

# Optimal Transmit Spectra for Communication on Digital Subscriber Lines

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## Abstract

We present a general framework for designing optimal transmit spectra for symmetric bit-rate Digital Subscriber Line (DSL) services, in particular HDSL2. Using the channel and interference transfer functions and SNR estimates, we set up and solve an optimization problem to maximize the capacity. Sizable gains in performance margins (or bit rates) result. Furthermore, by design, the spectra are spectrally compatible with other services. While the framework is quite general — it does not depend on the exact choice of modulation scheme, for example — it is also extremely simple and of low computational complexity. Our results can be used either for dynamically adapting the signaling spectra to account for changing noise or interference conditions or for the design of new fixed transmit spectral masks using worst-case analysis.

## 1 Introduction

*Digital Subscriber Lines* (DSLs) are standard telephone lines (twisted pairs) configured to carry high bit-rate traffic (> 1 Mbps). DSL modems along with cable modems form a growing market for wire-based high-speed modems. These modems exploit large transmission bandwidths (> 1 MHz for DSLs) in order to deliver high data rates. Several DSL services (xDSL in general) exist and are categorized according to the bit rates they deliver. Some of the key ones are:

**ADSL — Asymmetric DSL.** This service is designed to provide a full-duplex high-speed (on the order of 6 Mbps) downstream (from central office to subscriber) channel and a low-speed (on the order of 640 kbps) upstream (from subscriber to the central office) channel over each twisted pair.

**VDSL — Very high bit-rate DSL.** This yet-to-be-standardized service will provide a full-duplex symmetric or asymmetric high-bit-rate (on the order of 50 Mbps) channel over a single twisted pair less than 3 to 6 kft long.

**HDSL2 — High bit-rate DSL 2.** This soon-to-be-standardized service will provide a full-duplex 1.544 Mbps signal transmission service over a *single twisted pair* (< 18 kft long) without repeaters.

DSLs or twisted pairs are bundled together into groups in a binder and carry different xDSL services. Crosstalk (near-end (NEXT) and far-end (FEXT)) results due to proximity of different lines in a binder (see Figure 1) and significantly limits achievable bit-rates [1]. In this paper we develop optimal transmit spectra using joint signaling techniques for *crosstalk avoidance* and optimal power distribution to maximize bit rates. By design, we maintain spectral compatibility with existing neighboring services. This problem was first solved in [2] but only for self-NEXT (NEXT from same service lines) and AWGN. In this paper, we solve the problem in presence of self-NEXT, self-FEXT, AGN (Additive Gaussian Noise), and interference from other services.

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The transmit spectra for Digital Subscriber Lines (DSLs) need to achieve high uncoded performance margins. High performance margins ensure that the DSL service remains robust to noise and cross-talk interference (NEXT and FEXT). Simultaneously these transmit spectra must be spectrally compatible with existing services. Spectral compatibility ensures that the deployment of DSL service will not adversely affect existing services on neighboring lines.

In this paper we will focus on the general framework for distributing transmit power over frequency. While the techniques developed here are general (and not limited to xDSL services), we will present our results in the context of the HDSL2 service.

Several T1E1.4 contributions have addressed the issue of transmit spectra for HDSL2 ([3] and [4] are some recent ones). To date, all proposed transmit spectra have been fixed (i.e., they do not vary with the interference and noise combinations) and have been defined in terms of fixed PSD masks (e.g., OPTIS [3] and MONET-PAM [4]). The PSD masks have been obtained as a design compromise after studying the performance of the fixed transmit spectra in the presence of a variety of worst-case interferers.

Instead of fixed transmit spectra, *we propose optimal transmit spectra that adjust to the given interference and noise combination in order to maximize performance (margins or bit rates).*

The key advantages of the techniques developed here are:

1. For the same input power, optimal transmit spectra yield higher performance margins or bit rates than any fixed transmit spectra [5].
2. Optimal transmit spectra are inherently spectrally compatible with other services.
3. Optimal spectra can be used with any modulation scheme, including DMT, CAP, QAM, PAM, etc.
4. Near-optimal spectra can be computed almost trivially and with a low computational complexity. The algorithms can easily be implemented on a DSP chip.
5. Optimal transmit spectra exist and can be efficiently computed even for complicated loops with bridged taps.

Our results can be used in a number of ways, including

1. full dynamic (on-line) adaptation of the signaling spectrum to account for changing noise or NEXT and FEXT conditions, changing line characteristics due to temperature, etc.
2. design of new fixed transmit spectral masks using worst-case analysis (the new masks would supercede OPTIS and MONET-PAM due to their improved performance).
3. optimization of the transmit spectrum “underneath” a standardized spectral mask (see contribution [6]).

Section 2 outlines the definitions and notation used. Details on obtaining optimal transmit spectra are presented in Section 3. We discuss simulation results in Section 4 and present conclusions in Section 5.

## 2 Definitions and Notation

Due to the close proximity of the lines within a binder, there is considerable amount of crosstalk interference between different neighboring telephone lines. Physically, there are two types of interference (see Figure 1):

**Near-end crosstalk (NEXT):** Interference between neighboring lines that arises when signals are transmitted in opposite directions. If the neighboring lines carry the same type of service then the interference is called self-NEXT; otherwise, we will refer to it as different-service (DS) NEXT.

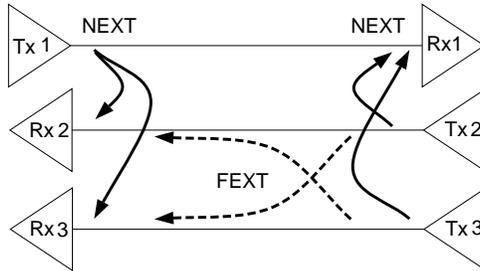


Figure 1: *NEXT* and *FEXT* between neighboring lines in a binder. *Tx*'s are transmitters and *Rx*'s are receivers.

**Far-end crosstalk (FEXT):** Interference between neighboring lines that arises when signals are transmitted in the same direction. If the neighboring lines carry the same type of service then the interference is called self-FEXT; otherwise, we will refer to it as different-service (DS) FEXT.

The term *self-interference* refers to the combined self-NEXT and self-FEXT. Channel noise is modeled as additive Gaussian noise (AGN).

We will denote the upstream and downstream transmit frequency spectra by  $S^u(f)$  and  $S^d(f)$ , respectively. An *Equal PSD* (EQPSD) signaling scheme in frequency band  $B$  is one for which  $S^u(f) = S^d(f) \neq 0$  for all  $f$  in  $B$  (that is, both upstream and downstream transmissions occupy the band  $B$  in the same way). An *Frequency Division Signaling* (FDS) scheme in frequency band  $B$  is one for which  $S^u(f) = 0$  when  $S^d(f) \neq 0$  for all  $f$  in  $B$  and vice versa (that is, both transmissions occupy orthogonal frequency bands within  $B$ .)

### 3 Optimal transmit spectra

#### 3.1 Absence of self-interference

In the absence of self-NEXT and self-FEXT, the interference combination consists of different service interferers (such as HDSL, T1, ADSL, etc.) and AGN. We obtain the optimal power distribution in each direction of transmission by the classical water-filling solution [7].

#### 3.2 Presence of self-interference

Self-NEXT and self-FEXT severely limit the achievable bit-rates of HDSL2. The optimal signaling scheme in the presence of self-NEXT and AGN is given in [2]. We have extended these results to account for self-NEXT, self-FEXT, different service interferers, and AGN (see [8] for the details on the solution method).

*A hallmark of the optimal solution is the use of FDS to separate the upstream and downstream transmissions in frequency regions facing high self-NEXT.*

For example, in Figure 2 we plot the CSA loop 6 channel transfer function and self-interference transfer functions from 39 self-NEXT and 39 self-FEXT interferers. Note that the self-NEXT transfer function rises with frequency (indicating that NEXT is more intense at high frequencies).

For a monotonic channel transfer function (as shown in Figure 2), the upstream and downstream transmit spectra can be conveniently (yet optimally) separated into two distinct signaling regions: a low frequency region using EQPSD signaling and a high frequency region using FDS [8]. Interestingly, this continues to hold even for many non-monotonic channel transfer functions (such as those arising due to bridged taps).

To find the optimal transmit spectra, we maximize the Shannon capacity of the HDSL2 line given the various interferences and an average power constraint. A simple iterative algorithm yields the optimal solution [8]:

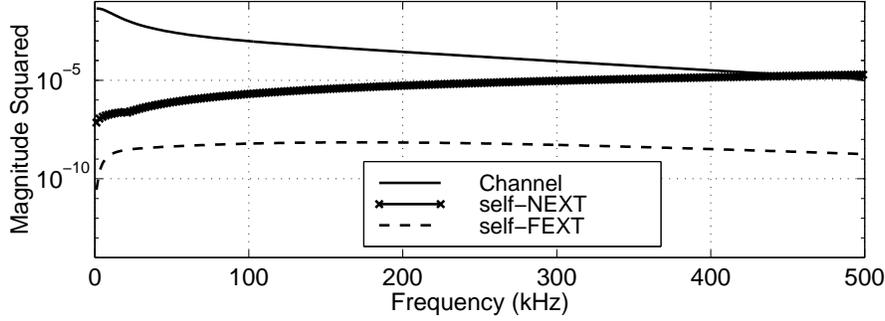


Figure 2: Magnitude-squared transfer functions of the channel CSA loop 6 with 39 self-NEXT interferers and 39 self-FEXT interferers.

1. Estimate the switch-over frequencies between EQPSD signaling and FDS.
2. Perform optimal power distribution within the EQPSD and FDS regions using water-filling [7, 9].
3. Loop between 1 and 2 until convergence is reached.

In addition, we have found a simple test condition that closely approximates the optimal switch-over frequencies for Step 1 [8]. This approximation reduces the above algorithm to a single, computationally simple step of water-filling.

It is key to note that the optimal transmit spectra do not dictate any specific modulation scheme, but rather simply describe how a modulation scheme should optimally distribute its power over frequency. Thus, optimal transmit spectra can be used with a number of different modulation schemes, including but not limited to DMT, CAP, QAM, PAM, etc.

## 4 Simulation Results and Discussion

### 4.1 Simulation Details

Bit rate fixed at 1.552 Mbps.

Total average input power (one-sided) in each direction  $P_{\max} = 16.78$  dBm.

Different service interference models obtained from Annex B of T1.413-1995 (from [10], the ADSL standard), with exceptions as in [11].

Self-NEXT interference modeled as a 2-piece Unger model [12].

Margins are calculated according to [13].

OPTIS transmit spectra are obtained by tracking 1 dBm/Hz below the OPTIS PSD masks (see [3]).

OPTIS performance margin numbers are from [3] and spectral compatibility numbers are from [11].

AGN of  $-140$  dBm/Hz added to the interference.

DMT modulation scheme:

Sampling frequency  $f_s = 1000$  kHz.

Bin width  $W = 2$  kHz.

Number of bins  $K = 250$ .

Start frequency = 1 kHz.

Bit error rate (BER) =  $10^{-7}$ .

SNR gap = 9.8 dB.

No cyclic prefix. No limitation on maximum number of bits per tone.

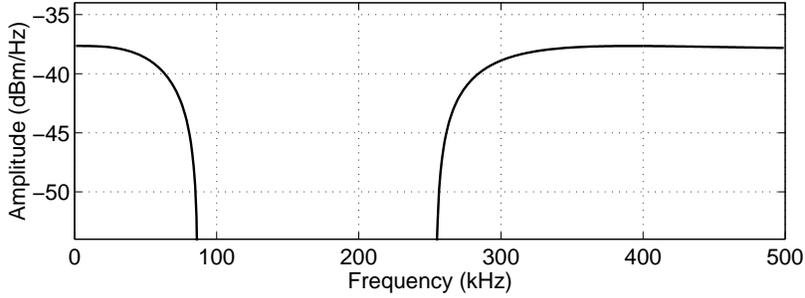


Figure 3: Optimal transmit spectra for HDSL2 on CSA loop 6 with 49 HDSL NEXT interferers and AGN of  $-140$  dBm/Hz. Since there is no self-interference, FDS is not required. The upstream and downstream transmissions employ the same spectrum.

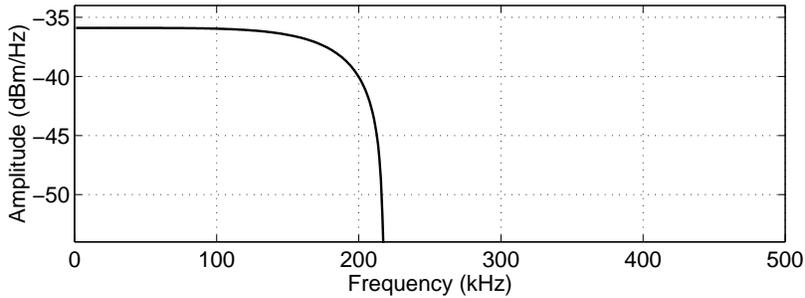


Figure 4: Optimal transmit spectra for HDSL2 on CSA loop 6 with 25 T1 NEXT interferers and AGN of  $-140$  dBm/Hz. Since there is no self-interference, FDS is not required. The upstream and downstream transmissions employ the same spectrum.

## 4.2 Examples

Figures 3, 4, and 5 illustrate the optimal transmit spectra on CSA loop 6 for HDSL2 in the presence of 49 HDSL, 25 T1, and 39 HDSL2 interferers, respectively. In the case of different service interferers (HDSL and T1 in Figures 3 and 4), the optimal upstream and downstream spectra are the same (EQPSD throughout). In the case of HDSL2 interferers (Figure 5), self-NEXT at high frequencies forces the optimal upstream and downstream spectra to separate in frequency giving rise to an FDS region. As an added bonus, no echo cancellation<sup>1</sup> will be required in the large FDS region.

*Note that the optimal transmit spectra vary significantly with the interference combination.*

In the case of bridged taps, the channel transfer function has nulls and varies with each loop. This strongly indicates the necessity of a transmit spectra that can adapt to the channel as well as interference conditions. Figure 6 illustrates the optimal transmit spectra for CSA loop 4 (having bridged taps) for HDSL2 in the presence of 39 self-NEXT and 39 self-FEXT HDSL2 interferers.

## 4.3 Performance Margins

The noise margin or performance margin of a channel for a fixed bit rate and BER measures the maximum degradation (from noise and interference) in performance that a channel can sustain before being unable to transmit at that bit rate and BER (see [14]).

Table 1 lists the performance margins of the optimal transmit spectra vs. those obtained using the (performance-standard) OPTIS transmit spectra<sup>2</sup> for CSA loop 6. For different service interferers (HDSL and T1) only the NEXT powers were considered; for HDSL2, “self” comprises both self-NEXT and self-FEXT. Equal performance margins

<sup>1</sup>Echo cancellation is used to reduce interference between upstream and downstream transmissions on the same line.

<sup>2</sup>OPTIS transmit spectra are obtained by tracking 1 dBm/Hz below the OPTIS PSD masks (see [3]).

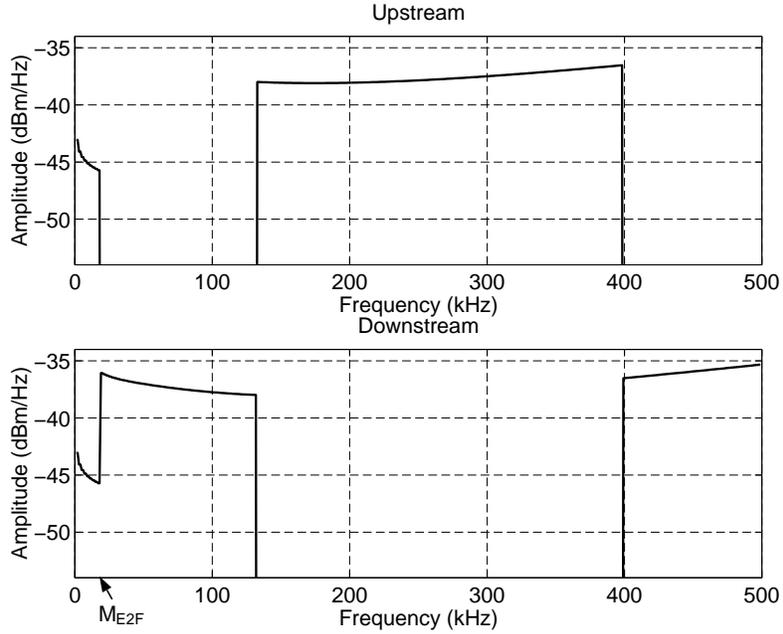


Figure 5: Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop 6 with 39 self-NEXT and 39 self-FEXT interferers. EQPSD signaling takes place to the left of switch-over frequency  $M_{E2F}$  and FDS to the right. (Note that the placement of the FDS regions is nonunique. Here we choose a split such that upstream and downstream transmissions use equal transmit powers and result in equal performance margins. Other splits are possible; see [8] for details.)

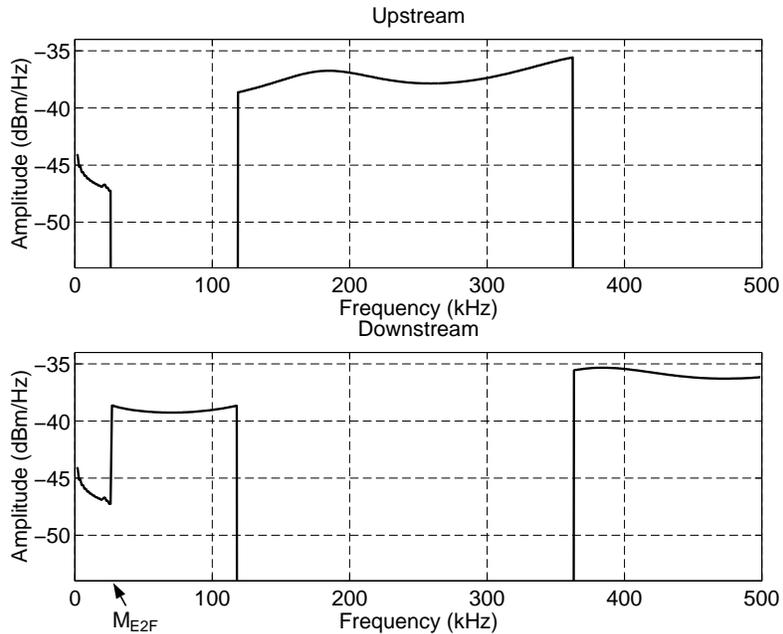


Figure 6: Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop 4 with 39 self-NEXT and 39 self-FEXT interferers. EQPSD signaling takes place to the left of switch-over frequency  $M_{E2F}$  and FDS to the right. (Again we choose a split such that upstream and downstream transmissions use equal transmit powers and result in equal performance margins. Other splits are possible; see [8] for details.)

Table 1: *Uncoded performance margins (in dB) for CSA loop 6: OPTIS vs. Optimal. OPTIS numbers were obtained from [3]. Diff = Difference between worst-case Optimal and worst-case OPTIS.*

Crosstalk source	xDSL service	OPTIS		Optimal		Diff
		Up	Dn	Up	Dn	
49 HDSL	HDSL2	2.7	12.2	<b>18.5</b>	<b>18.5</b>	<b>15.8</b>
25 T1	HDSL2	19.9	17.5	<b>21.3</b>	<b>21.3</b>	<b>3.8</b>
39 self	HDSL2	2.1	9.0	<b>18.3</b>	<b>18.3</b>	<b>16.2</b>
24 self+24 T1	HDSL2	4.3	1.7	<b>5.4</b>	<b>5.4</b>	<b>3.7</b>

Table 2: *Spectral-compatibility margins (in dB) for CSA loop 6: OPTIS vs. Optimal. OPTIS numbers were obtained from [11].*

Crosstalk source	xDSL service	OPTIS	Optimal	
			Up	Dn
49 HDSL	HDSL	7.86 (OPTIS and Optimal not involved)		
39 HDSL2	HDSL	7.84	<b>13.28</b>	<b>20.84</b>
49 HDSL2	HDSL	7.26	<b>12.71</b>	<b>20.15</b>

were obtained for upstream and downstream transmissions. We can clearly see that the optimal scheme outperforms OPTIS with large performance gains in all the cases.

#### 4.4 Spectral Compatibility

Whenever we optimize the bit rate of any xDSL service on a line we need to make sure that this service does not significantly interfere with the existing surrounding xDSL services. In other words, we need to check if the service being optimized is *spectrally compatible* with existing services. Spectral compatibility is measured in terms of noise margins (called spectral compatibility margins) of neighboring services in presence of the optimized service.

By design, the optimal transmit spectra achieve good spectral compatibility margins. Through water-filling, we distribute more HDSL2 power in regions of low interference and less HDSL2 power in regions of high interference. Thus, we avoid the transmission frequencies of neighboring lines and therefore simultaneously reduce the effect of HDSL2 transmissions on these neighboring lines.

To illustrate, consider the spectral compatibility between HDSL2 and HDSL. Table 2 lists the spectral compatibility margins of the optimal transmit spectra vs. OPTIS [11] for CSA loop 6. We compare the performance margins for HDSL in the presence of two types of interferers; other HDSL lines and HDSL2 lines. The column “OPTIS” lists the margins obtained using OPTIS transmit spectra and the column “Optimal” lists the margins obtained using optimal transmit spectra (different for each combination of interferers). We see that the optimal spectra have better spectral compatibility margins than OPTIS. A similar analysis could be carried out for T1 and ADSL services with similar results.

## 5 Conclusions

1. Optimal transmit spectra can yield large gains in performance margins compared to fixed-mask schemes. We can trade these increased performance margins for increased bit rates or decreased average transmission power.
2. Optimal transmit spectra are inherently spectrally compatible with existing services.
3. Optimal spectra are not bound to any particular modulation scheme.

4. There exist near-optimal transmit spectra that are trivially complex to compute, even for complicated loops such as those with bridged taps.
5. No echo cancellation is required in frequency bands employing FDS.
6. Equal performance margins can be obtained for upstream and downstream directions using optimal transmit spectra.
7. Transmit spectra can be adapted on-line to changes in line conditions (e.g., temperature variations, etc.).
8. This scheme requires knowledge of the characteristics of neighboring interfering services. These can either be estimated at start-up or analyzed in a worst-case manner for a particular line under consideration. This information could also be obtained from a central office database that specifies the type of services in each binder group in the telephone cable.

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