Spreading and Power Control for Multiple Antenna Transmit Diversity

Dinesh Rajan

Department of Electrical and Computer Engineering
Introduction

Multiple Access Channel
- different users collide

Signaling Scheme

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDMAM</td>
<td>FDMA</td>
</tr>
</tbody>
</table>

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CDMA

chip

$T_c$

User 1

User 2

$T$

bit period

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Motivation

Multipath Fading problem in wireless channel

Diversity controlled ‘multipath’ to overcome fading
Diversity

- temporal redundancy
  - coding
- spectral diversity
  - multiple carriers
- spatial redundancy
  - multiple receivers / transmitters
Transmission Scheme - Multiple Antennas

Tx signals $\rightarrow$ Antenna 1 $\rightarrow$ Channel 1 $\rightarrow$ noise $\rightarrow$ Channel 2 $\rightarrow$ + $\rightarrow$ received signal

$y_k = h_{1,k} x_{1,k} + h_{2,k} x_{2,k} + n_k$

$x_{1,k}$, $x_{2,k}$ are transmitted signals on antennas 1 and 2

$n_k$ is the additive white Gaussian noise

$y_k$ is the signal received at mobile at time $k$
Transmit Antenna Diversity

- orthogonal time (TSTD)
- orthogonal code (OTD)
- optimal antenna selection (STD)
Capacity and Outage

\[ C_{\text{single}} = \frac{1}{2} E \left[ \log \left( 1 + |A_1|^2 \text{SNR} \right) \right] \]

\[ C_{\text{STD}} = \frac{1}{2} E \left[ \log \left( 1 + \max \{ |A_1|^2, |A_2|^2 \} \text{SNR} \right) \right] \]

\[ P_{\text{single}} = \Pr \left( C_{\text{single}} < R \right) \]

\( A_1, A_2 \): channels between 2 transmit antenna and mobile
Increase in capacity with increasing number of antennas

- $M = 1$
- $M = 2$
- $M = 4$
- $M = 8$

Capacity

orthogonal
time/code

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Probability of Outage

![Graph showing the probability of outage for different antenna systems (single antenna, OTD, STD) against rate. The graph indicates that the probability of outage increases with the rate for all systems, with the single antenna system having the highest probability and the STD system having the lowest.](graph.png)
Selection Diversity

- choose best antenna available
- use feedback information from mobile
- throughput constrained to $1 \text{ bit/transmission}$
- number of spreading codes is fixed
- need scheme to transmit on both antennas simultaneously
New Transmission Scheme

- Gold codes for spreading
- Shifted Gold codes on multiple antennas
  - a b c d e f g h  user 1, antenna 1
  - g h a b c d e f  user 1, antenna 2
- Throughput limit: 2 bits/transmission
- No feedback information
Performance measures

- Probability of bit error: $P_e$
- 1 bit high accuracy vs 2 bits lower accuracy
- Throughput
  - Single antenna: $C_{\text{single}} = 1 - H(P_e)$
  - New scheme: $C_{\text{new}} = 2 - H(P_{e1}) - H(P_{e2})$
  - Maximal choosing: $C_{\text{mc}} = 1 - H(P_{e-mc})$
- Probability of outage
  - $\Pr(C_{\text{single}}) < R$
### Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>no of users</td>
<td>5</td>
</tr>
<tr>
<td>detector</td>
<td>Decorrelator</td>
</tr>
<tr>
<td>spreading code length</td>
<td>31</td>
</tr>
<tr>
<td>fading</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>correlation between antennas</td>
<td>0, 0.75</td>
</tr>
<tr>
<td>channel coding</td>
<td>no</td>
</tr>
</tbody>
</table>
Throughput

SNR

Single Antenna
New Scheme
Maximal Choosing

Throughput

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6

0 5 10 15 20 25

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6

0 5 10 15 20 25
Throughput - correlated antennas

The graph shows the throughput vs. SNR for different schemes:
- Single Antenna
- New Scheme
- Maximal Choosing

As SNR increases, the throughput for all schemes also increases, with the Maximal Choosing scheme consistently outperforming the others.
Probability of outage

![Graph showing probability of outage vs. SNR for different schemes: Single Antenna, New Scheme, Maximal Choosing - no error, Maximal Choosing - 10% error. The graph illustrates how the probability of outage decreases with increasing SNR for each scheme.](image-url)
Coding Scheme

- single antenna/maximal selection
  - no coding
  - R bits/transmission
- new scheme
  - rate 1/2 convolutional coding
  - 2R bits/transmission
Probability of Bit Error

- Single Antenna
- New Scheme
- Maximal Choosing - no error
- Maximal Choosing - 10% error

SNR (dB) vs. Probability of bit error
Optimum Scheme?

- power control perspective
  - maximal choosing $[P,0]$ or $[0,P]$
  - new scheme $[P/2,P/2]$
- 1 bit feedback
  - joint antenna selection and power control
- optimum at
  - low SNR maximal choosing
  - high SNR new scheme
Optimization

Throughput

\[
\max_{0 < \alpha < 1} \quad \mathbb{E} \left[ 2 - H \left\{ Q \left( \frac{A_1 \sqrt{\alpha \sqrt{\text{SNR}}}}{\sqrt{R^{-1}_{11}}} \right) \right\} - H \left\{ Q \left( \frac{A_2 \sqrt{1 - \alpha \sqrt{\text{SNR}}}}{\sqrt{R^{-1}_{22}}} \right) \right\} \right]
\]

given \ \text{SNR}, \ \text{sign} (A_1 - A_2)

A_1, A_2: channel between 2 transmit antenna and mobile

R: code correlation matrix

H: binary entropy function

E: expectation over fading process
Throughput

orthogonal codes
uncorrelated antennas
Optimal Power Distribution

Orthogonal codes uncorrelated antennas

maximizing alpha

SNR

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Throughput - optimal scheme

![Graph showing throughput vs. S/N ratio with lines for optimal alpha, equal power, and maximal choosing. The graph indicates that orthogonal codes and uncorrelated antennas are optimal.]
Throughput

non-orthogonal codes
uncorrelated antennas
Optimal Power Distribution

Optimal Power Distribution for non-orthogonal codes and uncorrelated antennas.
Throughput - optimal scheme

non-orthogonal codes
uncorrelated antennas
Optimal Power Distribution

non-orthogonal codes antennas corr = 0.75
Throughput - optimal scheme

non-orthogonal codes
antennas corr = 0.75
Outage Probabilities

- non-orthogonal codes
- uncorrelated antennas

Rate 0.8

SNR

Probability of outage

optimal alpha
equal power
Outage Probabilities

non-orthogonal codes
antennas correlation = 0.75

Rate 0.8
Extensions

- space time signaling scheme
- space time trellis coding and modulation
- single user, multi antenna decoder?
- dynamic power control
Decorrelator

\[ y = RAb + z \]
\[ \hat{b} = R^{-1}y \]

\[ R = \begin{bmatrix} s_1 & s_1 & s_2 & \ldots & s_1 & s_N \\ s_2 & s_1 & s_2 & \ldots & s_2 & s_N \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ s_{N} & s_{N} & s_{2} & \ldots & s_{N} & s_{N} \end{bmatrix} \]

- code correlation matrix
- \( s_i \) : spreading code of user \( i \)

- \( b \) : actual user bits transmitted
- \( y \) : received vector after chip matched filtering
- \( A \) : channel amplitudes
- \( z \) : noise
Capacity

The graph shows the capacity as a function of SNR (Signal-to-Noise Ratio) for both single antenna and maximal choosing scenarios.

- **Y-axis (Capacity):** The capacity is plotted on a logarithmic scale, ranging from 0.4 to 2.0.
- **X-axis (SNR):** The SNR is plotted from 0 to 10.

Two curves are depicted:
- The blue solid line represents the capacity with a single antenna.
- The blue dotted line represents the capacity with maximal choosing.

Both curves ascend as the SNR increases, indicating an increase in capacity with an increase in SNR.
Probability of Outage

![Graph showing the probability of outage versus SNR for different antenna configurations: single antenna, OTD, and STD.]
Results - Effect of Error

Effect of error on optimal antenna selection

- Single antenna
- Orthogonal code
- Optimal selection

Probability of Outage vs. Rate for different error percentages:
- Error = 0%
- Error = 20%
- Error = 40%
- Error = 60%
Throughput

non-orthogonal codes
uncorrelated antennas
Throughput

non-orthogonal codes
antennas corr = 0.75