RICE UNIVERSITY

Arterial Fluid Mechanics Computations with the Stabilized Space–Time
Fluid–Structure Interaction Techniques

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Master of Science

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APRIL 2007
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Abstract

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The stabilized space–time fluid–structure interaction (SSTFSI) techniques developed by the Team for Advanced Flow Simulation and Modeling (T*AFSM) are applied to the field of arterial fluid mechanics through the FSI modeling of a cerebral artery with a small, saccular aneurysm. All arterial structures are modeled with membrane elements, which are geometrically nonlinear. FSI computations of cardiovascular systems presently interest the scientific community as such types of analysis provide a non-invasive means of analyzing a patient’s condition and risk for aneurysm rupture, a potentially life-threatening condition. Test computations for varying arterial wall thickness and blood pressure are presented for this cerebral aneurysm, with the arterial geometries of the computations closely approximating patient-specific image-based data. Results show the T*AFSM’s ability to handle complex and realistic FSI simulations while demonstrating the capability and utility of FSI simulations in the field of cardiovascular fluid mechanics.
Acknowledgments

I would like to thank my advisor Dr. Tayfun Tezduyar for his guidance and valuable lessons as the leader of T*AFSM. I will strive to carry these into my profession and into my personal life. I am also grateful to Dr. Ed Akin and Dr. Andrew Meade for serving with him on my thesis committee and for their insight and suggestions.

I am extremely grateful to Dr. Sunil Sathe for providing me direction and sharing his experience with me during my research. I thank Luca Aureli for his friendship and professional example as I began my studies at Rice. I thank 2nd Lieutenants Jason Pausewang and Matt Schwaab and Dr. Ramakrishnan Srinivas for their encouragement and camaraderie. I also thank Dr. Brian Conklin for offering a medical perspective, and Tim Cragin for carrying my work another step forward.

Finally, to my friends in Houston, both air force and civilian, old and new, you helped me keep things in perspective as I worked toward achieving my goals. I will never forget our time together. To my parents, my brothers and sisters, my grandparents, and my extended family, thank you for your love and support. Without your encouragement, I would not be where I am today. To all of my family, blood or not, thank you for believing in me through every step of this challenging process.

This work was supported in part by the Rice Computational Research Cluster, NSF Grant CNS-0421109, and a Rice University partnership with AMD and Cray.
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Chapter 1

Introduction

One in 15 Americans eventually develops a cerebral aneurysm. Once this weak spot forms on a blood vessel in the brain, the aneurysm swells and fills with blood. This places increased pressure on the surrounding nerves and brain tissue, and if the aneurysm bursts or ruptures, the consequences are grave. With ruptured aneurysms occurring in as many as 27,000 people per year in the U.S. alone, the hardship caused by aneurysms is widespread. Of patients with ruptured aneurysms, 40 percent do not survive the first day of this trauma and up to 25 percent die within the first six months following rupture. Unfortunately, the prognosis does not significantly improve for those suffering a ruptured cerebral aneurysm. After a year, 60 percent are deceased, while only 20 percent of those with ruptured aneurysms are alive and functioning without serious disability. (1; 2)

1.0.1 Motivation

In addition to the significant danger posed by an aneurysm at risk for rupture, there are several complications involved with the brain surgery used to eliminate aneurysms. Operations to remove and correct cerebral aneurysms have the potential
to damage healthy blood vessels and lead to post-operative stroke. In addition, the potential remains for aneurysm recurrence and rebleeding after surgery. For these reasons, scientists seek to learn more about how aneurysms are formed, the factors leading to their growth, and how to prevent and treat them.

Researchers around the world are working to simulate and model flow through arteries in order to better understand and predict aneurysm growth and behavior. Early diagnosis of aneurysm sites is often the difference between life and death. By accurately modeling arterial flow and gathering useful data from these simulations, computational models can greatly aid the diagnosis process. Therefore, through simulations of this nature, the potential exists to save countless lives in the U.S. and across the globe.

Within an aneurysm, the flow and pressure dictate the deformation of the artery and likewise the deformations affect the flow. The connection between these phenomena demands a coupled approach for simulating the flow and the structural deformations. Previous work by Torii et al. consisted of simulations based on block-iterative methods (see (3; 4; 5)). Other researchers have done away with the structural computations entirely and simply dealt with the fluid problem. However, as noted in (4), arterial wall deformation has a “significant influence” on wall shear stress (WSS) distribution, “an important factor in creation, growth and rupture of cerebral aneurysm[s].” The research in this thesis consists of fluid-structure interaction (FSI) computations on a closed-flow artery with aneurysm, in which the movement of the
arterial walls is more directly coupled to the flow and vice versa, thus providing a more robust and accurate model for the flow within a cerebral aneurysm and the forces on the arterial walls.

The goal of this research is to build a foundation for computationally modeling and characterizing the risks associated with cerebral aneurysms, in an effort to reduce the mortality rate of patients with brain aneurysms. In order to accurately characterize the flow within an aneurysm, several computational methods are used to improve upon previous research.

This FSI analysis will help establish modeling that may one day assist medical professionals weighing the risks of surgery against the risk of rupture. This work focuses on the preliminary results of FSI computations carried out on a small, saccular cerebral aneurysm model. Utilizing realistic fluid and structural properties and flow velocity, FSI studies of patient aneurysms like the one modeled in this thesis, provide a non-invasive means of assessing risk.

1.0.2 Overview

Chapter 2 gives the fundamental equations for the simulation cases studied. This section presents the equations governing the fluid and structural mechanics involved in the aneurysm FSI simulations.

Chapter 3 presents the finite element formulations for fluid flow and structural deformation. The Deforming-Spatial-Domain/Stabilized Space-Time (DSD/SST) formulation (6; 7; 8; 9) is used for the flow simulation, whereas a finite element formu-
lation based on the principle of virtual work is used for the structural deformation. The semi-discrete formulation of structural mechanics is used for the structure, and the Stabilized Space-Time Fluid-Structure Interaction (SSTFSI) method combines the other two techniques into a complete FSI method.

Chapter 4 provides numerical examples for flow through a specific cerebral aneurysm based on patient data. The four examples presented in this chapter illustrate the effects of variable aneurysmal wall thickness and/or high blood pressure on an aneurysm during the cardiac cycle.

Chapter 5 summarizes the conclusions drawn from this research in cardiovascular fluid mechanics. Furthermore, this section discusses possible paths for future work in this area.
Chapter 2

Governing Equations

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2.1 Fluid mechanics

Let $\Omega_t \subset \mathbb{R}^n$ be the spatial domain with boundary $\Gamma_t$ at time $t \in (0, T)$. The subscript $t$ indicates the time-dependence of the domain. The Navier–Stokes equations of incompressible flows are written on $\Omega_t$ and $\forall t \in (0, T)$ as

$$
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - \mathbf{f} \right) - \nabla \cdot \mathbf{\sigma} = 0 , \tag{2.1}
$$

$$
\nabla \cdot \mathbf{u} = 0 , \tag{2.2}
$$

where $\rho$, $\mathbf{u}$, and $\mathbf{f}$ are the density, velocity, and external force, respectively. The stress tensor $\mathbf{\sigma}$ is defined as

$$
\mathbf{\sigma}(\rho, \mathbf{u}) = -p \mathbf{I} + 2\mu \mathbf{\varepsilon}(\mathbf{u}) , \tag{2.3}
$$
with

\[ \varepsilon(u) = \frac{1}{2} \left( (\nabla u) + (\nabla u)^T \right), \]  

(2.4)

and

\[ \mu = \rho \nu. \]  

(2.5)

Here, \( p \) is the pressure, \( I \) is the identity tensor, \( \mu \) is the viscosity, \( \nu \) is the kinematic viscosity, and \( \varepsilon(u) \) is the strain-rate tensor. The essential and natural boundary conditions for Eq. (2.1) are represented respectively as

\[ u = g \text{ on } (\Gamma_t)_g \]

\[ \mathbf{n} \cdot \sigma = h \text{ on } (\Gamma_t)_h \]  

(2.6)

where \((\Gamma_t)_g\) and \((\Gamma_t)_h\) are complementary subsets of the boundary \( \Gamma_t \), \( \mathbf{n} \) is the unit normal vector, and \( g \) and \( h \) are given functions. A divergence-free velocity field \( u_0(\mathbf{x}) \) is specified as the initial condition.

### 2.2 Structural mechanics

The governing equations for the structural model, the corresponding finite element formulation, and how this formulation couples with the rest of the FSI system remain
the same as they were given in (10). For the arterial structural models covered here, the second Piola–Kirchhoff stress tensor \( \mathbf{S} \) is defined in a way that represents the membrane model.

Let \( \Omega_t^s \subset \mathbb{R}^{n_{xd}} \) be the spatial domain with boundary \( \Gamma_t^s \), where \( n_{xd} = 2 \) for membranes. The parts of \( \Gamma_t^s \) corresponding to the essential and natural boundary conditions are represented by \( \Gamma^s_{t g} \) and \( \Gamma^s_{t k} \). The superscript "s" indicates the structure. The equations of motion are written as

\[
\rho_s \left( \frac{d^2 \mathbf{y}}{dt^2} + \eta \frac{d\mathbf{y}}{dt} - \mathbf{f}^s \right) - \nabla \cdot \mathbf{\sigma}^s = \mathbf{0} ,
\]

(2.7)

where \( \rho_s, \mathbf{y}, \mathbf{f}^s \) and \( \mathbf{\sigma}^s \) are the material density, structural displacement, external force, and the Cauchy stress tensor (11; 12), respectively. Here, \( \eta \) is the mass-proportional damping coefficient. The damping provides additional stability and can be used where time-accuracy is not required, such as in determining the deformed shape of the structure for specified fluid mechanics forces acting on it. The stresses are expressed in terms of the 2nd Piola–Kirchoff stress tensor \( \mathbf{S} \), which is related to the Cauchy stress tensor through a kinematic transformation. Under the assumption of large displacements and rotations, small strains, and no material damping, the membranes are treated as Hookean materials with linear elastic properties. For membranes, under the assumption of plane stress, \( \mathbf{S} \) becomes (see (13; 10)):

\[
S^{ij} = (\tilde{\lambda}_m G^{ij} G^{kl} + \mu_m \left[ G^{il} G^{jk} + G^{ik} G^{jl} \right]) E_{kl} ,
\]

(2.8)
where for the case of isotropic plane stress,

\[ \bar{\lambda}_m = \frac{2\lambda_m \mu_m}{(\lambda_m + 2\mu_m)}. \]  \hspace{1cm} (2.9)

Here, \( E_{kl} \) are the components of the Green–Lagrange strain tensor, \( G^{ij} \) are the components of the contravariant metric tensor in the original configuration, and \( \lambda_m \) and \( \mu_m \) are the Lamé constants.
Chapter 3

Finite Element Formulations

3.1 DSD/SST formulation of fluid mechanics

The finite element formulations remain as given in (10). In the DSD/SST method (6; 7; 8; 9), the finite element formulation is written over a sequence of \( N \) space–time slabs \( Q_n \), where \( Q_n \) is the slice of the space–time domain between the time levels \( t_n \) and \( t_{n+1} \). At each time step, the integrations are performed over \( Q_n \). The space–time finite element interpolation functions are continuous within a space–time slab, but discontinuous from one space–time slab to another. The notation \((\cdot)_n^-\) and \((\cdot)_n^+\) will denote the function values at \( t_n \) as approached from below and above. Each \( Q_n \) is decomposed into elements \( Q_{n_e}^e \), where \( e = 1, 2, \ldots, (n_{el})_n \). The subscript \( n \) used with \( n_{el} \) is for the general case where the number of space–time elements may change from one space–time slab to another. The essential and natural boundary conditions are enforced over \((P_n)_g\) and \((P_n)_h\), the complementary subsets of the lateral boundary of the space–time slab. The finite element trial function spaces \((S^h_u)_n\) for velocity and \((S^h_p)_n\) for pressure, and the test function spaces \((V^h_u)_n\) and \((V^h_p)_n = (S^h_p)_n\) are defined by using, over \( Q_n \), first-order polynomials in space and time.
The DSD/SST formulation (from (9)) is written as follows: given \( (u^h)^-_n \), find \( u^h \in (S^h_u)_n \) and \( p^h \in (S^h_p)_n \) such that \( \forall w^h \in (V^h_u)_n \) and \( \forall q^h \in (V^h_p)_n \):

\[
\begin{align*}
&\int_{Q_n} w^h \cdot \rho \left( \frac{\partial u^h}{\partial t} + u^h \cdot \nabla u^h - f^h \right) dQ + \int_{Q_n} \varepsilon(w^h) : \sigma(p^h, u^h) dQ \\
&- \int_{(P_n)_h} w^h \cdot h^h dP + \int_{Q_n} q^h \nabla \cdot u^h dQ + \int_{\Omega_n} (w^h)^+_n \cdot \rho ((u^h)^+_n - (u^h)^-_n) d\Omega \\
&+ \sum_{e=1}^{(n_{el})_n} \int_{Q^n_e} \frac{1}{\rho} \left[ \tau_{SUPG} \rho \left( \frac{\partial w^h}{\partial t} + u^h \cdot \nabla w^h \right) + \tau_{PSPG} \nabla q^h \right] \cdot [L(p^h, u^h) - \rho f^h] dQ \\
&+ \sum_{e=1}^{(n_{el})_n} \nu_{LSIC} \nabla \cdot w^h \rho \nabla \cdot u^h dQ = 0 , \quad (3.1)
\end{align*}
\]

where

\[
L(q^h, w^h) = \rho \left( \frac{\partial w^h}{\partial t} + u^h \cdot \nabla w^h \right) - \nabla \cdot \sigma(q^h, w^h) . \quad (3.2)
\]

This formulation is applied to all space–time slabs \( Q_0, Q_1, Q_2, \ldots, Q_{N-1} \), starting with \( (u^h)^-_0 = u_0 \). Here \( \tau_{SUPG}, \tau_{PSPG} \) and \( \nu_{LSIC} \) are the SUPG, PSPG and LSIC (least-squares on incompressibility constraint) stabilization parameters. There are various ways of
defining these stabilization parameters. Here are the definitions given in (9):

\[ \tau_{\text{SUPG}} = \left( \frac{1}{\tau_{\text{SUPG12}}} + \frac{1}{\tau_{\text{SUPG3}}} \right)^{-\frac{1}{2}}, \]  
(3.3)

\[ \tau_{\text{SUPG12}} = \left( \sum_{a=1}^{n_{\text{en}}} \left| \frac{\partial N_a}{\partial t} + u^h \cdot \nabla N_a \right| \right)^{-1}, \]  
(3.4)

\[ \tau_{\text{SUPG3}} = \frac{h_{\text{RGN}}^2}{4\nu}, \]  
(3.5)

\[ h_{\text{RGN}} = 2 \left( \sum_{a=1}^{n_{\text{en}}} |\mathbf{r} \cdot \nabla N_a| \right)^{-1}, \]  
(3.6)

\[ \mathbf{r} = \frac{\nabla |u^h|}{\| \nabla |u^h| \|}, \]  
(3.7)

\[ \tau_{\text{SUPG}} = \tau_{\text{SUPG}}, \]  
(3.8)

\[ \nu_{\text{LSIC}} = \tau_{\text{SUPG}} \| u^h \|^2, \]  
(3.9)

where \( n_{\text{en}} \) is the number of (space–time) element nodes and \( N_a \) is the space–time shape function associated with the space–time node \( a \). As an alternative to the construction of \( \tau_{\text{SUPG}} \) as given by Eqs. (3.3)–(3.4), the option of constructing \( \tau_{\text{SUPG}} \) based on separate definitions for the advection-dominated and transient-dominated limits was proposed in (10):

\[ \tau_{\text{SUPG}} = \left( \frac{1}{\tau_{\text{SUPG1}}} + \frac{1}{\tau_{\text{SUPG2}}} + \frac{1}{\tau_{\text{SUPG3}}} \right)^{-\frac{1}{2}}, \]  
(3.10)

\[ \tau_{\text{SUPG1}} = \left( \sum_{a=1}^{n_{\text{en}}} \left| (u^h - \mathbf{v}^h) \cdot \nabla N_a \right| \right)^{-1}, \]  
(3.11)

\[ \tau_{\text{SUPG2}} = \frac{\Delta t}{2}, \]  
(3.12)
where \( v^h \) is the mesh velocity. It was noted in (10) that separating \( \tau_{SUGN12} \) into its advection-dominated and transient-dominated components as given by Eqs. (3.11)–(3.12) is equivalent to excluding the \( \left( \frac{\partial N_a}{\partial t} \bigg|_{\xi} \right) \) part of \( \left( \frac{\partial N_a}{\partial t} \right) \) in Eq. (3.4), making that the definition for \( \tau_{SUGN1} \), and accounting for the \( \left( \frac{\partial N_a}{\partial t} \bigg|_{\xi} \right) \) part in the definition for \( \tau_{SUGN2} \) given by Eq. (3.12). Here \( \xi \) is the vector of element (parent-domain) coordinates. The option of modifying the \( \nu_{LSIC} \) definition given by Eq. (3.9) to take the mesh velocity into account was also proposed in (10):

\[
\nu_{LSIC} = \tau_{SUPG} \| u^h - v^h \|^2 .
\]

For more ways of calculating \( \tau_{SUPG} \), \( \tau_{PSPG} \) and \( \nu_{LSIC} \), see (14; 9; 15; 16). References (9; 15; 16) also include the Discontinuity-Capturing Directional Dissipation (DCDD) stabilization, which was introduced as an alternative to the LSIC stabilization.

It was remarked in (14; 17; 9; 18; 15; 16) that in marching from time level \( n \) to \( n + 1 \), there are advantages in calculating the \( \tau \)s from the flow field at time level \( n \). That is

\[
\tau \leftarrow \tau_n ,
\]

where \( \tau \) is the stabilization parameter to be used in marching from time level \( n \) to \( n + 1 \), and \( \tau_n \) is the stabilization parameter calculated from the flow field at time level \( n \). One of the main advantages in doing that, as it was pointed out in (17; 9; 18; 15; 16),
is avoiding another level of nonlinearity coming from the way τs are defined. In
general, it is desirable to make τs less dependent on short-term variations in the flow
field. For this purpose, a recursive time-averaging approach was proposed in (15; 16)
for determining the τs to be used in marching from time level n to n + 1:

\[ \tau \leftarrow z_1 \tau_n + z_2 \tau_{n-1} + (1 - z_1 - z_2) \tau , \]  

(3.15)

where \( \tau_n \) and \( \tau_{n-1} \) are the stabilization parameters calculated from the flow field
at time levels n and n - 1, and the \( \tau \) on the right-hand-side is the stabilization
parameter that was used in marching from time level n - 1 to n. The magnitudes and
the number of the "averaging parameters" \( z_1, z_2, \ldots \) can be adjusted to create the
desired outcome in terms of giving more weight to recently calculated τs or making
the averaging closer to being a trailing average.

In addition, for high-aspect-ratio elements near solid surfaces, in (10) the option of
setting \( r = n \) was proposed. This would spare \( h_{RON} \) from undesirable fluctuations as
\( ||u^h|| \) gets smaller and smaller for elements that become thinner and thinner. As it was
noted in (10), setting \( r = n \) still produces the desired outcome from the computation
of \( h_{RON} \), but without hard-wiring the computation for any particular element type or
shape.

Several of the remarks from (10) concerning this chapter are relevant and are
reproduced in this thesis as Remarks 1–9:

**Remark 1** Strictly speaking, the DSD/SST formulation given by Eq. (3.1) was in-
roduced in (9) and is slightly different from the formulation given in (6; 7). The two formulations are equivalent if the stabilization parameters $\tau_{SUPG}$ and $\tau_{PSPG}$ are defined to be identical, $\nu_{LSIO} = 0$, and $\nabla \cdot (\mu \varepsilon (w^h))$ is zero (which will be the case for linear elements) or neglected.

**Remark 2** As an alternative to the way the SUPG test function is defined in Eq. (3.1), we propose the SUPG test function option of replacing the term \( \left( \frac{\partial w^h}{\partial t} + u^h \cdot \nabla w^h \right) \) with \( (u^h - v^h) \cdot \nabla w^h \). This replacement is equivalent to excluding the \( \left( \frac{\partial w^h}{\partial t} \right)_\xi \) part of \( \left( \frac{\partial w^h}{\partial t} \right)_\xi \). We call this option “WTSE”, and the option where the \( \left( \frac{\partial w^h}{\partial t} \right)_\xi \) term is active “WTSA”.

**Remark 3** With the function spaces defined in the paragraph preceding Eq. (3.1), for each space–time slab velocity and pressure assume double unknown values at each spatial node. One value corresponds to the lower end of the slab, and the other one to the upper end. The option of using double unknown values at a spatial node will be called “DV” for velocity and “DP” for pressure. In this case, we use two integration points over the time interval of the space–time slab, and this time-iteration option will be called “TIP2”. This version of the DSD/SST formulation, with the options set DV, DP and TIP2, will be called “DSD/SST-DP”.

**Remark 4** We propose here the option of using, for each space–time slab, a single unknown pressure value at each spatial node, and we will call this option “SP”. With this, we propose another version of the DSD/SST formulation, where the options set is
DV, SP and TIP2, and we will call this version “DSD/SST-SP”. Because the number of unknown pressure values is halved, the computational cost is reduced substantially.

**Remark 5** To reduce the computational cost further, we propose the option of using only one integration point over the time interval of the space–time slab, and we call this time-integration option “TIP1”. With this, we propose a third version of the DSD/SST formulation, where the options set is DV, SP and TIP1, and we will call this version “DSD/SST-TIP1”.

**Remark 6** As a third way of reducing the computational cost, we propose the option of using, for each space–time slab, a single unknown velocity value at each spatial node, and we will call this option “SV”. In the SV option, of the two parts of Eq. (3.1), the one generated by $(w^h)_n^+$ is removed, and we explicitly set $(u^h)_n^+ = (u^h)_n^-$, which makes the velocity field continuous in time. Based on the SV option, we propose a fourth version of the DSD/SST formulation, where the options set is SV, SP and TIP1, and we will call this version “DSD/SST-SV”. With this version of the DSD/SST formulation, we propose to use the SUPG test function option WTSE.

**Remark 7** In terms of computational cost the DSD/SST-SV formulation would be quite comparable to the ALE formulations. This makes the DSD/SST-SV formulation very competitive in computational efficiency.
3.2 Semi-discrete formulation of structural mechanics

With \( y^h \) and \( w^h \) coming from appropriately defined trial and test function spaces, respectively, the semi-discrete finite element formulation of the structural mechanics equations (see (19; 20; 21)) is written as

\[
\int_{\Omega_0^s} w^h \cdot \rho \frac{d^2y^h}{dt^2} \, d\Omega + \int_{\Omega_0^s} w^h \cdot \eta \frac{dy^h}{dt} \, d\Omega + \int_{\Omega_0^s} \delta E^h : S^h \, d\Omega = \int_{\Omega_t^s} w^h \cdot (t^h + \rho^s f^s) \, d\Omega. \tag{3.16}
\]

The fluid mechanics forces acting on the structure are represented by vector \( t^h \). This force term is geometrically nonlinear and thus increases the overall nonlinearity of the formulation. The left-hand-side terms of Eq. (3.16) are referred to in the original configuration and the right-hand-side terms in the deformed configuration at time \( t \). This formulation at each time step gives a nonlinear system of equations. In solving that nonlinear system with an iterative method, we use an incremental form (see (19; 20; 21; 22)), which is expressed as

\[
\begin{bmatrix}
M \\
\beta \Delta t^2 \\
\beta \Delta t \\
K
\end{bmatrix}
+ \begin{bmatrix}
(1 - \alpha) \gamma C \\
(1 - \alpha) K
\end{bmatrix}
\Delta d^i = R^i. \tag{3.17}
\]

Here \( M \) is the mass matrix, \( C \) is the artificial-damping matrix, \( K \) is the consistent tangent matrix associated with the internal elastic forces, \( R^i \) is the residual vector at the \( i^{th} \) iteration, and \( \Delta d^i \) is the \( i^{th} \) increment in the nodal displacements vector \( d \). The artificial-damping matrix \( C \) is used, as mentioned in Section 2.2, only in computations.
where time-accuracy is not required, and for spatially-constant $\eta$ it can be written as $C = \eta M$. All of the terms known from the previous iteration are lumped into the residual vector $R^i$. The parameters $\alpha, \beta, \gamma$ are part of the Hilber–Hughes–Taylor (23) scheme, which is the time-integration technique used here.

### 3.3 Stabilized Space–Time Fluid–Structure Interaction (SSTFSI) method

This section describes the SSTFSI method based on the finite element formulations given by Eqs. (3.1) and (3.16), with a slight change of notation and with a clarification of how the fluid–structure interface conditions are handled. In this notation subscripts 1 and 2 will refer to fluid and structure, respectively. Furthermore, while subscript $I$ will refer to the fluid–structure interface, subscript $E$ will refer to “elsewhere” in the fluid and structure domains or boundaries. Then the equations representing the SSTFSI method are written as follows:

\[
\begin{align*}
\int_{Q_n} w_{1E}^h \cdot \rho \left( \frac{\partial u^h}{\partial t} + u^h \cdot \nabla u^h - f^h \right) dQ + \int_{Q_n} \epsilon(w_{1E}^h) : \sigma(p^h, u^h) dQ \\
- \int_{(P_n)_{h}} w_{1E}^h \cdot h_{1E}^h dP + \int_{Q_n} q_{1E}^h \nabla \cdot u^h dQ + \int_{\Omega_n} (w_{1E}^h)^+ \cdot \rho \left( (u^h)^+ - (u^h)^- \right) d\Omega \\
+ \sum_{\varepsilon=1}^{(n_{el})_n} \int_{Q_{\varepsilon}} \frac{1}{\rho} \left[ \tau_{\text{SUPG}} \left( \frac{\partial w_{1E}^h}{\partial t} + u^h \cdot \nabla w_{1E}^h \right) + \tau_{\text{PSPG}} \nabla q_{1E}^h \right] \cdot [L(p^h, u^h) - \rho f^h] dQ \\
+ \sum_{\varepsilon=1}^{(n_{el})_n} \int_{Q_{\varepsilon}} \nu_{\text{LSIC}} \nabla \cdot w_{1E}^h \rho \nabla \cdot u^h dQ = 0,
\end{align*}
\]

(3.18)
\[ \int_{Q_n} q_{11}^h \nabla \cdot u^h \, dQ + \sum_{c=1}^{(n_{el})_n} \int_{Q_c} \frac{1}{\rho} \left[ \tau_{PSPG} \nabla q_{11}^h \right] \cdot \left[ L(p^h, u^h) - \rho f^h \right] \, dQ = 0 , \quad (3.19) \]

\[ \int_{(\Gamma_{11})_{REF}} (w_{11}^h)^- \cdot (u_{11}^h)^- - u_{21}^h ) \, d\Gamma = 0 , \quad (3.20) \]

\[ \int_{(P_{n})_{n+1}} (w_{11}^h)^- \cdot h_{11}^h \, dP = -\int_{(P_{n})_{n+1}} (w_{11}^h)^- \cdot p n \, dP + \int_{Q_n} 2\mu \varepsilon((w_{11}^h)^-) \cdot \varepsilon(u) \, dQ \\
+ \int_{Q_n} (w_{11}^h)^- \cdot \nabla \cdot (2\mu \varepsilon(u)) \, dQ \quad (3.21) \]

\[ \int_{(\Omega_{21})_{REF}} w_{21}^h \cdot (h_{21}^h + (h_{11}^h)_A + (h_{11}^h)_B) \, d\Omega = 0 , \quad (3.22) \]

\[ \int_{(\Omega_{2})_0} w_2^h \cdot \rho_2 \frac{d^2 y_2^h}{dt^2} \, d\Omega + \int_{(\Omega_{2})_0} w_2^h \cdot \eta \rho_2 \frac{dy_2^h}{dt} \, d\Omega + \int_{(\Omega_{2})_0} \delta E^h : S^h \, d\Omega \\
= \int_{\Omega_2} w_2^h \cdot \rho_2 f_2^h \, d\Omega + \int_{\Omega_{2E}} w_{2E}^h \cdot h_{2E}^h \, d\Omega + \int_{\Omega_{2i}} w_{2i}^h \cdot h_{2i}^h \, d\Omega . \quad (3.23) \]
Here \((\Gamma_{2t})_{\text{Ref}}\) and \((\Omega_{2t})_{\text{Ref}}\) represent some reference configurations of \(\Gamma_{2t}\) and \(\Omega_{2t}\), respectively. In reconciling the slightly modified notation used here with the notation used in Eqs. (3.1) and (3.16), \(\rho_2 = \rho^s\), \(f_2^s = f^s\), \((\Omega_2)_0 = \Omega^s_0\), \(\Omega_2 = \Omega^s_1\), and \(\Omega_{2e}\) indicate the partitions of \(\Omega_2\) corresponding to the interface and “elsewhere”.

Also, \(h_{2t}^h = t^h\), and \((h_{1i}^h)_{A}\) and \((h_{1i}^h)_{B}\) represent the values of \(h_{1i}^h\) associated with the fluid surfaces above and below the membrane structure. The symbol \(h_{2e}^h\) denotes the prescribed external forces acting on the structure in \(\Omega_{2e}\), which is separate from \(f_2^h\).

In this formulation, \((u_{1i}^h)_{n+1}\), \(h_{1i}^h\) and \(h_{2i}^h\) (the fluid velocity, fluid stress and structural stress at the interface) are treated as separate unknowns, and Eqs. (3.20), (3.21) and (3.22) can be seen as equations corresponding to these three unknowns, respectively.

The structural displacement rate at the interface, \(u_{2t}^h\), is derived from \(y^h\).

The formulation above is based on allowing for cases when the fluid and structure meshes at the interface are not identical. If they are identical, the same formulation can still be used. If the structure is represented by a 3D continuum model instead of a membrane model, the formulation above would still be applicable if the the domain integrations over \(\Omega_{2e}\) and \(\Omega_{2t}\) in the last two terms of Eq. (3.23) are converted to boundary integrations over \(\Gamma_{2e}\) and \(\Gamma_{2t}\). In such cases, \(h_{2e}^h\) would represent the prescribed forces acting “elsewhere” on the surface of the structure.

For constant viscosity, the term \(\nabla \cdot (2\mu \varepsilon(u))\) in Eq. (3.21) vanishes for tetrahedral elements and in most cases can be neglected for hexahedral elements. The same statement can be made also in the context of that term being a part of the expression.
L(p^h, u^h) appearing in Eqs. (3.18) and (3.19).

**Remark 8** The versions of the SSTFSI method corresponding to the DSD/SST-DP, DSD/SST-SP, DSD/SST-TIP1 and DSD/SST-SV formulations (see Remarks 3-6) will be called “SSTFSI-DP”, “SSTFSI-SP”, “SSTFSI-TIP1” and “SSTFSI-SV”, respectively.

**Remark 9** In terms of computational cost the SSTFSI-SV formulation would be quite comparable to the ALE FSI formulations. This makes the SSTFSI-SV formulation very competitive in computational efficiency.
Chapter 4

Numerical Examples: Modeling Flow Through a
Saccular Cerebral Aneurysm

4.1 Arterial geometry and physical properties

Blood pumps through a section of cerebral artery containing a small, saccular aneurysm. The flow rate of the blood circulating through this artery pulsates at one cycle per second (a typical period for the human heartbeat). The arterial section modeled in these simulations closely approximates the patient-specific image-based geometry used in (4). This geometry was extracted from a computed tomography model of a segment of the middle cerebral artery (location shown in Figure 4.1) from a 57 year-old male with aneurysm. As seen in this problem geometry (Figure 4.2), the diameter of both the inlet and outlet of the arterial section is 3.0 mm, and the length of the section is 15.0 mm, while the size of the aneurysm is approximately 6.0 mm. In addition to these initial geometric parameters, the arterial wall thickness is chosen to suit one of two cases, depending on the simulation chosen. The cases available for study are an aneurysm with uniform arterial wall thickness (UWT) or
one with variable wall thickness (VWT).

![Diagram of MCA and ICA](image)

**Figure 4.1:** Location of the middle cerebral artery (MCA) and the internal carotid artery (ICA). Picture taken from (27). Circle indicates modeled segment.

For the UWT simulations, the arterial wall thickness, density, stiffness, and Poisson’s ratio are 0.3 mm, 1000 kg/m$^3$, 500 kPa, and 0.45 respectively. The arterial stiffness value of 500 kPa is comparable to the values used in (24; 25; 3; 5; 4), where the comparison to experimental values determined the value of the stiffness used (see (4)). The stiffness value used in each test case is given in the individual description of that test case. For the VWT cases, the wall thickness decreases from 0.3 mm on the healthy sections of the artery to 0.1 mm on the aneurysm. The reduced wall thickness of the aneurysm assumes arterial wall stretching at the aneurysm site and helps prove the versatility of the simulation techniques. Apart from wall thickness, the physical properties for both cases are equivalent.

In addition to accurately modeling the properties of the artery and the blood flowing through it, the simulations account for the damping effects of the brain tissue.
Figure 4.2: Middle cerebral artery with aneurysm — single-artery segment. Problem geometry.

surrounding the artery. Dense cerebral matter encompasses arteries like the one studied, and this tissue has a damping effect on the structural dynamics of such arteries. Therefore, the addition of mass-proportional damping helps remove high-frequency modes in the structural deformation of the artery. The damping coefficient \( \eta \) used in each case is given in the individual description of that case. Preliminary results for these problems were reported in (26).
Figure 4.3: Middle cerebral-artery with aneurysm — single-artery segment. Wall-thickness distribution for the VWT artery segment.

4.2 Fluid properties

Although non-Newtonian in general, the blood in this thesis is assumed to behave like a Newtonian fluid. The justification provided here reproduces the explanation given in (27). With the arterial diameter being approximately 3.0 mm and the flow rate 2.0 ml/s, the average shear rate in the artery is approximately \( \dot{\gamma} = \frac{4Q}{\pi R^3} \sim 755 \text{ s}^{-1} \), where \( Q \) and \( R \) are the flow rate and radius. As noted in (27), the viscosity of the blood can be assumed constant if the shear rate is high enough (\( \geq 150 \text{ s}^{-1} \)) (28). Furthermore, the material density of the arterial wall (1000 kg/m\(^3\)) is known to be close to the density of blood. With these considerations, the fluid density and kinematic viscosity are set to 1000 kg/m\(^3\) and \( 4.0 \times 10^{-6} \text{ m}^2/\text{s} \).
4.3 Boundary conditions

In order to achieve a realistic simulation environment and results, several boundary conditions are imposed on the artery during computations. Accurately modeling the flow rate through the system is the top priority. From the flow rate, inflow velocity is calculated and input into the simulations. Not only are conditions specified at the inflow boundary, conditions are given at the outflow boundary for traction boundary condition. This condition, varying the pressure at the exit, is also derived from the flow rate. The traction condition is scaled and modified for two blood pressure scenarios: one for a healthy adult and one for a patient with high blood pressure.

In addition to varying the blood pressure through the artery with aneurysm, the wall thickness of the aneurysm is varied in the simulations. Calculations are performed on either an aneurysm with uniform arterial wall thickness (UWT) or one with variable wall thickness (VWT). Therefore, test computations are carried out for normal and high blood pressure in both UWT and VWT arterial segments. On the arterial walls, conditions specify a no-slip boundary condition for the flow. For the structural mechanics boundary condition at the ends of the artery, displacement is set to zero at the edges for the membrane elements.

4.3.1 Womersley inflow profile

In order to pulsate the inflow with the cardiac cycle, a flow rate curve following a simple sinusoidal wave is implemented to build up to the realistic flow rate model.
**Figure 4.4:** Middle cerebral artery with aneurysm — single-artery segment. Pulsating inflow velocity.

This inflow cycle has a period comparable to the cardiac cycle but its simplicity and smoothness make implementation easier for the initial simulations. Starting with this cycle also helps set the baseline computation parameters as the user gains experience dealing with these types of simulations. Once comfortable handling sinusoidal inflow profiles, the user implements the actual inflow profile of the cardiac cycle.

In all the test cases computed, a single inflow profile specifies the velocity profile as a function of time. This boundary condition, given by the Womersley pulsative flow model, prescribes the given flow rate into the artery and through the aneurysm during each phase of the cardiac cycle. The inflow rate given by this model closely approximates the flow rate used in (4), which can also be found in (3; 5). Figure 4.4 shows this pulsating inflow velocity profile as a function of time. Note this profile is
Figure 4.5: Middle cerebral artery with aneurysm — single-artery segment. Normal blood pressure profiles for the outflow traction boundary condition.

similar to the one obtained by using the Womersley solution of a pulsating flow (29).

To simplify computations, the simulations use an input file containing the digitized version of the Womersley solution. A plot-digitizing program (see (30)) extracts data points from the flow rate plots in (4). This program digitizes scanned plots of data and converts them into text files. Once digitized, plots are adjusted for minimum and maximum pressure and modified to suit the flow rate of blood to any artery in the cardiovascular system. All modifications are arrived at through simple scaling analysis of flow rate, arterial diameter, and blood pressure. This digitized information proves useful to all of the cardiovascular simulations.

After implementing this new inflow data, the computations more closely model actual cardiovascular conditions within the brain. However, the model is incomplete
Figure 4.6: Middle cerebral artery with aneurysm — single-artery segment. High blood pressure (hypertension) profiles for the outflow traction boundary condition.

without a traction boundary condition at the exit instead of zero pressure. With this boundary condition at the exit, the computations are a more accurate representation of flow through a cerebral artery with aneurysm. In order to accomplish this task, the flow rate values from the digitized Womersley model of pulsatile inflow are converted to inflow velocity values and used as the inflow boundary condition. The mathematical model used to convert inlet flow rate to outlet pressure is the Windkessel model.

4.3.2 Windkessel model

The traction boundary condition at the outflow boundary varies pressure at the exit according to inlet velocity and flow rate and is therefore coupled to the previous
Figure 4.7: Middle cerebral artery with aneurysm — single-artery segment. Normal and high blood pressure (NBP and HBP) profiles.

inflow condition. This new condition is based on one of two blood pressure profiles, depending on the simulated case. These pressure profiles are functions of time and are obtained from an approximate solution (3) of the Windkessel model (31). In this model, the pressure at the outlet is derived proportionally from the flow through the distal arterial network of the circulatory system. Equation 4.1d gives the final version of this modified solution to the Windkessel model. In this set of equations, Q represents the flow rate (taken from the inflow boundary condition) in m³/s, t is time in seconds, C is the compliance of the distal arterial network (m³/Pa), and R is the resistance of the distal arterial network (Pa·s/m³). These parameters are set so the range for the pressure profile is approximately 80 to 120 mm Hg for normal blood pressure (NBP). Table 4.1 shows the values used to generate this profile with
the Windkessel model. After creating the pressure profile, the units of pressure are converted from N/m² to mm Hg. This profile can now be adjusted according to produce profiles with varying systolic and diastolic blood pressure ranges. For example, by scaling the NBP pressure profile, pressure can be adjusted to vary from 100 to 170 mm Hg for high blood pressure (HBP) cases. The NBP and HBP pressure profiles are used to study the flow through an aneurysm under normal and hypertensive blood flow. The pressure boundary condition for each case is shown in Figures 4.5 and 4.6. Furthermore, Figure 4.7 shows the NBP and HBP profiles together.

| p₀ | 9.00×10⁸ N/m² |
|Δt | 0.005 s |
| R  | 1.83×10⁹ Pa·s/m³ |
| C  | 3.00×10⁻¹¹ m³/Pa |
| α  | 0.5 |

**Table 4.1:** Windkessel model parameters: These parameters are used to produce the pressure profile at the exit of the arterial section.
\[ p_n = e^{-t_n/RC} \int_0^{t_n} \frac{1}{C} Q(\tau)e^{\tau/RC} d\tau + p_o, \]  
\[ p_{n+1} = e^{-t_{n-1}/RC} I_{n+1} + p_o, \]  
\[ I_{n+1} = I_n + \Delta I_n, \]  
\[ I_n = \int_0^{t_n} \frac{1}{C} Q(\tau)e^{\tau/RC} d\tau, \]  
\[ \Delta I_n = \int_{t_n}^{t_{n+1}} \frac{1}{C} Q(\tau)e^{\tau/RC} d\tau, \]  
\[ (p_{n+1} - p_o) = e^{-\Delta t/RC} e^{-t_{n}/RC} I_n + e^{-\Delta t/RC} e^{-t_{n}/RC} \Delta I_n, \]  
\[ (p_{n+1} - p_o) = e^{-\Delta t/RC} (p_n - p_o) + \frac{\Delta t}{C} Q_{n+\alpha} e^{-\Delta t(1-\alpha)/RC} \] 

4.4 Discretization and mesh properties

The structural mesh for the artery consists of membrane elements. The fluid and structure meshes are generated on a single node of a PC cluster using automatic mesh generators. The mesh for the artery consists of 3,726 nodes and 7,388 three-node triangular elements (see Figure 4.8). The fluid mechanics mesh has 10,987 nodes and 53,645 four-node tetrahedral elements. Across the diameter, there are between 11 and 12 nodes, and the average element size is 0.296.

All computations are carried out in a parallel computing environment of PC clusters. In all cases computed, the fluid and structure meshes are compatible at the fluid–structure interface. All computations are completed without any remeshing.
In all cases, the fully-discretized, coupled fluid and structural mechanics and mesh-moving equations are solved with the quasi-direct coupling technique (see Section 5.2 in (10)). In solving the linear equation systems involved in every nonlinear iteration, the GMRES search technique (32) is used with a diagonal preconditioner. The computations are carried out with the SSTFSI-TIP1 technique (see Remarks 5 and 10 in (10)) and the SUPG test function option WTSA (see Remark 2 in (10)). The stabilization parameters used are those given by Equations (7)–(13) in (10) for the NBP-UWT case and Equations (7)–(12) and (17) in (10) for all other cases. The damping coefficient $\eta$ is set to $6.0 \times 10^3 \text{ s}^{-1}$. The time-step size is $3.333 \times 10^{-3} \text{ s}$. The number of nonlinear iterations per time step is 5, and the number of GMRES iterations per nonlinear iteration is 150 and 225, respectively, for the NBP and HBP cases. The F$\rightarrow$S$\rightarrow$FSI sequence described below is used in the computations.

In a majority of the simulations conducted by the T$\times$AFSM, the FSI computations are preceded by a set of pre-FSI computations that provide a good starting point for the FSI computations. These pre-FSI computations include the fluid-only and structure-only computations. Two options were proposed in (26) for the pre-FSI generation of the fluid mesh. It can be generated by starting with a mesh corresponding to the initial shape of the structure and updating it as the structure-only computation proceeds, or generated after the structure-only computation is completed. The simulation sequence used in these computations, proposed in (26), is as follows:
Figure 4.8: Middle cerebral artery with aneurysm — single-artery segment. Mesh elements at the inflow boundary and arterial wall.

4.5 Fluid→Structure→FSI (F→S→FSI) sequence

Step 1: Generate the fluid and structure meshes based on the shape of the unstressed structure.

Step 2: Compute a developed flow field while holding the structure rigid.

- The outflow traction is set to a value close to 80 mm Hg for NBP and 100 mm Hg for HBP.

- The inflow velocity is set to a value corresponding to the outflow traction.
**Step 3:** Compute the structural deformation, with the fluid stresses at the interface held steady at their values from Step 2, and simultaneously update the fluid mesh.

- Structural deformation can be determined with a steady-state computation or a time-dependent computation that eventually yields a steady-state solution.

- For the steady-state computation, \( \Delta t \to \infty \) and \( \alpha = 0 \) in Eq. (21) in (10), the number of time steps is one, and the initial displacement, velocity and acceleration are set to zero.

- The mesh quality obtained with the time-dependent computation is better than the one obtained with the steady-state computation.

**Step 4:** Compute the FSI with the inflow and outflow conditions held steady at the values used in Step 2.

- Sometimes, to prevent a sudden increase in the structural acceleration at the start of this step, it may be necessary to begin with an increased structural mass that would later be decreased back to its actual value. An unrealistically large acceleration can initiate an instability that is subsequently magnified.

**Step 5:** Compute the FSI with the inflow and outflow conditions pulsating.
4.6 Computation and results

Following the steps and methods outlined in previous sections, FSI computations are carried out for several different cases. This thesis focuses on four cases: NBP-UWT, NBP-VWT, HBP-UWT, and HBP-VWT, with most of the analysis covered in the NBP-UWT section. Studying the results from these cases involves processing velocity vectors, investigating pressure profiles, and discerning areas of weakness on the aneurysm. Additional parameters are observed and monitored throughout computations.

Vectors illustrating the magnitude of the velocity within the aneurysm provide insight into the flow field within the arterial section. This section contains several snapshots of the flow field at eight different times during a single cardiac cycle. These times are listed in Table 4.2 and correspond to important moments during the cardiac cycle. Point 1 and 8 are the start and end times, respectively. Points 2 through 6 occur during the first systolic peak, and point 7 lies on the second peak. More specifically, point 2 corresponds to a moment in time of great flow acceleration leading up to the maximum inflow velocity and arterial pressure. Points 3 and 4 occur when flow velocity is near maximum. In addition, point 4 closely corresponds to the peak systolic pressure (PSP). Point 5 takes place during the brief period of flow deceleration and pressure drop after this first pulse in the cycle. Point 6 marks the end of the first pulse where inflow velocity is nearly minimum and pressure levels off briefly.
<table>
<thead>
<tr>
<th>Point Label</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.060</td>
</tr>
<tr>
<td>3</td>
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</tr>
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<td>0.333</td>
</tr>
<tr>
<td>7</td>
<td>0.520</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**Table 4.2:** Velocity vector plots: vector plots illustrate the cardiac cycle at these points in time

4.6.1 Uniform wall thickness simulation

The first series of FSI simulations studies the artery/aneurysm model with uniform arterial wall thickness. Following the ramping process outlined in Section 4.5, the model undergoes a series of computations simulating the cardiac cycle. Data from these computations illustrates key information regarding aneurysm development, growth, and rupture. The plots of the magnitude of the velocity vectors give several details about the fluid dynamics within the aneurysm. Figure 4.12 shows the velocity vectors within the aneurysm at the eight given points in time. Note that the velocity vectors at the end of the cardiac cycle return to a profile nearly identical to the initial profile.

During the first snapshot, at the start of the cardiac cycle, the velocity vectors are very steady with only slight flow circulation visible in the center of the aneurysm. In the second snapshot, the highest velocity flow is seen impacting the neck of the aneurysm. In addition, flow circulation increases at the center of the aneurysm, with
increased velocity at the aneurysm walls. One can foresee the implications of such flow on aneurysmal growth or rupture at the region of impingement and through increased velocity at the walls. Torii equates high velocity near the wall and flow impingement to wall shear stress (WSS) in (4) stating, “impinging yields high velocity gradients” correspond to regions of “maximum wall WSS”. In the third instant, a vortex core clearly forms, and velocity increases along the entire perimeter of the aneurysm. This increased wall velocity and vorticity can increase WSS according to Torii’s observations. In the fourth snapshot, the vortex core appears to split while the high velocity flow continues to impinge on the neck of the aneurysm. Now, at point 5, on the backside of the flow rate spike, inflow velocity and pressure begin to drop, and the aneurysm reduces in size. During the next frame, the velocity bottoms out and pressure has decreased, but the flow circulation remains clear and distinct. At point 7, both the velocity and pressure are increasing and the vortex core tightens. At the final point, corresponding to the end of the cardiac cycle, the aneurysm returns to its original size with a velocity profile similar to the initial state seen at point 1.
Figure 4.9: Middle cerebral artery with aneurysm — single-artery segment. Pulsating inflow velocity with point labels.

Figure 4.10: Middle cerebral artery with aneurysm — single-artery segment. Normal blood pressure profiles for the outflow traction boundary condition with point labels.
Figure 4.11: Middle cerebral artery with aneurysm — single-artery segment. Pressure profiles for the outflow traction boundary condition with point labels (HBP case).
Figure 4.12: Blood flow through a cerebral aneurysm for the NBP-UWT case. Time history (left to right and top to bottom) showing the velocity vectors colored by their magnitude.
Figure 4.13: Middle cerebral-artery with aneurysm — single-artery segment. Computed with the membrane element. Verification of mass balance for the NBP-UWT case. Volumetric inflow rate, difference between the volumetric inflow and outflow rates, and rate of change for the artery volume.
4.6.2 Further examples

The NBP-UWT case studies the aneurysmal deformation for a cerebral artery of a person in good health (i.e. normal blood pressure, aneurysm with normal wall thickness). In addition to this case, three other cases are explored under less ideal circumstances to demonstrate more realistic results while testing the capability of the methods behind the numerical simulations.

Variable wall thickness simulation

With regard to flow circulation and areas of increased flow velocity near the arterial wall, the NBP-VWT case shows results similar to that of the NBP-UWT case. However, circulation becomes more evident and aneurysmal expansion is exaggerated due to the thinner wall. Nonetheless, the results from this simulation confirm those from the NBP-UWT case.

High blood pressure simulations

In (5), the author states that hypertensive blood pressure significantly changes the WSS distribution on the aneurysm. This change affects the growth of the aneurysm, and therefore, further simulations are conducted using high blood pressure.

Once again, the results from the HBP cases mirror those of the NBP-UWT case. However, there are a few important and noticeable results to consider. First, in the HBP-UWT case, the aneurysm’s reaction to the flow appears more violent, vortices quickly form, and the aneurysm expands more, due to the increased velocity. Likewise,
in the HBP-VWT case, the aneurysm expands greatly, more than any other case. In addition, the velocity vectors reveal forces orthogonal to the aneurysm walls, implying more stress on the aneurysm (see point 2 in Figure 4.18). Furthermore, in the third snapshot, the high velocity continues around entire aneurysm, hitting incoming flow. The leads to pressure build-up at multiple sites along the neck of the aneurysm (an area of potential weakness). Torii's statements about hypertension are confirmed by these simulations. Furthermore, it should be noted that people with hypertension are more susceptible to aneurysms, and therefore these simulations prove useful in understanding the true nature of brain aneurysms.

4.6.3 Pressure elevation plots

Studying areas of elevated pressure on the arterial walls provides a way to identify areas vulnerable to rupture. Pressure elevation plots are taken at six keys times during the cardiac cycle in order to identify these areas. These moments occur during the cycle at the extreme values of velocity, pressure, and acceleration. These points are seen for the NBP cases in Figures 4.20 and 4.21. The inflow velocity plot for the HBP cases is the same, and the minimum and maximum pressures occur at the same time due to the scaling used to render the HBP pressure profile. The HBP-UWT example is chosen to demonstrate findings at the six key points during the cycle.

The pressure imparted on the arterial walls results from changes in the inflow velocity, with a pressure gradient driving flow downstream, and from the "back pressure" created by the cardiovascular system as a whole. The magnitude of this baseline
pressure is much greater than the magnitude of the pressure gradient driving the flow (see Figure 4.22). Therefore, pressure elevation plots are offset by subtracting this baseline pressure. This gives a better picture of the weakened and vulnerable areas along the arterial wall and on the aneurysm.

Figure 4.23 shows the pressure elevation plot at the point where the pressure in the arterial section is minimum (see Figure 4.21). The pressures are taken from a plane central to the section. On the figure, the inlet is at the bottom left and the outlet at the right. This figure demonstrates how a small pressure gradient drives blood flow as pressure decreases from the beginning of the arterial segment, through the section of artery with aneurysm, and to the end of the segment. At the minimum pressure, there appears to be no areas with abnormally heightened pressure. However, as computations continue through the cardiac cycle, the inflow velocity increases sharply. Figure 4.24 shows the pressure elevation plot during the time of greatest flow acceleration. Once again, pressure decreases from inlet to outlet. However, pressure begins to accumulate in the aneurysm. Moving further in time, less than 10 percent through the cardiac cycle, maximum flow velocity is achieved (see Figure 4.25). At this point, pressure accumulates even more within the aneurysm, with higher pressures at the walls and an area of lesser pressure at the center of the aneurysm. This poses a risk in itself, but the danger does not end here. As noted previously, a region of flow impingement develops where the pressure spikes at the downstream neck of the aneurysm. Flow hitting this region imparts a greater force than elsewhere in the
flow. This increased pressure can cause the aneurysm to grow and eventually rupture. At this point in the cycle, the pressure along the arterial wall in this section varies the most, and maximum pressure has not yet been attained. Shortly after flow rate and velocity are maximum, maximum pressure in the blood vessel is also reached. At this moment (see Figure 4.26), a pressure spike in the elevation plot is again visible where flow impinges on the aneurysm neck. Concerning areas of vulnerability, this region is once again implicated. This trend continues through the end of the cardiac cycle (see Figures 4.27 and 4.28).

4.6.4 Artery deformation and comparison

FSI analysis allows for the observation of aneurysmal deformation under different conditions. Understanding arterial deformation is crucial in grasping an understanding of the stress on the aneurysmal wall and the possibility of further growth or rupture. Fluid studies alone fall short in this analysis, but FSI can paint a more complete picture. Volume measurements for the artery were taken at each time step, for all four cases. Figure 4.29 shows the deformed shapes of the aneurysm for all four cases at the time of maximum deformation. The order of the cases from minimum to maximum deformation is NBP-UWT, HBP-UWT, NBP-VWT and HBP-VWT. Due to the thinner wall layer at the site of aneurysm, the variable thickness cases deformed the most, with the HBP-VWT case deforming 32.03% (see Table 4.3). For the best-case scenario of a person with a healthy blood pressure and assuming arterial walls of normal stiffness and pliability, the arterial volume increases as much as
10.00%. With these large deformations taking place around 100,000 times per day, aneurysmal growth and rupture can foreseeably occur.

<table>
<thead>
<tr>
<th>Case</th>
<th>$V_{\text{init}}$ (mm$^3$)</th>
<th>$V_{\text{max}}$ (mm$^3$)</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBP-UWT</td>
<td>282.19</td>
<td>310.56</td>
<td>10.00</td>
</tr>
<tr>
<td>NBP-VWT</td>
<td>319.79</td>
<td>378.97</td>
<td>18.51</td>
</tr>
<tr>
<td>HBP-UWT</td>
<td>296.77</td>
<td>345.21</td>
<td>16.32</td>
</tr>
<tr>
<td>HBP-VWT</td>
<td>349.10</td>
<td>460.94</td>
<td>32.03</td>
</tr>
</tbody>
</table>

**Table 4.3:** Arterial volume comparison: the percent volume change differs for each case depending on the blood pressure and wall thickness.
Figure 4.14: Blood flow through a cerebral aneurysm for the NBP-VWT case. Time history (left to right and top to bottom) showing the velocity vectors colored by their magnitude.
Figure 4.15: Middle cerebral-artery with aneurysm — single-artery segment. Computed with the membrane element. Verification of mass balance for the NBP-VWT case. Volumetric inflow rate, difference between the volumetric inflow and outflow rates, and rate of change for the artery volume.
Figure 4.16: Blood flow through a cerebral aneurysm for the HBP-UWT case. Time history (left to right and top to bottom) showing the velocity vectors colored by their magnitude.
Figure 4.17: Middle cerebral-artery with aneurysm — single-artery segment. Computed with the membrane element. Verification of mass balance for the HBP-UWT case. Volumetric inflow rate, difference between the volumetric inflow and outflow rates, and rate of change for the artery volume.
Figure 4.18: Blood flow through a cerebral aneurysm for the HBP-VWT case. Time history (left to right and top to bottom) showing the velocity vectors colored by their magnitude.
Figure 4.19: Middle cerebral-artery with aneurysm — single-artery segment. Computed with the membrane element. Verification of mass balance for the HBP-VWT case. Volumetric inflow rate, difference between the volumetric inflow and outflow rates, and rate of change for the artery volume.
Figure 4.20: Middle cerebral artery with aneurysm — single-artery segment. Pulsating inflow velocity, key points notated.

Figure 4.21: Middle cerebral artery with aneurysm — single-artery segment. Normal blood pressure profiles for the outflow traction boundary condition, key points notated.
Figure 4.22: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at maximum inflow velocity (HBP-UWT).
Figure 4.23: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at minimum pressure (HBP-UWT).
Figure 4.24: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at maximum flow acceleration (HBP-UWT).
Figure 4.25: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at maximum velocity (HBP-UWT).
Figure 4.26: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at maximum pressure (HBP-UWT).
Figure 4.27: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at maximum flow deceleration (HBP-UWT).
Figure 4.28: Middle cerebral artery with aneurysm — single-artery segment. Pressure elevation plot at minimum velocity (HBP-UWT).
Figure 4.29: Middle cerebral-artery with aneurysm — single-artery segment. Computed with the membrane element. Deformation of the aneurysm for all four cases. The order of the cases from minimum to maximum deformation is NBP-UWT, HBP-UWT, NBP-VWT and HBP-VWT.
Chapter 5

Conclusions and Recommendations

The simulation results demonstrate the capability of FSI modeling in studying cardiovascular fluid mechanics while illustrating the value of such methods in the medical field. Further advancing the ideas and methods used in the UWT-NBP case, the variable wall thickness computations prove the utility of finite element computations in handling complex and realistic models. In the next step, increasing the reality of the computations, hypertensive simulations confirm the lethal effects of high blood pressure on delicate intracranial arteries. With each new case, the methods and accuracy of the FSI techniques improve, as FSI modeling moves closer to becoming a valuable diagnostic tool for doctors and patients. Even these initial simulations provide valuable insight into cerebral aneurysms, demonstrating the growing potential of this technique.

The simulations yield useful information about the possible progression of an aneurysm. Once an aneurysm develops at a weakened spot on an artery, results show that new stress points are created where the neck of the aneurysm impinges the flow. In addition, flow patterns emerge which lead to flow circulation, higher velocity near
the arterial wall, and greater WSS along the weakened aneurysmal wall. Such factors tend to further harm the damaged area, leading to the growth and possible rupture of the aneurysm. These ideas are confirmed by all cases, regardless of aneurysmal wall thickness or blood pressure.

The results presented in this thesis are promising, and through continued research and experimentation, scientists will continue to upgrade FSI techniques in this fascinating field. One method proposed in (33) uses continuum elements instead of membrane elements to model the artery with aneurysm. With this model based on continuum elements, the various layers of the arterial wall may be more accurately modeled by varying the physical properties, namely stiffness, at each layer. In addition, these properties may be altered on a patient-by-patient basis to account for additional factors. These modifications not only include personalizing blood pressure or varying arterial wall thickness (as seen in this thesis), but researchers could also account for arterial hardening and add a layer of plaque lining the artery due to artherosclerosis. With these modifications and others to improve the fidelity of the simulations, FSI modeling can provide a noninvasive means of assessing risk for a patient with aneurysm.
Bibliography


[18] T. Tezduyar, “Stabilization parameters and local length scales in SUPG and


