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REDUCED RANK ALGORITHMS FOR WIRELESS SPACE-TIME CHANNELS

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ABSTRACT

The estimation of channel coefficients constitutes a major function of the receiver. It is also an important step in the signal detection process at the receiver, in a wireless communication system. To describe the space-time channel from multiple transmitters to a single receiver, many parameters are required. This means that in order to perform channel estimation and signal detection, the order *P* say, of the required filter will be very large. Since the channel is time varying thus the filter adaptive, a large *P* implies a slow response to changing channel conditions. This is one motivation for reduced-rank adaptive filtering. In this project, several aspects of space-time channel parameter estimation, and signal detection, are examined. Three methods have been studied. A maximum likelihood channel estimation method and two different subspace-based, reduced-rank methods - the "cross-spectral" method and the "residual correlations" method.

1. INTRODUCTION

Recent trends in wireless communications feature a rapid growth in the number of wireless subscribers. The projection is that there will be about 300 million mobile handsets sold, by the year 2002. Also where previously, voice traffic constituted the dominant traffic over the wireless communication channel, recent trends show a rapid increase in the volume of multimedia traffic over a wireless channel. However, available channel bandwidth is always a limited resource, as service providers are allocated a fixed bandwidth by the FCC. Therefore the need arises for new techniques for efficient use of available bandwidth, increased system capacity and reliability.

One way in which to meet these demands would be to exploit the inherent diversity in a wireless channel. Diversity is a phenomenon whereby an independently faded replica of the transmitted signal is provided to the receiver. While there are several forms of diversity [19] (eg. temporal and frequency), a space-time channel achieves yet another form of diversity - antenna diversity, by employing multiple antennas both at the receiver and at the transmitter. To illustrate antenna diversity [22], say two antennas are seperated by some specified distance, or are differentially polarized, then one may receive a null, while the other receives a strong signal. By selecting the best signal at all times, the receiver can mitigate small-scale fading effects.

Previous work has already shown that multiple antennas at the receiver improves the reliability of the channel. However antenna diversity at the mobile handset is difficult to achieve for several reasons, one of which is the electromagnetic interaction of antenna elements on small platforms. More recently, the tendency is towards the use of multiple transmit antennas at the base station. The space-time channel considered in this project comprises multiple transmit antennas at the base station, and a single receive antenna at the mobile. Typically, a mobile handset would have more than one antenna at the receiver, and the reduced-rank methods studied in this project can be extended to a multiple receive antenna case.

1.1 REDUCED-RANK FILTERING

Reduced-rank filtering methods have been proposed for array processing and radar applications, and more recently, for linear inteference suppression in DS-CDMA communications systems. The goal of reduced-rank adaptive filtering is to find a lower dimensional filter that yields a steady-state performance that is as close as possible to that obtained by the full-rank solution. There are several motivations for this. First, the problem under consideration could be overmodelled. Second, it could be required that the order of the adaptive filter be some specific value, due to complexity or real-time implementation requirements. Finally the least squares class of algorithms converge as a function of the filter order, hence lower rank filters converge faster.

Much of the previous work in reduced-rank filtering has been directed towards overcoming the overmodelling problem. If the full rank problem is defined to be of dimension *P*, and *D* is the dimension of the reduced-rank equivalent, then previous schemes required that *D* be at least equal to the dimension of the signal subspace. First, an estimate of the covariance matrix of the observed data is obtained, then its singular value decomposition is obtained. The eigenvalues corresponding to the *D* largest singular values are then retained to form the rank *D* eigensubspace in which the reduced rank filter will operate. In the "principal components" method for instance, the received vector is projected onto this lower dimensional estimate of the signal subspace. This method is very effective if the dimension of signal subspace is known. If *D* is less than this dimension, the algorithm suffers greatly.

In this project we examine the problem of reduced-rank channel estimation in a space-time channel comprising multiple transmit antennas, a single receiver, multiple paths, and a single user. We assume the number of multipaths is *P*. For simplicity, we ignore the response of the receiving antenna to the different directions of arrival of the multiple paths. It is also assumed that all the transmit antennas send the same message simultaneously, such that "in effect", one symbol is being sent at any time, with some diversity gain.

We begin by obtaining the maximum likelihood estimate of the channel impulse response. Then we look at the full-rank LS solution for *w* [4], the weights of the adaptive filter.

The two reduced-rank methods studied do not require that the dimension of the projected subspace be greater than the signal subspace. They are the "cross-spectral" method [2], and the "residual-correlations" method [1]. In the cross-spectral method, an estimate of the signal subspace is obtained by specifying a metric which provides a measure of the cross-spectral energy projected along each basis vector. The *D* largest energies are retained, where *D* is the rank of the filter. Unlike the cross-spectral method, the residual-correlations (ResCor) method does not rely on an explicit estimate of the signal subspace. Rather, a set of basis vectors are generated by means of a multistage successive refinement procedure.

2. THE COMMUNICATION MODEL

We model a communication channel with *n* transmit antennas at the base say, and a single antenna at receiver. The received signal, r_t at time *t*, is a superposition of the *n* transmitted signals distorted by the impulse response of the channel.

$$r_t = \sum_{i=1}^{n} h_i s_{i,t} + \eta_t \qquad (1)$$

where η_t constitutes the noise quantity modelled as independent complex Gaussian random variables with zero mean, and variance $N_0/2$ per dimension. h_i is the impulse response of the channel. It is modelled as samples of independent complex Gaussian random variables with zero mean and variance 0.5 per dimension. In other words the signals from different antennas undergo independent Rayleigh fades. The physical propagation of the radio signal is dominated by multipath propagation, and often there is no direct line of sight between transmitter and receiver. Say there are *P* multipaths and each of these paths is associated with a complex attenuation α, and a delay δ, h may be further expressed as

$$h = \delta\,(t-\tau)\alpha \qquad (2)$$

Therefore a vector $\boldsymbol{H}$ whose elements h_p denote the channel coefficient describing the *p*th path, may be written as

$$H = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_P \end{bmatrix} = \begin{bmatrix} \delta_1(t-\tau_1) \\ \delta_2(t-\tau 2) \\ \vdots \\ \delta_P(t-\tau_P) \end{bmatrix} * \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_P \end{bmatrix}$$

where the operation (*) represents an element by element multiplication of the vector of path delays and the vector of complex attenuations. The received signal is then a convolution of the transmitted symbols with the impulse response of the channel and may then be written as

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{bmatrix} = \begin{bmatrix} c_1 & 0 & \cdots & 0 \\ c_2 & c_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_P & c_{P-1} & \cdots & c_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_p \end{bmatrix}$$

In compact form, the received signal may be written as

$$\boldsymbol{r} = C\,H + \eta \tag{3}$$

Clearly, since the structure of $\boldsymbol{C}$ is Toeplitz, it is possible to express **r** as

$$\boldsymbol{r} = H\,C + \eta \tag{4}$$

3.1 MAXIMUM LIKELIHOOD CHANNEL ESTIMATION

The maximum likelihood estimate of $\boldsymbol{H}$ is given by [3]

$$\hat{H} = \arg\max_{H} p(r \mid C, H, K) \tag{5}$$

The conditional density function is given as

$$p(r \mid C,H,K) = \frac{1}{(2\pi |K|)^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2}(r - CH)^H K^{-1}(r - CH) \right\}$$

where $\boldsymbol{K}$ is the estimated covariance matrix of the noise. The corresponding log likelihood function is

$$\Lambda = -\frac{1}{2}(r - CH)^H K^{-1}(r - CH)$$

We need to minimize Λ over all $\boldsymbol{H}$. Taking a derivative of the log likelihood function, and setting this equal to zero, we obtain an estimate of $\boldsymbol{H}$ as

$$\hat{H} = \left(C^H K^{-1} C\right)^{-1} C^H K^{-1} r \tag{6}$$

3.2 THE ILSE ESTIMATE

Without loss of generality, the full rank LS estimate involves minimizing the mean square error (MSE) between some desired signal s say, and a weighted version of the received signal y say.

$$\varepsilon = E\left\{ \left| s - w^H y \right|^2 \right\} \tag{7}$$

The iterative least squares (with enumeration) algorithm [4] is an algorithm that uses some estimate of the channel coefficients to estimate the signal transmitted. It does this by solving for C in the expression

$$\varepsilon = \min_{H,C} E\left\{ \left| r - CH \right|^2 \right\} \tag{8}$$

The idea behind this algorithm is to minimize the expression above by iteratively solving for $\boldsymbol{H}$ and $\boldsymbol{C}$. In other words, keeping one of the matrices constant, while solving for the other. The algorithm can be simply written as

- Assume that an initial estimate of the channel matrix $\boldsymbol{H}$ is available.

- Iterate until convergence of signal estimate:

  1. Solve for $\boldsymbol{C}$:
     This would involve taking a derivative of

     $$E[rr^H - r\hat{H}^H C^H - C\hat{H}r^H + C\hat{H}\hat{H}^H C^H]$$

     with respect to $\boldsymbol{C}$, and setting this equal to zero. We obtain

     $$\begin{aligned} \hat{C}^H &= \left(E[\hat{H}\hat{H}^H]\right)^{-1} E[\hat{H}r]^H \\ &= \left(\hat{R}_H\right)^{-1} \hat{R}_{Hr} \end{aligned} \tag{9}$$

  2. Update the estimate of the channel $\boldsymbol{H}$.

The phrase "with enumeration" in "ILSE" means that the above minimization is done only over a symbol set whose elements are elements of the finite alphabet. For instance, for a BPSK modulated signal, the symbol set would be {+1, -1}.

This algorithm provides excellent performance because the spatial diversity is fully exploited. However an inherent limitation of the algorithm is that the channel is assumed to be a flat fading channel whereas this is not always the case (eg. in a W-CDMA system). A channel is described as flat faded when the delay spread of the channel is less than a symbol period. Typical values of delay spread are 0.3 μs for indoor channels and 10 μs for outdoor channels. Therefore the fore-mentioned assumption is consistent with data rates of 1.5 Mbs indoors and 50 Kbps outdoors. However, it is note-worthy that a frequency-selective channel may actually be preferrable, as there is more diversity to benefit from.

For a multipath channel, the estimate of the channel covariance matrix $\boldsymbol{R_H}$, will typically be full rank. However, its effective rank will be smaller, meaning that $\boldsymbol{R_H}$ typically would have a few dominating eigenvalues. This implies that the transmitted signal lies in a low-dimensional subspace.

3.3 THE REDUCED-RANK CROSS-SPECTRAL ESTIMATE

As already stated before, using the cross-spectral method of reduced rank filtering, an estimate of the signal subspace is obtained. However, this estimate does not have to be of a dimension greater or equal to the dimension of the actual signal subspace. The estimate of the signal subspace is achieved through a singular value decomposition of some observed received data matrix ***r***. Say ***r*** is an $L \times N$ matrix, then

$$r = U\,S\,V^H \tag{10}$$

where
U = $L \times N$ orthonormal matrix of left singular vectors of ***r***
S = $N \times N$ diagonal matrix of singular values
V = $N \times N$ orthonormal matrix of right singular vectors

The column vectors of ***U*** form the orthonormal singular basis of the estimated covariance matrix $\boldsymbol{R_{rr}}$. ***V*** contains the eignevectors of $\boldsymbol{R_{rr}}$

$$\begin{aligned} R_{rr} &= V\,\Lambda V^H \\ &= V\,S^2\,\mathrm{V}^{\mathrm{H}} \end{aligned}$$

where S^2 is the corresponding diagonal matrix of eigenvalues. We wish to select a number $D < N$ of these columns such that the mean-square ε error is minimized.
We recall that in the maximum likelihood channel estimation and full rank ILSE symbol detection above, we proceeded by first estimating $\hat{H}$ then ***C.*** In the event that $\hat{H}$ is not known, we need to solve an equivalent of equation (7), suited to our model.

$$\varepsilon_{mmse} = E\left\{\left| C - w^H r \right|^2\right\} \tag{11}$$

From this we obtain the least squares weight w by taking the derivative of

$$E\left[CC^H - Cr^H w - w^H rC^H + w^H rr^H w\right]$$

and setting it equal to zero. We obtain

$$w = \left(R_{rr}\right)^{-1} R_{rc} \tag{12}$$

Note that this algorithm would also require a training sequence or an initialization of the filter.

The cross-spectral metric for selecting the signal subspace is developed by first assuming that $C = w^H r$. Then the signal contributions from the N-dimensional data space E_c can be quantified as

$$E\left[CC^H\right] = \left(w^H r\right)\left(w^H r\right)^H = w^H \left(R_{rr}\right)^{-1} w = R_{rc}{}^H V \Lambda^{-1} V_H R_{rc}$$

Setting the cross-correlation vector

$$\rho = V^H R_{rc} \tag{13}$$

with elements $\{\rho_{k,}\ 1 \le k \le N\}$, we have

$$E_c = \sum_{k=1}^{N} \frac{\rho_k{}^2}{\lambda_k} = \sum_{k=1}^{N} \theta_k \tag{14}$$

where λ_k $\{1 \le k \le N\}$, are the eigenvalues of the covariance matrix $\boldsymbol{R_{rr}}$.

If E_c is considered the total energy of the estimated signal, then θ_k is a projection of the signal energy along the k^{th} basis vector of the space spanned by the columns of $\boldsymbol{R_{rr}}$. Therefore the optimal reduced-rank basis consists of D of those eigenvectors that correspond to the D largest values of θ_k. The cross-spectral subspace is stable.

3.4 THE REDUCED-RANK RESIDUAL-CORRELATIONS ESTIMATE

The residual correlations technique on which this algorithm is based, was introduced by Goldstein and Reed [8]. Here in a procedure reminiscent of a Gram-Schmidt orthogonalization, a set of basis vectors is generated. If the reduced-rank signal subspace is denoted by S_D, then S_D is a $P \times D$ matrix whose columns form an orthonormal basis for the signal subspace. The projected received vector is then given by

$$\tilde{r} = S_D{}^H r \tag{15}$$

the filter output corresponding to the transmitted signal C is

$$\tilde{C} = \tilde{w}^H r \tag{16}$$

and the MMSE weight is given by

$$\tilde{w} = \left(\tilde{R}_{rr}\right)^{-1} \tilde{R}_{rc} \tag{17}$$

where

$$\tilde{R}_{rr} = E\left[\tilde{r}\,\tilde{r}^H\right]$$

$$\tilde{R}_{rc} = S_D{}^H R_{rc}$$

The training-based, block ResCor algorithm which provides S_D is :
Initialization :

$$d_0 = C_0\ ,\ \ Y_0 = r_0$$

For n = 1, ……..., D (*Forward recursion*):

$$\begin{aligned} p_n &= Y_{n-1}\, d^H{}_{n-1} \\ \delta_n &= \|p_n\| \\ h_n &= p_n / \delta_n \\ d_n &= h_n{}^H\, Y_{n-1} \\ B_n &= null\,(h_n) \\ Y_n &= B_n\, Y_{n-1} \end{aligned}$$

For n = D, …….., 1 (*Backward Recursion*) :

$$w_n = (\varepsilon_n\, d^H_{n-1}) / \|\varepsilon_n\|^2$$

$$\varepsilon_n = d_{n-1} - w_n{}' \varepsilon_n$$

where $\varepsilon_D = d_D$. The estimate of the transmitted data symbol is $w'_1\varepsilon_1$.

By studying the block algorithm above, we can appreciate the criterion governing the construction of the signal subspace in the ResCor method. First, the cross-correlation of the received data and the desired signal is obtained. The result is normalized and thus forms the first orthornormal basis for the signal subspace. The received data is projected onto this basis, to form the new desired signal. The received data is also projected onto the null space of this basis, to form the new lower dimensional data matrix. Again the new data matrix is cross-correlated with the new desired signal to form another basis vector (the residual correlation) for the signal subspace, and the cycle repeats. It is clear to see, that the most likely estimate of the signal can be obtained from the first step, by a series of backward recursions. The effectiveness of this algorithm is perhaps appreciated in the tracking mode. Recursive and blind versions of the algorithm exist. It is interesting to note that numerical results (from the example given in section 4), show that in a CDMA type system the ResCor algorithm achieved a substantially lower error rate than the matched filter receiver, and this error rate was very close to the MMSE error rate.

3.5 EXAMPLE

A lot of research has been done on channel estimation for CDMA systems. The example below provides a comparison of the three schemes discussed above (along with other schemes [21]), for a CDMA system. The other schemes involved in the example, but not discussed here are : Stochastic Gradient with Partial Despreading (SG-PD[1]), LS with Partial Despreading (LS-PD), and MMSE with Partial Despreading (MMSE-PD). The Cross-Spectral method is denoted CS. In this example, the number of users is 42, the processing gain is 128 and the received powers are log-normal with standard deviation 6 dB. Details of this simulation are available in [1]. The example is mentioned here only for the purpose of developing an intuition for the relative performance of the three algorithms in a scenario other than that examined in this project.

From Figure 1, we see that for the system considered, the ResCor algorithm outperformed all the others, except the MMSE-PD[1]. It also acheived its minimum error rate when $D = 8$. This implies a much lower computational complexity than for the full rank ($P = 128$) solution.

Figure 2 shows that for the system considered, a low-rank ResCor algorithm ($D = 4$) converged significantly faster than the full rank ($P = 128$) RLS algorithm, yet, with nearly the same asymptotic SINR.

[1] PD (partial despreading), is proposed in [21]. It refers to a despreading filter that is matched to segments of the spreading code. The outputs of the PD filter are linearly combined according to an LS or MMSE criterion.

4. 1 SIMULATIONS

A single user scenario is considered. We assume $P = 12$. Data is collected in a 12×12 matrix. The aim is to compare the performance of the three afore-mentioned algorithms to estimate the transmitted bits. The simulation is done in baseband. In other words we transmit ±1. Matlab is used for all simulations.

5. RESULTS

Figure 3 shows the maximum likelihood estimate of ***H***. In this case, the received data is a vector. The broken lines with asterisks (*) represent the estimate of ***H***, while the thick lines represent the actual ***H***.

Figure 4 shows the learning curve of the LMS estimate of an element of ***H***. The step size used is 0.1.

Figure 5 shows results from the comparison of all three algorithms. Since the ILSE is iterative, and the simulation was performed over a single block of data, a slight modification was made to the ILSE, in the sense that an LMS estimate of ***H*** was used to derive the estimate of C over one block of data.

It is seen that the ResCor algorithm performs better than the Cross-Spectral and ILSE algorithms. It achieves its minimum error rate at rank at $D = 3$. It also achieves a lower error rate than the ILSE algorithm.

6. CONCLUSIONS

In this project, various aspects of channel parameter estimation and signal detection were examined. Three algorithms were presented, two of which are reduced-rank algorithms that have been previously applied in other scenarios. In this project all three algorithms are applied in a space-time channel, single user scenario, and their performance compared. Numerical results showed that the ResCor algorithm achieved a lower error rate than the other two algorithms, and at a lower filter order. The ResCor is also computationally less expensive in that it does not require matrix inversions or the operation a singular value decomposition on the received data. Extension of this project can be made to a multi-user case, and a recursive version of the Cross-Spectral method can be developed so as to compare convergence plots for the three algorithms.

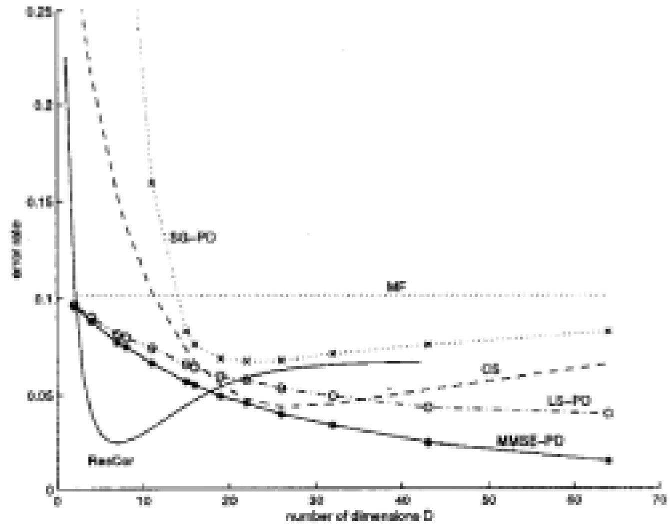

Figure 1: Error rate vs. number of dimensions for reduced-rank adaptive algorithms after training with 200 symbols.

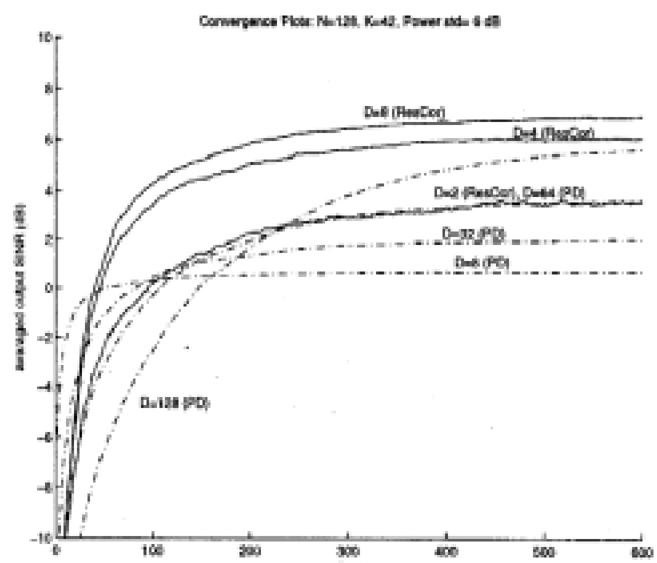

Figure 2: Convergence plots for recursive ResCor and RLS-PD algorithms

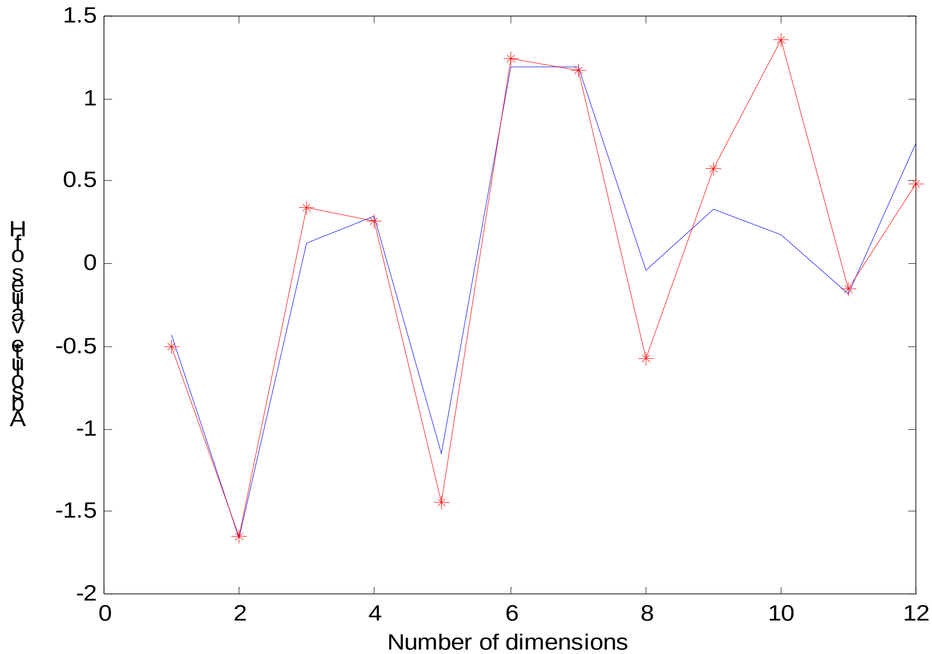

Figure 3 : Maximum likelihood estimate of ***H***

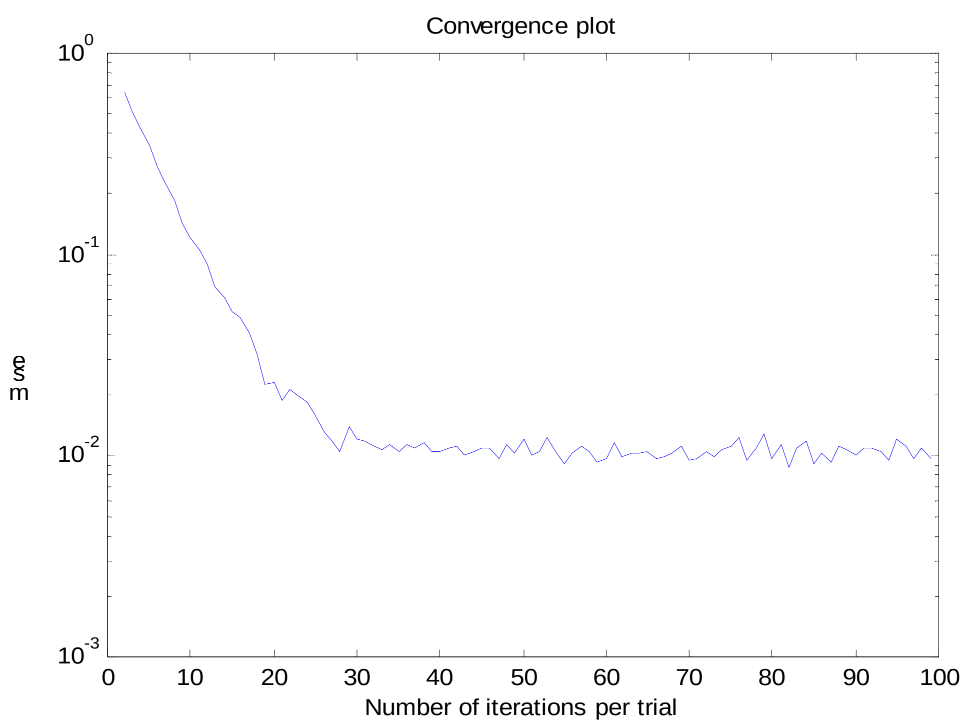

Figure 4: Plot of learning curve LMS estimate of *h* (one element of ***H***).

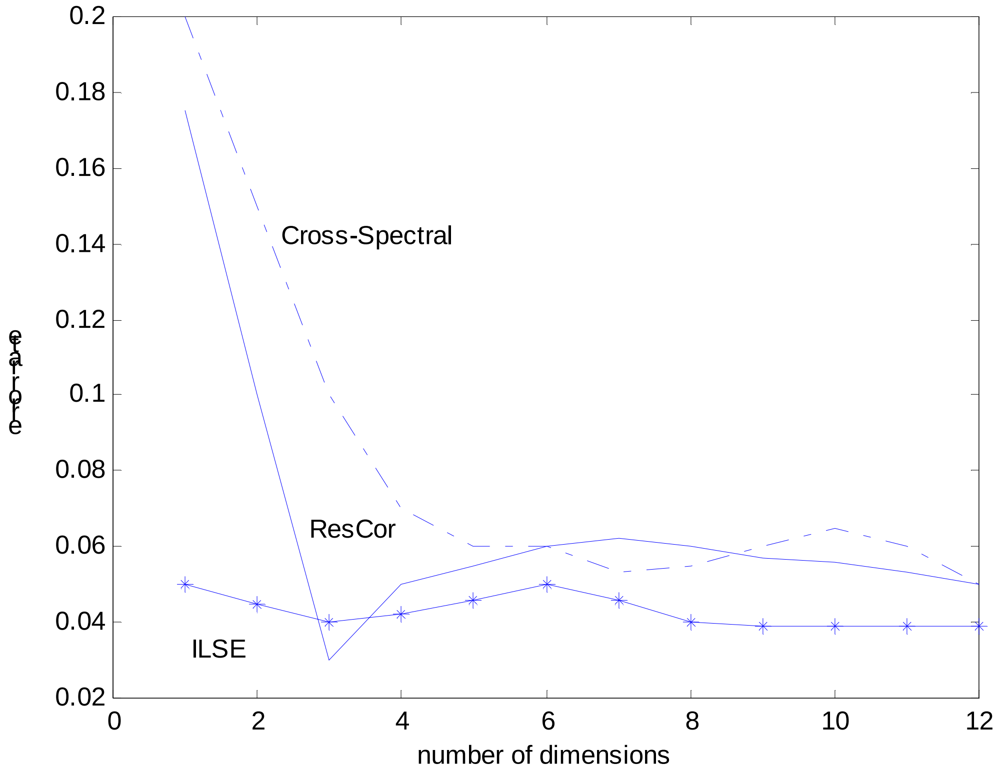

Figure 5: Error rate vs. number of dimensions, for the three algorithms.

REFERENCES

[1] Michael L. Honig., and J. Scott Goldstein "Adaptive Reduced-Rank Residual Correlation Algorithms for DS-CDMA Interference Suppression". *Signals, Systems and Computers*, 1998.

[2] J. Scott Goldstein and Irving S. Reed "Reduced-Rank Adaptive Filtering". *IEEE Transactions on Signal Processing* , 1997.

[3] Chaitali Sengupta "Algorithms and Architectures for Channel Estimation in Wireless CDMA Communication Systems". *Thesis : Doctor of Philosophy*, ECE Department, Rice University. 1998.

[4] Per Pelin "Space-Time Algorithms for Mobile Communications." *Doctoral Thesis for the Degree of Doctor of Philosophy*, Department of Signals and Systems, Chalmers University of Technology. Sweden. 1999.

[5] J. Scott Goldstein, Irving S. Reed and Louis L. Scharf "A Multistage Representation of the Wiener Filter Based on Orthogonal Projections." *IEEE Transactions on Information Theory*. 1998

[6] Chaitali Sengupta, Joseph Cavallaro and Behnaam Aazhang "Subspace-Based Tracking of Multipath Channel Parameters for CDMA Systems." *European Transactions on Telecommunications*. 1997

[7] Peter A. Zulch and J. Scott Goldstein " Comparison of Reduced-Rank Signal Processing Techniques." *Signals, Systems and Computers*. 1998.

[8] J. Scott Goldstein and Irving S. Reed "A New Method of Wiener Filter and its Application to Interference mitigation for Communications." *Signals, Systems and Computers*. 1997

[9] Akbar M. Sayeed, Andrew Sendonaris, and Behnaam Aazhang "Multiuser Detectors for Fast-Fading Multipath Channels" *Signals, Systems and Computers*. 1997

[10] Chaitali Sengupta, Ari Hottinen, Joseph Cavallaro and Behnaam Aazhang, "Maximum Likelihood Multipath Channel Parameter Estimation in CDMA Systems." ECE Department, Rice University. 1997.

[11] Erik Lindskog "Space-Time Processing and Equalization for Wireless Communications" *Dissertation in Signal Processing*, Signals and Systems. Uppsala University, Sweden. 1999.

[12] Stephen E. Bensely and Behnaam Aazhang "Subspace-Based Channel Estimation for Code Division Multiple Access Communication Systems." *IEEE Transactions on Communications*. 1996.

[13] Jonas Standell and Erik Lindskog "Separate Temporal and Spatial Parametric Channel Estimation." *Signals and Systems*, University of Uppsala, Sweden. 1997

[14] Chaitali Sengupta Joseph Cavallaro and Behnaam Aazhang "On Multipath Channel Estimation for CDMA Systems Using Multiple Sensors." *IEEE Transactions on Communications*. 1998

[15] Erik Lindskog and Claes Tidestav "Reduced_Rank Channel Estimation" *Signals and Systems*, University of Uppsala, Sweden. 1999

[16] Chaitali Sengupta Joseph Cavallaro and Behnaam Aazhang "Maximum Likelihood Multipath Channel Parameter Estimation in CDMA Systems Using Antenna Arrays." *In Proceedings International Sysposium Personal Indoor Mobile Radio Communications (PIMRC98)*. 1998

[17] Merlbourne Barton and Donald W. Tufts "Reduced-Rank Least Squares Channel Estimation." *IEEE Transactions of Acoustics, Speech and Signal Processing*. 1990

[18] Siavash M. Alamouti " A Simple Transmit Diversity Technique for Wireless Communications." *IEEE Journal on Select Areas in Communications*. 1998.

[19] Vahid Tarokh, Nambi Seshadri and A. R. Calderbank "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction." *IEEE Transactions on Information Theory*. 1998.

[20] Vahid Tarokh, Ayman Naguib, Nambi Seshadri and A. Robert Calderbank "Combined Array Processing and Space-Time Coding." *IEEE Transactions on Information Theory*. 1998.

[21] R. Singh and L. Milstein "Adaptive Interference Suppression in Direct-Sequence Spread-Spectrum CDMA. *IEEE Transactions on Communications*. 1998.

[22] Theodore S. Rappaport " Wireless Communications, Principles and Practice." *Prentice Hall NJ*. 1996.

[23] John G. Proakis "Digital Communications" *McGraw-Hill*. 1995.

[24] Kishan Shenoi "Digital Signal Processing In Telecommunications." *Prentice Hall NJ*. 1995.