OPTIMALLY WEIGHTED HIGHPASS FILTERS USING MULTISCALE ANALYSIS

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ABSTRACT

In this paper, we propose a general framework for studying a class of weighted highpass filters. Our framework, based on a multiscale signal decomposition, allows us to study a wide class of filters and to assess the merits of each. We derive an automatic procedure to optimally tune a filter to the local structure of the image under consideration. The entire algorithm is fully automatic and requires no parameter specification from the user. Several simulations demonstrate the efficacy of the proposed algorithm.

I. INTRODUCTION

Visual feature recognition is critical to human existence. For instance, blurred vision in infants may inhibit bonding between mother and child [7, pp. 159–160]. Recognition of image features depends on the local level and contrast near the feature. One of the primary steps in recognition is *edge* or *boundary extraction.* To aid in this task it is often desirable to enhance the edges using a highpass filtering scheme. Unfortunately, high pass filtering also amplifies noise present in the image.

The local intensity affects the eye's sensitivity to noise in images. Specifically, the visual system is much less sensitive to noise in bright areas of an image than it is in dark areas. This observation is commonly referred to as Weber's Law [2]. An obvious approach to image enhancement is to sharpen bright regions of an image more than darker regions. One very simple method to accomplish this is to weight the amount of highpass filtering proportional to the local mean. This gives rise to a class of nonlinear image enhancement filters known as *mean-weighted highpass filters* [5, 10].

Empirical evidence also suggests that the visual system is less sensitive to noise in the edges or highly structured regions of the image. This effect, called *masking* [8], has led to nonlinear *edge-weighted highpass filters* [1, 9].

One limitation of existing weighted highpass filters is that the filter structure is fixed. This means that the *scale* of the local mean or edge detector is fixed. Hence, the user must specify a local neighborhood for the mean or explicitly define what is meant by a local edge. Also, these algorithms typically require user specified weighting parameters and often threshold the nonlinear highpass image in an ad hoc fashion.

In this paper, we propose a general framework for studying the class of weighted highpass filters based on the multiscale signal decomposition of Mallat and Zhong [4]. The paper is organized as follows. In Section II, we review previous work on weighted highpass filters and discuss some of the limitations of existing methods. We also give a brief review of multiscale analysis. Section III introduces a novel weighted highpass filter based on multiscale analysis. Several simulations demonstrate the efficacy of the proposed filter in Section IV. Conclusions are drawn in Section V.

II. PREVIOUS WORK

A standard method of image enhancement is unsharp masking [2]. In unsharp masking, the original image is modified by subtracting a signal proportional to a smoother version of the original image. Equivalently, a signal proportional to a highpass filtered version of the original image can be added to the original. Let H denote a linear highpass filter, let $f(x, y)$ denote the image at hand, and consider

$$g = f + Hf. \tag{1}$$

Adding the highpass filtered image to the original enhances or emphasizes edges and structure in the image. Alternatively, suppose we have a blurred image f and a linear restoration filter R. We may consider the difference between f and the restored Rf as a highpass filter, that is, $Hf = Rf - f$. With this notation, linear deblurring can also be viewed as a form of unsharp masking as in (1).

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A. Weighted Highpass Filters

The enhanced or restored image g may be undesirable if noise in the original image f is amplified by H. Weber's Law and the masking effect suggest the following nonlinear approach to image enhancement. Let L denote a linear filter that is tuned to a specific type of local image feature. By "local" we mean that the output image Lf at the point (x, y) depends only on the local neighborhood of f about (x, y). By "tuned" we mean that $|Lf(x, y)|$ is large if a local image feature such as an edge or region of high intensity (high local mean) is near (x, y) in f. A weighted highpass filter is defined by

$$H_W f(x, y) = |Lf(x, y)|^p Hf(x, y), \quad p \geq 1. \tag{2}$$

The image $|Lf|^p$ "weights" the highpass filtered image Hf pointwise according to the strength of the local features associated with L. For example, if L corresponds to a local mean, then $H_W f$ is roughly proportional the output image obtained by applying H only in regions with high local mean [5, 10]. If L is a local edge-detector, then $H_W f$ is proportional to the output image obtained by applying H in regions where an edge is detected [1, 9].

B. Limitations of Previous Work

One important drawback to the mean-weighted and edge-weighted filters previously studied in [1, 5, 9, 10] is that the filter structure is fixed. Hence, such filters may only be appropriate for image structure at a fixed scale. Our idea is to wed the ideas of multiscale analysis and weighted highpass filters to produce an adaptive filter that automatically adjusts the filter to the local structure of the image at hand. Before discussing our method, we briefly review the multiscale analysis of images.

C. Signal Characterization Using Multiscale Edges

The notion of multiscale signal analysis is motivated by the need to detect and characterize the edges of small and large objects alike. In an image, different structures give rise to edges at varying scales — small scales correspond to fine detail and large scales correspond to gross structure. In order to detect all image edges, one must study the image at each scale. Mallat and Zhong use the scales of the wavelet transform to characterize the important edges in an image [4]. The wavelet ψ and smoothing function ϕ proposed in [4] and used throughout this paper are depicted in Figure 1.

Smoothed versions of the image f are obtained by convolution with ϕ in both x and y directions. Larger scales (smoother images) are obtained by dilating ϕ. Because we are working with discretized images, we have a discretized set of scales. The effective resolution is halved each time as we move up through scales, producing the discrete range of scales $1, 2, \ldots, 2^j, \ldots, 2^N$, where 2^N is the dimension of the image. At scale 1 we have the original image, and at scale 2^N we have a constant image equal to average pixel value. We denote the smooth image at scale 2^j by $S_{2^j} f$.

Edge and detail information in f is obtained by convolution with ψ. Detail information at larger scales is obtained by dilating ψ. At scale 2^j we have three detail images: $W^h_{2^j} f$, $W^v_{2^j} f$, and $W^d_{2^j} f$, where the superscripts h, v, and d denote the horizontal, vertical, and diagonal (=horizontal+vertical) applications of ψ, respectively.

In [4] it is shown that the modulus maxima of the wavelet transform provide a nearly complete characterization of an image. Mallat and Zhong characterize the image edges at scale 2^j by the maxima of

$$M_{2^j} f(x, y) = \sqrt{|W^h_{2^j} f(x, y)|^2 + |W^v_{2^j} f(x, y)|^2}. \tag{3}$$

In the next section, we use the local edge and mean information carried by the smooth and detail images at various scales to formulate a general class of weighted highpass filters.

III. OPTIMALLY WEIGHTED HIGHPASS FILTERS

Our algorithms utilize local edge and local mean information at different scales. What is a good definition of "locality" differs from image to image. Our goal is to choose the best scale or scales for the weighted highpass filter.

A. Multiscale Mean-Weighted Filters

We can easily formulate the mean-weighted highpass filter in the multiscale framework. Pointwise multiplication of the highpass image with $|S_{2^j} f|^p$

$$|S_{2^j} f|^p \, Hf \tag{4}$$

yields a $p+1$st order weighted highpass filter that produces the strongest response in regions where the local mean (at the scale 2^j) is large. Adjusting the scale 2^j is equivalent to adjusting the size of the local neighborhood used to compute the mean. We thus have the following collection of mean-weighted highpass filters

$$\{\, |S_{2^j} f|^p \, Hf \; : \; j = 1, \ldots, J, \; p = 1, \ldots, P \,\}. \tag{5}$$

The exponent p controls the relative weighting in light and dark regions; increasing p tends to emphasize areas of peak intensity.

B. Multiscale Edge-Weighted Filters

We define the detail modulus as

$$|D_{2^j} f(x,y)| = \sqrt{|W^h_{2^j} f(x,y)|^2 + |W^v_{2^j} f(x,y)|^2 + |W^d_{2^j} f(x,y)|^2}. \quad (6)$$

Our experiments have shown that $|D_{2^j} f|$ provides better results for our application than $M_{2^j} f$, possibly because it treats edges at different orientations more fairly. Pointwise multiplication of the highpass image with $|D_{2^j}|^p$ produces a $p+1$-order weighted highpass filter tuned to edges at the scale 2^j — an edge-weighted highpass filter. The multiscale analysis produces a set of edge-weighted highpass filters; each is tuned to edges at a prescribed scale:

$$\{\, |D_{2^j} f|^p \, Hf \; : \; j = 1, \ldots, J, \; p = 1, \ldots, P \,\}. \quad (7)$$

Increasing the exponent p tends to localize the weighting to areas where the detail image has local maxima.

C. Optimal Filter Design

Multiscale analysis provides a suite of weighted highpass filters, (5) and (7), suitable for image enhancement. The question now becomes: Which one is best for a given image? Even more generally we may consider the collection of filters

$$\mathcal{C} = \left\{ \sum_{j=1}^{J} \sum_{p=1}^{P} \alpha_{j,p} |D_{2^j} f|^p \, Hf + \beta_{j,p} |S_{2^j} f|^p \, Hf \right\}, \quad (8)$$

with arbitrary real coefficients $\{\alpha_{j,p}, \beta_{j,p}\}$. The collection $\mathcal{C}$ is simply the subspace of nonlinear highpass filters spanned by (5) and (7).

We now propose an automatic procedure for choosing the best filter in $\mathcal{C}$ for a given image. The idea is very straightforward. By design, all the filters in (5) or (7) suppress undesirable effects, such as noise amplification in uniform or low intensity regions of the image. However, not all filters treat actual image structure the same, as each is tuned to structure at a different scale. We find the optimally weighted highpass filter in $\mathcal{C}$ that minimizes the error between the linear highpass image and the weighted highpass image. The design of the weighted highpass image prevents it from matching the noise that is amplified in the linear highpass image. However, the weighted highpass image can match the enhanced image detail in the linear highpass image by properly adjusting the filter parameters $\{\alpha_{j,p}, \beta_{j,p}\}$. Mathematically we have

$$H_{\text{opt}} = \arg\min_{N \in \mathcal{C}} \|Nf - Hf\|_F^2, \quad (9)$$

where we have chosen the Frobenius norm for computational convenience.

The optimal filter H_{opt} is unique and can be computed in a simple fashion. First let

$$\mathbf{d}_{j,p} = \text{vec}(|D_{2^j} f|^p \, Hf), \quad (10)$$

$$\mathbf{s}_{j,p} = \text{vec}(|S_{2^j} f|^p \, Hf), \quad (11)$$

$$\mathbf{h} = \text{vec}(H\, f), \quad (12)$$

where vec is the operator that forms a column vector from a matrix by stacking its columns. Since the Frobenius norm coincides with the vector 2-norm, (9) can be rewritten as

$$H_{\text{opt}} = \arg\min_{N \in \mathcal{C}} \left\| \sum_{j=1}^{J} \sum_{p=1}^{P} \alpha_{j,p} \mathbf{d}_{j,p} + \beta_{j,p} \mathbf{s}_{j,p} - \mathbf{h} \right\|_2^2. \quad (13)$$

It is clear that the optimal filter is specified by the $2JP$ parameters $\{\alpha_{j,p}\}$ and $\{\beta_{j,p}\}$. Now define the matrix

$$\mathbf{X} = [\mathbf{d}_{1,1}, \ldots, \mathbf{d}_{J,P}, \mathbf{s}_{1,1}, \ldots, \mathbf{s}_{J,P}] \quad (14)$$

and the parameter vector

$$\gamma = [\alpha_{1,1}, \ldots, \alpha_{J,P}, \beta_{1,1}, \ldots, \beta_{J,P}]. \quad (15)$$

The optimal filter parameters are given by

$$\begin{aligned} \gamma_{\text{opt}} &= \arg\min_{\gamma \in \mathbb{R}^{2JP}} \|\mathbf{X}\gamma - \mathbf{h}\|_2^2 \\ &= (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{h}. \end{aligned} \quad (16)$$

The optimal filter in vectorized form is

$$\mathbf{h}_{\text{opt}} = \mathbf{X}\gamma_{\text{opt}}. \quad (17)$$

Note that by restricting the largest admissible scale J, we put an upper limit on the scale of local feature detection. This is most important in mean-weighted filters since at the highest possible scale the smooth image is simply a constant value, the mean of the image. At this extreme, the mean-weighted filter is proportional to the linear highpass filter and the optimal filter is returned as the linear highpass filter itself. Moreover, we may pose the optimization over any subspace spanned by a subset of the edge-weighted and/or mean-weighted filters.

IV. SIMULATIONS

In this section we present several examples to illustrate the performance and flexibility of our method.

A. Edge-Weighted Restoration

In this example, we consider a blurred and noisy image of one of the authors, denoted as f and pictured in Figure 2(a). The transfer function of the blur is assumed known, and hence the optimal linear inverse filter R is readily computed. A small amount of Gaussian white noise has been added to the blurred image. The highpass filter in this case is $Hf = Rf - f$, the difference between the blurred image and the linear restoration. The linearly restored image $f + Hf$ is shown in Figure 2(b).

The space of edge-weighted highpass filters we consider here is

$$\text{Span}\{\, |D_{2^j} f|^2\, Hf,\ |D_{2^j} f|^4\, Hf \,\}_{j=1}^{4}. \qquad (18)$$

The optimal filter parameters are

$$\gamma_{\text{opt}} = 0.0036 \times [1.00, 0.00, -0.17, 0.00, -0.06, 0.00, 0.26, 0.00]^T, \quad (19)$$

where the ordering is $(|D_{2^j} f|^2\, Hf,\ |D_{2^j} f|^4\, Hf,\ j = 1, \ldots, 4)$. The nonlinear restoration using the optimal filter above is pictured in Figure 2(c). For comparison, we show the original image in Figure 2(d). Since the true image is available in our simulation, we may also consider the squared error between the restorations and the original. In this case, the linear restoration squared error (normalized) is 1 and the optimally weighted restoration squared error is 0.82.

B. Edge-Weighted Enhancement

We now consider two examples of image enhancement. The key feature in both examples is that the images are processed by exactly the same optimally weighted highpass filter algorithm — with no tweaking of parameters to handle the drastically differing image structures. The original images are shown in Figure 3(a), bridge image, and Figure 4(a), PET (Positron Emission Tomography) brain image.[1] Both images are enhanced using a linear highpass filter H whose convolution mask is given by

$$\mathbf{H} = \begin{bmatrix} -0.5 & 0 & -0.5 \\ 0 & 2 & 0 \\ -0.5 & 0 & -0.5 \end{bmatrix}. \qquad (20)$$

The enhanced images, shown in Figures 3(b) and 4(b), are computed by $g = f + Hf$.

The space of edge-weighted highpass filters considered in this case is

$$\text{Span}\{\, |D_{2^j} f|\, Hf \;:\; j = 1, \ldots, 4 \,\}. \qquad (21)$$

The optimally weighted filter parameters for the bridge image are

$$\gamma_{\text{opt}} = 0.033 \times [1.00, -0.25, -0.02, 0.45]^T, \qquad (22)$$

where the ordering is $(|D_{2^j} f|\, Hf,\ j = 1, \ldots, 4)$. The optimal nonlinear enhancement of the bridge is shown in Figure 3(c).

The optimally weighted filter parameters for the PET image are

$$\gamma_{\text{opt}} = 0.057 \times [1.00, -0.80, 0.19, 0.60]^T. \qquad (23)$$

The optimal nonlinear enhancement of the PET image is shown in Figure 4(c).

Note that the optimal filters are quite different for the two images. However, in both cases the resulting nonlinear filter enhances the detail of the image while reducing the noise amplified by the linear highpass filter.

V. CONCLUSIONS

We have developed a family of optimally weighted highpass filters based on multiscale analysis. Two significant features distinguish our method from previous work. First, the filters do not have a fixed form like previously proposed filters. Therefore, the filters are capable of matching the structure of the image at hand. Secondly, the design of the optimal filter is fully automatic. Previously proposed filters often required user specified parameters and/or ad hoc thresholding schemes. Simulations have demonstrated that the proposed filter provides very good results for a wide variety of images with differing local structure.

There are many possible avenues for future work in this area. For example, spatial adaptation of the parameters within the image itself — as in the Jones Adaptive Multiplexer (JAM) [3] — will better match the local image structure. A related idea is considered in [6] to improve the performance of the weighted highpass filter proposed in [1]. One may also be interested in the performance using multiscale analyses other than that of [4]. It may be advantageous to decompose the linear highpass filter H at different scales as well. Furthermore, the adaptively weighted highpass filters belong to the class of nonlinear filters known as *Volterra filters*. The theory of Volterra filters should provide insight into the analysis, implementation, and design of nonlinear enhancement filters. On a final, more ambitious note, optimally weighted highpass filters could provide a plausible model for studying masking phenomena in the human visual system.

[1] Courtesy of Col. Brian W. Murphy, Director of Computing Operations, Center For Positron Emission Tomography, Department of Nuclear Medicine, School Of Medicine & Biomedical Sciences, State University of New York at Buffalo.

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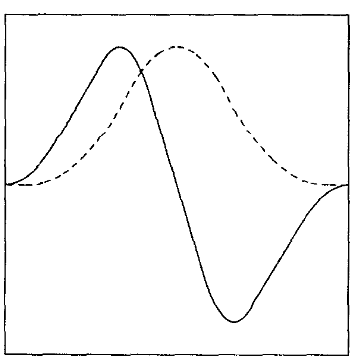
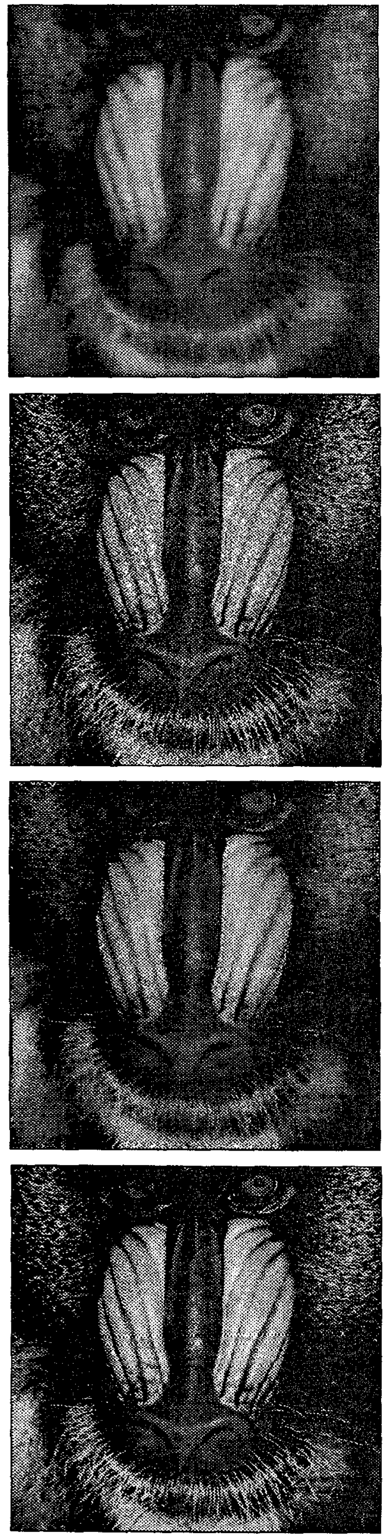
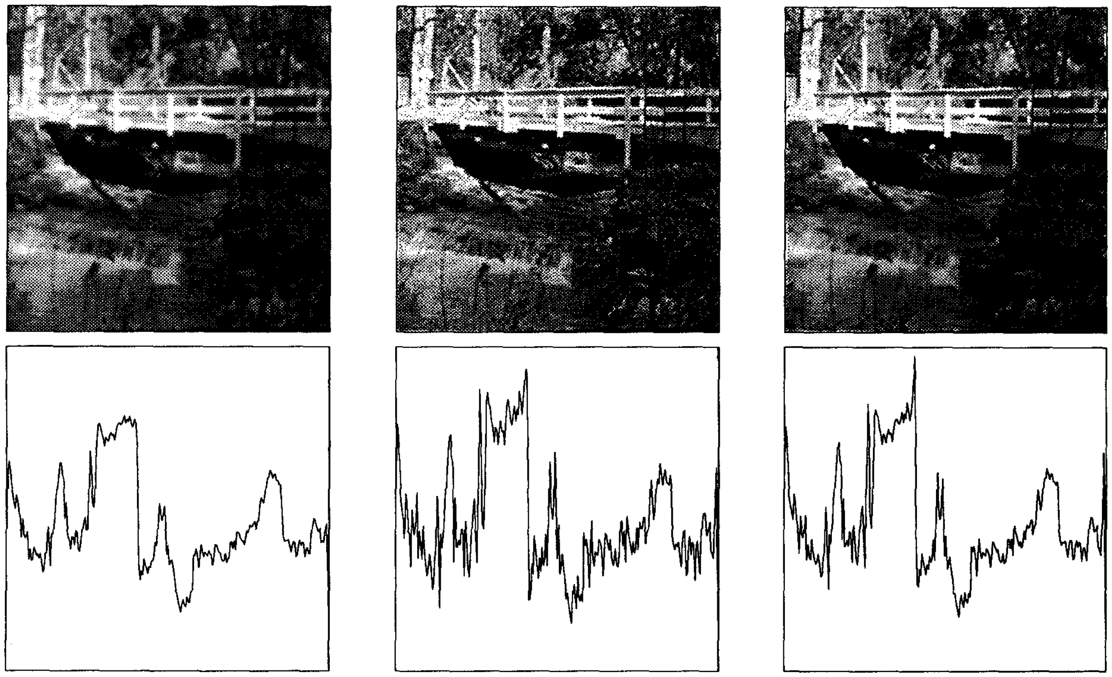


Figure 1: *Smoothing function ϕ (dashed) and wavelet ψ (solid) employed in the multiscale decomposition.*



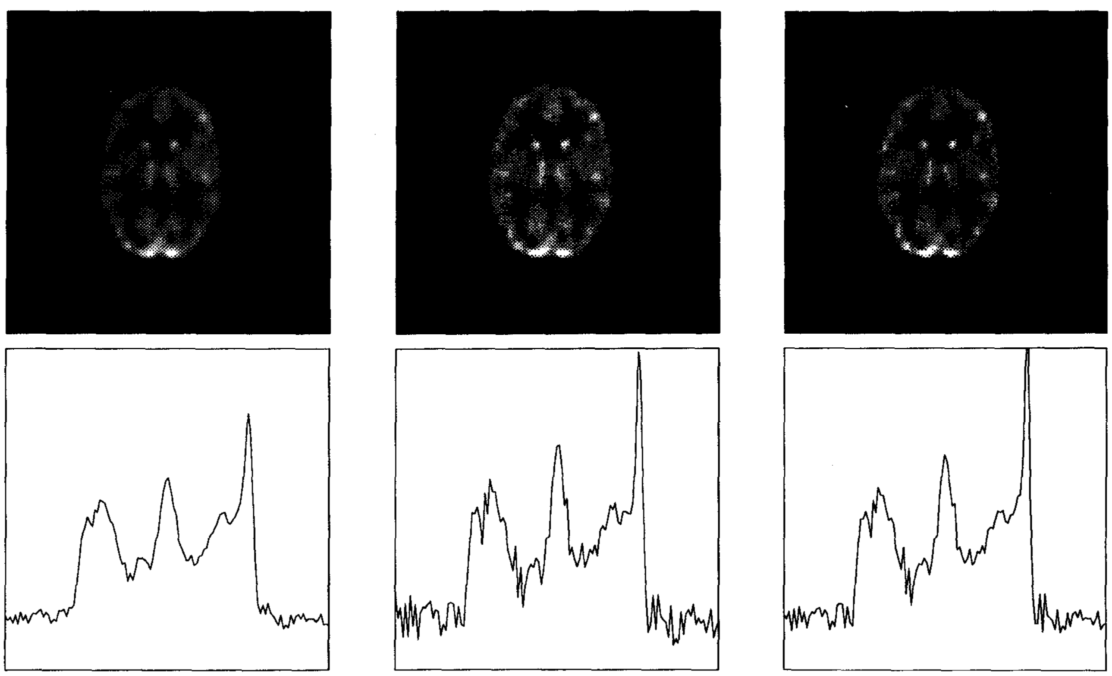
(a) (b) (c) (d)

Figure 2: *Optimally weighted image restoration. (a) Blurred, noisy image. (b) Image restored using linear highpass filter. (c) Image restored using optimal edge-weighted highpass filter. (d) Original image for comparison.*



(a) (b) (c)

Figure 3: *Optimally weighted enhancement. (a) Original image. (b) Image enhanced using linear highpass filter. (c) Image enhanced using optimal edge-weighted highpass filter. Above, we show the image; below, we show a vertical cross-section through the center of the image.*



(a) (b) (c)

Figure 4: *Optimally weighted enhancement. (a) Original image (PET reconstruction). (b) Image enhanced using linear highpass filter. (c) Image enhanced using optimal edge-weighted highpass filter. Note that the nonlinear filtering algorithm employed here is identical to that used in Figure 3.*