RICE UNIVERSITY

AN ANALYSIS OF THE MOTION OF PIGS THROUGH NATURAL GAS PIPELINES

by

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ABSTRACT

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The dynamics of propelling a pig through a natural gas pipeline, using the gas being transported, are analyzed. The gas flow is assumed to be isothermal, quasi-steady, and one-dimensional. The pig is modeled as a cylinder. For the cases considered, the pig may either obstruct the pipe and allow no gas to flow past it, or may permit gas to flow through a concentric hole in the pig, through an annulus formed by the pig and the inside of the pipe, or through both. The governing equations which describe the motion of the pig and the flow of gas in the pipe are developed. These equations form a system of nonlinear differential equations which must be solved numerically. Time derivatives are replaced by backward differences, and at each time step a system of nonlinear algebraic equations is solved. A FORTRAN program is included which solves for the important gas flow parameters and the position, velocity, and acceleration of the pig as it moves through the line.
Such a quantitative analysis provides a basis for pig design which could be applied to regulate pig velocities or to increase gas flow rates while the pig is in the line.
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NOMENCLATURE

\( d_p \) Diameter of pipe
\( d_l \) Diameter of hole at the upstream end of the pig
\( d_2 \) Diameter of hole at the downstream end of the pig
\( d_3 \) Outer diameter of the pig
\( F_f \) Frictional force for the pig with no annulus
\( F_f^* \) Dimensionless frictional force; \( 4F_f/(\pi d_p^2 P_e) \)
\( f_c \) Fanning friction factor for the pig surfaces
\( f_p \) Fanning friction factor for the pipe
\( \gamma \) Ratio of specific heats; \( c_p/c_v \)
\( k_1 \) Dimensionless diameter ratio; \( d_1/d_p \)
\( k_2 \) Dimensionless diameter ratio; \( d_2/d_p \)
\( k_3 \) Dimensionless diameter ratio; \( d_3/d_p \)
\( l \) Length of pig
\( l^* \) Dimensionless pig length; \( l/d_p \)
\( L \) Length of pipe
\( L^* \) Dimensionless pipe length; \( L/d_p \)
\( M_{a+} \) Annulus Mach number at upstream end of pig
\( M_{a-} \) Annulus Mach number at downstream end of pig
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INTRODUCTION

In natural gas pipelines, devices called "pigs" perform several maintenance operations. They may remove water and solid contaminants, apply protective or drag reducing coatings to the inside of the pipe, or inspect the pipe for damage caused by corrosion or relative shifting of the pipeline. The pigs are propelled through the line by the gas being transported, thus eliminating the need to shut the line down for routine cleaning and inspection.

The pigs usually consist of two or more polyurethane rings attached to a cylindrical steel hub, although cylindrical pigs made of foam rubber are also used. The outer diameter of most pigs is slightly larger than the inner pipe diameter. This oversize provides a tight seal which creates a differential pressure across the pig, propelling it through the pipe. Variations on the basic pig shape depend on the particular use for which a pig is designed. The cleaning pigs have brushes attached to the body which scrub the inside of the pipe, and tight seals which keep water and debris ahead of the pig. The inspection pigs may have calipers which detect diameter changes in the pipe.
The typical configuration, described above, blocks the flow of gas in the pipeline, which, unfortunately, reduces the gas flow rate. Furthermore, the pig velocity is determined by the operating conditions of the pipeline, and can be reduced only by increasing the interference between the pig and pipe diameters, which increases the friction. An inspection pig may have to travel very slowly in order to accurately assess the condition of the pipe. Increasing the friction on the pig to achieve this reduced velocity would result in drastically lower gas flow rates, as well as in excessive wear on the rubber rings. Thus, depending on the desired pig velocity, the amount of gas transported while the pig travels through the line may be appreciably less than that transported through the pipe without a pig. This solid configuration, which does not allow gas to flow past the pig, may be necessary for the cleaning pig, in order to keep water and debris ahead of the pig, but the inspection pig has no such constraint. Allowing some of the gas to flow through the pig, either through a hole or through an annular clearance between the pig and the pipe, could provide the required pig velocity, while keeping the gas flow rate high. Since the cost of routine pigging includes the amount of gas that is not transported, due to the decreased flow rate, any improvement in the gas flow rate lowers the cost of the operation and warrants investigation.
The problem of analyzing the flow of a cylindrical capsule through a pipe carrying an incompressible fluid has been studied extensively (2,3,4,5,9,10,14,15), but the problem posed here, of motion in a compressible fluid, has been discussed in the literature only qualitatively, with design recommendations based primarily on field experience. It has been mentioned (18) that the speed of travel of a pig may be controlled by installing ports or jets in the pig which would open to allow gas to flow through, but no quantitative analysis or support for this practice has been presented, and no reference has been made to the effect of this on gas flow rate through the pipe.

A quantitative model for the flow of gas through a pipeline containing a pig would provide a rational basis for pig design, which could result in improved gas flow rates. The objectives of the analysis to be presented are to propose a model for flow through a gas pipeline containing a pig, to derive the governing equations for the flow, and to provide a means of solving these equations for the flow parameters of interest. Variations from the typical solid pig will be considered, and will include pigs with a concentric annulus of uniform clearance, with a concentric hole of constant diameter, with a concentric hold of linearly decreasing diameter in the direction of flow, and with both an annulus and a hole.
ANALYSIS

A physical model must be established before the equations which describe the system mathematically can be developed. The analysis to be presented considers a pig of known mass and geometry moving through a pipe of length $L$ and diameter $d_p$. The pressures at the pipe inlet and exit, $P_o$ and $P_e$, respectively, are fixed and known. Frictional forces are exerted on the gas at the pipe wall and at the pig surfaces. As the pig travels the length of the pipe, the parameters of interest are the position, velocity and acceleration of the pig, the pressures immediately upstream and downstream of the pig, the inlet and exit Mach numbers, and the Mach numbers for the flow through the hole and annulus.

Assumptions

Any model of a physical system must include some assumptions which permit a reasonably uncomplicated analytical solution. These assumptions are useful in providing a first approximation to the system, and they may be relaxed in successive analyses. The purpose of the following analysis of the flow through a gas pipeline carrying a pig is to provide such an initial modeling of the problem. The assumptions made are that:
1. The flow is isothermal.
2. The flow is quasi-steady.
3. All properties are uniform across the cross section of the pipe.
4. Constant pressures are maintained at the pipe inlet and exit.
5. The gas is thermally ideal.
6. The pipe has a constant diameter.
7. Changes in elevation are negligible.
8. The pig is cylindrically symmetric, with the most general pig shape illustrated in figure 1. The pig is assumed to be concentric in the pipe.

A conservative estimate of the accuracy of the isothermal flow assumption may be obtained by examining the limiting cases for the flow, which are adiabatic flow, where no heat transfer takes place, and isothermal flow, where the maximum amount of heat is transferred. For adiabatic flow, the stagnation temperature is constant, and this condition may be written (7)

\[ T(1 + \frac{\gamma - 1}{2}M^2) = \text{constant} \]

For the isothermal case, \( T = \) constant. Considering the worst possible case, of the Mach number increasing from \( M=0 \) to \( M=\frac{1}{\sqrt{\gamma}} \), the error in assuming an adiabatic flow to be isothermal is only about 10\%. This gives an upper bound on the error in assuming a subcritical flow to be
isothermal. The actual error should be much lower, since neither the pig or the pipe are insulated, and the range of Mach numbers is smaller.

The next important assumption is that of quasi-steady flow. The flow of gas in the pipe is not truly steady, since the flow parameters change as the pig moves along the line. But if the pig were held fixed in any of its successive positions along the line, the flow would be invariant with time. This forms the basis for the quasi-steady assumption, which considers the flow to be steady at each pig location as it travels the length of the pipe.

The other assumptions are fairly obvious, and require no further explanation. Along with the first two, they provide a realistically simplified model of the actual system.

![FIGURE 1: General Pig Configuration](image-url)
The Governing Equations

The equations that describe the flow will be non-dimensionalized, in order to provide a more general solution. In general, a capital letter will represent a dimensional parameter, and its lower case counterpart will represent the corresponding dimensionless parameter. The dimensionless quantities corresponding to the pipe length, L, and the pig length, l, will be denoted by L* and l*, respectively. All dimensionless diameters will be expressed as k, with a subscript denoting to which dimensional diameter they correspond. The dimensionless velocities of the gas will be represented by the Mach numbers, M, rather than v.

A characteristic length, pressure, and time must be defined. The characteristic length is taken to be the pipe diameter, d_p. The characteristic pressure is the pressure at the pipe exit, P_e. The characteristic time is d_p/\sqrt{\gamma RT} . Using the definitions of characteristic length and time, a characteristic velocity, \sqrt{\gamma RT}, and a characteristic acceleration, \gamma RT/d_p, may be defined.

The dimensionless forms of the physical parameters are obtained by dividing the dimensional parameter by the characteristic parameter, so that

\[ p = \frac{P}{P_e} \quad x = \frac{x}{d_p} \quad \dot{x} = \frac{\dot{x}}{\sqrt{\gamma RT}} \quad \ddot{x} = \frac{d_p \ddot{x}}{\gamma RT} \quad \text{etc.} \]
The nine equations characterizing the flow are the continuity and momentum equations for the gas in the upstream portion of the pipe, in the downstream portion of the pipe, in the hole and in the annulus, along with a force balance on the pig. The isothermal condition for the flow is used implicitly. The equations listed above relate the velocities of the gas at the pipe inlet and exit, the pressures immediately upstream and downstream of the pig, the position, velocity, and acceleration of the pig, and the upstream and downstream velocities of the gas flowing through the hole and annulus. The quasi-steady assumption, which considers the flow to be steady at each pig location, allows the steady flow form of the governing equations to be used. The following equations describe the steady flow of gas in a pipe containing a pig located a distance $X$ from the pipe entrance.

1. Upstream continuity equation

Mass must be conserved in the portion of the pipe extending from the inlet to the upstream end of the pig. This requires that the mass flow rate, $m$, be constant. For steady, isothermal flow of an ideal gas past a circular cross section of diameter $d$, this principle may be written

$$Pd^2V = \text{constant} \quad (1.1)$$
Taking a cross section of the pipe at the inlet, and a cross section at the upstream end of the pig, and applying equation (1.1), the result is

\[ P_0 d_0^2 V_0 = P^+ d_1^2 V^+ + P^+(d_3^2 - d_2^2)V_a^+ + P^+(d_3^2 - d_1^2)\dot{x} \]  \hspace{1cm} (1.2)

Dividing by \( P \frac{d^2}{d^2} \sqrt{\gamma RT} \), the equation is nondimensionalized; and is rearranged to obtain

\[ F_1 = P_0 M_0 - P^+ [k^2_{1,2} M_1^+ + (1-k^2_3)M_2^+ + (k^2_3-k^2_1)\dot{x}] = 0 \]  \hspace{1cm} (1.3)

2. Downstream continuity equation

The mass flow rate must also be constant in the section of pipe between the downstream end of the pig and the pipe exit. Applying equation (1.1) to a cross section at the downstream end of the pig and a cross section at the pipe exit yields

\[ P_e d_2^2 V_e = P^- (d_3^2 - d_2^2)\dot{x} + P^- d_2^2 V_n^- + P^- (d_3^2 - d_1^2)V_a^- \]  \hspace{1cm} (2.1)

Nondimensionalizing equation (2.1) by dividing by \( P_e \frac{d^2}{d^2} \sqrt{\gamma RT} \), and rearranging terms, the final equation is obtained.

\[ F_2 = M_e - P^- [(k^2_2-k^2_3)\dot{x} + k^2_2 M_n^- + (1-k^2_3)M_a^-] = 0 \]  \hspace{1cm} (2.2)

3. Force balance on the pig

Pressure forces and frictional forces act on the pig. The resultant force must equal the rate of change of linear momentum of the pig. The general equation for this
balance is

\[ m_\text{c} \ddot{x} = P^+ A^+ - P^- A^- + \bar{P}_h (A^- - A^+) - F_h - F_a \]  

(3.1)

where

- \( A^+ \) = the upstream cross sectional area of the pig
- \( A^- \) = the downstream cross sectional area of the pig
- \( F_a \) = the frictional force exerted on the outside of the pig, either by gas in the annulus or by the pipe wall
- \( F_h \) = the frictional force exerted on the pig by the gas in the hole
- \( \bar{P}_h \) = an average pressure of the gas in the hole, taken to be \( \frac{1}{2} (P^+ + P^-) \). This average pressure is used to approximate the pressure forces exerted in the direction of flow by the gas in the hole.

The frictional force in the hole is given by

\[ F_h = \tau_{\text{ch}} A_h \]  

(3.2)

where \( \tau_{\text{ch}} \) = the wall shear stress in the hole
- \( A_h \) = the wall surface area in the hole.

For an ideal gas, the average wall shear stress in the hole is

\[ \tau_{\text{ch}} = \frac{s_h (4f_c) (P^+ + P^-) [\dot{x} - \bar{v}_h]^2}{16RT} \]  

(3.3)
where

\[ s_h = \begin{cases} 
  +1 & \text{for } \dot{x} - \bar{v}_h > 0 \\
  -1 & \text{for } \dot{x} - \bar{v}_h < 0 
\end{cases} \]

\( \bar{v}_h \) = an average velocity for the gas flowing through the hole, taken to be \( \frac{1}{2}(v_h^+ + v_h^-) \). This assumption approximates the velocity distribution in the hole as linear.

The surface area of the hole is that of a truncated right circular cone, and is given by

\[ A_h = \frac{\pi}{2}(d_1 + d_2) \sqrt{\frac{1}{4}(d_1 - d_2)^2 + x^2} \]  

(3.4)

The value of the frictional force on the outside of the pig depends on whether an annulus exists or not. If the outside pig surface contacts the pipe wall, the frictional force, \( F_{a'} \), is assumed to be constant and known. Let \( F_f \) denote this force. For the pig with a clearance between its outer diameter and the pipe, the frictional force is a viscous shear force, as for the hole. The shearing stress is given by

\[ \tau_{ca} = \frac{s_a (4f_c) (p^+ + p^-) [\dot{x} - \bar{v}_a]^2}{16RT} \]  

(3.5)

where

\[ s_a = \begin{cases} 
  +1 & \text{for } \dot{x} - \bar{v}_a > 0 \\
  -1 & \text{for } \dot{x} - \bar{v}_a < 0 
\end{cases} \]

\( \bar{v}_h \) = an average velocity for the gas flowing through the annulus, taken to be
1/2(V^+_a+V^-_a). This assumption approximates
the velocity distribution in the annulus
as linear, as was done for the hole
velocities.

The definition of the average pressure across the pig,
1/2(P^+_p+P^-_p), was used in equations (3.3) and (3.5). The area
over which the shear stress given in equation (3.5) acts
is

\[ A_a = \pi d_3 l \] (3.6)

so that the total frictional force is

\[ F_a = \frac{s_a \pi d_3 l (4 f_c)(P^+_p+P^-_p)[\dot{X}-1/2(V^+_a+V^-_a)]^2}{16RT} \] (3.7)

Substituting equations (3.3) and (3.4) into equation
(3.2), substituting this result into equation (3.1),
and nondimensionalizing by multiplying the resulting
equation by \(4/\pi d^2 p_e\), the final equation is

\[ F_3 = \left(\frac{4m_c \gamma RT}{\pi d^3 p_e}\right) x - p^+(k_3^2-k_1^2) + p^-(k_3^2-k_2^2) \]

\[ -\frac{1}{2}(p^+_p+p^-_p)(k_1^2-k_2^2) + F^*_h + F^*_a = 0 \] (3.8)

where

\[ F^*_h = \frac{1}{8} s_h \gamma (4 f_c)(k^+_1+k^-_2) \sqrt{1/4(k^+_1-k^-_2)^2+(x^*_l)^2} \]

\[ (p^+_p+p^-_p)[\dot{X}-1/2(M^+_h+M^-_h)]^2 \]
The case where \( k_3 = 1 \) corresponds to the pig without an annulus, where the frictional force on the outside of the pig, \( F_f \), is known and constant.

The parameters which must be known initially, such as the mass of the pig and the frictional force, may now be specified only through the dimensionless quantities

\[
F_a^* = \begin{cases} \\
\frac{4F_f}{\pi d^2 P e} & \text{for } k_3 = 1 \\
\frac{1}{4} s_a \gamma (4f_c) \# k_3 (p^+ + p^-) \left[ \delta - \frac{1}{2} (M_a^+ + M_a^-) \right]^2 & \text{for } k_3 \neq 1
\end{cases}
\]

which are more general.

4. Downstream momentum equation

The momentum equation for the gas flowing between the downstream end of the pig and the pipe exit is given by the momentum equation for steady, isothermal, frictional flow of an ideal gas through a pipe of diameter \( d_p \) and length \( L-X \). This may be written in the following well known form (1):

\[
F_4 = (4f_p) (L^*-x) + \frac{1}{\gamma} \left[ \frac{1}{M_e^2} + \frac{1}{(M^-)^2} \right] - 2 \ln \left( \frac{\bar{M}^-}{M_e} \right) = 0 \quad (4.1)
\]

where

\[
\bar{M}^- = (1-k_3^2)M_a^- + (k_3^2 - k_2^2) \delta + k_2^2 M_n^-
\]
5. Hole continuity equation

Applying equation (1.1) to cross sections at the hole inlet and exit, and dividing by \( p_e \sqrt{gRT} \), this equation is found to be

\[
F_5 = k_1 p^+ M_h^+ - k_2 p^+ M_h^- \quad (5.1)
\]

6. Hole momentum equation

The differential form of this equation is obtained by considering the control volume shown in figure 2.

![Figure 2: Hole Control Volume](image)

The force balance on the fluid in the control volume is

\[
- \frac{\pi}{4} d^2 dP - \tau_{ch} \pi d \xi = \frac{P}{RT} \frac{\pi}{4} d^2 V dV \quad (6.1)
\]

where

\[
\tau_{ch} = \frac{s_h (4f_c) p (V-\dot{x})^2}{8RT}
\]
\[ s_h = \begin{cases} +1 & \text{for } V - \dot{\chi} > 0 \\ -1 & \text{for } V - \dot{\chi} < 0 \end{cases} \]

Equation (6.1) may be rearranged to obtain

\[ 0 = \frac{dP}{P} + \frac{s_h (4 f_c) (V - \dot{\chi})^2 d\xi}{2 dRT} + \frac{1}{RT} V dV \]  \hspace{1cm} (6.2)

From the ideal gas law for isothermal flow,

\[ \frac{dP}{\rho} = \frac{dP}{P} \]  \hspace{1cm} (6.3)

And from continuity,

\[ \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \]  \hspace{1cm} (6.4)

Using the definition of Mach number,

\[ \frac{dV}{V} = \frac{dM}{M} \]  \hspace{1cm} (6.5)

Substituting equations (6.3) and (6.5) into (6.4),

\[ \frac{dP}{P} + \frac{dA}{A} + \frac{dM}{M} = 0 \]  \hspace{1cm} (6.6)

Combining equations (6.2), (6.5), and (6.6), and non-dimensionalizing gives

\[ 0 = -\frac{dA^*}{A^*} - \frac{dM}{M} + \frac{s_h (4 f_c) \gamma (M - \dot{\chi})^2 d\xi^*}{2k} + \gamma M^2 \frac{dM}{M} \]  \hspace{1cm} (6.7)

For a hole with a linear taper,

\[ k = k_1 + \left( \frac{k_2 - k_1}{\xi^*} \right) \xi^* \]  \hspace{1cm} (6.8)
\[ A^* = \frac{\pi}{4} k^2 \quad (6.9) \]
\[ dA^* = \frac{\pi}{2} k \left( \frac{k_2 - k_1}{\xi^*} \right) d\xi^* \quad (6.10) \]

Substituting equations (6.8), (6.9), and (6.10) into (6.7) gives

\[ (1-\gamma M^2) \frac{dM}{M} = \left[ \frac{1}{2} s_h (4f_c') \gamma (M-\hat{\chi})^2 - 2\left( \frac{k_2 - k_1}{\xi^*} \right) \right] \]
\[ \left[ k_1^+ \left( \frac{k_2 - k_1}{\xi^*} \right) \xi^* \right] \quad (6.11) \]

Separating variables:

\[ \int_{0}^{M^*} \frac{d\xi^*}{k_1^+ \frac{k_2 - k_1}{\xi^*} \xi^*} = \int_{M_h^-}^{M_h^+} \frac{(1-\gamma M^2) dM}{M} \]
\[ \left[ \frac{1}{2} s_h (4f_c') \gamma (M-\hat{\chi})^2 - 2\left( \frac{k_2 - k_1}{\xi^*} \right) \right] \]

The final equation is obtained by integrating both sides of the above.

For \((k_1-k_2)s_h>0:\)

\[ F_6 = \frac{\xi^*}{k_1-k_2} \ln \left( \frac{k_1}{k_2} \right) + \frac{1}{a} \ln \left( \frac{M_h^+}{M_h^-} \right) - \frac{1}{2} \left[ \frac{1}{a} + \frac{\gamma}{c} \right] \]
\[ \ln \left[ \frac{a+bM_h^+ + c(M_h^+)^2}{a+bM_h^- + c(M_h^-)^2} \right] + \frac{b}{\sqrt{q}} \left[ \frac{\gamma}{c} - \frac{1}{a} \right] \left[ \tan^{-1} \left( \frac{2cM_h^+ + b}{\sqrt{q}} \right) \right] \]
\[-\tan^{-1} \left( \frac{2cM_h^- + b}{\sqrt{q}} \right) \]
\[ = 0 \quad (6.12a) \]

For \((k_1-k_2)s_h<0:\)
\[ F_6 = \frac{k^*}{k_1 - k_2} \ln \left( \frac{k_1}{k_2} \right) + \frac{1}{a} \ln \left( \frac{M^+_h}{M^-_h} \right) - \frac{1}{2} \left[ \frac{1}{a} + \frac{Y}{c} \right] \]

\[
\ln \left[ \frac{a + bM^+_h + c(M^+_h)^2}{a + bM^-_h + c(M^-_h)^2} \right] + \frac{b}{2\sqrt{-q}} \left[ \frac{Y}{c} - \frac{1}{a} \right]
\]

\[
\ln \left[ \frac{(2cM^+_h + b - \sqrt{-q})(2cM^-_h + b + \sqrt{-q})}{(2cM^+_h + b + \sqrt{-q})(2cM^-_h + b - \sqrt{-q})} \right] = 0 \quad (6.12b)
\]

where

\[
a = \frac{1}{2} s_h (4f_c) \gamma \dot{x}^2 - 2 \left( \frac{k_2 - k_1}{k^*} \right)
\]

\[
b = -s_h (4f_c) \gamma \ddot{x}
\]

\[
c = \frac{1}{2} s_h (4f_c) \gamma
\]

\[
q = 4ac - b^2
\]

When the hole has a constant diameter, a much simpler form is available. This equation is

\[
F_6 = \frac{-s_h (4f_c) \gamma l^*}{2k_1} + \left[ \frac{1}{\dot{x}^2} - \gamma \dot{x} \right] \left[ \frac{1}{M^+_h - \dot{x}} - \frac{1}{M^-_h - \dot{x}} \right]
\]

\[
+ \left[ \frac{1}{\dot{x}^2} + \gamma \right] \ln \left[ \frac{M^+_h - \dot{x}}{M^-_h - \dot{x}} \right] - \frac{1}{\dot{x}^2} \ln \left( \frac{M^+_h}{M^-_h} \right) = 0 \quad (6.12c)
\]

It may be possible, under some circumstances, for the average velocity of the gas in the hole to be the same as the pig velocity. For this situation, the momentum
equation becomes

\[ F_6 = 2 \ln \left( \frac{k_2}{k_1} \right) - \ln \left( \frac{M_h^+}{M_h^-} \right) + \frac{1}{2} \gamma [(M_h)^2 - (M_h)^2] = 0 \] (6.12d)

7. Annulus continuity equation

Applying equation (1.1) to the inlet and exit cross sections of the annulus, and nondimensionalizing, this equation is found to be

\[ F_7 = p^+ M_a^+ - p^- M_a^- = 0 \] (7.1)

8. Annulus momentum equation

The differential form of the equation may be obtained by considering an annular control volume of infinitesimal length \( d\xi \). The balance of forces on the fluid in this control volume is given by

\[
- \frac{\pi}{4} \left( d_2^2 - d_3^2 \right) dP - \tau_p \frac{\pi}{d_3} d_d d\xi - \tau_{ca} \frac{\pi}{d_3} d_3 d\xi
\]

\[ = \frac{P}{RT} \frac{\pi}{4} \left( d_2^2 - d_3^2 \right) \nu \ dv \] (8.1)

where

\[ \tau_p = \frac{(4f_p)PV^2}{8RT} \] (8.2)

\[ \tau_{ca} = \frac{s_a (4f_p)P (V - \dot{x})^2}{8RT} \] (8.3)

\[ s_a = \begin{cases} 
+1 & \text{for } V - \dot{x} > 0 \\
-1 & \text{for } V - \dot{x} < 0 
\end{cases} \]
Equation (6.6) reduces to

\[ \frac{dp}{P} + \frac{dM}{M} = 0 \]  \hspace{1cm} (8.4)

for this constant area annulus. Substituting equations (6.5), (8.2), (8.3), and (8.4) into equation (8.1), nondimensionalizing and rearranging, yields

\[ 0 = -(1-k_3^2)\frac{dM}{M} + \frac{1}{2}\gamma(4f_p)M^2d\xi^* + \frac{1}{2}s_a\gamma(4f_c)k_3(M-\hat{x})^2d\xi^* \]

\[ + (1-k_3^2)\gamma M^2 \frac{dM}{M} \]  \hspace{1cm} (8.5)

Separating variables,

\[ \int_{0}^{\xi^*} d\xi^* = \int_{M_a^-}^{M_a^+} \frac{(1-k_3^2)(1-\gamma M^2) \frac{dM}{M}}{\left[\frac{1}{2}\gamma(4f_p)M^2 + \frac{1}{2}s_a\gamma(4f_c)k_3(M-\hat{x})^2\right]} \]

The final equation is obtained by integrating both sides of this equation.

For \( s_a > 0 \)

\[ F_8 = \xi^* + (1-k_3^2) \left\{ \frac{1}{2} \ln \left( \frac{M_a^+}{M_a^-} \right) - \frac{1}{2} \left[ \frac{1}{a} + \frac{1}{c} \right] \ln \left[ \frac{a+bM_a^++c(M_a^+)^2}{a+bM_a^-+c(M_a^-)^2} \right] + \frac{b}{\sqrt{q}} \left[ \frac{\gamma}{c} - \frac{1}{a} \right] \tan^{-1} \left( \frac{2cM_a^++b}{\sqrt{q}} \right) - \tan^{-1} \left( \frac{2cM_a^-+b}{\sqrt{q}} \right) \right\} = 0 \]  \hspace{1cm} (8.6a)
For \( s_a < 0 \)

\[
F_8 = \varepsilon^* + (1-k_3^2) \left\{ \frac{1}{a} \ln \left( \frac{M^+_{a}}{M^-_{a}} \right) - \frac{1}{2} \left[ \frac{1}{a} + \frac{1}{c} \right] \right. \\
\ln \left[ \frac{a+bM^+_{a}+c(M^+_{a})^2}{a+bM^-_{a}+c(M^-_{a})^2} \right] + \frac{b}{2\sqrt{-q}} \left[ \frac{\gamma}{c} - \frac{1}{a} \right] \\
\ln \left[ \frac{(2cM^+_{a}+b+\sqrt{-q}) (2cM^-_{a}+b+\sqrt{-q})}{(2cM^+_{a}+b-\sqrt{-q}) (2cM^-_{a}+b-\sqrt{-q})} \right] \right\} = 0
\]  

(8.6b)

where

\[
a = \frac{1}{2} s_a \gamma (4f_c) k_3 x^2
\]

\[
b = -s_a \gamma (4f_c) k_3 x
\]

\[
c = \frac{1}{2} \gamma (4f_p) + \frac{1}{2} s_a \gamma (4f_c) k_3
\]

\[q = 4ac-b^2\]

For the case where the average gas velocity in the annulus is equal to the pig velocity, the equation is

\[
F_8 = (4f_p) \varepsilon^* - \frac{(1-k_3^2)}{\gamma} \left[ \frac{1}{(M^+_{a})^2} - \frac{1}{(M^-_{a})^2} \right] - 2\ln \left( \frac{M^+_{a}}{M^-_{a}} \right) = 0
\]

(8.6c)

9. Upstream momentum equation

The momentum equation for the gas in the section of pipe between the inlet and the upstream end of the pig
is given by the momentum equation for the isothermal, frictional, steady flow of an ideal gas through a pipe of diameter \( d_p \) and length \( x-\ell \). This is

\[
F_g = (4f_p)(x-\ell^*) - \frac{1}{\gamma} \left[ \frac{1}{M_o^2} - \frac{1}{(M^+)^2} \right] - 2\ln \left( \frac{M_0}{M^+} \right) = 0 \tag{9.1}
\]

where

\[
\tilde{M}^+ = (1-k_3^2)M_a^+ + (k_3^2-k_1^2)\dot{x} + k_1^2M_h^+
\]

The nine equations (1.3), (2.2), (3.8), (4.1), (5.1), (6.12), (7.1), (8.6), and (9.1) may be solved for the nine unknowns, which are \( M_o^+ \), \( M_e^+ \), \( p^+ \), \( p^- \), \( x \), \( M_a^+ \), \( M_a^- \), \( M_h^+ \), and \( M_h^- \). If no hole exists, \( M_h^+ \) and \( M_h^- \) are removed from this list of unknowns, and equations (5.1) and (6.12) are not used. If an annulus is absent, \( M_a^+ \) and \( M_a^- \) are not solved for, and equations (7.1) and (8.6) are discarded. For either the case of the pig without a hole or of the pig without an annulus, the number of equations to be solved reduces to seven. If neither a hole nor an annulus is present, the number of equations to be solved is five.

The equations derived above form a system of non-linear equations. In addition, these equations contain derivatives of \( x \) with respect to time. Because of their complexity, these equations cannot be solved analytically. Their solution must be found using some numerical technique.
Constraints on the Solution

A number of constraints that are often taken for granted when a gas dynamics problem is solved by hand must be enforced explicitly when the problem is to be solved by a computer. Since the direction of flow does not reverse, the condition that the Mach number be greater than zero must be imposed. The gas flow in the annulus and hole should be in the same direction as the gas flow in the rest of the pipe, so these Mach numbers must also be positive. The pig must move in the direction of the gas flow, so its velocity must be positive. Since all pressures are absolute, they must be positive.

The Mach numbers, pressures, and pig velocity may be forced to be greater than zero by employing the following technique. Let $M_{\ln} = \ln(M)$, $p_{\ln} = \ln(p)$, and $x_{\ln} = \ln(x)$. These new parameters may assume any real value. By changing variables in the equations, and solving for the new parameters, the constraint is automatically satisfied, since $M = \exp(M_{\ln})$, $p = \exp(p_{\ln})$, and $x = \exp(x_{\ln})$.

Next, consider the problem of forcing the gas in the hole to flow in the same direction as the gas in the pipe. This is equivalent to forcing a pressure drop across the pig. Combining equations (6.6), (6.9), (6.10), and (6.11) yields
Since the flow is subcritical, \(1-\gamma M^2\) is positive, and forcing the constraint that \(dP<0\) reduces to forcing

\[
s_h(4f_c)(M-x)^2 + \left(\frac{k_1-k_2}{\xi^*}\right)M^2 > 0
\]  \hspace{1cm} (C.1)

For the constant diameter hole, this reduces to the condition that \(s_h>0\), so the velocity of the gas in the hole must be greater than the pig velocity. This condition is not necessarily true for the tapered hole.

Before considering the method of imposing this constraint, examine the same constraint for the annulus, i.e. \(dP<0\). Combining equations (8.4) and (8.5) gives

\[
\frac{-dP}{P} = \frac{\gamma[(4f_p)M^2 + s_a(4f_c)k_3(M-x)^2]}{2(1-k_3^2)(1-\gamma M^2)} \frac{d\xi^*}{k_1 - \left(\frac{k_1-k_2}{\xi^*}\right)\xi^*}
\]

The condition that \(dP<0\) is met if

\[
(4f_p)M^2 + s_a(4f_c)k_3(M-x)^2 > 0
\]  \hspace{1cm} (C.2)

The constraints on the flow through the hole and annulus are imposed by employing a quasi-penalty method. Let \(c_h\) be the value of the constraint given in equation
(C.1), and \( c_a \) be the value of the constraint given in equation (C.2). The conditions that \( c_h \) and \( c_a \) be greater than zero must be forced. This may be done by modifying the hole and annulus momentum equations. Define a function \( F_6' \), such that

\[
F_6' = |F_6| + |1 - \frac{c_h}{|c_h|}|
\]

Replace \( F_6 \) by the function \( F_6' \) in the set of equations to be solved. This new function can be zero only if \( F_6 = 0 \) and \( c_h > 0 \).

Similarly, define \( F_8' \) such that

\[
F_8' = |F_8| + |1 - \frac{c_a}{|c_a|}|
\]

This replaces \( F_8 \) in the set of equations, and can be zero only if \( F_8 = 0 \) and \( c_a > 0 \).

Using the methods described in this section, the constraints are incorporated naturally into the system of equations. These new equations may then be solved as an unconstrained problem, since the constraints will automatically be satisfied for any solution that is obtained.

Another constraint on the flow is the physical impossibility of converting a subcritical flow to a supercritical one. Choking will occur in the hole or annulus when the Mach number reaches \( \frac{1}{\sqrt{\gamma}} \). This
constraint cannot be as easily integrated into the system of equations as the previous constraints. It will be enforced only after the equations are solved.
METHOD OF SOLUTION

The governing equations presented describe the flow state for a pig located at a particular position in the pipe. The motion of the pig through the pipe may be simulated by solving these equations at successive pig locations along the pipe. The pig starts from the pipe entrance at a time arbitrarily set to zero. After a time increment, \( \Delta t \), the pig is at some new position, a distance \( X \) from the pipe inlet. The governing equations are solved at this point for the inlet and exit Mach numbers, the pressures immediately upstream and downstream of the pig, the position, velocity, and acceleration of the pig, and the Mach numbers in the upstream and downstream ends of the hole and annulus. After the solution at this time step is found, the time is incremented again, and the equations are solved at the new pig position, farther along the pipe.

The set of equations to be solved may be converted to a system of nonlinear algebraic equations by approximating the time derivatives of \( x \) by backward differences. This difference scheme allows the velocity and acceleration of the pig to be expressed in terms of the current position, the previous positions, and the time step. The approximations to the derivatives are
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\[
\begin{align*}
\dot{x}_i &= \frac{x_i - x_{i-1}}{\Delta t} \\
\ddot{x}_i &= \frac{x_i - 2x_{i-1} + x_{i-2}}{(\Delta t)^2}
\end{align*}
\]

where

\begin{align*}
i &= \text{number of the current time step} \\
x_i &= \text{dimensionless current position} \\
x_{i-1} &= \text{dimensionless position at the previous time step} \\
\Delta t &= \text{size of the time step (assumed constant)}
\end{align*}

For very small time steps, successive values of \(x\) may be very nearly equal. Subtracting them as shown above would result in excessive roundoff error. To eliminate this problem, the following formulation is developed.

Define

\[
\gamma_i = x_i - 2x_{i-1} + x_{i-2}
\]

Then

\[
\begin{align*}
x_i &= \gamma_i + 2x_{i-1} - x_{i-2} \\
\dot{x}_i &= \frac{\gamma_i + \sigma_i}{\Delta t} \\
\ddot{x}_i &= \frac{\gamma_i}{(\Delta t)^2}
\end{align*}
\]

where

\[
\sigma_i = \gamma_{i-1} + \gamma_i + \ldots + \gamma_1 + \gamma_0
\]
These expressions are substituted for $x$ and its derivatives in the governing equations. At each time step, $y_i$ replaces the previous unknowns $x_i$, $\dot{x}_i$, and $\ddot{x}_i$. Once $y_i$ is known, these other parameters are readily obtained, using equation (1). The definition of $\sigma_i$ given in equation (2) is useful, since only a running sum of the previous $\gamma$'s must be stored.

Since the equations to be solved are second order in $x$, two initial conditions must be specified. These conditions are

$$
\begin{align*}
  x_0 &= 0 \quad \text{at } t = 0 \\
  \dot{x}_0 &= 0
\end{align*}
$$

In other words, the pig starts from rest at the origin of a coordinate system attached to the pipe inlet. Using these initial conditions:

$$
\begin{align*}
  \gamma_0 &= 0 \\
  x_1 &= \gamma_1 \\
  x_2 &= \gamma_2 + 2x_1
\end{align*}
$$

At the last time step, the pig must be just at the pipe exit, so that $x=L^*$. It is improbable that using the previous value of $\Delta t$ will accomplish this, so the last time step will have some smaller value $\Delta t_f$. Since the last time step is a different size, the difference
equations must be reformulated. Denoting the last time step by the subscript \( n \), the new substitutions for \( x \) and its derivatives become

\[
x_n = \gamma_n + 2x_{n-1} - x_{n-2}
\]

\[
\dot{x}_n = \frac{1}{(\Delta t_f)} (\gamma_n + \sigma_n)
\]

\[
\ddot{x}_n = \frac{1}{(\Delta t_f)^2} \left[ \gamma_n + \left(1 - \frac{\Delta t_f}{\Delta t}\right) \sigma_n \right]
\]

Since the position is known at the last time step, \( \gamma_n \) is also known. Also \( P^- = P_e \), and \( M_e = M^- \). For this step, delete \( \gamma, M_e \) and \( p^- \) from the list of unknowns, and add \( \Delta t_f \) to this list. The downstream continuity equation and the downstream momentum equation are not used, since the pig is at the pipe exit, so the number of equations to be solved is reduced by two.

Now that the governing equations have been reduced to a system of \( N \) nonlinear algebraic equations, a method must be found to solve these. Most methods devised to solve a system of nonlinear equations have some of the same basic characteristics. They require initial guesses for the \( N \) unknowns. Using these guesses, they evaluate the functions and obtain a new guess for the solution. This process of revising the current guess continues until a solution is reached which satisfies some previously
specified convergence criteria. The methods differ from each other in the way that they generate the next guess. The approximation to the solution may be thought of as a point in N-dimensional space. The current guess and the next guess can be considered to be connected by a vector, and this vector is called a "step" in the methods considered here.

One method of solving these equations, called the "steepest descent method", takes a step in the direction of the negative gradient, which is evaluated at the current point. This is the direction in which the values of the functions are decreasing most rapidly. The step size is the largest possible while still forcing the functions to continue to decrease. This method becomes very slow in converging once the approximation becomes close to the solution.

Another broad class of methods is comprised of the quasi-Newton methods. The basis for these is Newton's method, which is basically an N-dimensional generalization of the familiar one-dimensional Newton method. The variations on this basic method allow solutions to be obtained with less computer work than would be possible using the standard Newton method. These methods converge rapidly when the approximation is close to the solution, but often require a good initial guess.
Because of the convergence characteristics of the steepest descent and quasi-Newton methods, it seems reasonable to combine them in a method which would use each to its best advantage. In addition, the number of problems for which convergence occurs should be greater than that for either method used separately, making the method a better general equation solver. This type of "hybrid" method was proposed by Powell (12), and he published a FORTRAN subroutine which uses the method (13). This subroutine was used with only minor modifications to solve the equations presented earlier. A detailed explanation of the method and subroutine algorithm is presented in the two articles by Powell, referenced above.
The program used to solve the problem of a pig moving through a gas pipeline is composed of several subroutines. The main routine reads the input parameters and initial guess for the solution, prints the results, and performs, primarily, bookkeeping operations in between. It keeps track of the time step, stores the pig positions, checks for choking in the hole and annulus, and calls the equation solver, subroutine NS01A. This subroutine solves the system of equations by the method that has been simplistically outlined here, and described more realistically in references (12) and (13) by Powell. At each iteration, the functions representing the governing equations must be evaluated. The subroutine NS01A calls the subroutine GENFUN to evaluate the functions. GENFUN takes the approximation to the solution, calculates the position, velocity, and acceleration of the pig, and calls the functions corresponding to the equations that must be solved for each of the special cases. Each equation is represented by a different function, for example FUN1, FUN2, etc. Some of the governing equations are sufficiently different for the different special cases to warrant two separate function subroutines. The special case
considered determines which of these is to be used. The function values, evaluated at the approximation to the solution, are then returned to NS01A and the iteration is continued until convergence is obtained.

The parameters used in the main routine are defined in the program listing. All of these parameters are dimensionless. When the program prompts the user to enter the initial guess, it asks for the "residual". This quantity is just the expected ratio of the first pig position to the pipe length. The parameters FC and FP are equal to $4f_c$ and $4f_p$, respectively.

The parameters used in the subroutine NS01A are discussed in the papers by Powell, but will be briefly defined here also.

**ACC** = accuracy parameter. Convergence is assumed if the sum of squares of the function values is less than ACC.

**AJINV** = NxN array containing the elements of the inverse Jacobian.

**DMAX** = estimate of the maximum distance of the initial guess from the solution.

**DSTEP** = length used to approximate the first partial derivatives of the functions by finite differences.

**F** = vector of length N containing the values of the functions.
IPRINT = parameter which allows printing of the approximation, resulting function values, and sum of squares of the functions at each iteration if set to one. If it is set to zero, only error messages will be printed.

MAXFUN = maximum number of function evaluations to be allowed.

N = number of equations to be solved. (Which must also equal the number of unknowns.)

X = vector of length N containing the initial guess when the subroutine is called, and containing the final solution when the subroutine finishes.

W - workspace vector; the number of elements required in W is N(2N+5).

The choice of the parameter DSTEP is very important, since the subroutine will never take a step smaller than DSTEP. In general, DSTEP should be set to a value as small as possible without allowing roundoff errors to dominate the differencing process. A value of $10^{-8}$ was usually satisfactory for the equations solved in this work. A value of DSTEP that is too large will usually cause an error message stating that a certain number of function evaluations (less than N+4) did not decrease the sum of squares of the function values.
confronting the user is that of obtaining a good guess for the first time step. This is basically a trial and error procedure. The case of the solid pig requires the least accurate initial guess for convergence, so it can be run first to establish a base for comparison. The guesses for the other cases may be obtained by changing the solution for the solid pig in a way that is compatible with some intuitive knowledge of the system. For example, a pig with a hole in it should travel more slowly than the solid pig, and should give a slightly greater exit Mach number. The function values may be printed out for the initial guess simply by setting IPRINT = 1. The program has a prompt which asks whether this printing is desired. This feature is extremely useful in obtaining a first guess and in checking initial convergence.
RESULTS AND DISCUSSION

The dimensionless parameters that must be specified for the system are $p_0$, $m^*$, $\gamma$, $4f_c$, $4f_p$, $l^*$, $L^*$, $k_1$, $k_2$, $k_3$, and $F_f^*$. Thus, the nondimensionalized problem is only slightly more general than the dimensional formulation, with its fifteen required parameters. Since eleven dimensionless parameters are required to characterize the system, a comprehensive sampling of all possible flow conditions is not feasible. The results to be presented represent only a fraction of the possible conditions, but will serve to illustrate some possible effects. The dimensionless input parameters that were used in all of the cases considered were:

\[
\begin{align*}
  p_0 &= 3.3 \\
  m^* &= 570 \\
  \gamma &= 1.3 \\
  4f_c &= 0.0001 \\
  4f_p &= 0.0001 \\
  l^* &= 3.0 \\
  L^* &= 160,000
\end{align*}
\]

These were chosen to correspond approximately to the dimensional parameters:
\( P_0 = 330 \text{ psia} \)
\( P_e = 100 \text{ psia} \)
\( T = 50^\circ F \)
\( R = 96.6 \text{ ft-lbf/lbm}^\circ R \)
\( d_p = 1 \text{ foot} \)
\( m_c = 100 \text{ lbm} \)
\( l = 3 \text{ feet} \)
\( L = 30 \text{ miles} \)

The values of \( k_1, k_2, k_3, \) and \( F^*_f \) were varied, while keeping the dimensionless parameters listed above constant. The pig mass was assumed to be independent of pig configuration in the results to be presented. For a more accurate approximation, the value of \( m_c^* \) should be varied, in order to be consistent with the values of \( k_1, k_2, \) and \( k_3. \) When all other flow parameters are constant, decreasing the mass of the pig increases the pig velocity and the gas flow rate. Thus, an analysis such as the one to be presented, which takes \( m_c \) to be the mass of the solid pig, regardless of the pig geometry, will underestimate the exit Mach number and average pig velocity for all cases except for the case of the solid pig.

The average pig velocity and the gas flow rate are the two most important results to be obtained from the analysis. Since the pressure and cross sectional area
at the pipe exit are constant, the mass flow rate at the
exit is directly proportional to the exit Mach number,
M_e. Thus, changes in exit Mach number will be interpreted
as changes in gas flow rate in the cases to follow.

The velocity of the solid pig may be regulated only
by adjusting the amount of friction between the outside
of the pig and the pipe wall. This is achieved by
changing the amount of interference between the outer
diameter of the pig and the inner diameter of the pipe.
Figures 3 and 4 illustrate the effect of friction on
the exit Mach number and on the dimensionless average
velocity of the solid pig. The exit Mach number for
the pipe containing no pig is 0.64322. At low values of
friction between the solid pig and the pipe, the exit
Mach number for the pipe carrying the pig is slightly
less. The average pig velocity and exit Mach number are
nearly constant over the range \( F^*_f = 0 \) to \( F^*_f = 0.1 \).
Increasing the amount of friction beyond \( F^*_f = 0.1 \)
results in a rapid drop in both exit Mach number and
average pig velocity, so that for an average pig
velocity of 0.23, the exit Mach number is about half of
that for the clear pipe. This method of slowing
the pig, by increasing the friction between the pig and
the pipe, is obviously unsatisfactory, due to the large
decreases in gas flow rate.
FIGURE 3: Effect of Friction on Exit Mach Number for the Solid Pig

FIGURE 4: Effect of Friction on the Average Velocity of the Solid Pig
Drilling a concentric, constant diameter hole through the solid pig would provide a better means of decreasing the pig velocity. Figures 5 and 6 illustrate the effect of friction on the exit Mach number and on the average pig velocity for a pig having a constant ratio of hole diameter to pipe diameter of 0.3. The shapes of these curves are similar to those obtained for the solid pig. But by adding the hole, the pig velocity of 0.23 is obtained with a small amount of friction between the pig and the pipe, and the exit Mach number is only slightly less than that for the pipe containing no pig. In addition, the exit Mach number is slightly higher than that for the solid pig with the same amount of friction.

A wide range of pig velocities may be obtained, simply by varying the size of the hole. Figures 7 and 8 demonstrate the effect of hole size on exit Mach number and on average pig velocity for $F^*_f = 0.001$. As the hole size increases, the exit Mach number increases and the pig velocity decreases.

At the same value of $F^*_f$, the exit Mach number for the case of a pig with any size hole is greater than that for the case of the solid pig, but the pig with a hole travels more slowly through the pipe, obstructing the flow longer. Because the pigs with different hole sizes require different amounts of time to travel the
FIGURE 5: Effect of Friction on Exit Mach Number for the Pig with a Constant Diameter Hole (k=0.3)

FIGURE 6: Effect of Friction on the Average Velocity of the Pig with a Constant Diameter Hole (k=0.3)
FIGURE 7: Effect of Constant Hole Diameter on Exit Mach Number \((F_f=0.001)\)

FIGURE 8: Effect of Constant Hole Diameter on Pig Velocity \((F_f=0.001)\)
length of the pipe, the effect of the hole size on the total amount of gas transported cannot be obtained by merely comparing the exit Mach numbers. The same time length must be used to compare the different pigs. Let $t_{\text{max}}$ be the dimensionless time needed for the slowest pig to travel through the pipe. A pig with a different hole diameter travels through the pipe in a faster dimensionless time, $t$. Until time $t$, the exit Mach number is that obtained for the pipe carrying this pig, but between times $t$ and $t_{\text{max}}$, the exit Mach number is that for the unobstructed pipe. The expression for the total mass of gas transported over the length of time $t_{\text{max}}$ is

$$ m_{\text{TOT}} = \frac{\pi q^2 P e}{4RT} [M_e t + M_{ep}(t_{\text{max}} - t)] $$

where $M_e$ is the exit Mach number for the pipe carrying the pig, $M_{ep}$ is the exit Mach number for the clear pipe, $t$ is the time required for a pig to move through the pipe, and $t_{\text{max}}$ is the amount of time for the slowest pig considered to travel through the line. A comparison of the total amounts of gas transferred, for the hole sizes presented earlier, is given in Table 1. $F_\text{f}^*$ is 0.001, as before.
The total amount of gas transported decreases as the hole size increases from k=0 to k=0.5, but the amount of this decrease is not very significant when it is compared to the total amount transferred. Reducing the mass of the pig when the hole size is increased should make the differences in mass transferred even more negligible, since the lighter pig travels faster, and since the exit Mach number is greater for the lighter pig.

The results presented indicate that a pipe containing a pig with a constant diameter hole has a higher gas flow rate than the pipe carrying a solid pig, and the total amount of gas transferred over comparable time lengths is only slightly less than that for the pipe carrying the solid pig. This may be important in long pipelines where a consistently high flow rate may be required. More importantly, varying the diameter of the hole in the
pig allows considerable versatility in regulating the pig velocity, while barely affecting the total amount of gas transported.

Other configurations for the pig may be considered. The cases of a pig with an annulus and of a pig having a variable diameter hole presented special numerical problems that were not encountered in the cases of the solid pig and of the pig having a constant diameter hole. These problems involved the convergence of the nonlinear equation solver to a solution. The convergence characteristics for each of the cases considered will be discussed in more detail later in this section.

Table 2 gives a comparison of pig velocity and exit Mach number for the solid pig, with $F_k = 0$, and for a pig with an annulus. The ratio of pig diameter to pipe diameter is 0.9 for the pig with a clearance. Convergence for this case was difficult to obtain, so the results presented are based on only partial movement of the pig through the pipe. Due to these convergence problems, no results were obtained for the cases where both a hole and an annulus exist.

<table>
<thead>
<tr>
<th>$k_3$</th>
<th>$M_e$</th>
<th>$\dot{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.64296</td>
<td>0.27896</td>
</tr>
<tr>
<td>0.9</td>
<td>0.64302</td>
<td>0.15208</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Solid Pig and a Pig with an Annulus.
The case of a pig having a variable diameter hole also presented convergence problems, although results could occasionally be obtained. Taking $F_f^* = 0.001$, Table 3 compares the pig velocity and exit Mach number for the solid pig, where $k_1 = k_2 = 0$, for the pig with $k_1 = k_2 = 0.3$, and for the pig with $k_1 = 0.31$, $k_2 = 0.3$. The average Mach number for the gas flowing through the hole is also presented.

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$M_e$</th>
<th>$\dot{x}$</th>
<th>$M_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.64267</td>
<td>0.278937</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>0.64273</td>
<td>0.23057</td>
<td>0.7215</td>
</tr>
<tr>
<td>0.31</td>
<td>0.3</td>
<td>0.64431</td>
<td>0.25735</td>
<td>0.0832</td>
</tr>
</tbody>
</table>

Table 3: Comparison of Solid Pig, Pig with a Constant Diameter Hole, and Pig with a Variable Diameter Hole.

Although the above solution for the variable diameter hole satisfies the constraints on the solution, and satisfies the governing equations to the specified accuracy (the sum of squares of the functions is less than $10^{-10}$), this solution initially appears unreasonable. The Mach number of the gas flowing through the variable diameter hole is dramatically lower than that through the constant diameter hole. The small change in diameter should not cause such a large decrease. The
solutions for the exit Mach number and pig velocity, however, do not look as bad. This behavior is probably due to the following numerical peculiarity of the equations. For small hole sizes and a low pig friction factor, \(4f_c\), the solutions for \(\dot{x}\) and \(M_e\) may be almost independent of the hole Mach number. Thus, convergence to an accurate pig velocity and exit Mach number may occur long before accurate hole Mach numbers are obtained. Depending on the flow parameters, then, the program may yield a very good approximation to the pig velocity and the exit Mach numbers, while giving a very poor approximation to the average Mach number in the hole. It may be possible to improve the solution by specifying a greater accuracy, although roundoff errors may become significant.

The results presented so far are applicable only to the particular flow situation considered. The basic trends may generalize to other systems, but each system that is described by different dimensionless parameters must be analyzed individually, in order to obtain precise results for that set of flow conditions. The remainder of this section will concern some aspects of the analysis that are more general.

Care must be taken to insure that the specified flow conditions do not cause choking to occur in the
pipe, since the program presented here does not consider this possibility, and convergence will not occur. Choking may occur in the hole or annulus, and the program attempts to deal with these cases. Choking is not detected before the solution is found, and this case is analyzed after a physically impossible solution has been obtained. Due to the averaging of velocities across the cross sections immediately upstream and downstream of the pig, the program may allow supercritical velocities in the hole or annulus, as long as the average velocity over the cross section is subcritical. Convergence to this solution will occur, and this solution is returned to the main program. The main program then tests the Mach numbers in the hole and annulus, to determine if they are supercritical, and thus impossible. If they are, the program assumes choked flow, sets the appropriate parameters, and calls the equation solver again to solve for the correct, choked, solution at that step. This method was successful in treating choking for the case of a pig with a hole only. Convergence problems prevented testing for the other pig configurations.

All of the problems encountered in the analysis of the system involved the nonlinear equation solver that was chosen, and not the equations to be solved. The method presented appears to provide a realistic mathematical modeling of the physical system, which was the primary
objective. The remaining problems to be solved are numerical, and of secondary importance.

Problems in obtaining convergence were significant for some of the cases considered. The speed and probability of obtaining a solution varied tremendously, depending on the pig geometry considered. For some of the pig configurations, much improvement in the convergence of the equations is needed before the method would be suitable for general use.

Convergence for the case of the solid pig was rapid, and the solution would usually be obtained with even a very bad initial guess. Because of these characteristics, it was often useful to run the program for the case of the solid pig before attempting other geometrical variations. The results obtained for the solid pig were then used as a guide in selecting the initial guesses for the other configurations.

Convergence was slower for the case of a pig having a constant diameter hole, and a better initial guess was usually required. The rate of convergence decreased as the hole size increased, and a more accurate initial guess was required with increasing hole diameter. In general, if the initial guesses were accurate to one significant figure, convergence occurred, but for small holes, the method often converged with a much worse guess.
The initial guesses had to be accurate to at least one significant digit for the method to initially converge in the case of a pig having an annulus. The rate of convergence was extremely slow, and convergence problems occurred after the initial time step. This later failure to converge was a problem that did not occur in the analysis of other pig configurations.

Convergence was erratic for the case of a pig having a variable diameter hole. The method sometimes converged with a bad initial guess and diverged with a good one. On the few occasions when convergence occurred, the rate was comparable to that obtained for the case of the pig having a constant diameter hole.

Results for the cases where both a hole and an annulus exist were never obtained, probably due to the same difficulties experienced in analyzing the pig with only an annulus. If the method did converge for the combination cases, the rate would be expected to be slower than that obtained for the other cases, since more equations must be solved.

Generally, the solution to the governing equations should be obtained more rapidly when the initial guess is very good, since fewer iterations are required. This suggests a possibly major improvement in the program. In the program listed, the initial guess for the solution at the first time step is read as input. The program uses
this initial guess to calculate the solution at the first step. At all time steps after the first one, however, the program uses the solution at the previous time step as the initial guess for the solution at the current step. This method has worked for even very large time increments, in all cases except those where an annulus was present. But the speed of convergence could be increased by improving the initial guesses at these later time steps. Employing some extrapolation technique to predict the solution currently sought would achieve this. Such a modification may also solve some of the convergence problems encountered in the analysis of a pig having an annulus.
CONCLUSION

Equations have been developed to describe the gas flow and pig motion in a gas pipeline. One dimensional, quasi-steady, isothermal flow with friction was assumed. The governing equations form a system of nonlinear differential equations requiring a numerical solution. Time derivatives were replaced by backward differences, and a system of nonlinear algebraic equations was solved at each time step, using the Powell hybrid method.

Eleven dimensionless parameters describe the physical system. Since so many parameters must be specified, the generality of any solution obtained is limited. Sample results were presented for one particular system which carried pigs of various configurations. These results indicated that a pig having a concentric, constant diameter hole could provide higher gas flow rates while in the line than the solid pig having the same amount of friction. Using a pig with a hole also allowed considerably more versatility in designing a pig which would travel at a specific speed, without decreasing the gas flow rate. Conservative estimates of the benefits that could be achieved by using a pig having a hole were generated for the system analyzed.
Numerical convergence problems arose for the cases of a pig with an annulus and of a pig having a variable diameter hole. These problems must be eliminated before the possible benefits of using pigs with these configurations can be evaluated.
REFERENCES


18. Whipple, L. Frank. "Price of On-Stream Pigging: 0.75 to 1.5% of Installation Cost," *Oil and Gas J.* (July 18, 1966).
APPENDIX - PROGRAM LISTING
This program solves the governing equations for flow of a compressible fluid through a pipe with a pig (capsule) in the line. The concentric hole and annulus configurations considered are:

Case 1: Concentric hole of linearly decreasing diameter along the length; no annulus
Case 2: No hole; annulus with constant diameters
Case 3: No hole; no annulus
Case 4: Constant diameter hole; annulus with constant diameters
Case 5: Constant diameter hole; no annulus
Case 6: Hole of linearly decreasing diameter along the length; annulus with constant diameters

Definition of terms:

ACC = Required accuracy of solution of N dimensional equation
ACCN = Dimensionless acceleration of pig
AEMN = Average exit Mach number
AEIP = Pressure at annulus exit
AJINV = Approximation to the inverse of the Jacobian matrix
AK1 = Ratio of upstream hole diameter to pipe diameter
AK2 = Ratio of downstream hole diameter to pipe diameter
AK3 = Ratio of outer pig diameter to pipe diameter
APV = Average pig velocity
B(1) = Inlet Mach number
B(2) = Exit Mach number
B(3) = Ratio of upstream pressure to exit pressure
B(4) = Ratio of downstream pressure to exit pressure
B(5) = Residual of position
B(6) = Upstream annulus Mach number
B(7) = Downstream annulus Mach number
B(8) = Upstream hole Mach number
B(9) = Downstream hole Mach number
DELTA = Time increment
DELTIF = Final time increment
DMAX = Estimate of the distance of the solution from the initial guess
EMN = Vector containing the exit Mach numbers at each step
FC = Pig friction factor
FF = Dimensionless frictional force parameter
FP = Pipe friction factor
GAM = Ratio of specific heats
GENFUN = Subroutine to evaluate governing equations
HEXITP = Pressure at hole exit
ICHOKA = Parameter set to zero if flow through the annulus is not choked; set to 1 if choked
ICHOKH= PARAMETER SET TO ZERO IF FLOW THROUGH THE
HOLE IS NOT CHOKED; SET TO 1 IF CHOKED
IFLAG= FLAG TO INDICATE LAST TIME STEP
MAXFUN= MAXIMUM NUMBER OF FUNCTION EVALUATIONS TO BE USED
IN SOLVING THE SYSTEM OF EQUATIONS
N= NUMBER OF EQUATIONS TO BE SOLVED
NSOLN= EQUATION SOLVER SUBROUTINE
PIGL= RATIO OF PIG LENGTH TO PIPE DIAMETER
PIPEL= RATIO OF PIPE LENGTH TO PIPE DIAMETER
PMASS= DIMENSIONLESS MASS PARAMETER
PO= RATIO OF INLET PRESSURE TO EXIT PRESSURE
POS= DIMENSIONLESS POSITION OF PIG
SIGMA= SUM OF GAMMAS
STEP= STEP LENGTH TO BE USED TO APPROXIMATE DERIVATIVES BY
FINITE DIFFERENCES
VEL= DIMENSIONLESS VELOCITY OF PIG
X= VECTOR CONTAINING THE DIMENSIONLESS POSITIONS OF
THE PIG AT EACH TIME STEP
Y= VECTOR CONTAINING THE VARIABLES TO BE SOLVED FOR

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION X(100),YLOG(9),Y(9),F(9),B(9),AJINV(9,9),
* W(207),EMN(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FP,PIGL,PIPEL,FF,
* DELTAT,X,SIGMA,DELTAT,POS,VEL,ACCN,HEXP,F,HEXIP,
* ICASE,IFLAG,ICHOKH,ICHOKA
EXTERNAL GENFUN
PRINT 1
1 FORMAT(' WHICH CASE NUMBER IS THIS?')
READ(5,2)ICASE
2 FORMAT(12)

READ APPROPRIATE INPUT PARAMETERS
GO TO (10,20,30,40,50,60),ICASE
N=7
AK3=1.
PRINT 11
11 FORMAT(' ENTER AK1,AK2,PO,PMASS,GAM,FC,FP,PIGL,PIPEL,
*FF,DELTAT')
READ(5,12)AK1,AK2,PO,PMASS,GAM,FC,FP,PIGL,PIPEL,FF,DELTAT
PRINT 17
A-3

17 FORMAT( 'ENTER INITIAL GUESSES FOR INLET MACH NUMBER, EXIT, MACH NUMBER, UPSTREAM PRESSURE RATIO, / \ DOWNSTREAM PRESSURE RATIO, RESIDUAL, UPSTREAM HOLE MACH NUMBER, DOWNSTREAM HOLE MACH NUMBER ')' )
READ(5,12)(Y(L),L=1,7)

12 FORMAT(15E15.8)
PRINT 13, CASE, PO, PMASS, GAM, FC, FP, PIGL, PIPEL, DELTAT
FORMAT( 'THE INPUT PARAMETERS ARE: ' CASE=' ',12/ ' ' INLET MACH NUMBER=' E15.8/ ' EXIT MACH NUMBER=' E15.8/ ' UPSTREAM PRESSURE RATIO=' E15.8/ ' DOWNSTREAM PRESSURE RATIO=' E15.8/ ' RESIDUAL=' E15.8/ ' UPSTREAM HOLE MACH NUMBER=' E15.8/ ' DOWNSTREAM HOLE MACH NUMBER=' E15.8')
PRINT 14, AK1, AK2, FF
FORMAT( 'K1=' E15.8/ ' K2=' E15.8/ ' FRICTIONAL FORCE PARAMETER=' E15.8)
PRINT 15, Y(L), L=1,5
FORMAT( 'THE FOLLOWING ARE THE INITIAL GUESSES FOR THE UNKNOWN VARIABLES: ' INLET MACH NUMBER=' E15.8/ ' EXIT MACH NUMBER=' E15.8/ ' UPSTREAM PRESSURE RATIO=' E15.8/ ' DOWNSTREAM PRESSURE RATIO=' E15.8/ ' RESIDUAL=' E15.8/ ' TIME INCREMENT=' E15.8)
PRINT 16, AK1, AK2, FF

20 N=7
AK1=0.
AK2=0.
PRINT 21
FORMAT( 'ENTER AK3, PO, PMASS, GAM, FC, FP, PIGL, PIPEL, DELTAT ')
READ(5,12)AK3, PO, PMASS, GAM, FC, FP, PIGL, PIPEL, DELTAT
PRINT 27

27 FORMAT( 'ENTER INITIAL GUESSES FOR INLET MACH NUMBER, EXIT, MACH NUMBER, UPSTREAM PRESSURE RATIO, / \ DOWNSTREAM PRESSURE RATIO, / \ RESIDUAL, UPSTREAM ANNULUS MACH NUMBER, DOWNSTREAM ANNULUS MACH NUMBER ')' )
READ(5,12)(Y(L),L=1,7)
FF=0.
PRINT 13, CASE, PO, PMASS, GAM, FC, FP, PIGL, PIPEL, DELTAT
PRINT 23, AK3
PRINT 15, Y(L), L=1,5
PRINT 24, Y(L), Y(7)

24 FORMAT( 'UPSTREAM ANNULUS MACH NUMBER=' E15.8/ ' DOWNSTREAM ANNULUS MACH NUMBER=' E15.8/ ' ' )
GO TO 70
N=5
AK1=0.
AK2=0.
AK3=1.
PRINT 31

FORMAT(' ENTER PO,PMass,GAM,FC,FP,PIGL,PIPEL,FF,DELTAT')
READ(5,12)PO,PMass,GAM,FC,FP,PIGL,PIPEL,FF,DELTAT
PRINT 37

FORMAT(' ENTER INITIAL GUESSES FOR INLET MACH NUMBER,EXIT
*MACH NUMBER,'/ UPSTREAM PRESSURE RATIO,DOWNSTREAM PRESSURE RATIO,
*RESIDUAL')
READ(5,12)Y
PRINT 13,ICASE,PO,PMass,GAM,FC,FP,PIGL,PIPEL,DELTAT
PRINT 33,FF

FORMAT(' FRICTIONAL FORCE PARAMETER=E15.8)
PRINT 15,(Y(L),L=1,5)
GO TO 70
N=9
PRINT 41

FORMAT(' ENTER AK,AK3,PO,PMass,GAM,FC,FP,PIGL,PIPEL,DELTAT')
READ(5,12)AK1,AK3,PO,PMass,GAM,FC,FP,PIGL,PIPEL,DELTAT
PRINT 47

FORMAT(' ENTER INITIAL GUESSES FOR INLET MACH NUMBER,
*EXIT MACH NUMBER,UPSTREAM PRESSURE RATIO,'/ DOWNSTREAM PRESSURE
*RATIO,RESIDUAL,UPSTREAM ANNULUS MACH NUMBER,DOWNSTREAM
*ANNULUS MACH NUMBER,UPSTREAM HOLE MACH NUMBER,DOWNSTREAM
*HOLE MACH NUMBER')
READ(5,12)(Y(L),L=1,9)
AK2=AK1
FF=0.
PRINT 13,ICASE,PO,PMass,GAM,FC,FP,PIGL,PIPEL,DELTAT
PRINT 43,AK1,AK3

FORMAT(' K=E15.8, K3=E15.8)
PRINT 15,(Y(L),L=1,5)
PRINT 24,Y(6),Y(7)
PRINT 16,Y(8),Y(9)
GO TO 70
N=7
PRINT 51

FORMAT(' ENTER AK,PO,PMass,GAM,FC,FP,PIGL,PIPEL,FF,DELTAT')
READ(5,12)AK1,PO,PMass,GAM,FC,FP,PIGL,PIPEL,FF,DELTAT
PRINT 57

FORMAT(' ENTER INITIAL GUESSES FOR INLET MACH NUMBER,
*EXIT MACH NUMBER,UPSTREAM PRESSURE RATIO,'/ DOWNSTREAM PRESSURE
*RATIO,RESIDUAL,UPSTREAM HOLE MACH NUMBER,DOWNSTREAM HOLE MACH
*NUMBER')
READ(5,12)(Y(L),L=1,7)
AK2=AK1
AK3=1.
PRINT 13,ICASE,PO,PMASS,GAM,FC,FP,PIGL,Pipel,DELTAT
PRINT 53,AK1,FF
53  FORMAT(' K=',E15.8/' FRICTIONAL FORCE PARAMETER=',E15.8)
PRINT 15,(Y(L),L=1,5)
PRINT 16,Y(6),Y(7)
GO TO 70
60 N=9
PRINT 61
61 FORMAT(' ENTER AK1,AK2,AK3,PO,PMASS,GAM,FC,FP,PIGL,PipeL,
*DELTAT')
READ(5,12)AK1,AK2,AK3,PO,PMASS,GAM,FC,FP,PIGL,Pipel,DELTAT
PRINT 67
67 FORMAT(/' ENTER INITIAL GUESSES FOR INLET MACH NUMBER,EXIT
*MACH NUMBER,UPSTREAM PRESSURE RATIO,DOWNSTREAM/' PRESSURE RATIO,
*RESIDUAL,UPSTREAM ANNUlus MACH NUMBER,DOWNSTREAM ANNUlus MACH
*NUMBER,UPSTREAM '/' HOLE MACH NUMBER,DOWNSTREAM HOLE MACH NUMBER')
READ(5,12)(Y(L),L=1,9)
FF=0.
PRINT 13,ICASE,PO,PMASS,GAM,FC,FP,PIGL,Pipel,DELTAT
PRINT 63,AK1,AK2,AK3
63  FORMAT(' 1(1=',E15.8/' 1(2=',E15.8/' 1(3=',E15.8)
PRINT 15,(Y(L),L=1,5)
PRINT 24,Y(6),Y(7)
PRINT 16,Y(8),Y(9)
C
C SET PARAMETERS FOR THE SUBROUTINE NS01A
70 PRINT 71
71 FORMAT(' ENTER STEP SIZE,ACCURACY,MAXIMUM NUMBER OF
*FUNCTION EVALUATIONS,'/' MAXIMUM PREDICTED DISTANCE FROM
*SOLUTION')
READ(5,72)STEP,ACC,MAXFUN,DMAX
72 FORMAT(2E15.8.I5,E15.8)
PRINT 73,STEP,ACC,MAXFUN,DMAX
73  FORMAT(//' STEP SIZE=',E15.8/' ACCURACY REQUIRED=',E15.8/ 
:(< 'MAXIMUM NUMBER OF FUNCTION EVALUATIONS=' ,I5/ 
*: MAXIMUM PREDICTED DISTANCE FROM SOLUTION=' ,E15.8)
PRINT 74
74  FORMAT(' IF YOU WANT THE FUNCTION VALUES PRINTED AT EACH 
*ITERATION,'/' ENTER 1; OTHERWISE ENTER 0')
READ(5,75)IPR
75 FORMAT(I2)
FLAG=0
CALCULATE UNKNOWNS AT EACH TIME STEP

CRITH=1./DSQRT(GAM)
ICHOKA=0
ICHOKH=0
SIGMA=0.
Y(3)=Y(3)/10.
Y(4)=Y(4)/10.
DO 80 L=1,N
YLOG(L)=YLOG(Y(L))
80 CONTINUE
I=1
IF(ICASE.EQ.3)PRINT 85
IF(ICASE.EQ.2)PRINT 86
IF(ICASE.EQ.1.OR.ICASE.EQ.5)PRINT 87
IF(ICASE.EQ.4.OR.ICASE.EQ.6)PRINT 88
85 FORMAT(///6X,' TIME',9X,' POSITION',4X,' INLET MACH NO.',
* 2X,' EXIT MACH NO.',3X,' UPSTREAM P',4X,' DOWNSTREAM P'),
* 2X,' ANNULUS M+',5X,' ANNULUS M-')
86 FORMAT(///6X,' TIME',9X,' POSITION',4X,' INLET MACH NO.',
* 2X,' EXIT MACH NO.',3X,' UPSTREAM P',4X,' DOWNSTREAM P',
* 4X,' ANNULUS M+',5X,' ANNULUS M-')
87 FORMAT(///6X,' TIME',9X,' POSITION',4X,' INLET MACH NO.',
* 2X,' EXIT MACH NO.',3X,' UPSTREAM P',4X,' DOWNSTREAM P',
* 5X,' HOLE M+',9X,' HOLE M-')
88 FORMAT(///6X,' TIME',6X,' POSITION ',5X,' INLET M+',8X,' P+',1OX,' P-',
* 6X,' ANNULUS M+',2X,' ANNULUS M-',4X,' HOLE M+',4X,' HOLE M-')
90 CALL NSO1A(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
IF(POS.GT.PIPEL)GO TO 100
GO TO C92.92.93,93,93',ICASE
92 IF(CCASE.EQ.7).LT.CRITH)GO TO 96
IF(CCASE.EQ.1.OR.CCASE.EQ.5)ICHOKH=1
IF(CCASE.EQ.2)ICHOKA=1
YLOG(7)=YLOG(4)
CALL NSO1A(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
YLOG(7)=YLOG(CRITH)
GO TO 96
93 Y(7)=DEXP(YLOG(7))
Y(9)=DEXP(YLOG(9))
IF(Y(7).LT.CRITH.AND.Y(7).LT.CRITH)GO TO 76
IF(Y(7).GT.CRITH.AND.Y(9).GT.CRITH)GO TO 94
IF(Y(7).GT.CRITH)GO TO 95
ICHOKH=1
YLOG(7)=YLOG(4)
CALL NS01A(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
YLOG(7)=DLOG(CRITH)
GO TO 96
94 ICHOKH=1
ICHOKA=1
YLOG(7)=YLOG(4)
YLOG(9)=YLOG(4)
CALL NS01A(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
YLOG(7)=DLOG(CRITH)
YLOG(9)=DLOG(CRITH)
GO TO 96
95 ICHOKA=1
YLOG(7)=YLOG(4)
CALL NS01A(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
YLOG(7)=DLOG(CRITH)
96 X(I)=POS
DO 97 L=1,N
Y(L)=DEXP(YLOG(L))
97 CONTINUE
Y(3)=Y(3)*10.
Y(4)=Y(4)*10.
OLDSIG=SIGMA
SIGMA=Y(5)*PIPEL
YLOG(5)=DLOG((2.*SIGMA-OLDSIG)/PIPEL)
TIME=I*DELTAT
EMN(I)=Y(2)
IF ICASE.EQ.3 PRINT 120,TIME,POS,(Y(L),L=1,4)
IF ICASE.EQ.1 OR ICASE.EQ.2 OR ICASE.EQ.5 PRINT 121,TIME,
* POS,(Y(L),L=1,4),(Y(6),Y(7)
* (Y(L),L=1,4),(Y(L),L=6,9)
ICHOKA=0
ICHOKH=0
I=I+1
GO TO 90

LAST TIME STEP

SINCE THE POSITION IS KNOWN AT THE LAST TIME STEP, USE POS=PIPEL,
AND SOLVE FOR DELTF.

100 IFLAG=1
N=N-2
YLOG(2)=DLOG(0.1)
DO 101 K=J+N
101 YLOG(K)=DLOG((PIPEL-X(I-1))/(X(I-1)-X(I-2)))
CALL NS01A(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
GO TO(102,102,110,103,102,103),ICASE
IF(ICASE.EQ.1.OR.ICASE.EQ.5)ICHOKH=1
IF(ICASE.EQ.2)ICHOKA=1
YLOG(5)=DLOG(0.1)
CALL NSOA(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
YLOG(5)=DLOG(CRITH)
GO TO 110
103    Y(5)=DEXP(YLOG(5))
    Y(7)=DEXP(YLOG(7))
    IF(Y(5).LT.CRITH.AND.Y(7).LT.CRITH)GO TO 110
    IF(Y(5).GT.CRITH.AND.Y(7).GT.CRITH)GO TO 104
    IF(Y(5).GT.CRITH)GO TO 105
ICHOKH=1
YLOG(7)=DLOG(0.1)
CALL NSOA(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
YLOG(7)=DLOG(CRITH)
GO TO 110
104    ICHOKH=1
        ICHOKA=1
        YLOG(5)=DLOG(0.1)
        YLOG(7)=DLOG(0.1)
        CALL NSOA(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
        YLOG(5)=DLOG(CRITH)
        YLOG(7)=DLOG(CRITH)
        GO TO 110
105    ICHOKA=1
        YLOG(5)=DLOG(0.1)
        CALL NSOA(N,YLOG,F,AJINV,STEP,DMAX,ACC,MAXFUN,IPR,W,GENFUN)
        YLOG(5)=DLOG(CRITH)
110    TIME=(I-1)*DELTAT+DEXP(YLOG(3))*DELTAT
    DO 115 L=1,N
    Y(L)=DEXP(YLOG(L))
    CONTINUE
    Y(2)=Y(2)*10.
    IF(ICASE.EQ.1.OR.ICASE.EQ.5)EMN(I)=(AK2**2)*Y(5)
    *+(1.-AK2**2)*VEL
    IF(ICASE.EQ.4.OR.ICASE.EQ.6)EMN(I)=(AK2**2)*Y(7)
    *+(1.-AK3**2)*Y(5)+(AK3**2-AK2**2)*VEL
    IF(ICASE.EQ.2)EMN(I)=(1.-AK2**2)*Y(5)+(AK2**2)*VEL
    IF(ICASE.EQ.3)EMN(I)=VEL
    PMINUS=1.0
    IF(ICASE.EQ.3)PRINT 120,TIME,POS,Y(1),EMN(I),Y(2),PMINUS
    IF(ICASE.EQ.1.OR.ICASE.EQ.2.OR.ICASE.EQ.5)PRINT 121,TIME, *
    POS,Y(1),EMN(I),Y(2),PMINUS,Y(4),Y(5)
    IF(ICASE.EQ.4.OR.ICASE.EQ.6)PRINT 122,TIME,POS,Y(1), *
    EMN(I),Y(2),PMINUS,(Y(L),L=4,7)
* C * C * C CALCULATE AVERAGE PIG VELOCITY
C APV=PIPEL/TIME
PRINT 140,APV
140 FORMAT(//' THE AVERAGE PIG VELOCITY WAS',E15.8)
C C C CALCULATE AVERAGE EXIT MACH NUMBER
C SUM=0.
IM=I-1
DO 141 J=1,IM
141 SUM=SUM+EMN(J)
SUM=SUM*DELTAT+EMN(I)*DELTF
AEMN=SUM/TIME
PRINT 145, AEHN
145 FORMAT(//' THE AVERAGE EXIT MACH NUMBER WAS',E15.8)
STOP
END
SUBROUTINE NSO1A(N,X,F,AJINV,DSTEP,DMAX,ACC,MAXFUN,
* IPRINT,W,CALFUN)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION X(N),F(N),AJINV(N,N),W(1)
EXTERNAL CALFUN
C SET VARIOUS PARAMETERS
MAXC=0
C MAXC COUNTS THE NUMBER OF CALLS OF CALFUN
NT=N+4
NTEST=NT
C NT AND NTEST CAUSE AN ERROR RETURN IF F(X) DOES NOT DECREASE
DTEST=FLOAT(N+N)-0.5
C DTEST IS USED TO MAINTAIN LINEAR INDEPENDENCE
NX=N*N
NF=NX+N
NW=NF+N
MW=NW+N
NDC=MW+N
ND=NDC+N .
C THESE PARAMETERS SEPARATE THE WORKING SPACE
C ARRAY W
FMIN=0.
C USUALLY FMIN IS THE LEAST CALCULATED VALUE OF F(X)
C AND THE BEST X IS IN W(NX+1) TO W(NX+N)
DD=0.
C USUALLY DD IS THE SQUARE OF THE CURRENT STEP LENGTH
DSS=DSTEP*DSTEP
DM=MAX*DMAX
DMM=4.*DM
IS=5
C IS CONTROLS A GO TO STATEMENT FOLLOWING A CALL OF CALFUN
TINC=1.
C TINC IS USED IN THE CRITERION TO INCREASE THE STEP LENGTH
C START A NEW PAGE FOR PRINTING
IF(IPRINT)11,1*85
85 PRINT 86
86 FORMAT(1H1)
C CALL THE SUBROUTINE CALFUN
1 MAXC=MAXC+1
CALL CALFUN(N,X,F,IOVER)
IF(IVAL, EQ, 1) RETURN
C TEST FOR CONVERGENCE
FSQ=0.
DO 2 I=1,N
FSQ=FSQ+F(I)*F(I)
2 CONTINUE
IF(FSQ-ACC)3,3,4
C PROVIDE PRINTING OF FINAL SOLUTION IF REQUESTED
3 IF(IPRINT)5,5,6
6 PRINT 7,MAXC
7 FORMAT(///5X,'THE FINAL SOLUTION CALCULATED BY NSO1A
* REQUIRED,'IS,' CALLS OF CALFUN AND IS')
PRINT 8,(I,X(I),F(I),I=1,N)
8 FORMAT(114X,'I',7X,'X(I),F(I),I=1,N)
PRINT 9,FSQ
9 FORMAT(/5X,' THE SUM OF SQUARES IS',E17.8)  
10 RETURN  
C  TEST FOR ERROR RETURN BECAUSE F(X) DOES NOT DECREASE  
11 GO TO (10+11,11,11,11),IS  
12 IF(FSQ-FMIN)15,20,20  
13 IF(ID-DS)12,12,11  
14 NTEST=NTST-1  
15 PRINT 16/NT  
16 FORMAT(/5X,' ERROR RETURN FROM NS01A BECAUSE',IS,  
* 'CALLS OF CALFUN FAILED TO IMPROVE THE RESIDUALS')  
17 DO 18 I=1,N  
18 XC I )=W(NX+I)  
19 F(I)=W(NF+I)  
18 CONTINUE  
20 FSQ=FMIN  
21 GO TO 3  
C  ERROR RETURN BECAUSE A NEW JACOBIAN IS UNSUCCESSFUL  
22 PRINT 19  
23 FORMAT(/5X,' ERROR RETURN FROM NS01A BECAUSE F(X)',  
* ' FAILED TO DECREASE USING A NEW JACOBIAN')  
24 GO TO 17  
25 NTEST=NT  
C  TEST WHETHER THERE HAVE BEEN MAXFUN CALLS OF CALFUN  
26 IF(MAXFUN-MAXC)21,21,22  
27 PRINT 23/MAC  
28 FORMAT(/5X,' ERROR RETURN FROM NS01A BECAUSE',IS,  
* ' THERE HAVE BEEN',IS, ' CALLS OF CALFUN')  
29 IF(FSQ-FMIN)3~17,17  
C  PROVIDE PRINTING IF REQUESTED  
30 IF(IPRINT)24,24,25  
31 PRINT 26/MAC  
32 FORMAT(/5X,' AT THE ',IS, ' TH CALL OF CALFUN WE HAVE')  
33 PRINT 8,(I,X(I),F(I),I=1,N)  
34 PRINT 9,FSQ  
35 GO TO (27,28,29,87,30),IS  
C  STORE THE RESULT OF THE INITIAL CALL OF CALFUN  
36 FMIN=FSQ  
37 DO 31 I=1,N  
38 W(NX+I)=X(I)  
39 W(NF+I)=F(I)  
31 CONTINUE  
C  CALCULATE A NEW JACOBIAN APPROXIMATION  
32 IC=0  
33 IS=3  
34 IC=IC+1  
35 X(IC)=X(IC)+DSTEP  
36 GO TO 1  
29 ,K=IC  
30 DO 34 I=1,N  
31 W(K)=(F(I)-W(NF+I))/DSTEP  
32 K=K+N  
33 CONTINUE  
34 X(IC)=W(NX+IC)  
35 IF(IC-N)33,35,35  
C  CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE  
C  DIRECTION MATRIX  
36 K=0  
37 DO 36 I=1,N  
38 DO 37 J=1,N
K=K+1
AJINV(I,J)=W(K)
W(ND+K)=0.
37 CONTINUE
W(NDC+K+I)=1.
W(NDC+I)=1.+FLOAT(N-I)
36 CONTINUE
CALL MB01B(AJINV,N)
C START ITERATION BY PREDICTING THE DESCENT AND
C NEWTON MINIMA
38 DS=0.
DN=0.
SP=0.
DO 39 I=1,N
X(I)=0.
F(I)=0.
K=1
DO 40 J=1,N
X(I)=X(I)-W(K)*W(NF+J)
F(I)=F(I)-AJINV(I,J)*W(NF+J)
K=K+N
40 CONTINUE
DS=DS+X(I)*X(I)
DN=DN+F(I)*F(I)
SP=SP+X(I)*F(I)
39 CONTINUE
C TEST WHETHER A NEARBY STATIONARY POINT IS PREDICTED
C IF(FMIN*FMIN-DMM*DS)41,41,42
C IF SO THEN RETURN OR REVISE JACOBIAN
42 GO TO (43,43,44),IS
44 PRINT 45
45 FORMAT(/S,X,'ERROR RETURN FROM NS01A BECAUSE A',
'NEARBY STATIONARY POINT OF F(X) IS PREDICTED')
GO TO 17
43 NTEST=0
DO 44 I=1,N
X(I)=W(NX+I)
46 CONTINUE
GO TO 32
C TEST WHETHER TO APPLY THE FULL NEWTON CORRECTION
41 IS=2
IF(DN-DD)47,47,48
47 DD=AMAX1.0,N,DSS)
DS=0.25*DN
TINC=1.
IF(DN-DSS)49,58,5B
49 IS=4
GO TO 80
C CALCULATE THE LENGTH OF THE STEEPEST DESCENT STEP
46 K=0
MULT=0.
DO 51 I=1,N
DW=0.
DO 52 J=1,N
K=K+1
DW=DW+W(K)*X(J)
51 CONTINUE
DMULT=DMULT+DW*DW  
CONTINUE  
DMULT=DS/DMULT  
DS=DS*DMULUDMULT  
C TEST WHETHER TO USE THE STEEPDEST DESCENT DIRECTION  
IF(DS-DD)53,54,54  
C TEST WHETHER THE INITIAL VALUE OF DD HAS BEEN SET  
54 IF(DD)55,55,56  
55 DD=AMAX1(DSS,AMIN1(DM,DS));  
DS=DS/(DMULT*DMULT)  
GO TO 41  
C SET THE MULTIPLIER OF THE STEEPDEST DESCENT DIRECTION  
56 ANMULT=0.  
DMULT=DMULT*SQRT(DD/DS)  
GO TO 98  
C INTERPOLATE BETWEEN THE STEEPDEST DESCENT AND THE NEWTON DIRECTIONS  
53 SP=SP*DMULT  
ANMULT=(DD-DS)/((SP-DS)+SQRT((SP-DD)**2+(DN-DD)*  
*(DD-DS))))  
DMULT=DMULT*(1.-ANMULT)  
C CALCULATE THE CHANGE IN X AND ITS ANGLE WITH THE  
FIRST DIRECTION  
98 DN=0.  
SP=0.  
DO 57 I=1,N  
F(I)=DMULT*X(I)+ANMULT*F(I)  
DN=DN+F(I)*F(I)  
SP=SP+F(I)*W(ND+I)  
57 CONTINUE  
DS=0.25*DN  
C TEST WHETHER AN EXTRA STEP IS NEEDED FOR INDEPENDENCE  
IF(SP-W(NDC+1))>TEST)58,59,58  
IF(SP-SP-DS)60,59,58  
C TAKE AN EXTRA STEP AND UPDATE THE DIRECTION MATRIX  
50 IS=2  
60 DO 61 I=1,N  
X(I)=W(NX+I)+DSTEP*W(ND+I)  
W(NDC+I)=W(NDC+1)+1.  
61 CONTINUE  
W(ND)=1.  
DO 62 I=1,N  
K=ND+I  
SP=W(K)  
DO 63 J=2,N  
W(K)=W(K+N)  
K=K+N  
63 CONTINUE  
W(K)=SP  
62 CONTINUE  
GO TO 1  
C EXPRESS THE NEW DIRECTION IN TERMS OF THOSE OF THE  
DIRECTION MATRIX, AND UPDATE THE COUNTS IN W(NDC+1)  
ETC.  
59 SP=0.
```
K=ND
DO 64 I=1,N
X(I)=0W
DW=0.
TO 65 J=1:N
K=K+1
DW=DW+X(J)*X(K)
65 CONTINUE
GO TO (68,66),66
66 W(NDC+I)=W(NDC+I)+1.
SP=SP+DW*DW
IF(SP-DS)464,67
67 IS=1
KK=1
X(K)=DW
GO TO 69
68 X(I)=DW
69 W(NDC+I)=W(NDC+I)+1.
64 CONTINUE
W(ND)=1.
C REORDER THE DIRECTIONS SO THAT #K IS FIRST
IF(KK-1),70,71
70 KS=NDC+KK*N
DO 72 I=1,N
K=K+1
SP=W(K)
DO 73 J=1:KK
W(K)=W(K-N)
K=K-H
73 CONTINUE
W(K)=SP
72 CONTINUE
C GENERATE THE NEW ORTHOGONAL DIRECTION MATRIX
DO 74 I=1,N
W(NW+I)=0.
74 CONTINUE
SP=X(I)**X(I)
K=ND
DO 75 I=2:N
DS=SQRT(SP*(SP+X(I)**X(I)))
DW=SP/DS
DS=X(I)/DS
SP=SP+X(I)**X(I)
75 CONTINUE
W(K)=SP**I
76 CONTINUE
C CALCULATE THE NEXT VECTOR X* AND PREDICT THE RIGHT HAND SIDES
XMP=0.
K=0
DO 78 I=1:N
K(I)=W(K)*X(I)
78 CONTINUE
```
DO 79 J=1,N
   K=K+1
   W(NW+I)=W(NW+I)+W(K)*F(J)
   CONTINUE
FNP=FNP+W(NW+I)**2
DO 78 CONTINUE
CALL CALFUN USING THE NEW VECTOR OF VARIABLES
GO TO 1
C UPDATE THE STEP SIZE
DMULT=0.9*FMIN+0.1*FNP-FSQ
IF(DMULT)82,81,81
DD=AMAX1(DS+0.25*DD)
TINC=1.
IF(FSQ-FMIN)83,28,28
C TRY THE TEST TO DECIDE WHETHER TO INCREASE THE
C STEP LENGTH
SP=0.
DO 84 I=1,N
   SP=SP+ABS(F(I)-W(NW+I))
SS=SS+(F(I)-W(NW+I))**2
CONTINUE
FJ=1.+DMULT/(SP+SQRT(SP*SP+DMULT*SS))
SP=AMIN1(FJ*TINC,FJ)
TINC=FJ/SP
DD=AMAX1(DM+SP*DD)
GO TO 83
C IF F(X) IMPROVES STORE THE NEW VALUE OF X
IF(FSQ-FMIN)83,50,50
FSQ=FSQ
DO 88 I=1,N
   X(I)=W(NFI+I)
   W(NFI+I)=SP
   F(I)=W(NFI+I)
   W(NFI+I)=SP
   W(NW+I)=-W(NW+I)
CONTINUE
IF(IS=1)28,28,50
C CALCULATE THE CHANGES IN F AND IN X
DO 89 I=1,N
   X(I)=X(I)-W(NX+I)
   F(I)=F(I)-W(NF+I)
CONTINUE
C UPDATE THE APPROXIMATIONS TO J AND TO AJINV
K=0
DO 90 I=1,N
   W(NWI+I)=X(I)
   W(NWI+I)=F(I)
   DO 91 J=1,N
      W(NWI+I)=W(NWI+I)-AJINV(I,J)*F(J)
      K=K+1
   WINW+I)=W(NW+I)-W(K)*X(J)
CONTINUE
DO 90 CONTINUE
SP=0.
SS=0.
DO 92 I=1,N
DS=0.
DO 93 J=1,N
DS=DS+AJINV(J,I)*X(J)
CONTINUE
SP=SP+DS*F(I)
SS=SS+X(I)*X(I)
F(I)=DS
92 CONTINUE
DMULT=1.
IF(ABS(SP)-0.1*SS)94,95,95
94 DMULT=0.8
95 PJ=DMULT/SS
PA=DMULT/(DMULT*SP+(1.-DMULT)*SS)
K=0
DO 96 I=1,N
SP=PA*W(MW+I)
SS=PA*W(MW+I)
DO 97 J=1,N
K=K+1
W(K)=W(K)+SP*X(J)
AJINV(I,J)=AJINV(I,J)+SS*F(J)
97 CONTINUE
96 CONTINUE
GO TO 38
END
SUBROUTINE MB01B(C,N)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION C(N,N),J(100)
PD=1.
DO 124 L=1,N
DD=0.
DO 123 K=1,N
123 DD-DD+C(L,K)*C(L,K)
124 PD=PD*DD
DETM=1.
DO 125 L=1,N
125 J(L+20)=L
DO 144 L=1,N
CC=0.
M=L
IF((ABS(CC)-ABS(C(L,K)))GE.0.) GO TO 135
126 M=K
CC=C(L,K)
135 CONTINUE
127 IF(L.EQ.M) GO TO 138
128 K=J(M+20)
J(M+20)=J(L+20)
J(L+20)=K
DO 137 K=1,N
S=C(K,L)
C(K,L)=C(K,M)
137 C(K,M)=S
138 C(L,M)=1.
DETM=DETM*CC
DO 139 M=1,N
C(L,M)=C(L,M)/CC
DO 142 M=1,N
IF(L.EQ.M)GO TO 142
CC=C(M,L)
IF(CC.EQ.0)GO TO 142
C(M,L)=0.
DO 141 K=1,N
C(M,K)=C(M,K)-C(M,K)*C(L,K)
CONTINUE
CONTINUE
DO 143 L=1,N
IF(J(L+20).EQ.L)GO TO 143
M=L
M=M+1
IF(J(M+20).EQ.L)GO TO 133
IF(N.GT.M)GO TO 132
J(M+20)=J(L+20)
DO 163 K=1,N
CC=C(L,K)
C(L,K)=C(M,K)
C(M,K)=CC
CONTINUE
RETURN
END
SUBROUTINE GENFUN(N,Y,F,IOVER)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION Y(N),F(N),B(9),X(100)
COMMON P0,AK1,AK2,AK3,MASS,GAM,FC,FP,P1GL,P1PL,FF,
* DELTAT,SIGMA,DELTF,POS,VEL,ACCN,HEXITP,AEXITP,
* ICASE,IFLAG,ICHOKH,ICHOKA
IOVER=0
F(1)=DEXP(Y(1))

C C CALCULATE POSITION, VELOCITY, ACCELERATION OF PIG

IF(IFLAG.EQ.1)GO TO 4
B(2)=DEXP(Y(2))
B(3)=DEXP(Y(3))**10.
B(4)=DEXP(Y(4))**10.
B(5)=DEXP(Y(5))**PIPL-SIGMA
GAMMA=B(5)
IF(I.EQ.1)GO TO 2
IF(I.EQ.2)GO TO 3
POS=GAMMA**2.*X(I-1)-X(I-2)
VEL=(GAMMA+SIGMA)/DELTAT
ACCN=(GAMMA+SIGMA)/DELTAT**2)
GO TO (20,30,40,10,20,10),ICASE

2 POS=GAMMA
IF(POS.GT.PIPL)GO TO 51
VEL=(GAMMA+SIGMA)/DELTAT
ACCN=(GAMMA+SIGMA)/DELTAT**2)
GO TO (20,30,40,10,20,10),ICASE

3 POS=GAMMA**2.*X(1)
IF(POS.GT.PIPL)GO TO 51
VEL=(GAMMA+SIGMA)/DELTAT
ACCN=(GAMMA+SIGMA)/DELTAT**2)
GO TO (20,30,40,10,20,10),ICASE

4 DELTF=DEXP(Y(3))**DELTAT
B(3)=DEXP(Y(2))**10.
POS=PIPL
IF(I.EQ.1)GAMMA=PIPL
IF(I.EQ.2)GAMMA=PIPL-2.*X(I-1)
IF(I.LE.1.AND.I.NE.2)GAMMA=PIPL-2.*X(I-1)+X(I-2)
VEL=(GAMMA+SIGMA)/DELTAT
ACCN=(GAMMA+SIGMA+SIGMA)/DELTAT**2)
B(4)=1.0
B(5)=GAMMA
GO TO (20,30,40,10,20,10),ICASE

C C
EVALUATE FUNCTIONS FOR GENERAL CASE AND CASE WITH CONSTANT DIAMETER HOLE

10 IF(FLAG.EQ.1) GO TO 16
   DO 11 J=6,9
11 B(J)=DEXP(Y(J))
   IF(ICHOKA.EQ.0 .AND. ICHOKH.EQ.0) GO TO 14
   IF(ICHOKA.EQ.1 .AND. ICHOKH.EQ.1) GO TO 12
   IF(ICHOKA.EQ.1) GO TO 13
   AEXITP=B(4)
   HEXITP=B(9)*10.
   B(9)=1./DSQRT(GAM)
   GO TO 15
12 HEXITP=B(9)*10.
   AEXITP=B(7)*10.
   B(7)=1./DSQRT(GAM)
   B(9)=1./DSQRT(GAM)
   GO TO 15
13 AEXITP=B(7)*10.
   HEXITP=B(4)
   B(7)=1./DSQRT(GAM)
   GO TO 15
14 HEXITP=B(4)
   AEXITP=B(4)
15 F(2)=FUN2(B)
   F(3)=FUN3(B)
   F(4)=FUN4(B)
   JSUB=5
   GO TO 19
16 DO 17 J=6,9
17 B(J)=DEXP(Y(J-2))
   IF(ICHOKA.EQ.0 .AND. ICHOKH.EQ.0) GO TO 184
   IF(ICHOKA.EQ.1 .AND. ICHOKH.EQ.1) GO TO 182
   IF(ICHOKA.EQ.1) GO TO 183
   AEXITP=B(4)
   HEXITP=B(9)*10.
   B(9)=1./DSQRT(GAM)
   GO TO 185
182 HEXITP=B(9)*10.
   AEXITP=B(7)*10.
   B(7)=1./DSQRT(GAM)
   B(9)=1./DSQRT(GAM)
   GO TO 185
183 AEXITP=B(7)*10.
   HEXITP=B(4)
B(7) = 1./DSQRT(GAM)
GO TO 185

184 HEXITP = B(4)
AEXITP = B(4)
185 B(2) = (AK2**2)*B(9) + (1.-AK3**2)*B(7) + (AK3**2- 
*AK2**2)*VEL
F(2) = FUN3(B)
JSUB = 3
19 F(JSUB) = FUN5(B)
IF(ICASE.EQ.4) F(JSUB+1) = FUN6B(B)
IF(ICASE.EQ.5) F(JSUB+1) = FUN6(B)
F(JSUB+2) = FUN7(B)
F(JSUB+3) = FUN8(B)
F(JSUB+4) = FUN9(B)
GO TO 50

C

EVALUATE FUNCTIONS FOR CASES WITH HOLE AND WITHOUT ANNULUS

20 B(6) = 0.
B(7) = 0.
IF(IFLAG.EQ.1) GO TO 25
B(8) = DEXP(Y(6))
IF(ICHOKH.EQ.1) GO TO 21
HEXITP = B(4)
B(9) = DEXP(Y(7))
GO TO 22
21 HEXITP = DEXP(Y(7))*10.
B(9) = 1./DSQRT(GAM)
22 F(2) = FUN2(B)
F(3) = FUN3B(B)
F(4) = FUN4(B)
JSUB = 5
GO TO 29
25 B(9) = DEXP(Y(4))
IF(ICHOKH.EQ.1) GO TO 26
HEXITP = B(4)
B(9) = DEXP(Y(5))
GO TO 27
26 HEXITP = DEXP(Y(5))*10.
B(9) = 1./DSQRT(GAM)
27 B(2) = (AK2**2)*B(9) + (1.-AK2**2)*VEL
F(2) = FUN3B(B)
JSUB = 3
29 F(JSUB) = FUN5(B)
IF(ICASE.EQ.1) F(JSUB+1) = FUN6(B)
IF(ICASE.EQ.5) F(JSUB+1) = FUN6B(B)
F(JSUB+2) = FUN9(B)
GO TO 50
EVALUATE FUNCTIONS FOR CASE WITH ANNULUS AND WITHOUT HOLE

30 B(8)=0.
B(9)=0.
IF(IFLAG.EQ.1)GO TO 35
B(6)=DEXP(Y(6))
IF(ICHOKA.EQ.1)GO TO 31
AEXITP=B(4)
B(7)=DEXP(Y(7))
GO TO 32
31 AEXITP=DEXP(Y(7))*10.
B(7)=1./DSQRT(GAM)
32 F(2)=FUN2(B)
F(3)=FUN3(B)
F(4)=FUN4(B)
JSUB=5
GO TO 39
35 B(6)=DEXP(Y(4))
IF(ICHOKA.EQ.1)GO TO 34
AEXITP=B(4)
B(7)=DEXP(Y(5))
GO TO 37
36 AEXITP=DEXP(Y(5))*10.
B(7)=1./DSQRT(GAM)
37 B(2)=(AK3**2)*VEL+(1.-AK3**2)*B(7)
F(2)=FUN3(B)
JSUB=3
39 F(JSUB)=FUN7(B)
F(JSUB+1)=FUN8(B)
F(JSUB+2)=FUN9(B)
GO TO 50

EVALUATE FUNCTIONS FOR SOLID PIG CASE

40 45 J=6,9
45 B(J)=0.
IF(IFLAG.EQ.1)GO TO 46
F(2)=FUN2(B)
F(3)=FUN3(B)
F(4)=FUN4(B)
F(5)=FUN9(B)
GO TO 50
46 B(2)=VEL
F(2)=FUN3S(B)
F(3)=FUN9(S)

EVALUATE FUNCTION COMMON TO ALL CASES

50 F(1)=FUN1(B)
RETURN
51 IOVER=1
RETURN
END
REAL FUNCTION FUN1*8(B)

* UPSTREAM CONTINUITY EQUATION *

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FF,PIGL,PIPEL,FF,
* DELTAT,X,SIGMA,DELT,POS,VEL,ACCN,HEXIP,AEIXIP,
* ICASE,I,IFLAG,ICHOKH,ICHOKA
IF (ICASE.EQ.1.OR.ICASE.EQ.3.OR.ICASE.EQ.5) AN=0.
IF (ICASE.EQ.2.OR.ICASE.EQ.4.OR.ICASE.EQ.6) AN=1.-AK3**2
FUN1=PO*B(1)-B(3)*AK1**2*B(5)+AN*B(6)
* +(AK3**2-AK1**2)*VEL) RETURN
END

REAL FUNCTION FUN2*8(B)

* DOWNSTREAM CONTINUITY EQUATION *

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FF,PIGL,PIPEL,FF,
* DELTAT,X,SIGMA,DELT,POS,VEL,ACCN,HEXIP,AEIXIP,
* ICASE,I,IFLAG,ICHOKH,ICHOKA
IF (ICASE.EQ.1.OR.ICASE.EQ.3.OR.ICASE.EQ.5) AN=0.
IF (ICASE.EQ.2.OR.ICASE.EQ.4.OR.ICASE.EQ.6) AN=1.-AK3**2
FUN2=B(2)-B(4)*AK3**2-AK2**2)*VEL+(AK3**2-AK1**2)*VEL+AN*B(6)
* +(AK3**2-AK1**2)*VEL) RETURN
END
REAL FUNCTION FUN3*SCB)
************************ C * FORCE BALANCE ON PIG *
************************ C
IMPLICIT REAL*SCA-H.O-Z)DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FP,PIGL,PipeL,FF,
* DELTAT,X,SIGMA,DELF,PO9,VEL,ACCN,HEXITP,AEXITP,
* ICASE,IFLAG,ICOKH,ICOKA
HVEL=VEL-0.5*(B(8)+B(9))
AVEL=VEL-0.5*(B(6)+B(7))
GO TO (1,2,2.2,2,1),ICASE
1 HO=AK1-AK2
HOSQ=AK1**2-AK2**2
GO TO 3
2 HO=0.
HOSQ=0.
CONTINUE
FUN3=PMASS*ACCN-B(3)*(AK3**2-AK1**2)+B(4)*(AK3**2-AK2**2)
* -0.5*(B(3)+B(4))*HOSQ+GAM*FC*(AK1+AK2)*DSGRT(0.25
* *(HOSQ**2)+PIGL**2)*B(3)+B(4)*HVEL*DABS(HVEL)/8.
* +GAM*FC*PIGL*AK3*(B(3)+B(4))*VEL*DABS(VEL)/4.
RETURN
END

REAL FUNCTION FUN3*S8CB)
C ********************************************************** C
C ** FORCE BALANCE ON PIG - NO ANNULUS **
C ********************************************************** C
IMPLICIT REAL*8(A-H,Z-O)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FP,PIGL,PipeL,FF,
* DELTAT,X,SIGMA,DELF,PO9,VEL,ACCN,HEXITP,AEXITP,
* ICASE,IFLAG,ICOKH,ICOKA
HVEL=VEL-0.5*(B(8)+B(9))
GO TO (1,2,2.2,2,1),ICASE
1 HO=AK1-AK2
HOSQ=AK1**2-AK2**2
GO TO 3
2 HO=0.
HOSQ=0.
CONTINUE
FUN3=PMASS*ACCN-B(3)*(AK3**2-AK1**2)+B(4)*(AK3**2-AK2**2)
* -0.5*(B(3)+B(4))*HOSQ+GAM*FC*(AK1+AK2)*DSGRT(0.25
* *(HOSQ**2)+PIGL**2)*(B(3)+B(4))*HVEL*DABS(HVEL)/8.
* +GAM*FC*PIGL*AK3*(B(3)+B(4))*VEL*DABS(VEL)/4.
RETURN
END
REAL FUNCTION FUN4*B(B)

********************************
* DOWNSTREAM MOMENTUM EQUATION *
********************************

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FP,PIGL,PIPEL,FF,
* DELTAT,X,SIGMA,DELTF,POS,VEL,ACCN,HEXITP,AEXITP,
* ICASE,I,IFLAG,ICHOKH,ICHOKA
IF(ICASE.EQ.1.OR.ICASE.EQ.3.OR.ICASE.EQ.5)AN=0.
 IF(ICASE.EQ.2.OR.ICASE.EQ.4.OR.ICASE.EQ.6)AN=1.-AK3**2
 DUM=AN*B(7)+(AK3**2-AK2**2)*VEL+(AK2**2)*B(9)
 FUN4=FP*PIGL-POS)+(1./C**2-1./DUM**2)-GAM-2.*
 DLOG(DUM)/B(2)
RETURN
END

REAL FUNCTION FUN5*B(B)

***************************
* HOLE CONTINUITY EQUATION *
***************************

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FP,PIGL,PIPEL,FF,
* DELTAT,X,SIGMA,DELTF,POS,VEL,ACCN,HEXITP,AEXITP,
* ICASE,I,IFLAG,ICHOKH,ICHOKA
 FUN5=(AK1**2)*B(3)*B(8)-(AK2**2)*HEXITP*B(9)
RETURN
END
REAL FUNCTION FUN6*8(B)

************************************************************************

* HOLE MOMENTUM EQUATION *

************************************************************************

IMPLICIT REAL*8(A-H,O-Z)

DIMENSION B(9),X(100)

COMMON PO,AK1,AK2,AK3,PMAS,GM,FC,FP,PIGL,PIPL,FF,
*
DELT,A,CGM,DETF,POS,VEL,ACCN,HEXIP,AEXIP,
*
ICASE,ICL,ICHOKH,ICHOKA

HVEL=0.5*(B(8)+B(9)-VEL)

IF(ICHOKH.EQ.1)CNST=1./DSQRT(GAM)-B(9)

IF(ICHOKH,NE,1)CNST=1.

IF(DABS(HVEL).LT.1.E-9)GO TO 3

S=HVEL/DABS(HVEL)

AA=0.5*FC*GAM*(VEL**2)-2.*AK2-2/PIGL

BB=-S*FC*GAM*VEL

CC=0.5*FC*GAM

Q=4.*ACCC*(B**2)

DUM1=AA+BB*B(8)+CC*(B(9)**2)

DUM2=AA+BB*B(9)+CC*(B(9)**2)

CNSTRT=FC*HVEL*DABSCHVEL)/CAK1-AK2)*C(HUEL+UEL)

**2)/PIGL

IF(0.1,3,2)

1 DUM3=(2.*CC*B(8)+BB-DSQRT(-Q))**2.*CC*B(9)+BB-DSQRT(-Q)

DUM4=(2.*CC*B(8)+BB+DSQRT(-Q))**2.*CC*B(9)+BB+DSQRT(-Q)

FUN=PIGL*DLOG(AK1/AK2)/(AK1-AK2)+DLOG(B(8)/B(9))/AA

*-(1./2.*AA)+GAM/(2.*CC))**DLOG(DUM1/DUM2)+BB*GM/CC-1.,/AA

*DSQRT(DUM3/DUM4)**(DUM3/DUM4)/(2.*DSQRT(-Q))

FUN=ABS(FUN)+ABS(1.-CNSTRT/ABS(CNSTRT))+ABS(1.-CNST/ABS(CNST))

RETURN

2 FUN=PIGL*DLOG(AK1/AK2)/(AK1-AK2)+DLOG(B(8)/B(9))/AA

*-(1./2.*AA)+GAM/(2.*CC))**DLOG(DUM1/DUM2)+BB*GM/CC-1.,/AA

*DSQRT(2.*CC*B(8)+BB))/DSQRT(0)-DSQRT((2.*CC*B(9)+BB)/

DSQRT(0))

FUN=ABS(FUN)+ABS(1.-CNSTRT/ABS(CNSTRT))+ABS(1.-

CNST/ABS(CNST))

RETURN

3 PRINT 4

4 FORMAT(' THE RELATIVE VELOCITY IN THE HOLE IS ZERO')

FUN=-2.*DLOG(AK2/AK1)+DLOG(B(8)/B(9))-0.5*GAM*(B(8)**2-

B(9)**2)

FUN=ABS(FUN)+ABS(1.-CNST/ABS(CNST))

RETURN
REAL FUNCTION FUN6S*(B)

************
* HOLE MOMENTUM EQUATION *
* CONSTANT DIAMETER HOLE *

IMPLICIT REAL*(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GM,FC,FP,PIGL,PIPEL,FF,
* DELTAT,XSIGMA,DELT,F,POS,VEL,ACCN,HEXITP,AEXITP,
* ICASE,IFLAG,ICHOKH,ICHOKA
HVEL=0.5*(B(8)+B(9))-VEL
IF(ICHOKH.EQ.1)CNST=1./DSQRT(GM)-B(8)
IF(IFCHOKH.NE.1)CNST=1.
IF(DABS(HVEL).LT.1.E-8)GO TO 1
FUN=-FC*PIGL/(2.*AK1)+(1./VEL-GM*VEL)*(1./
* (B(8)-VEL)-1./(B(9)-VEL))+1./(VEL**2)+GM)*DLOG((B(8)
* -VEL)/(B(9)-VEL))-(1./(VEL**2))*DLOG(B(8)/B(9))
FUN6S=DABS(FUN)+DABS(1.-HVEL/DABS(HVEL))+DABS(1.-
* CNST/DABS(CNST))
RETURN
1 PRINT 2
2 FORMAT(' THE RELATIVE VELOCITY IN THE HOLE IS ZERO')
FUN=DLOG(B(8)/B(9))-0.5*GM*(B(8)**2-B(9)**2)
FUN6S=DABS(FUN)+DABS(1.-CNST/DABS(CNST))
RETURN
END

REAL FUNCTION FUN7S*(B)

************
* ANNULUS CONTINUITY EQUATION *

IMPLICIT REAL*(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GM,FC,FP,PIGL,PIPEL,FF,
* DELTAT,XSIGMA,DELT,F,POS,VEL,ACCN,HEXITP,AEXITP,
* ICASE,IFLAG,ICHOKH,ICHOKA
FUN7=B(3)**B(6)-AEXITP*B(7)
RETURN
END
REAL FUNCTION FUN8(B)
**********************************************************************
* ANNULUS MOMENTUM EQUATION *
**********************************************************************
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FP,PIGL,IPEL,FF,
* DELTA,T,X,STMA,DELTA,POS,VEL,ACC,N,HEXITF,AEXITF,
* ICASE,I,IFLAG,IPIEH,IICOKA
IF(IICOKA.EQ.1)CNST=1./DSQRT(GAM)-B(8)
IF(IICOKA.NE.1)CNST=1.
VEL0=0.5*(B(6)+B(7))'-VEL
IF(DABS(VEL0).LT.1.E-8)GO TO 3
S=VEL0/DABS(VEL0)
AA=S*FC*AK3*GAM*VEL**2)*0.5
BB=-S*FC*AK3*GAM*VEL
CC=0.5*FP*GAM+0.5*S*FC*AK3*GAM
Q=0.4*S*S*CC=(BB)**2
DUM1=AA+BB*(6)+CC*(B(6)**2)
DUM2=AA+BB*B(7)+CC*B(7)**2
CNSTRT=FP*1.)C2.*CC*B(6)+BB*DSQRT(-Q)*)*(2.*CC*B(7)+BB+DSQRT(-Q))
DUM3=(2.*CC*B(6)+BB+DSQRT(-Q))***(2.*CC*B(7)+BB+DSQRT(-Q))
FUN8=PIGL*C(AK3**2)*B(6)/B(7)**2+GAM/C-1./AA)*DLOG(1.-CNSTRT/DABS(CNSTRT))
RETURN
FUN8=DABS(FUN)+DABS((1.-CNSTRT/DABS(CNSTRT)))+DABS(1.-
* CNST/DABS(CNST))
RETURN
IF(Q).LT.13.2
FUN8=PIGL*1./CNSTRT)DLOG(1.-CNSTRT/1.DABS(CNST))
RETURN
3 PRINT 4
4 FORMAT(' THE RELATIVE VELOCITY IN THE ANNULUS IS ZERO')
FUN8=FPIGL*(1.-AK3**2)*B(6)/(B(7)**2)-1./B(7)**2)*GAM
* +2.*DLOG(5.B/7))
RETURN
END
REAL FUNCTION FUN9*8(B)

*********************************************************************
* UPSTREAM MOMENTUM EQUATION *
*********************************************************************

IMPLICIT REAL*S(A-H,Q-Z)
DIMENSION B(9),X(100)
COMMON PO,AK1,AK2,AK3,PMASS,GAM,FC,FP,PIGL,PIPEL,FF,
* DELTAT,X,SIGMA,DELT,F,POS,VEL,ACCN,HEXITP,AEXITP,
* ICASE,I,IFLAG,ICHOKH,ICHOKA
IF(ICASE.EQ.1 OR ICASE.EQ.3 OR ICASE.EQ.5)AN=0.
IF(ICASE.EQ.2 OR ICASE.EQ.4 OR ICASE.EQ.6)AN=1.-AK3**2
DUM=AN*B(6)+(AK3**2-AK1**2)*VEL+(AK1**2)*B(8)
FUN9=FP*(POS-PIGL)-(1./(B(1)**2-1./(DUM**2)/GAM-2.*DLOG(B(1)/DUM)
RETURN
END