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MAXIMUM LIKELIHOOD MULTIPATH CHANNEL PARAMETER ESTIMATION IN CDMA SYSTEMS

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ABSTRACT

The problem addressed in this paper is the estimation of the channel parameters in a Code Division Multiple Access (CDMA) communication system, in the presence of multipath effects. Maximum likelihood estimation of these parameters has been investigated in the past with the main drawback being the complexity of the multi-dimensional algorithms. The algorithm presented in this paper elegantly decomposes the multiuser problem into a series of single user problems. The algorithm first estimates a composite channel impulse response of each user and then extracts the channel parameters of all the paths of each user from the channel impulse response. We evaluate the performance of the algorithm through simulation studies.

1. INTRODUCTION

In a *Code Division Multiple Access (CDMA)* communication system, a communication channel with a given bandwidth is accessed by all the users simultaneously. The different mobile users are distinguished at the base station receiver by the unique *spreading code* assigned to the users to modulate their signals. Hence, the CDMA signal transmitted by any given user consists of that user's data which modulates the unique spreading code assigned to that user, which in turn modulates a carrier (the frequency of which is the same for all users), using any well-known modulation scheme such as *binary phase shift keying (BPSK)*. The receiver receives a linear superposition of the signals transmitted by all the users, attenuated by arbitrary factors and delayed by an arbitrary amount. The goal of channel parameter estimation is to determine these unknown and time varying attenuation factors and delays by processing the received signal, to facilitate recovery of the data transmitted by each user.

Channel estimation is one of the major problems in radio communications particularly when the mobile system is subject to multipath fading, that is, the transmission channel consists of more than one distinct propagation path for each user's signal. Moreover, when the CDMA technique is used to allow multiple users access to a single channel, the system is susceptible to the *near-far* effect. The near-far problem arises when the signals from the different users arrive at the receiver with widely varying power levels. The near-far problem has been shown to severely degrade the performance of standard single user techniques (e.g., matched filters, correlators, etc.) in conventional CDMA systems.

Conventional CDMA systems try to limit the near-far problem with power control. However, even a small amount of the near-far effect can drastically degrade the performance of conventional receivers. For many years this was thought to be an inherent limitation of CDMA until Verdú developed the optimum multiuser detector [1]. Verdú's work was followed by many suboptimal schemes of lower computational complexity [2, 3, 4], all of which are near-far resistant. However, these methods deal only with detection and assume that the timing of the spreading waveforms is known.

Most of the initial work done on timing acquisition for CDMA systems focused on jointly estimating the necessary parameters for all users [5, 6]. While these techniques produce excellent results, they can be computationally intense since they involve solving a multidimensional optimization problem for a large number of parameters.

In this paper we focus on a maximum likelihood (ML) approach that decomposes the multiuser problem into a series of single user ones. The algorithm draws upon certain computationally elegant features of the maximum likelihood approach presented in [7] and [8]. The main contribution of this paper is the development of an algorithm which can work in a multipath environment and estimate the delays and amplitudes of all the significant propagation paths of all the users in a computationally efficient manner. The algorithm assumes the transmission of training sequences by all the users being acquired. The algorithm we describe makes no assumption whatsoever on the individual delays. The delays are estimated modulo N, where

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N is the length of each spreading code. The additive noise is assumed to be a circularly complex zero mean Gaussian random vector, but no *a priori* assumption is made on its covariance.

The paper is organized into the following sections. Section 2 presents the CDMA system under study and a model for the multipath channel. In Section 3 we describe the maximum likelihood algorithm. Section 4 analyzes the complexity and Section 5 presents the results of our simulations. We conclude the paper in Section 6 with a summary and directions for future research.

2. CDMA SYSTEM AND MULTIPATH CHANNEL MODEL

We assume a K-user narrow band direct sequence CDMA system with BPSK (Binary Phase Shift Keying) modulation with each transmitted signal selected from a binary alphabet and limited to $[0, T]$, where T is the symbol period. Each user transmits a zero mean stationary bit sequence with i.i.d. components and different users are independent of each other.

The complex baseband representation of the k^{th} user's transmitted signal is given by

$$s_k(t) = \sqrt{2P_k} \sum_i b_{k,i} c_k(t - iT) \ , \qquad (1)$$

where P_k is the transmitted power, $b_{k,i} \in \{+1, \ -1\}$ is the i^{th} transmitted bit and $c_k(t)$ is the spreading waveform. The spreading or code waveform is composed of N chips and if we assume BPSK for the spreading modulation we have $c_k(t) = \sum_{n=0}^{N-1} c_{k,n} \Pi(t - nT_c)$, where $c_{k,n} \in \{+1, \ -1\}$ and the chip pulse waveform $\Pi(t)$ is a rectangular pulse of duration T_c. We will assume that the extent of the spreading code is one bit period and hence we have $T = NT_c$.

The received signal at the base station is a superposition of attenuated and delayed signals transmitted by all the K users and is given by

$$r(t) = \sum_{k=1}^{K} w_k s(t - \tau_k) + \nu(t) \qquad (2)$$

where w_k is the complex amplitude with which the k^{th} signal is received and includes contributions from the channel attenuation and the phase offset; τ_k is the relative delay with respect to a reference at the receiver. The noise component $\nu(t)$ is assumed to be Gaussian with zero-mean and double-sided spectral density of $\mathcal{N}_0/2$.

The continuous time signal at the receiver is discretized by sampling the output of a chip-matched filter at the chip rate. The observation vector at time i, $\mathbf{r}_i \in \mathbb{C}^N$, is then formed by collecting N such outputs together. Since the system is asynchronous, each observation vector can be viewed as a linear combination of $2K$ *signal vectors* – 2 components from each user due to the past and current bits as shown in figure 1. We can now write $\mathbf{r}_i$ as :

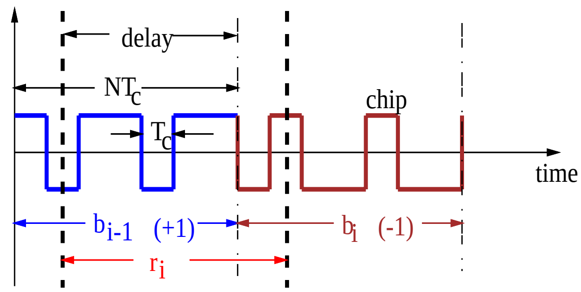

Figure 1: System model - received signal

$$\mathbf{r}_i = \mathbf{A}\mathbf{W}\mathbf{b}_i + \boldsymbol{\nu}_i \ , \qquad \boldsymbol{\nu}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{K}) \qquad (3)$$

where $\mathbf{A}$ is the matrix of the "signal vectors" which depend only on the spreading codes (known) and delays (unknown) of the users, $\mathbf{W}$ is a diagonal matrix of complex amplitudes (unknown), $\mathbf{b}_i$ contain the users' data bits, and $\mathbf{K}$ is the unknown noise covariance.

$\mathbf{W}$ is a $2K \times 2K$ diagonal matrix of the form :

$$\mathbf{W} = \text{diag}[w_1, w_1, \dots, w_K, w_K],$$

and the $2K$ length vector $\mathbf{b}_i$ is of the form :

$$[b_{1,i-1}, b_{1,i}, \dots, b_{k,i-1}, b_{k,i}]^T,$$

where $b_{k,i}$ is the i^{th} bit of the k^{th} user. The code response matrix $\mathbf{A}(\tau) \in \mathbb{C}^{N \times 2K}$ has columns corresponding to two adjacent bits of each user :

$$\mathbf{A}(\tau) \triangleq [\mathbf{a}_1^R(\tau_1) \quad \mathbf{a}_1^L(\tau_1) \quad \cdots \quad \mathbf{a}_K^R(\tau_K) \quad \mathbf{a}_K^L(\tau_K)] \qquad (4)$$

If T_c is the chip period, let $\tau_k/T_c = q + \gamma$, $q \in \{0, 1, \cdots, N-1\}$, $\gamma \in [0, 1)$, we have :

$$\mathbf{a}_K^R(\tau_K) = (1-\gamma)\mathbf{c}_K^R[q] + \gamma \mathbf{c}_K^R[q+1],$$

$$\mathbf{a}_K^L(\tau_K) = (1-\gamma)\mathbf{c}_K^L[q] + \gamma \mathbf{c}_K^L[q+1] \qquad (5)$$

where $\mathbf{c}_K^R[q]$ and $\mathbf{c}_K^L[q]$ are the spreading codes shifted by integer (multiples of chips) delays.

$$\mathbf{c}_K^R[q] = [c_{K,N-q} \cdots c_{K,N-1} \ 0 \cdots 0]^T$$

$$\mathbf{c}_K^L[q] = [0 \cdots 0 \ c_{K,0} \cdots c_{K,N-q-1}]^T \qquad (6)$$

In order to efficiently model the multipath effect we will rearrange equation 3. We write the product of matrices $\mathbf{A}$ and $\mathbf{W}$ as:

$$\begin{aligned} \mathbf{A}\mathbf{W} &= [w_1\mathbf{a}_1^R \quad w_1\mathbf{a}_1^L \quad \cdots \quad w_K\mathbf{a}_K^R \quad w_K\mathbf{a}_K^L] \\ &= [\mathbf{u}_1^R \quad \mathbf{u}_1^L \quad \cdots \quad \mathbf{u}_K^R \quad \mathbf{u}_K^L] \qquad (7) \end{aligned}$$

.

$$\mathbf{u}_k^R = w_k\{(1-\gamma)\mathbf{c}_k^R[q] + \gamma\mathbf{c}_k^R[q+1]\}$$

$$\mathbf{u}_k^L = w_k\{(1-\gamma)\mathbf{c}_k^L[q] + \gamma\mathbf{c}_k^L[q+1]\}$$

i.e. $\mathbf{u}_k$-s are linear combinations of $\mathbf{c}_k[q]$-s.

So the $\mathbf{u}_k$-s can be rewritten as the product of a matrix ($\mathbf{U}_k$) containing all possible $\mathbf{c}_k[q]$-s, or spreading codes shifted by all possible integer delays between 0 and $(N-1)$ and a vector ($\mathbf{h}_k$) which provides the appropriate weights:

$$\mathbf{u}_k^R = \mathbf{U}_k^R \mathbf{h}_k, \quad \mathbf{u}_k^L = \mathbf{U}_k^L \mathbf{h}_k, \quad \mathbf{U}_k \in \mathbb{C}^{N\times N}, \quad \mathbf{h}_k \in \mathbb{C}^{N\times 1}$$
$$\mathbf{U}_k^{(R)} = [\mathbf{c}_k^{(R)}[0] \cdots \mathbf{c}_k^{(R)}[N-1]],$$
$$\mathbf{U}_k^{(L)} = [\mathbf{c}_k^{(L)}[0] \cdots \mathbf{c}_k^{(L)}[N-1]],$$
$$\mathbf{h}_k = \begin{bmatrix} 0 \\ \vdots \\ w_k(1-\gamma) \\ w_k\gamma \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow q^{th} entry \\ \leftarrow \quad (q+1)^{th} \\ \\ \\ \end{matrix} \tag{8}$$

Rewriting matrix $\mathbf{AW} = [\mathbf{u}_1^R \;\; \mathbf{u}_1^L \;\; \cdots \;\; \mathbf{u}_K^R \;\; \mathbf{u}_K^L] = [\mathbf{U}_1^R\mathbf{h}_1 \;\; \mathbf{U}_1^L\mathbf{h}_1 \;\; \cdots \;\; \mathbf{U}_K^R\mathbf{h}_K \;\; \mathbf{U}_K^L\mathbf{h}_K]$. Hence we can write $\mathbf{AW} = \mathbf{UH}, \quad \mathbf{U} \in \mathbb{C}^{N\times 2NK}, \quad \mathbf{H} \in \mathbb{C}^{2NK\times 2K}$:

$$\mathbf{U} = [\mathbf{U}_1^R \quad \mathbf{U}_1^L \quad \cdots \quad \mathbf{U}_K^R \quad \mathbf{U}_K^L];$$
$$\mathbf{H} = diag(\mathbf{h}_1, \mathbf{h}_1, \cdots \mathbf{h}_K, \mathbf{h}_K)$$

This allows us to rewrite equation 3 as :

$$\mathbf{r}_i = \mathbf{UHb}_i + \boldsymbol{\nu}_i, \tag{9}$$

where $\mathbf{U}$ is a known matrix of spreading codes and $\mathbf{H}$ has all the unknown parameters of all users. The goal of this paper is to estimate the matrix $\mathbf{H}$.

The advantage to be gained from rewriting equation 3 as equation 9 is that it allows easy modeling of multiple propagation paths without increasing the size of any of the matrices involved which is directly related to the computational load of the algorithm. In equation 3, as the number of paths, P, increases, the size of matrices $\mathbf{A}$ and $\mathbf{W}$ also increases. However in equation 9, the size of matrices $\mathbf{U}$ and $\mathbf{H}$ does not increase, as P increases; instead matrix $\mathbf{H}$ becomes more dense, as shown below.

Let each user have P paths, with corresponding delays and amplitudes of $\tau_{k,p}$, $w_{k,p}$; $\tau_{k,p}/T_c = q_{k,p} + \gamma_{k,p}$ where $q_{k,p}$ is the integer part and $\gamma_{k,p}$ is the fractional part of the delay. The multiple paths can now be incorporated in the channel impulse response vector $\mathbf{h}_k$, of user k as :

$$\mathbf{h}_k = \begin{bmatrix} 0 \\ \vdots \\ w_{k,1}(1-\gamma_{k,1}) \\ w_{k,1}\gamma_{k,1} \\ \vdots \\ w_{k,P}(1-\gamma_{k,P}) \\ w_{k,P}\gamma_{k,P} \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow q_{k,1}^{th} \\ \leftarrow (q+1)_{k,1}^{th} \\ \\ \leftarrow q_{k,P}^{th} \\ \leftarrow (q+1)_{k,P}^{th} \\ \\ \\ \end{matrix} \tag{10}$$

Now $\mathbf{H}$ has all the unknown parameters of all the paths of all the users and will be estimated from the observations $\mathbf{r}_i$-s, the known sequence of transmitted bits (preamble), and the knowledge of the spreading codes of the different users.

3. PROPOSED CHANNEL ESTIMATION ALGORITHM

Given L observations $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_L$ the log-likelihood function (Λ) of these observations can be expressed as

$$-\ln|\mathbf{K}| - \operatorname{tr}\left\{\frac{1}{L}\sum_{i=1}^{L}(\mathbf{r}_i - \mathbf{UHb}_i)^H\mathbf{K}^{-1}(\mathbf{r}_i - \mathbf{UHb}_i)\right\}$$
$$= -\ln|\mathbf{K}| - \operatorname{tr}\left\{\mathbf{K}^{-1}\frac{1}{L}\sum_{i=1}^{L}(\mathbf{r}_i - \mathbf{UHb}_i)(\mathbf{r}_i - \mathbf{UHb}_i)^H\right\} \tag{11}$$

where $|\cdot|$ represents the determinant operator and $\mathrm{tr}(\cdot)$, the trace operator. Maximization of the log-likelihood function is to be carried out with respect to $\{\boldsymbol{\tau}, \mathbf{w}, \mathbf{K}\}$. This problem is ill-defined if $\mathbf{b}_1, \ldots, \mathbf{b}_L$ are unknown and hence the need for training sequences or preamble. The maximum with respect to $\mathbf{K}$ is easily found to be

$$\widehat{\mathbf{K}}(\mathbf{U}, \mathbf{H}) = \frac{1}{L}\sum_{i=1}^{L}(\mathbf{r}_i - \mathbf{UHb}_i)(\mathbf{r}_i - \mathbf{UHb}_i)^H. \tag{12}$$

Substituting this into equation 11, we find that we need to maximize $-\ln|\widehat{\mathbf{K}}|$ or minimize $|\widehat{\mathbf{K}}|$ over all $\{\boldsymbol{\tau}, \mathbf{w}\}$ or $\{\mathbf{H}\}$. Direct minimization of $|\widehat{\mathbf{K}}|$ with respect to $\{\boldsymbol{\tau}, \mathbf{w}\}$ is rather intractable and hence, it is carried out in three steps [7], [8]:

(i) Capture the effect of all the unknowns in a single $N \times 2K$ complex matrix $\mathcal{Y} \stackrel{\Delta}{=} \mathbf{UH}$. Form the unconstrained ML estimate of $\mathcal{Y}$, given by $\widehat{\mathcal{Y}}$.

(ii) Having obtained $\widehat{\mathcal{Y}}$, obtain the estimates $\hat{\mathbf{h}}_k$ by minimizing the weighted least squares fit between $\mathcal{Y}$ and its unstructured estimate, $\widehat{\mathcal{Y}}$.

(iii) Having obtained $\hat{\mathbf{h}}_k$, extract the $\hat{\boldsymbol{\tau}}$ and, $\hat{\mathbf{w}}$ through a least squares fit of $\hat{\mathbf{h}}_k$ to our parametric channel model in equation 10.

3.1. Step 1: Covariance approximation

Let us define correlation matrices: $\widehat{\mathbf{R}}_{rr} = \frac{1}{L}\sum_{i=1}^{L}\mathbf{r}_i\mathbf{r}_i^H$, $\widehat{\mathbf{R}}_{br} = \frac{1}{L}\sum_{i=1}^{L}\mathbf{b}_i\mathbf{r}_i^H$, $\widehat{\mathbf{R}}_{bb} = \frac{1}{L}\sum_{i=1}^{L}\mathbf{b}_i\mathbf{b}_i^T$. In terms of the sample correlation matrices, $\widehat{\mathbf{K}}(\mathcal{Y})$ can be written as:

$$\widehat{\mathbf{K}}(\mathcal{Y}) = \widehat{\mathbf{R}}_{rr} - \mathcal{Y}\widehat{\mathbf{R}}_{rb}^H - \widehat{\mathbf{R}}_{rb}\mathcal{Y}^H + \mathcal{Y}\widehat{\mathbf{R}}_{bb}\mathcal{Y}^H.$$

So, $\widehat{\mathcal{Y}}$, the unconstrained ML estimate of $\mathcal{Y}$ is:

$$\widehat{\mathcal{Y}} = \arg\min_{y}|\widehat{\mathbf{R}}_{rr} - \mathcal{Y}\widehat{\mathbf{R}}_{rb}^H - \widehat{\mathbf{R}}_{rb}\mathcal{Y}^H + \mathcal{Y}\widehat{\mathbf{R}}_{bb}\mathcal{Y}^H|$$

This can be shown to be :

$$\widehat{\mathcal{Y}} = \widehat{\mathbf{R}}_{rb}\widehat{\mathbf{R}}_{bb}^{-1}. \tag{13}$$

Substituting back,

$$\widehat{\mathbf{K}}(\widehat{\mathcal{Y}}) = \widehat{\mathbf{R}}_{rr} - \widehat{\mathcal{Y}}\widehat{\mathbf{R}}_{rb}^H \tag{14}$$

Now we can rewrite the cost function using expressions for $\widehat{\mathcal{Y}}$ and $\widehat{\mathbf{K}}$.

3.2. Step 2: Channel impulse response estimation

Using expressions for $\widehat{\mathcal{Y}}$ and $\widehat{\mathbf{K}}$, estimation of $\mathbf{H}$ after some simple algebraic steps can be shown to be ($\widehat{\mathcal{Y}} = \mathbf{U}\widehat{\mathbf{H}}$):

$$\widehat{\mathbf{H}} = \arg\min_{\mathbf{H}} \operatorname{tr}\left\{\widehat{\mathbf{R}}_{bb}(\mathcal{Y} - \widehat{\mathcal{Y}})^H \widehat{\mathbf{K}}^{-1}(\mathcal{Y} - \widehat{\mathcal{Y}})\right\}$$

Since the received signals of the different users are "uncorrelated", $\widehat{\mathbf{R}}_{bb}$ is block diagonal for large L. This diagonal form helps to dramatically separate the estimation process of each user. So, we can estimate the k^{th} user's channel impulse response to be:

$$\widehat{\mathbf{h}}_k = \arg\min_{\mathbf{h}_k} \Big[(\mathbf{y}_{2k-1} - \hat{\mathbf{y}}_{2k-1})^H \widehat{\mathbf{K}}^{-1}(\mathbf{y}_{2k-1} - \hat{\mathbf{y}}_{2k-1}) + (\mathbf{y}_{2k} - \hat{\mathbf{y}}_{2k})^H \widehat{\mathbf{K}}^{-1}(\mathbf{y}_{2k} - \hat{\mathbf{y}}_{2k})\Big]$$

For each user we have contributions for right and left signal vectors, i.e., the $2k^{th}$ and $(2k-1)^{th}$ columns of $\mathcal{Y}$. Let us recall that the columns of $\mathcal{Y}$ are nothing else but the $\mathbf{u}_k$-s in equation 7:

$$\mathbf{y}_{2k-1} = \mathbf{u}_k^R = \mathbf{U}_k^R \mathbf{h}_k, \quad \mathbf{y}_{2k} = \mathbf{u}_k^L = \mathbf{U}_k^L \mathbf{h}_k$$

Hence, the estimate of $\mathbf{h}_k$ is :

$$\widehat{\mathbf{h}}_k = \arg\min_{\mathbf{h}_k} \Big[(\mathbf{U}_k^R\mathbf{h}_k - \hat{\mathbf{y}}_{2k-1})^H \widehat{\mathbf{K}}^{-1}(\mathbf{U}_k^R\mathbf{h}_k - \hat{\mathbf{y}}_{2k-1}) + (\mathbf{U}_k^L\mathbf{h}_k - \hat{\mathbf{y}}_{2k})^H \widehat{\mathbf{K}}^{-1}(\mathbf{U}_k^L\mathbf{h}_k - \hat{\mathbf{y}}_{2k})\Big]$$

This can be shown to be :

$$\widehat{\mathbf{h}}_k^H = (\hat{\mathbf{y}}_{2k-1}^H \widehat{\mathbf{K}}^{-1}\mathbf{U}_k^R + \hat{\mathbf{y}}_{2k}^H \widehat{\mathbf{K}}^{-1}\mathbf{U}_k^L)(\mathbf{U}_k^{R^H} \widehat{\mathbf{K}}^{-1}\mathbf{U}_k^R + \mathbf{U}_k^{L^H} \widehat{\mathbf{K}}^{-1}\mathbf{U}_k^L)^{-1} \tag{15}$$

3.3. Step 3: Channel parameter extraction

The last step is the least squares fit of $\widehat{\mathbf{h}}_k$ to our parametric channel model to extract the strongest P paths. For each pair of adjacent coefficients of $\widehat{\mathbf{h}}_k$

$$[\mathrm{w}_q, \gamma_q] = \arg\min_{w\in\mathbb{C},\gamma\in[0.1)} ||\widehat{h}_{k,q} - (1-\gamma)w||^2 + ||\widehat{h}_{k,q+1} - \gamma w||^2.$$

We then search for the strongest path :

$$\hat{q} = \arg\max_{q\in\{0,\cdots,N-1\}} |w_q|$$

$$\hat{\tau} = (\hat{q} + \gamma_q)T_c, \qquad \hat{w} = w_{\hat{q}}$$

The estimated path is subtracted from $\widehat{\mathbf{h}}_k$ and the process is repeated to find the next strongest path, until either a specified number of paths have been identified or $|\hat{w}|$ falls below some predetermined significant level [9].

4. COMPUTATIONAL COMPLEXITY

The computational complexity of the various steps of the algorithm are :

- Covariance approximation
  - Calculation of sample correlation matrices, $\widehat{\mathbf{R}}_{rr}$, $\widehat{\mathbf{R}}_{br}$, and $\widehat{\mathbf{R}}_{bb}$ — $O(N^2)$, $O(2NK)$, $O(4K^2)$
  - Calculation of $\widehat{\mathcal{Y}}$, $\widehat{\mathbf{K}}(\widehat{\mathcal{Y}})$ (equations 13 and 14) — $O(8K^3)$, $O(4NK^2)$, $O(2KN^2)$
- Channel impulse response estimation (equation 15) — $O(N^2)$, $O(N^3)$, $O(N^3)$
- Extraction of parameters (Section 3.3) — $O(N)$

However the complexity can be reduced further if $\widehat{\mathbf{K}}$ is assumed to be identity (the noise being additive white Gaussian) instead of explicitly calculating it. This whiteness assumption is practical not only because thermal noise is assumed Gaussian but also because the residual noise due to other-cell interference can be considered spatially and temporally white.

Other than eliminating the computation of $\widehat{\mathbf{K}}$, this will also reduce the complexity of the channel impulse response estimation step to $O(N^2)$. The inverse of $\widehat{\mathbf{R}}_{bb}$ can be estimated using matrix inversion update algorithms ($O(N^2)$, using the Sherman-Morrison-Woodbury formula [10]), or can be pre-calculated as the preamble is a known sequence of bits. Also the various matrix multiplications in the algorithm can benefit greatly using well-known parallelization techniques on a number of processors [10].

5. SIMULATION RESULTS

We briefly describe the preliminary simulations that we conducted to evaluate the performance of the proposed estimators. A code length of $N = 31$ was used in all the simulations. The delays of all the users were assumed uniformly distributed in $[1, 31)$ chips. The multiple access interference presented by each interferer, which is the ratio of the interferer's and desired user's received energies, was uniformly distributed in $[0, MAI]$dB. The default values of the system parameters, except when they are varied along the x-axis: the number of observations is $L = 200$, $MAI = 20dB$, the signal-to-noise ratio of the background noise is $SNR = 6dB$, and the number of users is $K = 25$. The number of paths for each user is denoted by P. Each point in our plots corresponds to 1000 Monte-Carlo trials.

The results (figure 3) presented are for the weakest user. Since a large number of interdependent parameters are being estimated, it is not very revealing to look at the estimation error of each individual parameter. Instead, the channel impulse response is constructed (equation 10) using the actual parameters and the estimated ones and the 'loss' due to the estimation procedure is calculated as:

$$loss = \left(\frac{\mathbf{h}_k}{||\mathbf{h}_k||}\right)^T \left(\frac{\widehat{\mathbf{h}_k}}{||\widehat{\mathbf{h}_k}||}\right) \qquad (16)$$

To gain some insight into how the proposed algorithm affects the bit error rate (BER), we simulated a RAKE receiver (figure 4), and calculated the BER using the actual as well as the estimated channel parameters. For comparison, we have also included BER plots with channel estimates from a subspace based algorithm [9] and a 'sliding correlator' similar to that used in current generation systems [11].

We have also compared (figure 2) the proposed algorithm to a similar ML scheme proposed in [12], called large scale maximum likelihood algorithm (LSML). The LSML algorithm has been designed for single path situations and models all other users except the user of interest as colored Gaussian noise. The figure shows the root-mean-square-error in the estimate of the delay of the weakest user after acquisition has occured.

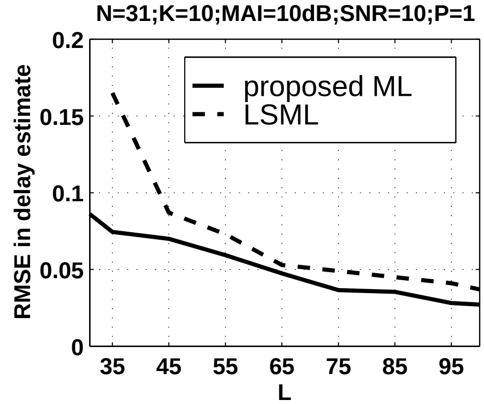

Figure 2: Proposed algorithm vs. LSML

6. CONCLUSION

We have developed a maximum likelihood algorithm for multipath channel delay and amplitude estimation for a set of transmitting users in the reverse link of a wireless CDMA communication system. This algorithm generalizes a single-path model based ML algorithm presented in [8] to include handling of multiple propagation paths without increasing the sizes of the matrices involved and hence without significantly increasing the computational load. The algorithm elegantly decomposes the multiuser problem into a series of single-user ones.

The additive noise in the system is assumed to be zero-mean, Gaussian but no assumption is made on its covariance, which is estimated within the algorithm. Our simulations verify that the algorithm is near-far resistant. Also, the estimators are not dimensionally limited; making it ideal for acquisition of a large number of users. Furthermore, the preamble required is not prohibitively large. Our simulation results show that acquisition occurs with 150-200 bits.

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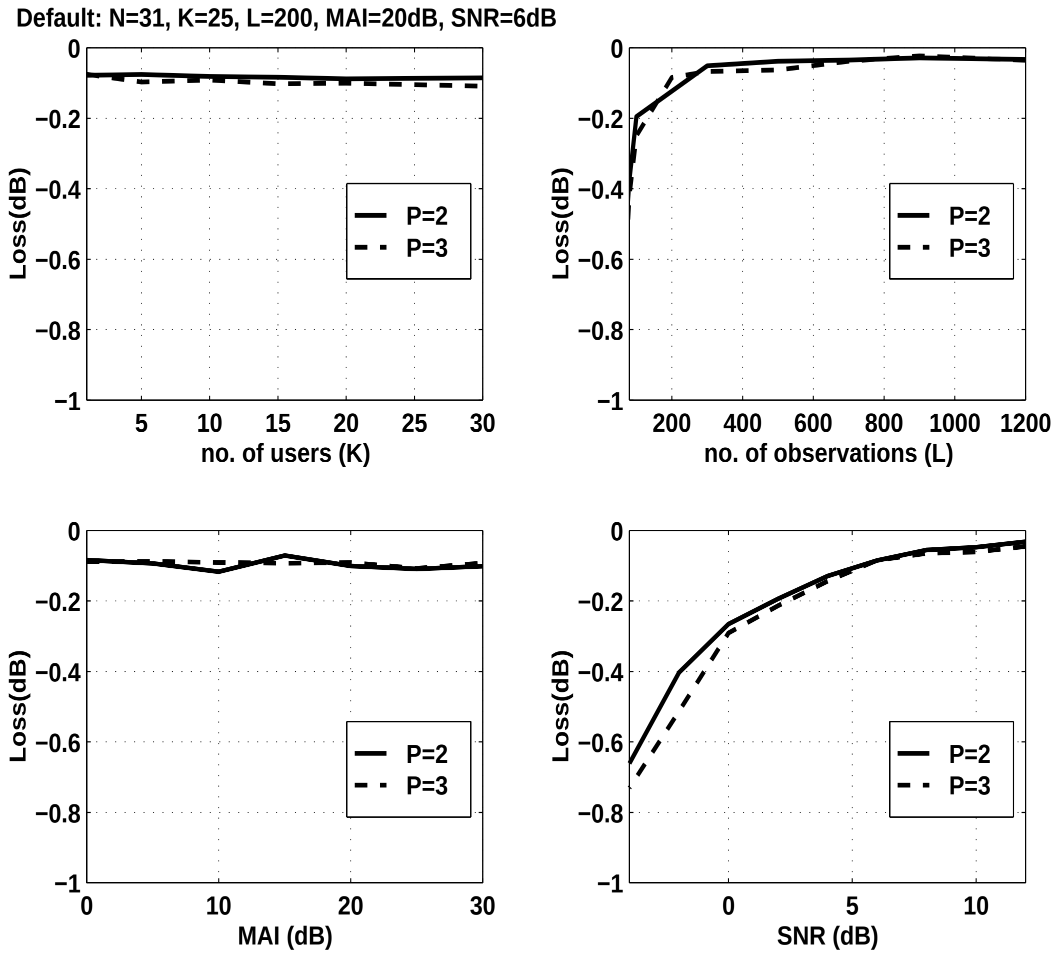

Figure 3: Loss (defined in equation 16) due to ML estimation vs. various system parameters

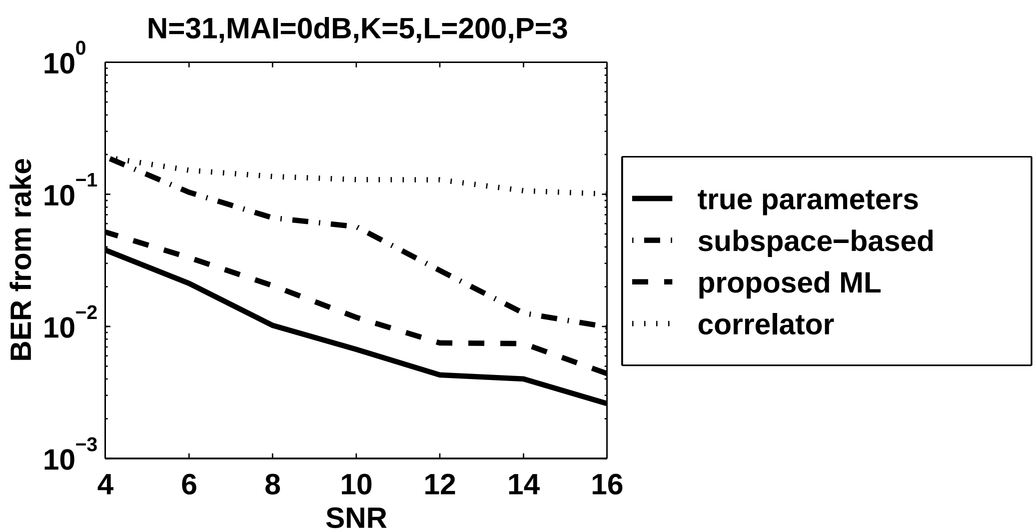

Figure 4: BER using a RAKE receiver and multipath channel parameters from: true parameters or perfect channel knowledge, estimates from a subspace-based algorithm, estimates from the proposed ML algorithm, and estimates using a sliding correlator.