Time-Frequency Analysis of Seismic Sequences [1]

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Summary

Reflection patterns play an important role in seismic sequence stratigraphy, therefore making their quantitative description essential for the construction and validation of sequence stratigraphic models from seismic data. The characteristics of a seismic reflection pattern can be elicited from the data by representing the data as a joint function of time and frequency. Developments in the field of time-frequency analysis have led to $t - f$ representations with better resolution than can be obtained with classical methods for local frequency analysis. This justifies a study of the application of these new representations to the analysis of seismic data. Some examples of the $t - f$ representation of the seismic response of layered sequences are given. They clearly show the contribution of the stratigraphic sequence to the spectral content of the data. As an illustration of how sequence stratigraphic models can be constructed from a $t - f$ representation it is shown how the observed pattern in real seismic data can be matched to a sequence model.

Introduction

Seismic facies analysis consists of the delineation and characterization of reflection patterns in seismic sections. Although seismic facies analysis is nowadays incorporated in virtually all interpretations of seismic data, a good method for a quantitative description of seismic reflection patterns has not yet been reported in the literature. A quantitative description of seismic facies characteristics may aid in the construction and validation of sequence stratigraphic models. Complex-trace attributes (Taner et al. 1979) can be used in the description of waveforms, but are not always suited for the characterization of the larger pattern. Since reflection patterns can be characterized by the distribution of amplitude and frequency of the signal over the facies unit (Sangree & Widmier 1977), a natural approach towards a quantitative description of seismic facies is to study the frequency content of the data as a function of time and space. In recent years important developments in the field of the joint time-frequency analysis of signals have been made (Cohen 1989). We apply these time- frequency analysis methods to the study of one-dimensional reflection sequences in synthetic and real seismic data.

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Theory

In our study we used a representation of the so-called quadratic shift-invariant class. Prominent members of this class of t-f representations are the power spectrum of a sliding window of data (spectrogram) and the Wigner distribution. The Wigner distribution $W_{ss}(t,f)$ of a signal $s(t)$ is defined by:

$$W_{ss} = \int_{-\infty}^{+\infty} \exp(j2\pi f\tau) R_{ss}(t,\tau)\ d\tau, \qquad (1)$$

where t is time, f is frequency, τ is a time-lag variable and $j = \sqrt{(-1)}$. The r.h.s. of Eq. (1) is the Fourier transformation with respect to time-lag of the instantaneous auto-correlation function:

$$R_{ss}(t,\tau) = s(t+\tau/2)s^*(t-\tau/2), \qquad (2)$$

where the asterisk denotes complex conjugation. A drawback when using the Wigner distribution for analysis of multi-component signals (such as seismic signals) is the occurrence of cross-terms in the representation. These cross-terms obscure the time-frequency structure of the signal, making the Wigner distribution of a seismic signal useless for visual interpretation. By smoothing the Wigner distribution over a window $K_P(t,f)$ one obtains a representation $P_{ss}(t,f)$ that has much higher resolution than a spectrogram, but does not suffer as much from the cross-terms as a Wigner distribution. This smoothing operation is a 2D convolution over time and frequency [3]:

$$P_{ss}(t,f) = \int\int K_P(t-t', f-f') W_{ss}(t',f')\ dt'df'. \qquad (3)$$

Making use of the convolutional property of the Fourier transformation, the smoothing can be replaced by a window operation in the Fourier transformed domain. The Fourier transformation of the instantaneous auto-correlation with respect to time t is the Ambiguity function $A_{ss}(\tau,\nu)$ of the signal:

$$A_{ss}(\nu,\tau) = \int \exp(j2\pi\nu\tau) R_{ss}(t,\tau)\ d\tau, \qquad (4)$$

where ν is a frequency-lag variable. The Ambiguity function is a Fourier transform dual of the Wigner distribution:

$$W_{ss}(t,f) = \int\int \exp(j2\pi(f\tau - \nu t)) A_{ss}(\nu,\tau)\ d\nu d\tau. \qquad (5)$$

The smoothed $t-f$ representation can now be found by multiplying the Ambiguity function with a window $\tilde{K}_P(\nu,\tau)$, followed by a Fourier transformation over time-lag τ and an inverse Fourier transformation over frequency-lag ν:

$$P_{ss}(t,f) = \int\int \exp(j2\pi(f\tau - \nu t)) \tilde{K}_P(\nu,\tau) A_{ss}(\nu,\tau)\ d\nu d\tau. \qquad (6)$$

To find a window that optimally separates cross-terms from auto-terms while retaining as much as resolution as possible, is an optimization problem. The best way

[3] all integrals run from $-\infty$ to $+\infty$

to solve this problem is to adapt the smoothing window in the Ambiguity $(\nu - \tau)$ domain. The auto-terms of the Wigner distribution map to the region around the origin in the Ambiguity domain. The cross-terms between signal components occur at increasing time lags τ and frequency lags ν, and are thus located away from the origin. The window is chosen in such a way that (1) it is located around the origin of the Ambiguity function, (2) passes maximum energy and (3) has a fixed size. The size of the window determines the resolution of the representation. In our study we use a representation that is a Wigner distribution, smoothed with a time-varying optimum window, adapted from Baraniuk and Jones (1993).

Modeling

The acoustic response of horizontally stratified impedance models was calculated with a propagator matrix method (Vermaas and Drijkoningen 1993). The calculations are done in the slowness-frequency domain $(p - f)$, followed by a transformation to the time domain. We used a minimum phase source wavelet with a center frequency of 40 Hz. The time-domain data are transformed to the $t - f$ domain with the method described in the theory. The stratigraphic sequences were constructed as a sum of random and Gaussian distributed thicknesses, velocities and densities superposed on linear gradients.

Results

Figure 1 shows the results of the modeling for four models of layered sequences. The density ρ as a function of depth z, was taken proportional to the velocity: $\rho \sim c^{1/4}$. The first sequence (Sequence No. 1) has layers of constant layer thickness, $h(z) = 5\ m$, and alternating velocities $1650 \pm 25\ m/s$. Sequence No. 2 has 50 layers of randomly distributed thicknesses and velocities around a mean thickness $\bar{h}(z)$, of 6 m and a mean velocity of $\bar{c}(z) = 1650\ m/s$, with a randomness of 60 % around $\bar{h}(z)$ and 8 % around $\bar{c}(z)$. The velocities in Sequence No. 3 and Sequence No. 4 are a combination of Gaussian distributed and alternating velocities superposed on a linear velocity gradient. Their main feature is an increase of layer thickness with depth for Sequence No. 3 and a decrease of layer thickness with depth for Sequence No. 4 (24 layers with thickness ranging between 0.5 m and 12 m). All models have a water layer of 200 m on top. The synthetic data are shown in the middle of Fig. 1. In order to show the reflection pattern the synthetic trace is plotted 20 times. The $t - f$ representations are shown on the right, with on the horizontal axis frequency and on the vertical axis time. The contours represent energy densities.

The $t - f$ representations of the modeled data clearly reveal the characteristics of the impedance distribution in the underlying stratigraphic sequences. The seismic response of the nearly uniform layered Sequence No. 1 is tuned to the frequency of the (travel-time) thicknesses of the layers. For Sequence No. 2, the randomly chosen layer thicknesses result in an interference pattern with no evident relation between time and frequency localization of energy. Sequences No. 3 and No. 4 have upward and downward thinning layer thicknesses respectively. The representations clearly show how the increasing layer thinness gives rise to tuning towards higher frequencies.

Figure 2 shows a part of a stacked marine seismic data set and the representations of two traces from the section. The images of the synthetic reflection patterns can be observed in real data as well. In the upper part of the section there is an increase of frequency with time. It was interpreted to be the result of frequency tuning caused by a sequence of downward thinning layers, comparable with synthetic Sequence No. 4. The first 40 traces of the upper part of the section are shown in Fig. 3, together with the synthetic data for Sequence No. 4. The reflection patterns of synthetic and real data are in excellent agreement with each-other.

Conclusion

In this paper we have shown how the $t - f$ representation can be used for the investigation of the earth's contribution to the spectral content of seismic data. The time-frequency representation effectively maps reflection patterns of synthetic seismic data. The observed pattern in the $t - f$ representation of real data can also be obtained from synthetic data. A more detailed investigation of seismic facies models and their image in the $t - f$ domain is needed before the relation between seismic facies and impedance distribution can be thoroughly assessed. However, it is clear that the $t - f$ representation may be well suited to the construction and validation of sequence stratigraphic models.

References

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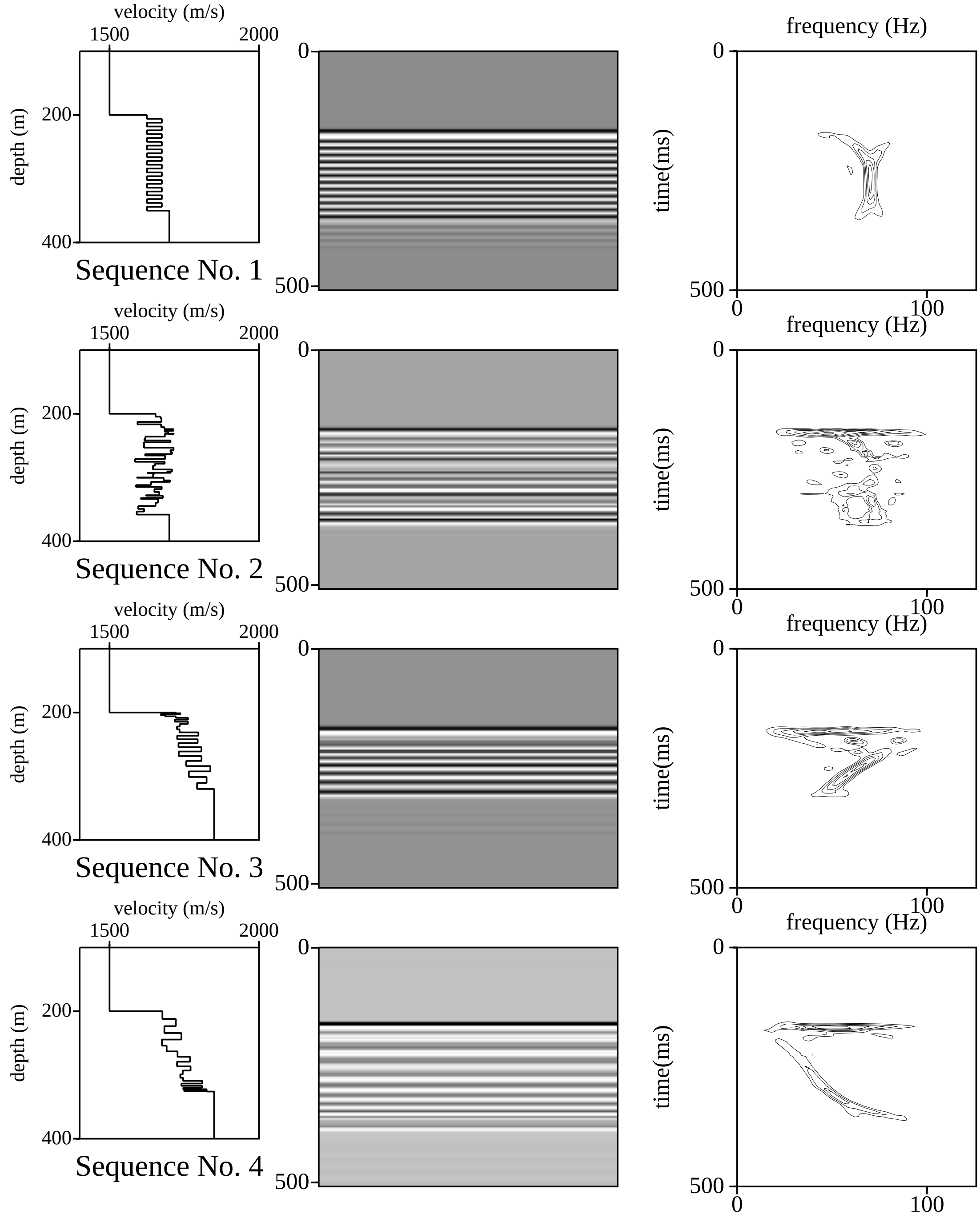

Figure 1: (left) Velocity models, (middle) synthetic seismic data (trace repeated 20 x), (right) t-f representation of synthetic data

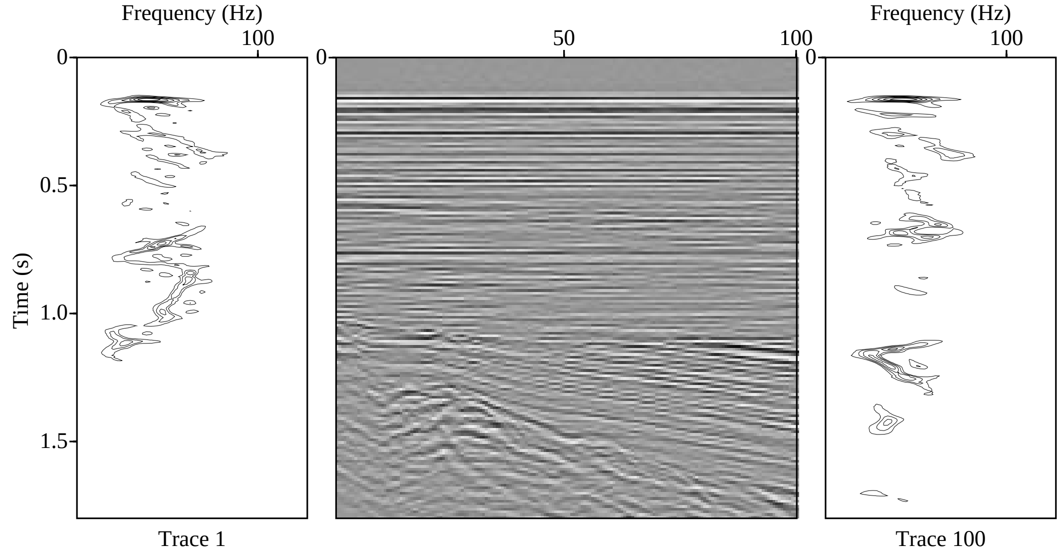

Figure 2: (left) t-f representation of trace 1, (middle) stacked seismic data (Alboran Sea, SW-Mediterranean), (right) t-f representation of trace 100.

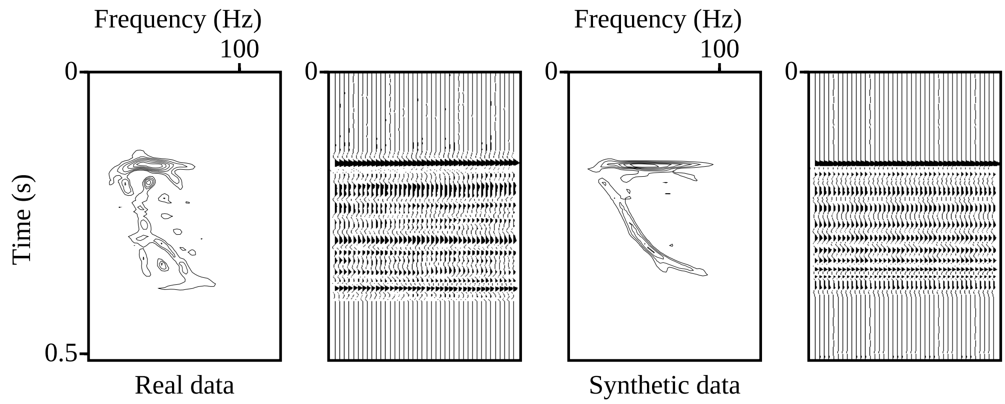

Figure 3: (left) Time-frequency representation of the upper 0.4 (s) of trace 1 of the real data set, first 40 traces of the data set; t-f representation of the synthetic trace of Sequence No. 4, (right) synthetic trace Sequence No. 4 (repeated 40 x).