Impact of Multiple Access on Uplink Scheduling

Dinesh Rajan  Ashutosh Sabharwal  Behnaam Aazhang
Department of ECE, Rice University, Houston, Texas 77005.
\{dinesh,ashu,aaaz\}@rice.edu

Abstract — In this paper, we consider uplink scheduling for bursty traffic. We characterize the achievable rate region for Gaussian multiple access in terms of minimum required powers, with a constraint on average transmission delay for all users. We show that delay and rate constrained, power minimizing schemes perform scheduling accompanied with power control. Further, for the class of randomized stationary schedulers, it is shown that the achievable region is a convex polytope. We highlight that power requirements of a user can be reduced by either allowing additional delay (time scheduling gain) or increasing the power of another user (multiuser power exchange). Results are presented for two user additive white Gaussian noise channel and can be extended to finite state fading channels.

I. INTRODUCTION

In this paper, we consider transmission of bursty sources over noisy channels with delay constraints. Several sources of interest (e.g., compressed video, file transfer and web access) are labeled as bursty sources because they produce bursts of data with variable number of bits per unit time. Furthermore, most of these bursty sources have delay deadlines to allow meaningful communication. Typically, the delay due to queuing is excluded from analysis of physical layer algorithms, partly due to asymptotic information theoretic techniques. Though the lack of a unified framework to address delay in communicating over noisy channels is well appreciated in [1, 2], limited efforts have been made to rectify the situation [3]. The work in [3] combines queuing theoretic methods with information theoretic limits to formulate a basic unified theory. As noted in [3], the simplified communication model is used only to demonstrate the possibility of a unified framework.

A more practical time-slotted system was used in [4], to highlight the impact of queuing delay on average power requirements for single-user communication. In [4], the utility of time scheduling of packets was shown in time-varying channels; fewer packets are transmitted during poor channel conditions and more during good channel conditions or when the delay tends to increase beyond the acceptable limits. To accommodate for variable number of packets, power control was used in [4]. In [5], it was shown that power reduction can be achieved even over time-invariant Gaussian channels by increasing average delay deadlines of the packets, i.e., not only channel variations allow power savings via scheduling, but also source burstiness.

In systems with multiple users, scheduling of user data is employed to share the same channel [6, and references therein]. Most work to date in scheduling treats the channel as a fixed capacity link (for AWGN case) or a link with time-varying capacity (fading channels), and thus abstracts away the physical layer parameters like power. The above abstraction, though suitable for wired links, does not fully utilize the capabilities of wireless communication. For instance, as we showed in [5], power control is not only effective in mitigating channel errors but also in meeting delay deadlines for bursty sources.

In this paper, we study uplink communication, where multiple users communicate with a single receiver. Our objective is to meet delay deadlines for all users, with maximal power efficiency. We consider a time-slotted system where each user sends a bursty source with finite average delay (measured in number of time-slots). By using mutual information approximation [7], we reduce any transmission scheme to two variables per user: determining the number of packets to be transmitted in any given time slot (scheduling), and associated power required for their zero outage reception (power control). We do not restrict ourselves to orthogonal multiple accessing schemes (like TDMA) to overcome the multiple access interference. Instead, we find the best performance possible using any access method [10]. Our major contribution is identification that in multiple access channels, the power gain due to scheduling can be decomposed into two components, time scheduling gain and multiuser power exchange. The time scheduling gain is available by allowing additional delay, much like in single user communication [5]. The multiuser tradeoff uses the fact that for a given rate pair, power of one user can be increased to reduce the power required for another user. We completely characterize the capacity region in terms of minimal required powers for the two-user case.

Finally, we note that our work is fundamentally different from the notion of delay limited capacity in [8], which was introduced to guarantee worst case performance in fading channels without queuing.

We formalize the problem and provide a method of solution in Section II and conclude the paper is Section III.

II. PROBLEM FORMULATION AND SOLUTION
A schematic of uplink communication with queuing is shown in Figure 1. Assume a time slotted system with $K$ users. Each user has a queue of capacity $L_i$ packets. The packet arrivals $a_{i,n}$ into these queues are independent and identically distributed with respect to user index $i = 1, 2, \ldots, K$ and time index $n$. The distribution of $a_{i,n}$ is assumed to have support $0, M_i$. Each user communicates with a central base station to meet an average packet delay of $D_i$ while ensuring no buffer overflow.\(^1\) In order to clearly see the effect of source burstiness in designing the scheduler, we assume an additive white Gaussian noise (AWGN) channel between all users and the base station, of bandwidth $W$. The proposed procedure can be extended to finite state fading channels in a straightforward manner.\(^2\) The received signal $Y_n$ during the $n$th time slot is given by

$$Y_n = \sum_{i=1}^{K} P_{i,n} X_{i,n} + Z_n, \quad (1)$$

where, $Z_n$ is the Gaussian noise of variance $N$, $X_{i,n}$ is the transmitted signal of user $i$ which depends on the number of packets transmitted $(u_{i,n})$ in time slot $n$, and $P_{i,n}$ is the transmitted power for user $i$ in time slot $n$.

**Achievable region:** Given average arrival rates $R_i$, and average delay constraints $D_i$ for user $i$, $i = 1, 2, \ldots, K$, the achievable region, $\mathcal{P}$, is defined as the set of all $K$-tuples $(P_1, P_2, \ldots, P_K)$ where $P_i = E_n(P_{i,n})$ is the average transmitted power, such that reliable transmission is feasible under the given rate and delay constraints.\(\Box\)

Our objective is to characterize $\mathcal{P}$ and the transmission policies which achieve the boundary of $\mathcal{P}$. We will assume $K = 2$ to simplify the subsequent treatment. The class of policies considered choose the number of packets $(u_{1,n}, u_{2,n})$ to be transmitted in slot $n$ at powers $(P_{1,n}, P_{2,n})$ based on the queue states $(x_{1,n}, x_{2,n})$. Our main result is based on the following observation.

**Observation:** For given constant rate arrivals for the two users, the boundary of the capacity region in the power plane is piecewise linear. For (1), the capacity region is given by \(10\)

$$P_1 > NW \left( \exp \left( \frac{R_1}{W} \right) - 1 \right) \quad (2)$$

$$P_2 > NW \left( \exp \left( \frac{R_2}{W} \right) - 1 \right) \quad (3)$$

$$P_1 + P_2 > NW \left( \exp \left( \frac{R_1 + R_2}{W} \right) - 1 \right). \quad (4)$$

The achievable region is just a restatement of the well known multiple access channel capacity region \(10\) which gives the set of achievable rates $(R_1, R_2)$ for a given power $(P_1, P_2)$. Similarly, when users have a finite delay constraint and bursty arrivals, the achievable region $\mathcal{P}$ is characterized in Theorem 1. We note that $\mathcal{P}$ is convex and piecewise linear. The proof is based on a time sharing argument.

**Theorem 1 (Two user delay limited power region)** Consider the model in (1) with two users. Let the buffer sizes at the two users be $L_1$ and $L_2$ packets and the corresponding input process have no more than $M_1$ and $M_2$ packet arrivals, respectively, in each time slot. Consider a class of coordinated schedulers with the knowledge of the system parameters (queue lengths, channel states) available to both users and achieve zero channel outage and zero buffer overflow. For any given pair of average delay constraints $(D_1, D_2)$, the set of all achievable average powers $\mathcal{P}$ is given by

$$P_1 > p_1^* \quad (5)$$

$$P_2 > p_2^* \quad (6)$$

$$P_1 + P_2 > p^* \quad (7)$$

where $p^*, p_1^*, p_2^*$ depend on $(D_1, D_2)$ and the distributions of the two input processes. The average delays and powers at user $i$ are denoted by $D_i$ and $P_i$.

**Sketch of Proof:** At each time slot, the coordinated scheduler transmits $r_1$ and $r_2$ packets from user 1 and 2 respectively, to the central receiver. Let $\Theta$ represent the set of all pairs of transmitted rates $r = (r_1, r_2)$. Each $r$ is transmitted with some probability $\alpha_r$ depending on $(D_1, D_2)$ and the distributions of the two input processes.

To find the boundary, let us assume that at each rate $r$, we transmit at powers so that $P_1 + P_2 = NW \left( \exp \left( \frac{r_1 + r_2}{W} \right) - 1 \right)$. Now, time sharing between the policies, the total average powers satisfy

$$P_1 + P_2 = \sum_{r \in \Theta} NW \left( \exp \left( \frac{r_1 + r_2}{W} \right) - 1 \right) \alpha_r = p^*. \quad (8)$$

It follows in a straightforward manner that the achievable region under average delay constraint $(D_1, D_2)$ is given by (7) where $p_1^*$ is the average power required to maintain average delay $D_1$, if user $i$ was operating in a single user channel without multiple access interference.\(\Box\)

Although in practical systems, information about a user’s buffer is not available at all other users, Theorem 1 assumes that this information is available to find absolute limits on the power requirements. The ultimate goal is to find decentralized schedulers that approach the performance of the centralized scheduler.

A consequence of Theorem 1 is that to calculate $\mathcal{P}$, we only need to find the two vertex points corresponding to lowest possible powers $p_1^*$ and $p_2^*$ respectively. We state without proof that the extreme vertex points of the achievable region for any finite delay can be computed using the following steps.

1. Set $u_1(x_1, x_2) = u_1(x_1)$ and evaluate $u_1(x_1)$ using the single user scheduling approach \(5\) to minimize power $P_1$ under average delay constraint $D_1$.

2. Using this scheduler action for user $i$, i.e., $u_1(x_1, x_2)$ we can find the scheduler action for the other user to minimize its own transmit power using the Value Iteration Algorithm \(11\).
The basic idea used in our solution relies on the observation that if one user transmits the same power in a multiple access environment as it would in a single user channel, then the other users will have to transmit at much higher powers than their respective single user Shannon limits. This ability to tradeoff the powers between the users is what we refer to as multiaccess power exchange. In addition, additional delay for a user leads to possible power reduction for all the users.

In our analysis, we have assumed that all packets that arrive during any time slot, are available for transmission at the beginning of the next time slot. Thus the lowest delay that can be achieved for any user is 1, and in this case the scheduler action is straightforward: the scheduler transmits all the packets that arrive in time slot $n$ during time slot $n+1$. This is used as a baseline case to compare the impact of delay in our results.

An illustrative two-user capacity region is given in Figure 2. Each user receives either 0 or $M_i$ packets in a time slot with equal probability, where $M_1 = M_2 = 4$. The capacity region for a delay pair is given by all the power tuples above the corresponding boundaries. The dotted line is the boundary when both users have delays of 1 time-slot (the baseline case), while dashed line is the boundary for delays of 2 time-slots. The lowest solid straight line represents the Shannon limit in Equations (2)-(4). It is clear from the figure that increasing the delay from 1 to 2 time slots, achieves substantial reduction in the average powers required. Also, points on the boundary correspond to different power tuples for a fixed average rate and delay constraint, verifying the multiuser power exchange effect.

III. CONCLUSIONS

The importance of uplink scheduling is in realizing that by suitable coordination among the users, one of the users could get lower powers at the cost of increasing the other users power. Clearly, any users power cannot be reduced below what is needed to maintain successful communication at that rate and delay in a single user environment. However, the single user power value can only be achieved for a particular user at the cost of increasing the other users powers. This could be of potential benefit in situations where one of the mobiles could have access to a higher power source (e.g., in cars). Of course this leads to issues about fairness which need to be addressed. A problem of immediate interest is to find optimal distributed schedulers, where each user does not have access to the state of the other users queue.

REFERENCES