Multiuser Detectors for Fast-Fading Multipath Channels

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Abstract

We propose a new framework for multiuser detection in fast fading channels that are encountered in many mobile communication scenarios. The detectors proposed in this paper employ a novel multiuser receiver structure based on time-frequency processing that exploits joint multipath-Doppler diversity. Performance analysis shows that the proposed time-frequency multiuser receivers, due to their inherently higher level of diversity, can potentially deliver substantial performance gains relative to conventional multiuser RAKE receivers.

1. Introduction

Code division multiple access (CDMA) has emerged as one of the most promising systems for multiuser wireless communication. Two of the most significant factors limiting the performance of CDMA systems are multipath fading caused by multiple mobile/non-mobile scatterers, and multiaccess interference caused by multiple users simultaneously using the channel. The RAKE receiver structure is used in practice to combat multipath fading [9], and various multiuser detection schemes have been proposed [8] to overcome multiaccess interference. Recently, multiuser RAKE receivers have been proposed to combat multiaccess interference in the presence of multipath fading [11]. However, such schemes are applicable only in slow fading scenarios in which the channel characteristics change slowly over time.

Fast fading is encountered in many mobile communication scenarios and is known to degrade the performance of RAKE receivers due to less reliable channel estimation. Recently, a single-user spread-spectrum communication scheme has been proposed for fast fading channels that exploits the temporal channel variations to provide another means for diversity — Doppler diversity — to counter such degradation in performance [9]. The methodology uses joint time-frequency processing which is a powerful approach to time-varying signal processing [3]. At the heart of our approach is a fundamental time-frequency channel model that is exploited via joint multipath-Doppler processing to achieve a substantially higher level of diversity. The higher level of diversity achieved by time-frequency processing can potentially provide additional performance gains beyond the compensation for the degraded performance of the conventional RAKE receiver [12, 13, 11].

In this paper, we leverage the time-frequency formulation to propose a new multiuser detection framework in the context of CDMA systems to combat multiaccess interference in fast fading channels. Analytical and simulated results presented in this paper show that the proposed multiuser time-frequency RAKE receivers can potentially provide significant performance gains over existing multiuser RAKE receivers.

2. Time-Frequency-Based Channel and Signal Models

In this section, we provide a brief description of the fundamental time-frequency channel model [12, 13] in the context of multiuser CDMA systems. As depicted in Figure 1, the complex baseband signal \( x(t) \) at the output of the channel is related to the transmitted complex baseband signal \( s(t) \) by \( x(t) = \int h(t, \tau)s(t - \tau)d\tau \), where \( h(t, \tau) \) is the time-varying impulse response of the channel [9]. An equivalent representation central to our discussion is in
terms of the channel spreading function $H(\theta, \tau)$:

$$x(t) = \int_0^{T_m} \int_0^{B_d} H(\theta, \tau) s(t - \tau) e^{j2\pi \theta t} d\theta d\tau$$  \hspace{1cm} (1)

where $H(\theta, \tau) \overset{def}{=} \int h(t, \tau) e^{-j2\pi \theta t} dt$. Therefore, the output signal $x(t)$ is a linear combination of time-frequency shifted copies of $s(t)$. In (1), $T_m$ is the multipath spread of the channel, and denotes the maximum delay produced by the channel. Similarly, $B_d$ is the Doppler spread and denotes the maximum Doppler shift introduced by the channel.

The dynamics of the channel are best described statistically, and the wide-sense stationary uncorrelated scatterer (WSSUS) model [9, 1] is widely used, which assumes that $H(\theta, \tau)$ is a two-dimensional uncorrelated Gaussian process:

$$E[H(\theta, \tau)H^*(\theta', \tau')] = \Psi(\theta, \theta') \delta(\theta - \theta') \delta(\tau - \tau'),$$  \hspace{1cm} (2)

where $\delta(x)$ denotes the Dirac delta function. The function $\Psi(\theta, \tau) \geq 0$ is called the scattering function. The multipath spread $T_m$ is the maximum (essential) support of $\Psi(\theta, \tau)$ in the $\tau$ direction, and the Doppler spread $B_d$ is its maximum support in the $\theta$ direction.

For a spread spectrum signal $s(t)$ of duration $T$ and chip period $T_c$, the time-varying wireless channel admits the following fundamental finite-dimensional representation [13, 12]

$$x(t) \approx \frac{T_c}{T} \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} H_{lm}^m (t - lT_c) e^{j2\pi pm/T}, \hspace{0.5cm} 0 \leq t < T,$$

with $L = \lfloor T_m/T_c \rfloor$, $M = \lfloor B_d T_c \rfloor$, and $H_{lm}^m \overset{def}{=} H(m/T_c, lT_c) = \int h(t, lT_c) e^{-j2\pi pm/T} dt$, where $h(t, \tau)$ is a bandlimited approximation (w.r.t. $\tau$) to $h(t, \tau)$ [13, 12]. In (3), $L$ denotes the number of resolvable multipath components, and $M$ denotes the number of resolvable Doppler components.

We should note that in practical fast fading scenarios, even though the Doppler spread is significant enough to degrade the performance of conventional RAKE receivers, it is not large enough to fully exploit Doppler diversity directly by resolving a Doppler component ($M=2$). There are several approaches for maximal exploitation of Doppler diversity that may be employed in practice [12, 13]. One particularly promising scheme is the use of long overlapping spreading waveforms [11] to make the channel maximally time-selective.

For a CDMA system with $K$ users and employing synchronous coherent BPSK signaling, such as may be encountered in the downlink of a mobile communication system [14], the signal at the input of the receiver is given by

$$r(t) = x(t) + n(t) = \sum_{i=-I}^{I} \sum_{k=1}^{K} b_k(i) x_k^i(t) + n(t)$$  \hspace{1cm} (4)

where $b_k(i) \in \{-1, 1\}$ is the $i$-th bit of the $k$-th user, $x_k^i(t)$ is the unmodulated received baseband signal for the $i$-th bit of the $k$-th user, $I$ is the size of the detection window, and $n(t)$ is the complex baseband additive white Gaussian noise (AWGN) with power spectral density $N_0$. In terms of the representation (3), the signal $x_k^i(t)$ can be expressed as

$$x_k^i(t) = \frac{T_c}{T} \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} H_k^{ml}(i) s_k(t - lT - iT_c) e^{j2\pi pm/T},$$

where $s_k(t)$ is the spreading waveform of the $k$-th user, and $H_k^{ml}(i)$ are the channel coefficients corresponding to the $i$-th bit of the $k$-th user. Note that we have absorbed the signal powers and the carrier phases for the different users in the channel coefficients $H_k^{ml}(i)$.

For simplicity of exposition, we introduce a vector notation for the signals. Let us start by defining

$$s_k^m(t) \overset{def}{=} s_k(t - lT_c) e^{j2\pi mp/T},$$

and let $s(t)$ denote the $KLM \times 1$ vector

$$s(t) \overset{def}{=} [s_1^T(t), s_2^T(t), \ldots, s_K^T(t)]^T,$$

where the $LM \times 1$ vector $s_k(t)$ is given by

$$s_k(t) \overset{def}{=} [s_k^0(t), s_k^1(t), \ldots, s_k^{M-1}(t)]^T, \hspace{1cm} k = 1, \ldots, K$$

in terms of the $L \times 1$ vectors

$$s_k^m(t) \overset{def}{=} [s_k^m(t), s_k^{m+1}(t), \ldots, s_k^{M-1}(t)]^T, \hspace{1cm} m = 0, \ldots, M-1.$$  \hspace{1cm} (9)

Similarly as $s_k(t)$ and $s_k^m(t)$, define the $LM \times 1$ vectors $h_k^{m0}(i)$ in terms of the $L \times 1$ vectors $h_k^{n0}(i)$, which are in turn defined in terms of $H_h^{m0}(i)$. Finally, define the $KLM \times K$ channel matrix for the $i$-th symbol

$$H(i) \overset{def}{=} \begin{bmatrix} h_1(i) & 0 & \cdots & 0 \\ 0 & h_2(i) & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & h_K(i) \end{bmatrix}$$

and the $K \times 1$ vector for the $i$-th bit

$$b(i) \overset{def}{=} [b_1(i), b_2(i), \ldots, b_K(i)]^T.$$  \hspace{1cm} (11)
In terms of the above notation, the received signal \( r(t) \) can be expressed as
\[
    r(t) = \sum_{i=1}^{I} s_i^d(t-iT)H(i)b(i) + n(t). \tag{12}
\]

Thus, the received signal is a linear combination of the time-frequency shifted signals \( s_i^d(t) \) which also define the front end time-frequency matched filters for realizing the sufficient statistics for detecting the bits of different users. For negligible intersymbol interference (\( T_m \ll T \)), the output of the time-frequency matched filters for the \( p \)-th bit is given by the \( KLM \times 1 \) vector
\[
    z(p) \overset{\text{def}}{=} \int r(t)s^*(t-pT)dt = RH(p)b(p) + w \tag{13}
\]
where \( R \overset{\text{def}}{=} \int s^*(t)s^*(t)dt \) and \( w \overset{\text{def}}{=} \int s^*(t-pT)n(t)dt \) is a zero-mean complex Gaussian noise vector with \( \mathbb{E}[ww^H] = N_0R \). It follows that the “one-shot” detector suffices in which the decision about the \( p \)-th bit is based on the received waveform for the corresponding bit only. Thus, in subsequent sections, we suppress the index \( p \) in the notation and focus on the 0-th bit without loss of generality.

3. Multiuser Detectors for Fast Fading Channels

In this section, we develop a multiuser detection framework that incorporates the fundamental multipath-Doppler channel model of Section 2 to deliver near-far resistant receiver structures for fast fading channels. Due to the prohibitive computational complexity of the optimal receiver, lower complexity, suboptimal approaches are sought in practice. Next, we discuss an approach for designing suboptimal near-far resistant receivers that are computationally tractable.

3.1. Computationally Tractable TF RAKE Receivers

The structure of the suboptimal receivers that we derive is motivated by the single-user maximal-ratio-combiner (MRC) detector. Recall the definition of \( z \) in (13), and decompose it as \( z = [z_1, z_2, \ldots, z_K] \) analogous to the decomposition of \( s \). In the absence of multiaccess interference the optimal receiver for each user is the time-frequency maximal-ratio-combiner (MRC) \[9\]
\[
    \hat{b}_k = \text{sign} \left\{ \text{Re} \left[ H_k^H z_k \right] \right\} = \text{sign} \left\{ \text{Re} \left[ \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} H_{kl}^m z_k^m \right] \right\}, \quad k = 1, \ldots, K \tag{14}
\]
which coherently combines the different multipath-Doppler shifted signal components to achieve \( LM \)-order diversity. Note that the MRC requires the knowledge of the channel coefficients \( H_k^m \), which may be estimated through a pilot transmission, for example. The signal component of \( z_k \) in this case is \( \mathbb{E}[s_k h_k b_k] \), where \( \mathbb{E}[s_k h_k b_k] = \int |s_k(t)|^2 dt \), resulting in \( \mathbb{E}[|h_k b_k|^2] = b_k \) as the real-valued signal component of the test statistic in (14). Of course, in the presence of other users, the MRC detector in (14) is not near-far resistant due to the presence of multiaccess interference.

As evident from (14), the MRC detector makes the decision by coherently combining the matched filter outputs \( z_k \) corresponding to the different multipath-Doppler shifted signal components. As mentioned before, in the absence of other users, the signal component of \( z_k \) is (up to a constant)
\[
    \int r(t)s_k^*(t)dt \bigg|_{r(t)=x(t)} = h_k b_k. \tag{15}
\]
In our suboptimal approach, the idea is to obtain an estimate of the noise free matched-filter outputs, \( h_k b_k \), and then to coherently combine them as in (14) to obtain the bit estimates. The estimation procedure for \( h_k b_k \) should be such that the resulting detector is near-far resistant. In terms of the \( K \) users, we seek an estimate of
\[
    y = \int r(t)s^*(t)dt \bigg|_{r(t)=x(t)} = Hb. \tag{16}
\]
The nature of the estimate of \( y \) determines the structure of the receivers. For computational efficiency, both the receiver structures that we propose employ a linear estimate. Generically, the \( y \) estimate takes the form
\[
    \hat{y} = Fz = FRHb + Fw = y_s + y_n \tag{17}
\]
where \( y_n \overset{\text{dist}}{\sim} \mathcal{N}(0, Q) \) with \( Q = N_0 FRF^H \). The matrix \( F \) is chosen to yield a near-far resistant estimate of \( y \). Following the application of \( F \), maximal ratio combining is applied to the different multipath-Doppler components \( \hat{y}_k \) of each user. However, since the noise in the estimate \( \hat{y} \) is colored, a prewhitening operation is needed. The general form of the overall multiuser TF RAKE receiver is
\[
    \hat{b} = \text{sign} \left\{ \text{Re} \left[ H^H D \hat{y} \right] \right\} = \text{sign} \left\{ \text{Re} \left[ H^H DFz \right] \right\}, \tag{18}
\]
where the block-diagonal prewhitening matrix \( D \) is given by
\[
    D = \begin{bmatrix}
        Q_{11}^{-1} & 0 & \cdots & 0 \\
        0 & Q_{22}^{-1} & \cdots & \cdots \\
        \vdots & \vdots & \ddots & \vdots \\
        0 & \cdots & 0 & Q_{KK}^{-1}
    \end{bmatrix}. \tag{19}
\]
where \( Q_{lk} \) is the block of \( Q \) corresponding to the \( l \)-th user. In the following subsections, we discuss two special cases of the family based on the choice of the estimator matrix \( F \).
3.1.1 Decorrelating TF RAKE Receiver

If the ML estimate of $y = Hb$ is employed, the resulting estimate is linear and we obtain a generalization of the decorrelating receiver [6, 15]. The estimator matrix $F$ in this case is given by

$$F_{ML} = R^{-1}$$

(20)

since

$$\hat{y}_{ML} = \arg \max_y \left[ 2\text{Re}[z^H y] - y^H R y \right] = R^{-1} z.$$  

(21)

From (18), it follows that the resulting decorrelator TF RAKE receiver takes the form

$$\hat{b}_{dec} = \text{sign} \left\{ \text{Re} \left[ H^H D_{dec} R^{-1} z \right] \right\},$$

(22)

where $D_{dec}$ is defined as in (19), with $Q = N_0 R^{-1}$. The decorrelator TF RAKE receiver in (22) combats fast multipath fading by exploiting joint multipath-Doppler diversity via the fundamental channel model (3), and attains near-far resistance via the ML estimate in (21), analogous to the conventional decorrelating detector [6].

3.1.2 Minimum Mean-Squared-Error TF RAKE Receiver

If a linear minimum mean-squared error (MMSE) estimate of $y = Hb$ is employed before coherent combination, a generalization of the MMSE detector [7, 5] is obtained. In this case, the estimator matrix $F$ is the solution to

$$F_{mmse} = \arg \min_{F} E \left[ \| Hb - F z \|^2 \right],$$

(23)

and can be shown to take the form

$$F_{mmse} = \left( R + N_0 \Psi^{-1} \right)^{-1},$$

(24)

where $\Psi \overset{def}{=} E[H H^H]$. The resulting MMSE TF RAKE receiver is given by

$$\hat{b}_{mmse} = \text{sign} \left\{ \text{Re} \left[ H^H D_{mmse} F_{mmse} z \right] \right\}$$

(25)

where $D_{mmse}$ is defined as in (19). Note that $\Psi$ is a function of the powers $p_k$ of different users, and the second-order channel statistics of the corresponding users. Specifically, for the WSSUS channel model, $\Psi$ has a diagonal structure, with the $k$th diagonal block being equal to $p_k \Psi_k$, where $\Psi_k$ is a diagonal matrix corresponding to the powers in the different multipath-Doppler channel coefficients of the $k$th user.

4. Performance Analysis

In this section, we assess the performance of the proposed time-frequency multiuser detectors in fast fading scenarios. Of the two particular multiuser detectors proposed in the last section, only the decorrelator receiver lends itself to tractable performance analysis. Proceeding analogous to the standard performance analysis for the decorrelator detector [6, 15], we first derive an expression for the probability of bit error for the decorrelator receiver, and then provide simulated results for both the decorrelator and MMSE TF RAKE receivers.

Recall from (22) that the decorrelator receiver is of the form

$$\hat{b} = \text{sign} \left\{ \text{Re} [g] \right\}$$

(26)

where the test statistic $g$ is given by

$$g = H^H D_{dec} H b + H^H D_{dec} R^{-1} w.$$  

(27)

It follows that the test statistic $g_k$ corresponding to the $k$th user is

$$g_k = h_k^H Q_{kk}^{-1} h_k b_k + h_k^H Q_{kk}^{-1} \tilde{w}_k = g_{k,k} + g_{n,k},$$

(28)

where $\tilde{w}_k$ is the component of the vector $R^{-1} w$ corresponding to the $k$th user. It can be readily verified that $E[g_{n,k}^2 | h_k] = h_k^H D_{dec} R^{-1} w$. Thus, the probability of bit error for the $k$th user, conditioned on the knowledge of the channel coefficients $h_k$, is [9]

$$P_k | h_k = Q \left( \sqrt{2 h_k^H Q_{kk}^{-1} h_k} \right)$$

(29)

where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx$. Note that since $h_k$ consists of independent complex Gaussian random variables,

$$h_k^H Q_{kk}^{-1} h_k = \sum_{l=1}^{LM} \lambda_l \gamma_l$$

(30)

where the $\lambda_l$’s are the eigenvalues of $p_k Q_{kk}^{-1} \Psi_k$, and the $\gamma_l$’s are independent $\chi^2$ random variables each with two degrees of freedom. The unconditional probability of error can be obtained by averaging $P_k | h_k$ with respect to the probability density function of $h_k^H Q_{kk}^{-1} h_k$, and is given by [9, pp. 801–802]

$$P_k = \frac{1}{2} \sum_{l=1}^{LM} \left[ \frac{\lambda_l}{\lambda_l - \lambda_1} \right] \left[ 1 - \sqrt{\frac{\lambda_l}{1 + \lambda_l}} \right].$$

(31)

Note that the eigenvalues $\lambda_l$’s include the dependence on the signal powers $p_k$ and the noise power $N_0$.

We next provide some numerical results on the performance of the decorrelator and MMSE time-frequency (TF)
multiuser receivers. All the results are based on a spreading gain of \( N = T/T_c = 31 \) and employ M-sequences as the spreading codes. We also assume that one multipath component \((L = 2)\) is resolvable. In fast fading scenarios, we assume that one Doppler component \((M = 2)\) is resolvable.

### 4.1. Numerical Results

First, we calculate the analytical probability of error \((P_e)\) based on (31) for the decorrelating TF RAKE receiver and the conventional multiuser RAKE receiver as a function of signal-to-noise ratio (SNR). To demonstrate the potential performance gains achievable through Doppler diversity, we compare the performance of the conventional multiuser RAKE receiver operating in a slow fading environment with that of the TF RAKE receiver operating in a fast fading scenario. Figure 2 shows the results for \(K = 2\) and 4 users. Evidently, by exploiting Doppler diversity, the multiuser TF RAKE receiver has the potential of significantly outperforming the conventional multiuser RAKE receiver. For example, for \(K = 4\), the TF receiver achieves a 3\(dB\) gain in SNR over the conventional multiuser RAKE receiver. As expected, as the number of users increases, a higher SNR is needed to achieve a prescribed value of \(P_e\) for the desired user. Note that the performance gains of the multiuser TF RAKE receiver over the conventional multiuser RAKE receiver increase monotonically as the SNR increases.

Next, we assess the performance of the MMSE TF RAKE receiver based on Monte Carlo simulations. Figure 3 compares the performance of the MMSE TF RAKE receiver and the conventional RAKE receiver as a function of SNR. The results are similar to those obtained for the decorrelating receiver, suggesting potentially superior performance of the time-frequency detector due to Doppler diversity.

![Figure 2. Analytical probability of bit error versus SNR for decorrelating TF RAKE and conventional RAKE receivers.](image1)

![Figure 3. Simulated probability of error versus SNR for the MMSE TF RAKE receiver and the conventional multiuser RAKE receiver.](image2)

![Figure 4. Near-far resistance of the multiuser receiver structures.](image3)

Figure 4 demonstrates the near-far resistance of the proposed MMSE TF RAKE receiver by plotting the probability of bit error for the first user (at SNR=10\(dB\)) as a function of the SNR of the second user relative to that of the first user. We expect similar behavior for the TF decorrelator receiver.
Finally, Figure 5 seems to suggest that there is no significant difference in the performance of the multiuser TF RAKE receiver as a result of using M-sequences versus randomly generated spreading sequences.

5. Conclusions

In this paper, we have proposed a novel framework for multiuser detection in fast fading channels. At the heart of our approach is a fundamental finite dimensional time-frequency channel model that facilitates the exploitation of joint multipath-Doppler diversity. Analytical and simulated results demonstrate that the proposed multiuser time-frequency RAKE receivers can potentially provide significant performance gains relative to existing multiuser RAKE receivers.

The multiuser time-frequency RAKE receivers proposed in this paper espouse the paradigm of removing multiaccess interference first, followed by maximal ratio combining of the multipath-Doppler components. However, receiver structures that employ diversity combining first are also possible. According to a study of multiuser RAKE receivers [4], employing diversity combining first yields better performance with perfect knowledge of channel parameters, whereas executing interference suppression first, as in our approach, may be more robust if channel estimates are used. Performance comparison of the two approaches in the context of fast fading is a useful direction for future work.

Finally, we note that we restricted our discussion to the synchronous case for clarity of exposition. However, the results of this paper can be readily extended to the asynchronous scenarios as well.

References
