstrate the performance of our two step approach with two experiments. In the first experiment we use non-parametric estimators, in the second one we use parametric MLE estimators.

2.4.1. Non-parametric Monte Carlo

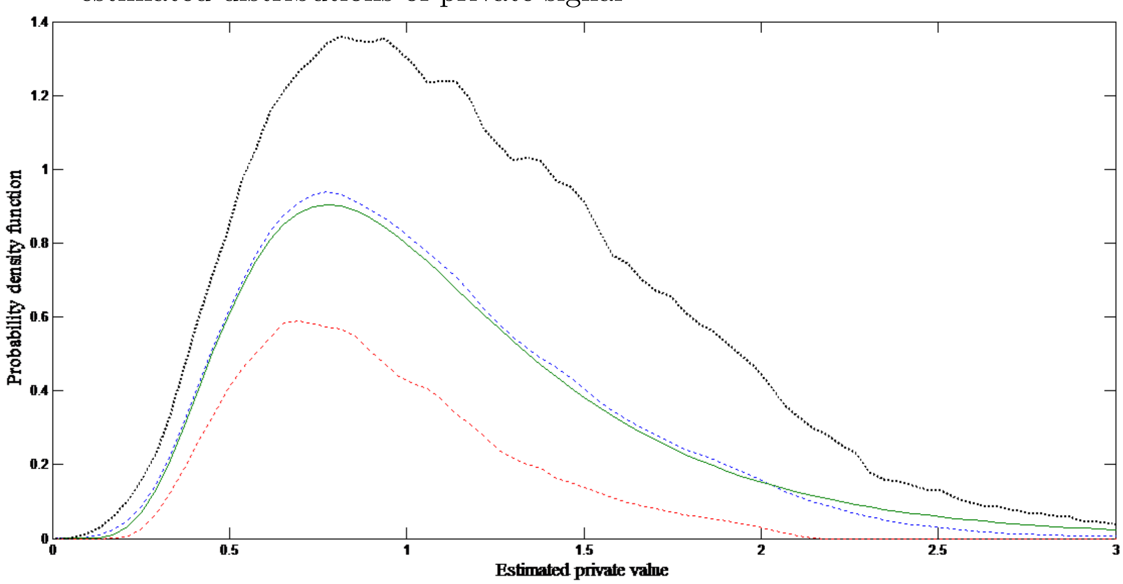
In the first experiment in which we tested the non-parametric version of our two step estimator, we assume that there are no covariates and auctions are homogenous. We consider $L = 400$ auctions, which give us 400 informed and 400 uninformed bids to estimate the distribution of the private signal. Our Monte Carlo experiment consists of 1000 replications. The true distribution F of private signals of the informed bidders is lognormal with location and scale parameters of 0 and 0.5,respectively. We assume that the reservation price is 0.1. For each replication, we first generate L private signals from this distribution. We then generate the bids of the informed and uninformed firms using the equilibrium bid functions 2.2 and 2.3.

Next, we apply the non-parametric version of our estimation procedure for each replication. First we estimate the distribution and density functions of the uninformed bid using 2.4 and 2.5, respectively. Second, we compute the pseudo private signals following 2.1. Finally, from this pseudo private signal data we estimate the density function of the private signal using 2.7.

As for the kernels used in estimation, we follow Guerre et al (2000). For K_g and K_f we choose the triweight kernel such that $K_g(u) = K_f(u) = \frac{35}{32}(1-u^2)^3$. The bandwidths are chosen as $h_g = 1.06\widehat{s}_b T_b^{-\frac{1}{5}}$ and $h_f = 1.06\widehat{s}_h T_h^{-\frac{1}{5}}$ where $\widehat{s}_b$ and $\widehat{s}_h$ are the estimated standard deviations of bids and pseudo signals, respectively. The factor 1.06 comes from the so-called rule of thumb (Hardle,1991).

The simulation program is written in MATLAB. In each replication we get three estimated functions: the density and the distribution functions of the uninformed bid and the density function of the private signal. Figure 2.1 displays the true density of the private signal in plain line. For each evaluation point, the mean, 5% percentile and 95% percentile of the 1000 estimates of $\widehat{f}(\cdot)$ are plotted.

Figure 2.1. MC experiment with non-parametric estimation: true and estimated distributions of private signal



As seen in Figure 2.1, the true density lies between the 5% and 95% percentiles all over its support. Moreover, the mean of the estimated densities follow the true density very closely, with a slight bias. We conclude that our non-parametric two stage estimator works fairly well in estimating the private signal distribution[8].

2.4.2. Parametric Monte Carlo

In the second experiment we applied the parametric MLE estimators as explained before. We run the experiment in OCS drainage auctions setting for which we have data of covariates for the auctioned objects, which are the tracts in this case. We proxy the private signal of the informed firm with the ex-post profitability of the tract, which is denoted by Π. This is defined as the discounted revenues less drilling costs and less royalty payments. Our proxies for the public information variables, are the average of the ex-post profitability of the tract (Π) and of neighbor tracts (V), which is denoted by val[9]; the number of neighbor tracts (N); and tract acreage (a). Therefore, we approximate Z_t with the following

$$Z_t' = (1, val_t, N_t, a_t)$$

[8]It is important to note that, our estimates are reliable in a certain interval on the support. This is due to the fact that we are estimating a bounded distribution for the uninformed bid in the first stage, hence the parts of the distribution close to the boundaries are estimated with a bias (points outside the interval $[\mathrm{R}+h_g, b_{\max}-h_g]$.) In the second stage estimation, we exclude the values of the pseudo private signals which correspond to these biased points coming from the first stage, as seen in Equation 2.6. Hence, estimates of the private value distribution outside the interval $\left[\widetilde{h}(\mathrm{R}+h_g)+h_f\ ,\widetilde{h}(b_{\max}-h_g)-h_f\right]$ are not reliable. In our Monte Carlo experiment, the average of end points of this interval over the 1000 simulations gives us $[0.3291, 2.3591]$.

[9]We found this average more explanatory than the average ex-post value of the neighbor tracts for the bidding behavior of the non-neighbor firms. It is possible that the profitability of the neighboring tracts are not yet revealed at the time of the auction, yet the non-neighbor firms might be getting more information about the tract from other resources which are not covered by the variables included in this study.

In this experiment we assumed tract specific characteristics are also stochastic. For each of the covariates used in the experiment, we simply determined their underlying distributions and dependency structure among them, then took random draws in accordance with their stochastic nature. For val, we fitted a normal distribution to determine the mean and variance. For N and a[10], we simply found their empirical distributions. We tested for correlations, but we could not find significant relationship between the covariates.

For the parametric Monte Carlo experiment, our sample size is $L = 200$ and the simulation size is $S = 1000$. So, we simulate 1000 data samples of size 200.

We assume that the underlying distribution of the private signal H conditional on the publicly available information Z_t is lognormal, as $F_t(h_t|Z_t) = \log normal(\mu_{2t}, \sigma_t^2)$ where μ_t is a linear function of the public information variables, and σ^2 is a constant as the following:

$$\mu_t = \alpha_2' Z_t = \alpha_{21} + \alpha_{22} val_t + \alpha_{23} N_t + \alpha_{24} a_t, \qquad \sigma_t^2 = \beta_2 \tag{2.8}$$

So, for tract t, we took random draws from the underlying distributions of the covariates val, N and a , and we assigned values to the parameters $(\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \beta)$[11]. Then we calculate the parameters (μ_{2t}, β_2) of the lognormal distribution $F_t(h_t|Z_t)$,

[10]Acreage takes several different values in our data sample. We determined the values that occure most often, and built a discrete support form those.We found the empirical distribution of acreage on this discrete support.

[11]We assigned the parameters α and β the values we got from the estimation of the same econometric model with data from OCS auctions. We assigned the same parameter values for each tract and for each simulation.

according to the equation 2.8. After this, we drew a random private signal from this distribution and applied equations 2.2 and 2.3 to get the simulated uninformed and informed bids, b_t and σ_t, for tract t. We repeat this for $t = 1, ..., L$, and obtain one simulated sample of size L. Finally, we replicate this simulation 1000 times to obtain 1000 simulated samples.

For each of the simulated samples in our experiment, which consisted of L uninformed and L informed bids, we applied our two stage estimator explained in the previous section, to see if it is able to recover the initially assumed parameters of the private signal distribution $F(h|Z)$. At the first stage, the distribution of the uninformed bids, $G(b|Z)$ is assumed to be

$$G_t(b_t|Z_t) = \log normal(\mu_{1t}, \sigma_{1t}^2)$$

where $\mu_{1t} = \alpha_1' Z_t = \alpha_{11} + \alpha_{12} val_t + \alpha_{13} N_t + \alpha_{14} a_t \;\; and \; \sigma_{1t}^2 = \beta_1$.

We use the simulated uninformed bids b_t and public information Z_t in a maximum likelihood estimation to obtain the parameters $(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}, \beta_1)$ of $G(b|Z)$, hence $\widehat{G}$. Then, we plug $\widehat{G}$ and $\widehat{g}$ in equation 2.1 to obtain the pseudo private signals $\widetilde{h}$. Finally, using these pseudo private signals, we estimate the distribution of the private signal $F(h|Z)$ by MLE, assuming a lognormal specification with parameters linearly dependant on the public information Z as the following:

$$F_t(b_t|Z_t) = \log normal(\mu_{2t}, \sigma_{2t}^2)$$

where $\mu_{2t} = \alpha_2' Z_t = \alpha_{21} + \alpha_{22} val_t + \alpha_{23} N_t + \alpha_{24} a_t \;\; and \; \sigma_{2t}^2 = \beta_2$.

Finally, we take the average of the estimates from the 1000 simulated data samples to arrive at the ultimate estimates of the parameters of the distribution of private signal.

Table 2.1 gives the results of our parametric Monte Carlo experiment. We present the mean of the estimated parameters of the private signal distribution from the 1000 simulated samples and the mean squared errors (MSE's) of estimations for sample sizes of 50, 100, and 200 to get an idea about the asymptotic performance of our estimator.

Table 2.1. Results of Monte Carlo experiment with parametric estimation

$F_t(b_t \mid Z_t) = \log normal(\mu_{2t}, \sigma^2_{2t})$, $\mu_{2t} = \alpha_{21} + \alpha_{22} val_t + \alpha_{23} N_t + \alpha_{24} a_t$ *and* $\sigma^2_{2t} = \beta_2$						
	Sample size	α_{21}	α_{22}	α_{23}	α_{24}	β_2
True parameters		7.4816	0.0237	0.4791	0.8528	5.3069
Estimated parameters	50	7.423	0.0237	0.476	0.8879	5.1239
	100	7.4470	0.0243	0.486	0.8439	5.2351
	200	7.4091	0.0235	0.494	0.8497	5.2896
MSE	50	25.8833	0.0005	0.4046	0.4183	0.2848
	100	11.6398	0.0002	0.1822	0.1816	0.1457
	200	5.7742	0.0001	0.092	0.0845	0.0679

As seen in Table 2.1, our two stage parametric estimator delivers pretty good results. Estimated values of the parameters are fairly close to the true parameter values. Moreover, the MSE of estimation decreases significantly when we increase the sample size in our simulations. Hence, we conclude that our two stage estimator is performing better in larger samples.

2.5. Conclusion

In this chapter, we show the identification and estimation of the private signal of the informed bidder in a common value asymmetric information auction environment. The critical assumptions in the model which leads to the identification result are the single informed bidder's signal being sufficient for the signal of the uninformed bidders. By this way, winner's curse correction does not interfere in his bid function, and his private signal is identified.

We propose a two stage estimation method, which follows the identification of the private signal. We test the performance of this estimator for non-parametric and parametric cases with Monte Carlo experiments. In the non-parametric case, the true density lies between the 5% and 95% percentile lines all over the support, and our estimator performs well. In the parametric case, we can estimate the true parameters very closely, with a decreasing mean square error as we increase the sample size. So, our estimator performs better in terms of mean square error as the sample size increases.

CHAPTER 3

An Empirical Application for OCS Drainage Leases

3.1. Introduction

In this chapter we introduce the case of OCS Drainage Leases as an example of the asymmetric information common value auction model, based on the work of Hendricks and Porter (1988) (HP hereafter). The main purpose of this chapter is to apply the estimation method developed in the previous chapter to estimate the private signals of the informed bidders in the OCS drainage lease auctions, who are the neighbor bidders in this case,and the underlying distribution for them. Since the characteristics of the tracts and the other parameters at each auction are different, we have to use covariates to account for these heterogeneities. As a result, we find it more convenient to use the parametric version of the two stage estimator proposed in the previous chapter.

After estimating the underlying distribution of the private signals, we conduct a counterfactual experiment for a symmetric information case. We simulate the bidding game for the existing case of asymmetric information, in which there is one neighbor bidder (who is more informed about the value of the auctioned drainage tract), and a number of other non-neighbor bidders who have less precise information. We follow the bidding model and equilibrium in HP in doing this. On the other side, we simulate another bidding game where each bidder receives a signal about the value of the tract from the same distribution as the informed bidder, hence a symmetric value environment. We base this simulation on the equilibrium of the symmetric information common value bidding game introduced by Smiley (1979). Smiley (1979) shows that under certain conditions, the equilibrium bid function of this bidding game is proportional to the signal. This result proves to be very practical for simulation purposes.

We compare the results of the two different informational structures on the auction revenue of the government. We find that the government benefits more from the asymmetric informational structure among the bidders, when the tract value is low. On the other hand, the probability that a symmetric information environment results in higher government revenue is higher for high value tracts.

In section 2, we introduce the OCS drainage lease auctions and the data we have. In section 3, we estimate the asymmetric information model and present the results. In section 4, we discuss the counterfactual case of a symmetric information model and we compare the effects of the two informational structures in terms of the government's auction revenue. Section 5 concludes this chapter.

3.2. OCS Mechanism and the Data

In this section, we apply the proposed estimation procedure to data of the federal lands off the coasts of Louisiana and Texas which were leased between 1959 and 1973. Although we have data for a longer period, we restricted our study to this period since some structural changes occurred in 1974[1]. In the sample each lease is sold via

[1]In 1974 the eight biggest oil firms are banned from submitting joint bids. Moreover, prices get more volatile starting from 1974.

a first price sealed bid auction. The winning bidder pays the amount he bid at the time of the sale, which is called a *bonus payment*. One sixth of the revenues from any oil or gas extracted accrue to the government on an annual basis as a *royalty payment*. A nominal rental fee is paid by the firm each year until either the lease expires or production begins. If no exploratory work is done after five years have elapsed, ownership of the lease reverts to the government. If sufficient amount of oil and/or gas is discovered so that the firm begins producing, the lease is automatically renewed for as long as it takes the firm to extract the hydrocarbons.

In the sample the reservation price that the government announced was $25 per acre on most drainage leases. Additionally, it has the right to reject the high bid, if it finds the bid too low compared to its own private estimate of the value of the tract. The high bid was rejected in around %16 of the drainage tracts[2].

For each tract in our dataset, we have the information on tract number, the date it was auctioned. whether it was a wildcat or drainage tract, its location, tract acreage, submitted bids, names of the bidder firms, its neighbor tracts; the firms which have previously owned those neighbor tracts, the number and date of any wells that were drilled; and annual production through1980 if any oil or gas was extracted

The data set also includes the ex-post tract values calculated by HP for each tract[3].

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[2]HP uses the data from the auctioned drainage tracts between 1959 and 1969. High bid in 16% of the drainage tract auctions in their data set was rejected. We assume there would be a slight difference in our case since we use a larger data set which includes HP's.

[3]Future production paths were converted into revenues by using the real wellhead prices at the date of sale and discounting to the auction date at a 5% per annum rate. Cost data was obtained from American Petroleum Institute, then subtracted from discounted revenues. See Hendricks, Porter and Boudreau (1987) for details about how the ex-post values were calculated.

HP indicates several facts against competitive bidding. In their data, HP found 74 tracts with multiple informed firms, but only 17 tracts had multiple informed bids. Moreover, net profits were not significantly lower on tracts with multiple informed firms than on tracts with one informed firm, and the bid of informed firms were strictly decreasing in the number of informed firms. Finally, the informed firms have large positive profits implying large information rents, most of which would most probably be eliminated if there were competition among informed bidders. All these facts are also supported by the absence of a law prohibiting firms from forming a bidding consortium. Therefore,on tracts where there are more than one informed bid submitted, the maximum of these is taken as the coordinated bid of the informed firms.

In our study, we include 69 tracts which were auctioned in drainage sales during this period by the government and which have positive ex-post value as calculated by HP. From the sample of 69 drainage tracts, 49 has more than one neighbor firm. However, only 6 of these 49 tracts received more than one neighbor bid throughout the sample period, confirming the finding of HP regarding the coordinated bidding behavior of the neighbor firms. Therefore, we focus on the model with one informed firm and an arbitrary number of uninformed firms, as in HP Table 3.1 gives a summary of the data that is used in estimation of the parameters of the model.

Table 3.1. Summary statistics

	Mean	**Std. Dev.**	**Min.**	**Max.**
σ: maximum bid by neighbor	18.25	35.43	0	236.29
b: maximum bid by non-neighbor	14.77	26.08	0	179.89
N: number of neighbor tracts	7.39	1.3	4	9
Π: ex-post tract profitability	52.6	66.57	0.011	295.98
V: average ex-post profit of neighbor tracts	32.37	47.08	-6.65	194.51
a: tract acreage	2,530	1,337	0,625	5,544

Dollar figures are in millions of 1972 dollars.

3.3. Estimation

In the following estimation, we proxy the private signal of the neighbor firm with the ex-post profitability of the tract, which is denoted by Π. This is defined as the discounted revenues less drilling costs and less royalty payments. Our proxies for the public information variables, since we do not have the seismic survey data, are the average of the ex-post profitability of the tract (Π) and of neighbor tracts (V), which is denoted by val[4], the number of neighbor tracts (N), and tract acreage (a). Therefore, we approximate Z_t with the following

$$Z_t' = (val_t, N_t, a_t)$$

In the first stage of estimation, we assume that b_t has lognormal distribution conditional on the publicly available information, as the following

$$G_t(b_t|Z_t) = \log normal(\mu_{1t}, \sigma_{1t}^2), where\ \mu_{1t} = \alpha_1' Z_t\ and\ \sigma_{1t}^2 = \beta_1$$

[4]We found this average more explanatory than the average ex-post value of the neighbor tracts for the bidding behavior of the non-neighbor firms. It is possible that the profitability of the neighboring tracts are not yet revealed at the time of the auction, yet the non-neighbor firms might be getting more information about the tract from other resources which are not covered by the variables included in this study.

with $\mu_{1t} = \alpha_1' Z_t = \alpha_{11} + \alpha_{12} * val_t + \alpha_{13} * N_t + \alpha_{14} * a_t$ is the log-scale parameter and β_1 is the shape parameter of the lognormal density.

Looking at the equilibrium strategies, we see that $G(\cdot)$ is a piecewise distribution function. Therefore, to estimate the parameters of $G(\cdot)$ we maximize the following likelihood function:

$$\log L_1(\alpha_1, \beta_1 | \mathbf{b}, \mathbf{Z}) = \sum_{t=1}^{T} [I(b_t > 0) \cdot \log(\phi(b_t | Z_t)) +$$

$$I(b_t \leq 0) \cdot \log(\Phi(res_t | Z_t))]$$

where ϕ and Φ are the normal density and distribution functions respectively, and res_t is the reservation price used by the government at the auction for tract t. We get the estimates of the parameters of the distribution of uninformed bids, $G(\cdot)$, as the following:

$$\{\widehat{\alpha}_1, \widehat{\beta}_1\} = \arg\max_{\alpha_1, \beta_1} \log L_1(\alpha_1, \beta_1 | \mathbf{b}, \mathbf{Z})$$

Now that we have the maximum likelihood estimators $\widehat{G}(\cdot)$ and $\widehat{g}(\cdot)$ of $G(\cdot)$ and $g(\cdot)$ respectively, we can plug them in equation 2.1 to find the *pseudo signal* of the informed bidder at tract t as the following:

$$\widetilde{h}_t = \sigma_t + \frac{\widehat{G}(\sigma_t)}{\widehat{g}(\sigma_t)}$$

At the second stage of the estimation, we use these pseudo signals $\widetilde{h}_t$ for all tracts $t = 1, ..., T$ to estimate the distribution of H conditional on the publicly available information. We basically apply the same estimation method as in the first stage and assume that H has a lognormal distribution as the following:

$$F_t(h_t|Z_t) = \log normal(\mu_{2t}, \sigma_{2t}^2), where\ \mu_{2t} = \alpha_2' Z_t\ and\ \sigma_{2t}^2 = \beta_2$$

where $\mu_{2t} = \alpha_2' Z_t = \alpha_{21} + \alpha_{22} * val_t + \alpha_{23} *\ N_t + \alpha_{24} * a_t$ is the log-scale parameter and β_2 is the shape parameter of the lognormal density. We estimate the parameters of this density by maximizing the following likelihood:

$$\log L_2(\alpha_2, \beta_2|\widetilde{\mathbf{h}}, \mathbf{Z}) = \sum_{t=1}^{T} \Big[I(\widetilde{h}_t > \widehat{h}_t) \cdot \log(\phi(\widetilde{h}_t|Z_t)) +$$

$$I(\widetilde{h}_t \leq res_t) \cdot \log(\Phi(res_t|Z_t)) \Big]$$

The estimates for the parameters α_2 and β_2 of the distribution of H are obtained as the following:

$$\{\widehat{\alpha}_2, \widehat{\beta}_2\} = \arg\max_{\alpha_2, \beta_2} \log L_2(\alpha_2, \beta_2|\widetilde{\mathbf{h}}, \mathbf{Z})$$

Table 3.2 shows the effect of the selected public information variables on parameters of the lognormal distribution of non-neighbor bids. While average of the ex-post values and the acreage of the tract have positive effects on the location parameter

μ, we see that as the number of neighbor tracts increase, the location parameter decreases. However, number of neighbor tracts seems to be an insignificant variable for explaining the location of the non-neighbor bid distribution. Acreage of the tract has a positive effect, implying that the bigger the tract, the more value is expected from that tract. Moreover, the coefficient of the acreage is significant at 1% level, while the variable val is significant at 10%. The scale parameter σ^2 is estimated to be highly significant and pretty large.

The estimated mean and standard deviation of the log-normal distribution of the non-neighbor bids for a tract with average values of val, N and a are \$10.64 millions and \$123 millions respectively.

Table 3.2. Results of first stage estimation

$\mu_{1t} = \alpha_1' Z_t = \alpha_{11} + \alpha_{12} * val_t + \alpha_{13} * N_t + \alpha_{14} * a_t$, $\sigma_{2t}^2 = \beta_1$		
variable	**estimated parameter**[5]	**std. dev.**
constant	14.81^{***}	1.58
val	0.012^{*}	0.007
N	-0.23	0.22
a	0.58^{***}	0.22
$\boldsymbol{\sigma_2^2}$	2.21^{***}	0.21
Log Likelihood	-145.05	

Table 3.3 gives the results from the second stage, in which we estimate the distribution of the private signal using the pseudo private signals calculated at the end of the first stage.

We obtained the standard deviations of the parameters estimated at the end of the second stage by bootstrap.

Table 3.3. Results of second stage estimation

$\mu_{2t} = \alpha_2' Z_t = \alpha_{21} + \alpha_{22} * val_t + \alpha_{23} * N_t + \alpha_{24} * a_t$, $\sigma_{2t}^2 = \beta_2$		
variable	**estimated parameter**	**std. dev.**
constant	7.48	5.29
val	0.024	0.019
N	0.48	0.7
a	0.85*	0.44
$\boldsymbol{\sigma_2^2}$	5.31***	0.51
Log Likelihood	−158.38	

We see that all the variables have positive effect on the location parameter of the log-normal distribution of the private signal, although only the acreage is weakly significant. The estimate for the scale parameter is significant with a high value.

Next section gives the setting of a symmetric informational structure and investigates the effects of informational symmetry on the results of the bidding game.

3.4. A counterfactual simulation

3.4.1. The Symmetric Information Common Value Auction Model

In this part, we introduce the equilibrium of the bidding game in an environment where all bidders receive signals from the same signal distribution as the neighbor bidders do, i.e. symmetric information environment, as presented by Smiley (1979). Then, we estimate distribution of the signal conditional on the value of the tract v, $F(h|v)$, whose parameters are in the equilibrium bid function.

Smiley (1979) has found that, under certain conditions, the equilibrium bid function of the symmetric information common value bidding game, s, is proportional to the signal h as in

$$s(h) = \rho \cdot h \tag{3.1}$$

where the proportionality constant ρ depends on the parameters of the distribution of h as well as the amount of competition. This result is especially useful in empirical applications like this one because of the simplicity of calculating the equilibrium bid function[6]. To obtain this result, Smiley (1979) gives the sufficient conditions upon $F(h|v)$ (the distribution of the signal conditional on value of the tract) and $k(v)$ (the prior belief of bidders about the distribution of the value of the tract) as the following:

$$F(\lambda h|\lambda v) = F(h|v) \text{ for } \lambda > 0 \tag{3.2}$$

and

$$k(v) \propto \frac{1}{v^2} \tag{3.3}$$

The homogeneity condition upon $F(h|v)$ defines the way the distribution function shifts as a result of shifts in v. The condition on the prior $k(v)$ says that it is a neutral prior.

[6]As shown in Paarsch (1992), the equilibrium bid function of the most general specification of the common value symmetric information bidding game is a highly nonlinear function.

Smiley (1979) has also shown that Gumbel, Log-Normal and Weibull specifications for $F(h|v)$ satisfy condition 3.2, and lead to equilibrium bid function in the form of 3.1 under condition 3.3.

For the purpose of our symmetric information counterfactual simulation, we adopt the Log-Normal specification for $F(h|v)$. Therefore, following Smiley (1979), we assume the signal has a density function conditional on the value of the tract as in the following:

$$f(h_i|v) = \frac{1}{h_i \sigma_3 \sqrt{2\pi}} \exp\left[-\frac{(\ln h_i - \ln v - \mu_3)^2}{2\sigma_3^2}\right]$$

Since Smiley (1979) assumes that $E(h_i|v) = v$, μ_3 and σ_3 are related as in

$$\mu_3 = -\frac{\sigma_3^2}{2} \tag{3.4}$$

Consequently, it is shown that the proportionality constant ρ in the equilibrium bid function 3.1 is equal to

$$\rho = \frac{e^{\sigma_3^2/2} \int_0^\infty \frac{\ln u}{u} \exp(-\ln^2 u)[1 + D(\ln u)]^{nb-1} du}{\int_0^\infty u^{(\sigma_3\sqrt{2}-1)} \ln u \exp(-\ln^2 u)[1 + D(\ln u)]^{nb-1} du} \tag{3.5}$$

where $D(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$, and nb is the number of bidders in the auction.

In the parametric estimation section, we have estimated the parameters of $F(h|Z)$,the distribution of H conditional on the public signal Z. Here, we need to estimate the parameter σ_3^2 of $F(h|v)$, the distribution of H conditional on the value of the tract, in order to find the value of the proportionality constant ρ and use it in equation 3.1 to get equilibrium bids under the symmetric information scenario.

In estimating the parameters of the distribution $F(h|v)$, we once again use the pseudo private signals $\widetilde{h}$, which were calculated at the end of the first stage of parametric estimation of $F(h|Z)$. As the empirical counterpart of the value of the tract v, we use the ex-post profitability variable Π. We maximize the following likelihood function:

$$\log L_3(\mu_3, \sigma_3 | \widetilde{\mathbf{h}}, \mathbf{\Pi}) = \sum_{t=1}^{T} \Big[I(\widetilde{h}_t > 0) \cdot \log(\phi(\ln \widetilde{h}_t | \Pi_t)) +$$

$$I(\widetilde{h}_t \leq 0) \cdot \log(\Phi(\ln res_t | \Pi_t)) \Big]$$

subject to the condition in equation 3.4.

The estimates for the parameters σ_3 and μ_3 of $F(h|v)$ are obtained as the following:

$$\widehat{\sigma}_3 = \arg\max_{\widehat{\sigma}_3} \log L_3(\sigma_3, -\frac{\sigma_3^2}{2} | \widetilde{\mathbf{h}}, \mathbf{\Pi}), \quad \widehat{\mu}_3 = -\frac{\widehat{\sigma}_3^2}{2}$$

σ_3 is estimated to be $\widehat{\sigma}_3 = 3.0292$ with a standard error of 0.1314. It is plugged in Equation 3.4 to get $\widehat{\mu}_3 = -4.5879$.

3.4.2. Simulation of the bidding game under two informational structures

Next, we simulate bidding game in symmetric and asymmetric information environments separately to compare the government's revenue under two different informational structures. In the OCS drainage lease auction setup, symmetric information corresponds to a case where all bidders receive the private signal of the neighbor bidder for the auctioned tract. Asymmetric information environment corresponds to the real case of OCS drainage auctions, which include both neighbor and non-neighbor bidders.

3.4.2.1. Symmetric Information. We first simulate the symmetric information case. For each tract t in our sample of data, we calculate the proportionality constant ρ_t by plugging $\widehat{\sigma}_3^2$ and number of bidders for tract t, nb_t in equation 3.5. Then we simulate 5000 symmetric information winning bids. The symmetric information is signaled in the form of the ex-post profitability of the tract, same as the private signal of the neighbor bidder in the asymmetric information case.

To simulate 5000 winning bids for tract t, first we draw n_t signals from $F(h|v)$ with the estimated parameters $\widehat{\sigma}_3 = 3.0292$ and $\widehat{\mu}_3 = -4.5879$, using the ex-post profitability Π_t. Next, we multiply these n_t draws of the signal with the proportionality constant ρ_t to end up with n_t symmetric information bids for tract t. We find the highest of the n_t bids and record it as the winning bid, hence the government revenue, for tract t. We repeat this procedure 5000 times to simulate 5000 winning bids for tract t. Finally, we take the average of these 5000 winning bids to record as the simulated symmetric information government revenue for tract t. We do this for every tract in our sample.

3.4.2.2. Asymmetric Information. As for the asymmetric information case, for each tract t in our sample of data, we first simulate 5000 highest neighbor and 5000 highest non-neighbor bids using the estimated parameters in tables 2 and 3 respectively. More specifically, using the values of the covariates for tract t and estimated coefficients of those covariates, we calculate the parameters of the non-neigbor bid distribution $(\widetilde{\mu}_1, \widetilde{\sigma}_1)$ and the private signal distribution $(\widetilde{\mu}_2, \widetilde{\sigma}_2)$. Then we draw values from $\log normal(\widetilde{\mu}_1, \widetilde{\sigma}_1)$ until we get 5000 simulated non-neighbor bids which are at most as large as the highest non-neighbor bid in our data sample[7].

In simulating the neighbor bids, we draw 5000 values from the estimated distribution of the private signal, which are not greater than the highest pseudo private signal $(\widetilde{h})$ we calculate at the end of the first stage of estimation[8].Then, we plug these values in 2.3.

We end up having 5000 pairs of highest neighbor and non-neighbor bids. For each pair, we take the higher one and record it as the government revenue. Finally, we take the average of these 5000 government revenues and record it as the government revenue in asymmetric information case for tract t. We do this for all the tracts in our sample.

One important thing to notice is that, the σ_3 parameter in ρ (equation 3.5) accounts for the uncertainty in the signal distribution. As shown in Smiley (1979), $\lim\limits_{\sigma_3 \to 0}$

[7]The reason for the truncation is explained in the following paragraphs.
[8]The reason for the truncation is explained in the following paragraphs.

$\rho(\sigma_3, nb) = 1$, hence, when $\sigma_3 = 0$, the bidder bids the signal he receives. When σ_3 is high, hence the precision of the signal is low and it is not trustworthy, the bidder cuts down his bid significantly. For example, for a tract with three bidders (the average number of bidders on a tract in our sample is 3.5), $\widehat{\sigma}_3 = 3.0292$ results in an equilibrium bid of 1% of the signal received.

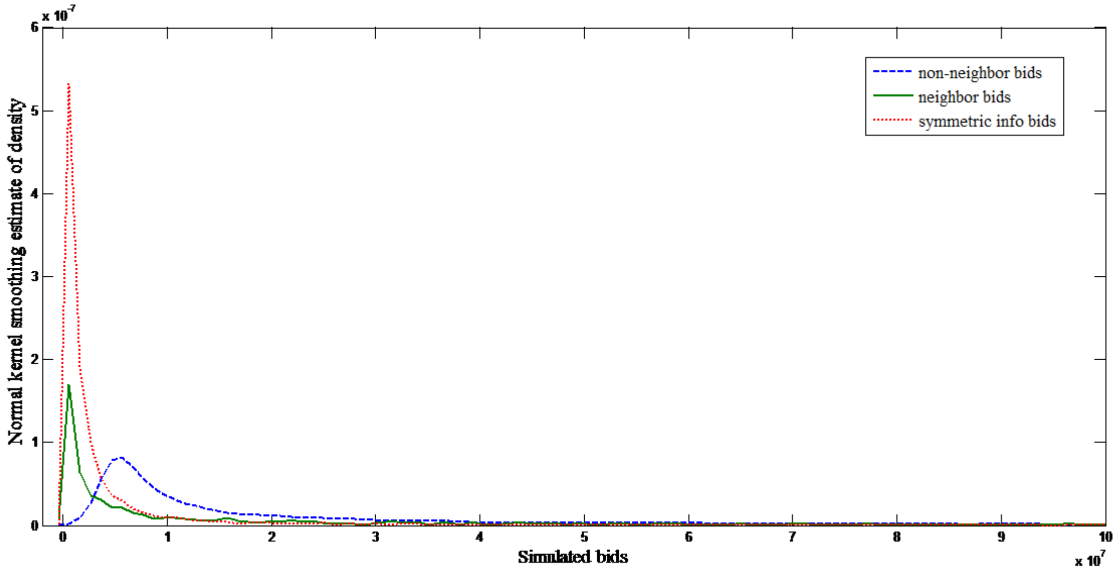
However, bid functions in the asymmetric information case in equation 2.3 and especially equation 2.2 do not have the direct mechanism to respond to the level of uncertainty in the signal distribution as we have in ρ. Moreover, the variances of the non-neighbor bid and the signal distributions are estimated to be very high. Consequently we draw many values from the tails of these distributions. As a result, asymmetric information simulation for the government revenue gives abnormally high revenue levels, which are not quite comparable with what we have in our data and what we get from the symmetric information simulation.

As a result, we found it necessary to draw from the trimmed non-neighbor bid and trimmed private signal distributions for the asymmetric information case. We accept the simulated non-neighbor bids and signals only if they are lower than or equal to their highest sample counterparts.

3.4.3. Discussion of experiment results

Results with trimming are quite interesting. In 17 of the 69 tracts, the simulated government revenue in symmetric information environment is higher than that in the asymmetric information environment. 14 of these 17 tracts have an ex-post profitability higher than the average ex-post profitability of the tracts in our data set,

Figure 3.1. Simulated bids for a tract with value $16.3 million



which is $52.6 million in 1972 dollars.(Only 25 tracts have ex-post profitability above the average) This tells us that high valued tracts result in higher government revenue with a higher probability when all bidders are informed, whereas low valued tracts benefit the government ,more often when there are both informed and uninformed bidders in the auction.

In the case of low valued tracts, when all bidders are informed, the winning bid hardly goes above the value of the tract. Less informed bidders in the asymmetric information simulation are driving the winning bid up above the symmetric information government revenue.

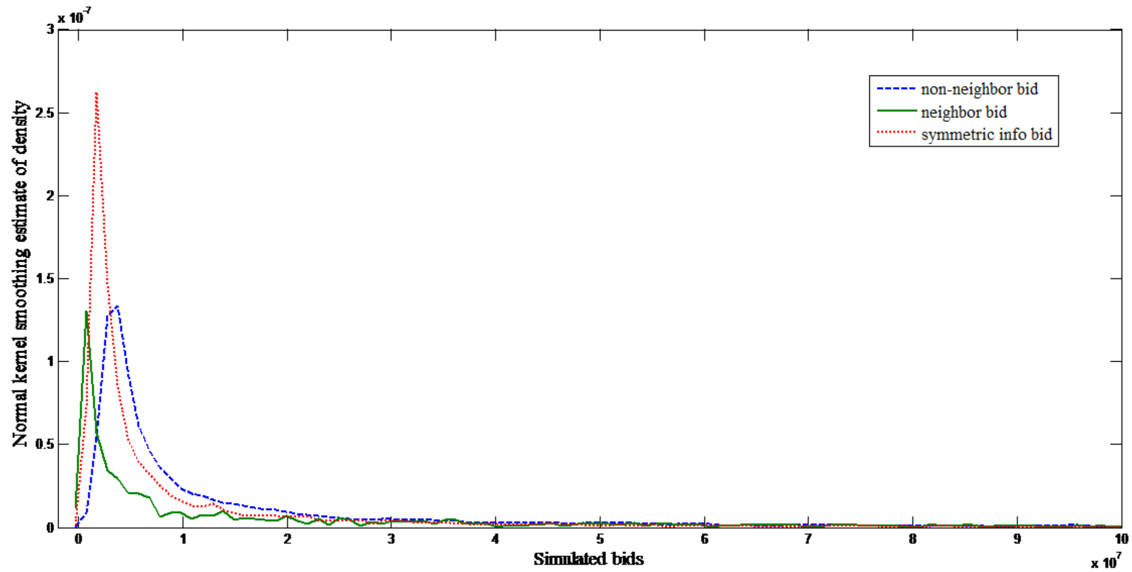
The average value of the tracts in the data set is $52.6 million in 1972 dollars. Figure 3.1 depict the kernel smoothing density estimates of the neighbor and non-neighbor bids in the asymmetric information simulation and the bids in the symmetric information simulation for a tract with a value of $16.3 million.

As seen in the Figure 3.1, the non-neighbor bids of the asymmetric information simulation have a higher mean (the sample mean of the 5000 simulated non-neighbor bids is \$116.8 millions), with a heavier tail and higher variance (the sample standard deviation of the non-neighbor bids is approximately ten times higher than that of neighbor bids) when compared to the neighbor bids (the sample mean of the 5000 simulated neighbor bids is \$34.6 millions). In 3491 of the 5000 simulations for this tract, the non-neighbor bid beats the neighbor bid, as expected looking at the distributions of the bids.

More interestingly, the symmetric information simulation bids for the same tract has a much thinner tail, hence having a sample mean of \$9 millions. Their variance is also much less than the bids in the asymmetric information case. The symmetric information bid beats the winner of the asymmetric information case in only 754 of the 5000 simulations.

For the low valued tracts, we can conclude the following: if there are both non-neighbor and neighbor bidders in the auction, the non-neighbor bidder might be bidding higher than the value of the tract due to the imprecision of public signal. Actually, the seismic survey information which is available to every potential bidder in the auction reveals the hydrocarbon potential of a very large area including the auctioned tract. This can be misleading. On the other hand, when all the bidders have the kind of information a neighbor bidder has, they make a much reliable guess about the productivity of the tract, since they know the results of explorations done in very close approximity of the auctioned tract. Therefore,they hardly bid higher than

Figure 3.2. Simulated bids for a tract with value $125.7 million



the value of the tract. This results in a higher government revenue in the asymmetric information case.

Next, we pick a tract with a value above the average tract value in the data set. 25 of the 69 tracts in our data set have higher ex-post value than the sample average. In 14 of these 25 tracts, the symmetric information revenue of the government is higher than that of the asymmetric information case. Hence, it is observed that the government has a greater chance of getting higher revenue in a symmetric information environment as the value of the tract increases.

In Figure 3.2, we depict the kernel smoothing density estimates of the neighbor and non-neighbor bids in the asymmetric information simulation and the bids in the symmetric information simulation for a tract with a value of $125.7 million. It is observed that the sample mean of the 5000 simulated non-neighbor and neighbor bids are $46 million and $27.4 million, respectively. The sample standard deviation of the non-neighbor bids is almost three times higher than that of the neighbor bids. This is also obvious from the thicker tail of the non-neighbor bid. In $3,397$ of the $5,000$ simulations for this tract, the non-neighbor bid beats the neighbor bid. This time, the symmetric information revenue of the government is higher than the asymmetric information revenue in 1551 of the 5000 simulations. This is more than double the number for the low value tract. We conclude that, for a high valued tract, the symmetric information bidders are better able to compete with the dispersed bids of the non-neighbor bidders, when compared to the case in a low valued tract.

In conclusion, we observe that the non-neighbor bids being highly dispersed is the primary determinant of the results of the counterfactual experiment. High variation in the non-neighbor bids is expected due to the less informative signal (public signal) non-neighbor bidders receive about the value of the tract. However, we find the estimated variance of the non-neighbor bids much higher than the levels it would be expected. This results from the highly dispersed bid data we have from the OCS auctions.

3.5. Conclusion

The OCS drainage lease auctions, as held by the Department of the Interior until 1982, are a very typical example of the asymmetric information common value auction model. In their empirical study, HP has revealed the facts and findings for the asymmetric information structure among the bidders of drainage leases for the period 1954 -1969. The previous chapter of this dissertation discusses the identification of private values in the asymmetric information common value auction models and proposes the first structural estimation method for this model in the literature. In this chapter, we apply this method to estimate the structural parameters of underlying distribution of signals of the neighbor bidders in the auctions of OCS drainage leases between 1954 and 1972. Having estimating the structural parameters, we run counterfactual simulations to see how the government's auction revenue would be effected were the bidders symmetrically informed.

We find that the revenue in the asymmetric information case is significantly above the revenue in symmetric information case for low value tracts. As the tract value increases, the symmetric information revenue starts to beat the asymmetric information revenue more often. This is explained with the finding that non-neighbor bidders in an asymmetric information environment bid with high variation and low precision (which leads to significant negative profits for them most of the time) when compared to symmetric information bidders. We expect the non-neighbor bids to be dispersed due to the less informative nature of the public signal, only which they have access to. However, the extremely high variations in their bids seen in the simulated data is a result of the dispersed data we have from the OCS auctions.

CHAPTER 4

An Empirical Analysis of ERCOT Balancing Market

4.1. Introduction

The electricity spot market auctions have been analyzed extensively in the literature. A wide variety of models based on varying behavioral assumptions, have been used. Klemperer and Meyer's (1989) supply function equilibrium (SFE) model has been regarded as a good model of electricity spot markets, since it captures the underlying structure of the market well. This model is based on the idea that under demand uncertainty, a strategy in the form of a supply function (multiple price-quantity pairs) constitute the optimal strategy (as opposed to a single price-quantity pair strategy) There has been many applications of this model to predict market performance. Green and Newbery (1992), Newbery (1998) and Green (1999) apply supply function equilibrium model to predict markups in England and Whales wholesale electricity. Another model that has been a good base for the investigation of the spot market auctions is the divisible good (share) auction theory. Hortacsu and Puller (2008) is one of the recent studies building its analytical approach on Wilson (1979), which is a seminal work in the share auctions literature.

While asymmetric environments have proved to be difficult to analyze, for the spot market auctions, it is essential to consider models in which different bidders exhibit different characteristics. For example, investment decisions will generate asymmetries endogenously even if the generator firms start out as ex-ante symmetric. Important asymmetries that affect the positions of the bidders in the market and their strategies significantly are their cost structures and forward contract quantities. Wolak (2000) develops a methodology that yields insight about generation unit level cost functions using the bid and the contracted quantity data from the National Electricity Market (NEM1) in Australia. He shows that contract obligations significantly affect bidders' incentives to exercise market power. De Frutos and Fabra (2008) found that the scope of contracts to improve market performance crucially depends on both their volume and distribution across firms. Therefore inference about this asymmetry about the participants of the market should give enormous insight about strategies of the players and resulting prices that are being observed in the market currently. In their study investigating the performance of the bidders in spot BES market of ERCOT, Hortacsu and Puller (2008) also treat the contract quantity of bidders as private information and they use a behavioral assumption to identify the contract positions of bidders. Sioshansi and Oren (2007) first calculate the ex-post optimal supply functions of firms in the BES market of ERCOT, then pick the bidders who bid closest to their ex-post optimal supply functions. They then derive the more general Nash Equilibrium set of ex-ante optimal supply functions and they test if the behavior of those bidders satisfy a Nash Equilibrium. Niu, Baldick and Zhu (2005) propose a linear asymmetric SFE model of the BES with transmission constraints to develop firms' optimal bidding strategies considering forward contracts, and they evaluate the market power mitigation effects of forward contracts.

In this study, we work on Hortacsu and Puller (2008)'s share auction model of the BES market in ERCOT. Hortacsu and Puller (2008) has assumed that the supply schedules of bidders are additively separable. They also have built the marginal cost functions of the bidders using the heat rate and fuel information of plants. Using the additive separability assumption and the marginal cost information they find the optimal supply schedules implied by their model very conveniently. Moreover, they determine the contract quantities of bidders using a behavioral assumption, which is basically derived from the first order optimality condition of the bidders' profit maximization problem.

First point to notice in Hortacsu and Puller's solution is that their assumption of additive separability is an a priori restriction imposed on the supply schedules. In our study, we do not assume any functional form for the supply schedules. Then, we use a resampling based estimation technique to infer belief of bidders regarding the distribution of the market clearing price given their contract quantity and bid. This, then, enables us to use the first order optimality condition to find the implied marginal cost of the bidders.

One good aspect of the resampling based estimation we conduct is that we do not need to pool data from different auctions on different hours or different days. As a result, we do not need to control for in our estimation the changing factors between different auctions. A potential shortcoming of not imposing any structure on the supply schedules is that the implied marginal costs are not necessarily monotone functions, which is actually the expected in the electricity industry.

Second thing we point out in Hortacsu and Puller (2008) is that they assume that the bidders behave according to the first order optimality condition to find their contract quantities. Then, they use these quantities to build the bidders' optimal supply schedules, which ends up conflicting the former behavioral assumption to find the contract quantities. In our study of the BES, we approach the model from a different direction. We assume at the very beginning that only the largest bidders in the market behave according to our model, which is supported by the findings of previous studies on the BES market of ERCOT. Then, using the optimality condition, we find the implied marginal cost structure of the largest bidders. Finally, we compare these implied marginal costs with the marginal cost schedules we build based on the heat rate and fuel type information of these bidders.

So, in this study, our purpose is to analyze the behavior of bidders in BES market with less restrictive and more consistent assumptions.

The rest of this chapter continues as the following. In section 2, we talk about the general characteristics of power markets, ERCOT and the Balancing Energy Services market of ERCOT. Section 3 introduces the model. Section 4 explains the resampling based estimator we apply to the bid data to infer the belief of bidders regarding the equilibrium price, and our empirical strategy. Section 5 introduces the data, and explains how we build the cost functions. Section 6 discusses results of the estimation,compares the actual costs to the estimated marginal costs of the Bayesian Nash Equilibrium of the model. and concludes.

4.2. Power Markets, ERCOT and Balancing Energy Services Auctions

One of the main factors that guided the restructuring of the electricity power industry was the concerns about reliability of the power supply. While the authorities aimed at pushing the decisions and risks associated with electricity generation, distribution and retailing to the market, reliability concerns always required some structure to be imposed upon the system.

It is the characteristics of the electricity supply and demand that renders this industry impossible to operate reliably as a deregulated industry with competitive supply. These characteristics can be listed as (a) large variations in demand over the course of a year; (b) nonstorability of electricity; (c) the need to physically balance supply and demand at every point on the network continuously to meet physical constraints on voltage, frequency, and stability; (d) the inability to control power flows to most individual consumers; (e) limited use of real time pricing by retail consumers, and (f) that even under the best of circumstances (i.e. with effective real time pricing of energy and operating reserves) non-price mechanisms (blackouts) will have to be relied upon from time to time to ration imbalances between supply and demand to meet physical operating reliability criteria because markets cannot clear fast enough to do so[1] (Joskow, 2006).

The term *reliability* is defined by the NERC (National Electricity Reliability Council) as "the degree to which the performance of the elements of the technical

[1] In response to questions about why demand response was not relied upon to respond to the sudden loss of 1,100 MWs of generating capacity that led to rolling blackouts in Texas on April 17, 2006, a representative of the ISO is reported to have said: “In this case, when four generators tripped, it was just bang-bang-bang-bang.” *Electric Transmission Week*, April 24, 2006, pages 1 and 12, SNL Financial LC.

system results in power being delivered to consumers within accepted standards and in the amount desired", which can be decomposed into the two aspects of the electricity system: *security* and *adequacy*. Security describes the ability of the system to withstand disturbances (contingencies), and is provided by means of protection devices and operation standards and procedures that include security constraint dispatch and the requirement for ancillary services such as voltage support, regulation capacity, spinning reserves, black start capability, etc. The notion of generation adequacy on the other hand represents the systems ability to meet demand, on a longer time scale basis, considering the inherent fluctuation and uncertainty in demand and supply, the nonstorability of power and the long lead time for capacity expansion.

Although quality and the quantity of the existing generation capacity sets the stage for the operation of the electricity markets, design of the market to facilitate demand response and the spot market are also crucial for provision of a reliable services.

4.2.1. ERCOT

ERCOT is the independent system operator (ISO) overseeing majority of Southern and Central Texas. The area operated by ERCOT is named as the ERCOT region. ERCOT covers the 85% of the load in Texas. ERCOT region has about 23 million consumers, 550 generating units with total generation capacity of $84,000$ MW. Market. There are approximately $1,150$ active market participants which generate, move, buy, sell or use wholesale electricity[2].

[2]ERCOT Quick Facts. Retrieved from ercot.com on 2.13.2012.

In 1996, PUCT endorsed ERCOT to become an ISO to oversee the equitable access to the power grid among the competitive market participants. This made ERCOT the first ISO in the US. When the Texas Legislature restructured the Texas electric market in 1999 by unbundling the investor owned utilities and creating retail customer choice, ERCOT was assigned four primary responsibilities: System reliability (planning and operations), open access to transmission, retail switching process for customer choice, wholesale market settlement for electricity production and delivery.

ERCOT employs an *energy-only* market design. Unlike markets with capacity market design, which have a long term capacity market where the resources are paid fixed capacity payments during the year regardless of the relationship of supply and demand, the purpose of the energy-only market design is to allow energy prices to rise significantly during shortage conditions. This way, appropriate price signal is provided for demand response and new investment, when the available supply is insufficient to meet both energy and minimum operating reserve requirements.

Pricing of the electricity consumption is a vital part of the debate between the competing wholesale electricity market designs for resource adequacy. A series of papers by Borenstein investigate the effects of various types of real time pricing mechanisms on peak demand, spot market prices, investment on electricity generation and efficiency of generation[3].

The importance of price elastic customers (demand response) is emphasized in an energy-only market design. In energy only markets, at least some price elastic

[3] See forexample Borenstein (2005a, 2005b and 2010)

consumers are required for price to clear the market when capacity constraint binds; otherwise non-price rationing could result in the form of blackouts.

The other part of the energy-only market design of ERCOT that allows the market participants to respond to scarcity conditions is the Balancing Energy Services (BES) market. In the BES market, expectation of high prices during shortage conditions and the frequency of the occurrence of shortage conditions attract new capacity investment in the market. The higher the electricity price in shortage conditions, the lower the frequency of shortages. In this way, system is expected to settle down at the capacity level which results in the optimal frequency of power shortages.

4.2.1.1. Demand Response. Ability of the demand side to respond to the prices or scarcities eases the provision of system security and resource adequacy. Withdrawal of demand at times of scarcity or peaking price acts as additional resource connected to the system So, demand response provides cheaper security and adequacy measures. In the long run, demand response is also expected to decrease the power costs of industrial and residential customers due to decreased costs of investments for reliability measures (which are practically passed on to the consumers in a competitive retail electricity sector)

Prior to the introduction of retail competition in January 2002, ERCOT relied upon 3500 MW of interruptible load, group load curtailment programs, residential direct load control, and other load management programs to maintain reliability. With the redesign of the ERCOT market between 1999 and 2001 to foster competition, the market lost a planning reserve resource of nearly 3000 MW, and participants were left confused over who would assume responsibility for overall resource adequacy.

PUCT requires ERCOT to integrate back the existing demand response potential systematically into the market. While debates in ERCOT about the appropriate role of demand response is going on, there is agreement with the general principle that demand side resources should be permitted to compete with generation resources to the extend that the demand side resource can provide a service similar to the service supplied by generator resources (Zarnikau, 2010).

In today's ERCOT design, demand response can participate in the market under three categories: *Loads acting as a Resource (LaaR)*: commercial and industrial customers with interruptible loads that can meet certain performance requirements can qualify to become Load Resources and provide operating reserves in the ERCOT ancillary services (AS) markets. In the AS markets, the value of a Load Resource's load reduction is equal to that of an increase in generation by a generating plant. Load Resources that are scheduled or selected in the ERCOT Day-Ahead AS Markets are eligible to receive a capacity payment regardless of whether they are actually curtailed.

Emergency Interruptible Load Service (EILS): ERCOT procures (EILS) by selecting qualified loads to make themselves available for interruption in an electric grid emergency. EILS is an emergency load reduction service designed to decrease the likelihood of the need for firm load shedding (a.k.a, "rolling blackouts"). Customers meeting EILS criteria may bid to provide the service through their qualified scheduling entities (QSEs). *Voluntary Load Response*: refers to a customer's deviation from its scheduled or anticipated load level in response to price signals (e.g., balancing energy prices or peak demand periods) in situations where the customer has not formally offered its response to the market as a "resource".

In 2010, over 2,200 MW of capacity were qualified as LaaRs. These resources regularly provided reserves in the responsive reserves market, but never participated in the balancing energy market[4].

These opportunities are available mostly to industry level customers. Industry level customers make around one fourth of the total load in the ERCOT region. The rest of the load belongs to residential customers. Demand response in the residential customer level is only possible if the LSE's offer real time tariffs to their customers.

4.2.1.2. Spot Market. A vast majority of the power transactions in the United States take place in the form of bilateral contracts between agents in the electricity industry. (In the ERCOT region, around 95% of the power is sold through bilateral contracts). However, with such limited demand response, it is the spot market (the Balancing Energy Services market as named in ERCOT) that provides the needed flexibility to the system which results from real time demand uncertainty. Unexpectedly high levels of demand occurring at a point in time creates scarcity. Generators, who are willing to provide the demanded power at that point in time at the lowest price are called to do so. The design of the spot market should allow the bidders to cover their costs and make a reasonable amount of profit, while not letting them exploit the scarcity and charge extremely high prices. Another target that the spot

[4] 2010 State of the Market Report for the ERCOT Wholesale Electricity Markets, August 2011, Potomac Economics, Ltd. (Independent Market Monitor for the ERCOT Wholesale Market)

market design is expected to achieve is to provide necessary price signals for the building of the new generation capacity, so that shortage conditions in the system do not occur more often than the optimal frequency. Bilaterally contracted quantities of generation firms, types and level of available capacity ,and existing regulations like price caps are some of the factors that affect the performance of the spot market crucially.

In this chapter we analyze the performance of bidders in the electricity spot market in Texas. We use the hourly bid data from BES to find bidders' marginal costs implied by a Bayesian Nash equilibrium model. Then we compare their actual costs to the estimated costs.

4.2.2. Bidding in ERCOT's Balancing Energy Services Market

Majority of power transactions in ERCOT take place in the form of bilateral contracts in an "over the counter" market. Contract quantities and prices are private information to the firms. However, between 2% to 5% of the power transactions take place at the Balancing Energy Services market. This is the market overseen by the independent system operator ERCOT to assure the real time balancing of supply and perfectly inelastic demand.

In the Day-Ahead Market, which occurs one day before the demand and generation for a specific time, each market participant submits their generation schedules and forecasted load schedules for the next day, through a qualified scheduling entity (QSE) to ERCOT. The submitted generation schedules do not need to be balanced in the sense that they do not need to match submitters' bilateral contract amounts[5]. Then, BES bids can be submitted or changed until one hour before the hour for which the bids are submitted. BES participants can submit at most 20 monotonically increasing price-quantity pairs to increase (Up Balancing Energy Services, UBES in ERCOT's terms) and 20 monotonically increasing pairs to decrease (Down Balancing Energy Services, DBES) their scheduled quantities in the day-ahead market.

ERCOT aggregates all the bids. Every fifteen minutes, ERCOT observes the realized demand and calls bidders to generate the power they have bid at the current market clearing price. Market is cleared this way four times during the hour, but bidders submit one bid for the whole hour.

Although the main purpose of the BES market is to assure the real time balancing of generation and load in the case of unexpected events like generator outages and extremely hot or cold weather, since firms are not required to submit balanced schedules in the day-ahead market, they may be strategically planning to fulfill their contract obligations by purchasing some power in the BES. If at the end of the BES market for a 15 minute interval, if a firm's total generation is short of its obligation, then it purchases the missing generation amount from the market at the market clearing price. On the other hand, firms who exceed their obligations sell their excess generation at that price. Load serving entities can also purchase the power they need if they have not already secured it through bilateral contracts with power generators.

[5] Until mid 2002, ERCOT required market participants to submit balanced schedules to prevent them from using the BES to purchase their base-load.

In case firms over schedule at the day ahead market, Public Utility Commission of Texas (PUCT) wants to make sure that they are available to decrease their supply at the BES market. In order to facilitate this, PUCT has imposed a regulation on the firms telling that they have to offer at least 15% of their scheduled energy to DBES at any price within the offer caps[6]. However, it is voluntary to participate in UBES.

Power generating companies have a good amount of information about their competitors. First, in Texas, power plants have similar production technologies. Data on fuel efficiency of each generation plant is publicly available. Therefore, cost structures of each market participant can be estimated with a very good precision. Second, competitors can purchase information on real time online-offline status of each generator in the market. (Hortacsu and Puller (2008)) This helps them guess their competitors' bids, and develop bidding strategies. Finally, the aggregated bids in the BES are made publicly available after a certain time. Therefore, bidders can deduce their residual demand curves by subtracting their own bid from the aggregated supply schedule. Since bidders participate in the BES market repeatedly, bidders can get a very good idea about the shape of their residual demand curve to the extend that the strategies of competitors and the other conditions in the market do not change.

One important unknown to the bidders in the market is the contract quantities of their competitors. As shown by Wolak (2000), contract position is a strong determinant for market power incentives of participants. Contract positions constitute

[6]System wide offer cap was $\$1,500$ per MWh starting from March 1, 2007. It was increased to $\$2,250$ per MWh on March 1, 2008.With the switch from zonal to nodal market on December 1, 2010, the offer cap was increased to $3000 per MWh.

the private information of bidders, which play a significant role in determining their strategies.

4.3. A Multi-Unit Auction Model of BES Market

We adopt Hortacsu and Puller (2008)'s model of the BES auctions in ERCOT.

We assume there are N bidder firms in the market. $\{C_{it}(q),\ i = 1, ..., N\}$ represent the cost of generating q amount of electricity at time t for a bidder i in the auction. Demand is assumed to be the sum of a deterministic price elastic component and a stochastic constant term as

$$\widetilde{D}_t(p) = D_t(p) + \varepsilon_t$$

for the sake of generality, although we will work with a price inelastic demand in our application.

QC_{it} is the amount firm i has contracted to generate at time t in previous bilateral agreements with other agents in the power market. The price of this contracted quantity is PC_{it}.

In each time period (auction) t, each firm submits a supply schedule $S_{it}(p, QC_{it})$. We assume that the supply schedules are continuously differentiable and have bounded derivatives.

The auctioneer aggregates the simultaneously submitted supply schedules for time t and finds the market clearing price p_t^c given as

$$\sum_{i=1}^{N} S_{it}(p_t^c, QC_{it}) = \widetilde{D}_t(p_t^c) \tag{4.1}$$

Due to the uniform price rule in the BES auctions, firms are paid market clearing price times the quantity they have bid at that price: $S_{it}(p_t^c, QC_{it})p_t^c$. Therefore, after realization of the market clearing price, firms get the following ex-post profit:

$$\pi_{it} = S_{it}(p_t^c, QC_{it})p_t^c - C_{it}(S_{it}(p_t^c)) - (p_t^c - PC_{it})QC_{it}$$

The submitted quantities at different prices have to include the contracted quantity QC_{it}. Since these quantities were contracted to be delivered at an agreed upon price PC_{it}, the bidder i has to reimburse the difference $p_t^c - PC_{it}$ back to its customer for all the contracted quantity (assuming market clearing price is higher than the contract price.). Hence the term $(p_t^c - PC_{it})QC_{it}$ is subtracted from total profit. (If the market clearing price p_t^c is lower than the contracted price PC_{it}, this will be an extra charge rather than a reimbursement.)

From the point of view of bidder i, the market clearing price p_t^c is the most important source of uncertainty. A bidder cannot foresee the market clearing price because of two things that he does not know: first, bidders cannot know for certain what the exact demand is going to be at time t; second, bidders cannot know what their rivals are bidding for time t: no one knows each other's contract quantity. It is important to note that, it would be possible to guess each other's strategy if contract quantities were public information, due to the fact that costs are fairly well known in the electricity market. Therefore, the market demand $\widetilde{D}_t$ and the unobserved components of the rivals' profit maximization problems which are the contract positions and prices of rival firms $\{(QC_{jt}, PC_{jt}),\ j \in -i\}$, are the main sources of uncertainty for bidder i.

Hortacsu and Puller (2008) define a probability measure over the realizations of the market clearing price from the perspective of bidder i, to characterize a Bayesian Nash equilibrium. This probability measure is conditional on QC_{it}, which is a private information of bidder i, and bidder i's supply schedule $\widehat{S}_{it}(p)$. The rivals of bidder i are assumed to play their equilibrium strategies $\{S_{jt}(p, QC_{jt}),\ j \in -i\}$.

$$H_{it}(p, \widehat{S}_{it}(p)|QC_{it}) \equiv \Pr(p_t^c \leq p|QC_{it}, \widehat{S}_{it}(p))$$

Utilizing the fact that $p_t^c \leq p$ is possible only if there is excess supply at price p, we can write

$$H_{it}(p|\widehat{S}_{it}(p), QC_{it}) = \Pr\left(\sum_{j\in -i} S_{jt}(p, QC_{jt}) + \widehat{S}_{it}(p) \geq \widetilde{D}_t(p)|QC_{it}, \widehat{S}_{it}(p)\right)$$

$$= \int\limits_{QC_{-it} x \varepsilon_t} 1\left\{\sum_{j\in -i} S_{jt}(p, QC_{jt}) + \widehat{S}_{it}(p) \geq D_t(p) + \varepsilon_t\right\} dF(QC_{-it}, \varepsilon_t|QC_{it})$$

using the definition of the market clearing price in Equation 4.1. $F(QC_{-it}, \varepsilon_t|QC_{it})$ is the joint distribution of the contract quantities of rivals of firm i and the demand noise ε_t conditional on the contract quantity of firm i. Note that $H_{it}(p|\widehat{S}_{it}(p), QC_{it})$ is the belief of bidder i about the distribution of the market clearing price.

The expected utility maximization problem of bidder i for a general utility formulation which can take on both risk averse and risk neutral forms is written as the following:

$$\underset{\widehat{S}_{it}(p)}{max.} \int_{\underline{p}}^{\overline{p}} U\left(p\widehat{S}_{it}(p) - C_{it}(\widehat{S}_{it}(p)) - (p - PC_{it})QC_{it}\right) dH_{it}(p|\widehat{S}_{it}(p), QC_{it})$$

Note that the expectation is taken over all possible realizations of the market clearing price with respect to the probability density $dH_{it}(p|\widehat{S}_{it}(p), QC_{it})$. It is important to point out here that other bidders' contract positions are not in bidder i's objective function. Hence, Hortacsu and Puller (2008) do not model this environment as a common value auction.

Hortacsu and Puller (2005) shows that the Euler-Lagrange necessary condition for the pointwise optimality of the supply schedule $S^*_{it}(p)$ is given by

$$p - C'_{it}(S^*_{it}(p)) = (S^*_{it}(p) - QC_{it})\frac{H_s(p|S^*_{it}(p), QC_{it})}{H_p(p|S^*_{it}(p), QC_{it})}{}_7 \tag{4.2}$$

where

$$H_p(p|S^*_{it}(p), QC_{it}) = \frac{\partial}{\partial p}\Pr(p^c_t \leq p|QC_{it}, S^*_{it}(p))$$

$$H_s(p|S_{it}^*(p), QC_{it}) = \frac{\partial}{\partial S} \Pr(p_t^c \leq p|QC_{it}, S_{it}^*(p))$$

$H_p(p|S_{it}^*(p), QC_{it})$ is the density of the market clearing price when firm i submits the supply schedule $S_{it}^*(p)$, while $H_s(p|S_{it}^*(p), QC_{it})$ denotes how much the probability distribution of the market clearing price shifts when bidder i shifts its supply $S_{it}^*(p)$. Note that this term is always non-negative[8].

Hortacsu and Puller (2008) point out to the following observations from Equation 4.2:

- H_S can be seen as an expression that captures the market power of bidder i: the more bidder i can shift the distribution of the market clearing price with his supply schedule, the more market power he has.
- If we see Equation 4.2 as a markup expression, the markup bidder i makes depends on the market power of bidder i: as $H_S \to 0$, price equals marginal cost.
- When bidder i is a net buyer ($S_{it}^*(p) < QC_{it}$), i.e. it has to purchase power from other agents to fulfill its contract requirements, then it bids below marginal cost.
- Moreover, $S_{it}^*(p) - QC_{it} = 0$ implies $p = C_{it}^{'}(S_{it}^*(p))$. In other words, at the quantity level which exactly covers its contract quantity, bidder i bids its marginal cost.

Next, we discuss how the contract quantities of firms can be inferred.

[8] An increase in $S_{it}^*(p)$ weakly decreases the market clearing price, which in turn increases the probability that the market clearing price is less than price p.

4.3.1. Unobserved contract quantities

It is proposed by Hortacsu and Puller (2008) that if $C'_{it}(S^*_{it}(p))$ is observed, the contract quantity of bidder i can be found at the intersection of its marginal cost and supply schedule. This is a direct result of the last observation at the end of the previous section. This can be interpreted as when bidders are optimizing their strategies, at their contract quantity level, they bid their marginal cost. Also, when a bidder is short of its contract quantity, it bids below its marginal cost. When bidding for production above its contracted level, the bidders bids above its marginal cost.

However, one should keep in mind that Equation 4.2 is the conclusion of a model of strategic behavior. Therefore, Hortacsu and Puller (2008) find the contract quantity bidders assuming that they behave according to their model. However, they later use this contract quantity to find the optimal bidding strategies of the bidders. At this point, we agree with Oren and Sioshansi (2007) arguing that since their aim is to analyze the extent to which firm behavior is consistent with the model, they would prefer a contract quantity estimate which does not require them to assume that agents behave as dictated by the model. So, they follow Niu et al (2005) and simply assume that the contract quantity of a bidder is equal to quantity it scheduled at the day ahead market.

In this study, we aim to find the implied marginal costs of bidders using the optimality condition. For every price-quantity pair bid by a power generator, we can solve for the marginal cost of generating that marginal quantity assuming that we have the contract quantity information and the supply and price derivatives of H.

However, since we use Equation 4.2 for finding the marginal costs, contract quantity of a firm remains unidentified. However, we need this contract quantity information to estimate the implied marginal cost levels, as will be explained in the next section. As a result, we follow Niu et al (2005) in assuming that the bidders exactly cover their contracted level of production in their day ahead market schedule. Therefore, contract quantity of bidders is equal to their day ahead market schedule[9].

4.4. Finding the marginal cost functions implied by the model

Note that if we assume that the bids we observe in the data are generated by a Bayesian Nash equilibrium of the uniform price multi-unit auction game satisfying the necessary condition in Equation 4.2, we can non-parametrically identify the marginal cost functions of the bidders up to their contract quantity. Once we plug the bid of bidder i and its contract quantity at a specific auction t, in Equation 4.2,what remains to calculate the marginal cost function $C_{it}(\widehat{S}_{it}(p))$ is $H_{it}(p, \widehat{S}_{it}(p)|QC_{it})$, the belief of bidder i about the distribution of market clearing price p_t^c conditional on his private information and strategy, and its derivatives with respect to S and p.

Hortacsu and Puller (2008) walk around the need to estimate H by restricting the functional form of the supply function strategies to a class of strategies that are additively separable in the private information possessed by bidders: $S_i(p, QC_i) = \alpha_i(p) + \beta_i(QC_i)$. They show that under this restriction, Equation 4.2 takes the form $p - C_{it}'(S_{it}^*(p)) = \frac{S_i^*(p,QC_i)-QC_i}{-RD_i'(p)}$ where $RD_i'(p)$ is the price derivative of the

[9]Throughout the analyses, we will also look at the implications of Hortacsu and Puller (2008)'s assumption regarding the contract quantity. After all, they argue that talks with industry professionals also confirm that bidders bid lower than their MC when they are short of their contract quantity, and higher than their MC when they are bidding for quantities higher than their contract quantity.

ex-post realization of the residual demand curve, which is faced by bidder i , and $RD_i'(p) = D'(p) - \sum_{j \neq i} \alpha_j'(p)$. In summary, since demand is also additively separable in its random component ε, residual demand, which is the difference between the demand and the aggregated supply schedules of rival firms, is additively separable in private information of rivals (QC_j) and the random component of the demand (ε). Therefore, in the expression for the price derivative of the residual demand, there is no ε or QC_j, which implies that knowing what the rivals have bid, or what the realized value of ε is, does not change the strategy of a given bidder. In other words, what is ex-ante optimal is also ex-post optimal. So, Hortacsu and Puller (2008) can calculate the ex-post optimal supply schedule $S_{it}^*(p, QC_{it})$ for a given cost function and the realized residual demand.

Although this is a very convenient way of finding the optimal supply schedules, additive separability is an a priori restriction on the supply functions. Hortacsu and Puller (2008) test this restriction by regressing the slope of a linear fit of the calculated supply schedules on the calculated contract quantity of bidders and some other variables in a panel regression model. They find that the contract quantity is significant at 5% level, although the economic impact of the contract quantity on the slope is very little. When they also control for the auction fixed effects in their panel regression, the economic and statistical significance diminishes. However, regressions for individual firms reveal that additive separability is violated by some firms, while some satisfy this restriction. They find that firms who perform best in terms of profitability benchmarks come close to satisfying additive separability (for example Reliant, which operates under the name NRG now). On the other hand, there are also firms for which the variation in contract quantity explains 50% of the variation in bid function slope. In conclusion, they have heterogenous results from the testing of additive separability restriction.

As a result, we would like to proceed with no restrictive assumptions on the functional form of the supply schedules to find the marginal costs implied by optimality. We estimate H and plug it in the first order optimality condition together with the supply schedules submitted by the bidders and the day ahead scheduled quantity as the contract quantity to solve for the marginal cost functions of bidders. One point to keep in mind is that a potential consequence of not imposing any functional forms on the supply schedules is that the estimated marginal costs may not come up as non-decreasing functions. Next, we elaborate on the identification and estimation of H.

4.4.1. Identification of market clearing price distribution H

We follow the empirical strategy used in Hortacsu (2000), which estimates the valuations of bidders in a divisible good auction model of the Turkish treasury bills.

First, we discuss identification of the distribution of the market clearing price from the bid and demand data. Let $G(p, S)$ be the probability that at a given price p, the quantity S is less than or equal to the *stochastic* residual demand faced by bidder i:

$$G(p, S) = \Pr(S \leq \widetilde{D} - \sum_{j \neq i} S_j(p, QC_j))$$

Note that if we can estimate the joint distribution of the bids of rivals of bidder i and the stochastic demand from the data, we could estimate G for all p and S pairs. This probability is also equal to the probability of market clearing price being less than or equal to a given price p, when the quantity bid at the price p by bidder i is S:

$$H_{it}(p|S_{it}(p), QC_{it}) = G_i(p, S)|_{S=\widehat{S}_{it}(p)} \tag{4.3}$$

Therefore, the distribution of the market clearing price p^c_t is identified if we can estimate the $G(p, S)$ using the bid and the demand data.

4.4.2. Empirical Strategy

In the equilibrium, bidder i knows its rivals' bid schedules up to their contract positions QC_{-i} (since each firms' cost structure is well known due to the regulations in the electricity market, contract positions are the only unknown to bidder i). Therefore, assuming that the contract positions have a distribution which is common knowledge to all market participants, the residual demand faced by bidder i is a function of the $N-1$ random variables, QC_{-i} and the stochastic demand.$\widetilde{D}$. Therefore, bidder i can get a good inference about the distribution of the residual demand by repeating the following procedure many times: take $N-1$ random draws from the distribution of the contract positions and evaluate the rivals' bids. Take a random draw from the distribution of the demand. Take the difference of the demand and the aggregated bids of rivals. Find the market clearing price at the intersection of the residual demand and bidder i's supply function. Looking at the empirical distribution of the market clearing prices simulated in this way gives a good idea to bidder i about the distribution of the market clearing price.

In our application to the BES auctions in ERCOT, we do not know the distribution of the contract positions or the functional form that links the contract position to the supply function. However, if we assume that

(1) the contract positions QC_{it}, $i = 1, ..., N$ are distributed *iid*

(2) the supply function strategies of bidders are *symmetric*[10]

(3) the contract positions QC_{it}, $i = 1, ..., N$ and the random component of the demand ε_t are independent[11],

then, we can apply this logic to simulate market clearing prices by resampling from the bid data and drawing from the demand distribution, which can be estimated from the demand data that we have. We propose the following resampling procedure[12]:

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[10]Note that we do not impose a specific functional form, which is the case in Hortacsu and Puller (2008). They restrict the supply functions to a class of additively separable functions to simplify the first order condition in Equation 4.2.

[11]The contract quantites QC_{it} are generally determined long before the spot market for time t opens. On the other hand, bidders have to bid in the BES as late as only 1 hour before the realization of the demand. With the highly developed forecasting techniques, bidders can forecast the demand with very little error within one hour of its realization. Therefore, the random component of the demand $\widetilde{D}(p) = D(p) + \varepsilon_t$ is due to last minute changes in the conditions of the market like the weather. As a result, independence of ε_t and the contract positions of bidders is a realistic assumption.

[12]Hortacsu (2000, 2002) applies this resampling based approach to estimate the distribution of the market clearing price from the point of view of bidder i in a Bayesian-Nash equilibrium of the divisible good auction model of treasury bill auctions, under the assumptions 1 and 2, for the case of deterministic supply. He has proved the consistency of this estimator which resamples from bids of a single auction. He also shows that the estimator performs well in a Monte-Carlo experiment. In an extension where he allows supply to be stochastic, he shows that the estimators performs well with resampling from bid data of a single auction and simulating from an AR(1) fit of the supply data.

(1) Fix bidder i among the total N_t bidders in auction t

(2) From the sample of N_t bids in the data set, draw a random sample of $N_t - 1$ bid vectors with replacement, each with $\frac{1}{N_t}$ probability.

(3) Draw from the distribution of demand (the load) for time t (which can be estimated using the demand data)

(4) Using the $N_t - 1$ resampled bids and the simulated demand for time t, construct the residual supply function of bidder i

(5) Intersect bidder i's bid with the constructed residual demand and find the market clearing price.

(6) Repeat steps 2, 3, 4 and 5 B times, to get B market clearing prices for bidder i.

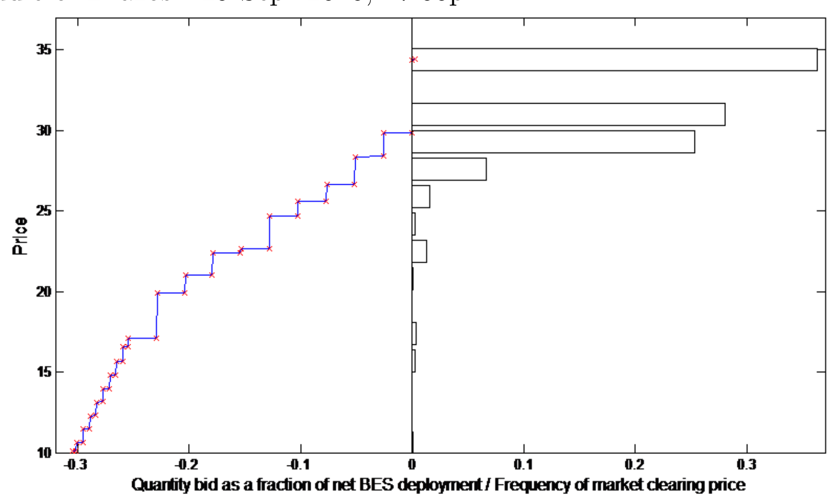
Then, a simple way to estimate $H_{it}(p|\widehat{S}_{it}(p), QC_{it})$ can be to build the empirical distribution of the B market clearing prices.

Note that with QC_{it}, $i = 1, ..., N$ independently and identically distributed, and bids generated by symmetric equilibrium strategies, the bids $S_{it}(p, QC_{it})$ will also be $i.i.d.$ The intuition behind the proof of consistency of this resampling based estimator is the following: resampling can be seen as a way to make random draws from the empirical distribution of bid vectors. By Glivenko-Cantelli Theorem, the empirical distribution converges (almost surely) to the true distribution as the sample size grows large.

Below are preliminary findings from application of this resampling based estimator to three firms in our data set, for non-stochastic demand. Here, demand is taken as known to all bidders relying on the fact that bidders in the BES market are experienced in load forecasting and they have a very precise guess about the load one hour ahead by looking at the trend of load. Bids are resampled 1000 times.

During the hour depicted in the figures, average load realized as 45,669 MW's. Net deployment of BES was 1,542, 1,728, 1,778, 1,679 MW's for the respective 15 minute intervals. Average BES deployment was 1,682 MWs. Market was cleared at \$41.16, \$47.92, \$47.92, \$46.38 at the respective15 minute intervals. Average price of the hour was \$45.85.

Figure 4.1. Distribution of resampled market clearing prices and supply schedule of Brazos - 29 Sep. 2010, 17:00pm



Note that we have found only a finite number of data points at which the market clears. This is due to the discrete number of price-quantity pairs bidders submit.

Brazos is one of the smaller players in ERCOT market. What we observe from 4.1 is that Brazos has bid on a price range between \$10 and \$34.39. Brazos has almost only bid to decrease its day ahead scheduled quantity, with only one pair to increase its generation by 4 MW's if the price hits \$34.39. It seems that Brazos has bid almost all its capacity in the day ahead market, and plays to decrease its generation in case of a low market clearing price. On the other hand, as per our estimation, it gives 50% chance to price hitting $34.39.

Figure 4.2. Distribution of resampled market clearing prices - Luminant - 29 Sep. 2010, 17:00 pm

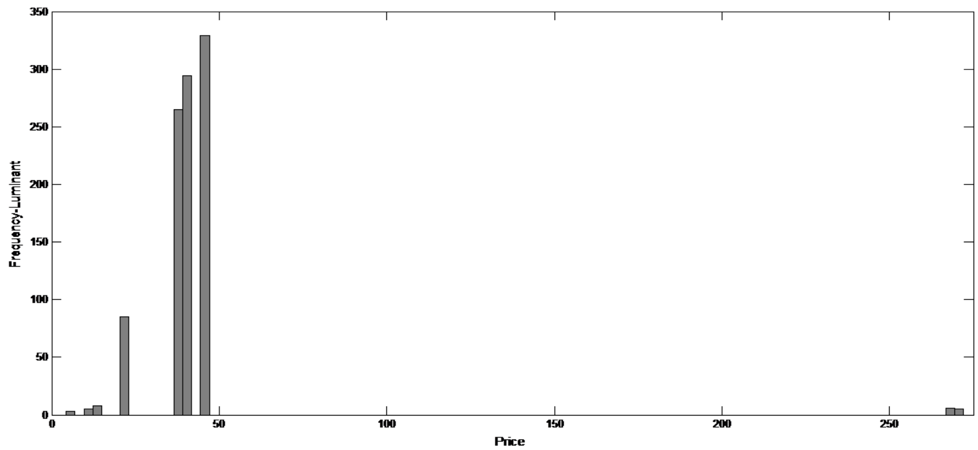
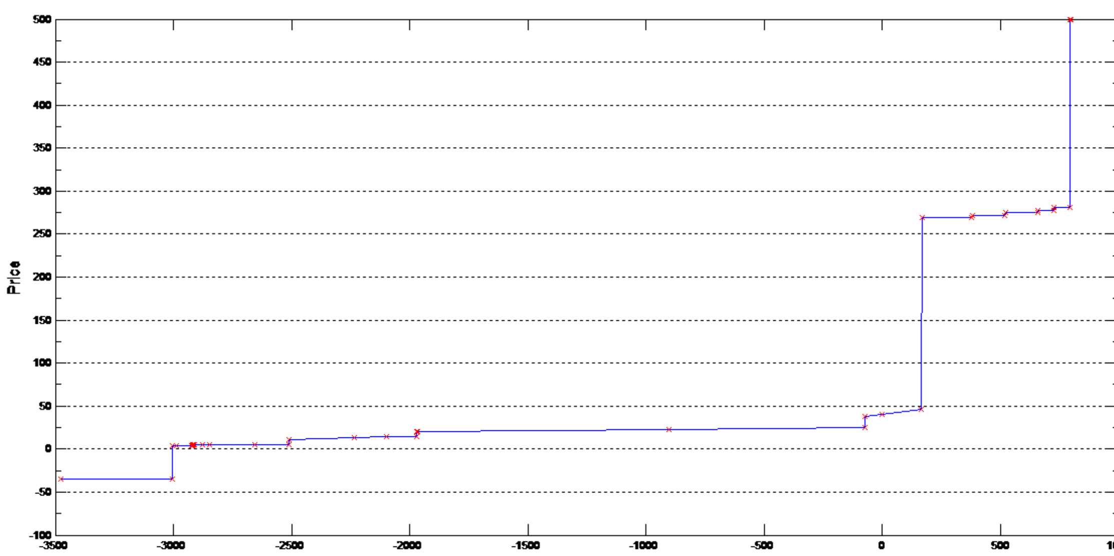


Figure 4.3. Luminant Supply Schedule - 29 Sep. 2010, 17:00 pm



Luminant and NRG are two big players in the market. They have bid for a wide range of prices and quantities[13].

Figure 4.4. Distribution of resampled market clearing prices - NRG - 29 Sep. 2010, 17:00 pm

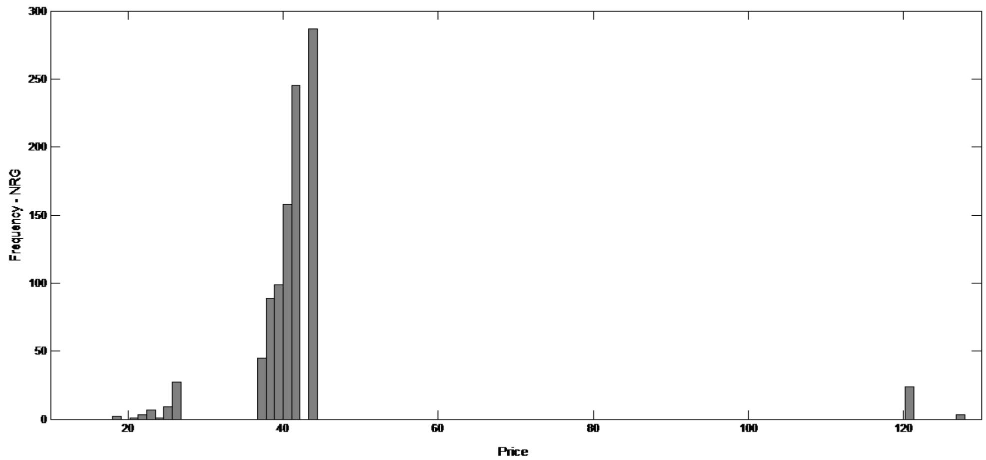
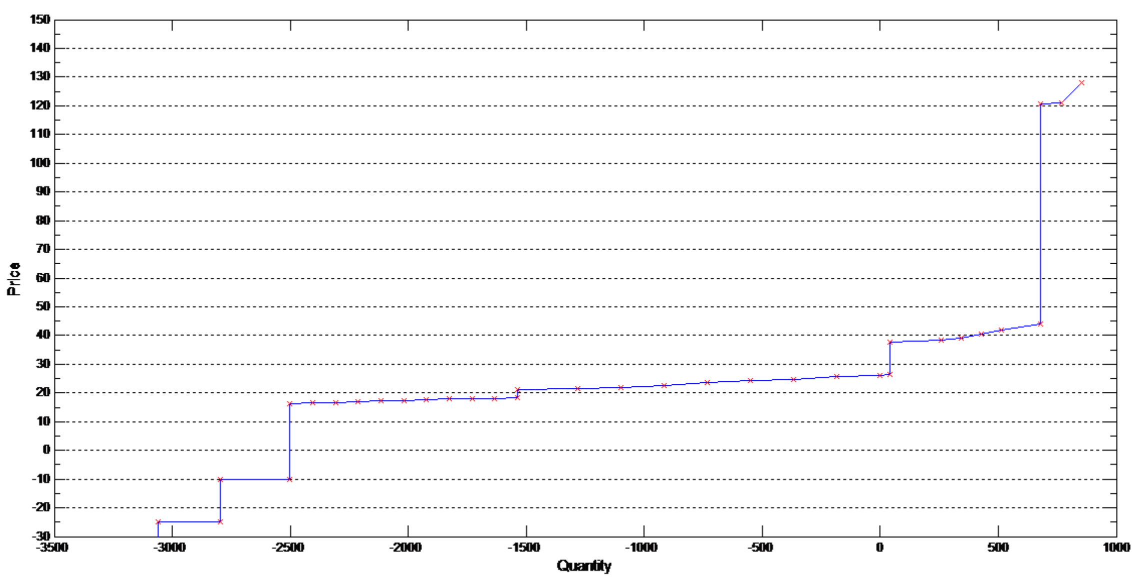


Figure 4.5. NRG supply schedule - 29 Sep. 2010, 17:00 pm



[13]We did not include the bids of NRG at prices $-\$291$ $-\$292$ and $-\$293$, which were to decrease its supply by 3061, 3062 and 3063MW's respectively in 4.5, in order to provide a more visible figure of the rest of the schedule.

4.5. Analysis of Observed Bid Schedules

4.5.1. Data

In this section we introduce the data used in the analysis of the bidding behavior of the companies participating in the BES in ERCOT. Among the data available to us are the individual BES bids, energy schedules of firms at the day ahead market and the resource plan details of the generation plants on an hourly basis from September 12, 2010 to November 30, 2010[14]. Energy schedules data provides us the quantity of electricity submitted to be generated by each firm at the day ahead market. Individual bid data has the bids to increase or decrease the day ahead market quantities in the real time market. Resource plan details let us know which generation units in a firm's generation fleet are available at a specific point in time, their generation capacities and their planned utilization. Since the method we use to estimate the marginal cost functions does not require pooling bids from different hours, we have the opportunity to work on a single balancing energy auction each time. Therefore, the first task is to chose which auction to analyze. While on one hand, working on a single auction saves us the trouble of controlling for the changing factors from one time period to the next, on the other hand conclusion of the analysis will be applicable only to the specific period being analyzed. This also renders the analysis vulnerable to even small

[14]These data sets have been published on ERCOT website for the past two years, 60 days after the operating day until the system switched from zonal to nodal on December 1, 2010. After this date, all the historic data was removed from public availability and no more individual bid data is made available to public by ERCOT. We thank Ozgur Inal for sharing his compiled data set with us.

size anomalies occurring in the ERCOT system. Therefore, it is crucial to choose a time period which is representative and in which ERCOT functions normally[15].

For the purpose of this study, by normal activity of the system we mean mainly three things: First, there should be no inter-zonal congestion during the time period we analyze. Congestion of the power lines connecting zones in ERCOT results in different market clearing prices in different zones. We would like to analyze whole ERCOT region as a single market, hence we looked for the periods in which there were no inter-zonal congestion[16].Second, we would like to capture the behavior of the bidder firms at the peak demand hours of the day during which the peaker generators would be on and available to go online instantly. For this, we tried to understand the peaking times of the day throughout the period for which we have data. It turns out that during the month of September in 2010, the peak demand mostly occurs on 5 pm. Third, since we do not take into account the start-up and shut-down costs of the plants while building the variable cost, we would like to look at the periods during which start-ups and shut-downs do not occur for the plants we analyze. For this purpose, we check the resource plan details to see which plants are on and off by period, and chose a period in which on/off status of the plants do not change from previous to the chosen time period and from chosen to the next time period.

[15]However, it is always possible to analyze all the auctions, one auction at a time, and interpret the averaged results, or look at the trend of the statistics throughout time.

[16]We found the congestion cost information from the Zonal Grid Information Report Archives which are available at http://www.ercot.com/gridinfo/congestion/summary/. To double check, we also reached the balancing energy services daily reports at http://ercot.com/mktinfo/services/bal/2010/index and confirmed that market clearing price for energy is same for all zones implying no congestion. We picked from the days on which there were no congestion.

It is possible that a bidder turns into a monopoly in a certain zone due to the congestion of a grid line connecting that zone to the others. Therefore, bidders can have significantly different strategies when they are in anticipation of a congestion. Acknowledging this fact, we followed the strategy of Sioshansi and Oren (2007) and looked for the days congestion did not occur in any of the time periods. We found that during the one week period from Monday, September 27 to Sunday October 3, there were no congestion. Hence, we concluded that a one hour period picked from this week could be a good candidate which would be free from anticipation of congestion.

The peak demand started at around $38,000$ MWh's occurring at 5 pm on Monday September 27, 2010, climbed gradually up to around $46,000$ MWh's at 5 pm on Thursday, and went down to around $33,000$ MWh's at 5 pm on Sunday. During the weekdays from Tuesday to Friday, the peak demand varied between $40,000$ and $45,000$ MWh's. We decided that this cluster of days during which there is no congestion and the peak demand is relatively stable is a good candidate. Finally, we checked the occurrence of start-up and shut-down of plants for the selected companies for this time period. It turns out that only very limited number of start-up or shut-downs have occurred at the beginning and end of the 5 pm period on Wednesday, September 29, 2010. As a result, we chose to study the BES auction which took place at this specific time and date.

4.5.1.1. Building Cost Functions. We use the resource plan details data to see which generators of a bidding firm are online at a given time period. We have data on their hourly available capacity and the day ahead scheduled generation for each generator in the portfolio of the bidder firms. We assume that the marginal cost of a generation unit is constant up to its hourly capacity. To drive each firm's cost function, we assume that it deploys its generation portfolio starting from the generator with the lowest marginal cost. When finding the variable cost of providing UBES and DBES generation, wind, hydroelectric and solar plants are assumed to be excluded from the generation portfolio for strategic bidding. due to the difficulty of estimating their availability.

Environmental Protection Agency (EPA) 's National Electric Energy Data System (NEEDS) v.4.10 has the heat rate, fuel type and SO_2 permit rate data for all the generators. To find a generator's variable cost, we multiply the heat rate of that generator (Btu/kWh) with the price of the related fuel . We used the average natural gas price for September 2010 reported in Information Administration's (EIA) Electric Power Monthly Archives. We follow Hortacsu and Puller (2008) and Oren and Sioshansi (2007) in adding \$0.1/mmBtu for transportation on top of the fuel price. Subbituminous price for coal plant and cost of uranium for nuclear plants were retrieved from snl.com.

After finding the fuel cost of the plants we added operating and maintenance costs[17].

Coal fired plants in Texas are required to hold federal emission permits for each ton of SO_2 emission. Permit rate (lbs/mmBtu) for each generator is available in NEEDS v.4.10. We found the permit price (\$/ton) for SO_2 at the Federal Energy Regulatory Commission (FERC)'s Emission Allowance Archives.

[17]We used a report for 2010 dated May, 2011 by the Nuclear Energy Institute (http://www.nei.org/). Per this report, gas, coal and nuclear plants, have operating and maintenance costs of 23%, 11% and 70% of total production costs respectively.

Having calculated the marginal cost functions of bidders, we shift the centers to the day ahead scheduled quantity to find variable costs of providing balancing energy services. This is interpreted as the marginal cost of providing UBES or the cost savings of providing DBES.

4.5.2. Estimation of the marginal cost functions implied by the model

In order to resample from the pool of bids at the given hour, we first aggregated the supply schedules of QSE's for the four different zones of ERCOT. By this way, we built the system-wide supply schedule of each QSE which are monotone non-decreasing step functions of quantity. We have 68 different QSE's bidding for 5 pm on September 29, 2010[18]. For each resampling, we drew with replacement 67 bids each with probability $\frac{1}{68}$. We then aggregated these 67 bids into a non-decreasing step function. We repeated this procedure 5000 times.

Next, we model the load L_t at hour t using a simple linear model: $L_t = \alpha + \beta L_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma^2)$. We chose this autoregressive type model since a strong determinant of the hourly load is the load level at the previous hour. The other explanatory variable of common use in load forecasting is the temperature. Also for long time series data, yearly, seasonally, monthly, weekly, weekend-weekdays and daily seasonality of the load should be accounted for. We believe bidders participating in the BES market have such detailed load forecasting tools. However for our purpose, the simple linear model is sufficient.

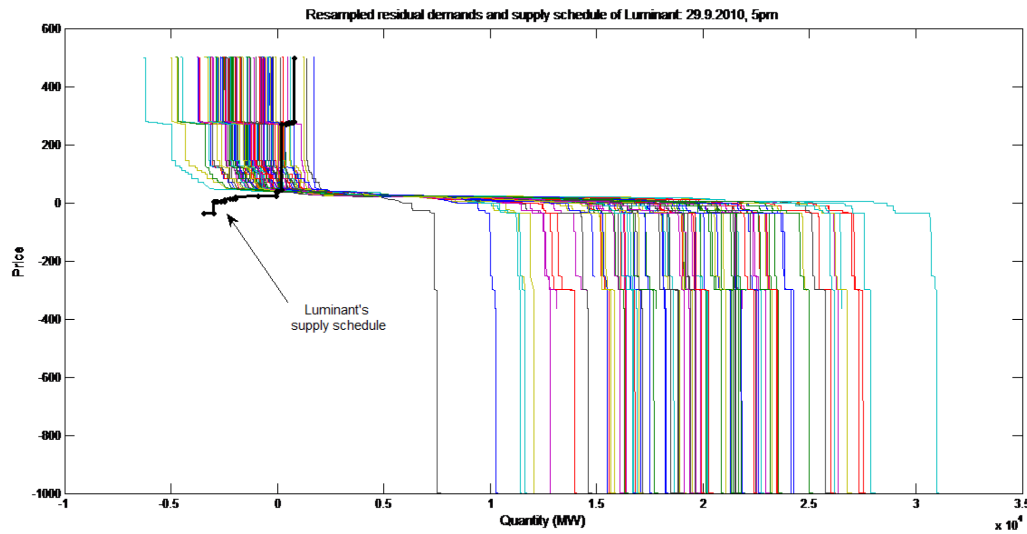
[18]However, this number includes several sub-QSE's bearing the same company name. Overall, there are 39 different companies.

We estimated the model with hourly load data between 4:00 to 17:00 during the weekdays of the week starting with September 27, 2010, which sum up to 65 observations. During these portions of the days we chose, the load has an increasing pattern. The estimate of the coefficients are $(\widehat{\alpha}, \widehat{\beta}, \widehat{\sigma}^2) = (1023.3, 1.0137, 786.4)$. The predicted load for the hour we study (17:00 on Wednesday, September 29,2010) is $46,160$ MWs. Realized load at this hour is $45,669$ MWs.

For each of the 5000 resampled aggregate bids of others, we also took a random draw from the estimated distribution of the error term and added this noise to the predicted level of the load. So we obtained one simulated load level for each resampling. We know from ERCOT that aggregate day ahead scheduled generation at this hour was $43,937$ MWs. We subtract this quantity from the simulated load levels to find the simulated net BES demands. We subtract the aggregated 67 bid schedules from the simulated BES demands to get 5000 simulated residual BES demands in the form of non-increasing step functions. Figure 4.6 is a depiction of 100 simulated residual BES demands together with the supply schedule of Luminant.

Next, we find the simulated market clearing prices at the intersection of the supply schedule of the bidder we are interested in with each of the simulated residual demands. Since we have discrete data points for the supply schedules and the residual demands, we interpolate both of these to find the values at a finer grid. This way, we get a smoother estimate of the $H_i(p|S_i(p), QC_i)$, belief of bidder i about the market clearing price distribution.

Figure 4.6. Simulated residual demands



Previous studies of ERCOT's BES market show that larger participants in the market come closer to the optimal strategies implied by a profit maximization assumption. Hence, we estimate the market clearing price distributions H for the three largest companies bidding at the BES market: Luminant, NRG and Calpine.

We estimate H non-parametrically with a normal kernel. For all the companies, we choose the smallest window width that gives a non-zero estimate for H_p (since it is in the denominator in Equation 4.2) for all the price levels that company has bid for. (selected window widths are 15, 10 and 20 for Luminant, NRG and Calpine respectively)

Figures 4.7 and 4.8 show the non-parametric estimates of the believes of three big companies regarding the distribution and density of the market clearing price, conditional on their own contract quantity and bid.

Finally, to estimate H_S, we shift the supply schedules $S(p)$ of the three companies by 20 MWs in a range of -200 and $+200$ MWs around their true position. For each

Figure 4.7. Estimated distribution of market clearing price for three large companies

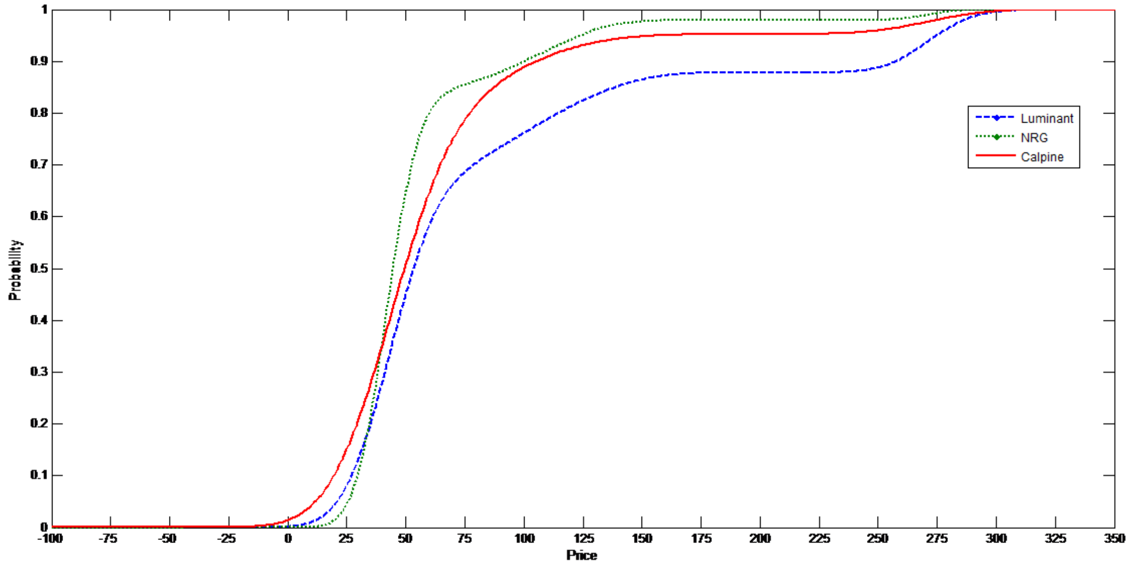
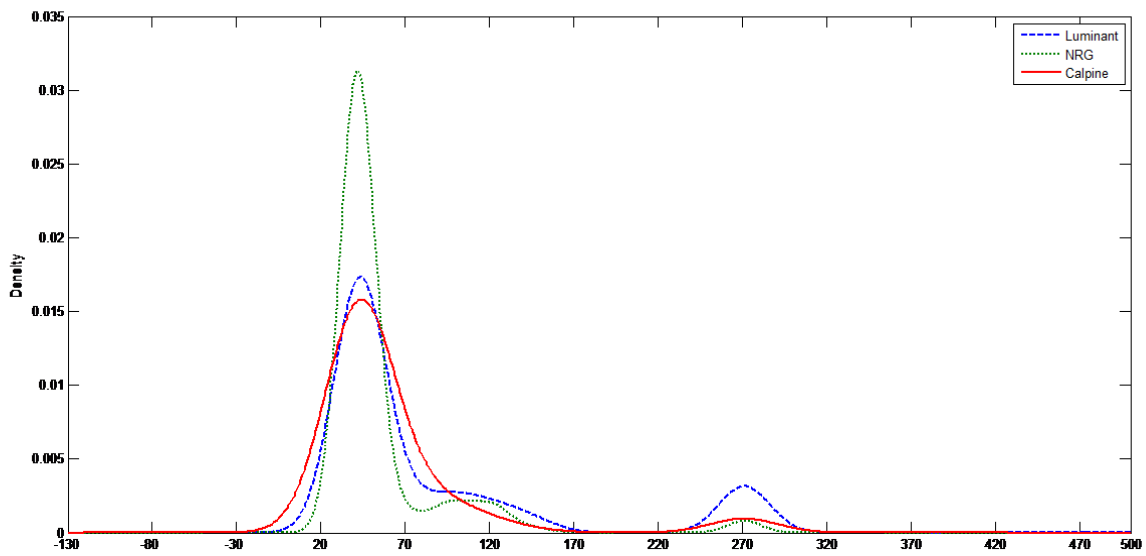


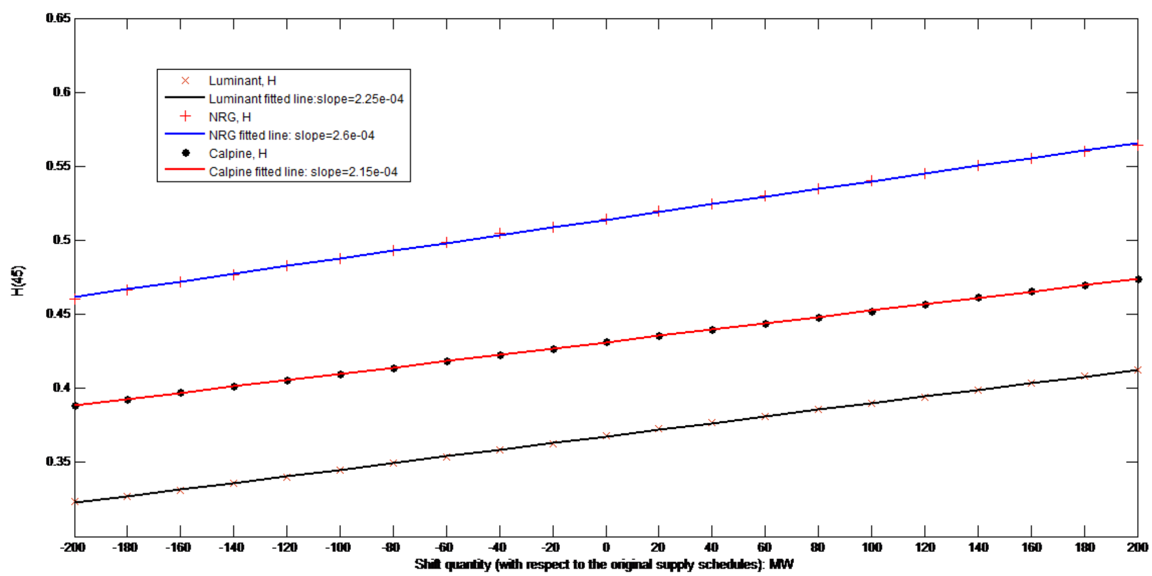
Figure 4.8. Estimated densities of market clearing price for three large companies



of the shifted supply schedules we find the new intersections with the 5000 simulated residual BES demand, then estimate H using the 5000 market clearing prices. In other words for each shift, we re-estimate H. Then, at a given price level p, we look at how much a shift in $S(p)$ has changed $H(p)$.

For a given company, we regress the values of $H(p)$ for shifted supply schedules on a constant and a vector of shift quantities $(-200, -180, -160, ..., 0, ..., 160, 180, 200)$. The estimated slope parameter of this linear fit is our estimate for $H_S(p)$ [19]. Figure 4.9 depicts the estimated values of $H(45)$ (the average market clearing price occurred as \$45.85 during the hour) for the original and shifted supply schedules of Luminant, NRG and Calpine. (shift quantity being zero represents the original supply schedule of the company).

Figure 4.9. $H_S(p)$ at $p = \$45$: Luminant, NRG and Calpine

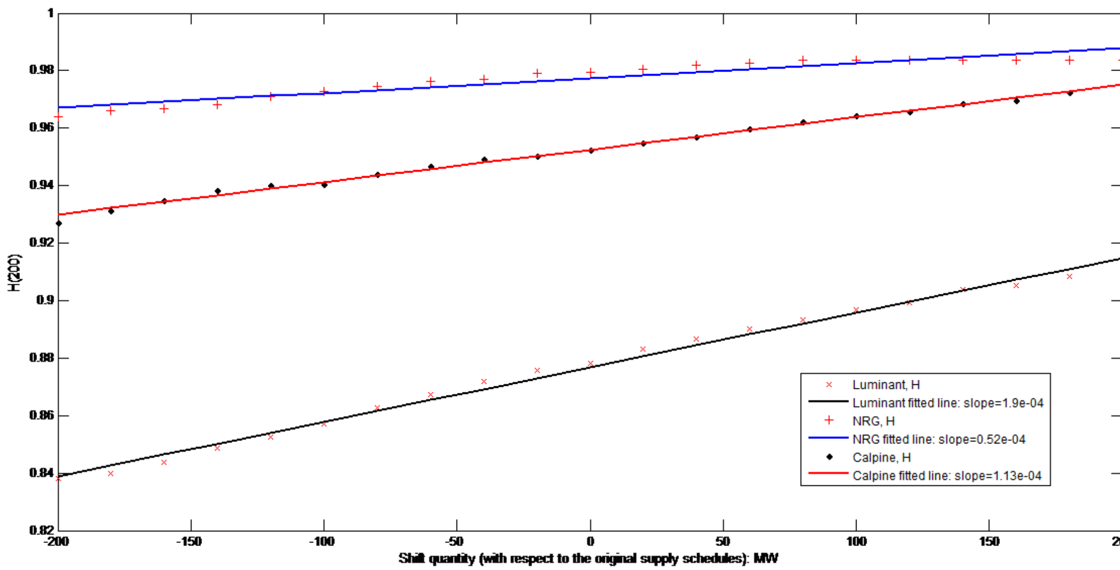


Regression analysis reveals that at the market clearing price $p = \$45$, Luminant, NRG and Calpine have close estimates of $H_S(p)$: $2.25e-04$, $2.6e-04$, $2.15e-04$ respectively. $H_S(p)$ is the ability of a bidder to manipulate the distribution of market clearing price by changing its supply. Therefore, as has also been argued by Hortacsu

[19]Looking at the figures, it is obvious that these points have a linear pattern for the given price levels.

and Puller (2008), this term captures the market power of the bidder. Estimated numbers say that NRG has the highest H_S around the real market clearing price at the 5 pm auction on 29 September 2010, while Calpine has the lowest. On the other hand, at price \$200, Luminant has the highest and NRG has the lowest H_S with the estimates $1.9e-04$, $1.13e-04$ and $0.52e-04$ in decreasing order.

Figure 4.10. $H_S(p)$ at $p = \$200$: Luminant, NRG, Calpine



At both price levels, the distributions increase as the supply schedules of the bidders are shifted to the right. This is expected, since increasing the supply at every price level (i.e. shifting the supply curve to the right) can only weakly decrease the market clearing price, hence increase its distribution. ($H_S(p)$ is always non-negative for all bidders, for all p)

Moreover, note that $\frac{H_S(p.S(p))}{H_p(p,S(p))} = \frac{\frac{\partial H}{\partial p}\cdot\frac{\partial p}{\partial S}}{\frac{\partial H}{\partial p}} = \frac{\partial p}{\partial S}$. Hence, the ratio can be interpreted as the ability of a bidder to change the market clearing price by changing its supply. The estimates of this ratio for the price level $p = 45$ are 0.0134, 0.0087 and 0.015

for Luminant, NRG and Calpine respectively. For $p = 200$ the estimates are 9.3243, $1.49e + 011$, and 4.5834 for the same order of the three bidders. We observe that all the bidders have much higher ability to change the market clearing price at the high price level $p = 200$. This is expected as the bidders have bid much steeper supply schedules for the prices around $200, while they have bid almost horizontal around $p = 45$. (This is called *hockey-stick* bidding).

Finally, we plug estimated $H_S(p)$ and $H_p(p)$ in the markup expression derived from the first order optimality condition Equation 4.2, together with the price-quantity $(p, S_i^*(p))$ pairs bid by the three bidders we are interested in:

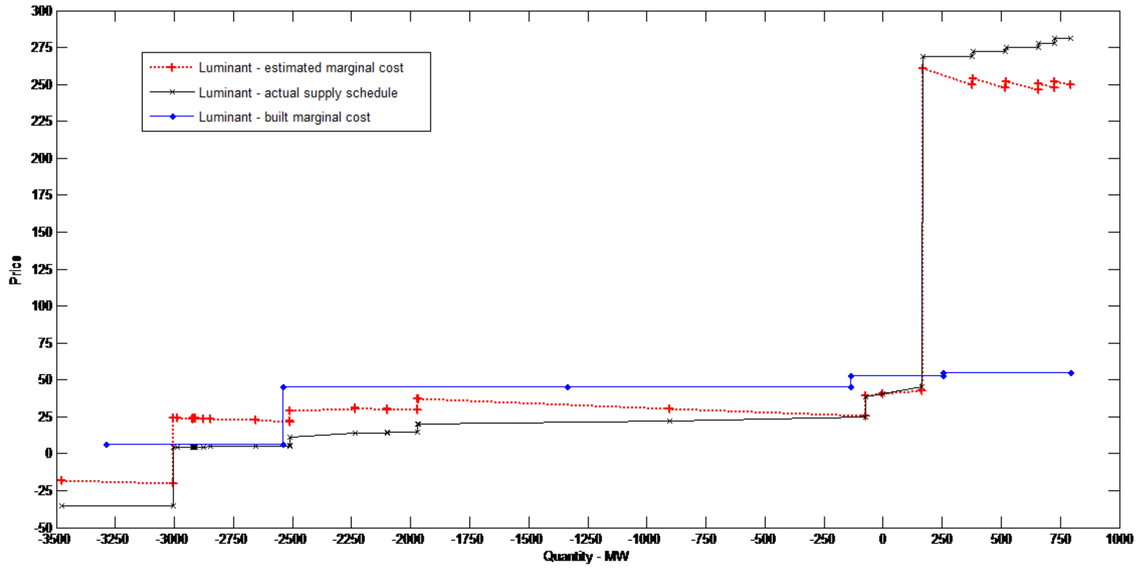
$$p - C_i'(S_i^*(p)) = (S_i^*(p) - QC_i)\frac{H_s(p|S_i^*(p), QC_i)}{H_p(p|S_i^*(p), QC_i)}$$

We solve for the marginal cost $C_i'(S_i^*(p))$ of generating $S_i^*(p)$ for all p that exists in the supply schedule of bidder i. Results for the three bidders can be seen in figures 4.11 to4.15.

4.5.2.1. Discussion of the estimation results. Figures 4.11 to 4.15 depict the bids, estimated marginal costs and actual marginal costs of the three largest bidders, Luminant, NRG and Calpine, in BES market. One thing that is easy to notice is that the estimated marginal costs are not necessarily non-decreasing. This is a result of not imposing any functional form or structure on the supply schedules.

Next, note that the magnitude of the positive markup to the right of the contract quantity (and the mark-down to the left) depends on the difference of the quantity bid at that price with the contract quantity level (i.e. long or short position), and

Figure 4.11. Luminant's bid, estimated MC and actual MC

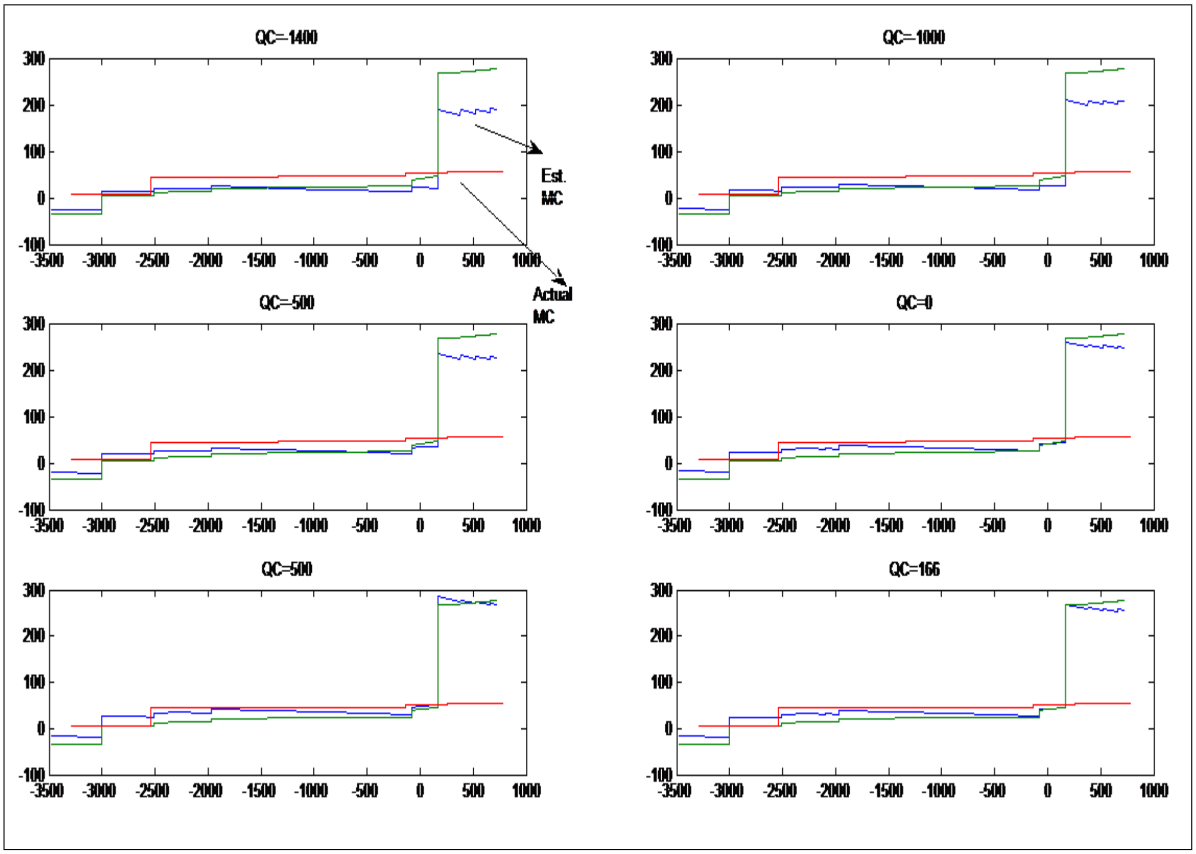


the $\frac{H_S}{H_p}$ ratio. So, the bidder puts a higher markup on its cost when it has exceeded its contracted quantity level, and when H_S is higher. As we have discussed earlier that the term H_S captures the market power of the firm, we can say that equilibrium strategies of the model take into account market power of bidders.

Figure 4.11 depicts Luminant's estimated MC for QC=0. Figure 4.12 gives the estimated MC for several different assumed QC levels. This demonstrates how the choice of the contract quantity changes the estimated markup. As implied by the First order condition in equation 4.2, the assumed contract quantity level determines the point of intersection of the estimated marginal cost and the supply schedule of the bidder.

Focusing only on two contract quantity levels, $QC = 0$, which is the level assumed at the beginning of the study, and $QC = 166$ which is implied by the actual cost, we see that at the realized market clearing price, the 0 contract quantity level implies

Figure 4.12. Luminant's bid, actual MC and estimated MC for different QC's



a small profit, while the other one implies that the generator just covers its cost. However, since these contract levels are close to each other, the general look of the markups and markdowns do not differ a lot. We estimate a negative mark-up around \$20-\$25, that diminishes to zero at the contract quantity level. On the long position side we estimate a positive markup aroud \$25-\$30. However, the actual cost implies mark downs as large as double the magnitude of the mark-downs implied by the estimated cost. And on the mark-up side, actual cost implies close to ten times the estimated mark-up.

Figure 4.13. NRG's bid, estimated MC and actual MC

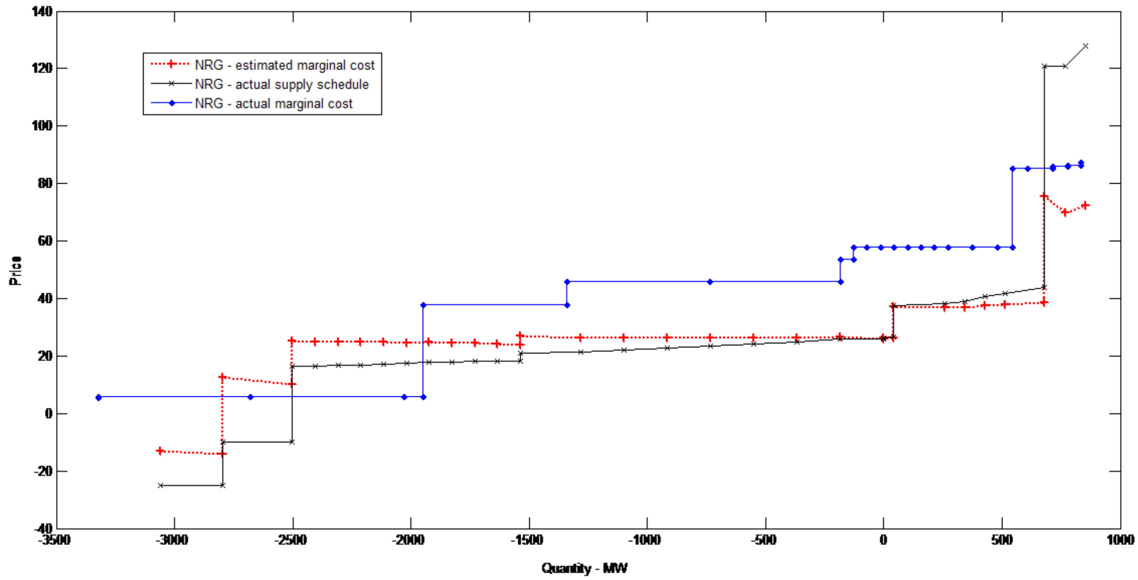
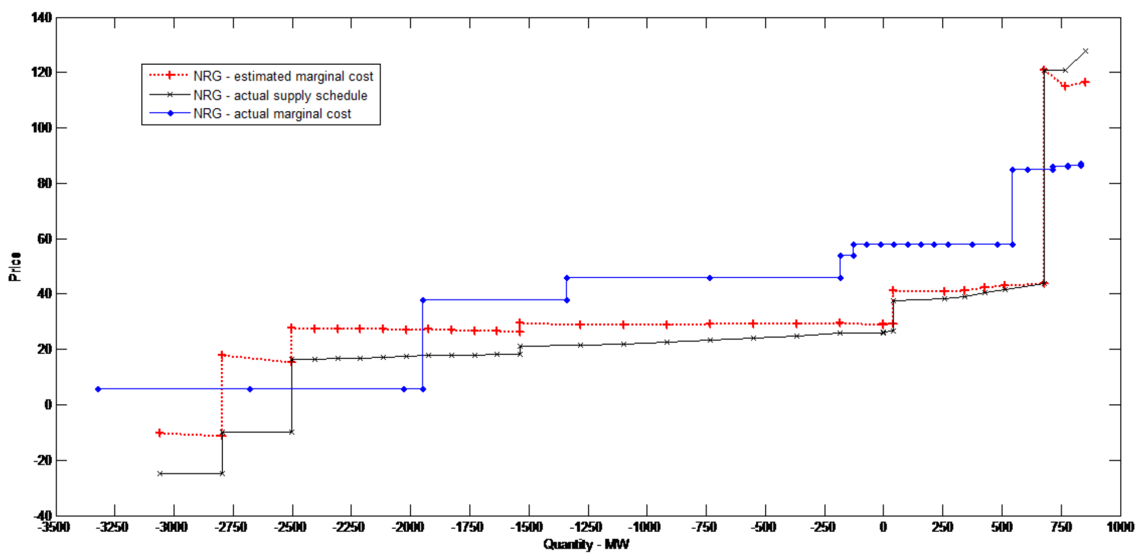
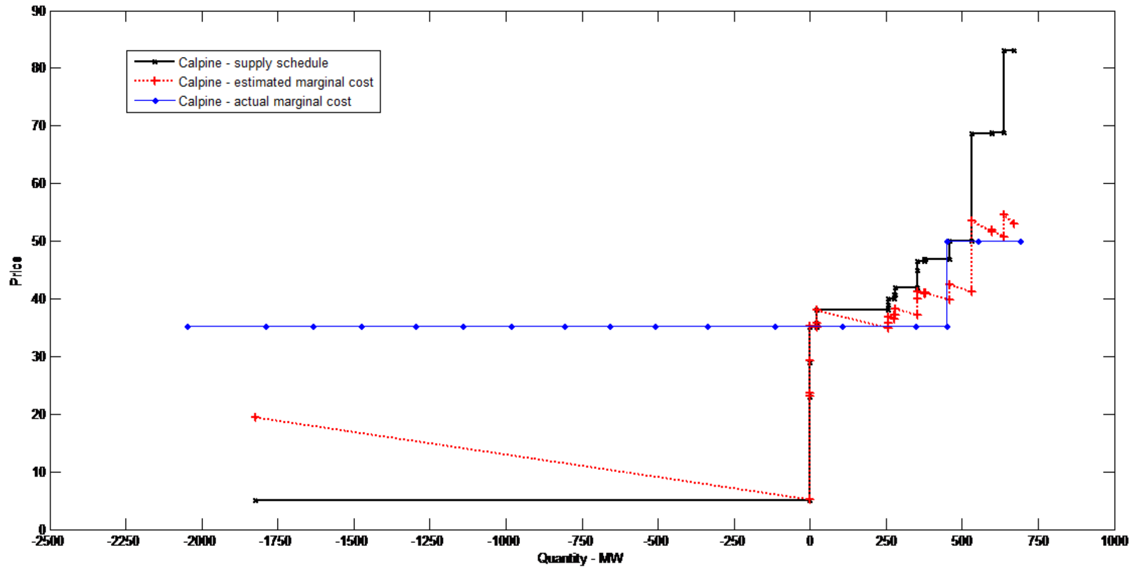


Figure 4.14. NRG's bid, estimated MC and actual MC: QC=678 MW's



Figures 4.13 and 4.14 give two estimates of the implied marginal cost for NRG for the cases of day ahead schedule exactly covering the contract quantity, and of day ahead schedule's being 678 MWs short of contract quantity respectively. 678 MW

Figure 4.15. Calpine's bid, estimated MC and actual MC



is the quantity where NRG's supply schedule. intersects with its actual marginal cost. This is the contract quantity level implied by Hortacsu and Puller's behavioral assumption deduced from the first order optimality condition in Equation 4.2: firms bid lower than their marginal cost for the quantities which still are not enough to cover their contract quantity, and they bid with a positive markup for the quantities above their contract quantity, and finally, they bid their marginal cost for the quantity that exactly covers their contract level. As can be observed from the two figures, assuming a contract quantity of 678 MW results in a marginal cost estimate which is more comparable with the actual marginal cost. Although our purpose in this study is to find the marginal costs implied by the model, the cost built on the heat rate and fuel type information can be taken as the actual cost.

On the side where the bidder is short of its contract quantity, taking the contract quantity level implied by the actual cost, we estimate a mark-down starting from around \$15 and diminishing to zero at the contract quantity level. On the side where NRG is bidding for more than its contracted quantity, we estimate a markup around \$10-\$15 on top of its marginal cost.

However, when we compare these estimated markups with the markups implied by actual costs, actual cost implies greater markups (in absolute value) The magnitude of the actual markup and markdowns can go up to 6-8 times the estimated ones.

Therefore, at the auction we analyze, both Luminant and NRG have larger markdowns and markups than the optimal levels implied by the Bayesian-Nash equilibrium of the multi-unit auction model of the BES market.

Calpine is the smallest of the three firms we are analyzing. It also has the least number of price-quantitiy pairs in its supply schedule for the period we look at. The estimated cost of Calpine shows a dramatic level of non-monotonic behavior, mostly due to the few price-quantity pairs in its submitted supply schedule. Calpine's actual cost intersects its supply schedule at quantity 0, hence implying a 0 contract quantity. For this level of the contract quantity figure 4.15 shows that on the negative side, Calpine has an estimated mark-down of close to \$7 on average. On the positive side, we estimate varying levels of mark-up from \$0 to \$20, increasing with the quantity. But the general trend of estimated cost is similar to the other two. When we look at the actual cost, on the negative side it implies a mark-down around \$30, which is 4 to 5 times higher than the estimated one. On the positive side, the actual cost generally follows the estimated cost very closely. However, as not seen with the other two firms, actual markup is smaller than the estimated markup for some quantity levels. The average difference between the actual and estimated markup for Calpine is around \$5, which makes Calpine the bidder coming closest to bidding the optimal implied by the model.

4.6. Conclusion

In this chapter, we estimated the costs in a multi-unit auction model of the BES. In doing so, we wanted to improve the model of Hortacsu and Puller (2008) by removing the restrictive a priori assumption of additive separability of the supply schedules with respect to the private information of bidders. We also tried to draw attention to the identification of contract quantities and how it changes the interpretation of findings. We found that, although not necessarily non-decreasing, the marginal cost functions implied by the Bayesian-Nash equilibrium of the multi-unit auction model of the BES are informative about the actual cost levels.

The non-parametric resampling based estimator we used in this study does not require us to pool data from different auctions. This saves us having to control for covariates and worry about possible unobserved heterogeneity, which could render our estimator biased.

In summary, we find that, for the BES auction we study, the three largest bidders, Luminant, NRG and Calpine, have marked-down their bids more than the optimal amount implied by the model for the quantities where they were short of their contractual obligations, while they have put a mark-up larger than the optimal level implied by the model for quantities in excess of their contract obligations. Among the three bidders we studied, Calpine has come closest to bidding its optimal implied by the Bayesian-Nash equilibrium of the multi-unit auction model of the BES market.

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