RICE UNIVERSITY

Stochastic Fatigue Analysis of FPSO Topside Structures with Linear and Nonlinear Supports

by

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ABSTRACT

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Floating Production, Storage, and Offloading (FPSO) Systems are quite often subjected to stochastic sea wave loadings. In this thesis, a methodology is developed for estimating the fatigue life of topside structures. Proper Response Amplitude Operator (RAO) of the FPSO system, and the well-known Ochi-Hubble sea wave elevation spectrum are combined to provide the design spectrum at the deck level on topside FPSO. For ordinary Single-Degree-of-System (S-D-O-F) piece of equipment, the dynamic response is simulated by a time series model. A non-recursive Rainflow cycle counting method is applied to the equipment stress time history to identify significant cycles that produce fatigue damage in the time domain. The results of the Rainflow cycle counting method are supplemented by results from a power spectrum based, exclusively, approach. Further, pipe systems with/without limit stops on topside FPSO are modeled as Bernoulli-Euler beam. A Galerkin method is therefore employed in conjunction with the beam random vibration theory. Specifically, the statistical linearization technique is adopted to derive the equivalent linear system for the pipe example with nonlinear constraints. The applicability
of the proposed approach is demonstrated by the analysis of both a simple S-D-O-F piece of equipment and by an illustrative example of pipeline conveying fluid with/without nonlinear constraints subject to sea wave loading. The proposed in this thesis integrated approach can be used for the stochastic fatigue analysis of structures on topsides FPSO during preliminary design when piping system responses must be estimated analytically. Further work can consider the coupling between the transverse and the longitudinal response of the pipelines on topside of FPSO and related issues.
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Nomenclature

$R_x(\tau)$  Autocorrelation function of stochastic process $x(t)$

$S_x(\omega)$  The spectral density or power spectrum of stochastic process $x(t)$

$m_n$  The $n$th order moments of power spectrum $S_x(\omega)$

$\sigma_x$  Standard deviation of stochastic process $x(t)$

$p(x)$  Probability density function for stochastic process $x(t)$

$a$  Amplitude of stochastic process $x(t)$

$p(a)$  Probability density function of the amplitude $a$

$\varepsilon$  Spectral width parameter

$S_y(\omega)$  The spectral density or power spectrum of stochastic process $y(t)$ which is the output or response of a linear system under the input or excitation $x(t)$

$H(\omega)$  Frequency transfer function relates the input $x(t)$ and output $y(t)$

$H_{MA}(\omega)$  Transfer function of the MA difference equation

$q$  MA filter order

$b_v$  MA filter coefficient

$W(n)$  Stationary zero-mean white noise process with power spectrum $S_w(\omega) = 1$

$P_{MA}(\omega)$  Simulated acceleration or displacement spectrum through AR model

$m_k, M_k$  Local minimum, and local maximum

$m_k^-$  Backward (left) minimum
\( m_k^+ \)  
Forward (right) minimum

\( t_k \)  
Time when there is \( k \)th maximum

\((m_k^{RF}, M_k)\)  
\( k \)th Rainflow cycle pair

\( S_k^{RF} \)  
\( k \)th stress level (amplitude) for damage calculation

\( F^{RF} \)  
Rainflow cycles matrix

\( F \)  
Min-max matrix

\( \hat{F} \)  
Max-min matrix

\( N(s) \)  
Number of cycles at stress amplitudes \( s \) when failure occurs obtained from the S-N curve.

\( \varepsilon, \beta \)  
Material constant

\( D_i \)  
The fraction of damage suffered by the structure component due to \( n_i \) cycle at stress level \( s_i \),

\( T_f \)  
Service lifetime of the component

\( \Delta D_i \)  
Damage rate associated with a sample time \( t \)

\( N^+(s; T) \)  
The expected number of crossings of the level \( s \) with positive slope in the interval \([t, t + T]\).

\( S_{sea}(\omega) \)  
Sea wave elevation spectrum

\( S_{wind\,sea}(\omega) \)  
Individual wind generated sea wave spectrum, the high frequency part of \( S_{sea}(\omega) \)

\( S_{swell}(\omega) \)  
Swell spectrum, the low frequency part of \( S_{sea}(\omega) \)
$H_s$  Significant Wave, the average of the highest $1/3$ $th$ wave population

$\omega$  Radian frequency

$\omega_p$  Peak radian frequency

$\Gamma$  Gamma function.

$\lambda$  Peak enhancement factor

$H_o^{(i)}(\omega, \theta_i)$  Elementary wave direction transfer functions for $j$ locations

$H_o^{mean}(\omega, \theta_i)$  Average elementary wave direction transfer functions of various locations

$H_o^{mean}(\omega, \bar{\theta})$  Average elementary wave direction transfer functions of various locations in the heave direction

$\bar{\theta}$  Main wave direction

$D(\omega, \theta_i)$  Spreading function

$\theta_i$  Spreading directions

$S^{mean}(\omega)$  Mean sea wave spectrum at the FPSO topside, associated with the heave degree-of-freedom

$S_v^{mean}(\omega)$  FPSO deck level velocity mean spectrum

$S_a^{mean}(\omega)$  FPSO deck level acceleration mean spectrum

$\sigma_{S_a(\omega)}$  Standard deviation of FPSO deck level acceleration mean spectrum

$S^{max}(\omega), S_{\text{design}}(\omega)$  FPSO deck level acceleration max spectrum which is the design spectrum $S_{\text{design}}(\omega)$

$S_{d \_\text{structure}}(\omega)$  1 SDOF piece of equipment displacement spectrum in heave direction
$S_{a_{\text{structure}}} (\omega)$ 1 SDOF piece of equipment acceleration spectrum in heave direction

$\sigma_{\text{equipment}}$ 1 SDOF piece of equipment cross section stress

$S_{\sigma} (\omega)$ 1 SDOF piece of equipment stress spectrum

$U$ Fluid constant axial velocity $U$ relative to the pipeline

$L$ Span-length of the pipeline

$d_e, D_e$ Internal and external diameter of the cross-section of the pipeline

$E$ Modulus of elasticity of the pipeline material

$I$ Moment of inertia of the cross-section of the pipeline

$m_p$ Mass of the pipeline per unit length

$m_f$ Mass of the fluid per unit length

$\ddot{w}(x,t)$ Total displacement of the pipeline

$w(x,t)$ Pipeline relative displacement to the supports in heave direction

$h(t)$ Random displacement response at the supports due to the FPSO hull undergoes sea wave loading in the same direction

$p_{\text{eff}} (t)$ Effective random load due to the support motion

$R$ Integer number of degree-of-system of a lumped parameter system

$\phi_i (x)$ Basis functions satisfying the boundary conditions

$q_i (t)$ Generalized coordinates

$\delta_{ij}$ Kronecker delta symbol

$\Xi, \hat{\Omega}$ R th order square matrices
\( \hat{\omega}_k \)  
- \( k \)-th natural frequency of the discretized pipeline with fluid inside

\( \hat{\omega}_i \)  
- 1st order natural frequency of the discretized pipeline with fluid inside

\( S_q(\omega) \)  
- Power spectrum matrix of the stationary random vector process \( q(t) \)

\( \alpha(\omega) \)  
- Frequency-response transfer functions matrix

\( I \)  
- \( r \)th order identity matrix

\( S_u(\omega; \bar{x}) \)  
- Power spectrum of the pipeline displacement at the coordinate \( x = \bar{x} \)

\( s_{\text{max}}(t) \)  
- Maximum stress in the cross-section placed at the coordinate \( x = \bar{x} \)

\( M(x,t) \)  
- Random bending moment along the pipeline

\( S_{s_{\text{max}}}(\omega; \bar{x}) \)  
- Power spectrum of the maximum stress of the pipeline

\( \sigma^2_{s_{\text{max}}}(\bar{x}), \sigma^2_{\dot{s}_{\text{max}}}(\bar{x}) \)  
- Stationary variance of \( s_{\text{max}}(t) \) and \( \dot{s}_{\text{max}}(t) \)

\( K \)  
- Spring stiffness introduced by limit stops located on each side of pipeline

\( \omega_{\text{ef}}^2 \)  
- Equivalent natural frequency introduced by the limit stops

\( a \)  
- Distance between the limit stops and the pipeline
Chapter 1 Introduction

1.1 Motivation

The steadily increasing need for oil and gas in the past several years has stimulated marine offshore exploration in increasingly deeper waters. As a result, it has created the design and erection of a wide range of new offshore platforms in deep sea regions around the world. Floating Production, Storage and Offloading (FPSO) Systems have gained growing importance in the development of offshore oil fields. The use of FPSO systems dates back approximately to the mid-1970’s, and these systems have already been installed in more than 50 offshore fields. In fact, they represent an economic solution to the processing, storage and periodical offloading of oil and gas from the nearby sub-sea wells in deepwater fields. A large percentage of existing FPSOs are constructed from the conversion of tankers, and the number of new-built units is growing dramatically worldwide. It is expected that five to ten new FPSO systems will be put into use each year in the near future [ref. 1, 2].

Compared to other traditional fixed offshore platforms, FPSO systems have considerable advantages, such as easy decommissioning after a completed field production, option to support crude oil treatment facilities, and ability to support crude oil storage, thereby eliminating the need for long pipelines to the shore.

Typical FPSO systems, as shown in Fig 1.1, are mostly ship shaped floating structures, which are coupled by means of rotary bearings to a cylindrical structure, the turret. The turret is anchored to the sea bottom by means of a mooring line spread. A system of flexible risers connects the turret to the sea bottom equipment, and to a system
transferring fluids, energy, and control between the turret and the whole floating structures for all its possible headings. Therefore, FPSO systems are capable of "weathervaning" around the turret in order to minimize the environmental loads acting on them.

![Image of FPSO systems among offshore platforms](image)

Fig. 1.1 FPSO systems positioned among many offshore platforms (Courtesy of Aker Marine Company)

Once reservoir fluids enter the FPSO, they will be moved up and transferred into the topside modules. In the separation and compression modules, the fluids are separated into oil, water and gas streams. The processed oil is then routed to the storage tanks, from where it will eventually be loaded onto tankers and shipped to the market.

Being a relative new and complicated concept for production of oil and gas, and breaking new ground in terms of the environmental conditions, a thorough design process is required for floating production systems.

FPSO systems usually operate in a very complex and hostile environment, and are subjected to a number of loadings and stresses during their lifetime. There are mainly two kinds of loadings acting on these huge floating platforms: functional loadings, and
environmental loadings. Functional loadings are quasi-static, which vary slowly over time, and are due to the steel weight, the ballast, the desk loads, and the reaction forces. Environmental loadings are caused by sea waves, currents, and wind, which in most cases are stochastic in nature. In particular, the sea wave loadings are dominant for the main parts of these offshore systems. Therefore, in this thesis only the sea wave loadings are considered into the computation.

Sea waves are one of the most complex and ever-changing phenomena in nature. Loosely speaking, the sea is a large body of water bounded by irregular shorelines, an undulating permeable bottom, and a wavy free moving surface. Waves at the sea surface are generated primarily by the wind blowing over the water surface, but continue to exist after the wind has ceased to affect them. When the wind generated waves move out of the generation area into an area of calm winds, or if the wind ceases to blow, they change to swells, start to decay, and slowly change shape. A swell travels a very long distance without losing its identity, while the life of an individual wave is limited to a much shorter period. Therefore, swells are not easy to ignore if the magnitude of sea waves is comparable to that of the swells in the circumstances of a mild wave climate [ref. 3]. This is the case studied in this thesis, and the sea wave loadings are composed of both the wind wave (high frequency part) and the swell wave (low frequency).

Specifically, a sea wave relates to the profile between two successive up (or down) crossings of the zero line. The wave period, $T_z$, is the time interval between the two successive zero-up-crossings. The wave height, $H$, is the vertical distance between the highest and the lowest points between the two successive zero-up-crossings. Significant Wave ($H_s, T_s$) is the average of the highest $1/3$ th of the wave population. Usually, the
sea state is defined in terms of the significant wave height and the mean zero crossing rates. In deep sea [ref. 4] \((d/L > 0.5)\), where FPSO systems are generally operated, the sea bottom does not influence the surface waves. Here, the symbol \(L\) denotes the wave length or horizontal distance between successive crests, and \(d\) is the distance from the mud line to the mean wave line.

Waves of different periods and heights are superimposed on one another to form the wave spectrum. The wave spectrum throughout this study is the energy density computed in terms of the water surface elevation. The sea waves have a stochastic nature, the long-term behavior of which is generally considered non-stationary, and non-Gaussian. In a short time interval (hours), the statistical properties of the sea state may be considered stationary. Specifically, gross observations in the past have, generally, confirmed a Gaussian distribution for the water surface elevation. Thus, in this study, the theory of stationary stochastic processes is extensively used for their description.

All loads that are time-varying in magnitude and/or direction will cause stress variations in structures, and finally may lead to fatigue damage. The relationship of sea waves to FPSO systems can not be an exception to this.

The most catastrophic scenario for an FPSO system includes structural failure of hull girders due to extreme bending moments. A complete methodology for the time variant reliability assessment of FPSO hull girders subjected to degradations due to corrosion and fatigue has been presented early in 2001 by Sun and Bai [ref. 5-6]. Lotsberg has described the probabilistic inspection planning of FPSO hull structures with respect to fatigue [ref. 7]. The FPSO system is still treated as a whole structure in the fatigue orientated analysis study conducted [ref. 8-10]. This study lacks detailed analysis
of equipment/structures as a separated component undergoing the sea wave loadings transferred from the hull. Recently, due to the popularity of FPSO systems, up-to-date reliability analysis results have been presented again by Bai [ref. 11-12] as the expert in this field. Carlos De Lemos has formulated a fatigue calculation methodology of the flexible risers connected to FPSO systems. An integrated model of a ship body, mooring lines and risers to define the distribution of ship headings for fatigue analysis by using an irregular bi-directional sea approach are evaluated in his paper [ref. 13]. Certain industrial codes have integrated the hull hydrodynamic analysis and the load predictions for the design and fatigue life calculation based upon some 3D finite element models, as shown in Fig. 1.2. Dynamics of a large moored floating body in ocean waves involves frequency dependent added mass and radiation damping, as well as the linear and nonlinear mooring line characteristics. Therefore, coupled hull-mooring-riser system analyses dealing with non-linear response in the time domain have also been conducted using these codes. In addition, some researchers have also derived certain numerical results for coupled analysis of FPSO systems [ref. 14].

Fig. 1.2 FEM model of FPSO hull
Specifically, the fatigue problems of an FPSO also include the fatigue damage of supporting structures on topside. The problems are caused by the sea wave loadings transferred to the hull. Basically, the high-pressure gas/oil process equipment and piping systems are originally not designed for excessive deformations. This fatigue damage brings huge costs to the oil production industry in consideration of the expensive offshore work rate, since repair or replacement will require shutdown of the entire production. More seriously, there may be unexpected casualties. Nevertheless, there is a lack of a comprehensive mathematical formulation for analyzing the dynamic response of topside modules, i.e., surge, sway, yaw, heave, roll and pitch, under complicated sea wave loadings, and for analyzing wave-induced fatigue damage of the topside structure modules. In fact, ABS [ref. 15] has outlined the general guide on fatigue analysis of FPSO systems based on the spectral method. It specifies that the stress transfer function is determined by the finite element method (FEM) of structural analysis using a three dimensional (3-D) model. This 3-D model incorporates the entire hull structure, the topside equipment support structure, and the interface with the mooring system. For a specific structure, more fine mesh FEM analysis should be performed to obtain a more precise stress distribution. The demanding FEM computation of relatively many different topside structures is not affordable or practical for many industry design companies, especially in the preliminary design period. By contrast, the dynamic structure analysis can give the direct and economic solution which is elucidated in this thesis.

Specifically, the topside facilities consist of several super-modules (processing, wellhead, mud, utilities, and accommodation), as well as some topside mounted structures (helideck, flareboom, pipe rack, main and auxiliary lifeboat stations, and two
drilling modules) as shown in Fig. 1.3. The helideck can be treated as the truss elements as in Fig. 1.4, and the pipelines with fluid inside can be treated as single span beam element subjected to support motion. This study is devoted to formulating a methodology to estimate sea wave induced fatigue life of structures located on topside of FPSO systems.

![Fig. 1.3 FPSO topside (Courtesy of Fluor Corporation)](image1) ![Fig. 1.4 FPSO topside modules](image2)

1.2 Objectives

This thesis aims to provide an integrated and affordable approach for the fatigue life analysis of linear and nonlinear structures on topside FPSO systems subjected to sea wave excitations, which is transferred through the hull body to the main deck floor. To achieve this objective, the research study follows specific steps starting with analysis of the uni-axial fatigue of simple linear structures towards the determination of the complex fatigue life of pipelines conveying fluid on topside of FPSOs. Specifically, the Rainflow cycle counting based fatigue life prediction approach in the time domain, and the stress crossing based method in the frequency domain, accomplishes the following objectives.

The classical Rainflow counting algorithm involved in uni-axial fatigue is first reviewed, and a new non-recursive algorithm is introduced which is more efficient with
regard to computing cost. This non-recursive algorithm is followed by the post procedure damage calculation. In addition, the frequency domain approach based on stress crossing theory is presented, and is used to compare to the one used in the time domain to verify the effectiveness and correctness of the new algorithm.

The sea wave loading data on the main deck floor of the FPSO systems, regarded as the base acceleration to topside structures, is generally given as power spectra in the frequency domain. An appropriate Monte Carlo algorithm, the Moving Average (MA) algorithm, is employed to generate a stationary random process simulating the sea wave loading since the fatigue damage cycle counting procedure is conducted in the time domain. The Galerkin method is involved in this study to discretize the equation of motion of a simple supported beam on both ends, simulating the pipeline structure. Furthermore, statistical linearization technique is used to obtain the nonlinear system response when limiting devices are contemplated for the middle of the pipe.

To assess the applicability of the proposed method, both the fatigue life of a simple generic piece of equipment and of the linear and non-linear pipeline structure examples are calculated. The results pertaining to all the numerical examples are also delivered.

1.3 Thesis Organization

There are seven chapters in the thesis including the Introduction and the Summary of Results. The first chapter is devoted to the introduction and discussion about the motivation and the objectives of this work. A short review of FPSO systems is presented to introduce the basic characteristics of FPSO systems and to illustrate random sea wave
loading conditions. Past related research work on FPSO systems is presented so as to introduce some background knowledge in this field, and the necessity and applicability of this research work is elucidated. The technical steps involved in this context are simply described.

Chapter 2 presents background information regarding the methodology for calculating the random un-axial fatigue life which is crucial to achieve the research objectives of this study. The first part of Chapter 2 elucidates the so called Rainflow algorithm and the damage rule involved in the time domain uni-axial fatigue estimation. A new non-recursive cycle counting algorithm is then introduced in detail, which is more efficient regarding the computing cost. For completeness, a frequency domain fatigue life calculating approach based on the stress crossing theory is then briefly reviewed in the remaining part of Chapter 2.

The third chapter presents the dynamic characteristic analysis of FPSO systems, which is applicable to both linear and non-linear structures on topside of FPSOs. In this context, discrete on-site transfer function values at different locations (modules) on the topside of the FPSO systems for various spreading directions are obtained directly from industry codes, and incorporated in this analysis. Further, the Ochi-Hubble sea wave spectrum, which can represent various sea conditions, is used, and is multiplied by the modulus of the averaged proper elementary wave transfer functions to derive the loading spectrum in the main wave direction at the deck level. The acceleration power spectrum of the deck level is next obtained by using the relationship between displacement and acceleration. The design spectrum was chosen by adding 3 times the standard deviation of the obtained spectrum at each sampling frequency to accommodate the most
unfavorable condition of the structure and is used in the following chapters to calculate the fatigue life of different topside equipment/structures.

The content continues with the application of the methods developed in conjunction with the dynamic characteristic knowledge of FPSO systems. In Chapters 4, 5 and 6, the application of these methods to specific linear and non-linear FPSO topside structures and the pertinent results are presented, respectively.

For the linear case, a generic SDOF piece of equipment model is first considered in Chapter 4. The response acceleration spectrum is then obtained by using the dynamic system transfer function. Next, the stress time series of the obtained spectrum is simulated by the MA algorithm, and is used to conduct the fatigue damage cycle counting. Finally, the equipment fatigue life is calculated in both time and frequency domains.

A straight uniform single-span pipeline containing fluid flowing with constant axial velocity is considered in Chapter 5. The sea wave induced excitation is modeled as the random support motion. Both end supports are assumed to be fixed in order to simplify the dynamic interaction with the supporting structure. Since the mass and the stiffness of the FPSO are much greater than those of the pipeline, this assumption is quite reasonable. The equation of motion of the fluid-carrying pipeline is derived based on several assumptions, such as small amplitude displacements, modeling the empty pipeline as a Bernoulli-Euler beam, and adopting the so-called “plug-flow” approximation for the fluid.

A brief review of the previous work on pipelines conveying fluid is first outlined. The fluid flow complicates the dynamic behavior of the pipeline as well as the pertinent
mathematical model. Therefore, the mathematical formulation of the problem is next carefully examined relying on some well-established theoretical results.

The Galerkin method is employed to obtain the random response of the pipeline in terms of appropriate basis functions, and the mathematical equation of motion is therefore discretized. In the present formulation, the natural modes of a beam are selected as basis functions which have the same boundary conditions as the pipe structure. Once the discretized equations of motion are obtained, fatigue damage assessment is performed, resorting to a frequency domain approach. For this purpose, the stress spectrum for a generic section of the pipeline is computed using the well-known random vibration theory input-output relationships and the design spectrum. Finally, this spectrum is used to estimate fatigue life for the pipeline by relying on the linear damage Miner's accumulation rule, and by incorporating into the analysis an appropriate S-N fatigue curve of the material. An application of the proposed approach regarding a pipeline simply supported at both ends is presented, and the fatigue life of an illustrative example is calculated.

Chapter 6 extends the work of Chapter 5 to analyze the effect of the nonlinearity of a fluid-conveying pipe introduced by the limit stops on each side. After considerable mathematical manipulations and appropriate simplifications, closed-form solutions to calculate the fatigue life have been obtained by applying the statistical linearization technique.

The investigation of the questions and issues in this thesis is concluded in Chapter 7, which summarizes the research findings, and proposes future research steps. It is emphasized that the dissertation develops an integrated and practical approach for the
stochastic fatigue analysis of structures on topside of FPSO systems. The results of this research may be applied for the preliminary design of engineering structures, and for the reliability assessment and the control of structures subjected to sea wave induced random fatigue loadings. Specifically, the technique developed through this study could be used in those situations where piping system responses have to be estimated analytically, in preliminary and even final design cycles of pipelines on FPSO structures.
Chapter 2 Random Uni-axial Fatigue Life Calculation

The random uni-axial loading data are given either as time serials in the time domain or power spectra in the frequency domain; and the latter case occurs mostly in the offshore industry, which is the area of interest of this study. In the case that the uni-axial loading spectra are given, the data can be treated as realizations of a stationary random process through Monte Carlo simulation, thus the existing fully developed uni-axial fatigue life prediction method can be used for the continuing study. Another option is that the data can be directly incorporated into the spectral analysis of fatigue life based on stress crossing theory, which can be used as a calibration to compare with the results obtained from the Rainflow algorithm.

2.1 Random Vibration and Vibration Induced Fatigue - A Review

The theory of random vibrations has been developed in order to predict the response of structures subject to random environments, such as the response of bridges and buildings subject to the action of the wind or to earthquakes, the response of boats or offshore platforms subject to the random sea wave action, the response of vehicle suspensions and chassis subject to road turbulences, and the spatial promoters subject to harsh acoustic environments at the time of launch.

The development of this random vibration theory requires much knowledge and it works in different domains such as structural structures dynamics, random signals analysis, and probability theory. With the development of civil and military spatial
exploration programs, the first theory of random vibration appears in the 1950s. Among these works, taken in chronological order, the major works of Crandall & Mark [ref. 16] in 1963 have established a milestone. Further, Lin has made important contribution to random vibration mathematical theory in 1967 [ref. 17]. Simultaneously, important progress has been realized in the field of random data measurement. Bendat & Piersol [ref. 18] have published the reference book on this topic in 1966. With the development of high speed computers, and by means of signals processing techniques, the practice of the random vibration theories has become more and more involved in all industries. Recent work [ref. 19] shows the developments in progress during last decades.

In the classical theory of random vibrations, the excitations are assumed to be Gaussian. This hypothesis is often justified by the central limit theory which stipulates that a resultant random variable of a superposition of a large number of independent elementary variables statistically stretches to be Gaussian, whatever the distributions of the elementary variables. Such processes are completely characterized by their statistical properties. Specifically, power spectral density gives the information about the signal frequency content as well as the variance of the process. One of the other hypotheses in random vibrations is that the structure is deterministic; its geometry and the constitutive materials characteristics are given.

In the time domain, the relation between the input and the output expresses itself by a convolution operation, characteristic of a linear system. Consequently, the calculation of the structural response in the time domain returns a very costly analysis. On the other hand, in the frequency domain, the operation of convolution corresponds to
a simple multiplication. Consequently, the response of the structure to a random
excitation is calculated much more quickly by using the spectral method. Specifically, for
linear systems, the structural response to a Gaussian excitation is equally Gaussian.
Therefore, the responses are also characterized by power spectral densities, in the same
way as the excitations. For non-linear systems, several linearization techniques were
introduced to transfer the system to an easily solved linear system. Spanos & Robert have
made a considerable contribution, and are continually active in this field with their
benchmark book [ref. 20].

The other topic addressed quite frequently in the area of random vibrations is the
fatigue problem. Fatigue analysis is an integral part of the mechanical design cycle for
systems operating in a sustained vibration environment, most of which is quite
complicated. This thesis is devoted to developing the methods of fatigue life prediction
for offshore structures subject to random vibrations excitations.

Fatigue is the phenomenon that materials, or structures, are found to be "aging" or
"degrading", and finally moving into "failure", due to the cumulative effect of many
time-varying external loading cycles. This phenomenon has been studied as early as the
18\textsuperscript{th} Century, by Wohler, who has characterized for the first time materials fatigue by
establishing a curve (S-N curve) to specify the life or the cycles to failure according to
different stress amplitudes.

With regard to fatigue life, the so-called "low-cycle fatigue" refers the regime that
the cyclic stress-strain response and the material behavior are best modeled under strain-
controlled conditions. On the other hand, for the well known high-cycle fatigue, stress is
dominant. The linear or non-linear structures subjected to random vibrations and of which the response is calculated by means of spectral analysis, belong to the second category.

Fatigue life is generally divided into three steps: the initiation of a crack, the slow propagation of the crack, and the fast propagation to failure due to the resulting instability. There is no any universally accepted definition of initiation, nevertheless, we will define it here as the appearance of a crack of length 0:01 mm. In the framework of this study, we are interested principally in the first phase of fatigue damage, for two reasons. The first reason is that, in the high-cycle fatigue case, the longest period is the initiation phase, which can represent up to 90% of the structure life, notably so in the case of hard metals. The initiation life, therefore, can be a comparatively good indication of the total structure life. The second reason is that the propagation of the crack induces strong non-linearity in the structural behavior; the classical spectral analysis is no longer valid. In addition, the crack propagation falls in the research category of fracture mechanics, which is a completely separate problem.

The generally used approach in the time domain for uni-axial fatigue life calculation is based on the well-known S-N curve. Specifically, this approach is adopted in the uni-axial fatigue life calculation in this context in conjunction with the linear damage rule. Further, this result is compared to the one derived by using the spectral method in the frequency domain, which is based on stress crossing theory.
2.2 Monte-Carlo Simulation Realized Loading Data Treatment

2.2.1 Stochastic Process

The theory of stochastic processes has been extensively studied in a number of works. A brief introduction of some basic terms is given in the following context.

Assume that the value of a stochastic process at time $t$ is denoted by $x(t)$. This process is generally described by its statistical properties, such as mean value, standard deviation etc. Specifically, a process is said to be stationary if these statistical properties do not vary with time. Similarly, many processes may be treated as stationary if the time interval is thought short enough. In this study, in a short time interval (hours), the statistical properties of the sea wave may be considered stationary.

The auto-correlation function of a stationary stochastic process $x(t)$ is defined as:

$$R_x(\tau) = E[x(t) \cdot x(t + \tau)] .$$  \hspace{1cm} (2.1)

Here, $\tau$ is any time interval, and $E$ denotes the operator of mathematical expectations. Since the sea surface variation about the mean water level is zero, the auto-correlation function for $\tau = 0$ is equal to the variance of the process:

$$R_x(0) = E[x(t)^2] = \sigma_x^2 .$$  \hspace{1cm} (2.2)
The one-sided spectral density, or power spectrum, is related to the auto-correlation by the equation:

\[
S_x(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} R_x(\tau) e^{-i\omega \tau} d\tau.
\] (2.3)

Here, \( \omega \) is the angular frequency. The energy spectrum shows how the energy is distributed over the various frequencies, as shown in Fig. 2.1.

\[\text{Fig. 2.1 Stochastic process } x(t), \text{ and its energy spectrum } S_x.\]

The moments of the energy spectrum are defined as:

\[
m_n = \int_0^\infty \omega^n S_x(\omega) d\omega,
\] (2.4)
where, $m_n$ denotes the $n$th order moments. The zero-order moments give the area below the spectral curve. Consequently, it represents the total energy of the process. The zero order moment is equal to the variance of the process. Specifically:

$$m_0 = \int S_x(\omega) d\omega = \sigma_x^2,$$

(2.5)

where, $m_0$ is the zero order moments, and $\sigma_x$ is the standard deviation.

The random process involved in many engineering problems is assumed to be Gaussian, with zero mean. The probability density function of any random process is then expressed as:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}}.$$  

(2.6)

The amplitude of this random process is distributed according to the Rice distribution, and the probability density function of the amplitude is:

$$p(a) = \frac{1}{\sqrt{2\pi}\sigma_x} \left[ \varepsilon \cdot \exp\left(-\frac{a^2}{2\sigma_x^2 \cdot \varepsilon^2}\right) + \sqrt{1-\varepsilon^2} \cdot a \cdot \exp\left(-\frac{a^2}{2\sigma_x^2}\right) \cdot \int_{\varepsilon \sigma_x}^{\sigma_x} \exp\left(-\frac{1}{2} y^2\right) \cdot dy \right],$$  

(2.7)

Where, $a$ is the amplitude, and $\varepsilon$ is the spectral width parameter defined by the equation:
\[ \varepsilon = \left( 1 - \frac{m_2^2}{m_0 \cdot m_4} \right)^{1/2} \quad \text{(2.8)} \]

For simplicity, many engineering problems in industry are treated as linear systems, where the relation between the input or excitation \( x(t) \) and the output or response \( y(t) \) is described by a linear differential equation with constant coefficients. A good example of such a system is the sea wave loading transferred from the hull body to the deck level, which is regarded as a base acceleration excitation, and the resultant stress response in the offshore platform topside structures. The amplitudes of the excitation and the response are related through the transfer function \( H(\omega) \). The response spectrum of a linear system is therefore given by the equation:

\[ S_y = |H(\omega)|^2 \cdot S_x(\omega). \quad \text{(2.9)} \]

### 2.2.2 Loading Time Series Simulation

The sea wave loadings are treated as realizations of a stationary random process. It is sought to approximate the target spectrum by the power spectrum \( P(\omega) \) of the output of a digital filter to the band-limited white noise input.

#### 2.2.2.1 Stochastic Time Series Modeling

Time series modeling seeks to formulate an algorithm by which a sequence of values are generated representing possible observations of a stochastic process at discrete
values of the indexing parameter, which is usually time \( t \). Specifically, the model parameters are estimated by comparing the estimated statistics of the generated sequence with those of the actual observations of the stochastic process, which are treated as sample functions drawn out of an ensemble.

The fundamentals of time series analysis and modeling have been introduced in books such as [ref. 21, 22], in which the basic characteristics of the various models, their identification, estimation, and validation procedures are covered in considerable detail. Of particular importance are the stationary time series models: Auto Regressive (AR), Moving Average (MA), and Auto Regressive Moving Average (ARMA).

Application of time series modeling and simulation techniques to random vibration, of which fatigue is one part of the complete analysis, is only of relatively recent origin. Gersh [ref. 23] is perhaps the first to systematically extend the concepts of time series modeling to the application of mechanical and structural engineering problems. Later, a multivariate model is published in his series of papers. Reed and Scanlan [ref. 24] have made use of scalar ARMA models to characterize and simulate fluctuating wind velocity time histories. In addition, an AR model has been used by Wyatt and May [ref. 25] for the fluctuating velocity component. Further, Spanos [ref. 26, 27] have employed scalar AR, MA, ARMA models to simulate ocean wave surface elevation. They propose a least square procedure, and the model parameters is determined to minimize the sum of the square of the difference between the spectrum of the time series derived from the ARMA models and that of the target spectrum. Moreover, Spanos and Mignolet [ref. 28, 29] present a unified approach for the development of simulation algorithms to approximate a
target spectral matrix by the response of an AR discrete dynamic system to white noise excitation. The ARMA procedures have been exemplified by many spectra applications encountered in various technical areas, such as earthquake engineering (Kanai-Tajimi spectrum), ocean engineering (Pierson-Muskowitz spectrum), and wind engineering.

The AR model is best suitable for treating all pole spectra, otherwise the numerical results will have some predictably erratic features. Therefore, for the spectra considered in this study, the Moving Average (MA) algorithm is more appropriate, and is thus adopted in this study.

2.2.2.2 Moving Average (MA) Algorithm

Any stationary random process can be regarded as the output of an infinite order filter, which has the pulse transfer function expressed in the form:

$$H_{MA}(\omega) = \sum_{v=-q}^{q} b_v z^{-v}$$  (2.10)

where $q$ is the filter order, and $b_v$ is the filter coefficient, determined by the equation:

$$b_v = \frac{T}{\pi} \int_{-\pi}^{\pi} S(\omega) \cos(\nu \omega) d\omega$$  (2.11)
where \( S(\omega) \) is the given spectrum, and \( T \) pertain to the cut-off frequency.

To find the unknown coefficients, the condition:

\[
\frac{T}{2\pi} \int_T^0 \left| \sqrt{S(\omega)} - H_{\text{MA}} \right|^2 d\omega = \min \text{imum}
\]  

must be satisfied.

In addition, a time series \( x_n \) can be synthesized by using recursive numerical procedure:

\[
x_n = \sum_{n-q}^q b_j W_{n-j} \quad , \quad q \rightarrow \infty
\]

where, \( W(n) \) is a stationary zero-mean white noise process with power spectrum \( S_W(\omega) = 1 \).

In fact, the stochastic process obtained though the MA method can be thought of as a weighted average of \( 2q + 1 \) white noise deviates moving in time.

In this respect, by using the mathematical simulation algorithm, the sea wave loading time series \( x_n \) can be simulated as a random process which possesses a power spectrum \( P_{MA}(\omega) \) identical to the given design spectrum \( S(\omega) \). The sea wave loading time series \( x_n \) of the deck level can be considered as the excitation or input for any specific structure module. Obviously, this time history can be used for fatigue calculation.
of both linear and non-linear mechanical systems. The response or the output \( y_n \), which is necessary for further fatigue analysis, must be obtained by considering various frequency transfer functions, which include the information about the module natural frequency, damping coefficients etc by using Eq. (2.9) and again through a Monte Carlo simulation. However, for the case of linear systems, the previous digital synthesis can be used to produce any time series of linear system response due to the effectiveness of the MA approach. It is used exclusively here for the synthesis of a time series, which represents the response of an FPSO topside structure module.

2.3 Time Domain Random Uni-axial Fatigue Theory

2.3.1 Rainflow Cycle Counting Method

2.3.1.1. Cycle Counting and Related Methods

The three basic cycles counting methods used in the fatigue life estimation are: crossing count, peak count, and range count. Numerous variations of each method have been developed to reduce cyclic time histories to some simple forms for analysis and testing purposes, or to match some preconceived theory of fatigue damage accumulation or crack propagation [ref. 30]. In fact, all these methods basically count one parameter to characterize the complete information of the loading sequences. Since defining a cycle needs two independent parameters, these methods are sometimes found to be inadequate in which assumptions must be made on the other parameter. As a result, two-parameter counting methods, such as the range-pair-range, Rainflow and racetrack method [ref. 31-
34], have become popular, and are extensively used in conjunction with local stress-strain fatigue damage calculation schemes. Rainflow counting, which is introduced by Endo [ref. 35] in 1969, is perhaps the most widely accepted method for the identification of fatigue critical events.

For some time the industry has put effort into developing techniques for converting time series into Rainflow cycle count matrices. A number of numerical techniques are available from these previous studies. The technique most widely used is the Rainflow counting algorithm presented by Downing and Socie [ref. 36]. A complete description of this algorithm has been provided by Rice et al. [ref. 37]. Further, Wu and Kammula [ref. 38] have developed a real-time algorithm for Rainflow counting techniques that reduces memory requirements and speeds computations. Finally, an alternative definition of the Rainflow cycle equivalent to the original one has been given by Rychlik [ref. 39-61].

This study introduces Rychlik's definition of the Rainflow cycles. For completeness of the discussion, the procedure is next briefly reviewed.

### 2.3.1.2 Rainflow Counting Procedure

The critical step of the Rainflow counting procedure is to identify the peaks and valleys in the loading history. Specifically, a peak is defined as the point at which the first derivative of the load history changes from positive to negative. A valley is defined as the point at which the first derivative of the load history changes from negative to positive. All these peaks and valleys can also be called turning points.
Fig. 2.2 Stress hysteresis loop for a fatigue cycle

To avoid counting the small cycles resulting from numerical jitter in the time series that have little influence on the structural damage, a range filter is typically first used. This filter requires that successive local extremes must differ by a minimum value, typically called the threshold, before they are considered to be extremes that should be retained by the filter. In fact, the overall sequence of loading is still retained after the filter [ref. 31]. Thus, the input to the Rainflow counting procedure is a simple series of peaks (local maxima) and valleys (local minima) that form hysteresis loops as shown in Fig.2.2. Consequently, the Rainflow counting algorithm proceeds by matching peaks and valleys to form closed hysteresis loops.

The classical Rainflow algorithm is illustrated in Fig.2.3. Specifically, the stress time history is plotted as a shape where the axis is vertically downward, and the lines connecting the stress peaks are imagined to be a series of roofs. The rain drips down these roofs according to a specific rule to capture the complete cycles and half cycles. As a rule, rain begins sequentially at the inside of a stress peak. If the rain initiates at a local minimum, the rain starting at each peak is allowed to drip down, and continue, until it
meets opposite another local minimum smaller than the minimum from which it initiated, where the rain must stop. By contrast, if the rain initiates at a local maximum, the rain stops until it meets opposite another local maximum larger than the maximum from which it initiated. For example as shown in Fig. 2.3, the rain begins at peak 1 and stops opposite to peak 9, just because peak 9 is smaller than peak 1. And a half cycle is counted between peak 1 and peak 8. Fig 2.4 shows the cycles and half cycles for the Rainflow counting example in Fig. 2.3. This algorithm is a recursive procedure.

2.3.1.3 Rychlik Rainflow Algorithm

This study introduces the algorithm proposed by Rechlik, a non-recursive algorithm. It is more tractable for mathematical and statistical analysis. Moreover, it is easy to see the connection between Rainflow cycles and crossings of the intervals.

![Fig. 2.3 Rainflow counting method](image1)

![Fig. 2.4 Full cycles and half cycles results](image2)
Specifically, consider a stress load series \( \{y_t\} \) of duration \( T \) with local maximum of magnitude \( M_k \) occurring at \( t_k \) for any \( t < T \) as shown in Fig 2.5.

For simplicity, it is assumed that the first local extreme is a minimum, therefore, the sequence of valleys and peaks is denoted by:

\[
\{Y_i, Y_2, Y_3, \ldots \} = \{m_0, M_0, m_1, M_1, \ldots \}.
\]  

(2.14)

For the \( k \)th maximum at time \( t_k \), the corresponding "Rainflow" minimum \( m_k^{RF} \) can be obtained by comparing the backward (left) minimum \( m_k^- \), and the forward (right) minimum \( m_k^+ \), which are determined by the following steps as shown in Fig. 2.6:

1. For each local maximum \( M_k \), determine \( t_k^- \) and \( t_k^+ \) by drawing the dashed horizontal line through \( M_k \);
2. Between \( t_k^- \) and \( t_k^- \), determine the minimum value \( m_k^- \);
3. Between \( t_k \) and \( t_k^+ \), determine the minimum value \( m_k^+ \);
(4) Compare \( m_k^- \) and \( m_k^+ \), choose the larger one, and define it as \( m_k^{RFC} \) for any \( t_k^+ < T \); when \( t_k^+ = T \), \( m_k^{RFC} = m_k^- \).

(5) Select the \( k \)th Rainflow range pair as \( (m_k^{RFC}, M_k) \).

![Fig. 2.6 Rychlik’s definition of Rainflow cycle](image)

(6) Save the \( k \)th stress level (amplitude) for damage calculation:

\[
S_k^{RFC} = \left( M_k - m_k^{RFC} \right)/2. \tag{2.15}
\]

This procedure is repeated until all the peaks are treated.

The Rainflow cycle is a pair consisting of the minimum \( m_k^{RFC} \), and the maximum \( M_k \). The amplitude is the most important characteristic for fatigue evaluation. Generally in fatigue analysis applications, a cycle is represented as a range-mean pair. The definition of this amplitude is shown in Fig. 2.7.
Fig. 2.7 The amplitude, range and mean of a cycle

When the algorithm reaches the end of a time-series data record, a series of unmatched peaks and valleys remains unclosed and, therefore, are not counted by this algorithm. These so-called "half-cycles" typically include the largest peak and valley in the record, and they may also include other large events. Therefore, the potentially most damaging events (the largest cycles) contained in the time series are not counted by the rainflow counting procedure. Various researchers have proposed several techniques for handling these half-cycles in fatigue applications. Some ignore them, some count them as half of a complete cycle, and others count them as full cycles. In this study, for safety reasons, all the half cycles are conservatively counted as full cycles.

The output of the Rainflow counting algorithm is a characterization of the stress cycle by its maximum and its minimum value. After the Rainflow counting, this characterization of those fatigue cycles must be converted for practical use in the fatigue analysis. The output file can take many forms, depending on the analysis and numerical techniques being used to determine the damage. From the discretized turning points, min-max and max-min cycles can also be extracted. Furthermore, the cycle count can be
summarized in a two-dimensional histogram, and be represented by a matrix. The Rainflow cycles matrix, $F^{RFC}$, the min-max matrix $F$, and the max-min matrix $\hat{F}$ is summarized in the following:

$$F^{RFC} = (f^{RFC}_{i,j})_{i,j=1}^n;$$

(2.16)

$$F = (f_{i,j})_{i,j=1}^n;$$

(2.17)

$$\hat{F} = (\hat{f}_{i,j})_{i,j=1}^n.$$  

(2.18)

And all the terms are defined as:

$$f^{RFC}_{i,j} = \text{number of } \{m^{RFC}_k = u_i, M_k = u_j\};$$

(2.19)

$$f_{i,j} = \text{number of } \{m_k = u_i, M_k = u_j\};$$

(2.20)

$$\hat{f}_{i,j} = \text{number of } \{M_k = u_i, m_{k+1} = u_j\}.$$  

(2.21)

In Fig. 2.8, the matrices $F^{RFC}$, $F$, and $\hat{F}$ are illustrated.
Fig. 2.8 Rainflow cycle $F^{RFC}$, min-max $F$, max-min $\hat{F}$ for equipment time history

For both the Rainflow and min-max cycles counts, the counting distribution contains information about level crossings.

### 2.3.2 Damage Accumulation

The equivalent stress cycles generated by the Rainflow counting procedure are used for damage calculation and fatigue life prediction. For variable amplitude loads, the S-N curve is used in conjunction with the cycle counting method and the well recognized standard, Miner's linear damage accumulation rule.

#### 2.3.2.1 S-N curves and Palmgren-Miner Linear Cumulative Damage Rule
The fatigue life of a structural component is found to be dependent in a complex manner on a large number of factors; such as, the material properties, the load sequence, the geometry size and surface finish of the component, and the environmental factors.

The basic theory of fatigue is based on the macroscopic observation of the relationship between the applied stress and the number of cycles to failure, i.e., S-N curves. Although the S-N curves exhibit a large scatter which is primarily due to the inherent uncertainties underlying the fatigue phenomenon, as a result of the variability of the microscopic influence-dislocation, lattice defects, and grain boundaries, the bulk of fatigue test data is presented even today in the form of S-N curves; which is the so called Wohler [ref. 62] diagram due to its relative ease of obtaining data in this form, and its easy application to the fatigue design.

In laboratory experiments, the structure is often subjected to a constant amplitude load, and the number of cycles (periods) is continuously counted until the specimen breaks. The number of cycles \( N(s) \), as well as the stress amplitudes \( s \) is recorded. For small amplitudes \( s < s_\infty \), the fatigue life is often very large, and is set to infinity \( N(s) = \infty \), which means no damage will be observed even during an extended experiment. The amplitude \( s_\infty \) is called the fatigue limit or the endurance limit. The relationship between \( N(s) \) and \( s \) is expressed as:

\[
N(s) = \begin{cases} 
K^{-1}s^{-\beta} & s > s_\infty \\
\infty & s \leq s_\infty 
\end{cases}
\]  

(2.22)
Here, $K$ is a material dependent random variable, usually log-normally distributed, with:

$$K^{-1} = E \varepsilon^{-1}. \quad (2.23)$$

Here, $\ln E \in N\left(0, \sigma^2_E\right)$ and $\varepsilon, \beta$ are fixed constants related to the characteristics of the structure materials. $\varepsilon, \beta$ and $\sigma^2_E$ can be estimated from an S-N experiment, or by fitting a certain given S-N curve.

The prediction of fatigue life based on the S-N curves rests on the use of a suitable cumulative damage criterion representing the progressive deterioration of the structure during the cyclic loading. The concept of fatigue damage has been introduced for this purpose. Basically, it assumes that the structure component under repeated cyclic loading suffers an amount of damage which can be expressed as:

$$d(D) = f(n,s,N(s)) \quad (2.24)$$

where $D$ denotes the damage function, and $n$ is the number of cycles at a constant stress amplitude $s$. As mentioned in the preceding part, $N(s)$ is the number of cycles to failure at stress amplitude $s$ obtained from the S-N curve.

It is assumed that the damage which has occurred is permanent, and further applications of stress cycles at varying stress levels cause additional cumulative damage. The total damage is the sum of all damage increments accrued at each stress level, and
when the cumulative damage reaches a critical value, fatigue failure occurs. The cumulative damage is simple to apply; but considerable difficulty is encountered in practice in defining a damage function incorporating the stress sequencing and stress interaction effects.

The first cumulative damage law is proposed by Palmgren [ref. 63], and has been developed by Miner [ref. 64]. The linear damage theory is still widely used in spite of many limitations because of its simplicity in application, and because of the fact that many other theories developed to account for the limitations do not give more accurate estimates of fatigue life.

The first assumption made for the Palmgren-Miner damage rule (Miner's rule) is that each alternate stress cycle inflicts certain damage in the material equal to the reciprocal of the number of the cycles of failure at the stress level given by the S-N curve. This damage is irrespective of the relative position of the stress cycle in the whole stress history, and is independent of the stress magnitude before or after it. The second assumption is that the total damage due to the whole stress history is the sum of all damages of each individual cycle. The third hypothesis is that the fatigue failure of the material occurs when the cumulative damage reaches a value unity.

The fraction of damage suffered by the structure component due to \( n_i \) cycles at stress level \( s_i \) is expressed by \( D_i \), which is computed by the equation:

\[
D_i = \frac{n_i}{N_i}.
\]  
(2.25)
Here, $N_i$ is the number of cycles to failure at $s_i$ given by S-N curve. The structural component fails when the total cumulative damage:

$$D = \sum_i D_i = \sum_i \frac{n_i}{N_i}.$$  

(2.26)

reaches the value $D = 1$.

Typically, the fatigue cycles imposed on a structure are analyzed over some fixed period of time. Therefore, Eq. (2.26) should be expressed as the damage rate $\Delta D_i$ associated with the sample time $t$. This damage rate can be treated as the average damage rate over the service lifetime of the component $T_f$ ($\Delta D_i = \Delta D_f$), especially for static and ergodic processes, and then the service lifetime of the structure $T_f$ is the reciprocal of $\Delta D_f$, that is:

$$T_f = \frac{1}{\Delta D_f} = \frac{1}{\Delta D_f}.$$  

(2.27)

Again, Eq. (2.27) is predicated on the assumption that failure will occur when the damage equals one, and that the damage rate computed over time $t$ is representative of the average damage rate imposed upon the structure during its service lifetime. Namely, the number and distribution of fatigue cycles contained in the sample time are essentially identical to the number and distribution over the structure's service lifetime.
2.3.2.2 Damage Calculation for Rainflow Counted Data

Stationary and Ergodic loadings

For the time series in the sample time $t$, the damage rate is used to calculate the fatigue life. If the $k$th cycle has amplitude $s_k$, then it is assumed that it causes a damage equal to $1/N(s_k)$. Therefore, the total damage at time $t$ is:

$$D(t) = \sum_{i \leq t} \frac{1}{N(s_k^{RFC})} = \sum_{i \leq t} K(S_k^{RFC})^B = KD_\beta(t).$$  \hspace{1cm} (2.28)

where the sum contains all rainflow cycles which have been completed up to time $t$, and:

$$S_k^{RFC} = \left(M_k - m_k^{RFC}\right)/2. \hspace{1cm} (2.29)$$

The damage rate $\Delta D$, is defined by:

$$\Delta D_t = \frac{D(t)}{t}. \hspace{1cm} (2.30)$$

Therefore, the fatigue life $T_f$ for stationary and ergodic loads can be obtained using Eq.(2.27). A very simple predictor of $T_f$ is obtained by replacing $K^{-1} = E \varepsilon^{-1}$ in Eq. (2.28) by a constant, for example, the median value of $K$, which is equal to $\varepsilon$. 
Fatigue Life Distribution

For non-ergodic loadings, the sampled loading time series after rainflow count is not enough to represent all the damage during the service time. The fatigue life obtained is simply a statistical distribution.

The Palmgren-Miner hypothesis states that fatigue failure occurs when the damage exceeds one. Thus, the probability for fatigue failure is:

\[ P(T_f \leq t) = P(D(t) \geq 1) = P(K \leq \varepsilon D_\beta(t)). \]  

(2.31)

Here, \( K \) accounts for the uncertainty in the material. In the previous section, a lognormal distribution has been used for the fluctuation of \( K \) about \( \varepsilon \), by assuming that:

\[ \ln K = \ln \varepsilon - \ln E \]  

(2.32)

is a normal variable with mean \( \ln \varepsilon \) and standard deviation \( \sigma_E \).

The cycle sum \( D_\beta(t) \) is the sum of a large number of damage terms, only dependent on the cycles. For loads with short memory, it is assumed that \( D_\beta(t) \) is approximately normal, and the following can be defined:

\[ d_\beta = \lim_{t \to \infty} \frac{D_\beta(t)}{t} \quad \text{and} \quad \sigma_\beta^2 = \lim_{t \to \infty} \frac{V(D_\beta(t))}{t}. \]  

(2.33)
Thus, the normal distribution for $D_\beta(t)$ can be expressed in the form:

$$D_\beta(t) \approx N(d_\beta, \sigma_\beta^2 t).$$

(2.34)

The fatigue life distribution can be computed by combining the lognormal distribution $K$ with the normal distribution for $D_\beta(t)$.

2.4 Spectral Method to Calculate Damage in Frequency Domain

2.4.1 Rayleigh Approximation

Of particular interest in offshore structure analysis is how to find extreme quantities and extreme significant values for the sea wave loading time series. This implies that to predict how the extreme might be when the observed or simulated value exceeds the data range obtained from a limited number of observations. This kind of analysis is generally known as Weibull analysis, from the name of a well-known extreme value distribution. If the process is Gaussian, the Weibull distribution becomes a Rayleigh distribution given by the equation:

$$p(a) = \frac{a}{\sigma_s} \exp\left(\frac{-a^2}{2\sigma_s^2}\right).$$

(2.35)
Specifically, comparing Eq. (2.35) with Eq.(2.7), it shows that the Rayleigh distribution is exactly a Rice distribution with a spectrum width parameter equal to zero.

2.4.2 Direct Fatigue Life Calculation Method without Rainflow

Counting for Narrow band Process

With the help of the Rayleigh distribution, it is appropriate to directly apply Miner’s rule to the stress time history if the time history is a narrow band process, without resorting to the Rainflow procedure. For a narrow band process, the expected number of cycles is nearly equal to the expected number of maximum values.

Consider the situation in which the structure/module response time series \( y(t) \) is narrow band. The expected number of peaks occurring between the stress levels \( s \) and \( s + ds \) in the time interval \([t, t + T]\) is:

\[
N^+(s;T) - N^+(s + ds;T).
\]  

(2.36)

where \( N^+(s;T) \) denotes the expected number of crossings of the level \( s \) with positive slope in the interval \([t, t + T]\).

The expected incremental damage at the stress level \( s \), using Miner’s rule, is:

\[
E[D_3(t, t + T)] = \frac{N^+(s;T) - N^+(s + ds;T)}{N(s)}.
\]  

(2.37)
where \( N(s) \) is the number of cycles for fatigue failure at stress level \( s \). The Eq. (2.37) can be written as:

\[
E[D_s(t, t + T)] = -\frac{1}{N(s)} \frac{\partial N^+(s; T)}{\partial s} ds = -\frac{1}{N(s)} \frac{\partial}{\partial s} \left[ \int_0^T N^+(s; t) dt \right] ds .
\] (2.38)

For a stationary process, \( N^+(s, t) \) is independent of \( t \). Thus, the total expected damage at all stress levels in the same interval can be obtained by integral:

\[
E[D(T)] = -\int_0^T \frac{T}{N(s)} \frac{\partial}{\partial s} \left[ N^+(s) \right] ds .
\] (2.39)

Eq. (2.39) can be alternatively written by using the probability density of the peaks in the form:

\[
E[D(T)] = \int_0^T \frac{T}{N(s)} \left[ N^+(0) p(s) \right] ds .
\] (2.40)

where \( N^+(0) \) is the expected rate of zero crossings with positive slope.

Substituting Eq. (2.22) into Eq. (2.40) obtains:

\[
E[D(T)] = N^+(0) \cdot T \cdot K \int_0^T p(s) \cdot s^\theta ds .
\] (2.41)
For a narrow band process, the function $p(s)$ is Rayleigh distribution; therefore, the total expected damage can be written as:

$$E[D(T)] = \frac{N^+(0) \cdot T \cdot K}{\sigma_s^2} \int_0^\infty \exp \left( -\frac{s^2}{2\sigma_s^2} \right) \cdot s^\beta ds = N^+(0) \cdot T \cdot K \left( \sqrt{2}\sigma_s \right)^\beta \Gamma \left( 1 + \frac{\beta}{2} \right).$$ \hspace{1cm} (2.42)

According to Miner’s rule, the average time to failure $T_f$ can be calculated by setting the expected fatigue damage equal to 1. That is:

$$E[T_f] = \frac{1}{N^+(0) \cdot K \left( \sqrt{2}\sigma_s \right)^\beta \Gamma \left( 1 + \frac{\beta}{2} \right)}.$$ \hspace{1cm} (2.43)

In this equation, the value $N^+(0)$ can be determined by the expression:

$$N^+(0) = \frac{1}{2\pi} \frac{\sigma_s}{\sigma_v}.$$ \hspace{1cm} (2.44)

where $\sigma_s$ is the standard deviation of the structure response stress spectrum under sea wave loadings, and is given by the equation:

$$\sigma_s^2 = \int_0^\infty S_\sigma(\omega) d\omega.$$ \hspace{1cm} (2.45)
where $S_\sigma(\omega)$ is the stresses spectrum of the FPSO topside structures.

$\sigma_i$ is the standard deviation of the topside structure response velocity spectrum, and is given by the equation:

$$\sigma_i^2 = \int_0^\infty \omega^2 \cdot S_\sigma(\omega) d\omega.$$  

(2.46)
Chapter 3 Topside Dynamic Characteristic Analysis of FPSO Systems

The frequency domain analysis of topside structure random vibrations requires knowledge of the acceleration spectrum at the FPSO topside. The procedure adopted to define this spectrum for the FPSO under consideration will be outlined in this chapter.

In fact, a number of design parameters such as the mooring stiffness influence the behaviour of the floating structures when exposed to the action of the environmental loadings. Other parameters that vary as a natural consequence of operational conditions (such as the vessel draughts) or environmental changes (such as intensity and direction of wind, current, and waves) play an important role in determining the relative and absolute positioning and motions of the floating system. Therefore, selection of the best parameters in design is clearly related to the behaviour they exhibit under critical environmental conditions. Generally, it is anticipated that linear models may still be sufficient to predict ordinary structure dynamic behaviour. As a result, analysis of motion and sea wave loadings transferred to the deck level of FPSO systems, and investigation of the design parameters influence on the static and dynamic behaviour of the system can rely on the “linear” transfer functions which still capture the dynamic characteristic of the floating systems and their mooring facilities.

The environment loadings and the pertinent response motion of the floating system during operating lifetime are as follows: wind: surge, sway, yaw and roll; wave: surge, sway, yaw, heave, roll and pitch; current: surge, sway and yaw. All these random environmental loadings will cause stress variations in offshore structures, and may lead to
fatigue damage, especially since the sea wave loadings are dominant. This chapter will focus on the analysis of dynamic characteristic behaviour of FPSO systems under sea wave loadings.

Generally, fatigue damage problems of FPSO systems include the topside structure failure due to transference of the hull body stress response under sea wave loading into the topside deck level structure. Specifically, the components of the dynamic response of the FPSO system in the six degrees of freedoms (surge, sway, heave, roll, pitch and yaw) under the unit amplitude of sea waves, the so called elementary wave direction transfer functions $H_o(\omega, \theta)$, are obtained directly from an industrial finite element analysis code resulting from a global hydrodynamic analysis, and are thus incorporated into this study. This elementary transfer function describes the dynamic characteristic of the FPSO systems on the main deck level under the excitation of random sea waves.

3.1 Deck Level Design Spectrum on the Topside of FPSO

3.1.1 Spectral Properties of Sea Waves

In the literature, several formulae have been proposed to represent the main features of the sea wave elevation frequency spectra [ref. 65]. In this study the Ochi-Hubble sea elevation wave spectrum is used because of its capability of capturing multi-peaks in the energy spectrum as they pertain to a wind generated sea waves mixed with swell waves, which matches that of the FPSO systems studied in this context. In fact, it is a modified Bretschneider spectrum with the parameter $\lambda$ being a peak enhancement factor.
Specifically, the Ochi-Hubble spectrum decomposes the sea states into high and low frequency components each with the form:

\[
S(\omega) = \frac{1}{4} \left( \frac{4\lambda + 1}{\Gamma(\lambda)} \right)^{\frac{\lambda}{4}} \frac{H_s^2}{\omega^{(4\lambda+1)}} \exp \left[ -\left( \frac{4\lambda + 1}{4} \right) \left( \frac{\omega_p}{\omega} \right)^4 \right]. \tag{3.1}
\]

where, \( \omega \) denotes the circular frequency; \( \Gamma \) is the Gamma function; \( \omega_p \) and \( H_s \) are the peak frequency and the significant wave height, respectively; \( \lambda \) represents a peak enhancement factor, which controls the sharpness of the peak. The parameters \( \omega_p \), \( \lambda \) and \( H_s \) are both for the swell (low frequency), and the wind generated sea waves (high frequency). By adding the individual wind generated sea wave \( S_{\text{wind sea}}(\omega) \) and swell spectrum \( S_{\text{swell}}(\omega) \) together, the total sea wave spectrum is obtained by:

\[
S_{\text{sea}}(\omega) = S_{\text{wind sea}}(\omega) + S_{\text{swell}}(\omega). \tag{3.2}
\]

If the amount of energy in the two spectra is comparable, the resulting spectrum of sea waves, \( S_{\text{sea}}(\omega) \), is bimodal spectrum, i.e., it exhibits two distinct peaks. The FPSO system under consideration in this thesis operates in a relatively mild environment, with the result that the \( S_{\text{wind sea}}(\omega) \) part is quite small, and it is, thus, reasonably ignored.

The parameters \( \omega_p \), \( \lambda \) and \( H_s \) must be properly specified for the wind wave and swell spectra. Fig.3.1 shows the Ochi-Hubble spectrum for various values of the
parameters $\lambda$. In the present study, the following parameters are adopted to characterize the swell spectrum $S_{swel}(\omega) = S_{sw}(\omega)$: $\omega_p = 0.6065 \text{ rad/s}$; $H_s = 1.5 \text{ m}$ and $\lambda = 6$.

Fig.3.1 Ochi-Hubble sea wave elevation for various peak enhancement factors

The spectrum of sea wave at the FPSO topside can be obtained by using the pertinent Response Amplitude Operator (RAO). For this purpose, it is first necessary to evaluate the components of the dynamic response of the FPSO system in the six degrees-of-freedom (surge, sway, heave, roll, pitch and yaw) under unit amplitude of sea wave loadings, at different locations of the deck and for each spreading direction $\theta_i$, the so-called elementary wave direction transfer functions. Since vertical vibrations of either
piping systems or other simple structures located on the FPSO deck are of concern, only the heave degree-of-freedom is herein considered. The associated elementary wave direction transfer functions are obtained directly by using an industrial code, SESAM, resulting from the global hydrodynamic analysis of FPSO systems. These functions are denoted by $H_o^{(j)}(\omega, \theta_i)$, where the subscript stands for the heave degree-of-freedom, the superscript indicates the various location on the FPSO deck level and $\theta_i$ is the spreading direction. The on-site transfer functions, denoted as $H_o^{\text{mean}}(\omega, \theta_i)$, are evaluated for nine spreading directions, $\theta_i$, ($i = 1, 2, ..., 9$) equally spaced in the range $[0, 2\pi]$, by averaging the functions $H_o^{(j)}(\omega, \theta_i)$ pertinent to ten different locations at the FPSO topside ($j = 1, 2, ..., 10$). The values of $H_o^{(j)}(\omega, \theta_i)$ are computed in the frequency range $[2\pi/35, 2\pi/3]$ at the discrete frequencies $\omega_i = (2\pi/35) + i\Delta\omega, i = 1, 2, ..., 32$, being: $\Delta\omega = 2\pi/105$.

Once the averaged elementary wave direction transfer functions, $H_o^{\text{mean}}(\omega, \theta_i)$ ($i = 1, 2, ..., 9$) are known, the RAO for the FPSO, pertaining to the heave direction can be obtained by multiplying the spreading function weights and expressed as:

$$\left|H_o^{\text{mean}}(\omega, \bar{\theta})\right|^2 = \sum_{i=1}^{9} D(\omega, \theta_i) \left|H_o^{\text{mean}}(\omega, \theta_i)\right|^2. \quad (3.3)$$

where $\bar{\theta}$ denotes the main wave direction, and $D(\omega, \theta_i)$ is a suitable directional spreading function. Such function is normalized to ensure that the total energy represented by the directional spreading is conserved and satisfies condition:
\[ \sum_{i=1}^{9} D(\omega, \theta) = 1. \] (3.4)

Among the various expressions of the wave spreading functions available, the cosine-squared type is employed herein due to its simple form and wide applications [66]. That is:

\[
D(\omega, \theta) = \begin{cases} 
\frac{2}{\pi} \cos^2 \theta & |\theta| \leq \frac{\pi}{2} \\
0 & |\theta| > \frac{\pi}{2}
\end{cases} .
\] (3.5)

Finally, the mean sea wave spectrum at the FPSO topside, associated with the heave degree-of-freedom, can be obtained by simply multiplying the Ochi-Hubble spectrum (Eq. 4.1) by the RAO (Eq. 4.3). Specifically:

\[
S_{\text{mean}}^{\text{wave}}(\omega) = \left| H_{\omega}^{\text{mean}}(\omega, \theta) \right|^2 \cdot S_{\text{sea}}(\omega) .
\] (3.6)

### 3.1.2 Design Spectrum in Heave Direction for Further Fatigue Analysis

As mentioned in preceding section, the on site transfer functions are for ten different locations at the FPSO topside for each spreading direction in the frequency range \([2\pi/35, 2\pi/3]\). The coordinates of the ten locations are listed in Table 3.1, and the coordinates axes are shown in Fig.3.2.
### Table 3.1 Coordinates of ten locations for transfer function

<table>
<thead>
<tr>
<th>Joint number</th>
<th>Module Number</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
<th>Story</th>
</tr>
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<td>187.3</td>
<td>12.6</td>
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<tr>
<td>14490</td>
<td>e</td>
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<td>15.75</td>
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<td>f</td>
<td>90</td>
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<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>20137</td>
<td>k</td>
<td>190</td>
<td>47.25</td>
<td>24.25</td>
<td>2</td>
</tr>
<tr>
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<td>15.75</td>
<td>48.5</td>
<td>2</td>
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<tr>
<td>23090</td>
<td>n</td>
<td>160</td>
<td>47.25</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>28069</td>
<td>r</td>
<td>80</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>28065</td>
<td>s</td>
<td>120</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>30626</td>
<td>u</td>
<td>150</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>30065</td>
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<td>195</td>
<td>31.5</td>
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</tr>
</tbody>
</table>

Certain dynamic analyses of offshore structures may require the determination of flow induced excitation forces. The classical method for the calculation of these forces is based on the Morison Equation. This approach requires knowledge of the component of water velocity and acceleration spectra \( S_v \) and \( S_a \). Specifically, the velocity and accelerations can be deduced from Eq. 3.6 in the following:

\[
S^\text{mean}_v (\omega) = \omega^2 \cdot S^\text{mean}_a (\omega),
\]  

(3.7)
Fig. 3.2 Coordinate system for the ten locations involved in the calculation
and:

\[ S_a^{\text{mean}}(\omega) = \omega^4 \cdot S_a^{\text{mean}}(\omega) \]  

(3.8)

where \( S_a^{\text{mean}}(\omega) \) and \( S_a^{\text{mean}}(\omega) \) is the FPSO deck level velocity mean spectrum and acceleration mean spectrum, respectively. All these mean values are obtained by averaging the 10 different locations.

Since the maximum acceleration spectrum is the one, which generally, produces the most unfavorable condition of the structure, in order to account for the various sources of uncertainties involved in the described procedure, the maximum spectrum, \( S_{a}^{\max}(\omega) \), is used as the design spectrum which has the form:

\[ S_{a}^{\max}(\omega) = S_{\text{design}}(\omega) = S_a^{\text{mean}}(\omega) + 3 \cdot \sigma_{S_a(\omega)} \]  

(3.9)

where \( \sigma_{S_a(\omega)} \) denotes the standard deviation of the spectrum at each sampling frequency, deduced from the on site spectra \( S_a^{j}(\omega) \), \( j = 1,2,...,10 \), as shown in Fig. 3.3, pertaining to the ten locations on the FPSO topside. Eq. (3.9) fully characterizes in the frequency domain the random motion prescribed to the structure as a consequence of the FPSO heaving.

In Fig. 3.4 the mean and the design acceleration spectrum at the deck of the FPSO under consideration are displayed. It can be seen that the power is spread over the interval: \([2\pi/15, 46\pi/105]\).
The design spectrum can be considered as an excitation or input for any specific topside structures. The response necessary for further fatigue analysis must be obtained by considering various frequency transfer functions, which include the information about the structure natural frequency, damping coefficients etc. Obviously, the design spectrum, which has been produced, can be used for fatigue calculation of both linear and non-linear structures of mechanical systems. For the case of a simple linear system with a finite degree of natural frequency, by using the digital synthesis technique presented in Chapter 2, any time series of linear system response can be produced due to the
effectiveness of the MA approach. For pipelines treated as beams supported at the both ends with infinite natural frequency, the Galerkin method is employed to obtain a series of discretized equations of motion. In the next three chapters, the base acceleration design spectrum, $S_{\text{design}}(\omega)$, will be used in conjunction with the dynamic analysis of both a simplified 1SDOF structure, linear and nonlinear piping system conveying fluid to evaluate the stress spectrum at a given location for further fatigue life estimation.

Fig. 3.4 The design and mean acceleration spectrum
Chapter 4 Fatigue Life Estimation of Simple Linear Structure on Topside FPSO and Results

Many engineering problems in industry can be simplified to linear systems, where the relation between the input or excitation $x(t)$ and the output or response $y(t)$ is described by a linear differential equation with constant coefficients.

The wave loading time series results $x_n$ can be considered as the excitation or input for any specific structure (equipment) module. The response or the output $y_n$, which is necessary for further fatigue analysis, must be obtained by considering various frequency transfer functions, which include the information about the module natural frequency, damping coefficients etc. Obviously, the time history, which has been produced, can be used for fatigue calculation of both linear and non-linear structures of mechanical systems. However, for the case of linear systems, the previous digital synthesis in Chapter 2 can be used to produce any time series of linear system response due to the effectiveness of the MA approach. It is used exclusively for the synthesis of a time series, which represents the response of an equipment module on topside of FPSOs.

In this chapter, the FPSO topside acceleration power spectrum will be considered in conjunction with the response of a dynamic model for a generic piece of structure or equipment located on the topside of FPSO systems. Next, the Rainflow counting algorithm for identifying the cycles that produce fatigue damage has been applied. Finally, the fatigue life has been obtained by using either the cycle counting procedure or the stress up-crossing statistics.
4.1 S-D-O-F Linear System Stress Calculation

Generally, the amplitudes of the excitation $x(t)$ and the response $y(t)$ are related through the transfer function $H(\omega)$. The energy spectrum of the response process is given by the equation:

$$S_y = |H(\omega)|^2 \cdot S_x(\omega).$$  \hspace{1cm} (4.1)

For the case of S-D-O-F linear systems, consider the following simple model with mass $m$, stiffness $K_{eq}$, and cross section area $A$:

![Diagram of a generic piece of equipment](image)

**Fig. 4.1 A generic piece of equipment**

Attention is next focused on the significant natural modes of the equipment. Therefore, the equipment motion will be represented by the linear equation:

$$\ddot{y} + 2\zeta \omega_n \dot{y} + \omega_n^2 y = -\ddot{x}(t).$$  \hspace{1cm} (4.2)
Here, $x$ is the input sea wave loading, $y$ is the response displacement of a generic piece of equipment, and $\xi$ and $\omega_n$ are the damping coefficient, and the natural frequency of the equipment, respectively. For linear systems, apply Eq. (4.1), and the response spectrum is expressed as:

$$S_{d\_structure}(\omega) = \frac{S_{\text{max}}(\omega)}{(-\omega^2 + \omega_n^2) + (2\xi\omega\omega_n)^2},$$  \hspace{1cm} (4.3)

where $S_{d\_structure}(\omega)$ is the equipment displacement spectrum in heave direction, and $S_{\text{max}}(\omega)$ is the deck level design spectrum and has the relationship with the mean sea wave spectrum $S_{\text{mean}}(\omega)$ on the FPSO topside through $S_{\text{max}}(\omega) = S_{\text{design}}(\omega) = S_{\text{mean}}(\omega) + 3 \sigma_{S_x(\omega)}$. Applying Eq. (3.8), the equipment acceleration spectrum $S_{a\_structure}(\omega)$ is written as:

$$S_{a\_structure}(\omega) = \omega^4 S_{\text{max}}(\omega) = \omega^4 \frac{S_{\text{design}}(\omega)}{(-\omega^2 + \omega_n^2) + (2\xi\omega\omega_n)^2}. \hspace{1cm} (4.4)$$

Clearly, the stresses at the equipment cross section area are given by the equation:

$$\sigma_{\text{equipment}} = \frac{ma}{A}. \hspace{1cm} (4.5)$$
Therefore, the equipment stress spectrum $S_\sigma(\omega)$ is given by the equation:

$$S_\sigma(\omega) = \frac{m}{A} S_{\sigma\_structure}(\omega).$$  \hfill (4.6)

For further fatigue analysis and to apply the Rainflow algorithm elucidated in Chapter 2, which is carried out in the time domain, the next significant step becomes the digital generation of a time series bearing a compatible power spectrum with the loading $S_\sigma$.

### 4.2 Moving Average (MA) Algorithm Numerical Results

The main idea of the MA method is to generate the time series with the power spectrum $P_{MA}(\omega)$ as the output of an infinite order filter and identical to the target spectrum $S_\sigma(\omega)$.

To determine the filter coefficients $b_v$ in Eq. 2.11, the following equation is used:

$$b_v = T \int_0^T \sqrt{S_\sigma(\omega)} \cos (\nu T \omega) d\omega, \quad \nu = [-q, q].$$ \hfill (4.7)

where $q$ is the filter order. Detailed derivation has been clearly delivered in Chapter 2.

Furthermore, a time series can be synthesized by using the recursive numerical procedure:
\[ y_n = \sum_{v=-q}^{q} b_v W_{n-v}, \quad q \to \infty. \] (4.8)

Herein, \( y_n \) is the simulated generic piece of equipment stress, which possesses a power spectrum \( P_{Ma}(\omega) \) identical to the design spectrum, and \( W(n) \) is stationary zero-mean white noise process with power spectrum \( S_W(\omega) = 1 \).

Pertinent results for the piece of equipment module with \( \xi = 0.02 \) and \( \omega_n = 0.8 \omega_p \) are shown in Fig.4.2 and Fig. 4.3. Fig.4.2 shows good matching between the simulated equipment stress spectrum (marked with *) and the real stress spectrum. The filter order used here is \( q = 20 \). The time series generated with compatible spectrum is also shown in Fig 4.3.

Fig.4.2 MA simulation of equipment stress spectrum
The filter coefficients $b_i$ are listed in Table 4.1.

Fig. 4.3 MA simulation of wave induced FPSO topside equipment stress time series
Table 4.1 MA filter coefficients: filter order 20

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### 4.3 Rainflow Counted Equipment Time Series Results

For the specific linear SDOF equipment studied in this chapter and shown in Fig. 4.1, the Rainflow algorithm formulated in Chapter 2 is next applied. After the Rainflow counting, the Rainflow range pairs are represented in matrices, and a stress amplitude histogram is obtained. The effectiveness of this algorithm can be shown by comparing the “Rainflow” counted cycles, and the simple “min-max” counted cycles. Specifically, Fig. 4.4 & Fig.4.5 show the “Rainflow” counted cycles, and the simple “min-max” counted cycles, respectively, of the equipment shown in Fig. 4.1., Fig. 4.6 & Fig.4.7 show the corresponding amplitude histograms. It can be clearly seen that the “Rainflow” procedure yields more cycles with large amplitudes than the simple “min-max” procedure.
Fig. 4.4 Rainflow cycles for the equipment stress time series
Fig. 4.5 Min-max cycles for the equipment stress time series
Fig. 4.6 "Rainflow" counted equipment stress amplitude histogram
Fig. 4.7 “Min-max” counted equipment stress amplitude histogram
4.4. **Linear SDOF Equipment Fatigue Life Estimation**

For the particular SDOF steel equipment on topside of FPSO, which has the material parameters $\beta = 3.2$, $\varepsilon = 5.5E-10$, $m = 1200$ kg, cross section area $A = 30000$ mm$^2$ is selected.

### 4.4.1 Cycle Counting Based Method Results

The equivalent stress cycles generated by the Rainflow counting procedure are used for damage rate calculation and fatigue life prediction. The fatigue life is calculated by applying Eq. (2.28) to Eq. (2.30), and it is found to be:

$$T_f = 46.2 \text{ years} \quad (4.9)$$

### 4.4.2 Stress Crossing Based Solution Results

For several applications, such as the equipment fatigue analysis considered in this study, it can be assumed that the stress of interest is a narrow band ergodic random process. In this case, the expected number of cycles is approximately equal to the expected number of maximum values. Therefore, Miner's rule, mentioned in Chapter 2, is directly applied to the stress time history without resorting to the Rainflow procedure.

By applying Eq. 2.43 and keeping the preceding values for the problem parameters, it was found that $N^+(0) = 0.28$, and that the expected equipment fatigue life was found as:
$T_r = 35.5 \text{ years}.$

(4.10)

The deviation of this value from the one derived by using the Rainflow counting algorithm should be deemed reasonable, since Eq. 2.46 is based on certain simplified assumptions, such as the uni-modal, and the narrow band features of the equipment stress history.
Chapter 5 Fatigue Life Estimation of Pipeline Structure on Topside FPSO

In this Chapter, a straight uniform single-span pipeline containing fluid with constant axial velocity is considered. First, previous research efforts on pipeline conveying fluid is briefly reviewed to deliver some background knowledge. Since the fluid flow complicates the dynamic behavior of the pipeline as well as the pertinent mathematical model, the mathematical formulation of the problem is next carefully elucidated relying on some well-established theoretical results. The Galerkin method is then employed to obtain the random response of the pipeline by selecting natural modes of a beam as the appropriate basis functions, and the mathematical equation of motion is therefore discretized. Next, fatigue damage assessment is performed resorting to a frequency domain approach. For this purpose, the stress spectrum for a generic section of the pipeline is computed using well-known random vibration theory input-output relationships and the design spectrum. Finally, the stress spectrum is used to estimate fatigue life of the pipeline by applying linear damage accumulation rule and by incorporating an appropriate S-N fatigue curve of the material. An application of the proposed approach regarding a pipeline simply supported at both ends is presented, and the fatigue life of the illustrative example is calculated.

5.1 A Brief Review on Fluid Conveying Pipeline Research
Vibration analysis of pipeline conveying fluid has been a subject of numerous investigations due to their wide application in many industrial fields. It is more important for oil industry to formulate a design methodology and to develop practical assessment procedures due to high repair and installation cost of offshore operations. Therefore, many research efforts have been pursued towards the vibration analysis of the so called "free-span" pipelines which are suspended between two points on an uneven seafloor.

A considerable amount of work has been conducted by researchers and engineers to develop methods for response prediction of Vortex Induced Vibrations (VIV) of free span pipelines in the oil industry. Larsen in his paper [ref. 67] present a new strategy based on the combination of an empirical linear frequency domain model, and a non-linear time domain structural model. Yttervik has obtained an estimation of the VIV fatigue damage by using an extended VIV model on a typical free span pipeline [ref. 68]. Methods for combining stresses from multiple active modes have been proposed and tested [ref. 69] for both IL and CF VIV. Reid [ref. 70] use a 3-D finite element modeling in the hydrodynamic fatigue assessment of free spanning pipelines through analysis of three different pipelines on an uneven seabed. He has performed the fatigue analysis on the stress series measured in the model tests and successfully used his interpretation to verify and validate the presented computational procedure. The fatigue analysis approaches from the point view of reliability have also been developed in some recent works [ref. 71-73].

Besides the VIV related study, wave-induced fatigue problem of multi-span pipelines has been extensively studied by Xu [ref. 74] with the objective of developing a
design methodology in both time and frequency domain to determine the pipeline free span lengths.

Recently, the American Bureau of Shipping (ABS) has provided a guideline which includes the topics of designing the pipeline on topside of FPSO. It specifies that the finite element method (FEM) with fine mesh must be involved. In this respect, the demanding computation of FEM is not affordable or practical for many industry design companies in their preliminary design stage.

Several researchers have made significant progress on analyzing the fluid-conveying pipe vibration based on the beam theory which can be incorporated into the analysis of practical engineering problems. The early contributions to the literature are due to Feodos’ev [ref. 75], Housner [ref. 76], and Niordson [ref. 77], respectively. The equation of motion of fluid conveying pipes has been developed by using the kinematics of the Euler Bernoulli beam theory. Later, Long has done the complementary experimental study of transverse vibration of pipeline containing fluid [ref. 78]. The linear dynamic analysis of the pipe systems has been confirmed by the experiments earlier in the 1960s. A comprehensive review of the various models and solution methods for the dynamic analysis of pipelines with internal fluid flow has been provided by Païdoussis and his coworkers. His publication [ref. 79] includes a complete bibliography of all the important works in this field. His study is also based on the assumption that the pipe is treated not only as the Euler-Bernoulli beam but also the Timoshenko beam.

Nevertheless, there is a lack of a comprehensive closed-form mathematical formulation for analyzing fatigue problems of fluid-conveying pipelines on topside of FPSO under random sea wave loadings, especially for preliminary design reference. Next,
a methodology for estimation the fatigue life of a linear piping system conveying fluid is presented to provide an affordable solution.

5.2 Mathematical Modeling of Linear Piping System Conveying Fluid on FPSO Topside

5.2.1 Single-span Pipeline Conveying fluid on the FPSO Topside

Piping system on topside FPSOs includes: pipe, fittings, and supports parts, as in Fig. 5.1. The piping considered is supported at both ends to prevent damage which is different from the widely referred “free span pipelines”. Free span pipelines are usually suspended between two points on an uneven seafloor. The fluid flowing through a pipe can impose pressures on the inside wall to deflect the pipe and cause it to fail about. The broken of pipelines on topside FPSO due to expanded fatigue cracks are as same dangerous as the failure due to the excessive bending moment of the hull structure.

Fig. 5.1 Piping system on topside of an FPSO (courtesy of Fluor Corporation)
Certain assumptions are made in order to simplify the pipeline-modeling problem and yet must capture the fundamental characteristics of the dynamics of the system. Consider a straight uniform single-span pipeline with arbitrary boundary conditions at the ends. The pipeline contains a flowing fluid as shown in Fig. 5.2 and is placed on the topside of a FPSO system. Assuming there is no any other external action, the pipeline is subjected only to the excitation at the supports passed by the FPSO hull under the sea wave random loading in the heave direction. Without loss of generality, the present analysis focuses on the simple case of identical motion at both the supports. The end supports are assumed fixed to simplify the dynamic interaction with the supporting structure. Since the mass and the stiffness of the FPSO are much greater than those of the pipeline, this assumption is quite reasonable.

![Diagram of pipeline conveying fluid](image)

*Fig. 5.2. Single-span pipeline conveying fluid on the FPSO topside subjected to support motion*

In Fig. 5.2, the fluid has a constant axial velocity $U$ relative to the pipeline. $L$ is the span-length and $d_e, D_e$ denote the internal and external diameter of the cross-section of the pipeline, respectively.
5.2.2 Equation of Motion

In the present analysis, it is assumed that the coupling between the transverse and the axial motion is negligible and with small amplitude displacements. The equation of motion of the fluid-carrying pipeline is derived based on several other assumptions. The empty pipeline is treated as a Bernoulli-Euler beam and the material behavior is assumed linear-elastic. In addition, the fluid inside the pipeline is treated to be incompressible and the so-called "plug-flow" approximation is adopted which is reasonable for small Reynolds numbers. It is also assumed that the pipe deflections are of a long wavelength compared to the external diameter \( D_e \) of the cross-section, and the pipeline has a large slender ratio \( (L/D_e) \). Therefore, unsteady secondary effects may be disregarded and the equivalent of a slender-body approximation to the flow is allowed. Based on all the previous assumptions, the equation of motion of the pipeline conveying fluid is given in its simplest form,

\[
EI \frac{\partial^4 \tilde{w}(x,t)}{\partial x^4} + m_f U^2 \frac{\partial^2 \tilde{w}(x,t)}{\partial x^2} + 2m_f U \frac{\partial^2 \tilde{w}(x,t)}{\partial x \partial t} + (m_f + m_p) \frac{\partial^2 \tilde{w}(x,t)}{\partial t^2} = 0, \tag{5.1}
\]

where \( t \) and \( x \) denote the time and the coordinate measured along the axis of the pipeline, respectively; \( E \) is the modulus of elasticity of the material; \( I \) is the moment of inertia of the cross-section; \( m_p \) is the mass of the pipeline per unit length; \( m_f \) is the mass of the fluid per unit length; and \( \tilde{w}(x,t) \) is the total displacement of the pipeline, defined as
\[ \ddot{w}(x,t) = w(x,t) + h(t). \] (5.2)

where \( w(x,t) \) is the pipeline relative displacement to the supports in heave direction, and \( h(t) \) is the random displacement response at the supports due to the FPSO hull undergoes sea wave loading in the same direction. As explained clearly and in detail in Chap3, the support motion is modeled as a stationary random process with the power spectrum derived from the sea wave spectrum by using the appropriate Response Amplitude Operator (RAO) of the FPSO systems.

Eq. (5.1) only holds when the gravity effects, internal damping, externally imposed tension and pressurization effects are either absent or negligible. The formulation developed in this study all based on Eq. (5.1), since it properly describes the problem to present. Therefore, the extension of the motion equation to more general expressions is straightforward.

Since \( U \) is constant velocity, and the assumed small rotation, the fluid acceleration for linearized flow is expressed as [ref. 79]

\[
\left\{ \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right\} \left[ \frac{\partial \ddot{w}}{\partial t} + U \frac{\partial \ddot{w}}{\partial x} \right] = U^2 \frac{\partial^2 \ddot{w}(x,t)}{\partial x^2} + 2U \frac{\partial^2 \ddot{w}(x,t)}{\partial x \partial t} + \frac{\partial^2 \ddot{w}(x,t)}{\partial t^2} \] (5.3)

Compare Eq.(5.3) and Eq. (5.1), it is clear that how the fluid acceleration plays a role in Eq. (5.1). By using either the Newtonian or the Hamiltonian approach, the equation of motion of the pipeline can be formulated by incorporating the fluid acceleration and still keep the consistency with the above fluid model.
Substituting Eq. (5.2) into Eq.(5.1), the motion equation of the pipeline conveying fluid is rewritten as

$$ EI \frac{\partial^4 w(x,t)}{\partial x^4} + m_f U^2 \frac{\partial^2 w(x,t)}{\partial x^2} + 2m_f U \frac{\partial^2 w(x,t)}{\partial x \partial t} + (m_f + m_p) \frac{\partial^2 w(x,t)}{\partial t^2} = p_{\text{eff}}(t) \quad (5.4) $$

with

$$ p_{\text{eff}}(t) = -(m_f + m_p) \frac{d^2 h(t)}{dt^2} \quad (5.5) $$

where $p_{\text{eff}}(t)$ is the effective random load due to the support motion. Obviously, Eq. (5.4) must be supplemented by the appropriate boundary and initial conditions.

The first term in Eq. (5.4) is the flexural restoring force. There are three different inertia forces acting on the pipeline. Specifically, $m_f U^2 \frac{\partial^2 w(x,t)}{\partial x^2}$ is associated with the centrifugal acceleration experienced by the fluid as it flows along curved portions of the pipeline; $2m_f U \frac{\partial^2 w(x,t)}{\partial x \partial t}$ is related to the Coriolis acceleration arising because the fluid flows with axial velocity $U$ relative to the pipeline while the pipeline itself has an angular velocity $\frac{\partial^2 w(x,t)}{\partial x \partial t}$ at any point; $(m_f + m_p) \frac{\partial^2 w(x,t)}{\partial t^2}$ is the inertia force due to the vertical acceleration of the fluid conveying pipeline. It is clear that all these inertia terms are the results of the fluid acceleration Eq. (5.3), but only the last term in Eq. (5.3) produces the effective load in Eq. (5.5), since the support motion $h(t)$ is assumed to be only a function of time. Obviously, if the spatial correlation of base
acceleration is taken into account, additional contributions to \( p_{\text{eff}}(t) \) will be given by the Coriolis and the centrifugal terms in Eq. (5.1).

### 5.2.3 Galerkin Discretization

For further fatigue analysis in the frequency domain, the pipeline which is modeled as a beam with infinite degree-of-freedom must be simulated by a lumped parameter system with identical vibration. Therefore, the Galerkin method is next applied to discretize the equation of motion of the conveying-fluid pipeline subjected to the random support motion passed by the FPSO hull from the sea wave loading. Specifically, the solution of the fourth-order partial differential governing equation (5.4) of pipeline vibration is expressed as

\[
w(x,t) = \sum_{j=1}^{R} \phi_j(x)q_j(t),
\]

where \( R \) is the integer number of degree-of-system of a lumped parameter system. \( \phi_j(x) \) are the appropriate basis functions satisfying the boundary conditions, and \( q_j(t) \) are generalized coordinates. In this thesis, the mode shapes of a Bernoulli-Euler beam with length \( L \) and mass per unit length \( m_f + m_p \) with the same boundary conditions as the pipeline are chosen as the basis functions, which in fact are the eigenfunctions of the
boundary value problem associated with the equation of motion of a free un-damped system.

\[ E I \frac{\partial^4 w(x, t)}{\partial x^4} + (m_f + m_p) \frac{\partial^2 w(x, t)}{\partial t^2} = 0, \]  

(5.7)

Substituting Eq. (5.6) into Eq. (5.4) gives:

\[ E I \sum_{j=1}^{N} \phi_j''(x)q_j(t) + m_f U^2 \sum_{j=1}^{N} \phi_j''(x)q_j(t) + 2m_f U \sum_{j=1}^{N} \phi_j'(x)\dot{q}_j(t) + (m_f + m_p) \sum_{j=1}^{N} \phi_j(x)\ddot{q}_j(t) = -(m_f + m_p)\ddot{h}(t) \]  

(5.8)

in which the primes and the over dots denote the differentiation with respect to \(x\) and \(t\), respectively. Further, multiply both sides of Eq. (5.8) by \(\phi_k(x)\) and integrate from 0 to \(L\), the set of linear second-order ordinary differential equations for the generalized coordinates is obtained as in the following,

\[ \ddot{q}_k(t) + \omega_k^2 q_k(t) + \mu U^2 \sum_{j=1}^{R} b_{kj} q_j(t) + 2\mu U \sum_{j=1}^{R} c_{kj} \dot{q}_j(t) = -\tau_k \ddot{h}(t), \quad (k = 1, 2, \ldots, R). \]  

(5.9)

where \(\omega_k\) are the natural frequencies of the beam associated with the eigenfunctions \(\phi_k(x)\). The coefficients are
\[ \mu = \frac{m_f}{m_f + m_p}, \quad (5.10) \]

and

\[ b_{kj} = \frac{I_{kj}^{(2)}}{I_{kk}^{(1)}}, \quad c_{kj} = \frac{I_{kj}^{(3)}}{I_{kk}^{(1)}}, \quad \tau_k = \frac{I_k^{(4)}}{I_{kk}^{(1)}}, \quad (5.11) \]

In the previous relationships, the integrals

\[ I_{kk}^{(1)} = \int_0^L \phi_k^2(x) \, dx; \quad I_{kj}^{(2)} = \int_0^L \phi_k(x) \phi_j'(x) \, dx; \quad (5.12) \]

\[ I_{kj}^{(3)} = \int_0^L \phi_k(x) \phi_j'(x) \, dx; \quad I_k^{(4)} = \int_0^L \phi_k(x) \, dx. \quad (5.13) \]

have been introduced. Eq. (5.9) has been derived using the orthogonality properties

\[ \int_0^L \phi_k(x) \phi_j(x) \, dx = I_{kk}^{(1)} \delta_{kj}; \quad (5.14) \]

and
\[ EI \int_0^L \phi_k(x) \phi_j''(x) dx = \delta_{kj} \omega_k^2 (m_f + m_p) \int_0^L \phi_k^2(x) dx \]

(5.15)

of the beam eigenfunctions, the \( \delta_{kj} \) being the Kronecker delta symbol.

The coupled ordinary differential equations (5.9) can be rewritten in a matrix form as

\[
\ddot{\mathbf{q}}(t) + \mathbf{\Xi} \dot{\mathbf{q}}(t) + \mathbf{\Omega}^2 \mathbf{q}(t) = -\mathbf{\tau} \ddot{\mathbf{h}}(t),
\]

(5.16)

where

\[
\mathbf{q}(t) = \{q_1(t), q_2(t), \ldots, q_n(t)\}^T,
\]

(5.17)

\[
\mathbf{\tau} = \{\tau_1, \tau_2, \ldots, \tau_R\}^T,
\]

(5.18)

while \( \mathbf{\Xi} \) and \( \mathbf{\Omega}^2 \) are \( R \) th order square matrices with single element being expressed respectively as

\[
\mathbf{\Xi}_{kj} = 2 \mu U c_{kj}^2;
\]

(5.19)

and
\[ \Omega_{kj}^2 = \omega_k^2 \delta_{kj} + \mu U^2 b_{kj}. \]  

(5.20)

The eigenfunctions have different expressions for different boundary conditions such as the pinned-pinned pipe span and the cantilevered pipe clamped at \( x = 0 \), and free at \( x = L \).

The example considered here is the case of a pipeline supported at both ends. The eigenfunctions \( \phi_j(x) \) and the associated natural frequencies \( \omega_j \) are given by the equations

\[ \phi_j(x) = \sin \left( \frac{j \pi x}{L} \right); \quad \omega_j = j^2 \pi^2 \sqrt{\frac{EI}{(m_f + m_p)L^4}}. \]  

(5.21)

Therefore, the integrals in Eqs. (5.12) and (5.13) are in the following specific expressions.

\[ I_{kj}^{(1)} = \int_0^L \phi_j(x)dx = \frac{L}{2}; \]  

(5.22)

\[ I_{kj}^{(2)} = \int_0^L \phi_j(x)\phi'_j(x)dx = -\frac{k^2 \pi^2}{2L} \delta_{kj}; \]  

(5.23)

\[ I_{kj}^{(3)} = \int_0^L \phi_j(x)\phi'_j(x)dx = 0, \text{ for } k = j; \]  

(5.24)
\[ I_{kj}^{(3)} = \int_0^L \phi_k(x) \phi_j(x) dx = -j_{jk}^{(3)} = \frac{2jk}{k^2 - j^2} \left[ (-1)^{k+j} - 1 \right], \text{ for } k \neq j; \quad (5.25) \]

\[ I_k^{(4)} = \int_0^L \phi_k(x) dx = -\frac{L}{k\pi} \left[ (-1)^k - 1 \right]. \quad (5.26) \]

For the boundary condition considered in this study, in Eq. \( \Xi \) is a skew-symmetric matrix, and \( \hat{\Omega}^2 \) is a diagonal matrix in the form with elements

\[ \Omega_{kk}^2 = \hat{\omega}_k^2 = \omega_k^2 - \frac{k^2 \pi^2}{L^2} \mu U^2 \quad (5.27) \]

where \( \hat{\omega}_k \) is the \( k \)-th natural frequency of the discretized system.

It can be seen that the Coriolis force in Eq. (5.4), \( 2m_f U \partial^2 w(x,t)/\partial x \partial t \), gives rise to a damping term which induces the coupling of the discretized equations of motion (5.16). Secondly, the inertia force associated with the centrifugal acceleration in Eq. (5.4) acts like a compressive load and has effect on the natural frequencies. From Eq. (5.27), it is clear that when the fluid velocity \( U \) increases, the stiffness of the pipeline diminishes. If the velocity of the fluid reaches the critical value

\[ U_c = \frac{\pi}{L} \sqrt{\frac{EI}{m_f}}, \quad (5.28) \]
the lowest natural frequency, \( \hat{\omega}_i \), vanishes and the associated configuration becomes unstable.

Blevins in [ref. 85] has derived the approximate expression for the change in natural frequency due to steady internal flow for a uniform single-span pipeline conveying fluid, which can be obtained herein by manipulating Eq. (5.27), that is

\[
\beta_k = \frac{\hat{\omega}_k}{\omega_k} \left( 1 - \frac{m_i U^2 L^2}{EI} \frac{1}{(k\pi)^2} \right)^{1/2}
\]  

(5.29)

Once the discretized equations of motion (5.16) have been derived, the response statistics of the pipeline can be readily evaluated by applying linear random vibration theory either in the time or the frequency domain.

### 5.3 Frequency domain Fatigue Analysis

As shown in Eq. (5.5) in the previous section, the effective random load \( p_{\text{eff}}(t) \) is proportional to the FPSO deck level acceleration \( \ddot{h}(t) \). The pipeline supports associated motions \( h(t) \) is the results of the FPSO sea wave induced vibration in heave direction. The acceleration design spectrum \( S_{\text{design}}(\omega) \) on the FPSO topside is derived in detail in Chapter 3, which is exactly the deck level acceleration \( \ddot{h}(t) \) spectrum. Therefore, denotion \( S_b(\omega) \) is used to replace the \( S_{\text{design}}(\omega) \) in the following section for the reason of
consistence. Further, by applying the mode superposition method and well-known linear dynamic system input-output relationships, the stress spectrum for a generic section of the pipeline is derived which is required for the frequency domain analysis of pipeline random vibrations. Specifically, $S_{\dot{y}}(\omega)$ is used in conjunction with the discretized equations of motion (5.16) to evaluate the stress spectrum at a given location along the pipeline length for further fatigue life estimation.

5.3.1 Stress Spectrum for Pipeline on FPSO Topside

The order of partial differential equation (5.4) governing the pipeline vertical vibrations is lowered and transformed into Eq. (5.16), which represents the equation of motion of a $R$th order lumped parameter system, by applying the Galerkin method. According to the linear random vibration theory, the power spectrum matrix $S_q(\omega)$ of the stationary random vector process $q(t)$ can be evaluated by the following input-output relationship [20]

$$S_q(\omega) = \alpha(\omega) \tau \tau^T S_{\dot{\rho}}(\omega) \alpha^T(\omega).$$

where the * denotes complex conjugate, $S_{\dot{\rho}}(\omega)$ is the power spectrum of the acceleration at the base of the pipeline as discussed in previous chapters. The matrix of frequency-response transfer functions $\alpha(\omega)$ is given by the equation
\[ \alpha(\omega) = \left[ -\omega^2 I + i \omega \Xi + \hat{\Omega}^2 \right]^{-1}, \]  

where \( I \) is the \( R \) th order identity matrix.

The power spectrum of the pipeline displacement at the coordinate \( x = \bar{x} \) can be expressed in terms of \( S_q(\omega) \) by using Eq. (5.6), that is

\[ S_w(\omega; \bar{x}) = \phi^\top(\bar{x}) S_q(\omega) \phi(\bar{x}). \]  

(5.32)

here, \( \phi(x) \) is the \( R \) th order vector containing the eigenfunctions \( \phi_j(x) \).

Since the contribution of the cross-spectral densities of the generalized coordinates, \( S_{q,\alpha_i}(\omega), j \neq k \), is often negligible, Eq. (5.32) can be simplified as follows,

\[ S_w(\omega; \bar{x}) = \sum_{j=1}^{R} \phi_j^2(\bar{x}) S_{q,\alpha_j}(\omega), \]  

(5.33)

where \( S_{q,\alpha_j}(\omega) \) are the diagonal elements of the matrix \( S_q(\omega) \).

The maximum stress in the cross-section placed at the coordinate \( x = \bar{x} \) is a stationary random process given by the equation

\[ s_{\max}(t) \equiv s_{\max}(x,t)\big|_{x=\bar{x}} = \frac{M(x,t)\big|_{x=\bar{x}}}{I} z; \quad z = \pm \frac{D_s}{2}, \]  

(5.34)
where

\[ M(x, t) = -EI \frac{\partial^2 w(x, t)}{\partial x^2} \]  \hspace{1cm} (5.35)

is the random bending moment along the pipeline. Substitute Eq. (5.6) and take into account Eq. (5.33), the power spectrum of the maximum stress is

\[ S_{\text{max}}(\omega; \tilde{x}) = \left( \frac{D_E}{2} \right)^2 \sum_{j=1}^{q} \phi_j^*(\tilde{x})S_{\xi_j\xi_j}(\omega). \]  \hspace{1cm} (5.36)

Further, the stationary variance of \( s_{\text{max}}(t) \) and \( \dot{s}_{\text{max}}(t) \) can be obtained by using the following relationships

\[ \sigma^2_{\text{max}}(\tilde{x}) = \int_{-\infty}^{\infty} S_{\text{max}}(\omega; \tilde{x}) d\omega, \]  \hspace{1cm} (5.37)

and

\[ \sigma^2_{\dot{\text{max}}}(\tilde{x}) = \int_{-\infty}^{\infty} \omega^2 S_{\text{max}}(\omega; \tilde{x}) d\omega. \]  \hspace{1cm} (5.38)

In the next section, the above formulations are used for fatigue damage analysis of a specific pipeline conveying fluid.
5.3.2 Fatigue Life Estimation

In this section, the conventional frequency domain procedure is employed again for fatigue damage assessment of the pipeline on topside of FPSO. Classical textbooks [ref. 16, 17] have provided a detailed description of this method which is also introduced in the fatigue review part of this study (Chapter 2). A summary using appropriate denotation is briefly explained next.

To make use of Eq. (2.39) to Eq. (2.43) for fatigue life estimation, the maximum stress \( s_{\text{max}}(t) \) at any section of the pipeline under stochastic loading is assumed a narrow-band stationary process. According to the Miner’s linear damage accumulation rule, the damage is increased in each cycle. This implies that each stress peak causes a damage increase. Therefore, for a time interval \( T \), the expected damage due to all stress cycles, with peaks between the interval \([s_{\text{max}}, s_{\text{max}} + ds_{\text{max}}]\), is given by the equation

\[
\frac{n(s_{\text{max}})}{N(s_{\text{max}})} = \nu_c^T p(s_{\text{max}}) ds_{\text{max}}.
\]  

(5.39)

Here, \( n(s_{\text{max}}) \) represents the expected number of peaks with amplitude between \( s_{\text{max}} \) and \( s_{\text{max}} + ds_{\text{max}} \), while \( N(s_{\text{max}}) \) is the number of cycles to failure with constant amplitude \( s_{\text{max}} \). \( p(s_{\text{max}}) \) denotes the probability density function of peak amplitude, and \( \nu_c^T \) is the average number of zero crossings with positive slope per unit time. Further, the total expected damage in the time interval \( T \) can be computed as
\[ E[D(T)] = v_0^+ T \int_0^s \frac{P(s_{\text{max}})}{N(s_{\text{max}})} \, ds_{\text{max}}. \]  
\((5.40)\)

The number of cycles to fatigue failure with constant amplitude \(s_{\text{max}}\) is evaluated by incorporating the appropriate \(S-N\) curve parameters. That is,

\[ N(s_{\text{max}})s_{\text{max}}^b = c \Rightarrow N(s_{\text{max}}) = cs_{\text{max}}^{-b}, \]  
\((5.41)\)

in which \(b\) and \(c\) are positive constants and represents the characteristic of the material.

Substitute Eq. (5.41) into Eq. (5.40), Eq. (5.40) is rewritten as

\[ E[D(T)] = Tc^{-1}v_0^+ \left( \sqrt{2\sigma_{\text{max}}} \right)^b \Gamma\left( \frac{b + 2}{2} \right), \]  
\((5.42)\)

where

\[ v_0^- = \frac{1}{2\pi} \frac{\sigma_{\text{max}}}{\sigma_{\text{max}}}, \]  
\((5.43)\)

Here, \(\sigma_{\text{max}}\) and \(\sigma_{\text{max}}\) denote the stationary standard deviation of the maximum stress and its time derivative, and has been given by Eqs. (5.37) and (5.38), respectively.
Similarly to Eq. (2.43), based on the Miner’s Rule \( E[D(T_F)] = 1 \), the estimated fatigue life duration \( T_F \) is expressed as

\[
T_F = \frac{1}{c^{-1} \nu_0^{(\sqrt{2} \sigma_{\text{rms}})} \Gamma \left( \frac{b + 2}{2} \right)}. \tag{5.44}
\]

### 5.4 Numerical Application and Results

The proposed method is applied next to estimate the fatigue life of a pipeline example simply supported at both ends conveying a fluid with constant velocity \( U = 15 \text{ m/s} \). The power spectrum of the base acceleration \( S_x(\omega) \) has been obtained in Chapter 3.

The pipeline example considered in this thesis has the following geometrical parameters: length \( L = 16 \text{ m} \), external diameter \( D_e = 0.25 \text{ m} \) and wall thickness \( d = 0.005 \text{ m} \). The material is steel AISI C1020 with modulus of elasticity \( E = 203 \text{ GPa} \) and ultimate strength \( S_u = 441 \text{ MPa} \). The densities of the fluid and the pipeline material are \( \rho_f = 800 \text{ Kg/m}^3 \) and \( \rho_p = 7850 \text{ Kg/m}^3 \), respectively.

The appropriate S-N curve for the pipeline material considered here is defined by the following empirical equation [85],

\[
S = 1.62 S_u N^{-0.085} \Rightarrow N = (1.62 S_u)^{1/0.085} S^{-1/0.085}. \tag{5.45}
\]
Numerical investigations conducted during the present study have demonstrated that the dynamic response of the pipeline for the selected design and environmental conditions is adequately described by no more than three eigenfunctions of a simply-supported beam as in Eq. (5.6) of Galerkin expression of pipeline displacement in heave direction. Therefore, the original continuous system can even be replaced by a single-degree-of-freedom model without loosing a certain degree of accuracy, that is

$$\ddot{q}_1(t) + \dot{\omega}_i^2(t) q_1(t) = -\left(4 / \pi\right) \ddot{h}(t), \quad (5.46)$$

where $\dot{\omega}_i = 8.866 \text{ rad/s}$. According to Eq.(5.27), due to the presence of the fluid flow, the resulted natural frequency is correct since $\dot{\omega}_i < \omega_i = 9.208 \text{ rad/s}$. The critical fluid velocity value here is $U_c = 55.578 \text{ m/s}$ based on Eq. (5.28). Obviously, the single-mode approximation of the response greatly simplifies the frequency domain analysis. Particularly, the inversion of the matrix in Eq. (5.31) is avoided since the frequency-response matrix reduces to a function, which exactly is the frequency-response function of the single-degree-of-freedom system. Further, it should be mentioned that there is no contribution from the damping term associated with the Coriolis force shown in Eq. (5.4) since $\Xi_{11} = 2 \mu U c_{11} = 0$ for this case. It can be verified that this result applies also to clamped-clamped pipelines but not applicable to cantilevered beam models.
The power spectrum of the maximum stress at midspan, $s_{\text{max}}(t)$, evaluated through Eq. (5.36) by setting $\bar{x} = L/2$ and using the deck level design acceleration spectrum derived in Chapter 3 is shown in Fig. 5.3.

By using Eqs. (5.37) and (5.38), the following values of the standard deviations of $s_{\text{max}}(t)$ and its time derivative are obtained. In this case, $\sigma_{s_{\text{max}}}^2 = 3.137 \times 10^{15}$ N$^2$/m$^4$ and $\sigma_{s_{\text{der}}}^2 = 1.5587 \times 10^{15}$ N$^2$/s$^3$m$^4$. Finally, replacing these results in Eq. (5.44) and using the S-N curve (5.45), a fatigue life $T_f = 41.9584$ yrs is calculated.

![Figure 5.3. Power spectrum of maximum stress at midspan of the pipeline example](image-url)
Therefore, without the FEM analysis, a closed form methodology for estimating
the fatigue life of pipeline on topside of FPSO is completed. It must be emphasized that
the present example is not intended to provide binding results for designers, but to
provide an affordable and practical tool and guide for offshore industry engineers to
conduct preliminary design and to perform readily parametric analyses. Nevertheless,
since a relatively small amount of calculations is involved compared to the FEM oriented
methods, the present method can be potentially employed to develop preliminary design
criteria of different FPSO topside equipment/structures. Furthermore, more modes might
be necessary in the Galerkin solution for real world complex problems instead of the
fortuitous capturing of the dynamic response of the pipeline in present study.
Chapter 6 Methodology on Fatigue Life Estimation of Non-linear Structure on FPSO Topside

Conceivably, the fatigue life of the piping system can be increased by introducing limit stops in the span of the pipe. In this context, the statistical linearization technique is employed herein to estimate the pipe response. Specifically, S-D-O-F pipe system derived from previous contexts by using Galerkin methods is used as a reasonable example to show the effectiveness of the statistical linearization technique and the determination of the enhanced fatigue life of the pipe.

6.1 Piping System Non-linear Problem Study and Method of Statistical Linearization

6.1.1 Nonlinear Dynamic Study of Pipes Conveying Fluid

Non-linear dynamic analysis of both cantilevered and pinned-pinned pipes conveying fluid has been developed by Semle et al [ref. 80-83]. They have derived a complete set of geometrically nonlinear equations of motion of fluid conveying pipes. These equations have taken into account large strains and the assumption of Euler-Bernoulli beams kinematics. However, they are unduly complicated for practical applications especially when there is no large deformation happened in the pipes.

The geometric nonlinear problem of submarine pipe has been accurately modeled [in ref. 90], and numerical solutions have been provided. Lam et al [ref. 88] has incorporated a negative lift force (attraction) into the near-bed pipeline analysis in a horizontal current to solve the nonlinear fluid-structure interaction problem by
employing an iteration procedure. Qiao has investigated the bifurcations and chaotic motions of a fluid-conveying curved pipe restrained with nonlinear constraints [ref. 89]. Lee and Chung [ref. 84] have proposed a new non-linear modeling method of straight pipe conveying fluid with both ends fixed for vibration analysis. They have derived both the longitudinal and transverse displacements by solving the nonlinear coupled equation of motion using finite difference method. All investigations have focused on modeling various nonlinearity aspects on pipeline structures under sea wave or sea current loadings in the time domain. For the analysis of the nonlinearity appearing in FPSO topside pipeline structures, there are no closed-form analytical solutions to calculate the fatigue life. This thesis is to develop a procedure for practical use in preliminary design by incorporating the fully developed random vibration theory and statistical linearization technique.

6.1.2 Method of Statistical Linearization

It has been shown in previous chapters that an important tool for the computation of the statistics of the response of a linear system to stationary stochastic excitation is the input-output relationship of the power spectra expressed by Eq. (2.9). However, for a nonlinear system excited by a stationary random process, there is no exact solution for most cases. The method of statistical linearization [ref. 20, 87] is therefore introduced and analyzed in conjunction with a simple example. It is a method to replace the nonlinearity element in a system by an equivalent linear element.

First consider a S-D-O-F system with nonlinear stiffness to illustrate the key idea of this technique. The system is described by the 2\(^{nd}\) order differential equation
\[
\ddot{y} + 2\zeta \omega_n \dot{y} + \omega_n^2 \left[ y + \lambda G(y) \right] = x(t),
\]

(6.1)

where \(\omega_n\) is the natural frequency of the system, \(\zeta\) is the damping ratio, \(x(t)\) is the excitation, and \(G(y)\) is an odd non-linear function of \(y\). It is assumed that \(x(t)\) is a zero-mean stationary Gaussian process with power spectrum \(S_x(\omega)\). The goal is to derive an equivalent linear equation

\[
\ddot{y} + 2\zeta \omega_n \dot{y} + \omega_{eq}^2 y = x(t),
\]

(6.2)

where \(\omega_{eq}\) is selected such that a certain minimization criterion is met. That is, the minimization of the expected value of the error in the least square sense, which is expressed by the following equation

\[
\delta = \omega_n^2 \left[ y + \lambda G(y) \right] - \omega_{eq}^2 y = g(y) - \omega_{eq}^2 y.
\]

(6.3)

Therefore,

\[
E\{\delta^2\} = \text{min\ imum} \Rightarrow \\
\Rightarrow \frac{d}{d\omega_{eq}} E\{\delta^2\} = 0.
\]

(6.4)
From Eq. (6.4) one derives the equation

\[ \omega_{eq}^2 = \frac{E\{yg(y)\}}{E\{y^2\}} = \omega_n^2 \left(1 + \lambda \frac{E\{yG(y)\}}{\sigma_y^2}\right). \] (6.5)

where \( \sigma_y^2 \) is the variance of the output \( y \). It is assumed that the input has zero-mean.

The output, as well, is a zero-mean process, and \( \sigma_y^2 = E\{(y(t)-\mu)^2\} = E\{y(t)^2\} \). In evaluating the expected value of \( yG(y) \) in Eq. (6.5), the PDF of \( y(t) \) is required, which is unknown. The key idea is to approximate the process \( y(t) \) by a Gaussian process and, thus, describe its probability density in terms of only its variance \( \sigma_y^2 \). This approximation leads to a simplified version of Eq. (6.6) as shown in [ref. 20]. Specifically,

\[ \omega_{eq}^2 = \omega_n^2 \left(1 + \lambda E\left[\frac{dG}{dy}\right]\right). \] (6.6)

After determining \( \omega_{eq} \), one can proceed by calculating the variance of the response by using the equation of the equivalent linear system described by Eq. (6.5). This can be achieved by employing Eq. (2.9), which yields
\[ S_x(\omega) = |H(\omega)|^2 S_x(\omega) = \left| \frac{1}{-\omega^2 + \omega_{eq}^2 + i 2\zeta \omega_n \omega} \right|^2 S_x(\omega) \],

(6.7)

where \( H(\omega) \) is the frequency response of the equivalent system. Finally, one derives

\[ \sigma_y^2 = \int_{-\infty}^{\infty} S_y(\omega) d\omega = \int_{-\infty}^{\infty} \left| \frac{1}{-\omega^2 + \omega_{eq}^2 + i 2\zeta \omega_n \omega} \right|^2 S_x(\omega) d\omega. \]

(6.8)

Eq. (6.6) and (6.8) are nonlinear equations involving the unknowns \( \omega_{eq}^2 \) and \( \sigma_y^2 \); they can be solved numerically by using iterative methods.

### 6.2 Mathematical Modeling of Geometric Nonlinearities of Piping

**System on FPSO topside**

For safety reasons, limit stops are generally placed on the middle point (the location the largest relative displacement observed) of the piping systems on topside FPSO, which allow the piping system to move a certain amount in either direction before the support becomes active. That means the flexural vibration of the fluid inside pipeline is often limited through the introduction of a buffer, as illustrated in Fig. 6.1.
Fig. 6.1 Single-span pipeline conveying fluid on the FPSO topside with limit stops on each side

The linear springs with stiffness $K$ at each side is activated only when $|w(x,t)| > a$; where $w(x,t)$ is the pipeline relative displacement to the supports in the heave direction, and $a$ is the gap between the fluctuating beam and the springs. The pipeline as simulated Bernoulli-Euler beam is relatively long and thin; therefore it is assumed that the rotary inertia may be neglected in the moment equation.

To consider the spring support resulted nonlinear effect, Eq. 5.4 must be modified and rewritten as

$$
EI \frac{\partial^4 w(x,t)}{\partial x^4} + m_i \frac{\partial^2 w(x,t)}{\partial x^2} + 2m_J U \frac{\partial^3 w(x,t)}{\partial x \partial t} + (m_f + m_p) \frac{\partial^3 w(x,t)}{\partial t^2} = -(m_f + m_p) \frac{d^2 h(t)}{dt^2}
$$

for $|w(x,t)| \leq a$ (6.9)
\[ EL \frac{\partial^4 w(x,t)}{\partial x^4} + m_j U^2 \frac{\partial^2 w(x,t)}{\partial x^2} + 2m_j U \frac{\partial^2 w(x,t)}{\partial x \partial t} + (m_j + m_p) \frac{\partial^2 w(x,t)}{\partial t^2} + K(w(x,t) - a) = -(m_j + m_p) \frac{d^2 h(t)}{dt^2} \]

for \( w(x,t) > a \) \hspace{1cm} (6.10)

and

\[ EL \frac{\partial^5 w(x,t)}{\partial x^5} + m_j U^2 \frac{\partial^3 w(x,t)}{\partial x^3} + 2m_j U \frac{\partial^3 w(x,t)}{\partial x^2 \partial t} + (m_j + m_p) \frac{\partial^3 w(x,t)}{\partial t^2} - K(-w(x,t) - a) = -(m_j + m_p) \frac{d^2 h(t)}{dt^2} \]

for \( w(x,t) \leq -a \) \hspace{1cm} (6.11)

Using the same technique applied in Chapter 5 to lower the order of the partial differential equation, \( \phi_k(x) \) is multiplied and integrated from 0 to \( L \). The second order differential equation is obtained:

\[ \ddot{q}_k(t) + \omega_k^2(q_k(t) + F(q_k(t)) + \mu U^2 \sum_{j=1}^{R} b_{kj} q_j(t) + 2 \mu U \sum_{j=1}^{R} c_{kj} \dot{q}_j(t) = -\tau_k \dot{h}(t), \quad (k = 1, 2, \ldots, R). \]  

(6.12)

The same parameter definition of \( \mu, b_{kj}, c_{kj}, \tau_k \) still holds. \( F(q_k(t)) \) has the following expression,
\[
F(q_k(t)) = \begin{cases} 
\frac{K}{m_f + m_p} [q_k(t) + a] & \text{for } q_k(t) \leq -a \\
0 & \text{for } -a \leq q_k(t) \leq a \\
\frac{K}{m_f + m_p} [q_k(t) - a] & \text{for } q_k(t) \geq a 
\end{cases}
\]

(6.13)

It can be seen that the new term in Eq. (6.12) is the extra piecewise linear elastic force in heave direction comparing with Eq. (5.9). In Chapter 5, the detailed procedure to solve Eq. (6.9) has been presented. Further, the Galerkin method is again used to solve the fourth order partial differential equation. With the target to obtain the fatigue life estimation in frequency domain, it is demonstrated that the original continues piping system can be replaced by a single degree of freedom without loosing a certain degree of accuracy as shown in Eq. (5.46). To adopt the nonlinear effect the limit stops introduced and small damping, the piecewise linear motion of equation can be written as,

\[
\ddot{q}_1(t) + 2\zeta_1 \omega_1 \dot{q}_1(t) + \left[ \omega_1^2(t) + \omega_{eq}^2(t) \right] q_1(t) = -\left( \frac{4}{\pi} \right) \dot{\bar{h}}(t)
\]

(6.14)

where \( \dot{\omega}_1 = 8.866rad/s \) for the system studied in Chapter 5. \( \omega_{eq} \) is the limit stop introduced equivalent natural frequency. Next, the technique of statistical linearization is used to solve the nonlinear pipe problem by using the same design spectrum considered in Chapter 3.
6.3 Nonlinear Piping System Stress Response via the Statistical Linearization Procedure

The statistical linearization procedure is adopted next to solve the nonlinear equation of piping system on topside FPSO, i.e. Eq. (6.14).

The expected value of error is

\[
\delta = \frac{F}{m_f + m_p} - \omega_{eq}^2 q_i = g(q_i) - \omega_{eq}^2 q_i
\]  

(6.15)

Thus, introducing the operator of mathematical expectation \(E\), one derives:

\[
E\{\delta^2\} = \min \text{imum} \Rightarrow \\
\Rightarrow \frac{d}{d\omega_{eq}^2} E\{\delta^2\} = 0.
\]  

(6.16)

The equivalent natural frequency is expressed as

\[
\omega_{eq}^2 = \frac{E\{q_i g(q_i)\}}{E\{q_i^2\}} = \frac{1}{m_f + m_p} \frac{E\{q_i F(q_i)\}}{\sigma_{q_i}^2} = \frac{1}{m_f + m_p} E\left\{\frac{\partial F(q_i)}{\partial q_i}\right\},
\]  

(6.17)

where \(\sigma_{q_i}^2\) is the standard deviation of \(q_i(t)\).
The Gaussian distribution is used to approximate the distribution of \( q_i(t) \).

Thus, the equivalent natural frequency is

\[
\omega_{eq}^2 = \frac{1}{m_f + m_p} \frac{K}{\sqrt{2\pi} \sigma_{q_i}} \left[ \int_{-\infty}^{\alpha} e^{-q_i^2/2\sigma_{q_i}^2} \, dq_i + \int_{\alpha}^{\infty} e^{-q_i^2/2\sigma_{q_i}^2} \, dq_i \right],
\]

which leads to

\[
\omega_{eq}^2 = \frac{K}{m_f + m_p} \left[ 1 - \text{erf} \left( \frac{\alpha}{\sqrt{2}\sigma_{q_i}} \right) \right],
\]

Similarly to Eq. (5.30), the spectral equation

\[
S_{q_i}(\omega) = \left| \frac{1}{-\omega^2 + \omega_{eq}^2 + \bar{\omega}_i^2 + i2\gamma \bar{\omega}_i \omega} \right|^2 \frac{4}{\pi} S_{\text{design}}(\omega)
\]

is obtained.

Further, by using Eq. (5.36) and Eq. (5.37), the power spectrum of maximum stress and its standard deviation are obtained. Eq. (6.18) is nonlinear involving the unknowns \( \omega_{eq}^2 \) and \( \sigma_{q_i}^2 \), and can be solved iteratively. The initial approximation is from the linear equation derived parameters \( \sigma_{q_i}^2 = 3.137 \times 10^{14} \text{ N}^2/\text{m}^4 \) and \( s_{\text{max}}(t) \) in Figure 5.3. At each iteration, one computes the equivalent natural frequency by using Eq. (6.19), and updates the transfer function in Eq. (6.20), then the updated standard deviation of the
maximum stress power spectrum at the midspan is obtained. After \( n \)th iteration, the computation will be stopped when \( \left| \sigma_{\max}^2 (n+1) - \sigma_{\max}^2 (n) \right| \leq tol \).

Since the target of the study is to obtain the fatigue life of the FPSO topside pipes, the most important parameters used for the calculation is the standard deviation of the maximum stress spectrum \( \sigma_{\max}^2 \) and the expected zero-crossings of equivalent system. Next the numerical iteration results are used to obtain the fatigue life of the pipe example analyzed in Chapter 5.

### 6.4 Nonlinear Piping System Fatigue Life Numerical Results

The fatigue life of the simplified S-D-O-F pipe example used in Chapter 5, but with limit stops, is here calculated by using Eq. (5.44).

The power spectrum of the maximum stress at midspan \( s_{\max}(t) \) of the equivalent system evaluated through Eq. (5.37) by setting \( \bar{x} = L/2 \) and using the deck level design acceleration spectrum derived in Chapter 3 is shown in Fig. 6.2. The standard deviations of \( s_{\max}(t) \) and its time derivatives are \( \sigma_{\max}^1 = 2.9585 \times 10^{5} \text{ N}^2/\text{m}^4 \) and \( \sigma_{\max}^2 = 1.4699 \times 10^{5} \text{ N}^2/\text{s}^2/\text{m}^4 \), respectively.

For the FPSO topside pipe examined in this thesis with limit stops, the fatigue life \( T_f = 59.2224 \text{ yrs} \) is obtained by choosing appropriate parameters, \( a = 0.12 \text{ m} \), \( K = 7.9788 \text{e}^4 \text{ N/m} \) and damping coefficient \( \zeta = 0.04 \). The limit stops yielded equivalent natural frequency is \( \omega_{eq} = 1.5234 \text{ rad/s} \) which induced the increase of the original
stiffness. It is clearly shown that by putting limit stops on each side, the fatigue life of the pipe example is largely extended.

![Power spectrum of maximum stress at midspan of the pipeline example with limit stops](image)

**Figure 6.2.** Power spectrum of maximum stress at midspan of the pipeline example with limit stops

Further, the study is extended to optimize the spring stiffness $K$ and gap value $a$ introduced by the limit stops. The ratio between the equivalent system natural frequency and the original linear system pertaining to various limit stop gap values are plotted in Figure 6.3. By choosing appropriate $K$ value and gap value $a$, the target pipeline fatigue life can be obtained (Figure 6.4). It is clearly shown that when the gap values $a$ reaches a certain value, the nonlinear force introduced by the limit stop can be neglected, and the fatigue life of the equivalent system is equal to that of the linear system. By increasing
stiffness $K$ value, the fatigue life can be largely extended provided that the gap value $a$ is appropriately chosen.

* -- $K = 3.9894 \times 10^4$ N/m
o -- $K = 7.9788 \times 10^4$ N/m
+ -- $K = 15.958 \times 10^4$ N/m
x -- $K = 19.947 \times 10^4$ N/m

Figure 6.3. Equivalent system natural frequency value of the pipe example
Figure 6.4. Equivalent system fatigue life of the pipe example
Chapter 7 Concluding Remarks

The deterioration of engineering structures due to fatigue has been a challenging problem for engineers for many decades. In this context, the oil industry has formulated design methodologies for practical assessment procedures for traditional offshore structures. However, there is scarce literature focusing on the fatigue analysis of topside structures. The American Bureau of Shipping (ABS) has provided a guide on how to design FPSO topside structures. It specifies that the finite element method (FEM) with a fine mesh must be used. However, to perform some preliminary design analysis, the computation demand of FEM is not practical for many industry design companies. Therefore, an integrated approach for fatigue life analysis of equipment/structures located on the topside of FPSO systems has been proposed in this thesis. This approach, hopefully, be a useful tool in this field.

The applicability of the proposed approach has been demonstrated by the analysis of both a quite simple S-D-O-F piece of equipment, and a pipeline conveying fluid on topside of an FPSO with/without limit stops subject to sea-wave spectrum. The particular data used pertain to the Kizomba FPSO.

Discrete transfer functions at different locations (modules) in the topside of the FPSO for various spreading directions have been incorporated into the analysis. Combined with the Ochi-Hubble sea wave elevation spectrum, a design spectrum on deck level has been determined for fatigue studies.

Specifically, for the generic piece of equipment, the stress spectrum has been obtained by using the well-known dynamic system transfer function and the design
spectrum. A Moving-Average algorithm has been adopted to generate the stress time history, which has provided good matching between the simulated and the target spectra. A new recursive Rainflow counting algorithm has been employed to identify significant cycles. The fatigue life has been estimated by using the cycle counting based method. The stress crossing based fatigue life calculation method has also been used to verify and validate the presented computational procedure. Both methods have yielded quite reasonable results.

For the more complex pipeline example considered in this study, the sea wave induced excitation has been modeled as a random motion acting on both supports ends by ignoring the dynamic interaction between the pipeline and the supporting fittings. Note that the fluid flow complicates the dynamic behavior of the pipeline, as well as the pertinent mathematical model. The equation of motion of the pipeline conveying a fluid with constant velocity has been derived relying on certain existing formulations and some assumptions. Specifically, the empty pipeline has been modeled as a Bernoulli-Euler beam, and the so-called “plug-flow” approximation has been adopted for the fluid. The Galerkin method has been employed to obtain the random response of the pipeline in terms of appropriate basis functions, and to discretize fourth-order partial differential equation of motion. The natural modes of a beam have been selected as basis functions of the Galerkin formulation. The multi-degree-of-freedom random vibration solution has been derived in conjunction with the design spectrum. Finally, the fatigue life has been determined by employing the frequency domain method, which is also used to estimate the fatigue life of a simple piece of equipment.
For the non-linearity introduced by the limit stops for pipeline structures on topside of FPSOs, the statistical linearization technique has been employed in conjunction with random vibration theory in the frequency domain. The procedure for calculating the fatigue life of such structures has been provided based on the results derived from the linear pipeline structure. After extensive mathematical manipulations and appropriate simplifications, closed-form solutions are provided for practical use. It has been show that using limit stops in the mid-span of the pipeline can significantly increase its fatigue life.

All the fatigue life calculations have been performed by using the Miner’s linear damage accumulation rule. The complete procedures involve small amounts of calculation compared to those of FEM based formulations. As the results presented in this study show, the technique developed through these studies could be used in those situations where piping system responses must be estimated analytically, accompanied by an evaluation of the reliability of the estimates. It is hoped that the developed approach will provide an affordable and practical tool for preliminary design of not only FPSO topside structures, but also for other engineering applications which induce random vibration loading.

Further, it is noted that if it is desired to take into account the coupling between the transverse and the longitudinal displacement through the non-linear terms in equation of motion, or to treat the support loading power spectrum as correlated, more complex random vibration analysis will be required. Of course in this case it will provide more realistic predictions of the dynamic response of structures and equipment on FPSO topside, and it will be perhaps attempted in the future.
Reference:


