RICE UNIVERSITY

Essays on Investment Planning in Electricity Generating Capacity

by

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ABSTRACT

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In the first part of this study we develop and analyze two mathematical models that incorporate a time changing demand for electricity and uncertainty of input prices. The first model highlights the shortcomings in assuming a constant plant utilization under uncertainty of input prices and the effects of such assumption on the optimal investment in electricity generating capacity in a simple two period model. The second model presents sufficient restrictions to the optimal investment in electricity generating capacity problem to allow for a recursive solution. The necessary restrictions are extremely limiting to the extend that we found a solution for very simple scenarios. In our opinion, the problem is better handled in a case by case basis rather than under a general dynamic framework. Following the spirit of our conclusions of the first part of our study, in the second part we provide a methodology to simulate long-term natural gas prices, we analyze the investment prospects of nuclear and natural gas generating capacity in Mexico and provide a constraint approach for the optimal generation of hydroelectric plants in the Mexican hydroelectric system. These three problems belong to the solution of the optimal investment in electricity generating capacity in Mexico. To simulate the uncertainty of natural gas prices, we assume that natural gas prices are the sum of two stochastic processes: short-term and long-term variability. We characterize the short-term variability of natural gas prices using an Exponential General Autoregressive Conditional Heteroskedastic (EGARCH) model. The uncertainty of the long-term variability of natural gas
prices is based on the long-term natural gas prices scenarios of the National Energy Modeling System of the Energy Information Administration. Equipped with a methodology to simulate long-term natural gas prices, we investigate the investment prospects of nuclear and natural gas generating capacity in Mexico using the levelized cost methodology. Finally, we derive an algorithmic solution for a constraint version of the optimal generation of hydroelectric plants, then we provide a guide for its application to the Mexican hydroelectric system.
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Clearly, this work is not only mine, it represents the effort of all those who helped along my path. This dissertation is my way to say thanks for their trust, support.

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Preface

The increase in natural gas prices in recent years coupled with concerns about \( \text{CO}_2 \) emissions from coal has renewed interest in nuclear power. In addition, the volatility of natural gas prices has raised concerns about fuel security. Several authors have tried to tackle the problem by including uncertainty of natural gas prices in their models. For example, Awerbuch and Berger ([2]) use a mean-variance analysis of optimal portfolios of technologies for the generation of electricity. Roques et. al. ([14]) provide some evidence on the value of “the nuclear option” for merchant operators in deregulated markets. Murto and Nese ([12]) include the risk of changes in fossil fuel prices into their analysis of the decision to invest in biomass. Gollier et al. ([7]) examine the effect of modularity under input price uncertainty. Unfortunately, none of these studies provides a clear treatment of the load curve problem which we consider to be a distinctive feature of electricity markets. The load curve problem refers to the optimal dispatching of generating units and, ultimately, determines the capacity factor of the electricity generating capacity. Each of the mentioned studies assumes a constant capacity factor and, therefore, disregards the optimal dispatch and utilization of the generating capacity. In our view, this is a critical aspect of choosing an optimal mix of technologies and is of central importance for the efficiency of marginal cost pricing and competitive deregulated markets. The lack of attention to the load curve problem might be due to the fact that, until recently, combined cycle plants allowed natural gas to compete with nuclear for base load generation. On the other hand, the disregard of the load curve affects only the recent literature, since the study of optimal generating capacity started with the study of marginal pricing and
the load curve problem by Williamson ([17]) and Boiteux ([4]). Joskow ([9]) provides a summary of the earlier work in peak load pricing and capacity investment. Chao ([5]) extended the simple model introducing demand and supply uncertainty. We have not encountered any study that introduces generation cost uncertainty into the load curve problem.

This work addresses the load curve problem under stochastic input prices. It is composed of four chapters. The first chapter examines optimal investment in electricity generating capacity under stochastic input prices in a simple two period model. The second chapter then analyzes optimal investment in generating capacity in a dynamic setting. Unfortunately, relatively tight restrictions are needed to facilitate an analytical recursive solution. Chapters 3 and 4 present our attempt to provide a more practical approach to solving the optimal generating problem. The expectation of future natural gas prices plays a crucial role in determining the optimal investment in generating capacity. In Chapter 3, we present a methodology for characterizing natural gas prices in the long term (including a realistic representation of their possible volatility) that we apply to Henry Hub natural gas prices. In Chapter 4, we provide some practical considerations governing investment in the Mexican electricity system. First, we briefly describe the Mexican system. Next, we compare the distribution of total costs for combined cycle natural gas plants to levelized costs of nuclear plants. Finally, we present an algorithm for obtaining to the optimal generation from hydroelectric capacity in Mexico.

In summary, in the first two chapters, the thesis raises two serious concerns about the current methodology used to assess the merits of nuclear versus combined cycle plants: the assumption of a constant capacity factor and the avoidance of complex dynamic behavior inherent in electricity generation capacity investments. In the last two chapters, we provide a solution to some of the major obstacles obtaining
the optimal investment in electricity generation capacity. Although major obstacles remain, these will be left for future research.
Chapter 1

Input Price Uncertainty and Optimal Investment in Power Markets

1.1 Introduction

In this chapter, we present a simple two period model that allows for generation cost uncertainty and the load curve problem. The latter arises when the demand for electricity is affected by the marginal cost of generation. Under this simple setting, we evaluate the effect that a mean preserving spread (MPS) on the distribution of generation cost, according to the definition of Rothschild and Stiglitz ([15]), has on the optimal investment strategy of the central planner and the option value of nuclear power.\footnote{The value that the central planner gives to the ability to use nuclear power when the costs of natural gas would have been higher.}

Optimal investment in liberalized markets based on marginal cost pricing requires consideration of the load curve problem. Unfortunately, the commonly used levelized costs comparisons assume constant capacity factors for all plants. We show that the pricing mechanism of the central planner changes the effect that a MPS has on the optimal investment strategy, implying that ignoring the endogeneity of the load
curve in liberalized markets will yield the wrong estimates of the benefits associated with each technology. Specifically, we will analyze two possible pricing mechanisms: marginal cost pricing and pricing independently from marginal cost.

The introduction of uncertainty forces us to address the behavior of the central planner. We believe that uncertain electricity prices have impacts on the economy similar, but lower in magnitude, to the impact that oil prices have on the economy. We approximate the behavior of the central planner facing such consequences of variations in electricity prices by assuming the planner can be risk averse in the sense of disliking an MPS in electricity prices.

For tractability, we focus on three aspects of the electricity industry: the load curve problem, the uncertainty of generation cost and the pricing mechanism of electricity used by the central planner. We acknowledge that the investment strategy is dynamic in nature in that it depends on the investment opportunities faced by the central planner in the future. We believe that modelling the dynamic investment problem of the central planner will only obscure and make more difficult the presentation of the results from the effect of MPS without adding much to the discussion. Therefore, we choose to analyze the investment problem in a simple two period static model.

The paper is divided as follows. Section 1.2 develops the standard model to analyze the effect of a MPS in costs on the optimal investment in electricity generation capacity. Section 1.3 solves the optimal investment strategy when the central planner prices at marginal cost and evaluates the effects that a MPS on the distribution of generation costs has on the optimal investment strategy and the option value of a nuclear plant. Section 1.4 solves the central planner's problem when the central planner prices electricity independently from marginal cost. We then evaluate the effects of MPS under this new assumption. Our conclusions are summarized in Section 1.5. Finally, the Appendix provides the necessary results to make our presentation as
concrete as possible.

1.2 The Model

For simplicity we assume that there are only two technologies: a nuclear technology, \( N \), and a natural gas technology, \( G \). We assume that investment in both technologies is perfectly divisible. \(^3\)

We assume the nuclear technology has a higher investment cost than the natural gas technology but no generation cost. Let \( k_i \) be the per unit cost of investment in capacity for technology \( i = N, G \) with \( k_N > k_G \). On the other hand, let the generation cost of the natural gas technology be denoted by \( \rho > 0 \). Specifically, assume that \( \rho \in [\underline{\rho}, \overline{\rho}] \) is a random variable with density \( f(\rho) \) and cumulative distribution function \( F(\rho) \) and \( \rho > 0 \).

Denote the demand for electricity at instant \( j \in [0, 1] \) as \( d(p, j) \) where \( p \) denotes the price of electricity. Given our simple cost structure, generation cost is independent on the order of demand, implying that without loss of generality we can assume that \( d(p, j') > d(p, j) \) for all \( p \geq 0 \) and all \( j' < j \). We assume that the demand function is continuous and differentiable with respect to both \( p \) and \( j \). Finally, if we denote the inverse demand for electricity as \( p(g, j) \) then \( p(g, j') > p(g, j) \) for all \( g > 0 \) and \( j' < j \).

The problem of the central planner is to choose investment in natural gas and nuclear capacity in period 1 before the generation cost of natural gas technology is realized. Then, in period 2 the central planner decides how much to generate from

---

\(^2\)Although we name the second technology a natural gas technology, it can represent any fossil fuel technology: including plants that can utilize two different fossil fuels.

\(^3\)This assumption is not problematic in large integrated power systems where increases of demand are high relative to the size of the indivisibility of plants. For example, a .5% increase in capacity per month (6.16% annually) and a 3 month rule for requiring half of indivisibly capacity, \( i/2 \), to be required implies a total installed capacity of \( (1.005^3 - 1)q = i/2 \). For a 1,300 MW nuclear plant, 750 MW half of indivisibility capacity, it will require a total installed capacity of approximately 50,000 MW.
each technology at each instant after the generation cost of the natural gas technology is realized.

The central planner ranks different investment strategies according to the expected utility derived from the investment strategy. We ignored general equilibrium effects and compute the benefits from electricity generation at instant \( j \) using the consumer surplus:

\[
CS(q, j) = \int_0^q p(s, j) ds.
\] (1.1)

Finally we assume that the utility function is time separable with a discount factor \( \beta \).

### 1.3 Optimal investment strategy under MCP

In this section we derive the optimal investment in electricity generation capacity under marginal cost pricing. In Subsection 1.3.1 we simplify the central planner’s problem by solving for the optimal generation schedule. In Subsection 1.3.2 we solve the simplified central planner’s investment problem. In Subsection 1.3.3 we evaluate the effect of a MPS on the distribution of generation cost on the optimal investment strategy and the value of the nuclear option for a risk neutral central planner. Finally, Subsection 1.3.4 evaluates the effect of a MPS for a risk averse central planner.

#### 1.3.1 Optimal generation under MCP

Denote the available capacity in each technology as \((q_N, q_C)\), the realization of the generation cost for the natural gas technology as \(p\) and fix the instant of demand at \( j \).

By assumption, the generation cost for the nuclear technology is zero. If the demand is not bounded, \( d(p, 0) > 0 \) for all \( p > 0 \), then the inverse demand will always
be positive implying that generation from the nuclear technology should be equal to its capacity, specifically, \( g_N = q_N \). If the demand is bounded by \( M > 0 \) and \( M < q_N \), then unless there is free disposal, \( g_N = q_N \) is not an optimal solution.\(^4\) Since it will simplify the presentation, we will assume optimal generation using the nuclear technology is given by \( g_N = q_N \).\(^5\)

The situation is different for the natural gas technology. If \( \rho \geq p(q_N, j) \), then the marginal value of electricity when demand equals nuclear output is lower than the natural gas generation cost, implying that the natural gas capacity should not be used. On the other hand, if the marginal value of electricity when demand equals output from all plants exceeds the generation costs, \( p(q_N + q_G, j) \geq \rho \), then the optimal generation from natural gas should be equal to its capacity, \( g_G(j) = q_G \). Finally, if \( p(q_N + q_G, j) < \rho < p(q_N, j) \), then it is optimal to generate electricity using natural gas until the marginal value of electricity is equal to the generation cost, implying that \( g_G(j) = d(\rho, j) - q_N \). In summary, the optimal generation from nuclear technology is always to produce at full capacity, \( g_N(j) = q_N \), while the optimal generation from natural gas is given by:

\[
g_G(j) = \begin{cases} 
0 & \text{if } p(q_N, j) \leq \rho \\
 d(\rho, j) - q_N & \text{if } p(q_N, j) < \rho \leq p(q_N + q_G, j) \\
 q_G & \text{if } p(q_N + q_G, j) > \rho.
\end{cases}
\tag{1.2}
\]

More complex dispatching schedules might affect the optimal generation decision and make the generation decision to be dependent on the investment decision but this is beyond the scope of the paper.

\(^4\)If there is free disposal, then the optimal generation from nuclear technology is given by the set \([M, q_N]\).

\(^5\)If the upper bound \( M \) exists, then it should be the same for every instant, \( j \), since the installed equipment that uses electricity is fixed. Therefore, unless the optimal investment strategy is to only invest in nuclear technology, we will expect \( q_N < M \).
1.3.2 The central planner’s problem under MCP

The central planner’s problem is simplified by the separation between the generation decision, or optimal dispatch, and the investment decision. Denoting the investment strategy as \((q_N, q_G)\), the investment costs at period 1 are given by \(k_Nq_N + k_Gq_G\) and the expected generation net benefits are given by:

\[
\int_0^\rho \left\{ \int_0^1 u(CS(g_N(j) + g_G(j), j) - g_G(j))dj \right\} f(\rho)d\rho.
\]

Where \(u(\cdot)\) denotes the continuous and differentiable utility function that the central planner uses to rank per period instant net benefits. Changing the order of integration and imposing the optimal generation schedule for marginal cost pricing we obtain:

\[
\int_0^1 \left\{ \int_{\rho(q_N + q_G, j)}^{\rho(q_N + q_G, j)} u(CS(q_N + q_G, j) - \rho q_G)f(\rho)d\rho
\right.
\]
\[
+ \int_{\rho(q_N + q_G, j)}^{\rho(q_N + q_G, j)} u(CS(d(\rho, j)) - (d(\rho, j) - q_N)\rho)f(\rho)d\rho
\]
\[
+ \int_{\rho(q_N + q_G, j)}^{\rho(q_N,j)} u(CS(q_N, j))f(\rho)d\rho \right\} dj. \tag{1.3}
\]

We assume the range of \(\rho\) to be sufficiently large to avoid the obvious minor boundary problems with expression 1.3. Hence, the central planner solves

\[
\max_{q_N \geq 0, q_G \geq 0} u(-k_Nq_N - k_Gq_G) + \beta \int_0^1 \left\{ \int_{\rho(q_N + q_G, j)}^{\rho(q_N + q_G, j)} u(CS(q_N + q_G, j) - \rho q_G)f(\rho)d\rho
\right.
\]
\[
+ \int_{\rho(q_N + q_G, j)}^{\rho(q_N + q_G, j)} u(CS(d(\rho, j)) - (d(\rho, j) - q_N)\rho)f(\rho)d\rho
\]
\[
+ \int_{\rho(q_N, j)}^{\rho(q_N,j)} u(CS(q_N, j))f(\rho)d\rho \right\} dj. \tag{1.4}
\]

Proposition 7 in the Appendix shows that for linear and concave utility functions, \(u(\cdot)\), the maximization problem has a unique well defined maximum. We assume that
the exogenous parameters of the model imply an interior solution. The uniqueness of the solution of the maximization problem implies that the first order conditions (FOC) characterize the optimum.

The FOC with respect to \( q_N \) is given by:

\[
-u'(\cdot)k_N + \beta \int_0^1 \left\{ \int_{\rho}^{p(q_N + q_G, j)} u'(\cdot) p(q_N + q_G, j) f(\rho) d\rho + \int_{p(q_N + q_G, j)}^{p(q_N)} u'(\cdot) \rho f(\rho) d\rho \right\} dj = 0. \tag{1.5}
\]

In an abuse of notation \( u'(\cdot) \) denotes the first derivative of \( u(\cdot) \) evaluated properly.

The first order condition with respect to \( q_G \) is given by:

\[
-u'(\cdot)k_G + \beta \int_0^1 \int_{\rho}^{p(q_N + q_G, j)} u'(\cdot)(p(q_N + q_G, j) - \rho) f(\rho) d\rho dj = 0. \tag{1.6}
\]

We now characterize the option value of nuclear power. We understand the value of the nuclear option as the value of having the ability to use nuclear energy when the costs of natural gas is high. Let

\[
U(q_N, q_G) = u(-k_N q_N - k_G q_G) + \beta \int_0^1 \left\{ \int_{\rho}^{p(q_N + q_G, j)} u(CS(q_N + q_G, j) - p q_G) f(\rho) d\rho + \int_{p(q_N + q_G, j)}^{p(q_N)} u(CS(\rho, j)) - (d(\rho, j) - q_N) \rho f(\rho) d\rho \right\} dj.
\]

Denote the solution of the central planner's problem as \((\bar{q}_N, \bar{q}_G)\), and the solution to the optimal investment problem when the nuclear capacity is fixed at \( q_N \) as \( \bar{q}_G(q_N) \). Then the option value of the nuclear technology is given by:

\[
U(\bar{q}_N, \bar{q}_G) - U(0, \hat{q}(0)). \tag{1.7}
\]
Obviously under an interior solution the option value of the nuclear power is positive.

Once we have characterized the optimum, the next step is to impose a MPS over the distribution of the generation cost of the natural gas technology and to evaluate its effects on the optimal investment strategy. As we previously noted, we separate the effects of a MPS for a risk neutral and risk averse central planner.

1.3.3 Effects of a MPS for a risk neutral central planner with MCP

In this subsection we derive the effects that a MPS on the distribution of generation cost has on the optimal investment strategy of a risk neutral central planner and on the value of nuclear power.

A risk neutral central planner has a linear utility function. The first order condition with respect to \( q_N \) can then be simplified by integrating by parts to:

\[
-k_N + \beta \int_0^1 \left[ p(q_N, j) - \int_{p(q_N+q_G, j)}^{p(q_N, j)} F(\rho) d\rho \right] dj = 0. \tag{1.8}
\]

Similarly, the first order condition with respect to \( q_G \) can be simplified by integrating by parts to:

\[
-k_G + \beta \int_0^1 \int_{\rho}^{p(q_N+q_G, j)} F(\rho) d\rho dj = 0. \tag{1.9}
\]

We present our first result.

**Proposition 1.** A mean preserving spread on the distribution of the generation cost of natural gas increases the optimal investment capacity in natural gas technology and decreases it in the nuclear technology.

**Proof.** Denote the MPS on \( f(\rho) \) as \( h(\rho) \). Notice that equation 1.9 solves for the total
investment in capacity $q_T = q_N + q_G$. Denote the optimal investment strategy under $f(\rho)$ as $(q^f_N, q^f_G)$.

Evaluate equation 1.9 using $h(\rho)$ at $(q^f_N, q^f_G)$ and subtract the same equation using $f(\rho)$ evaluated at the same point. We obtain the following expression

$$\beta \int_0^1 \int_\rho^{p(q^f_N + q^f_G,j)} H(\rho) - F(\rho) d\rho dj \geq 0.$$  

Where the inequality follows from the definition of a MPS. The inequality implies that total capacity should increase since the derivative of the marginal value of natural gas’ capacity with respect to total capacity is:

$$\beta \int_0^1 F(p(q_T,j)) \frac{\partial p(q_T,j)}{\partial q_T} dj < 0.$$  

Therefore, we only need to show that $q_N$ will decrease to prove our proposition.

Simplify equation 1.8 by subtracting equation 1.9 to obtain

$$-k_N + k_G + \beta \int_0^1 \left\{ p(q_N,j) - \int_\rho^{p(q_N,j)} F(\rho) d\rho \right\} dj = 0. \quad (1.10)$$

Evaluating equation 1.10 using $h(\rho)$ at $(q^f_N, q^f_G)$ and subtracting the same equation using $f(\rho)$ evaluated at the same point. We obtain that

$$-\beta \int_0^1 \int_\rho^{p(q^f_N,j)} H(\rho) - F(\rho) d\rho dj \leq 0,$$

where again the inequality follows from the definition of a MPS. This implies that the investment in nuclear power decreases with a MPS since the derivative of the marginal excess value of nuclear capacity (the second term in 1.10) with respect to
$q_N$ is:

$$
\beta \int_0^1 \frac{\partial p(q_N, j)}{\partial q_N} \left[ 1 - F(p(q_N, j)) \right] dj < 0.
$$

Therefore under a MPS the investment in nuclear power decreases while the investment in natural gas increases, as we needed to show. 

The economic rationale of our first result is quite easy to understand. Under marginal cost pricing, natural gas capacity is used only if the generation cost is low enough, at a high generation cost, the high price reduces the demand of electricity. Therefore, natural gas plants are an option to be exercised when generation costs is low. Under a MPS, the marginal value of natural gas increases since the probability of a low generation cost increases. The increment in the probability of high natural gas generation cost does not have any effect since the option is not exercised at high generation cost. The decrease in the investment in nuclear energy is a consequence of the reduction in the marginal value of nuclear power. This is due to the fact that the probability that nuclear power substitutes for natural gas (for generation cost $p(q_N + q_G, j) < p < p(q_N, j)$) decreases under a MPS. A similar result was obtained by Murto and Nese ([12]) using a real option approach to a similar problem.

Moreover, not only is the optimal investment strategy to decrease investment in nuclear power technology, but also the value of the nuclear power option decreases. The next proposition formally shows this.

**Proposition 2.** The option value of nuclear power decreases following a MPS over the distribution of generation cost of the natural gas technology.

**Proof.** Let $h(\rho)$ be a MPS over $f(\rho)$. Then the difference in the option value of nuclear power is given by:

$$
U_h(\hat{q}_N, \hat{q}_G) - U_h(0, \hat{q}_G(0)) - U_f(\hat{q}_N, \hat{q}_G') + U_f(0, \hat{q}_G'(0)).
$$

(1.11)
Where $U_h(\cdot)$ and $U_f(\cdot)$ denote the objective function of the central planner when the distributions of generation costs are $h(\cdot)$ and $f(\cdot)$ respectively. Moreover, $(\tilde{q}_N^h, \tilde{q}_G^h)$ and $(\tilde{q}_N^f, \tilde{q}_G^f)$ denote the solution of the central planner’s problem when the distribution of generation costs are $h(\cdot)$ and $f(\cdot)$ respectively.

Notice that the following inequality follows from the definition of a maximum:

$$U_h(\tilde{q}_N^h, \tilde{q}_G^h) - U_h(0, \tilde{q}_G^h(0)) - U_f(\tilde{q}_N^f, \tilde{q}_G^f) + U_f(0, \tilde{q}_G^f(0)) < 
\left[U_h(\tilde{q}_N^h, \tilde{q}_G^h) - U_f(\tilde{q}_N^h, \tilde{q}_G^h)\right] - \left[U_h(0, \tilde{q}_G^h(0)) - U_f(0, \tilde{q}_G^f(0))\right].$$

We show that the right hand site in the inequality is less than or equal to zero. Consider the first term. From the linearity of the utility function we have that:

$$U_h(q_N, q_G) - U_f(q_N, q_G) = b\beta \int_0^1 \left\{ \int_{p(q_N + q_G, j)}^{p(q_N, j)} (CS(q_N + q_G, j) - \rho q_G) s(\rho) d\rho 
+ \int_{p(q_N + q_G, j)}^{p(q_N, j)} (CS(d(\rho, j), j) - (d(\rho, j) - q_N)\rho) s(\rho) d\rho + \int_{p(q_N, j)}^{p(q_N + q_G, j)} CS(q_N, j) s(\rho) d\rho \right\} dj$$

Where $b$ is the linear parameter in the utility function and $s(\rho) = h(\rho) - f(\rho)$. Integrating twice by parts and simplifying we obtain:

$$U_h(q_N, q_G) - U_f(q_N, q_G) = b\beta \int_0^1 \int_{p(q_N + q_G, j)}^{p(q_N, j)} \frac{\partial d(\rho, j)}{\partial \rho} T(\rho) d\rho dj \leq 0$$

Where the inequality follows from $\partial d(\rho, j)/\partial \rho < 0$ and $T(\rho) = \int_{\rho}^{\rho} H(s) - F(s) ds \geq 0$ follows from the definition of a MPS. Our last inequality specifically implies that $U_h(\tilde{q}_N^h, \tilde{q}_G^h) - U_f(\tilde{q}_N^h, \tilde{q}_G^h) \leq 0$. 
With respect to \( U_h(0, q_G^f(0)) - U_f(0, q_G^f(0)) \) notice that:

\[
U_h(0, q_G) - U_f(0, q_G) = b \beta \left\{ \int_{p}^{p(q_G,j)} (CS(q_G, j) - q_G \rho) s(\rho) d\rho + \int_{p(q_G,j)}^{\delta} (CS(d(\rho, j)) - \rho d(\rho, j)) s(\rho) d\rho \right\}.
\]

Integrating twice by parts we obtain:

\[
U_h(0, q_G) - U_f(0, q_G) = -b \beta \int_{0}^{1} \int_{p(q_G)}^{\delta} \frac{\partial d(\rho, j)}{\partial \rho} T(\rho) d\rho dj \geq 0
\]

Therefore, \( U_h(0, q_G^f(0)) - U_f(0, q_G^f(0)) \geq 0 \) and the option value of nuclear power is lower than zero as desired.

The intuition of our result is similar to our result in the optimal investment strategy. Specifically, marginal cost pricing implies that natural gas will be used only in low realizations. Hence, a MPS increases the value of natural gas. In contrast, the value of nuclear power depends negatively on the value of natural gas which increases under a MPS.

In the next subsection we analyze the problem of allowing for a risk averse central planner.

1.3.4 Effect of a MPS on a risk averse central planner with MCP

Unfortunately, we cannot sign the effect of a general MPS on either the optimal investment strategy or the option value of nuclear power. To simplify our presentation assume that the amount of nuclear power capacity is fixed and cannot be changed. Therefore the only relevant decision is the amount of natural gas capacity and the relevant first order condition is given by equation 1.6.
Let $h(\rho)$ be a MPS over $f(\rho)$ and let $q^f_G$ denote the optimal investment in the natural gas technology. Evaluate equation 1.6 using $h(\rho)$ at $q^f_G$ and subtract the same equation using $f(\rho)$ evaluated at the same point to obtain:

$$\beta \int_{q}^{p(q_N + q_G, j)} l(\rho) s(\rho) d\rho, \quad (1.12)$$

where

$$l(\rho) = u'(CS(q_N + q^f_G, j) - \rho q^f_G)(p(q_N + q^f_G, j) - \rho).$$

Integrating equation 1.12 twice by parts, we obtain

$$-l'(p(q_N + q_G, j)) T(p(q_N + q^f_G, j)) + \int_{q}^{p(q_N + q^f_G, j)} l''(\rho) T(\rho) d\rho, \quad (1.13)$$

where $T(\rho) = \int_{q}^{\rho} H(s) - F(s) ds$. Also $l'(\cdot)$ and $l''(\cdot)$ denote the first and second derivatives of $l(\cdot)$ given by:

$$l'(\rho) = -u''(\cdot)(p(q_N + q^f_G, j) - \rho)q^f_G - u'(\cdot)$$

$$l''(\rho) = u'''(\cdot)(p(q_N + q^f_G, j) - \rho)(q^f_G)^2 + 2u''(\rho)q_G.$$

The sign of $l''(\cdot)$ depends on the curvature of the utility function and the type of MPS. Notice that the first term of equation 1.13 is positive since:

$$-l'(p(q_N + q^f_G, j)) T(p(q_N + q^f_G, j)) = u'(\cdot) T(p(q_N + q^f_G, j)) \geq 0.$$

The second term of the equation can be either positive or negative. For quadratic utility functions $l''(\rho) = 2u''(\rho)q_G < 0$ implying that we cannot sign the effect of the MPS. Although the probability that the central planner uses the natural gas technology increases as before, the outcomes that might benefit are very low generation cost
which have low marginal values, raising the possibility that the expected marginal value of the natural gas technology actually decreases.

In our explanation of Proposition 1 we didn’t highlight the role of risk neutrality in yielding a definite result. But from our previous discussion its importance becomes clear. Our explanation focused instead on the option value of natural gas. Therefore, it is fair to ask ourselves what would happen if the central planner cannot decrease consumption of electricity when there are high realizations of the generation cost. Our next subsection deals with the optimal investment in nuclear and natural gas technology under the assumption of a fixed load curve.

1.4 Optimal investment strategy under a fixed load curve

In this section we solve the optimal investment strategy when the central planner faces a fixed load curve. We then evaluate the effects that a MPS has on the optimal investment strategy and the option value of nuclear power. A central planner faces a fixed load curve when the price of electricity does not depend on the generation cost. In order to simplify our notation we drop the price of electricity as an argument in the demand for electricity. Hence, the demand for electricity at instant \( j \) is denoted as \( d(j) \) and again without loss of generality we order the demand so it is decreasing in \( j \).

Again, as in the previous section, in order to simplify the optimal investment problem we solve for the optimal generation schedule first in Subsection 1.4.1. Then, Subsection 1.4.2 solves the optimal investment strategy. Subsection 1.4.3 and Subsection 1.4.4 analyze the effect of a MPS on the optimal investment strategy and the option value of nuclear power for a risk neutral and a risk averse central planner.
1.4.1 Optimal generation schedule under a fixed load curve

In this subsection, we solve for the optimal generation schedule under a fixed load. Denote the invested capacity as \((q_N, q_G)\) and denote the realization of the marginal cost of the natural gas technology as \(\rho\).

Since the nuclear technology has no generation cost, it is always optimal for the central planner to use all the nuclear capacity before using the natural gas technology. Therefore, if \(q_N \leq d(1)\), then the central planner is forced to use the natural gas technology for every \(j\). If \(d(0) > q_N > d(1)\), then by continuity of the demand function there exist a \(j(q_N)\) such that \(d(j(q_N)) = q_N\). Finally, if \(q_N \geq d(0)\), then the natural gas technology is never used. Henceforth we will assume that \(d(0) > q_N > d(1)\).

The natural gas technology is used for all \(j < j(q_N)\), and for all \(j \geq j(q_N)\) only the nuclear technology is used. Hence, the generation costs are given by:

\[
\int_0^{j(q_N)} (d(j) - q_N) \rho \, dj.
\] (1.14)

The next step is to specify and solve the central planner’s optimal investment strategy.

1.4.2 The central planner’s problem under a fix load

In this subsection we specify and solve the central planner’s problem. Notice that a fixed load curve implies fixed social benefits of electricity consumption. Therefore, the central planner minimizes the cost of investment plus the generation cost. Since the objective function of the central planner is not maximum social benefits but minimum costs, we introduce a different utility function for the central planner. We assume that the central planner ranks social cost according to \(v(\cdot)\) where \(v'(\cdot) > 0\) and \(v(0) = 0\).
The central planner minimizes the expected valuation of cost and solves:

\[
\min_{q_N, q_G} v(k_Nq_N + k_Gq_G) + \beta \int_{\rho} \int_{0}^{f(q_N)} v((d(j) - q_N)\rho)dj f(\rho)d\rho,
\]

subject to

\[q_N \geq 0 \quad q_G > 0 \text{ and } q_N + q_G \geq d(0).\]

Notice that the central planner’s problem can be simplified if the total capacity restriction is binding, \(q_N + q_G = d(0)\). To show this, notice that if \(q_N + q_G > d(0)\), then for all \(\rho\) the excess capacity \(q_G - (d(0) - q_N)\) will never be used. Moreover, the extra capacity consists only in natural gas capacity since the natural gas technology is only used when the nuclear technology is used up to capacity. Therefore, the central planner can do better by decreasing the investment in the natural gas technology up to the point where \(q_N + q_G = d(0)\).

The simplified central planner’s problem is given by:

\[
\min_{q_N > 0} v((k_N - k_G)q_N + k_Gd(0)) + \beta \int_{\rho} \int_{0}^{f(q_N)} v((d(j) - q_N)\rho)dj f(\rho)d\rho.
\]

Proposition 8 in the Appendix shows that the simplified central planner’s problem has a unique minimum. Assuming an interior solution, the FOC characterizes the optimal investment in the nuclear technology and the total capacity restriction characterizes the optimal investment in natural gas technology. The FOC with respect to the nuclear technology is given by:

\[
v'() (k_N - k_G) - \beta \int_{\rho} \int_{0}^{f(q_N)} v'((d(j) - q_N)\rho)dj f(\rho)d\rho = 0.
\]

The option value of nuclear power is given by the reduction of the expected valuation of cost when the central planner has the option to invest in nuclear power.
Specifically, denote the expected valuation of cost as:

\[ V(q_N) = v((k_N - k_G)q_N + k_Gd(0)) + \beta \int_{0}^{\hat{\rho}} \int_{0}^{j(q_N)} v((d(j) - q_N)\rho)djf(\rho)d\rho. \] (1.18)

When the nuclear option is not available, all the generation is covered by the natural gas technology. The supply restriction implies that \( q_G = d(0) \). The expected valuation of cost is given by:

\[ V(0) = v(k_Gd(0)) + \beta \int_{0}^{\hat{\rho}} \int_{0}^{1} v(d(j)\rho)djf(\rho)d\rho. \] (1.19)

The option value of the nuclear technology is given by \( V(0) - V(\bar{q}_N) \) where \( \bar{q}_N \) denotes the optimal investment in nuclear capacity.

The next step is to characterize the effect that a MPS has on the optimal investment strategy and the option value of nuclear energy. The next two subsections analyze this.

### 1.4.3 The effect of a MPS with a risk neutral central planner and a fixed load curve

In this subsection we assume that the central planner behaves as a risk neutral agent. Risk neutrality implies that \( v(\cdot) \) is a linear function, \( v''(\cdot) = 0 \) and \( v'(\cdot) \) is a constant. Equation 1.17 can be simplified to:

\[ k_N - k_G - \beta j(q_N) \int_{0}^{\hat{\rho}} \rho f(\rho)d\rho = 0. \]

Integrating by parts we obtain:

\[ k_N - k_G - \beta j(q_N)\rho + \beta \int_{0}^{\hat{\rho}} F(\rho)d\rho. \] (1.20)
From the simplified first order condition we can easily obtain the following result.

**Proposition 3.** Under a fixed load curve and a risk neutral central planner a MPS has no effect on the optimal investment in nuclear technology.

*Proof.* Let $q^f_N$ denote the optimal investment in nuclear technology when the distribution of marginal cost is $f(.)$. Let $h(\rho)$ be a MPS over $f(\rho)$. Evaluate equation 1.20 using $h(\rho)$ at $q^f_N$ and subtract the same equation using $f(\rho)$ evaluated at the same point, we obtain:

$$\beta \int_{L}^{\bar{\rho}} H(\rho) - F(\rho) = 0$$

Where the equality follows from the definition of a MPS. Hence, $q^f_N$ also solves the FOC using $h(\rho)$ implying that the optimal investment strategy doesn’t change with a MPS as desired.

The intuition of our result is very simple. A risk neutral central planner would value total generation cost as a linear function of the generation cost of the natural gas technology. Therefore, the expected generation cost is a function of only the expected generation cost of the natural gas technology. Since a MPS maintains the expected generation cost of the natural gas technology fixed, the total expected generation cost is also fixed under a MPS and the optimal investment strategy doesn’t change.

The following proposition follows.

**Proposition 4.** Under a fixed load curve and a risk neutral central planner the option value of the nuclear technology doesn’t change.

*Proof.* It obviously follows from the linearity of the expected valuation of costs.

Notice that both propositions strongly depend on the linearity of the utility function. Hence, it is not surprising that the result will change under a central planner with a risk averse utility function. The next subsection analyzes this in detail.
1.4.4 The effect of a MPS with a risk averse central planner and fixed load curve

In this subsection we analyze the effect that a MPS has on the optimal investment strategy of a risk averse central planner. A risk averse central planner is assumed to have a convex dislike of costs, i.e. $v''(\cdot) > 0$.

Changing the order of integration in equation 1.17 we obtain:

$$v'(\cdot)(k_N - k_G) - \beta \int_0^{j(q_N)} \int_{\rho}^{\beta} w(\rho)f(\rho)d\rho dj = 0,$$

where $w(\rho)$ is given by:

$$w(\rho) = v'( (d(j) - q_N) \rho) \rho.$$

Let $q_N^f$ denote the optimal investment in nuclear technology when the distribution of marginal cost of generation is $f(\rho)$ and let $h(\rho)$ be a mean preserving spread with respect to $f(\rho)$. Evaluate equation 1.21 at $q_N^f$ using $h(\rho)$ and subtract the same equation using $f(\rho)$ evaluated at the same point. We obtain

$$-\beta \int_0^{j(q_N)} \int_{\rho}^{\beta} w(\rho)s(\rho)d\rho dj.$$

In order to sign the effect of a MPS integrate by parts twice the inner integral of equation 1.22.

$$\int_{\rho}^{\beta} w(\rho)s(\rho)d\rho = \int_{\rho}^{\beta} w''(\rho)T(\rho)d\rho$$

Where $T(\rho) = \int_{\rho}^{\beta} H(\rho) - F(\rho)d\rho$ and

$$w''(\rho) = (d(j) - q_N)[v'''(\cdot)(d(j) - q_N)\rho + 2v''(\cdot)].$$

Therefore the effect of a MPS depends on the curvature of the utility function.
The next result summarizes the characterization of the utility function to be able to sign the effect of a MPS on the optimal investment strategy.

**Proposition 5.** *The effect of a MPS on the optimal investment strategy depends on the curvature of the utility function. If*

\[ v''(w)w + 2v''(w) > 0, \quad (1.24) \]

*then the effect of a MPS is to increase investment in the nuclear technology. On the other hand if the inequality is reversed the effect of a MPS is to decrease investment in nuclear technology.*

*Proof.* If condition 1.24 is satisfied by the utility function of the central planner, then the change in the sign of the FOC of the MPS with respect to the original distribution is negative. Since the change on the left hand side of the FOC, equation 1.17 is given by:

\[ v''(\cdot)(k_N - k_0)^2 + \beta \int_0^{j(q_N)} \int_0^\delta v''(\cdot)\rho^2 f(\rho)d\rho dj > 0. \]

Therefore, the optimal investment in nuclear power increases as was to be proved. The second part of the proposition is straight forward. \(\Box\)

There are several aspects to discuss with respect to Proposition 5. The first one is the requirement on the curvature of the utility function. Notice that for the quadratic utility function, \( v''(\cdot) = 0 \) and \( v''(\cdot) > 0 \). For the constant absolute measure of curvature utility function, i.e. \( v(\cdot) \) is such that \( v''(w)/v'(w) = a \), \( v''(w) > 0 \), and again (1.24) holds. Finally, for the constant relative measure of curvature utility function, i.e. \( v(\cdot) \) is such that \( v''(w)w/v'(w) = r \), we again find \( v''(w) > 0 \). Therefore, for the quadratic, and constant absolute and relative risk averse utility functions, condition 1.24 is satisfied. Although the increase in the investment in
nuclear technology cannot be generalized to any convex utility function, we are fairly confident that it holds true for a large class of convex utility functions, and that it will hold specifically for those utility functions used most frequently to characterize risk aversion.

The intuition for the result in Proposition 5 at least when condition 1.24 is satisfied is simple. Under a fixed load curve, natural gas must be used independently of its cost. Therefore, a MPS increases the probability of bad outcomes which are more heavily disliked by the central planner than good outcomes, thus increasing the marginal value of nuclear capacity. Compared to our results on optimal investment under marginal cost pricing, under a fixed load curve the natural gas technology is no longer an option. Therefore, an increase in MPS decreases the marginal value of the natural gas technology.

With respect to the option value of nuclear power we get a similar result. Unfortunately, we cannot characterize the necessary curvature of the utility function but we provide a sufficient condition satisfied by the quadratic and the constant absolute and relative risk averse utility functions.

**Proposition 6.** Under a fixed load curve and a risk averse central planner the option value of the nuclear technology increases with a MPS over the distribution of generation cost of the natural gas technology if $v'''(.) > 0$.

**Proof.** From the definition a minimum,

$$(V_h(0) - V_h(\tilde{q}_N^{h})) - (V_f(0) - V_h(\tilde{q}_N^{f})) > (V_h(0) - V_f(0)) - (V_h(\tilde{q}_N^{f}) - V_f(\tilde{q}_N^{f})). \quad (1.25)$$

The first term of the right hand side of the inequality is given by

$$V_h(0) - V_f(0) = \beta \int_0^1 \int_\beta^\beta v(d(j)p)s(p)dpdj.$$
Integrating twice by parts we find that

\[ V_h(0) - V_f(0) = \beta \int_0^1 v''(d(j)\rho)(d(j))T(\rho)d\rho \, dj \geq 0, \quad (1.26) \]

where the inequality follows from \( v''(\cdot) > 0 \) and \( T(\rho) = \int_0^\rho H(s) - F(s)ds > 0. \)

On the other hand, notice that

\[ V_h(q_N) - V_f(q_N) = \beta \int_0^{j(q_N)} \int_\rho^\beta v((d(j) - q_N)\rho)\, s(\rho)d\rho \, dj. \]

Integrating twice by parts we obtain

\[ V_h(q_N) - V_f(q_N) = \beta \int_0^{j(q_N)} \int_\rho^\beta v''((d(j) - q_N)\rho)(d(j) - q_N)^2T(\rho)d\rho \, dj > 0. \quad (1.27) \]

Equations 1.26 and 1.27 together with the assumption that \( v''(\cdot) > 0 \) imply that

\[ V_h(0) - V_f(0) - V_h(q_N) + V_f(q_N) = \beta \int_0^1 v''(d(j)\rho)(d(j))T(\rho)d\rho \, dj - \beta \int_0^{j(q_N)} \int_\rho^\beta v''((d(j) - q_N)\rho)(d(j) - q_N)^2T(\rho)d\rho \, dj > 0. \]

Therefore the option value of the nuclear technology increases under a MPS and a risk averse central planner if \( v''(\cdot) > 0 \) as desired. \( \Box \)

The intuition of our results is again simple. An increase in the risk of the natural gas technology increases the option value of nuclear power since the risk averse central planner has no other way of dealing with risk (he cannot reduce the demand for electricity). Unfortunately, the results cannot generalize to all convex utility functions but we provided sufficient conditions on the curvature of the utility functions which in our opinion are not restrictive.

22
1.5 Conclusions

In this chapter we analyzed the effect that a MPS has on the optimal investment strategy of the central planner under marginal cost pricing and a fixed load curve. We found that a risk neutral central planner that prices at marginal cost will increase investment in the natural gas technology given the nature of its option value. We notice that introducing risk aversion does not necessarily change the behavior of the central planner, but we highlighted the reliance of the result on marginal cost pricing. Moreover, when we drop the marginal cost pricing assumption and introduce a fixed load curve, a MPS will have no effect if the central planner is a risk neutral agent and while in most cases will increase the investment in the nuclear technology if the central planner is risk averse. It certainly increases the option value of nuclear technology.

In our opinion the recent discussion of nuclear power as a way to hedge against the volatility of natural gas prices only holds if the central planner is risk averse and there is some level of electricity demand that must be provided. Quantifying the amount of electricity that must be supplied and that part of the demand for electricity that is elastic to electricity prices is the next step towards obtaining a valid estimate of the option value of nuclear power. Common levelized cost techniques cannot be used for assessing the true value of nuclear power under uncertain generation costs. Moreover, if the load curve is not modeled and a fixed load curve is implicitly assumed, then it is not surprising that this leads to the usual interpretation that nuclear capacity provides a hedge against uncertain generation costs.

1.6 Appendix

In this section we present the mathematical results to make the presentation of the paper as concrete as possible.
Proposition 7. Under marginal cost pricing the central planner’s problem has a unique maximum if \( u''(\cdot) \leq 0 \).

**Proof.** The central planner’s problem under marginal cost pricing is given by Equation 1.4. The objective function is reproduced here:

\[
\begin{align*}
&u(-k_Nq_N - k_Gq_G) + \beta \int_0^1 \left\{ \int_{\bar{p}}^{p(q_N+q_G,j)} u(CS(q_N + q_G, j) - \rho q_G)f(\rho)d\rho \\
&+ \int_{p(q_N+q_G,j)}^{p(q_N,j)} u(CS(d(\rho, j)) - (d(\rho, j) - q_N)\rho)f(\rho)d\rho \\&+ \int_{p(q_N,j)}^{\bar{p}} u(CS(q_N,j))f(\rho)d\rho \right\} dj. \tag{1.28}
\end{align*}
\]

In order to prove that the central planner’s problem has a unique maximum we will prove that the objective function is concave with respect to \((q_N, q_G)\). The second derivative of the objective function with respect to \(q_N\) is given by:

\[
A = u''(\cdot)k_N^2 + \beta \int_0^1 \left\{ \int_{\bar{p}}^{p(q_N+q_G,j)} u''(\cdot)p(q_N+q_G, j)^2 + \frac{\partial p(q_N + q_G, j)}{\partial q_N} u'(\cdot) \right\} f(\rho)d\rho + \\
\int_{p(q_N+q_G,j)}^{p(q_N,j)} u''(\cdot)p(q_N, j)^2 + u'(\cdot) \frac{\partial p(q_N, j)}{\partial q_N} \right\} f(\rho)d\rho < 0,
\]

where the inequality follows from \( u'(\cdot) > 0, u''(\cdot) \leq 0 \) and \( \partial p(q, j)/\partial q < 0 \).

Similarly, the second derivative of the objective function with respect to \(q_G\) is given by:

\[
B = u''(\cdot)k_G^2 + \beta \int_0^1 \int_{\bar{p}}^{p(q_N+q_G,j)} u''(\cdot)(p(q_N+q_G, j) - \rho)^2 + \frac{\partial p(q_N + q_G, j)}{\partial q_G} u'(\cdot) \right\} f(\rho)d\rho < 0,
\]

where the inequality again follows from \( u'(\cdot) > 0, u''(\cdot) \leq 0 \) and \( \partial p(q, j)/\partial q < 0 \). Therefore, the diagonal elements of the hessian are negative. Finally, the cross ca-
pacity derivative is given by:

\[ C = u''(\cdot)k_Nk_G + \beta \int_0^1 \int_\rho^{p(q_N + q_G, j)} \left\{ u''(\cdot)p(q_N + q_G, j)(p(q_N + q_G, j) - \rho) + \frac{\partial p(q_N + q_G)}{\partial q}u'(\cdot) \right\} f(\rho)d\rho, \quad (1.29) \]

and the determinant is given by \( AB - C^2 > 0 \) completing the proof. \( \square \)

**Proposition 8.** Under a fixed load curve the central planner’s problem has a unique minimum if \( v''(\cdot) \geq 0 \).

**Proof.** The simplified central planner’s problem under a fixed load curve is given by equation 1.16. We reproduce the objective function

\[ v((k_N - k_G)q_N + k_Gd(0)) + \beta \int_0^\rho \int_0^{j(q_N)} v((d(j) - q_N)\rho)dj f(\rho)d\rho. \]

To prove a unique minimum we need to prove that the objective function is convex. The second derivative of the objective function with respect to \( q_N \) is given by:

\[ v''(\cdot)(k_N - k_G)^2 - \beta \int_\rho^{j(q_N)} \left\{ v'(0)\rho \frac{\partial j(q_N)}{\partial q_N} - \int_0^{j(q_N)} (\cdot)\rho^2 dj \right\} f(\rho)d\rho > 0 \]

Where the inequality follows from \( v''(\cdot) \geq 0 \) and \( \partial j(q_N)/\partial q_N < 0 \). Therefore the objective function is convex and there exist a unique minimum as desired. \( \square \)
Chapter 2

Optimal Investment in Electricity Generation Capacity: A Recursive Formulation

2.1 Introduction

In this chapter we solve for the optimal investment in electricity generation capacity for a somewhat restricted but, potentially easy expandable, model in a recursive setting. We examine the decisions of a central planner concerned with maximizing social benefits. Since the load curve and stochastic input prices are the focus of our work, we pay special attention to those issues.

Our contribution to the literature in this chapter is twofold: (1) the available analysis for the optimal investment in capacity for the generation of electricity does not model generation cost as a stochastic variable; (2) most of the analysis to date has focused on finding the optimal capacity in a static framework rather than determining the path of optimal investment. Our second contribution is relevant in cases where the demand for electricity is expected to increase for some time in the future, there is technological changes that will make some of the old technology obsolete, or if the stochastic process of generation cost has a memory component, such as an auto-regressive process.
We first show that the optimal investment problem has a recursive structure. The characterization of the optimal investment is somewhat harder however, because it has a non-recursive part. Hence, Section 2.2 presents the recursive model and shows the solution is unique. Section 2.3 characterizes the optimal investment path. Section 2.4 presents the conclusion while the appendix provides some of the more technical proofs.

2.2 The Optimal Investment Problem

In this section we present the general problem of a benevolent central planner that maximizes social benefit minus generation and investment costs.\(^1\) We show that the central planner’s problem has a recursive structure, which we exploit to show that a unique solution exists. The characterization of the solution is somewhat harder and it is presented in a separate section. We separate the sections since numerical solution methods only require that the solution is unique. By contrast, characterization of the optimal solution for the general case requires more complex techniques that, although they greatly enhance understanding of the problem, are not necessary in an empirical implementation of an extended model.

For simplicity we assume that there are only two technologies: a nuclear technology, \(N\), and a natural gas technology, \(G\).\(^2\) We assume that investment in both technologies is perfectly divisible.\(^3\)

\(^1\)We use a partial equilibrium model to simplify the problem of the central planner. It is a matter for future research to compare the solutions of a partial and a general equilibrium model.

\(^2\)Although we name the second technology a natural gas technology, it can represent any fossil fuel technology: including plants that can utilize two different fossil fuels.

\(^3\)This assumption is not problematic in large integrated power systems where increases of demand are relatively high relatively to the size of the indivisibility of plants. For example, a .5% increase in capacity per month (6.16% annually) and a 3 month rule for requiring half of indivisibility capacity, \(i/2\), to be require implies a total installed capacity of \((1.005^3 - 1)q = i/2\). For a 1,300 MW nuclear plant, 750MW half of indivisibility capacity, it will require a total installed capacity of approximately 50,000 MW.
We model the nuclear technology as having a higher investment cost than natural gas, but no generation cost. Letting $k_i$ denote the per unit cost of investment in capacity for technology $i = N, G$, we assume that $k_N > k_G$. On the other hand, let the generation cost of the natural gas technology be denoted by $\rho$. We assume that $\rho \in (0, \infty)$ is a random variable that is independent between time periods.\(^4\) Denote the density function of $\rho$ as $f(\rho)$ and the cumulative distribution function as $F(\rho)$.\(^5\) We assume that $f(\rho)$ is continuous. Finally, let $\delta_i$ denote the per cycle depreciation of technology $i = N, G$.

With respect to the demand structure, we assume there are $J$ sub-periods per cycle. Denote the inverse demand of sub-period $j = 1, \ldots, J$ as $p_j(\cdot)$ and the demand as $d_j(\cdot)$. We assume that the demand and the inverse demand are continuous and differentiable for all sub-periods. Finally, let $w_j$ denote the duration of sub-period $j$. It is important to note that the demand in period $j$ is not stochastic and does not grow.\(^6\)

The problem of the central planner is to choose investment in capacity in each technology and the generation of electricity from each technology at each sub-period. The investment decision is taken before the generation cost for the natural gas technology is realized. In contrast, the generation decision, made after $\rho$ has been realized. In other words, the central planner chooses sequences $\{q_{Nj}, q_{Gj}\}_{t=1}^{\infty}$ and $\{g_{Nj}, g_{Gj}\}_{t=1}^{\infty}$ for $j = 1, \ldots, J$; where $q_{it}$ denotes the installed capacity of technology $i$ at the beginning of cycle $t$ and $g_{ij}$ denotes the generation of electricity by technology $i$ at cycle

\(^4\)We restrict $\rho > 0$ in order to have a simple and well defined optimal generation schedule. Although the inclusion of zero does not change the calculations, it will complicate the optimal generation schedule.

\(^5\)The restriction of $\rho$ to being independent between periods can easily be relaxed.

\(^6\)The assumption that demand is non-stochastic can be relaxed. In contrast, the assumption that demand does not grow through time is a central issue for the recursive structure of the model and the existence of a unique solution. Increments of the demand imply increments of the consumer surplus. Therefore, if the demand increases faster than the discount factor the objective function is ill defined.
In order to be able to rank the outcomes from the investment and generation decisions, we assume that the central planner has a mechanism, a price mechanism for example, to allocate the generated electricity to those who have the highest valuation. Hence, the central planner ranks the benefit from generated electricity at time $t$, sub-period $j$ by:

$$CS_j(g^j_t) = \int_0^{g^j_t} p_j(s)ds.$$  

(2.1)

Given the valuation of the benefits from generated electricity, the central planner can optimally choose a sequence of investment and generation by maximizing the discounted sum of expected per period generation benefits minus generation and investment cost. Specifically, the central planner solves:

$$\max_{\{q^i_t\}_{i=1}^\infty, \{g^i_{t,j}\}_{i=1}^\infty} \sum_{t=0}^\infty \beta^t \sum_{j=1}^J w_j E[CS_j(g^k_{N,j} + g^k_{G,j}) - \rho g^k_{G,j}]$$

$$i = N, G; j = 1, \ldots, J$$

$$-k_N(q^t_{N} - (1 - \delta_N)q^t_{N}) - k_G(q^{t+1}_{G} - (1 - \delta_G)q^t_{G}),$$  

(2.2)

subject to

$$q^{t+1}_i - (1 - \delta_i)q^t_i \geq 0 \text{ for all } t = 0, \ldots \text{ and } i = N, G.$$  

Notice that the non-negative investment constraint in the maximization program of the central planner implies that plant decommissioning can not be accelerated. It is also important to note that we can simplify the dynamic central planner's problem by solving the optimal generation decision since the latter only depends on the available capacity at the given sub-period.

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7This is as a consequence of excluding fixed operating cost from the model. Since then the central planner will never choose to decommission faster. Accelerated decommissioning can be an important part of the dynamic investment behavior but it is beyond the current scope of this chapter.
2.2.1 Optimal Generation

In order to simplify the notation we will henceforth in this subsection drop subscripts \( t \) and \( j \) denoting cycle and sub-period respectively. Denote the available capacity as \((q_N, q_G)\) and the realization of the generation cost for the natural gas technology as \( \rho \). By assumption, the generation cost for the nuclear technology is zero.

If the demand is not bounded below, i.e. as the price of electricity goes to zero the demand of electricity goes to infinity, then the inverse demand will always be positive implying that the generation of the nuclear technology should be equal to its capacity, specifically, \( g_N = q_N \). On the other hand, If the demand is bounded, i.e. as the price of electricity goes to zero the demand of electricity is bounded, and the bound on demand is lower than nuclear capacity and there is free disposal, then the optimal nuclear generation is given by the set \([M, q_N]\), where \( M \) denotes the upper bound in demand. Since it will simplify the presentation, we will assume the generation using the nuclear technology is given by \( g_N = q_N \).\(^8\)

The situation is slightly different for the natural gas technology. If \( \rho \geq p(q_N) \), then the marginal value of electricity when only the nuclear technology is running is lower than the gas generation cost, implying that the natural gas technology should not be run at all. On the other hand, if the marginal value of electricity when natural gas and nuclear technologies are both running at full capacity is higher than the generation cost, \( p(q_N + q_G) \geq \rho \), then the optimal generation from natural gas should be equal to its capacity, \( g_G = q_G \). Finally, if \( p(q_N + q_G) < \rho < p(q_N) \), then it is optimal to generate electricity using natural gas until the marginal value of electricity is equal to the generation cost, implying that \( g_G = d(\rho) - q_N \). In summary, the optimal generation from nuclear technology is always to generate at full capacity,

\(^8\)If the upper bound \( M \) exists, then it should be the same for all sub-periods since the installed equipment that uses electricity is fixed. Therefore, unless it is optimal to only use the nuclear technology, we will expect \( q_N < M \).
\(g_N(q_N, q_G) = q_N\), while the optimal generation from natural gas is given by:

\[
g_G(q_N, q_G) = \begin{cases} 
0 & \text{if } p(q_N) \leq \rho \\
d(\rho) - q_N & \text{if } p(q_N) < \rho \leq p(q_N + q_G) \\
q_G & \text{if } p(q_N + q_G) > \rho.
\end{cases}
\] (2.3)

It is important to note that the optimal generation problem can always be solved separately from the investment problem. More complex dispatching schedules can be solved and included into the model under fairly mild restrictions. Such restrictions will become apparent in the next subsection.

### 2.2.2 The Investment Problem

The central planner's problem is simplified by the separation between the generation decision, or optimal dispatch, and the optimal investment decision. Given the optimal generation schedule in equation 2.3 the expected consumer surplus for period \(t\), sub-period \(j\) is:

\[
ECS_j(q_N, q_G) = \int_0^{p_j(q_N + q_G)} CS_j(q_N + q_G) f(\rho) d\rho + \\
\int_{p_j(q_N + q_G)}^{p_j(q_N)} CS_j(d_j(\rho)) f(\rho) d\rho + \int_{p_j(q_N)}^{\infty} CS_j(q_N) f(\rho) d\rho. \quad (2.4)
\]

Obviously, the expected consumer surplus function does not depend on \(\rho\) because the expectation operator integrates over its support. Moreover, the optimal generation schedule is already implicit. Our characterization of the optimal investment schedule requires expected consumer surplus to be continuous and differentiable. Therefore, any different dispatching schedule must imply continuity and differentiability of the

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\(^9\)It should be clear that the optimal investment will be different, but the latter does not affect the former.
expected consumer surplus if our characterization of the optimal investment is to be implemented.

The expected generation cost for period $t$, sub-period $j$ is:

$$
EGC_j(q^t_N, q^t_G) = \int_0^{p_j(q^t_N + q^t_G)} q^t_G \rho f(\rho) d\rho + \int_0^{p_j(q^t_N)} (d_j(\rho) - q^t_N) \rho f(\rho) d\rho.
$$

(2.5)

As in the case of the expected consumer surplus, the expected generation cost does not depend on $\rho$ and the dispatch schedule is implicit. The expected generation cost is also continuous and differentiable. Again, a different dispatch schedule must imply continuous and differentiable expected generation costs for it to fit our framework. Finally, Proposition 1 in the Appendix shows that the per period net benefits function is continuous, differentiable and strictly concave. Therefore, for our subsequent analysis to remain valid under a different dispatch schedule, the expected net benefits function must remain strictly concave.

The recursive structure of the central planner's problem

Using equations 2.4 and 2.5 the central planner's problem can be rewritten as:

$$
\max_{\{q^t_N, q^t_G\}} \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{j=1}^{J} w_j (EC_S_j(q^t_N, q^t_G) - EGC_j(q^t_N, q^t_G)) \right. \\
\left. - k_N(q^{t+1}_N - (1 - \delta_N)q^t_N) - k_G(q^{t+1}_G - (1 - \delta_G)q^t_G) \right\},
$$

(2.6)

subject to

$$
q^{t+1}_N - (1 - \delta_N)q^t_N \geq 0 \quad q^{t+1}_G - (1 - \delta_G)q^t_G \geq 0 \quad \text{for all} \ t = 0, \ldots.
$$

The central planner's problem has a recursive structure that allows us to use dynamic programming to characterize the solution. Specifically, equation 2.6 has the
following functional equation counterpart:

\[ v(q_N, q_G) = \max_{q_N', q_G'} \sum_{j=0}^{J} w_j (ECS_j(q_N, q_G) - EGC_j(q_N, q_G)) \]

\[ - k_N(q_N' - (1 - \delta_N)q_N) - k_G(q_G' - (1 - \delta_G)q_G) + \beta v(q_N', q_G), \quad (2.7) \]

where the maximization is done subject to

\[ q_N' - (1 - \delta_N)q_N \geq 0 \text{ and } q_G' - (1 - \delta_G)q_G \geq 0. \]

Restating the problem in terms of the value function, \( v(\cdot, \cdot) \), changes the problem of finding an infinite sequence of capacities to that of finding the solution to a functional equation. Specifically, the problem is to find a function \( v(\cdot, \cdot) \) that solves the functional equation 2.7. Many solution methods have been devised to solve functional equations like that of equation 2.7. In particular, the fact that the per period expected net benefits minus investment cost is a bounded, continuous and strictly concave function implies the the solution to equation 2.7 is unique, the function \( v(\cdot, \cdot) \) is strictly concave and the optimal investment is single valued. Appendix 2.5.1 and 2.5.2 prove the necessary results.

Once we know that the solution for the functional equation 2.7 is unique and solves the central planner's problem in equation 2.6, we can apply common numerical methods to solve for it and analyze the optimal policy functions. But, we will go further than that given the simple structure of our problem. The next section characterizes the optimal investment policy. We think is an important step in understanding more thoroughly the central planner's problem and will allow us to obtain more insights from the problem without restricting ourselves to a specific application.
2.3 Characterization of the Optimal Investment

Given that the function $v(\cdot, \cdot)$ is continuous and strictly concave, characterizing the optimal investment plan is just an issue of solving the maximization problem and applying the envelope theorem. Unfortunately, the problem is not as easy as it seems given that the restrictions are binding for a large part of initial installed capacities. The function $v(\cdot, \cdot)$, although continuous, is not differentiable at the points where the restriction just becomes binding.\textsuperscript{10} Since the function $v(\cdot, \cdot)$ is part of the objective function, the usual Kuhn-Tucker technique for solving non-linear programs cannot be applied directly.

Our strategy for characterizing the optimal investment focuses on the region where we know the non-negative investment constraints are not binding. Then, we extend the region of characterization noticing that when the constraints are binding the optimal investment is given by the binding constraint and a maximization problem over the technology where the constraint is not binding. Performing this procedure iteratively for both technologies allows us to extend the region where the value function is characterized. Finally, if both constraints are binding, the optimal solution is simply to invest zero in both technologies.

We start our analysis by finding the region of installed capacities where the non-negative investment constraints are not binding.

2.3.1 Installed capacities where the non-negative investment constraint is not binding

In order to find the region in the capacity support where the non-negative investment constraints are not binding, we solve the following closely related functional equation\textsuperscript{10}

\begin{footnotesize}
\textsuperscript{10}The left hand derivative is different from the right hand derivative.
\end{footnotesize}
where the maximization over next period capacities is unconstrained:

\[
\nu(q_N, q_G) = \max_{q'_N, q'_G} \sum_{j=0}^{J} w_j (ECS_j(q_N, q_G) - EGC_j(q_N, q_G)) - k_N(q'_N - (1 - \delta_N)q_N) - k_G(q'_G - (1 - \delta_G)q_G) + \beta \nu(q'_N, q'_G). \tag{2.8}
\]

The characterization of functional equation 2.8 will help us to find the regions where optimal investment violates the non-negativity constraints.

**Characterization of the functional equation with unconstrained maximization**

Since the non-negative investment constraints are not present in functional equation 2.8, and the per period expected net benefit minus investment cost is bounded, continuous and strictly concave, the function \(\nu(\cdot, \cdot)\) is also bounded, continuous and strictly concave. More important, Appendix 2.5.3 proves \(\nu(\cdot, \cdot)\) is differentiable.

Once we know that the function \(\nu(\cdot, \cdot)\) is continuous, strictly concave and differentiable, the investment in capacity can be characterized using the first order conditions (FOCs) and the envelope theorem. Furthermore, the FOCs characterize the unique global optimum of the problem. Specifically, they are given by:

\[
-k_N + \beta \frac{\partial \nu(q'_N, q'_G)}{\partial q'_N} = 0,
\]

\[
-k_G + \beta \frac{\partial \nu(q'_N, q'_G)}{\partial q'_G} = 0.
\]
Applying the envelope theorem to equation 2.8 we have that:

\[
\frac{\partial \nu(q_N, q_G)}{\partial q_N} = \sum_{j=0}^{J} w_j \left\{ p_j(q_N + q_G) F(p_j(q_N + q_G)) + p_j(q_N) [1 - F(p_j(q_N))] + \int_{p_j(q_N + q_G)}^{p_j(q_N)} \rho f(\rho) d\rho \right\} - k_N(1 - \delta_N) \quad (2.9)
\]

and,

\[
\frac{\partial \nu(q_N, q_G)}{\partial q_G} = \sum_{j=0}^{J} w_j \left\{ p_j(q_N + q_G) F(p_j(q_N + q_G)) - \int_{0}^{p_j(q_N + q_G)} \rho f(\rho) d\rho \right\} + k_G(1 - \delta_G). \quad (2.10)
\]

Using equations 2.9 and 2.10, the FOCs of the unconstrained maximization problem are given by:

\[
\beta \sum_{j=0}^{J} w_j \left\{ p_j(q'_N + q'_G) F(p_j(q'_N + q'_G)) + p_j(q'_N) [1 - F(p_j(q'_N))] + \int_{p_j(q'_N + q'_G)}^{p_j(q'_N)} \rho f(\rho) d\rho \right\} - k_N(1 - \beta(1 - \delta_N)) = 0 \quad (2.11)
\]

and,

\[
\beta \sum_{j=0}^{J} w_j \left\{ p_j(q'_N + q'_G) F(p_j(q'_N + q'_G)) - \int_{0}^{p_j(q'_N + q'_G)} \rho f(\rho) d\rho \right\} - k_G(1 - \beta(1 - \delta_G)) = 0. \quad (2.12)
\]

Denote the solution of the FOCs as \((q_N^*, q_G^*)\). Notice that the FOCs do not depend on the initial installed capacities, \((q_N, q_G)\), implying that the optimal capacities also do not depend on them. Moreover, the optimal investment policy at period 0 is to
invest \( q_G^* - (1 - \delta_G)q_G^0 \) in the natural gas technology and \( q_N^* - (1 - \delta_N)q_N^0 \) in the nuclear technology. Therefore, the optimal investment of the unconstrained problem violates the non-negative investment constraint if:

\[
q_N^0 > \frac{q_N^*}{(1 - \delta_N)} \text{ or } q_G^0 > \frac{q_G^*}{(1 - \delta_G)}.
\]  

(2.13)

Finally, if the initial amount of capital, \((q_N^0, q_G^0)\), does not violate the non-negative investment constraint, then the optimal investment path, maintaining constant capacities \((q_N^*, q_G^*)\), never visits regions where the constraint is violated. This is most important since, if we restrict the initial installed capacities to the region where the optimal investment path does not violate the non-negative investment constraint, we let \( q_N^0 \in [0, q_N^*(1 - \delta_N)^{-1}] \) and \( q_G^0 \in [0, q_G^*(1 - \delta_G)^{-1}] \), then the constrained and the unconstrained problem coincide in value. Moreover, we have found the initial amount of capital where the non-negative investments constraints just become binding for at least one technology, all initial capacities \((q_N^0, q_G^0)\) where \( q_i^0 = q_i^*(1 - \delta_i)^{-1} \) for \( i = N, G \).

As a consequence, \( v(\cdot, \cdot) \) is differentiable in the region where the constrained and the unconstrained problem coincide excluding the points where the constraint just becomes binding.

Summarizing, the region where we have characterized the value function \( v(\cdot, \cdot) \) is given by:

\[
R_1 = \{(q_N, q_G) : q_N \leq \frac{q_N^*}{(1 - \delta_N)} \text{ and } q_G \leq \frac{q_G^*}{(1 - \delta_G)}\}.
\]  

(2.14)

We also have characterized kinks in the value function in the border of region \( R_1 \). Specifically, the kinks in the value function are given by:

\[
R_1^b = \{(q_N, q_G) : q_N = \frac{q_N^*}{(1 - \delta_N)} \text{ or } q_G = \frac{q_G^*}{(1 - \delta_G)}\}
\]  

(2.15)

The optimal investment in region \( R_1 \) is to invest \( q_N^* - (1 - \delta_N)q_N \) in capacity \( N \).
and $q'_G - (1 - \delta_G)q_G$ in capacity $G$ in period 0, and to maintain capacities $(q'_N, q'_G)$ thereafter.

**Extending the region where $v(\cdot, \cdot)$ is characterized**

To extend the characterization of $v(\cdot, \cdot)$ we use a very simple iterative procedure (it will be clear along the way that the procedure is not recursive). Consider that the installed capacity in the nuclear technology is such that $q_N^*(1 - \delta_N)^{-2} > q_N > q_N^*(1 - \delta_N)^{-1}$; and let $q_G = 0$. We know from the unconstrained problem that the restriction for technology $N$ will be binding for one period, but it won’t be binding for period 2. Hence,

$$v(q_N, q_G) = \max_{q'_G} \sum_{j=0}^J w_j(ECS_j(q_N, q_G) - EGC_j(q_N, q_G))$$

$$- k_G(q'_G - (1 - \delta_G)q_G) + \beta v((1 - \delta_N)q_N, q'_G). \quad (2.16)$$

Notice that we have already characterized $v((1 - \delta_N)q_N, q'_G)$ and we showed that it is continuous, strictly concave and differentiable. Hence, the solution for the maximization problem is unique and is characterized by the FOC and the envelope theorem. Therefore, we have extended the characterization of $v(\cdot, \cdot)$ in the range $(q_N^*(1 - \delta_N)^{-1}, q_N^*(1 - \delta_N)^{-2}]$ and $q_G = 0$.\footnote{Notice that the maximization function only needs to be differentiable with respect to $q_G$.} For each $q_N$ in the extended range, we can extend the range of $q_G$ so that the initial installed capacity is not larger than the optimal capacity over $(1 - \delta_G)$. In other words, $q_G$ must allow us to get to the optimal capacities without violating the non-negative investment constraint in the natural gas technology. Specifically, if the function $q^*_G(q_N)$ denotes the solution for the maximization part of equation 2.16, then the constraint in the initial amount of capacity in the natural gas technology is given by $q^*_G(q_N)(1 - \delta_G)^{-1}$. 

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Summarizing, we have extended the range of \( v(\cdot, \cdot) \) to the region \( R_2 \) given by:

\[
R_2 = \{(q_N, q_G) : \frac{q_N^*}{(1 - \delta_N)} < q_N \leq \frac{q_N^*}{(1 - \delta_N)^2} \text{ and } q_G < \frac{q_G^*(q_N)}{(1 - \delta_G)}\}.
\]

We also have increased the number of points where the value function is not differentiable in both arguments by the set:

\[
R_2^b = \{(q_N, q_G) : q_N = \frac{q_N^*}{(1 - \delta_N)^2} \text{ or } q_G = \frac{q_G^*(q_N)}{(1 - \delta_G)}\}
\]

Once we extend \( v(\cdot, \cdot) \), we can apply the same procedure iteratively until \( q_N \) is such that \( q_G^*(q_N) = 0 \). We cannot go further than that since the problem is not define for negative capacities. Notice that \( q_G^*(q_N) \) is being defined in each extended period by the FOC and the envelope theorem. But it is easy to notice that the difference between the FOC of the iterations is that they are being evaluated at different values of \( q_N \). Hence, the FOC is continuous and differentiable in the extended region implying that we can differentiate the FOC with respect to \( q_N \) and solve for \( \partial q_G^*(q_N)/\partial q_N \). The FOC with the envelope theorem imply that:

\[
\rho \sum_{j=0}^{J} w_j \left\{ p_j((1 - \delta_N)q_N + q_G)F(p_j((1 - \delta_N)q_N + q_G)) - \int_{0}^{p_j((1 - \delta_N)q_N + q_G)} \rho f(\rho)d\rho \right\} - k_G(1 - \beta(1 - \delta_G)) = 0. \tag{2.17}
\]

Equation 2.17 defines \( q_G^*(q_N) \). Differentiating equation 2.17 with respect to \( q_N \) and solving for \( \partial q_G^*(q_N)/\partial q_N \) implies that:

\[
\frac{\partial q_G^*(q_N)}{\partial q_N} = -(1 - \delta_N). \tag{2.18}
\]

By definition, the initial installed capacity that will violate the investment constraint
is given by \( q_G^n(q_N)(1 - \delta_G)^{-1} \). Taking the partial derivative with respect to \( q_N \) we obtain:

\[
\frac{\partial \{ q_G^n(q_N) \cdot (1 - \delta_G)^{-1} \}}{\partial q_N} = \frac{(1 - \delta_N)}{(1 - \delta_G)}.
\] (2.19)

Applying the same argument for the natural gas technology, the FOC with the envelope theorem of the maximization with respect to \( q_N \) leads to:

\[
\beta \sum_{j=0}^{J} w_j \left\{ p_j(q'_N + (1 - \delta_G)q_G)F(p_j(q'_N + (1 - \delta_G)q_G)) + p_j(q'_N)[1 - F(p_j(q'_N))\right\} + \\
\int_{p_j(q'_N + (1 - \delta_G)q_G)}^{p_j(q'_N)} \rho f(\rho)d\rho \right\} - k_N(1 - \beta(1 - \delta_N)) = 0 \] (2.20)

Which defines the function \( q_G^n(q_G) \). Differentiating on both sides and solving for \( \partial q_N^n(q_G)/\partial q_G \) we obtain:

\[
\frac{\partial q_N^n(q_G)}{\partial q_G} = -(1 - \delta_G) - \frac{A}{A + B} < 0.
\] (2.21)

Where

\[
A = \beta \sum_{j=1}^{J} p'_j(q'_N + q_G(1 - \delta_G))F(p_j(q'_N + (1 - \delta_G)q_G)) < 0,
\]

\[
B = \beta \sum_{j=1}^{J} p'_j(q'_N)[1 - F(p_j(q'_N))] < 0.
\]

The constraint on the initial installed nuclear capacity is given by \( q_N^n(q_G) \cdot (1 - \delta_N)^{-1} \). Differentiating with respect to \( q_G \) yields:

\[
\frac{\partial \{ q_G^n(q_G) \cdot (1 - \delta_N)^{-1} \}}{\partial q_G} = \frac{(1 - \delta_G)}{(1 - \delta_N)} \frac{A}{A + B}
\] (2.22)

Figure 2.1 summarizes our results characterizing the optimal investment in ca-
Figure 2.1: Optimal Investment in the $q_N - q_G$ space.
pacity given the initial condition \((q_N, q_G)\). In points 1-4 the optimal investment in capacity, point \(E\), can be reached without violating the non-negative investment constraint. In period zero, optimal capacities, \((q_N^*, q_G^*)\), are chosen and maintained in all other periods. In points 5-6, the situation is different. They can not reach point \(E\) without violating the non-negative investment constraint in technology \(G\). The optimal strategy is to invest zero in technology \(G\) and \(q_N^*(q_G) - (1 - \delta_N)q_N\) in technology \(N\), point 7. The behavior is repeated until point \(E\) can be reached without violating the non-negative investment constraint. Notice that \(E\) can be reached in a finite number of periods since \(q_G^0(1 - \delta_G)^{t-1}\) goes to zero as \(t\) goes to infinity. The situation is similar in points 8-9. In this case, point \(E\) cannot be reached without violating the investment constraint in the nuclear technology. Hence, the optimal investment is to invest zero in technology \(N\) and to invest \(q_G^*(q_N) - (1 - \delta_G)q_G\) in technology \(G\), point 10. Again, the behavior is repeated until point \(E\) can be reached without violating the investment constraint in the nuclear technology. As in the case for points 5 and 6, point \(E\) can be reached in a finite number of periods. Finally, points 11 and 12 characterize the situation when there is too much installed capacity in both technologies. The optimal strategy is clearly to invest zero in both technologies.

2.4 Conclusions

We characterize the optimal investment given stochastic shocks in generation cost and a deterministic demand. Although the analysis assumes a simple model, we believe that it can be easily extended to a more complex situation where a numerical version might be necessary.

In writing this note several aspects of the problem were discussed. We must stress our beliefs that this note provides a flexible and strong base for further research into the area.
The solution of the model is quite simple. With demands that are fixed at every period, it is not surprising that complex dynamic behavior does not arise. Again, we consider that the structure of the model will allow us to easily extend the model. Hence, the treatment of some extensions of the model (specifically, demand increases in the short run) will be an integral part of a later version of the present note.

2.5 Appendix

We start the appendix by characterizing the expected sum of sub-period consumer surplus and generation cost.

The expected consumer surplus at period $t$, subperiod $j$, equation 2.4, is:

$$ECS_j(q^t_N, q^t_G) = CS_j(q^t_N + q^t_G)F(p_j(q^t_N + q^t_G)) + CS_j(q^t_N)[1 - F(p_j(q^t_N))]$$

$$+ \int_{p_j(q^t_N + q^t_G)}^{p_j(q^t_N)} CS_j(d_j(\rho))f(\rho)d\rho.$$

And the expected generation cost at period $t$, sub-period $j$, equation 2.5, is:

$$EGC_j(q^t_N, q^t_G) = \int_{0}^{p_j(q^t_N + q^t_G)} \rho q^t_G f(\rho)d\rho + \int_{p_j(q^t_N)}^{p_j(q^t_N)} (d_j(\rho) - q^t_N)\rho f(\rho)d\rho.$$

Given the additivity of the expectation operator, the expected net benefits at period $t$, sub-period $j$ is simply $ECS_j(q^t_N, q^t_G) - EGC_j(q^t_N, q^t_G)$. We give our first result.

Proposition 1. The expected sum of per sub-period net benefits from capacities $(q_N, q_G)$ is continuous, differentiable and strictly concave.
Proof. The expected per period net benefits is simply given by:

\[ \sum_{j=1}^{J} w_j (ECS_j(q_N, q_G) - EGC_j(q_N, q_G)). \]

Continuity and twice differentiability follow from initial assumptions. Specifically, it follows from our assumption of continuous and differentiable demands and inverse demands.

To prove strict concavity, we will show that the Hessian of \( ECS_j(q_N, q_G) - EGC_j(q_N, q_G) \) is negative semi-definite for all \( j \) and negative definite for at least one \( j \).

The partial derivative of the per sub-period expected net benefits with respect to \( q_N \) is:

\[ \frac{\partial \{ECS_j - EGC_j\}}{\partial q_N} = p_j(q_N + q_G)F(p_j(q_N + q_G)) \]

\[ + p_j(q_N)[1 - F(p_j(q_N))] + \int_{p_j(q_N + q_G)}^{p_j(q_N)} \rho f(\rho) d\rho, \]

and with respect to \( q_G \) it is:

\[ \frac{\partial \{ECS_j - EGC_j\}}{\partial q_G} = p_j(q_N + q_G)F(p_j(q_N + q_G)) - \int_{0}^{p_j(q_N + q_G)} \rho f(\rho) d\rho. \]

Thus, the Hessian is simply given by:

\[ H_j = \begin{pmatrix}
  p_j'(q_N + q_G)F(p_j(q_N + q_G)) + \\
  p_j'(q_N)[1 - F(p_j(q_N))] \\
  p_j'(q_N + q_G)F(p_j(q_N + q_G)) + p_j'(q_N + q_G)F(p_j(q_N + q_G))
\end{pmatrix} \]

If \( F(p_j(q_N + q_G)) = 0 \) for all \( j \), then the marginal benefits from investment in capacity
$G$, while the investment cost is positive. Obviously, the optimum will not belong to such a region, and it will belong to a region where $F(p_j(q_N + q_G)) > 0$ for at least one $j, \hat{j}$.

Thus, $H(1,1) \leq 0, H(2,2) \leq 0$ and the determinant of $H$ is higher or equal to zero, with strict inequalities for $\hat{j}$. Then, the sub-period expected net benefits are concave and strictly concave for at least one $j$.

The Hessian of the expected sum of per sub-period net benefits is given by:

$$H = \begin{pmatrix} \sum_j w_j a_j + \sum_j w_j b_j \\ \sum_j a_j & \sum_j w_j a_j \end{pmatrix},$$

(2.23)

where

$$a_j = \frac{\partial p_j(q_N + q_G)}{\partial q} F(p_j(q_N + q_G)),$$

$$b_j = \frac{\partial p_j(q_N)}{\partial q} [1 - F(p_j(q_N))].$$

Notice that the Hessian is negative definite since $H(1,1) < 0, H(2,2) < 0$ and $|H| > 0$ implying that the expected sum of per sub-period net benefits is strictly concave as desired.\hfill\Box

2.5.1 Existence and uniqueness of the value function

We will rely on results from Stokey and Lucas [16] to prove the necessary results. Hence we will adjust our notation as close as possible to their notation.

Rewrite the functional equation 2.7 as

$$v(q_N, q_G) = \max_{(q'_N, q'_G) \in \Gamma(q_N, q_G)} F(q_N, q_G, q'_N, q'_G) + \beta v(q'_N, q'_G),$$
where,

\[
F(q_N, q_G, q'_N, q'_G) = \sum_{j=0}^{J} w_j (ECS_j(q_N, q_G) - EG_C_j(q_N, q_G))
- k_N(q'_N - (1 - \delta_N)q_N) - k_G(q'_G - (1 - \delta_G)q_G),
\]

and,

\[
\Gamma(q_N, q_G) = \{ (q'_N, q'_G) : q'_N \geq q_N(1 - \delta_N) \text{ and } q'_G \geq q_G(1 - \delta_G) \}.
\]

**Proposition 2.** The solution to the functional equation 2.5.1, \( v \), is the solution for the non-recursive problem and \( v \) is unique.

**Proof.** Notice that \( \Gamma(q_N, q_G) \) is non-empty. The expected sum of sub-period net benefits minus the investment cost is well defined for all the support of \( (q_N, q_G) \). Finally, notice that since the consumer surplus is bounded by assumption, so is the discounted sum of per period expected sum of sub-period net benefits minus the investment cost, implying that

\[
\lim_{n \to \infty} \beta^n v(q^n_N, q^n_G) = 0,
\]

for all investment paths and initial conditions. Hence, we apply Theorem 4.3 page 72 in Stokey and Lucas [16] which proves the proposition. \( \square \)

### 2.5.2 Strict concavity of the value function

We have proved uniqueness of the solution. Our next step is to prove strict concavity of the value function and that the optimal investment is single valued. We will prove the result by means of Theorem 4.8 page 81 in Stokey and Lucas [16]. Before we are able to applied the theorem we need two small results.
Let \( x = (q_N, q_G) \) and let \( y = (q'_N, q'_G) \) then

**Proposition 3.** The function, \( F(x, y) \) is strictly concave; that is,

\[
F\left[\theta(x, y) + (1 - \theta)(x', y')\right] \geq \theta F(x, y) + (1 - \theta)F(x'y') \tag{2.24}
\]

all \( (x, y), (x', y') \in \{(x, y) \in X \times X : y \in \Gamma(x)\} \), and all \( \theta \in (0, 1) \) \( \tag{2.25} \)

and the inequality is strict if \( x \neq x' \).

**Proof.** Notice that \( F(x, y) \) is the expected sum of per sub-period net benefits minus the investment cost. Since \( F(x, y) \) is the sum of a strictly concave function with respect to \( x \) and a linear function with respect to \( y \), it should be clear that \( F(x, y) \) satisfies condition 2.24 as desired. \( \square \)

The second result is also very simple.

**Proposition 4.** \( \Gamma(x) \) is convex in the sense that for any \( 0 \leq \theta \leq 1 \), and \( x, x' \in X \),

\[
y \in \Gamma(x) \text{ and } y' \in \Gamma(x') \text{ implies } \\
\theta y + (1 - \theta)y' \in \Gamma[\theta x + (1 - \theta)x'].
\]

**Proof.** For the constrained functional equation:

\[
\Gamma(q_N, q_G) = \{(q'_N, q'_G) \in \mathbb{R}_+^2 : q'_N \geq q_N(1 - \delta_N) \text{ and } q'_G \geq q_G(1 - \delta_G)\}.
\]

Since the constraints are linear it is easy to see that \( \Gamma(q_N, q_G) \) is convex as desired. \( \square \)

**Proposition 5.** The function \( v \) is strictly concave and the optimal investment correspondence is single valued.

**Proof.** We have already proved that the function \( F(q_N, q_G, q'_N, q'_G) \) is continuous and bounded (by the maximum consumer surplus for example). Moreover, we also proved
that the function $F(\cdot)$ is strictly concave and that $\Gamma(x)$ is convex. Hence we can apply Theorem 4.8 in page 81 in Stokey and Lucas [16] which proves the result.

2.5.3 Differentiability of the $\nu(\cdot, \cdot)$

We have shown that the constraint problem has a unique solution. Moreover, we also prove that the value function is strictly concave, and that the optimal investment is a single valued function.

The only difference between $\nu$ and $\nu$ is the constraint over the maximization, $\Gamma(q_N, q_G)$ and $\Gamma_\nu(q_N, q_G)$, where $\Gamma_\nu(q_N, q_G) = \mathbb{R}_+^2$ for all $(q_N, q_G)$. $\Gamma_\nu(q_N, q_G)$ clearly is a non-empty continuous correspondence. Hence, $\nu(\cdot, \cdot)$ is continuous, strictly concave and the optimal investment policy is single valued. Thus, we only have to prove that $\nu(\cdot, \cdot)$ is differentiable.

**Proposition 6.** $\nu(\cdot, \cdot)$ is everywhere differentiable with derivatives given by:

$$
\nu_l(q_N, q_G) = F_l[q_N, q_G, q_N^*, q_G^*], \quad l = q_N, q_G
$$

**Proof.** Notice that the optimal investment policy for the unconstrained problem is always in the interior of $\Gamma(q_N, q_G)$ for all $(q_N, q_G)$. Therefore, the differentiability of $\nu(\cdot, \cdot)$ follows from the continuity and differentiability of $F(q_N, q_G, q_N^*, q_G^*)$. Stokey and Lucas [16] using a result from Benveniste and Scheinkman [3] prove the proposition in Theorem 4.11 page 85.

---

Here it should be emphasized that at no point have we checked the conditions under which $q_N^* = 0$ or $q_G^* = 0$. It has been implicitly assumed that the parameters of the model are such that the optimal capacities for the unconstrained problem are always positive.
Chapter 3

Long Term Simulation of Natural Gas Prices at Henry Hub

3.1 Introduction

Uncertainty about the evolution of natural gas prices is critical for deciding whether to invest in natural gas, nuclear or coal generating capacity. The value of our results in the previous two chapters depends on the specification of such uncertainty. Consequently, this chapter undertakes the task of characterizing the uncertainty in natural gas price movements. Specifically, this chapter of the dissertation focuses on characterizing the uncertainty of natural gas prices at Henry Hub.

We assumed that natural gas prices are composed of two independent elements: a long term component and a short term component. We characterize uncertainty about the short term component by analyzing the historical weekly prices at Henry Hub from the last week of 1993 to the last week of 2006. Unfortunately, past information is not enough to completely characterize the uncertainty about future natural gas prices. In particular, uncertainty about the structural elements determining the long term component of natural gas prices is also a significant part of the long run uncertainty. Therefore, we use the long term estimates of natural gas price at Henry Hub from the Energy Information Administration (EIA) together with their scenario projections
for alternative possible paths as a basis for our long term component.

The EIA produces its long term forecast from the National Energy Modeling System (NEMS). The NEMS is a structural model of energy markets that relies on different scenarios that vary structural parameters, such as technology growth, the effect of projected large scale projects, behavioral expectations in oil markets, etc., to provide plausible paths of natural gas prices in the future. The Department of Energy does not assign a probability distribution over the scenarios. Without further information, we decided to assign the same probability to each scenario.

In order to control the maximum natural gas price produced by our simulation, we exogenously placed an upper bound or "choke price" for the demand, on the simulated natural gas prices. We use a double-logarithmic transformation of natural gas prices to impose this limit. Since we do not have good empirical evidence on an appropriate value for the choke price, we analyze three choices: 15, 20 and 25 dollars of 2005.

In order to estimate the short term component of the natural gas price from the historical data, we use the frequency domain characterization of natural gas price movements to separate the range of frequencies into long term and short term components. Since we do not have good empirical evidence to separate the frequencies into the long term and short term components we tried different approximations of the long term and short term components.

This chapter is divided as follows. Section 3.2 analyzes the historical record of natural gas prices and decomposes the price series into its two components. Section 3.3 characterizes uncertainty about the short term component of natural gas prices. Section 3.4 reviews the main aspects of the EIA’s long term forecast for natural gas prices and proposes a characterization for the uncertainty of the long term component of natural gas prices. Section 3.5 analyzes our simulated prices. Finally, Section 3.6 concludes the chapter with some final comments and caveats about our
simulation methodology. Derivations of key analytical results have been relegated to an appendix.

3.2 Decomposing Natural Gas Prices

In this section we review the historical behavior of natural gas prices. We decompose natural gas prices into two components: a long term and a short term component. We illustrate how frequent changes in the long term trend make it difficult to find a suitable model to characterize the uncertainty of natural gas prices. Although modeling natural gas prices as a non-stationary process might work well for short term simulations, it is catastrophic for long term simulations where long term market forces control the variance of the process. Therefore, we decompose prices into a slowly changing mean (taken to reflect long term fundamental factors) with a stationary error around it. We borrow the Hodrick-Prescott (HP) filter, Hodrick and Prescott ([8]), from the business cycle literature to decompose the price into short term and long term components. The HP filter has being successfully used to distinguish factors affecting long term economic growth from business cycles and there is a large literature on its application to macroeconomic variables. We believe that it can be successfully applied to modelling natural gas prices.

3.2.1 Historical Behavior of Natural Gas Prices

We obtained information from Bloomberg(r) on daily nominal natural gas prices at Henry Hub from January 3rd, 1994 to June 29th, 2007. We deflated the prices using the monthly producer price index for all commodities (PPI) published by the Bureau of Labor Statistics. Given the difference in frequency between the PPI series and natural gas prices, we fit a cubic spline to the PPI series to estimate daily deflators
Figure 3.1: Effect of the Double Logarithmic Transformation

for natural gas prices. We constructed weekly prices by averaging the deflated closing prices for each trading day during each week.

We apply a double logarithmic transformation to natural gas prices in order to place an exogenous upper bound on the range of natural gas prices produced by our simulation.\(^1\) Specifically, the data series, \(p_t\), is transformed into \(y_t\) by:

\[
y_t = \log \left( -\log \left( \frac{p_t}{p^*} \right) \right). \tag{3.1}
\]

Where \(p^*\) denotes the choke price, the largest possible price that supports a demand higher than zero. Assuming that \(p^*\) is equal to 10. Figure 3.1 (a) shows the effect on the transformed data. Notice that the transformation has little effect on middle values but significantly alters small and very large prices. Figure 3.1(b) shows the density of the transformed random variable for different values of \(p^*\) when the original random variable is distributed normally with mean 5 and variance 1. As the

\(^1\)Our objective is to develop a methodology for the long term simulation of natural gas prices, without an upper bound and no market forces to bound natural gas prices these can explode, at least for some sample paths. Hence, the need of an exogenous upper bound.
choke price increases, the distribution becomes closer to symmetric with a bell shape. For lower values of the choke price the density becomes a little asymmetric although a general bell shape remains.

Although we believe it is quite reasonable to propose a choke price for natural gas consumption, there is scant information one can use to select a suitable value. We examined the effect of using different values: 15, 20 and 25 in real 2005 dollars.

Figure 3.2 plots the weekly real natural gas prices at Henry Hub and their double logarithmic transformation for choke prices of 15 and 25. From Figure 3.2 (a) it is clear that natural gas prices have experienced periods of large volatility. Moreover, given the large decreases of prices experienced after large increases the process appears to be mean reverting. Figure 3.2 (b) shows the double logarithmic transformation of natural gas prices using choke prices of 25 and 15. An increase in the choke price translates the data and, more importantly, decreases the relative size of the jumps and the volatility of the data.

By visual inspection, the natural gas process appears to be stationary around a
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<th>ADF - Test</th>
<th>PP - Test</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Statistic 1</td>
<td>Statistic 2</td>
</tr>
<tr>
<td>6</td>
<td>-4.1075</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>19</td>
<td>-3.7907</td>
<td>.01946</td>
</tr>
</tbody>
</table>

1 Regression with a constant, trend and lags.
2 Regression with a constant and trend, the Newey-West variance estimator computed with lags included.

Table 3.1: Unit Root Test on the Log (Weekly Real Natural Gas Prices)

changing mean. 2 Table 3.1 formally tests the null hypothesis that natural gas prices are nonstationary. We performed the Augmented Dickey-Fuller tests and the Phillips-Perron Test. Both tests reject the null hypothesis of nonstationarity even though both tests assume a linear time trend.

It is not clear to us whether real natural gas prices have followed a constant linear time trend in the last 13 years. It appears more reasonable to conclude that the trend has slowly changed perhaps as natural gas markets have increased in size and importance. In consequence, assuming a linear time trend would artificially increase the variance of the short term component. Hence, we decided to assume that the long term trend is a non-linear smooth function (although we will keep the linear trend assumption for comparability purposes). In the next subsection we use the Hodrick and Prescott (HP) filter to extract the long term component from the natural gas prices at Henry Hub.

3.2.2 The Hodrick and Prescott Filter

Although the exact relationship of the HP filter of a process to its spectral representation has being derived by King and Rebelo ([10]), and we will exploit their representation to adapt the HP filter for our specific needs, it is simpler to present...

2Theoretically, if the process is stationary, then the double log transformed process would also be stationary. The converse is also true.
the HP filter in its original form. The HP filtered series \( \{g_1, \ldots, g_T\} \) can be derived by solving the following optimization problem:

\[
\min_{\{g_t\}} \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2.
\]  (3.2)

Where \( \{y_1, \ldots, y_T\} \) represents the unfiltered data and \( \lambda \) represents the smoothing parameter of the HP filter. Notice that for large values of \( \lambda \), the filtered series, \( \{g_t\} \), approximates a linear function since then we would have:

\[
\sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 = 0.
\]

In contrast, for a value of the smoothing parameter close to zero, the optimal solution to the program is to set \( g_t = y_t \) and the filtered series is identical to the original series.

Values of \( \lambda > 0 \) allow for a non-linear but smooth behavior of the filtered series.

The business cycle literature has developed standard values of \( \lambda \), that depend on the frequency of the data, to distinguish long term growth from short term cycles. There is no reason to expect the long term component of real natural gas prices to follow the same process as the long term component of macroeconomic variables. Therefore, in order to choose the parameter \( \lambda \) we follow Maravall and Rio’s ([11]) suggestion to look at the effect of the HP filter on the population spectrum of the process in order to choose the right smoothing parameter. Rebelo and King ([10]) derive the time domain representation of the long term component (also called the trend component in the business cycle literature) by using the lag operator representation of the transformation 3.2:

\[
L(B) = \frac{1}{1 + \lambda[1 - B]^2[1 - B^{-1}]^2}.
\]  (3.3)
Where $B$ denotes the backshift or lag operator. The corresponding population spectrum of the filtered series is then given by:

$$s_y(\omega) = \frac{1}{2\pi} L(e^{i\omega})L(e^{-i\omega})\gamma_y(e^{-i\omega}).$$

Where $\gamma_y(.)$ denotes the autocovariance generating function of the covariance-stationary process $y_t$. Notice that $L(z) = L(z^{-1})$ which implies that we can analyze the effect of the HP filtered series by analyzing the real function $L(z)$:

$$L(e^{i\omega}) = \frac{1}{1 + \lambda[1 - e^{i\omega}]^2[1 - e^{-i\omega}]^2}$$

$$= \frac{1}{1 + 4\lambda[1 - \cos(\omega)]^2}$$

(3.4)

Figure 3.3 plots the values of equation 3.4 for different values of $\lambda$. Clearly, as $\lambda$ increases the population spectrum of the filtered series decreases at all frequencies greater than zero. This corresponds to the observation above that a very large value of $\lambda$ effectively makes the transformed series a straight line. Moreover, given the steepness of $L(.)$, we can view the HP filter as an approximation to a band pass filter for low frequencies. Therefore, we can set the smoothing parameter in such a way that frequencies above a certain range have values of $L(.)$ lower than some cut-off value such as .5.

For our application, we decided to assign all frequencies with periods lower than either 3 years (156 weeks) or 5 years (260 weeks) to the short term component. The smoothing parameters, $\lambda$ that are consistent with our rule are 380,099 and 2,932,353 for 3 and 5 years respectively. Figure 3.4 shows double logarithmic transformed data and the HP filtered long term component for the selected smoothing parameter values and choke prices of 15, 20 and 25.

It is clear from Figure 3.4 that the variance of the short term component increases
Figure 3.3: The effect of the smoothing parameter in the HP filter.

with the smoothing parameter. In the next subsection we will analyze and compare the short term component derived under the 3 and 5 years rules and linear long term component.

3.3 The Uncertainty of the Short Term Component

In this section we characterize, analyze and compare the stochastic structure of the estimated short term components derived in the previous section. Our proposed structure will serve as a building block for the long term simulation of natural gas prices.
Figure 3.4: Estimated Long Term Components: HP Filter
3.3.1 A comparison analysis of the estimated short term component

Figure 3.5 plots the estimated short term components using the HP filter for 3 and 5 years and the linear trend models for the double logarithm transformed data using a choke price of 20. The figure also shows the kernel density estimate for each short term component. We highlight two main aspects of the estimated short term components that are present for all three cases: the persistence of the short term component and the probability of extreme values. Notice that both characteristics are stronger as the smoothing parameter increases (with a linear trend being the limiting case).

Two explanations for high probabilities of extreme values are usually mentioned: (1) "fat tails" for the distribution of the error of the short term component; or (2) conditional heteroskedasticity. We believe that it makes more sense to account for conditional heteroskedasticity, which has a readily understood explanation in terms of the underlying fundamentals, before resorting to a non-standard distributional assumption, which does not admit so readily to such an explanation. Specifically, we use the Autoregressive Conditional Heteroskedastic family or models introduced by Engel ([6]) to model the possibility of conditional heteroskedasticity. In our application, we use the Exponential ARCH model developed by Nelson ([13]) to characterize the uncertainty of the short term component of weekly natural gas prices at Henry Hub. The EGARCH specification relaxes some of the more severe constraints of a standard ARCH, including the non-negativity of the estimated parameters. Finally, given that we used the HP filter consistent with 3 and 5 years to characterize the long term component of natural gas prices, monthly seasonality can still be important. Therefore, we include monthly seasonality in our specification of the EGARCH component.

The EGARCH($r$, $m$) model with an ARMA($p$, $q$) for the mean equation can be
Figure 3.5: Comparison of the estimated short term component
summarized as follows:

\[
y_t = bx_t + \rho_1(y_{t-1} - bx_{t-1}) + \ldots + \rho_p(y_{t-p} - bx_{t-p}) + \\
\epsilon_t + \phi_1\epsilon_{t-1} + \ldots + \phi_q\epsilon_{t-q},
\]

\[
\epsilon_t = \sqrt{\sigma_t^2}z_t,
\]

\[
\ln(\sigma_t^2) = \kappa + \beta x_t + \delta_1\ln(\sigma_{t-1}^2) + \delta_2\ln(\sigma_{t-2}^2) + \ldots + \delta_r\ln(\sigma_{t-r}^2) + \\
\theta_1z_{t-1} + \gamma_1(|z_{t-1}| - \sqrt{2/\pi}) + \theta_2z_{t-2} + \gamma_2(|z_{t-2}| - \sqrt{2/\pi}) + \ldots \\
+ \theta_{t-m}z_{t-m} + \gamma_{t-m}(|z_{t-m}| - \sqrt{2/\pi}).
\]

Here \( z_t \) is an independently identically distributed standard normal, \( y_t \) denotes the short term component of the double logarithm transformed real natural gas price and \( x_t \) is the vector of seasonal monthly dummies which are included in both the mean and the variance components of the equation. Specifically, we might expect real natural gas prices to differ systematically by season and also to be more variable in some seasons than in others.

### 3.3.2 Estimation of the stochastic model

We estimate the EGARCH-ARMA model using Maximum Likelihood. Since the model assumes \( z_t \) to be an independently and identically distributed standard normal, we construct the log-likelihood function from the normality assumption. We later test for the validity of the estimated model by examining the distribution of the estimated residuals. Since the main purpose of the exercise is to produce a long term simulation of future real natural gas prices, we wanted to avoid the risk of overfitting the data, and hence looked for a model as parsimonious as possible. After examining a number of alternatives, we settled for a EGARCH(1,1)×ARMA(1,1) specification with seasonal parameters for the mean and the variance equation.
We ran into convergence problems when estimating the monthly seasonal dummies. The problems ranged from not being able to maximize the likelihood function to not being able to include the seasonal dummies in the mean equation. It is important to mention that all models with the seasonality in the variance component produced residuals that satisfied the iid assumption. The main problem was that when seasonal dummies were included in the mean equation, these were significant and, unfortunately, when all the dummies were included, the maximization routine would not converge. However, we were able to estimate the seasonal parameters for the mean and the variance equation for transformations using a choke price of either 20 or 25 and where we estimate the long term component using the HP filter consistent with 5 years. After estimating the model, we conducted tests for consistency with our assumptions. In particular, we tested for autocorrelation of the standardized residuals and their squared values. We also tested the symmetry of the distribution of the standardized residuals and whether their mean was equal to zero and their variance to one. The selected specification passed all bar the symmetry test. Apparently, the left tail of the sample distribution of standardized residuals is heavier than a normal distribution, implying some misspecification of the short term component. The EGARCH specification performed better for a higher value of the choke price.

The estimation results are presented in Table 3.2. It is important to note that, as we previously mentioned, for some models we were not able to estimate the model with the seasonal dummies included in the mean equation. Moreover, the seasonal dummies in the variance equation turn out to be extremely important in making the standardized residuals independent through time. In our opinion, the fact that we have difficulties maximizing the likelihood function implies that our model might not fit every transformation equally well.

Only for the transformation using a choke price of 20 and 25 and using the HP
<table>
<thead>
<tr>
<th>Parameter</th>
<th>HP: 3 Years</th>
<th>HP: 5 Years</th>
<th>Linear Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>SE</td>
<td>Est.</td>
</tr>
<tr>
<td><strong>Choke Price 15</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_1 - \beta_{12}^{1}$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$\beta_2 - \beta_{12}^{1,2}$</td>
<td>178.83</td>
<td>&lt; .0001</td>
<td>193.17</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.91162</td>
<td>0.00964</td>
<td>0.93051</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.22460</td>
<td>0.02738</td>
<td>0.23213</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-2.4353</td>
<td>0.03436</td>
<td>-0.26310</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.47288</td>
<td>0.05393</td>
<td>0.42816</td>
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<tr>
<td>$\delta_1$</td>
<td>0.95109</td>
<td>0.01085</td>
<td>0.95373</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>-6.3536</td>
<td>0.05859</td>
<td>0.60562</td>
</tr>
<tr>
<td>Log-Like</td>
<td>891.685</td>
<td>890.644</td>
<td>887.355</td>
</tr>
<tr>
<td><strong>Choke Price 20</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_1 - \beta_{12}^{1}$</td>
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<td>N/A</td>
<td>27.86</td>
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<tr>
<td>$\beta_2 - \beta_{12}^{1,2}$</td>
<td>89.51</td>
<td>&lt; .0001</td>
<td>99.76</td>
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<tr>
<td>$\rho_1$</td>
<td>0.89989</td>
<td>0.01061</td>
<td>0.93647</td>
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<tr>
<td>$\phi_1$</td>
<td>0.25626</td>
<td>0.02621</td>
<td>0.22429</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-2.2189</td>
<td>0.03550</td>
<td>-0.24695</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-4.2858</td>
<td>0.05599</td>
<td>-0.38435</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.93495</td>
<td>0.01408</td>
<td>0.94311</td>
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<tr>
<td>$\kappa$</td>
<td>-6.4134</td>
<td>0.11549</td>
<td>-0.59520</td>
</tr>
<tr>
<td>Log-Like</td>
<td>1102.13</td>
<td>1112.855</td>
<td>1094.338</td>
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<td><strong>Choke Price 25</strong></td>
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<td>$b_1 - \beta_{12}^{1}$</td>
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<td>N/A</td>
<td>28.59</td>
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<td>$\beta_2 - \beta_{12}^{1,2}$</td>
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<td>$\rho_1$</td>
<td>0.89675</td>
<td>0.01762</td>
<td>0.93109</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.26342</td>
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<td>$\beta_1$</td>
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<td>-0.23263</td>
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<tr>
<td>$\gamma_1$</td>
<td>0.41161</td>
<td>0.08418</td>
<td>0.37519</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.92694</td>
<td>0.01894</td>
<td>0.93643</td>
</tr>
<tr>
<td>$\kappa$</td>
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<td>-0.61642</td>
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<tr>
<td>Log-Like</td>
<td>1217.35</td>
<td>1227.486</td>
<td>1209.097</td>
</tr>
</tbody>
</table>

1 LR test on every parameter being jointly equal to zero, p-val reported instead of Std. Err.
2 LR test does not include the seasonal dummies in the mean equation due to problems in MLE.
3 The model was estimated with a constant due to convergence problems in MLE.

Table 3.2: Estimates of the short term stochastic model
Figure 3.6: Seasonal Adjustments for the Mean and Variance: Two Year Cycle

filter for 5 years (transformations CH20HP5 and CH25HP5) were we able to estimate every parameter of the model.

Before continuing with our analysis of the seasonality of the variance and the mean levels of natural gas prices, we evaluate our assumption on the distribution of the standarized residuals based on their estimates. We present t-statistics on the moment conditions consistent with our statistical model. Specifically, under the model, the standardized residuals, $z_t$, are distributed with mean 0 and variance 1. Moreover, $z_t$ and $|z_t|$ shouldn't be correlated and $z_t$ and $z_t^2$ shouldn't be autocorrelated. Tables 3.3-3.5 present the t-statistic for each of the moment conditions using the estimated standardized residuals from the three models with three different choke prices.

Notice that the residuals are very well behaved for all combinations of transformations and filters. Therefore, as already mentioned, we will keep the CH20HP5 and CH25HP5 transformations for the long term simulation of natural gas prices.

For these two selected transformations, Figure 3.6 shows the estimated seasonality of the mean and variance components.

The seasonality in the mean levels is highly correlated with the pattern of under-
**Choke Price: 15**

<table>
<thead>
<tr>
<th>Moment</th>
<th>HP: 3 Years</th>
<th>Est.</th>
<th>t-Stat.</th>
<th>HP: 5 Years</th>
<th>Est.</th>
<th>t-Stat.</th>
<th>Linear</th>
<th>Est.</th>
<th>t-Stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[z_t]$</td>
<td>0.00008</td>
<td>0.00216</td>
<td>-0.00102</td>
<td>-0.02713</td>
<td>-0.00170</td>
<td>-0.04495</td>
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</tr>
<tr>
<td>$E[z_t^2 - 1]$</td>
<td>-0.00009</td>
<td>-0.00088</td>
<td>0.00130</td>
<td>0.01319</td>
<td>0.00161</td>
<td>0.01646</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[</td>
<td>z_t</td>
<td>- \sqrt{2/\pi}]$</td>
<td>-0.08899*</td>
<td>-3.34311</td>
<td>-0.07519*</td>
<td>-2.87828</td>
<td>-0.07037*</td>
<td>-2.71278*</td>
<td></td>
</tr>
<tr>
<td>$E[z_t(</td>
<td>z_t</td>
<td>- \sqrt{2/\pi})]$</td>
<td>-0.14478</td>
<td>-1.72469</td>
<td>-0.14535</td>
<td>-1.74863</td>
<td>-0.14306</td>
<td>-1.72962</td>
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</tr>
<tr>
<td>$E[z_t z_{t-1}]$</td>
<td>0.03118</td>
<td>0.87042</td>
<td>0.01076</td>
<td>0.29689</td>
<td>0.00480</td>
<td>0.13175</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$E[z_t z_{t-2}]$</td>
<td>-0.00358</td>
<td>-0.09957</td>
<td>0.00263</td>
<td>0.07307</td>
<td>0.00336</td>
<td>0.09335</td>
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<tr>
<td>$E[z_t z_{t-3}]$</td>
<td>0.04428</td>
<td>1.16882</td>
<td>0.04423</td>
<td>1.20721</td>
<td>0.04463</td>
<td>1.22707</td>
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</tr>
<tr>
<td>$E[z_t z_{t-4}]$</td>
<td>0.00500</td>
<td>0.12789</td>
<td>-0.00049</td>
<td>-0.01258</td>
<td>-0.00257</td>
<td>-0.06552</td>
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</tr>
<tr>
<td>$E[z_t z_{t-5}]$</td>
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<td>-0.00160</td>
<td>0.00280</td>
<td>0.07314</td>
<td>0.00258</td>
<td>0.06819</td>
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<td></td>
</tr>
<tr>
<td>$E[(z_t^2 - 1)(z_{t-1}^2 - 1)]$</td>
<td>-0.09675</td>
<td>-0.85354</td>
<td>-0.07818</td>
<td>-0.71657</td>
<td>-0.07001</td>
<td>-0.65363</td>
<td></td>
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</tr>
<tr>
<td>$E[(z_t^2 - 1)(z_{t-2}^2 - 1)]$</td>
<td>-0.09551</td>
<td>-0.41107</td>
<td>-0.09782</td>
<td>-0.44882</td>
<td>-0.09312</td>
<td>-0.43247</td>
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<tr>
<td>$E[(z_t^2 - 1)(z_{t-3}^2 - 1)]$</td>
<td>0.00833</td>
<td>0.04012</td>
<td>-0.05643</td>
<td>-0.37613</td>
<td>-0.06993</td>
<td>-0.51041</td>
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<tr>
<td>$E[(z_t^2 - 1)(z_{t-4}^2 - 1)]$</td>
<td>0.07082</td>
<td>0.34155</td>
<td>0.07876</td>
<td>0.44986</td>
<td>0.07864</td>
<td>0.47043</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$E[(z_t^2 - 1)(z_{t-5}^2 - 1)]$</td>
<td>0.09038</td>
<td>0.41260</td>
<td>0.02578</td>
<td>0.15149</td>
<td>0.00118</td>
<td>0.00766</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Null Hypothesis rejected at 90% confidence interval

Table 3.3: Moment t-test for difference to zero
Choke Price: 20

<table>
<thead>
<tr>
<th>Moment</th>
<th>HP: 3 Years</th>
<th></th>
<th></th>
<th>HP: 5 Years</th>
<th></th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[z_i]$</td>
<td>-0.00140</td>
<td>-0.03714</td>
<td>0.00301</td>
<td>0.07968</td>
<td>0.00217</td>
<td>0.05748</td>
</tr>
<tr>
<td>$E[z_i^2 - 1]$</td>
<td>-0.00081</td>
<td>-0.00821</td>
<td>0.00100</td>
<td>0.01045</td>
<td>0.00195</td>
<td>0.02063</td>
</tr>
<tr>
<td>$E[</td>
<td>z_i</td>
<td>- \sqrt{2/\pi}]$</td>
<td><strong>-0.08414</strong>*</td>
<td><strong>-3.18562</strong>*</td>
<td><strong>-0.06432</strong>*</td>
<td><strong>-2.50495</strong>*</td>
</tr>
<tr>
<td>$E[z_i({z_i</td>
<td>- \sqrt{2/\pi}})]$</td>
<td>-0.13488</td>
<td>-1.61387</td>
<td>-0.13026</td>
<td>-1.61705</td>
<td>-0.12711</td>
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<tr>
<td>$E[z_i z_{t-1}]$</td>
<td>0.01914</td>
<td>0.52432</td>
<td>-0.00658</td>
<td>-0.18334</td>
<td>-0.00801</td>
<td>-0.22297</td>
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<tr>
<td>$E[z_i z_{t-2}]$</td>
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<td>-0.01545</td>
<td>0.00597</td>
<td>0.16982</td>
<td>0.01048</td>
<td>0.29768</td>
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<tr>
<td>$E[z_i z_{t-3}]$</td>
<td>0.04208</td>
<td>1.14036</td>
<td>0.03657</td>
<td>1.01549</td>
<td>0.04011</td>
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<td>$E[z_i z_{t-4}]$</td>
<td>0.00056</td>
<td>0.01438</td>
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<tr>
<td>$E[z_i z_{t-5}]$</td>
<td>0.00864</td>
<td>0.22244</td>
<td>0.00198</td>
<td>0.05304</td>
<td>0.00441</td>
<td>0.11965</td>
</tr>
<tr>
<td>$E[(z_i^2 - 1)(z_{t-1}^2 - 1)]$</td>
<td>-0.0951</td>
<td>-0.52329</td>
<td>-0.09306</td>
<td>-0.87252</td>
<td>-0.09209</td>
<td>-0.87404</td>
</tr>
<tr>
<td>$E[(z_i^2 - 1)(z_{t-2}^2 - 1)]$</td>
<td>-0.11845</td>
<td>-0.53525</td>
<td>-0.13363</td>
<td>-0.75878</td>
<td>-0.13116</td>
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<tr>
<td>$E[(z_i^2 - 1)(z_{t-3}^2 - 1)]$</td>
<td>-0.04110</td>
<td>-0.21718</td>
<td>-0.08691</td>
<td>-0.66057</td>
<td>-0.08030</td>
<td>-0.66102</td>
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<tr>
<td>$E[(z_i^2 - 1)(z_{t-4}^2 - 1)]$</td>
<td>0.04995</td>
<td>0.26730</td>
<td>0.12374</td>
<td>0.70087</td>
<td>0.13097</td>
<td>0.78198</td>
</tr>
<tr>
<td>$E[(z_i^2 - 1)(z_{t-5}^2 - 1)]$</td>
<td>0.05683</td>
<td>0.28512</td>
<td>-0.02537</td>
<td>-0.17558</td>
<td>-0.05048</td>
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</table>

* Null Hypothesis rejected at 95% confidence interval

Table 3.4: Moment t-test for difference to zero
## Choke Price: 25

<table>
<thead>
<tr>
<th>Moment</th>
<th>HP: 3 Years</th>
<th></th>
<th></th>
<th>HP: 5 Years</th>
<th></th>
<th></th>
<th>Linear</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[z_t]$</td>
<td>-0.00817</td>
<td>-0.21617</td>
<td>-0.00661</td>
<td>-0.17468</td>
<td>0.03339</td>
<td>0.88357</td>
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</tr>
<tr>
<td>$E[z_t^2 - 1]$</td>
<td>0.00231</td>
<td>0.02334</td>
<td>0.00672</td>
<td>0.06846</td>
<td>0.00380</td>
<td>0.03978</td>
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<td></td>
</tr>
<tr>
<td>$E[</td>
<td>z_t</td>
<td>- \sqrt{2/\pi}]$</td>
<td>-0.08398*</td>
<td>-3.16995</td>
<td>-0.06963*</td>
<td>-2.67309</td>
<td>-0.06278*</td>
<td>-2.44353</td>
</tr>
<tr>
<td>$E[z_t(</td>
<td>z_t</td>
<td>- \sqrt{2/\pi})]$</td>
<td>-0.13384</td>
<td>-1.60190</td>
<td>-0.13397</td>
<td>-1.61376</td>
<td>-0.10242</td>
<td>-1.26842</td>
</tr>
<tr>
<td>$E[z_t z_{t-1}]$</td>
<td>0.00710</td>
<td>0.19404</td>
<td>-0.00792</td>
<td>-0.21619</td>
<td>-0.01136</td>
<td>-0.30951</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[z_t z_{t-2}]$</td>
<td>-0.00685</td>
<td>-0.19428</td>
<td>-0.00017</td>
<td>-0.00471</td>
<td>-0.00281</td>
<td>-0.07782</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[z_t z_{t-3}]$</td>
<td>0.03028</td>
<td>0.82999</td>
<td>0.03067</td>
<td>0.85941</td>
<td>0.03090</td>
<td>0.86399</td>
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</tr>
<tr>
<td>$E[z_t z_{t-4}]$</td>
<td>-0.00714</td>
<td>-0.18611</td>
<td>-0.00817</td>
<td>-0.21067</td>
<td>-0.01260</td>
<td>-0.32970</td>
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<tr>
<td>$E[z_t z_{t-5}]$</td>
<td>0.00948</td>
<td>0.24604</td>
<td>0.01299</td>
<td>0.34431</td>
<td>0.01227</td>
<td>0.32859</td>
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<td></td>
</tr>
<tr>
<td>$E[(z_t^2 - 1)(z_{t-1}^2 - 1)]$</td>
<td>-0.05405</td>
<td>-0.45390</td>
<td>-0.05600</td>
<td>-0.48969</td>
<td>-0.04974</td>
<td>-0.44528</td>
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<tr>
<td>$E[(z_t^2 - 1)(z_{t-2}^2 - 1)]$</td>
<td>-0.12293</td>
<td>-0.55790</td>
<td>-0.11517</td>
<td>-0.55094</td>
<td>-0.08526</td>
<td>-0.40057</td>
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<tr>
<td>$E[(z_t^2 - 1)(z_{t-3}^2 - 1)]$</td>
<td>-0.06153</td>
<td>-0.33930</td>
<td>-0.10516</td>
<td>-0.77247</td>
<td>-0.09800</td>
<td>-0.78643</td>
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<tr>
<td>$E[(z_t^2 - 1)(z_{t-4}^2 - 1)]$</td>
<td>0.03352</td>
<td>0.19029</td>
<td>0.05243</td>
<td>0.33333</td>
<td>0.02620</td>
<td>0.18311</td>
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</tr>
<tr>
<td>$E[(z_t^2 - 1)(z_{t-5}^2 - 1)]$</td>
<td>0.03974</td>
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<td>-0.00589</td>
<td>-0.03899</td>
<td>-0.02100</td>
<td>-0.15703</td>
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<td></td>
</tr>
</tbody>
</table>

* Null Hypothesis rejected at 95% confidence interval

Table 3.5: Moment t-test for difference to zero
ground storage. Underground storage is usually filled in the first days of December, which is usually neither the coldest month nor the month with highest heating demand. Both, the storage and the weather effects tend to reduce average prices. During the coldest times, the end of January and beginning of February uncertainty around the availability of natural gas is at its highest. Finally, in March when the spring starts and the cooling season has not yet started, average natural gas prices and their volatility decrease.

Another advantage of the EGARCH specification is the identification of an asymmetric effect of news on the volatility of gas prices. Figure 3.7 shows the effect of news on volatility for the estimated short term components for both transformations. Clearly, the effect of positive news (which reduces the price of natural gas, \( z > 0 \) in the double logarithmic transformed prices) is much smaller than the effect of negative news (which increases the price of natural gas, \( z < 0 \) in the double logarithmic transformed prices). Finally, notice that changes in volatility are relatively persistent since estimated values of \( \delta_1 \) are positive and relatively large: .94 and .93 for the CH20HP5 and CH25HP5 transformations respectively.
Finally, we test the normality assumption used in the estimation stage of the model. Notice that the rejection of the symmetry moment test for all specifications presented above already provides some preliminary evidence against the normality assumption, although this alone shouldn’t be an impediment for using the normal distribution in the simulation stage of the problem. Figure 3.8 produces Q-Q plots to compare the distribution of the standardized residuals of the selected transformations against a normal distribution. Clearly, both transformations have larger than normal left tails (corresponding to high prices), although the right tail is consistent with a normal distribution even at 99% probability. With respect to the left tail, the fat tail becomes significant at the 95% probability, although the deviation from normality is exponential thereafter. When compared to each other, it is clear that lower choke prices imply heavier left fat tails.

The conclusion that the estimated residuals have heavier left tails than the normal distribution is particularly troublesome from the simulation point of view. In order to explore the effect of heavier than normal tails in the long term simulation of natural gas prices, we will draw from a standard normal distribution but also bootstrap the estimated residuals to generate the simulated standard residuals. The latter procedure will allow some values to be drawn from the left tail as seen in the data.

3.3.3 Concluding comments

In this section we analyzed the structure of the estimated short term components of HHNG prices. The short term component was itself obtained using the HP filter and extracting fluctuations shorter than 3 years or 5 years, or using a straight line to define the long term component. Two main aspects were present in the three series of estimated short term components: conditional heteroskedasticity and the persistence of shocks. We estimated an EGARCH-ARMA specification for each of the
Figure 3.8: Normal Q-Q plots of the standardized residuals.
nine combinations of transformed prices and definition of the short term component. We found that using the HP filter consistent with a short term component of 5 years and transformed data using a choke price of 20 and 25 dollars of 2005 provided the most satisfactory model of the data. The estimated standardized residuals were basically white noise, although we reject the null hypothesis that their distribution was symmetrical using a simple t-test. We estimated and analyzed a seasonal component in the mean and variance of the short term deviations. Given the large response of the conditional variance to negative news, and non-symmetry of the standardized residuals, there was an obvious need to accurately describe the left tail in particular. Unfortunately, the sample distribution of the standardized residuals has heavier left tails than a normal distribution. Therefore for the simulation stage of our study we will simulate the short term deviations by bootstrapping the estimated standardized residuals in addition to generating them from a standard normal distribution.

3.4 Characterizing the Long Term Component

As we previously mentioned, we are convinced that time series models alone are inadequate for making a long term simulation of natural gas prices. We decided instead to use the Energy Information Agency's (EIA) model-based long term estimates for natural gas prices. The EIA's long term estimates for natural gas prices at Henry Hub are obtained from a non-stochastic general equilibrium model (National Energy Modeling System, NEMS), where different scenarios are solved and analyzed. Specifically, the EIA's Energy Outlook for 2007 includes three scenarios with respect to natural gas prices: the reference case, the low price case and the high price case.

Since the NEMS does not model the stochastic elements in natural gas markets, neither long term nor short term uncertainty were included in the model. We have instead turned to past data to capture the stochastic elements. The characterization
of short term uncertainty was the subject of the last section. In this section, we focus on the characterization of long term uncertainty of real natural gas prices.

We exploit the difference of the long term natural gas prices at Henry Hub under the different EIA scenarios to create a support for a probability distribution of the possible paths followed by the trend in natural gas prices. Unfortunately, the EIA does not assess the likelihood of each of their scenarios. We decided to follow a conservative (and also tractable) route by assuming that each of the paths is equally likely to occur. Technically, we assign a uniform distribution over the range of paths outlined by each of the possible EIA scenarios.

3.4.1 The EIA's long term natural gas prices at Henry Hub forecast

The 2007 Annual Energy Outlook (AEO) long term estimates for the energy sector in the United States issued by the Energy Information Administration (EIA) in the Department of Energy are derived from the National Energy Modelling System (NEMS). The NEMS covers the interaction and the determination of many variables that do not play a central part in our analysis such as the quantities and prices of oil, residual oil, electricity, and oil derived products. We will focus on the analysis and discussion of the EIA's long term forecast for natural gas prices alone. For a complete description of the assumptions behind the long term forecast of the EIA, the reader can consult the Annual Energy Outlook (2007), which can be freely downloaded. Moreover, the specifics of the NEMS model are also available over the internet.

It is important to note that the EIA highlights uncertainty about long term projections for world prices. Therefore, the AEO 2007 shows different scenarios for natural gas prices. We will use three scenarios simulated by the EIA: the reference case, the low price scenario and the high price scenario. We choose these scenarios instead of
the economic scenarios since they provide the largest range of possible price paths. The three scenarios are somewhat different with respect to their projections of natural gas prices. For 2030 there is a 27% difference between the high price scenario and the reference case; and there is a -15% difference between the low price scenario and the reference case. Over the entire sample paths, the maximum percentage differences are 27% and -20% for the high and low price scenario respectively.

For the natural gas price projections, the AEO takes into account the complex relationships between the cost of exploration, production and imports of natural gas. In particular, the AEO expects the price of exploration and production to fall from current levels as the supply of these services increases in response to the recent growth in their prices. Moreover, the AEO projects that an Alaska natural gas pipeline will go into operation in 2018. Even if that project does not eventuate, unconventional gas sources and LNG imports are expected to cope with the demand. The AEO also expects a decrease in natural gas consumption from the electric sector as natural gas fired power plants are replaced by coal fired power plants; although the decrease is offset by an increase in demand from the residential, commercial and industrial sectors. It is important to note that the AEO pays special attention to the price elasticity of natural gas for electricity generation.

The difference between the high and low price cases results from variability of the resource size and technology progress assumptions. We quote the EIA's comments on the difference between the price scenarios:

"...The low price case assumes greater world crude oil and natural gas resources that are less expensive to produce and a future market where all oil and natural gas production becomes more competitive and plentiful than the reference case. The high price case assumes that world crude oil and natural gas resources, including OPEC's, are lower and require greater cost to produce than assumed in the reference case."

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Figure 3.9: AEO:2007 Henry Hub Natural Gas Forecasted Price by Scenario

It is important to note that the EIA recognizes that the price scenarios do not bound the range of possible price paths. Unfortunately, however, any information above and below the price scenarios is not available.

Figure 3.9 shows the forecasted price of natural gas at Henry Hub for 2007 to 2030 under the three scenarios.

In the next subsection we propose to characterize uncertainty in the long term component of natural gas prices by drawing randomly from a uniform distribution over the range of possible prices forecasted by the EIA.

3.4.2 Uncertainty in the long term component of natural gas prices

The EIA acknowledges the uncertainty and complexity inherent in making a long term forecast of natural gas prices. Therefore, it would be a great coincidence if the realized long term component of natural gas prices coincided with one of the simulated EIA scenarios. Moreover, given that the short term component is stationary, if we only

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sample between three scenarios for the long term component of natural gas prices, the
distribution of prices at each forecasted year will be trimodal. Therefore, we decided
to generate a continuum of intermediate price paths by interpolating the yearly change
of the three scenarios performed by the EIA as follows. For each $t = 2007, \ldots, 2030$
and for each $\alpha \in [0, 1]$ let:

$$p_{t}(\alpha) = \begin{cases} 
p_{t-1}(\alpha) + 2[(\alpha - .5)\Delta p^{H} + (1 - \alpha)\Delta p^{R}] & \text{if } \alpha > .5 \\
p_{t-1}(\alpha) + 2[\alpha\Delta p^{R} + (.5 - \alpha)\Delta p^{L}] & \text{if } \alpha \leq .5.
\end{cases} \quad (3.8)$$

Where $p^{H}$, $p^{L}$, and $p^{R}$ denote the price paths for the high price, low price and reference
scenario respectively.

It is important to note that $\alpha$ cannot depend on $t$ since the scenarios are generated
under constant assumptions through the sample path. Therefore, we don't have in-
formation on the price growth if there is a change in the assumptions. On the positive
side, we would expect our short term component to pick up small changes in the long
term assumptions that characterize the long term component of natural gas prices.
Another tempting mistake would be to include the "economic growth" scenarios by
interpolation. Unfortunately, this is incorrect since the path of prices depends on the
complex relation of technological change and the availability of resources between oil
and natural gas. Figure 3.10 shows the generated price paths for $\alpha = .2, .4, .6, .8$.

Once the yearly prices have being obtained for selected values of $\alpha$, the next step is
to obtain monthly prices. Notice that our monthly long term price forecast shouldn't
include seasonality since this is part of the characterization of the short term devia-
tions. A common technique to generate monthly prices is to interpolate the points,
either linearly or with a cubic spline. Unfortunately, the problem is slightly more
complex since we are provided with yearly averages rather than monthly averages
with yearly frequency. Appendix 3.7 shows how to pick cubic spline interpolations
that are as smooth as possible (technically, have minimum variations) but which otherwise solve the averaging problem. Figure 3.11 shows the interpolated reference, high and low price scenarios.

3.4.3 Summary of the strategy to simulate the long term component of HHNG prices

Our proposed strategy can be summarized as follows:

1. We randomly draw the parameter $\alpha \in [0, 1]$ from a uniform distribution.

2. By interpolating the high price, low price and the reference case using Equation 3.8 we compute the average annual long term prices.

3. For each $\alpha$ we interpolate the annual forecast by choosing a continuous cubic spline with minimum variation.

The proposed strategy implies that the long term component is a continuous non-linear smooth function as assumed by the HP filter. In the next section we briefly
Figure 3.11: Minimum variation averaging cubic-spline monthly interpolation for selected scenarios

discuss the simulation of the short term component and hence the simulation of HHNG prices.

3.5 The Simulated Natural Gas Prices

In this section we simulate and analyze HHNG prices from 2007 to 2030 by adding the simulated short term component of Section 3.3 to the long term component of Section 3.4. Then, we analyze the simulated HHNG prices and compare the results using the two proposed specifications (CH25HP5 and CH20HP5).

3.5.1 Simulation Strategy

We perform 10,000 simulations of the natural gas price paths. Each price path is from January of 2007 to December of 2030 and the frequency of the simulated prices is monthly. We will focus on two aspects: (1) convergence of the simulated natural gas prices and (2) comparison of the simulated prices between the two selected
transformations: CH20HP5 and CH25HP5.

As our analysis of the short term component revealed, the distribution of the standardized residuals was not exactly normal. Specifically, we reject the symmetry moment test. Therefore, we decided to simulate the short term component using two techniques: we model the standardized residuals as a standard normal or we bootstrap using the estimate sample values. The simulations were initialized using the estimated residuals and variance from their respective short term component in December of 2006. The simulations of the short term component are done weekly and then they are averaged every four weeks to produce monthly simulations that can be used with the long term component. First, we analyze the simulations using the short term component estimated from the CH20HP5 transformation, then we analyze the simulation of the CH25HP5 transformation and finally we will compare them.

Besides analyzing the densities of the simulation through time, we will take special care to examine the maximums of the distribution. Given the large span of time over which prices are simulated together with the high autocorrelation of the short term component and its variance (higher than .9 both of them in both transformation), it is very likely that every price path simulated will contain some large outlying values.

**Simulation Results of the Long Term Component**

Figure 3.12 presents the kernel density estimate of the simulated prices for December 2030. We choose December of 2030 since it is the month with the highest distance between the high price and low price scenario.

Notice that the distribution for December 2030 of the average long term component is independent of whether we use CH20HP5 or CH25HP5 for the model of the short term component. This is not very surprising since the transformation only affects the forecasted prices by changing the initial price (the price in December of
2006). Moreover the shape of the densities is very easy to explain. In our formula to simulate the long term components the distance between the EIA low price scenario and the reference case is different than the distance between the high price scenario and the reference case. Therefore, our sampling is equivalent to sampling from two uniform distributions with different lengths with the same probability. It seems that our modified cubic spline interpolation does not significantly change the shape of the distribution.

Simulation using the CH20HP5 specification

We start with the case where we assume that the standardized error of the short term component is distributed as standard Normal. The left panel of Figure 3.13 presents evolution of the selected percentiles through time for the monthly simulated prices. Notice that the expected value shows a small effect of seasonality, in contrast to the tails of the distribution where the effect of seasonality is much larger. Moreover, the effect of seasonality in the tails is asymmetric: larger for high prices and smaller for
Figure 3.13: Selected percentiles and standard deviation through time: CH20HP5

low prices; this is a consequence of the asymmetric effect of news about variance. Another important feature is the extreme increase in the variance in the first two years. By 2008 the range of values is very similar to the range in subsequent years. The right panel of Figure 3.13 clarifies this aspect by presenting the evolution of the standard deviation with respect to time. Notice that the seasonality of the standard deviation is relatively large leading to a maximum difference of around 1.25 dollars.

The second aspect is also very clear: by 2008 we reach the maximum of the standard deviation.

Given the behavior of the standard deviation through time, it is not surprising that the histogram of the maximums per simulation follows the expected mean. The left panel of Figure 3.14 presents the histogram of the maximums by year while the right panel shows the box-plot of the size of maximums by year.

The majority of the maximums are within 13 to 17 2005 USD. Clearly, at each period there is a possibility to reach the maximum choke price of 20 dollars with the exception of the first year where the variance is considerably smaller and the chances of reaching 20 dollars are much lower.
Figure 3.14: Analysis of Price Hikes Per Year

The results using bootstrapped residuals are slightly different. Figure 3.15 (left panel) presents selected percentiles of the price distribution through time while Figure 3.15(right panel) presents the evolution of the standard deviation. Again, seasonality is evident in both the level and the standard deviation. Moreover, as in the case of the simulation using standard normal errors, the standard deviation increases relatively fast.

Notice that the simulation using bootstrapped errors has a lower variance than that using normal standard errors. This is a consequence of the fact that the standardized errors are not distributed exactly normal as shown in the Q-Q plots but rather they have fat tails.

The distribution of the maximums is very similar to that of the normal errors but the range of the common maximums is larger. The left panel of Figure 3.16 shows the box-plot of the maximums by year respectively and the right panel shows the histogram of the time of the maximum.

Although the variability of the simulated natural gas prices is lower when bootstrapping the errors, the variability of the maximums is higher.
Figure 3.15: Selected percentiles and standard deviation through time: Bootstrapped Errors

Figure 3.16: Analysis of Price Hikes Per Year: CH20HP5
Simulation using the CH25HP5 transformation

We again begin by analyzing the simulation using normally distributed errors. In Figure 3.17, the left panel and right panel show the evolution of selected percentiles and the standard deviation over time, respectively.

Again the simulations show our estimated seasonality effects in both the mean and the standard deviation. Notice that compared to the normal simulation using the CH20HP5 transformation, the 99th and the 1st percentile are very similar. Moreover, the variance is not very different between the simulations providing some preliminary evidence that our simulation strategy is relatively robust to the selected choke price. Not surprisingly however, the selection of the choke price does affect the distribution of the maximums. Figure 3.18 presents the histogram of the location of the maximums per simulation and the boxplot of the maximums by time respectively.

Notice that the range of the maximums and the histograms are similar to those of the transformation with a choke price of 20. The difference is that the large outliers above 20 dollars that didn’t exist in the CH20HP5 transformation.

Finally, we analyze the simulation results by bootstrapping the standardized resi-
duals from our estimation. Again, we present the evolution of the density of simulated prices through time and the evolution of the standard deviation.

Again the results are very similar to those of the CH20HP5 transformation: the standard deviation of simulated natural gas prices using the bootstrapped errors is lower than the standard variation using the normal errors, but then again, we expect higher maximums than the normal case.

The results of the distribution of the simulated maximums and the boxplot of the maximums by time is presented in Figure 3.20. As in the comparison of the simulation from the bootstrapped errors and the normal errors of the CH20HP5 transformation, the ranges of the bootstrapped errors using the CH25HP5 transformation are larger than those of the normal errors. Moreover, the extremes are somewhat larger.

### 3.5.2 Comparison of Selected Specifications

The results from the simulation are very promising and provide a relatively wide range of alternatives to model the possible evolution of natural gas prices. Notice that even within the 1st and the 99th percentile the simulation results are very similar across
Figure 3.19: Evolution of selected percentiles and Std. Dev. through time: Bootstrapped CH25HP5

Figure 3.20: Price Hikes: CH25HP5 Bootstrapped
specifications and between options for the standardized residuals. In contrast, the effect on the maximums does differ and we believe our presented alternatives cover several options. The most conservative option is given by choosing the CH20HP5 specification simulating the standardized residuals from a standard normal distribution. The simulation yielding the highest and most frequent maximums is given by the CH25HP5 specification with bootstrapping of the estimated standardized residuals.

### 3.6 Conclusions

In this chapter we proposed a methodology for the long term simulation H Hung prices. We relied on the long term scenarios of the EIA as a base line to create our scenarios. One major assumption is the additivity and independence of the short term and long term components of H Hung prices.

In order to control the maximums of our simulated natural gas prices, we use the double logarithmic transformation on natural gas prices with "choke price", the maximum allowed price. We selected three different choke prices, 15, 20 and 25 2005 USD/MMBtu, in order to investigate how this procedure affected the outcome.

In order to estimate the long term and short term components of the transformed H Hung prices we borrow the Hodrick and Prescott filter from the business cycle literature. We provide a brief analysis of the HP filter in order to determine the selection of the smoothing parameter. We use definitions of the smoothing parameter that are consistent with defining the long term trend as all frequencies higher than 3 or 5 years. Moreover, we also present the results assuming that the long term trend is linear.\(^3\) The short term component is then calculated as the residual between the transformed H Hung prices and the HP filtered series.

We model the short term component as an EGARCH\((1,1)\times ARCH(1,1)\). We

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\(^3\) Henry Hub natural gas prices are trend stationary
choose the EGARCH(1,1)×ARCH(1,1) as a generalization of a simple GARCH model. Moreover, the EGARCH specification allows for an asymmetric response of the variance to the type of shocks (positive shocks or negative shocks). We didn't find it necessary to include higher order terms in our specification, probably because we included seasonal dummies for the mean and variance equations. The model was estimated using Maximum Likelihood estimation (MLE) under the assumption of normality of standardized residuals.

We estimated the model for the nine combinations of choke price and definition of the short term component. Although most of the parameters were robust to these differences in specification, our stochastic model didn't perform equally well under all specifications. Moreover, it was relatively difficult to estimate the seasonal adjustment for the mean equation and for the variance equation. Only in two specifications, with choke prices of 20 and 25 using the HP filter consistent with long term variation lasting more than 5 years, were we able to estimate the seasonal adjustments. After estimating the stochastic model, we performed moment tests on the estimated standardized residuals to check our specification. The seasonal adjustments were critical for allowing the residuals to pass these tests. In particular, the estimated residuals were not autocorrelated. We passed the moment conditions test for autocorrelation of the standardized residuals and the square of the standardized residuals. We also passed the moment test for zero mean, and unit standard deviation. Unfortunately, we were not able to pass the symmetry test. Moreover, normal Q-Q plots of the estimated standardized residuals showed a fatter tail than normal. We concluded that we should simulate the standardized errors under the normality assumption and by bootstrapping the estimated standardized residuals.

We simulated monthly average HHNG prices in 2005 USD dollars from January 1, 2007 to December, 2030. We performed 10,000 simulations and analyzed the
evolution of the distribution and the standard deviation. Moreover, we also analyzed the distribution of the maximum. To simulate HHNG prices we simulated the long term component and the short term component separately.

The simulation of the long term component was based on the Annual Energy Outlook for 2007. We used three scenarios of the AEO 2007, namely the high price, low price and reference scenarios, to model the support for the probability distribution of the long term component. We interpolate the scenarios to provide a continuous support. In order to increase the sampling frequency of the long term forecast from annual to monthly we used a cubic spline interpolation. After simulating the long term component, we simulated the short term component and analyzed the simulations of the HHNG price.

We simulated the short term component under the standard normality assumption and by bootstrapping the estimated standardized residuals for the selected specifications. The four simulations were robust from the 1st to the 99th percentile, but they differed in the size of the maximums. In particular, the simulation that bootstrapped the estimated standardized residuals usually had larger maximums. The double logarithmic transformation prevented the simulated prices from exceeding the choke price. The simulations primarily differed in the upper tail.

Finally, we believe that we have provided a viable methodology for the long term simulation of natural gas prices. The selection of one of the four simulations presented depends primarily on the sensitivity of our application to high natural gas prices. The main difference between the simulations is the variance of the maximums.

3.7 Appendix

In this appendix we explain and develop the method we used to interpolate monthly average annual HHNG prices derived from the AEO 2007 HHNG price long term
forecast. Deriving monthly averaged data from annual averaged data is a different problem than interpolating annual observations as we will show.

The AEO 2007 HHNG prices long term forecast consists of 24 annual averages of HHNC prices for 2007 through 2030 in 2005 USD. Simply interpolating the forecasted average prices leaves a lot of room for arbitrary decisions: such as selecting the base month for the forecast. Since interpolating annual averages is not correct, some authors have renamed the problem of finding monthly averaged values as “time disaggregation”. The problem is characterized as follows.

Denote the data generation process by $f(x)$. We assume that $f(x)$ is a continuous differentiable function in $[0, T]$, and without loss of generality we assume that the time elapsed in a year is characterized by the unit interval. We are supplied with annual average data of $f(x)$ denoted as $f_i$, where $i = 1, \ldots, T$ and with the initial price $f(0)$. Therefore, $f_i$ satisfies:

$$\int_{i-1}^{i} f(x) dx = f_i \text{ for all } i = 1, \ldots, T.$$  

The problem consists in finding monthly averages, $f_{ij}$ for $i = 1, \ldots, T$ and $j = 1, \ldots, 12$ such that the sum of squares of the deviations between our estimation and the actual values is as close to zero as possible. Unfortunately, we do not have any information about the intra-annual variability. Therefore, we have to use the properties of the function $f(x)$ for determining the long term forecast at a monthly frequency. We will use the fact that $f(x)$ is a continuous differentiable function and the qualitative statement that the long term component of prices, $f(x)$, is a very smooth function at monthly frequencies.

We propose to solve the problem using splines. An average-cubic spline of the function $f(x)$ for annual averaged data is defined as the set of polynomials $S_i(x)$ for $i = 1, \ldots, T$ such that:
\( S_i(x) = a_i + b_i x + c_i x^2 + d_i x^3 \) \hspace{1cm} (3.9)

\[ \int_{i-1}^{i} S_i(x)dx = f_i \]

\( S_1(0) = f(0), S_i(i-1) = S_{i-1}(i-1) \) for \( i = 2, \ldots, T \)

\( S'_1(0) = 0, S'_i(i-1) = S'_{i-1}(i-1) \) for \( i = 2, \ldots, T. \)

Notice that we assumed that \( f'(0) = 0 \). We have \( 4t \) parameters to set and \( 3t \) equations. The remaining \( i \) equations are obtained by minimizing the square of the total variation of the average-spline subject to the restrictions imposed by the conditions 3.9. The problem is given by:

\[ \min_{\{a_i, b_i, c_i, d_i\}} \sum_{i=1}^{T} \int_{i-1}^{i} (S''_i(x))^2 dx \] \hspace{1cm} (3.10)

subject to

\[ \int_{i-1}^{i} S_i(x)dx = f_i \]

\( S_1(0) = f(0), S_i(i-1) = S_{i-1}(i-1) \) for \( i = 2, \ldots, T \)

\( S'_1(0) = 0, S'_i(i-1) = S'_{i-1}(i-1) \) for \( i = 2, \ldots, T. \)

In order to simplify the program we change the definition of the polynomials that we use to simplify our calculation. Let the set of cubic polynomials \( Z_i(x) \) defined on the interval \([0, 1]\) be such that:

\[ Z_i(x - i + 1) = S_i(x) \text{ for } x \in [i-1, i]. \]

Notice that we can simply recover \( S_i(x) \) from \( Z_i(x) \) by evaluating in four points.
and solving four equations, or by solving the system of equations generated by making the derivatives of $Z_i(x)$ equal to those of $S_i(x)$. Therefore, program 3.10 expressed in terms of the $Z_i(x)$ polynomials is given by:

$$\min_{\{a_i^z, b_i^z, c_i^z, d_i^z\}_{i=1}^T} \sum_{i=1}^T \int_{i=1}^i (Z_i''(x - i + 1))^2 dx$$

subject to

$$\int_{i-1}^i Z_i(x - i + 1) dx = f_i$$

$Z_1(0) = f(0), Z_i(0) = Z_{i-1}(1)$ for $i = 2, \ldots, T$

$Z_1'(0) = 0, Z_i'(0) = Z_{i-1}'(1)$ for $i = 2, \ldots, T$.

Notice that problem 3.11 is easier to solve than program 3.7. First, we simplify the restriction. Notice that $a_1^z = f(0)$ and $b_1^z = 0$, moreover:

$$\int_{i-1}^i Z_i(x - i + 1) dx = \int_0^1 Z_i(y) dy = a_i^z + \frac{1}{2} b_i^z + \frac{1}{3} c_i^z + \frac{1}{4} d_i^z,$$

while $Z_1'(0) = Z_{i-1}'(1)$ implies

$$b_i^z = b_{i-1}^z + 2c_{i-1}^z + 3d_{i-1}^z$$

for $i = 2, \ldots, T$.

Then, solve recursively to obtain:

$$b_i^z = b_1^z + 2 \sum_{j=1}^{i-1} c_j^z + 3 \sum_{j=1}^{i-1} d_j^z$$

for $i = 2, \ldots, T$.

Similarly, $Z_i(0) = Z_{i-1}(1)$ implies

$$a_i^z = a_{i-1}^z + b_{i-1}^z + c_{i-1}^z + d_{i-1}^z$$

for $i = 2, \ldots, T$. 
which we can solve recursively to obtain:

\[ a_i^z = a_1^z + \sum_{j=1}^{i-1} b_j^z + \sum_{j=1}^{i-1} c_j^z + \sum_{j=1}^{i-1} d_j^z \text{ for } i = 2, \ldots, T. \]

But notice that

\[ \sum_{j=2}^{i-1} b_j^z = (i - 1)b_1^z + 2 \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} c_k^z + 3 \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} d_k^z \text{ for } i = 2, \ldots, T. \]

Therefore, \( a_i^z \) for \( i = 2, \ldots, T \) is given by

\[ a_i^z = a_1^z + ib_1^z + 2 \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} c_k^z + 3 \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} d_k^z + \sum_{j=1}^{i-1} c_j^z + \sum_{j=1}^{i-1} d_j^z \text{ for } i = 2, \ldots, T, \]

and the average-annual constraint simplifies to:

\[ a_i^z + \frac{1}{2} b_i^z + \frac{1}{3} c_i^z + \frac{1}{4} d_i^z = f_i + 2 \sum_{j=2}^{i} \sum_{k=1}^{j-1} c_k^z + 3 \sum_{j=2}^{i} \sum_{k=1}^{j-1} d_k^z + \frac{5}{2} \sum_{j=1}^{i-1} d_j^z + \frac{1}{3} c_i^z + \frac{1}{4} d_i^z = f_i. \]

However, the double summation further simplifies to:

\[ 2 \sum_{j=1}^{i-1} (i - j)c_j^z + 3 \sum_{j=1}^{i-2} (i - j - 1)d_j^z + \frac{5}{2} \sum_{j=1}^{i-1} d_j^z + \frac{1}{3} c_i^z + \frac{1}{4} d_i^z = f_i - f(0) \text{ for } i = 2, \ldots, T. \]

Finally, we can also simplify the objective function since
\[
\sum_{i=1}^{T} \int_{0}^{1} (Z_i''(y))^2 \, dy = \sum_{i=1}^{T} \int_{0}^{1} (2c_i^2 + 6d_i^2 y)^2 \, dy \\
= \sum_{i=1}^{T} \int_{0}^{1} 4c_i^2 + 24c_i^2 d_i^2 y + 36d_i^2 y^2 \, dy \\
= \sum_{i=1}^{T} 4c_i^2 + 12c_i d_i + 12d_i^2.
\]

And the optimization can be rewritten as,

\[
\min_{\{c_i, d_i\}|_{i=1}^{T}} \sum_{i=1}^{T} 4c_i^2 = 12c_i d_i + 12d_i^2 \tag{3.12}
\]

subject to

\[
2 \sum_{j=1}^{i-1} (i - j)c_j + 3 \sum_{j=1}^{i-2} (i - j - 1)d_j + \frac{5}{2} \sum_{j=1}^{i-1} d_j + \frac{1}{3} c_i + \frac{1}{4} d_i = f_i - f(0) \text{ for } i = 2, \ldots, T
\]

\[
\frac{1}{3} c_1 + \frac{1}{4} d_1 = f_1 - f(0).
\]

Although a little messier, program 3.12 is a standard quadratic programming problem and efficient algorithms exist to solve them in a fast and reliable manner. Specifically, let
\[
x = \begin{bmatrix}
c_1 \\
d_1 \\
\vdots \\
c_T \\
d_T
\end{bmatrix}_{2T \times 1},
D = \begin{bmatrix}
8 & 12 & 0 & 0 & \ldots & \ldots & 0 & 0 \\
12 & 24 & 0 & 0 \\
0 & 0 & 8 & 12 \\
0 & 0 & 12 & 24 & \ddots \\
\vdots \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 12 & 24
\end{bmatrix}_{2T \times 2T},
\]
\[
b_0 = \begin{bmatrix}
f_1 - f(0) \\
\vdots \\
f_T - f(0)
\end{bmatrix}_{T \times 1}
\]

\[
A' = \begin{bmatrix}
\frac{1}{3} & \frac{1}{4} & 0 & 0 & \ldots & \ldots & 0 & 0 \\
2 & \frac{5}{2} & \frac{1}{3} & \frac{1}{4} & 0 & 0 \\
4 & \frac{5\frac{1}{2}}{2} & 2 & \frac{5}{2} & \frac{1}{3} & \frac{1}{4} & \ldots & 0 \\
2(i - 1) & 3(i - 2) + \frac{5}{2} & \ldots \\
2(T - 2) & 3(T - 3) + \frac{5}{2} & \ldots & 0 & 0 \\
2(T - 1) & 3(T - 2) + \frac{5}{2} & 2(T - 2) & 3(T - 3) + \frac{5}{2} & \ldots & \frac{1}{3} & \frac{1}{4}
\end{bmatrix}_{T \times 2T}
\]

Then program 3.12 can be simply expressed using linear algebra as:

\[
\min_{x} \frac{1}{2} x' Dx \text{ subject to } A' x = b_0
\]

Finally, denote the solution of the quadratic minimization problem as \(x^*\). Finally, the average monthly values can be easily obtained from the \(Z_i(x)\), polynomials. Let \(k(j)\) be the proportion of days that have passed to the beginning of month \(j = \)
1, \ldots, 12, where \( k(13) = 1 \); then for month \( j \), year \( i \) the average monthly value is given by:

\[
f_{i,j} = \frac{\int_{k(j)}^{k(j+1)} Z_i(y) \, dy}{k(j + 1) - k(j)}
\]

The solution is simply the difference of two polynomials.

Figure 3.11 shows the average cubic spline for the three selected cases of the annual HHNG price long term forecast of the EIA in the AEO 2007.
Chapter 4

Practical Elements of the Electricity Industry in Mexico

We start this chapter with a brief description of the electricity industry in Mexico in Section 4.1. Next, using the simulated Henry Hub natural gas prices of Chapter 3 we illustrate some aspects of the effect of uncertainty in the prospects of nuclear power in Mexico.

As it was shown in Chapter 1 for the case of a constant load curve, increasing uncertainty in natural gas prices increase the benefits from nuclear power\(^1\). In Section 4.2 we argument that for base load generation capacity a fix load curve can be used to simplify the problem. Then, we analyze the distribution total costs, the benefits from nuclear power and its costs. We also provide sensitivity analysis on the exogenous parameters of the problem to complete our analysis.

Finally, given the importance of the hydroelectric power in Mexico, in Section 4.3 we provide a methodology that approximates the solution of the optimal generation for hydroelectric plants that we believe can successfully applied to Mexico. Our methodology greatly simplifies the generation problem from hydroelectric plants by abstracting from the head effect.

\(^1\)Assuming a risk averse central planner.
4.1 The Mexican Electric System

The Mexican electric system is composed of two government own firms, Comisión Federal de Electricidad (CFE) and Luz y Fuerza del Centro (LFC), that own the transmission infrastructure and have the monopoly to sell electricity in delimited geographic areas. Private investment projects can sell their electricity to CFE, consume it within the enterprise or export it to surrounding countries. The electricity industry is regulated by the Comision Reguladora de Energy (CRE), while the Secretaria de Energia provides oversight and develops public policy for the heavy regulated (and mostly government own) energy markets in Mexico. Finally, given the relatively serious pollution problems in Mexico City, environmental laws are another important part of the institutional framework in Mexico.

In this section, we present the physical and legal structure of the Mexican electric system. Then, we comment on the expected growth in electricity consumption and the opportunities for nuclear power to help Mexico meet its energy requirements while simultaneously promoting its energy independence.\(^2\)

4.1.1 The Legal Structure

In 1992, the Mexican Constitution was amended to allow private investment in electricity generation in Mexico. Currently, private investors are allowed to participate in the electricity generating sector in five possible modes: autoabastecimiento (self-supplied), cogeneracion (cogeneration), productor independiente (independent producer), importacín y exportacín (importer and exporter) and pequea produccin (small producer). The independent producer mode was most successful in attracting private investment into the Mexican electricity market. Each of the modes was designed to

\(^2\)Although Mexico does not have uranium mining or enrichment facilities, the nature of nuclear fuel allows an opportunity to easily establish strategic stockpiles if they are thought to be necessary.
attract different private investors according to their needs and available technologies; specifically,

- Self-supplier: this is designed to allow investment in electricity generation by large industrial consumers.

- Cogeneration mode: this modality allows private investors to use residual heat and steam from their industrial process to generate electricity. The electricity generated can only be used by the producers or any joint firm that owns the project.

- Independent producer: this mode was designed to allow private investment in larger generation plants (a minimum of 30 MW capacity is necessary) with the sole purpose of selling electricity to CFE or exporting it.

- Small producer: this mode accommodates private investors in small capacity (up to 30 MW) plants with the sole purpose of selling electricity to CFE or exporting it. Rural communities can also use this mode (if capacity is lower than 1 MW) for self-supplied electricity.

- Exporter and importer: this mode is designed for private investors wishing to export and import electricity.

One producer can operate under more than one mode when these are not mutually exclusive. The legal framework goes beyond the simple generation license. For example, the producer can establish an interconnection contract with CFE to connect to the National Electric System (SEN). The interconnection contracts provide back-up electricity and the possibility of selling excess electricity to CFE. They may also specify required transmission augmentations. The CRE has published the exact methodologies it uses to determine the cost of each of the contracts. The intercon-
nection contracts will work for renewables (where the electricity produced cannot be regulated) in addition to more conventional generating capacity.

In 2006, the CRE signed 90 new licenses to generate or import electricity. This brought the total agreements signed since the inception of the new law to 580. The active licenses (approximately 90% of the signed ones) have an electric capacity of 19,245 MW. Independent producers represent 53.5% of the signed contracts, followed by self-suppliers with 25.7%, exporters with 9.5% and cogenerators with 7.9%. Independent producers have a total capacity of 12,557MW, which represents around one-third of the effective combined capacity of CFE and LFC of 38,382MW.

Combined cycle natural gas turbines (CCGT) are the technology of choice among independent producers. This technology represents 65.7% of the total electricity produced by private investors in Mexico.

With regard to nuclear power, the Constitution of Mexico established a monopoly for the Mexican government in matters pertaining to nuclear fuels and materials. Therefore, the government in Mexico has a monopoly on the generation of electricity from nuclear power.

4.1.2 The Demand for Electricity

The demand for electricity in Mexico is of two types. The largest component is the demand for electricity publicly supplied by the CFE and LFC, generated either by the CFE or LFC or by independent producers. Secondly, there is self-supplied electricity, which may also result in private parties trading electricity with the grid.

In Mexico, as in other countries, electricity consumption is positively correlated with economic activity. National consumption of electricity in 2006 was 197,435GWh representing a 3.2% increase over 2005.

The industrial sector is the largest consumer of electricity in the country with
a 58.8% share of the total followed by the residential sector with 25.3% and the commercial sector with 7.5%. Despite the importance of the industrial sector, in the last ten years the residential sector increased the fastest.

Given its geographical extent and diversity of economic structure and climates, Mexico also has a diversity of load curves. The north presents high variations between summer and winter (around 30% of the maximum demand). Moreover, there are two sets of peak hours from 10:00 to 18:00 and from 21:00 to 24:00. The first peak is during the hottest portion of the day, while the second one reflects the large number of air conditioners used at night. In the south, the differences between summer and winter are less pronounced, although the period from 20:00 to 23:00 hours represents the peak hours during the day.

The Mexican load curve is actually relatively flat compared to the curve usually encountered in many countries. This pattern may be partly explained by the introduction of peak hour pricing for industrial consumers. The figure shows the load curve patterns for typical working and non-working days. The set of graphs is divided by season, summer and winter, and by region, north and south.

The tariff structure for electric energy is divided according the usage and the voltage level of the line. The division is given as follows:

- **Residential**: 1, 1A, 1B, 1C, 1D, 1E, 1F and DAC (Residential of high consumption).

- **Public Services**: 5, 5A and 6.

- **Agriculture**: 9, 9M, 9CU and 9N.

- **Temporal**: 7.

- **Generals at low voltage**: 2 and 3.
Figure 4.1: Mexican load curve by season, region and type of day.
• Generals at medium voltage: OM, HM and HMC.

• Generals at high voltage: HS, HSL, HT and HTL.

• Voltage backup for medium voltage: HM-R, HM-RF and HM-RM.

• Voltage backup for high voltage: HS-R, HS-RF, HS-RM, HT-R, HT-RF and HT-RM.

• Interruptible Service: I-15 and I-30.

The tariff structure for medium and high voltage, and for high consumption residential service, is more complex than the normal residential tariff structure. The complex tariffs depend on marginal cost and a monthly automatic adjustment that depends on changes in fuel prices and inflation over the previous month. Moreover, the tariff structure also depends on geographical location, time and season.

Apart from the agriculture tariffs, which are adjusted annually, all other tariffs are adjusted monthly. The residential (except for DAC) and public service tariffs are adjusted by fixed factors. The remaining tariffs are adjusted by an "automatic monthly adjustment" that, as we mentioned earlier, includes variations in the fuel price and inflation.

As in many other countries, the commercial sector has the highest mean prices among all final users. Economists often speculate that this might reflect the fact that the elasticity of demand is lowest in this sector and the differential thus reflects price discrimination. Meanwhile the agriculture tariff is the lowest.

4.1.3 The Mexican Generation Capacity

The National Electric System (SEN) consists of a large interconnected system (SIN) together with the isolated systems in Baja California (although those are connected
to the Western Electricity Coordinating Council (WECC) in the Unites States and Canada). Moreover, the SEN can be further divided into electricity that serves the public, electricity generated by the CFE, LFC and private parties operating as independent producers, and electricity that serves private entities operating as self generators, cogenerators, and exporters.

In 2006, Mexico had a total generating capacity, including exports, of 56,337 MW, which represented a 4.6% increase over 2005. The total effective capacity managed by the CFE but constructed by independent producers rose from 8,251 MW in 2005 to 10,387 MW in 2006. As we mentioned the independent producer can also sell electricity as self supplied or exporters and some of the signed licensee haven't started producing.

Hydrocarbons generate 64.6% of the total electricity produced. Moreover, natural gas represents 42.6% of total electricity generated for public consumption (rising from 12.1% in 1996), compared with 21.6% for residual fuel oil and only 4.8% for nuclear power. Natural gas is mainly displacing residual oil, yielding substantial environmental benefits.

Hydroelectric plants also supply around 13.6% of the total electricity generated for public consumption. The CFE has invested in hydroelectric plants with large capacities, such as El Cajn (750MW) and La Yesca (750MW), and three investment projects, La Parota (900MW), the expansion of Villita (150MW) and Rio Moctezuma (114MW) that will be completed within the next five years.

In the private sector, total capacity supplied by independent producers, remote self-suppliers, exporters and cogenerators increased by 26.3%, 13.7%, 7% and 7.7% respectively over the period 2000-2006. Private investment thus has helped reduce the cost of electricity for big industrial consumers below what it would otherwise have been.
Exporters represented 7.7% of the privately owned electricity generated in Mexico. For international commerce, there are 9 interconnections between the US and Mexico and one between Belize and Mexico. Five of the nine connections with the US are high voltage direct current connections that operate only in emergency situations. Moreover, the main flows of electricity were between SEN and the Western Electricity Coordinating Council (WECC) where there is a medium voltage (230kV) connection capacity of 800MW. Flows of electricity between SEN and the Electric Reliability Council of Texas are very limited, and are mostly designed for emergencies. In 2006, the infrastructure for international commerce was unchanged from 2005 and international trade amounted to a net balance of 776GWh, an increase of 0.6% from 2005. The relation between the SEN and the WECC represents 82.5% of the net balance.

Electricity losses in Mexico are quite significant, amounting to 17.6% of the electricity generated. Technical losses arise in transmission, generation and self-use of electricity. Non-technical losses are usually from theft by the informal commercial sector and other clients who evade payment. In 2006, the non-technical losses represented 8.7% of total generation and around 50% of the total losses of electricity.

CFE and LFC own the entire transmission and distribution network. In 2006, there was an increment of 13,507km in the network, resulting in a total network length of 773,059km. The main components of the transmission lines are: 6.7% of 400kV and 230kV, 6.8% of 161kV and 69kV and 52.8% of 34.5kV and 2.4kV. The other 42.1% represents low tension lines.

4.1.4 Investment Prospects in Mexico

In principle, a number of factors make nuclear power an attractive option for Mexico. The relatively flat load curve with a reasonable quantity of hydroelectric capacity that can be used to shave remaining peaks off the load implies that Mexico has quite a
large base load that can be cost effectively served with nuclear power. The substantial natural gas capacity could also complement nuclear and hydroelectric power by serving the intermediate part of the load curve. In addition, the demand for electricity is expected to grow quite rapidly in the near future requiring a substantial expansion in generating capacity. A relatively large percentage of the demand, and the expected demand growth, also is concentrated in the Mexico city area and could be conveniently served by large base load nuclear power generators. Using nuclear power to supply more of the Mexico City load would also have the advantage of limiting air pollution in that region. Finally, as we mentioned in the analysis of the generating capacity, Mexico already has some experience with nuclear power. It currently has one nuclear power plant, Laguna Verde, which has two Boiling Water Reactors (BWR-5) with total capacity of 1,364.88 MWe. The steam cycle was constructed by General Electric and the generator by Mitsubishi Heavy Industries. The construction of Laguna Verde started in October of 1976 and the second unit achieved criticality in September of 1994. The first reactor entered online in July of 1990 and the second unit in April of 1995.

The officially expected increase in electricity demand for the period 2007-2016 is based on the relationship of electricity consumption to the Gross Domestic Product (GDP) and population. The latter is estimated to be 0.9% per annum for persons and 2.8% per annum for households. In summary, the planning scenario assumes a 3.6% annual increase in electricity demand for the nation as a whole. Reflecting the fact that more of the growth is expected in the currently urbanized areas, electricity consumption of the SEN is forecasted to have an annual growth rate of 4.8%.

Expansion of the SEN is planned to take place in phases. Not only is there a schedule of capacity under construction in the near term. The government has also produced a plan of future tenders to begin new construction projects.
Since hydroelectricity storage capacity is limited, electricity generation has to approximate demand at all times. The electric system thus also needs reserve capacity to satisfy the demand for electricity in case there are technical transmission problems or some generating capacity goes off-line. There are at least two techniques to determine the reserve capacity of a system and hence the future investment in electricity generation capacity: the deterministic approach and the probabilistic approach. In Mexico, the deterministic approach is used for investment purposes, and an Operational Reserve Margin (ORM) of 6% is targeted. In 2006, the ORM was 14.0% partly due to the lower than expected economic growth. It is expected that from 2011 onwards the ORM would come back to 6% and stay at this value.

For the period of 2007-2016 the CFE plans to add 22,153MW out of which 5,498MW is already under construction or tendered. Moreover, 5,867MW are planned to be retired. The most recently tendered construction projects are composed of 2,677MW of CCGT, 678 MW of coal, 416 of distributed capacity (small turbogas plants), 184 MW of wind power, 1,500MW of hydroelectric power, and 42MW of internal combustion. The last of these to be completed are expected to enter operation in 2012.

An additional capacity of 16,187 MW that has not yet been tendered will be installed in the period of 2009-2016. Although these projects haven’t been completely defined, CFE has suggested the location and technology for some of them. Specifically, they have suggested 8,385 MW (51.8%) of CCGT, 2,100 MW (13.95%) of coal capacity, 1,164 MW (7.2%) of hydroelectric capacity, 406 MW of wind power, 158 MW of geothermal capacity, 69 MW internal combustion leaving 3,826 MW (23.64%) with the technology currently unspecified. Clearly, the bulk of the anticipated investment in Mexico would be in natural gas, coal, hydroelectric and wind. At this moment there is no proposal to increase the nuclear generating capacity.
In particular, natural gas is expected to represent 53.6% of generating capacity by 2016. Unfortunately, a high proportion of natural gas, given the endowment of fuels in Mexico, impedes energy independence. Moreover, natural gas investment has been proposed under scenarios that do not take into account the uncertainty in natural gas prices. In Section 4.2 we provide some insight into the evaluation of natural gas investment under uncertain natural gas prices.

The state monopoly on the use of nuclear power remains a relatively large obstacle to its use. In contrast to coal, natural gas, wind and even hydroelectric plants, nuclear power plants can't be operated by private investors. Given the large up-front cost of constructing nuclear plants, and the many needs of the Mexican government for funds to invest in other infrastructure, it is not surprising that there aren't plans to increase nuclear generating capacity. Regrettably, institutional factors in this case can impede the diversification of generating capacity in Mexico.

4.2 Comparing Combined Cycle Natural Gas Plants to Nuclear Power Plants

In this section we compare the total costs of nuclear and combined cycle natural gas technologies for the generation of electricity. It is important to note that we assume a constant capacity factor and we analyze total expected costs rather than net social benefits. In favor of the constant capacity assumption we argue that: (1) the CFE is unlikely to retire or reduce the capacity factor of new to medium life plants due to technological advances given the take-or-pay structure of electricity supply contracts with independent private producers; (2) in the case that the industrial sector reacts to an increase in electricity prices by switching to other technologies, the partial deferral of investment and expected increases in non-industrial demand will ensure
a constant capacity factor for the new technologies used; (3) since it is optimal to use a new nuclear power plant or combined cycle natural gas plant for base load, the electricity generated by hydroelectric plants will not displace the generation of new base load capacity. Regrettably, our arguments will not apply for long term technological strategies; as we have shown, the constant capacity factor assumption is a major shortcoming when analyzing the optimal investment in electricity generation capacity.

The difference in the average economic life of nuclear and combined cycle natural gas plants requires us to adjust our methodology to analyze expected total costs. In particular, we will analyze the distribution of total costs on a per megawatt of generation basis, usually referred as a levelized costs. To calculate the levelized cost of combined cycle and nuclear plants, we will assume that both plants will enter online in 2014. For the case of combined cycle plants, we assume a construction period of 3 years while for nuclear plants it is 5 years. We also assume that combined cycle plants have an economic life of 30 years while for nuclear power plants it is 40 years. Since the long term simulation of natural gas prices presented in Chapter 3 lasts until the end of 2030, we present two options to simulate natural gas prices after 2030.

Finally, we also perform sensitivity analysis on the key parameters determining the levelized costs of nuclear and combined cycle natural gas technologies. While comparing both technologies, we will focus on the benefits of nuclear capacity for limiting the uncertainty in levelized total costs resulting from uncertain natural gas prices.

4.2.1 Levelized Costs

Denote the capacity factor for technology $i = N, CC$ as $\phi_i$, the overnight costs as $K_i$, the maintenance and operating costs per megawatt of capacity per year as $M_i$, fuel
costs per megawatt hour at time $t$ as $\rho_{i,t}$, the proportion of overnight costs expense at period $t$ for technology $i$ as $\gamma_{i,t}$ and the discount factor as $r$.\(^3\) Then, the discounted total cost per megawatt of capacity from technology $i = N, CC$ is given by:

$$ TC_i = \sum_{t=0}^{4} \frac{\gamma_{i,t} K_i}{(1 + r)^t} + \sum_{t=5}^{T_i+5} \frac{8760 \phi_i \rho_{i,t} + M_i}{(1 + r)^t} $$  \hspace{1cm} (4.1) $$

Denote the levelized costs of technology $i$, as $C_i$. Then, $C_i$ solves:

$$ TC_i = 8760 \phi_i C_i \sum_{t=5}^{T_i+5} (1 + r)^{-t} $$  \hspace{1cm} (4.2) $$

The fuel cost for combined cycle plants, $\rho_{CC}$, is given by heat rate times the natural gas price at Henry Hub minus .58 USD/MMBtu.\(^4\)

Then, the levelized costs for a mix of generating capacity is simply given by $(1 - q_N)C_{CC} + q_N C_N$. The rest of this section focuses on the analysis of the distribution of levelized costs.

### 4.2.2 Natural Gas Prices

Our methodology for long term simulation of natural gas prices is presented in Chapter 3. Unfortunately, the long term forecast of the Energy Information Administration ends in 2030. Therefore, we present two options to extend the forecast for the period between 2031 to 2043.\(^5\) In the first option we use the linear trend shown in the period 2026 to 2030 and extrapolate it to the period 2031 to 2043. In the second option, we maintain average prices for the extended period equal to the average price of 2030.

The second option represents a conservative scenario, while we refer to the first option

\(^3\)In the case of natural gas, the fuel costs or generation costs is a random variable implying a distribution of levelized costs.

\(^4\)This is the official assumption of the CFE with respect to natural gas prices.

\(^5\)For a natural gas combined cycle plants entering on line in 2014 and lasting for 30 years, natural gas prices should be simulated for the period 2031 to 2043.
as a pessimistic one for the long term mean of natural gas prices.

Figure 4.2 shows the expanded long term component of natural gas prices under the two possibilities. Although we provide a sensitivity analysis for the difference between options 1 and 2, we will use option 1 to analyze the distribution of levelized cost and the sensitivity analysis for the exogenous parameters.

![Graphs showing extended long term forecast](image)

Figure 4.2: Proposed Extensions of the Long Term Forecast.

We characterize the short term component of natural gas price variation using the two extreme positions presented in Chapter 3. Specifically, we use simulated prices using the CH25HP5 transformation with bootstrapped standard errors and the CH20HP5 transformation with normal standard errors. As in the case of the long term component, we provide a sensitivity analysis of the effect that the two options for short term components have on the benefits from nuclear power. We select the CH25HP5 transformation with bootstrapped standard errors to analyze the distribution of levelized costs and to perform the sensitivity analysis.
4.2.3 Costs Estimates

We used the costs estimates from the Nuclear Energy Agency and the International Energy Agency in 2005 ([1]) to obtain suitable values for the cost structure of nuclear and combined cycle natural gas plants.

Since Mexico has experience with combined cycle plants, we will use the overnight construction costs for combined cycle plants in the middle of the 430 to 860 USD/kWe range as the reference case scenario. Most of the time, overnight costs are spread over the last two construction years prior to commissioning. Therefore, we are going to assume than 10% of the costs are in the first year, with 45% being covered for each of the two subsequent years. For the heat rate we will use 6.83 MMBtu/MWh for the reference scenario and a range from 5.68 to 8.53 MMBtu/MWh for the sensitivity analysis.\(^6\) We will use 27.5 USD/kWe for the reference scenario for maintenance and operating costs. Finally, for natural gas prices we use the official assumption of Henry Hub natural gas prices minus .58 USD/MMBtu. The price corresponds to the delivered price at the new liquified natural gas terminal in the Pacific Ocean (Manzanillo).

In the case of nuclear, we assume a 5 year construction period with capital costs distributed equally at each year.\(^7\) Nuclear fuel costs in EIA’s 2005 study ranged from 3.5 USD/MWh in Canada to 13 USD/MWh in Japan. We will used 8.25 USD/MWh for the reference scenario.\(^8\) Maintenance and operation costs ranged from 50 to 115 USD/kWe. We will use 98.75 USD/MWh. Overnight capital costs ranged from 1,050 to 2,150 USD/kW. However, the ranges seemed obsolete given recent estimates of 2,500 to 4,000 USD/kW by Florida Power and Light and Progress Energy, EdF

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\(^6\)The selected range corresponds to a 40% to 60% range of efficiencies for combined cycle plants.

\(^7\)According to the NEA/IEA costs update, for projects taking more than 5 years, around 90% of the costs are within the assumed 5 year construction period, the extra time is usually spent in pre-construction studies.

\(^8\)It is important to mention that fuel costs include every stage of the fuel process: from mining and processing to final disposal.
Flamanville, Bruce Power Alberta, etc.\textsuperscript{9} We select 3,000 USD/MWh as the reference scenario. Finally, it is important to note that Mexico is expected to have lower nuclear power costs than United States given the large regulatory burden faced by public companies in the United States.

As we will observe in the sensitivity analysis, the discount factor is the single most important parameter determining the benefits of nuclear power. Moreover, the discount factor non-linearly affects the distribution of levelized costs. We provide a relatively large range for the real discount factor from 5.0\% to 9.0\%; we select a 7.0\% rate for the reference scenario given the higher capital costs in Mexico. It is important to mention that we are referring to the after tax real rate of return. Therefore, 9\% is relatively high even for a country such as Mexico.

Table 4.1 summarizes the range of the parameters and the reference scenario that we analyze.

\subsection*{4.2.4 Levelized Total Costs}

We use the reference scenario, the CH25HP5 transformation with bootstrapped standard error and increasing long term prices to analyze the levelized cost of a mixed investment in nuclear and combined cycle natural gas plants. Table 4.2 presents selected statistics of the distribution of levelized costs with respect to the proportion of nuclear power. The left panel of Figure 4.3 shows the kernel densities of the distribution of levelized costs for 50\% or 100\% combined cycle while the right panel shows selected percentiles of the distribution of levelized costs as the proportion of nuclear power increases.

The average levelized costs of combined cycle plants is 50.50 USD/MWh with a maximum in 10,000 observations of 68.23 USD/MWh and a minimum of 39.03 USD/MWh.

\textsuperscript{9}The information is publicly available from the World Economic Association, \url{http://www.world-nuclear.org/info/inf02.htm}. 

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Combined Cycle Natural Gas Plants</th>
<th>Nuclear Power Plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overnight construction costs (USD/kWe)</td>
<td>Range: 430 - 860</td>
<td>Range: 2,500 - 4,000</td>
</tr>
<tr>
<td></td>
<td>Reference: 645</td>
<td>Reference: 3,000</td>
</tr>
<tr>
<td>Annual operating and maintenance (USD/kWe)</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Heat Rate (MMBtu/MWh)</td>
<td>Range: 5.68 - 8.53</td>
<td>NA</td>
</tr>
<tr>
<td>Fuel Costs (USD/MWh)</td>
<td>NA</td>
<td>Range: 3.5 - 13</td>
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<tr>
<td>Capacity Factor (%)</td>
<td>NA</td>
<td>Reference: 8.25</td>
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<tr>
<td>Discount Rate</td>
<td>Range: 5.0 - 9.0</td>
<td>.80</td>
</tr>
<tr>
<td>Natural Gas Cost for CFE</td>
<td>NA</td>
<td>HHub - .58</td>
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</table>

*Common to Both Technologies*

Table 4.1: Reference scenario values and ranges for the sensitivity analysis.
Figure 4.3: Selected Statistics of the Distribution of Levelized Costs.
<table>
<thead>
<tr>
<th>Param</th>
<th>1</th>
<th>.9</th>
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<th>.5</th>
<th>.25</th>
<th>.1</th>
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<tr>
<td>Mean</td>
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<td>50.66</td>
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<td>52.05</td>
<td>52.83</td>
<td>53.45</td>
<td>53.60</td>
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<td>Std. Dev.</td>
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<td>4.29</td>
<td>3.39</td>
<td>2.26</td>
<td>1.13</td>
<td>0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>Min.</td>
<td>39.03</td>
<td>39.76</td>
<td>42.67</td>
<td>46.32</td>
<td>49.96</td>
<td>52.87</td>
<td>53.60</td>
</tr>
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Table 4.2: Selected Statistics of the Distribution of Levelized Costs (MMUSD/MW)

The estimated standard deviation equals 4.52 USD/MWh. On the other hand, the levelized costs of a nuclear plant in the reference scenario is 53.60 USD/MWh or around 3.10 USD/MWh higher than the natural gas plant. In a 50-50 system, the average levelized cost increases by 1.55 USD/MWh and the range is reduced to 46.32 to 60.92 USD/MWh. In contrast, the 90th percentile is reduced from 56.65 to 55.13 USD/MWh as a result of increasing nuclear proportion by 50%. Clearly, under the reference scenario, the gains from the safety value of nuclear power are only realized in the tail of the distribution.

Figure 4.3 clearly shows the decrease in variability of levelized costs as the proportion of nuclear power increases. For the reference scenario optimal investment lies between the two extremes.

In the following, we will analyze the sensitivity of our results to the selected parameters of the reference scenario.

**Overnight Costs**

The reference scenario assumes nuclear overnight or investment cost of 3,000 USD/kW. We use a range from 2,500 to 4,000 USD/kW to analyze the effect on the levelized
costs and benefits of nuclear power plants of changes in the nuclear overnight costs.

Figure 4.4 shows a set of three graphs that we'll use to analyze the sensitivity of the distribution of levelized costs and the benefits of nuclear power to changes in the reference parameters. Figure 4.4 (top) shows the mean levelized cost as a function of the overnight cost for three different proportions of combined cycle plants in the system. Figure 4.4 (bottom left) shows the probability that combined cycle levelized costs are lower than nuclear levelized costs.\textsuperscript{10} Finally, Figure 4.4 (bottom right) shows the expected loss due to high realizations of natural gas prices relative to nuclear levelized costs again for three different proportion of combined cycle plants in the system. In a nutshell, the bottom of Figure 4.4 shows the benefits from nuclear power, while the top shows its costs.\textsuperscript{11}

At a level of 2,717 USD/kW for nuclear overnight cost, nuclear power plants have the same levelized cost as natural gas plants. Unfortunately, nuclear levelized costs are very sensitive to overnight capital costs. For the selected range, levelized costs went from a low 48.13 USD/MWh to a high 64.54 USD/MWh. There is a probability of less than 1% for natural gas levelized costs to get as high as 64.54 USD/MWh. An increase in 60% in overnight costs (from 2,500 to 4,000 USD/MWh) implies a 25.42% increase in levelized costs from nuclear power. This is a large value compared to combined cycle plants as we will see below. Moreover, we call nuclear power \textit{profitable} if the expected loss of combined cycle plants relative to nuclear power is larger or equal to the difference in expected levelized costs.\textsuperscript{12} In this case, at a value lower than

\textsuperscript{10}Notice that this probability does not depend on the proportion of nuclear power since:

\[ \Pr(q_{CC}C_{CC} + (1 - q_{CC})C_N < C_N) = \Pr(C_{CC} < C_N). \]

\textsuperscript{11}It is important to mention that we are not considering the benefits from the reduction of the variance or volatility of levelized costs from increasing the proportion of nuclear plants in the system. We are strictly focusing in the benefits from decreasing total costs under high realizations of natural gas prices.

\textsuperscript{12}Given the tail of the distribution of levelized costs for natural gas combined cycle plants and the risk aversion of the central planner, it is still possible to see investments in nuclear power even
Figure 4.4: Costs and benefits from nuclear power by nuclear overnight capital costs.
of 2,972 USD/kWh nuclear power is *profitable*. We will keep revising the *profitable* level of overnight costs for nuclear plants as we perform further sensitivity analysis on other exogenous parameters of the model since, in our opinion, the overnight costs for nuclear power largely determine the desirability of investing in nuclear technologies.

For combined cycle plants, the reference scenario assumes overnight costs of 645 USD/kW for combined cycle plants. The overnight costs ranges from a minimum of 430 to maximum of 860 USD/kW. Figure 4.5 shows the levelized costs and benefits of nuclear power as functions of the combined cycle overnight cost. The benefits of nuclear power increase as the overnight costs of combined cycle plants increases. The levelized costs of combined cycle plant went from 47.91 to 53.09 USD/MWh, which implies that a doubling of the overnight costs of combined cycle natural gas plants implies an slight increase of 10.8% in the levelized cost. As Figure 4.5 shows, nuclear power has higher levelized costs than combined cycle plants for every value in the selected range. Moreover, the overnight cost of combined cycle plants has to reach a level of 903 USD/kW in order for nuclear power to have the same expected levelized costs as combined cycle plants. The expected loss of combined cycle plants ranges from 1.90 to 3.78 USD/MWh. At a overnight cost level of 670.40 USD/kW nuclear power is *profitable*. Similarly, if overnight costs for combined cycle plants reached the upper range of 860 USD/kW, then nuclear power has the same levelized costs as combined cycle plants for an overnight cost of 2,953 USD/kW while nuclear power is *profitable* until reaching a level of 3,208 USD/kW.

**Discount Factor Sensitivity Analysis**

The other parameter that significantly affects the levelized costs and the relative benefits of nuclear power is the discount factor. As the discount factor decreases, the

when those investments are not *profitable* in our expected value sense but it is unlikely.
Figure 4.5: Costs and benefits from nuclear power by combined cycle overnight capital costs.
levelized costs of nuclear and combined cycle plants will increase in value, but the increase will favor nuclear power plants. In the reference case we assume a relatively high discount factor of 7.0% as a real after-tax return. We selected a range from 5% to 9% to perform the sensitivity analysis. Figure 4.6 shows the benefits from nuclear power as a function of the discount factor. Clearly, the effect of the discount factor on nuclear power plants is much larger than for combined cycle plants. The levelized costs for nuclear power plants ranged from 45.28 to 63.11 USD/MWh while for combined cycle natural gas plants they ranged from 49.66 to 51.60 USD/MWh. Hence, a 44% increase in the discount factor implies a 4% increase in the levelized costs of combined cycle plants and a 28% increase in the levelized cost of nuclear power plants.

At a rate of 6.20%, the levelized costs of nuclear power plant and combined cycle plants are the same; while up to a discount factor of 6.92% nuclear power is profitable under the reference case scenario. On the other hand, if we take the lowest interest rate, 5%, as our reference case, then a nuclear overnight cost of 3,536 USD/kW equalizes the levelized costs of nuclear and combined cycle plants. Even at a nuclear overnight investment cost of 3,889 USD/kW nuclear power is profitable. On the other hand, if we take the highest interest rate of the range as the reference scenario, 9%, then nuclear power levelized increases significantly. Specifically, nuclear plants are profitable at overnight costs of 2,377 USD/kW, while nuclear plants have the same levelized costs as natural gas plants only if overnight investment costs reaches a very low 2,183 USD/kW. Clearly, the discount factor is as important as the overnight investment costs.
Figure 4.6: Costs and benefits from nuclear power by the discount factor.
4.2.5 Heat Rate

Since fuel costs for natural gas plants represent a large part of levelized costs, it is not surprising that changes in the heat rate have the most significant effect on the levelized costs of combined cycle natural gas plants. For the reference scenario, we assume a heat rate of 6.83 MMBtu/MWh from a range of 5.68 to 8.63 MMBtu/MWh. The heat rate will also affect not only the expected value of the distribution of levelized costs, but also the standard error of the distribution. Figure 4.7 shows the benefits and costs of nuclear power as function of the heat rate of combined cycle plants. As the heat rate decreases and combined cycle plants become more efficient, the levelized costs of natural gas plants decreases and the length of the 95th-5th range shrinks. The levelized costs for combined cycle plants ranged from 43.97 to 60.73 USD/MWh. Therefore an increase of 51% in the heat rate implies an increase of 38% in the levelized costs of combined cycle plants. The heat rate is by a large margin the most significant parameter affecting the levelized costs of combined cycle plants.

Nuclear power plants are profitable for heat rates equal or above 6.88 MMBtu/MWh, while the levelized costs of nuclear and combined cycle plants is the same at a higher heat rate of 7.38 MMBtu/MWh. On the other hand, assuming the lowest heat rate in the range, 5.68 MMBtu/MWh, implies that nuclear overnight costs have to fall to 2,331 USD/kW to be profitable and even more to 2,119 USD/kW to equalize the expected levelized costs of combined cycle plants with 60% efficiencies. In contrast, assuming a large heat rate of 8.63 MMBtu/MWh, 40% efficiency, makes nuclear power profitable for overnight costs as high as 3,974 USD/kW. Even at relatively high overnight costs of 3,651 USD/kW, the levelized costs of nuclear power and combined cycle plants are equal.
Figure 4.7: Costs and benefits from nuclear power by Heat Rate.
Nuclear Fuel Costs

The main focus of our research is the variability of natural gas prices. Therefore, we provide sensitivity analysis for the fuel cost of nuclear power. The reference case assumes nuclear fuel costs of 8.25 USD/MWh. The sensitivity analysis assumes a range from 3.5 to 13 USD/MWh. Figure 4.8 shows the benefit and costs of nuclear power as functions of nuclear fuel costs. Levelized costs for nuclear power ranged from a low 48.85 to a high 58.35 USD/MWh. Therefore, an increase of 271.4% in fuel costs produced an increase of almost 20% in levelized costs. Clearly, nuclear fuel costs have a limited effect on nuclear power levelized cost. At a level of 5.15 USD/MWh, nuclear and combined cycle plants have the same levelized costs, while nuclear power is still profitable up to 7.97 USD/MWh. Finally, if nuclear fuel costs reach 13 USD/MWh, then the overnight cost should fall to 2,283 USD/kW to equalize the levelized costs from combined cycle plants and to 2,538 USD/kW for nuclear plants to remain profitable. Although nuclear levelized costs are relatively insensitive to nuclear fuel costs, it is still important to set a reasonable range for them.

4.2.6 Sensitivity on the Natural Gas Prices

Thus far we have analyzed the benefits from nuclear power simulating natural gas prices with the CH25HP5 transformation and bootstrapped standard error. In this section, we analyze the results from using the CH20HP5 transformation with normal standard errors. This second option is more conservative with respect the variability of natural gas prices. Figure 4.9 shows the Q-Q plot of the distribution of levelized costs for combined cycle plants under the two transformations; it also shows the kernel densities. Clearly, the difference in levelized cost is minimal. Although there is strong autocorrelation for weekly prices, on an annual scale the correlation is much lower, hence producing prices that might be large during a couple months but in annual
Figure 4.8: Costs and benefits from nuclear power by nuclear fuel cost.
Figure 4.9: Comparison of the distribution of levelized costs by transformation of the short term component of natural gas prices.

Averages are not much larger than other years.

As we mentioned above, our views of the long term trend of natural gas prices after 2030 might also affect the levelized costs of combined cycle plants. Consequently, in Figure 4.10 we compare the distribution of levelized costs of combined cycle plants under the two different views of long term natural gas prices.

Although the difference between the two long term options is not as small as that of the short term component, it is clear that the differences are small: 1.33 and 1.34 USD/MWh in the 90th and 95th percentile respectively. Notice that the kernel density estimates are very close to each other.

4.2.7 Conclusion

In this section we compared the levelized costs of nuclear power plants to those of combined cycle plants and analyzed the sensitivity of our results to the key parameters of the comparison. The comparison assumed a constant capacity factor for combined cycle plans while natural gas prices were uncertain. We provided several
Figure 4.10: Comparison of the distribution of levelized costs by long term component.

arguments suggesting that for the case of Mexico and for deciding at the margin, our comparison is meaningful. On the other hand, in Chapter 1 we showed that the constant capacity factor assumption has important effects on the strategy for choosing optimal investment in generation capacity strategy and cannot be used to determined the long term strategy of technology diversification.

The sensitivity analysis showed that the overnight costs for nuclear plants, the discount factor and the heat rate largely determined the benefits of nuclear power relative to those of combined cycle plants. Clearly, nuclear power plants remain competitive for a large range of overnight costs, small discount factors and high heat rates for combined cycle plants (less than 50% efficiencies). Relative to the nuclear overnight costs, the discount factor and the heat rate, the benefits associated with limiting the uncertainty in levelized costs are small (at least for our simulation of long term natural gas prices). Although the benefits associated with limiting the uncertainty of total costs are present, they are not large enough to drive a new nuclear renaissance in Mexico. Finally, different views of natural gas prices after 2030 do not have a large effect on the levelized costs of combined cycle plants.
Without high confidence in nuclear overnight costs and the discount factor, it is impossible for us to make any kind of recommendation with respect to the future of nuclear power in Mexico. Moreover, our comparison exercise clearly shows that uncertainty around key parameter of nuclear power investment can easily overcome uncertainty of natural gas prices given the current cost structure.

4.3 Optimal Generation From Hydroelectric Capacity

In this section we propose a methodology for deriving optimal investment in hydroelectric generation capacity that we believe can be successfully applied to Mexico. We simplify and constrain the generation problem by abstracting from the fact that as the water in the reservoir of the dam decreases, the capacity of the generating units also decreases. This effect is commonly referred as the “head effect” for hydroelectric plants.\(^{13}\)

In 2006, the Mexican hydroelectric system had 79 hydroelectric plants with 222 generation units and a total capacity of 10,535.9 MW. The hydroelectric system represented 24.1% of the total installed capacity for Mexico. Moreover, in 2007 the new project “El Cajón” will add 744 MW of hydroelectric capacity. Between 2011 and 2016, two projects are scheduled: “La Yesca” (2011) with a planned capacity of 750 MW and “La Parota” (2015) with a planned capacity of 900 MW. Finally, the expansion of Villita by 400 MW (2014), Infiernillo by 200 MW (2014) and Zimapan by 566 MW (2016) will add a total of 3,560 MW of capacity by 2016.

Given the importance of the hydroelectric system in Mexico, careful modeling of

\(^{13}\)The exact relationship depends on the physical characteristics of the dam and the amount of sediments in the streams flowing into the reservoir and it is usually the relationship is complex and non-linear.
the mexican hydroelectric system is critical to choosing optimal investment for the
system. This is especially so when one recognizes the stochastic nature of natural
gas prices. It is also especially important since, at least theoretically, hydroelectric
systems can withhold water in low demand periods and release it in high demand
periods flattening the load curve and reducing the need for thermal peak capacity.
Flattening the load curve can also raise the demand for base load power and make
nuclear capacity more attractive.

4.3.1 The Mexican Hydroelectric System

Comisión Federal de Electricidad divides the Mexican hydroelectric plants in those
that serve a dual purpose of electricity generation and the regulation of water re-
lease and hydroelectric plants that have limited storage and thus are limited in their
generation capacity by the river conditions.

The first group is composed of the ten largest hydroelectric plants (LHP): An-
gostura, Chicoasén, Malpaso and Peñitas (Grijalva river); Caracol, Infiernillo and
Villita (Balsas river); Temascal (Tonto y Santo rivers); Aguamilpa (Santiago river),
and Zimapán (Moctezuma river). Table 1 shows important information on each of
these plants.

The second group is composed of 69 hydroelectric plants that, given their char-
acteristics, and the desire to minimize spillage, are constrained to provide output in
short time intervals (weekly or daily). On average, they produce a total of 6,639 GWh
per annum.

The capacity of hydroelectric plants depends on their ability to withhold water
during the rainy season to be released during peaks in the value of electricity. The
regulation index in Table 1 is calculated by dividing the reservoir capacity of each dam
by the independent yearly average water inflows to the dam. As the regulation index
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² Independent yearly average water inflows.

Table 4.3: Characteristics of the Largest Hydroelectric Plants México/¹.
Figure 4.11: Degradation of Potence for Selected Hydroelectric Plants.

increases, the amount of time the plant can last without spillage increases. Values above 50% represent a real possibility that the dam can be used for interannual regulation of water.

In Table 1 we observe that the Chicoasén, Penñitas and Villita plants cannot be used for long term storage and in general will produce according to the run on the river concerned. Villita solely depends on the water releases from Infiernillo. In contrast, the other LHP can be potentially used for longer term storage compromised only by the degradation of output as the level of the dam declines.

Figure 4.11 shows the degradation of output with respect to the water level at each of the remaining LHP. It is easy to see that the degradation in Malpaso, Infiernillo and to a lesser extent, Aguamilpa is relatively high. Therefore these plants should be kept at the maximum level of operational waters (MaxLOW). In contrast, Angostura, Temascal and Zimapán can potentially be used as storage units for some parts of the year. Finally, notice that Angostura has the largest reservoir and regulation index of the dams under consideration.
Generation from the LHP is also constrained by a minimum operating capacity. Table 4.3.1 shows this restriction by month for each of the LHP. Angostura is particularly important not only because of its ability to withhold water but also because the water released at Angostura will generate energy in Chicoaén, Malpaso and Peñitas. If it is optimal to release the water from Angostura when the own inflow in Chicoaén, Malpaso and Peñitas is low, then water withheld can generate up to 2,820 MW.

4.3.2 Optimal Generation from Hydroelectric Plants

The optimal scheduling of hydroelectric generation is a large mathematical problem that remains unsolved in a general form and in most cases is partially modeled and solved depending on the problem at hand. Uncertainty in water inflows and the value of electricity, the head effect, constraints on water releases for other purposes, the constraints on dam capacity and the dynamic nature of the choice variables all make it difficult, if not impossible, to find a general solution. Our specific focus at the moment is determining the optimal investment in natural gas generating capacity given the stochastic nature of gas-fired generation costs. Therefore, we decided to abstract from the head effect (at least in its general form) and focus on the inter-temporal distribution of water releases and its effects on the price of electricity and the required generation from natural gas.

In the following section, we introduce our simplified version of the hydroelectric generation problem. We analyze the structure of the problem and provide an algorithm to compute its solution. Then, we comment on the ability of the proposed methodology to deal with hydroelectric dams in sequence and multiple hydroelectric plants.
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/1 Table is part of “Cuadro A-4” page A-2 in “Programa de Inversiones Électricas 2007-2016”, CFE 2006.

Table 4.4: Minimum Generation Required for Operation (Gwh/month)/1
4.3.3 The simplified version of the hydroelectric generation problem

We abstract from the head effect, and from uncertainty in water inflows, demand and generation cost. The main aspects we consider are the optimal use of nuclear (or base load technologies) and natural gas capacity, dam capacity constraints including a minimum level of water,\textsuperscript{14} generation constraints, seasonality of water inflows, generation cost and demand.

Since nuclear plants are costly to shut-down and re-start, nuclear generation should be set equal to full capacity for all periods and any price of electricity.\textsuperscript{15} Therefore, optimal nuclear generation is included into the hydroelectric optimal generation problem by working with demands net of nuclear capacity.

To account for the seasonality of water inflows, we divide the total number of hours of the finite planning horizon, \( \theta_T \), into \( H \) hydrologic cycles. To allow for variation in demand, we divide each hydrologic cycle into \( N \) demand periods. Denote the duration of demand \( n = 1, \ldots, N \) at hydrologic cycle \( h = 1, \ldots, H \) in hours as \( \theta_{n,h} \) then,

\[
\sum_{n=1}^{N} \theta_{n,h} = \theta_{,h} \quad \text{and} \quad \sum_{h=1}^{H} \theta_{,h} = \theta_T.
\]

Water inflows per hour during hydrologic cycle \( h = 1, \ldots, H \) are denoted as \( \omega_h \). Water outflows that go through the turbines during hydrologic cycle \( h \) and demand period \( n \) are denoted as \( \omega_{n,h} \) and spillage during hydrologic cycle \( h \) is denoted as \( s_h \). The demand, inverse demand and consumer surplus are denoted as \( d_{n,h}, p_{n,h} \) and \( CS_{n,h} \) respectively. Finally, the natural gas generation cost is denoted as \( \rho_{n,h} \).

The dam constraints are denoted as follows. The dam capacity is denoted by \( \bar{k} \),

\textsuperscript{14} The minimum level of water constraint might be different than the minimum level of water for operation (MinLWO) since from a careful choice of the minimum level of water, it is possible to partially include the head effect.

\textsuperscript{15} This is also true for base load thermal capacity such as coal plants.
the minimum level constraint as \( k \). Although the generation capacity of a dam is constant, forced generation as a result of required water releases or other constraints can introduce differences in the hydroelectric generation per period. Denote the available generation capacity at demand period \( n \), hydrologic cycle \( h \) as \( q_{n,h}^H \). The generation capacity of natural gas is constant and is denoted as \( q_G \).

Water releases are transformed to generation according to the constant parameter \( \phi \), i.e. a water release of \( \omega \) will generate \( \phi \omega \) units of electricity.

The optimal generation problem for a hydroelectric plant is given by:

\[
\max \sum_{h=1}^H \sum_{n=1}^N \theta_{n,h} CS_{n,h} (\omega_{n,h} \phi + g_{n,h}^G) - \rho_{n,h} g_{n,h}^G \quad (4.3)
\]

subject to

\[
q_{n,h}^H / \phi \geq \omega_{n,h} \geq 0, q_G \geq g_{n,h}^G \geq 0
\]
\[
k \leq k_h \leq \bar{k}, s_h \geq 0
\]

for all \( h = 1, \ldots, H \) and \( n = 1, \ldots, N \).

where

\[
k_h = k_{h-1} + \theta_{n,h} w_h - \sum_{n=1}^N \theta_{n,h} \omega_{n,h} - \theta_h s_h
\]

for all \( h = 1, \ldots, H - 1 \) and \( k_0 = k_i, k_H = k_f \)

The constants \( k_i \) and \( k_f \) denote the initial and final water levels at the dam respectively.

Unfortunately, a unique solution of the optimal generation problem cannot be
guaranteed. To show this result, consider the following example. Assume that the natural gas capacity is very large and the generation cost is the same for all periods. Then, with no hydroelectric generation, the price of electricity will be the same for all time periods and the central planner is indifferent between releasing water at any period and the solution is not unique as desired. Moreover, the result is maintained even if a discount factor is included.

The number of constraints and choice variables also makes the analysis of the first order conditions cumbersome. Instead, we propose an algorithm to find a particular solution to the maximization problem.

4.3.4 The Algorithm

The algorithm is based on an arbitrage of marginal benefits between periods. Given the dam and generation constraints, each period can be active (if some water can be scheduled for release without breaking any constraint) or inactive (either a dam constraint or a generation constraint is already binding). The algorithm starts by checking whether there are any remaining active periods where water releases can be pre-scheduled and it continues until there are no more active periods.

If there are active periods the algorithm selects the period or periods with the highest marginal benefit in which to release water. If more than one period is selected, then they all have the same marginal benefit.

After selecting the tentative periods for pre-scheduling hydroelectric generation, the algorithm checks between the selected periods to see if there are periods where natural gas generation is displaced. Scheduling in periods where natural gas generation is being displaced is preferred since, given our assumption that natural gas costs are constant within each period, the marginal benefit of water in such periods is constant. In contrast, in periods where natural gas generation is not being displaced,
the marginal benefit of water is equal to the marginal willingness to pay, which has a negative slope.

Assuming that natural gas generation will be displaced for some periods, it is optimal to preschedule the minimum of the amount of extra hydroelectric generation required to completely displace natural gas and the hydroelectric generation capacity.

On the other hand, if natural gas generation is not being displaced, then there are two possible situations. Either there is an active period with lower marginal benefit or there isn’t an active period with lower marginal benefit. In the first case, the central planner will pre-schedule as much generation as possible in each period without violating several conditions. These include first that there be an equality between marginal benefits across periods. Second, the generation constraint needs to be respected. Third, generation from natural gas capacity needs to be constant. Finally, marginal benefits cannot be reduced below the next lowest marginal benefit. In the case where there are no active periods with a lower marginal benefit, the central planner will pre-schedule as much generation as possible without breaking the equality between marginal benefits or the generation constraint, while also maintaining a constant natural gas generation capacity.

Once a tentative preschedule is obtained, the algorithm checks the dam capacity constraints for all the periods, reducing prescheduled generation for any periods that make a dam constraint binding. It does this in chronological order.

The reduction of prescheduled water releases depends on whether the selected periods displace natural gas generation or increase the total generation of electricity. In the first case, we choose to make the reduction in each period proportional to the prescheduled generation. In this case, the marginal benefits are the same for all periods, and our solution is only one of the possible ones. The second case is somewhat more complicated since the active periods must be reduced in such a way
that the equality between marginal benefits is maintained. Whether it is preferable to solve the system of non-linear equations or to linearize the inverse demands around the total scheduled generation and solve the resulting linearized system depends on the computational resources available. In our implementation of the algorithm, we linearized the inverse demand at the scheduled generation.

The process of checking the dam constraints and reducing the prescheduled hydroelectric generation is done iteratively until no dam constraint is violated. Finally, the prescheduled generation is added to the scheduled generation and any periods where any constraint is binding are deactivated.

Figure 4.12 shows the flowchart of the algorithm. Notice that the algorithm schedules water releases where they have the most value and a period is deactivated only when no more water can be scheduled. Therefore, the algorithm reaches the optimal level of water generation as desired.

Unfortunately, the optimal generation from the algorithm as presented abstracts from important aspects of operating a real hydroelectric system, below we comment on the implementation of the algorithm in real hydroelectric systems.

4.3.5 Comments on the applicability of the algorithm

Most hydroelectric systems contain multiple hydroelectric plants. Some hydroelectric systems have single suitable location for hydroelectric plants, but others, commonly referred as hydroelectric systems for hydroelectric plants in a chain, have multiple suitable locations.

In the absence of transmission constraints, the optimal generation problem of separated hydrological plants can be solved by ordering the hydroelectric plants and then solving the single hydroelectric plant problem previously presented using net demands for each of them. The order does not affect the total generation of electricity.
Figure 4.12: Flowchart of the proposed algorithm.
by the hydroelectric system, the generation of electricity by natural gas capacity or
the price of electricity.

For the case of hydroelectric plants in a chain, the optimal generation problem
is quite different from the one hydroelectric plant case. Under some conditions, the
one hydroelectric plant problem can approximate the solution of the hydroelectric
system. Specifically, if the first plant of the system has a large reservoir of water, and
the downstream plants have either a low reservoir or high degradation of power and
negligible independent sources of water, then we can model the hydroelectric system as
one big dam. On the other hand, if the chain hydroelectric system does not satisfy this
requirement, or if the downstream plants have important independent water inflows,
then the one plant problem can still be used to approximate the solution of the
hydroelectric system. The hydroelectric plants are solved according to the ordering in
the hydrological system. In general, the solution is not correct, but if the downstream
plants are subject to rapidly diminishing output as the head falls or possess a low
reservoir, chances are that our proposed solution approximates the optimal solution.
Moreover, if the water releases are bound by the generation capacity constraint for all
periods, then our proposed solution may again approximate the optimal solution. The
optimal solution for hydroelectric plants in a chain is beyond the scope of our paper
and the sensitivity of the results from the algorithm to alternative configurations of
the generation from the hydroelectric system should be analyzed and documented.

4.3.6 Implementation to the Mexican Case

The two hydrology systems with chain hydroelectric plants in Mexico are the Grijalva
river and the Balsas river.

In the Grijalva river, only two dams (Angostura and Malpaso) have enough ca-
pacity to transfer water inter-annually. The degradation of power with decreasing
head is quite high for Malpaso, implying that it will be a run of river plant. Since
the three dams downstream from Angostura have either rapidly diminishing power
with decreasing head of water or a low reservoir, we are confident that our proposed
algorithm can be used to approximate the solution.

The case of the Balsas river does not present any problem since Infiernillo presents
high degradation of power with decreasing head, Villita doesn't have independent
inflows of water and Caracol cannot store water easily.

Finally, Temascal, Aguamilpa and Zimapán are distinct hydrology systems and
fit our proposed description of the problem. Hence, we are very confident that our
proposed solution of the optimal hydroelectric generation problem will closely ap-
proximate the actual solution.
Bibliography


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