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Transmitter Diversity and Coding Schemes

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Abstract

We analyze wireless communication systems employing multiple-transmit multiple-receive antennas in this thesis. Space-time codes have been proposed in the literature as an efficient means of coding over multiple transmit antennas. In particular, the rank and the determinant of code difference matrices have been shown to be important in the design of space time codes in quasi-static fading channels. In this thesis, we investigate the problem of maximizing the coding gain given by the minimum of the determinants of the code difference matrices. We show that equal singular values of the code difference matrices is a necessary and sufficient condition for obtaining optimal coding gain. We also show that equal singular values lead to robust codes. Finally, we present the construction of enhanced dimensional trellis coded modulation (EDTCM) codes based on equal singular values.
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Chapter 1

Introduction

There is an increasing demand for higher data rates in wireless communications to support different kinds of emerging applications such as wireless multimedia. The bandwidth cannot be increased correspondingly due to the premium on the bandwidth. Hence, it becomes necessary that the spectral efficiency of the existing bandwidth be increased. Multiple transmitter diversity has emerged as a promising solution in this direction.

Wireless channels are characterized by large attenuation and vagaries in the channel termed as fading [8]. It is well known that diversity in signaling is very effective in countering the effects of fading [4]. Diversity techniques are based on the notion that if the receiver can be provided with several replicas of the same information signal transmitted over independently fading channels, then the probability that all the signal components will fade simultaneously is reduced considerably. The popular forms of diversity are [7, 8]

1. Temporal diversity: Replicas of the information bearing signal are transmitted in different time slots, where the separation between the time slots is greater than the coherence time of the channel.

2. Frequency diversity: In this case, replicas of the information bearing signal are transmitted in different frequency bands, where the separation between the frequency bands is greater than the coherence bandwidth of the channel.
3. Antenna (spatial) diversity: It has been observed that antennas with a spacing of more than half a wavelength lead to spatially uncorrelated channels. The transmission of replicas of the information bearing signal over these uncorrelated spatial channels leads to spatial diversity.

Note that not all kinds of diversity are always feasible. For example a slowly fading channel (with a long coherence time) cannot support temporal diversity with practical interleaving depths. Similarly, frequency diversity is not feasible when the coherence bandwidth of the channel is comparable to the bandwidth of the signal employed. However, irrespective of the channel characteristics, antenna diversity can always be exploited as long as there is sufficient spacing between the antennas. It becomes very important to utilize spatial diversity at both the transmitter and the receiver especially when other forms of diversity are not feasible due to the nature of the channel available as well as the nature of the signaling used in the system.

In a practical communication system, the base-station is usually equipped with multiple antennas to exploit spatial diversity. It is not possible to provide the handset with many antennas due to size and cost constraints. It is common practice to use the multiple base-station antennas for receiver diversity in a reverse link (handset to base-station) transmission. Hence, the problem of using multiple antennas at the receiver is very well studied. In particular, techniques such as maximum likelihood combining, equal gain combining and selection of antennas are used depending on the extent of channel information available at the receiver[4, 8].

The performance of detectors at the receiver depends on the extent of the channel information available at the receiver. In many cases, the receiver is assumed to be possess the channel estimates without any error, to isolate the problem of channel estimation from the problem of detection. Further, this is not an unrealistic assump-
tion especially when the changes in the channel are not too fast. Training based on
preambles or pilot signals is a common way of estimating the channel at the receiver.
On the other hand, providing the transmitter with the channel estimates requires the
explicit use of a feedback channel and other additional resources (an exception to
this is time division duplex signaling, where channel estimation is possible at both
ends without a separate feedback channel). In this work, we assume that the trans-
mitter does not have any information about the channel realization. We will also
quantify the potential benefits possible if the transmitter does in fact have access to
the channel state information.

We will show that we can obtain substantial improvements in the bandwidth
efficiency by the use of multiple transmit antennas. In particular, we will investigate
the problem of designing optimal codes for a multiple input multiple output channel
arising from multiple transmit and multiple receive antennas. This problem is not
as well understood as the traditional problem of designing codes for a single input
multiple output channel.
Chapter 2

Related Work

In this chapter, we discuss related results from the literature, pertaining to multiple transmitter systems. We give a brief review of the evolution of the problem and overview some of the early results in this field. We also discuss the various coding design criteria proposed and also the codes constructed based on these criterion.

2.1 Background

The interest in the multiple transmitter system started with the base-station simulcast problem first studied in [3]. The simulcast problem, as the name suggests, involved multiple transmitters for a single intended receiver. The delay diversity scheme was proposed as an effective solution to this problem. In this scheme, replicas of one antenna are transmitted on the other antennas separated in time. Along similar lines of providing offset between the various transmissions in time, offset in frequency was also considered in [1, 19]. Seshadri and Winters [17] also analyzed the delay diversity scheme for multiple antennas and showed that this scheme provides the maximum possible diversity for a multiple antenna system. Coding for fading channels was first analyzed by Divsalar and Simon in [5]. The concept of the product distance was introduced by them. It was shown that the Hamming distance and the product distance of the codes play an important role in characterizing the error performance over fading channels. In particular, it was shown that the minimum Hamming distance is equivalent to the diversity and the product distance gives a measure of the
coding gain as opposed to the Euclidean distance over the additive white gaussian noise (AWGN) deterministic channels.

Tarokh et al. [21] analyzed the multiple input multiple output channel following the lines of [5]. They showed that the diversity gain of the code is characterized by the minimum of the ranks of the code difference matrices and that the coding gain is given by the minimum of the product of the non-zero eigen values of the code difference matrices. They also designed several codes following the design criteria of the rank and determinants of the code difference matrices and the concept of space time coding was introduced.

The rank criterion that determines the diversity gain of the system is relatively easy to satisfy and there are several simple observations (given in [21] and [12], among others) which help achieve the maximum diversity gain. It is widely believed that the diversity gain is more important than the coding gain, for reliable performance over fading channels and hence, the main focus so far has been to improve the diversity gain only. However, as we will show here, substantial gains can be achieved through coding gains. The role of Euclidean distance in code design for multiple antennas was discussed in [11] and a certain equal eigenvalues criterion has been introduced.

Transmit diversity systems can be broadly classified into the following categories based on the availability of the channel information at the transmitter and the receiver:

1. schemes with feed-back and feed-forward information (giving complete channel state information to both the transmitter as well as the receiver),

2. schemes with feed-forward information but no feed-back (receiver has the channel state information but the transmitter does not have any information),
3. schemes without any channel state information at the transmitter or the receiver.

Most past work assumes that the receiver has full channel state information with no such information available at the transmitter. The assumption that the receiver has good channel estimates is realistic for slowly time varying channels (like the slowly fading channel in an indoor environment). Recently, there has been some work for systems which can be classified under the third category above [10],[15]. Information theoretic analysis of these systems has been done in [6],[13],[16].

2.2 Examples of space-time codes

2.2.1 Delay diversity code

Delay diversity code was the first coding scheme proposed for a multiple transmit antenna system [17]. In this scheme, time delayed versions of the symbols are transmitted across the antennas. This multiple input channel has an equivalent representation as a single input channel with memory. In a fading channel, this is equivalent to transforming a flat fading channel into a frequency selective channel. The delay diversity scheme can achieve a diversity gain of the order of the number of transmit antennas. The coding gain is not optimized by this scheme.

2.2.2 Space-time trellis codes

The delay diversity scheme was further extended by Tarokh et al. in [21] to incorporate the coding gain to a certain extent. It was shown that the space-time trellis codes constructed in [21] have maximum diversity gain (given by the number of transmit antennas) and a coding gain greater than that of the delay diversity scheme. In particular, trellis codes of different complexities (measured by the number of states in
the trellis and the number of transitions from each state) and different constellations (QPSK, 8 PSK among others) were constructed. This was the first attempt in the direction of improving the coding gain of codes for multiple transmitter antennas apart from ensuring maximum diversity gain. However, it should be noted that no systematic approach has been laid out to maximize the coding gain or to ensure a minimum coding gain. The AT&T space-time codes were shown to have better performance than the delay diversity code under various channel conditions.

2.2.3 Orthogonal space-time block codes

Most of the space-time codes discussed in the literature were trellis coded modulation schemes for multiple transmit antennas. In [20], block code designs based on orthogonal structures of matrices were proposed. These orthogonal structures were motivated by the Radon-Hurwitz transformation for the construction of orthogonal matrices. A linear processing orthogonal design $\epsilon$ in variables $x_1, x_2, ..., x_n$ is an $n \times n$ matrix such that

1. the entries of $\epsilon$ are real linear combinations of the variables $x_1, x_2, ..., x_n$,

2. $\epsilon^*\epsilon = \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix with $(i, i)$th diagonal element of the form $l_1^i|x_1|^2 + l_2^i|x_2|^2 + ... + l_n^i|x_n|^2$, where $l_1^i, ..., l_n^i$ are all strictly positive numbers.

It has been shown that such orthogonal structures with complex entries exist for only $n=2, 4$ and $8$, where $n$ is the size of the orthogonal matrix. The case of $n=2$ was first explored by Alamouti in [2]. For $n=2$, the orthogonal structure takes the form

$$\epsilon = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$ (2.1)

The orthogonal structure for $n=2$, will be referred to as Alamouti transformation in the sequel. It was shown that this structure achieves a diversity of order 2 with
two transmit antennas. Further, the Alamouti transformation achieves performance comparable to maximal ratio combining at the receiver [2].

Thus, it can be seen that all the previous attempts at designing codes for multiple antennas were mostly aimed at achieving maximum diversity gain with little or no attention to the coding gain. In the next chapter, we will discuss ’equal eigenvalues’ criterion from [11] and then present a coding scheme based on this criterion. We will show that the equal eigenvalues lead to space-time codes with optimal coding gain, with a given Euclidean distance. Further, we will show that the codes designed by this criterion are robust to different channel realizations.
Chapter 3

Design of Codes With Optimal Coding Gain

In this chapter, we shall analyze the Chernoff bound on the pair-wise error probability of codewords and examine the rank and the determinant criteria for space-time coding. In particular, we shall derive an upper bound for the coding gain given by the determinant criterion and note the conditions under which this upper bound can be achieved. We shall give a meaningful definition to the Euclidean distance between codewords in the context of multiple antennas and relate the upper bound on the coding gain to the Euclidean distance.

3.1 System model

We consider a wireless communication system with $M$ transmit antennas and $N$ receive antennas. The fading on the channel is assumed to be frequency non-selective. We assume that the fading process is quasi-static, i.e., the channel coefficients do not vary significantly within a frame. Data is encoded and transmitted across the $M$ transmit antennas simultaneously. Let $c_i^t$ denote the symbol transmitted on antenna $i$ at time instant $t$. In general $c_i^t$ will be one of the symbols of the signaling constellation. Further, let $h_{ij}^t$ be the channel coefficient corresponding to the channel between the $i$th transmit antenna and the $j$th receive antenna while $n_j^t$ is the proper complex Gaussian noise with variance $1/2$ per dimension. Then, the signal received
at the $j$th receive antenna at time instant $t$, given by $y_{t}^{j}$, can be written as

$$y_{t}^{j} = \sqrt{E_s} \sum_{i=1}^{M} h_{t}^{i} c_{t}^{i} + n_{t}^{j}$$

(3.1)

where $E_s$ is the transmitted symbol energy. As pointed out in [21], the total diversity of a wireless communication system with $N$ receive antennas is equal to $N$ times the diversity of the rest of the system. Hence, we focus our attention on $N = 1$, i.e., the case of one receive antenna only. We shall drop the superscript in the received signal, channel coefficient and the additive noise component in the sequel. Denote by $l$, the number of symbol epochs, spanned by a codeword (also called frame length). A codeword is the collection of all the symbols transmitted over the $M$ antennas during the corresponding $l$ consecutive symbol epochs. Hence, a codeword $\mathbf{c}$ can be written out as a matrix of $M$ rows and $l$ columns (the rows correspond to the antennas and the columns correspond to time) given by

$$D_{c} = \begin{bmatrix}
    c_{1}^{1} & c_{1}^{2} & \cdots & c_{1}^{l} \\
    c_{2}^{1} & c_{2}^{2} & \cdots & c_{2}^{l} \\
    \vdots & \vdots & \ddots & \vdots \\
    c_{M}^{1} & c_{M}^{2} & \cdots & c_{M}^{l}
\end{bmatrix}$$

(3.2)

The channel vector $\mathbf{h}$ is given by (assuming one receive antenna)

$$\mathbf{h} = \begin{bmatrix}
    h_{1} & h_{2} & h_{3} & \ldots & h_{M}
\end{bmatrix},$$

The received vector $\mathbf{y}$ is given by

$$\mathbf{y} = \sqrt{E_s} \mathbf{hD}_{c} + \mathbf{n}$$

(3.3)

where $\mathbf{y} = \begin{bmatrix}
    y_{1} & y_{2} & y_{3} & \ldots & y_{l}
\end{bmatrix}$ and $\mathbf{n} = \begin{bmatrix}
    n_{1} & n_{2} & n_{3} & \ldots & n_{l}
\end{bmatrix}$. 
3.2 Upper bound on error probability

We define pair-wise error probability between any two codewords \( c \) and \( e \), \( Pr\{c \rightarrow e\} \), as the probability that the codeword \( c \) is wrongly decoded as codeword \( e \) and vice versa, at the receiver. From [12], the pair-wise error probability conditioned on the channel realization is given by

\[
Pr\{c \rightarrow e|\mathbf{h}\} \leq e^{-\left(\frac{d^2(c,e)E_s}{4N_0}\right)}
\]

(3.4)

where

\[
d^2(c,e) = \sum_{t=1}^{i} \left| \sum_{i=1}^{M} h_t(c_i^t - e_i^t) \right|^2
\]

(3.5)

which follows from an approximation to the \( Q \) function (Gaussian tail function).

The channel coefficients appear in \( d^2(c,e) \). In the sequel, the code difference matrix, \( \mathbf{D}_c - \mathbf{D}_e \), will be denoted by \( \mathbf{D}_{ce} \). The average pair-wise error probability over a quasi-static fading channel, at high SNR, is given by [21]

\[
Pr\{c \rightarrow e\} \leq \left( \prod_{i=1}^{r} \lambda_i \right)^{-1} \left( E_s/4N_0 \right)^{-r}
\]

(3.6)

where \( \lambda_i \) is the \( i \)th non-zero eigenvalue and \( r \) is the rank of the matrix \( \mathbf{A} = \mathbf{D}_{ce}^\dagger \mathbf{D}_{ce} \) (the superscript \( \dagger \) stands for matrix hermitian operation).

3.3 Analysis of error probability

From (3.6), the coding gain is given by \( (\prod_{i=1}^{r} \lambda_i)^{\frac{1}{2}} \) and the diversity gain is given by \( r \), the rank of the matrix \( \mathbf{A} \). When \( \mathbf{A} \) is full rank, the coding gain is equal to the determinant of \( \mathbf{A} \). We shall now investigate the condition under which the coding gain is optimized.
Definition 3.1  The squared Euclidean distance between any two codeword matrices, \( c \) and \( e \), is defined as

\[
d_E^2(c, e) = \sum_{i=1}^{M} \sum_{j=1}^{N} |c_i^j - e_i^j|^2
\]  

(3.7)

This is a natural extension of the Euclidean distance for a single transmitter system. It is to be noted that the Euclidean distance defined in this fashion is the trace of the matrix \( A \) defined earlier. By the properties of the eigenvalues, it is also equal to the sum of the eigenvalues of the matrix \( A \). We will show that the coding gain of the space time codes is related to the minimum Euclidean distance of the space time code.

The coding gain of a space time code with full diversity and a given minimum Euclidean distance between any two codewords is maximized if and only if every non-zero code difference matrix has equal singular values. The coding gain is further maximized by increasing the minimum Euclidean distance of the code.

This condition was referred to as equal eigenvalues criterion in [11]. The above observation can be easily proved as follows.

Let \( \mathcal{C} \) be a space time code with codewords \( c_1, c_2, \ldots, c_n \). Let the corresponding codeword matrices be represented as \( D_{c_1}, D_{c_2}, \ldots, D_{c_n} \). Define \( A_{ij} = (D_{c_i} - D_{c_j})(D_{c_i} - D_{c_j})^\dagger \), \( \forall 1 \leq i, j \leq n, i \neq j \). We know that the coding gain of the space time code \( \mathcal{C} \), with full diversity (full rank), is given by the minimum of the determinants of \( A_{ij} \). Let \( \lambda_1, \lambda_2, \ldots, \lambda_r \) be the eigenvalues corresponding to \( A_{ij} \). Let \( d_E(\mathcal{C}) \) be the minimum Euclidean distance of the code \( \mathcal{C} \). Further, let \( d_E(c_i, c_j) \) be the Euclidean distance between the codewords \( c_i \) and \( c_j \). Then, from the arithmetic mean - geo-
metric mean inequality, for every \( i, j \), we have,

\[
\det(A_{ij}) = \prod_{p=1}^{r} \lambda_p \\
\leq \left( \frac{\sum_{p=1}^{r} \lambda_p}{r} \right)^r \\
= \left( \frac{d^2_{E}(c_i, c_j)}{r} \right)^r
\]

(3.8)

where the equality is achieved if and only if all the eigen values are equal. Hence, for any two codewords \( c_i, c_j \), separated by a given Euclidean distance, the coding gain for the pair-wise error probability between the codewords is maximized if and only if all the singular values of the code difference matrix are equal. Further, when the singular values are all equal, the coding gain is determined by minimum of \( d_E(c_i, c_j) \) over all \( i, j, i \neq j \). We know that the minimum is given by \( d_E(C) \). Hence, the coding gain of the space-time code \( C \) can be increased only by increasing the minimum Euclidean distance.

### 3.4 Analysis of instantaneous error performance

The exponent in the pairwise error event probability given in (3.4), \( d^2(c, e) \), has the following equivalent representations [21]

\[
d^2(c, e) = hA h^\dagger
\]

(3.9)

where \( A = D_{ee} D_{ee}^\dagger \). Also, \( A \) is Hermitian so that there exists a unitary matrix \( V \) and a real diagonal matrix \( D \) such that \( VAV^\dagger = D \). The diagonal elements of \( D \), given by \( \lambda_i, i = 1, 2, \ldots M \), are the eigenvalues of \( A \). Define \( [\beta_1, \ldots, \beta_M] = hV^\dagger \) so that \( \beta_i = hv_i^\dagger \), where \( v_i \) is the \( i \)th row of
\textbf{V.} Then, we have

\[
d^2(\mathbf{c}, \mathbf{e}) = \sum_{i=1}^{M} \lambda_i |\beta_i|^2
\]  

(3.10)

Using the Cauchy-Schwarz inequality, we have

\[
|\beta_i|^2 = |\mathbf{h} \mathbf{v}_i\rangle^2 \\
\leq \|\mathbf{h}\|^2 \|\mathbf{v}_i\|^2 \\
= \|\mathbf{h}\|^2 \quad (3.11)
\]

Note that \(\|\mathbf{v}_i\|^2 = 1\), since \(\mathbf{v}_i\) is a row of a unitary matrix. We finally have,

\[
d^2(\mathbf{c}, \mathbf{e}) = \sum_{i=1}^{M} \lambda_i |\beta_i|^2 \\
\leq \sum_{i=1}^{M} \lambda_i \|\mathbf{h}\|^2 \\
= \|\mathbf{h}\|^2 \left( \sum_{i=1}^{M} \lambda_i \right) \quad (3.12)
\]

where we have made use of (3.11). Again, let \(d_E(\mathbf{c}, \mathbf{e})\) be the Euclidean distance between the codewords \(\mathbf{c}, \mathbf{e}\), i.e., \(\sum_{i=1}^{M} \lambda_i = d_E^2(\mathbf{c}, \mathbf{e})\). Then, the upper bound in (3.12) can be achieved if and only if

1. \(\mathbf{v}_i = \frac{\mathbf{h}}{\|\mathbf{h}\|^2}\), when \(\|\mathbf{h}\| \neq 0\), for some \(i\), along with \(\lambda_i = d_E^2(\mathbf{c}, \mathbf{e})\), and
2. \(\lambda_j = 0, \forall j \neq i\).

This is nothing but beam-forming at the transmitter. The implementation of a strategy such as this requires complete channel state information at both the transmitter and the receiver, in addition to adaptive coding and modulation techniques at the transmitter. Also, this is the best error
performance that can be obtained from the given channel realization, $h$, for a given $d_E(c, e)$.

We shall now compare the performance of the equal eigenvalues criterion with that of the beam-forming described above. With equal eigenvalues, we have

$$d^2(c, e) = \sum_{i=1}^{M} \lambda_i |\beta_i|^2$$

$$= \left( \frac{d_E^2(c, e)}{M} \right) \sum_{i=1}^{M} |\beta_i|^2$$

$$= \left( \frac{d_E^2(c, e)}{M} \right) \| h \|^2$$

(3.13)

where the last equality follows from the definition of $\beta_i$.

The implementation of the equal eigenvalues criterion does not require the channel state information at the transmitter. This result states that the performance of a code satisfying the equal eigenvalues criterion is always within a factor of $M$ of the best case performance of the given realization of the channel, or in other words, no more than $10 \log(M)$ dB less, although the channel state information is not available at the transmitter. This establishes the robust nature of the design criterion to different channel statistics. Note that the rank criterion and the determinant criterion discussed earlier, lead to a good average performance, as these criteria have been derived from average error probability. In this section, we have analyzed the error performance for any given realization of channel under quasi-static fading channel assumptions. By the above analysis, we have shown that the equal eigenvalues criterion leads to a good error performance for every realization of the channel which in turn leads to good performance when averaged over different channel realizations also).
We shall discuss the construction of codes based on the *equal eigenvalues* criterion and their performance, in the next chapter. We call the codes *enhanced dimensional trellis coded modulation* (EDTCM) codes to highlight that (a) dimensionality of the signaling constellation is increased by joint coding over time and space domains and (b) traditional TCM ideas have been used in designing the joint coding-modulation scheme.
Chapter 4

Enhanced Dimensional Trellis Coded Modulation

In this chapter, we address the problem of designing codes for multiple antennas based on the techniques discussed in the previous chapter. In a multiple transmit antenna system, the transmitter has additional degrees of freedom given by the number of transmit antennas. We investigate the advantages of utilizing the degrees of freedom available across space and time in a joint fashion. We make use of the equal eigenvalues criterion, in designing our codes across the space and the time domains, which we call enhanced dimensional trellis coded modulation (EDTCM).

4.1 Design of EDTCM codes

The equal eigenvalues criterion can be summarized as follows. Codes designed over \( l \) time epochs and \( M \) transmit antennas must satisfy the property that the code difference matrix must have equal singular values for every pair of codewords in the code set. Mathematically, for all \( c, e \in \mathcal{C} \), all the eigenvalues of \( \mathbf{D}_{ce} \mathbf{D}_{ce}^\dagger \) are equal or equivalently \( \mathbf{D}_{ce} \mathbf{D}_{ce}^\dagger = \lambda_{ce} \mathbf{I} \), where \( \lambda_{ce} \) is the common eigenvalue, specific to the codewords \( c \) and \( e \).
We achieve this condition via two sufficient conditions viz., If $X$ and $Y$ are two matrices such that (a) the dimensions of $X$, $Y$ are the same, (b) $XX^\dagger$ and $YY^\dagger$ have equal eigenvalues and (c) $XY^\dagger = -YX^\dagger$, then

1. the matrix $F$ formed by the difference $X - Y$ is such that $FF^\dagger$ has equal eigenvalues and
2. the matrix $G$ formed by the concatenation of the two matrices $X$ and $Y$ is such that $GG^\dagger$ has equal eigenvalues.

The two sufficient conditions listed above reduce the complexity of the design of the codewords. We need to search for matrices of lower dimensions to satisfy our criterion. The orthogonal designs discussed in [20] are useful in this regard. It can be easily shown that if $X$ and $Y$ have the form,

$$
\begin{bmatrix}
  C_1 & -C_2^* \\
  C_2 & C_1^*
\end{bmatrix}
$$

(4.1)

where $C_1$ and $C_2$ are any two complex numbers, then $X$ and $Y$ meet the sufficient conditions listed above. Hence, if we impose this structure on our codeword matrices, the *equal eigenvalues* criterion will be satisfied. This transformation was first presented in [2] and similar structures were later developed in [20] under the name of orthogonal designs using the Radon-Hurwitz transformation theory.

We need to make a distinction between the equal eigenvalues criterion and the orthogonal space time block code criterion from [20]. The latter criterion is given by $D_cD_c^\dagger = I$ for every codeword matrix $D_c$ whereas the equal eigenvalues criterion demands that the difference of code word
matrices have an orthogonal structure. The orthogonal block code design condition is a sufficient condition for the equal eigenvalues criterion and hence a stronger requirement on the structure of the codewords.

The orthogonal structure helps decompose our original goal of maximizing the common eigenvalue of the codeword difference matrix into the following independent tasks.

1. Make the eigenvalues of a codeword difference matrix equal for any given pair of codewords,

2. Maximize the common eigenvalue.

Note that the sum of the eigenvalues of $D_c D_c^\dagger$ is the squared Euclidean distance between the codewords $c$ and $e$. Hence, when the eigenvalues are all equal, maximizing the eigenvalues is equivalent to maximizing the Euclidean distance between the two codewords.

### 4.2 Construction of EDTCM code

We will present the construction of a EDTCM code for two transmit antennas using QPSK modulation. We implement a joint coding and modulation scheme over the two antennas. This is an extension of the Ungerboeck TCM scheme [9]. We construct a super-constellation $\mathcal{S}$ consisting of the Cartesian product of the two QPSK constellations. Note that each QPSK constellation is a 2 dimensional constellation containing 4 points. Hence, the superconstellation $\mathcal{S}$ contains 16 points in 4 dimensions.

We partition this set of 16 points thus obtained into subsets with fewer points. We ensure that the minimum Euclidean distance between any
two points in a set formed after partitioning a parent set is greater than that of the parent set. Finally, EDTCM code is constructed based on the partitioned subsets adhering to the rules

1. The distance between any two branches leaving a particular state should be maximum,

2. The distance between any two branches culminating in a particular state should be maximum.

3. All the points in the constellation should occur with equal probability.

The space-time trellis codes from [21], among others, did not allow parallel transitions it the trellis. Parallel transitions always lead to rank 1 for code difference matrices, thus limiting the maximum achievable diversity (given by the rank criterion). In our design, all the eigenvalues are equal and necessarily non-zero when the codewords are distinct due to the structure 4.1 imposed on the codeword matrices (we shall, henceforth, refer to this as Alamouti scheme). This always leads to full rank and hence full diversity. The presence of parallel transitions does not affect the diversity in the system.

4.3 Partitioning criterion

In this section, we will explain the process of partitioning with the example of a QPSK constellation. The QPSK points are labeled 0, 1, 2, 3 as shown in the Figure 4.1. The 16 points in the super-constellation are also labeled
from 0 through 15 as explained below. The QPSK constellation points and the elements of $\mathcal{S}$ can be easily distinguished from the context.

The elements of $\mathcal{S}$ are labeled as follows. The first column of any element of $\mathcal{S}$ is of the form $[C_1, C_2]^T$ (the superscript $T$ stands for matrix transpose operation), where $C_1, C_2$ are QPSK points. Note that $0 \leq C_1, C_2 \leq 3$.

The label of any element of $\mathcal{S}$ is given by $4 \times C_1 + C_2$. Hence, if the first column is $[2, 3]^T$, then the corresponding label would be $4 \times 2 + 3$, i.e., 11.

Table 4.1 lists the squared singular values for the codeword differences. Note that the eigenvalues of $\mathbf{D}_{\text{ce}}\mathbf{D}_{\text{ce}}^\dagger$ are the squared singular values of the code difference matrix $\mathbf{D}_{\text{ce}}$. The points are listed in groups based on the eigen values. Based on this table, we can divide the original set $\mathcal{S}$ containing 16 points into two subsets, $\mathcal{S}_1$ and $\mathcal{S}_2$ such that the minimum eigenvalue of the difference of any two points in a given subset is 4. $\mathcal{S}_1$, for example, comprises of $0, 2, 5, 7, 8, 10, 13, 15$. Note that in the original set $\mathcal{S}$, the minimum possible eigenvalue was 2. Hence, after the first stage of partitioning, the minimum eigenvalue is increased from 2 to 4.

The next step in the process would be to divide the set $\mathcal{S}_1$ (and also $\mathcal{S}_2$) into two subsets, such that the minimum eigenvalue is further increased. But we observe that this step does not exist with the partitioning process in this case. It follows that the set $\mathcal{S}_1$ can be divided into four subsets $\mathcal{S}_{11}, \mathcal{S}_{12}, \mathcal{S}_{13}, \mathcal{S}_{14}$ such that the minimum eigenvalue is now increased to 8 from 4 in the parent set $\mathcal{S}_1$. $\mathcal{S}_{11}$ would comprise of $0, 10$. Similarly, there exists a partitioning of $\mathcal{S}_2$ into subsets $\mathcal{S}_{21}, \mathcal{S}_{22}, \mathcal{S}_{23}, \mathcal{S}_{24}$, each with a minimum eigenvalue of 8. We have thus reached a stage where each set contains only two elements.
Figure 4.1  Labels in a QPSK constellation

<table>
<thead>
<tr>
<th>Codeword</th>
<th>Eigenvalue=2</th>
<th>Eigenvalue=4</th>
<th>Eigenvalue=6</th>
<th>Eigenvalue=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,3,5,12</td>
<td>2,5,7,8,13,15</td>
<td>6,9,11,14</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>6,9,11,14</td>
<td>2,5,7,8,13,15</td>
<td>1,3,4,12</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1,3,6,14</td>
<td>0,5,7,10,13,15</td>
<td>4,9,11,12</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>4,9,11,12</td>
<td>0,5,7,10,13,15</td>
<td>1,3,6,14</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1,4,6,9</td>
<td>0,2,7,8,10,13</td>
<td>3,11,12,14</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>3,11,12,14</td>
<td>0,2,7,8,10,13</td>
<td>1,4,6,9</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>3,4,6,11</td>
<td>0,2,5,8,10,15</td>
<td>1,9,12,14</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>1,9,12,14</td>
<td>0,2,5,8,10,15</td>
<td>3,4,6,11</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 4.1  Table showing the squared singular values of the code difference matrices. This table is used in partitioning the super-constellation formed by the QPSK symbols
4.4 Construction of trellis codes

We will discuss the construction of an eight state trellis code with QPSK modulation and two antennas. The rate of the code is 3/4 and the spectral efficiency is 1.5 bits/sec/Hz.

There are eight transitions from each state with two parallel transitions. The trellis diagram for the code is given in Figure 4.2. The notation followed in labeling this trellis is as follows. The labels of all the branches emerging from a given state are all listed next to the state. The labels within brackets represent parallel transitions. Hence, the code in Figure 4.2 with two labels in brackets indicates the presence of two parallel transitions. Each label in the figure, corresponding to an element in the super-constellation, represents a matrix as explained in the previous section. For example, the label 0 in the super-constellation represents the matrix

\[
\begin{bmatrix}
0 & 1 \\
0 & 2 \\
\end{bmatrix},
\]

where the entries in the matrix now represent QPSK symbols transmitted over the antennas. In above matrix, the columns represent the transmission over antennas whereas the rows represent different symbol durations. Hence, in this example, the QPSK symbols 0 and 1 are transmitted over the first antenna while the QPSK symbols 0 and 2 are transmitted on the second antenna in two successive symbol periods.

The minimum product of the eigenvalues for any two error events for this code is 64. This occurs for the parallel transitions and error events of length two. The Euclidean distance is 8. By construction, all the error events in this code have equal eigenvalues and hence adhere to our requirement for achieving maximum coding gain possible with a given
Euclidean distance (which is 4 in this case, determined by the constellation used for signaling (QPSK here) and the complexity of the code (8 states here)).

A similar problem of partitioning multi-dimensional constellations resulting from the Cartesian products of constellations like QPSK, 8-PSK and 16-QAM was addressed in [18]. A systematic way of partitioning such multi-dimensional constellations based on the Euclidean distance using the concept of multi-level codes was given.

4.5 Maximum likelihood decoder

The likelihood function for the received vector $y$ given that codeword $c$ is transmitted conditioned on the channel realization is given by

$$Pr(y|h, c) = \exp \left( -\frac{\left( \sum_{i=1}^{l} |y_i - \sum_{i=1}^{M} h_i c_i^i|^2 \right) E_s}{4N_0} \right)$$

(4.2)

Hence, the sufficient statistics for maximum likelihood decoding is given by

$$A(c|h) = \left( \sum_{i=1}^{l} |y_i - \sum_{i=1}^{M} h_i c_i^i|^2 \right)$$

(4.3)

We assume error free estimates of the channel at the receiver. We have implemented this maximum likelihood decoder for all our simulations with perfect channel estimates at the receiver.

4.6 Outage probability

In the case of quasi-static fading, where the channel realization is chosen randomly at the beginning of all time and is held fixed for all the uses of
Figure 4.2 8 state EDTCM code for 2 antennas and QPSK modulation transmitting at 1.5 bits/sec/Hz

the channel, the Shannon capacity of the channel is zero [6]. That is, there is a non-zero probability that the realized $h$ is incapable of supporting any rate greater than zero. Such a channel is known is non-ergodic channel. In such cases, the concept of outage probability is of more relevance. Given a rate $R$ and SNR $P$, one can find $P_{out}(R, P)$ such that for any rate less than $R$ and any positive $\delta$, there exists a code satisfying the power constraint $P$ for which the error probability is less than $\delta$ for all but a set of $h$ whose total probability is less than $P_{out}(R, P)$. This concept was originally introduced in [14].

Outage probability serves as a good reference for the performance of the codes designed here and elsewhere, in the case of quasi-static fading. $P_{out}(R, P)$ can be easily calculated in the case of two transmit antennas
and one receive antenna which has been simulated here.

\[ P_{\text{out}}(R, P) = Pr \left( \log_2(1 + \frac{P}{2} \|h\|^2) < R \right) \]

\[ = Pr \left( \|h\|^2 < \frac{2^R - 1}{(P/2)} \right) \quad (4.4) \]

\[ \|h\|^2 \] is a \( \chi^2 \) random variable with four degrees of freedom for two transmit and one receive antenna. Hence, we have

\[ P_{\text{out}}(R, P) = Pr \left( \|h\|^2 < \frac{2^R - 1}{(P/2)} \right) \]

\[ = 1 - e^x(1 + x) \quad (4.5) \]

where \( x = \frac{2^R - 1}{(P/2)} \). This is a fundamental limit which is plotted along with the simulations of frame error rates for comparison. In trellis codes, where the length of the codeword is the same as the length of the frame, the frame error rate is synonymous with the codeword error rate and hence the outage probability is a lower bound for the frame error rate under quasistatic channel conditions.

### 4.7 Higher rate codes

We construct codes transmitting at higher rates by relaxing the equal eigenvalues criterion. In particular, we ensure that the more probable error events confirm to the criterion proposed while the other error events have non-zero eigenvalues, each greater than a certain value fixed by the design.

We make use of the matrices of the form \( T_1(C_1, C_2) = \begin{bmatrix} C_1 & -C_2^* \\ C_2 & C_1^* \end{bmatrix} \) and \( T_2(C_1, C_2) = \begin{bmatrix} C_1 & C_2^* \\ C_2 & -C_1^* \end{bmatrix} \) for this purpose. Note that the two structures
Figure 4.3 8 state EDTCM code for 2 antennas and QPSK modulation transmitting at rate 2 bits/sec/Hz

together give rise to 32 elements with QPSK constellation. The 8 state QPSK code given in the Figure 4.3 transmitting at a rate of 2bps/Hz is constructed with these 32 elements. In the Figure 4.3, the matrices labeled 0 to 15 are obtained from the first structure $T_1(C_1, C_2)$ while the points labeled 16 to 31 are obtained from the second structure $T_2(C_1, C_2)$, where the convention for labeling given in section 4.3 is followed.

4.8 Simulation results

The codes proposed in this work have been simulated along with the 8 state AT&T code employing QPSK constellation [21]. The Alamouti scheme is also simulated for reference. Simulations have been done in quasi-static channel conditions as well as time varying channels with a doppler of 88 Hz and 176 Hz. The frame length for the simulation was 130
Figure 4.4 Performance of the codes in quasistatic fading conditions. Outage probability at 1.5bps/Hz and 2 bps/Hz has also been plotted for reference.
symbol transmissions from each antenna. In the case of quasi-static fading, the outage probability for 1.5 bps/Hz and 2 bps/Hz is also plotted. The performance of the different codes in quasistatic fading is given in Figure 4.4. The EDTCM code transmitting at 1.5 bps/Hz is 2dB away from the outage probability curve. The higher rate EDTCM code transmitting at 2 bps/Hz is also 2dB away from the corresponding outage probability curve.

We claim that the gap in performance can be bridged only by increasing the complexity of the codes. The Alamouti scheme has no coding gain and offers a diversity gain of order 2. The AT&T code has a diversity gain of 2 along with some coding gain. Hence, this code has a better performance than the Alamouti scheme. The EDTCM scheme has the best performance at the given rate of transmission.

Figure 4.5 gives the simulation results for a fading channel with a Doppler of 88 Hz while Figure 4.6 gives the performance of the codes in a fading channel with a Doppler of 176 Hz. It can be seen the EDTCM codes are consistently better by about 2dB-3dB than the other schemes in varying channel conditions.
Figure 4.5  Performance of the trellis codes in a fading channel with 88Hz Doppler
Figure 4.6  Comparison of EDTCM codes in a fading channel with 176Hz Doppler
Chapter 5

Conclusions

5.1 Contribution

In this thesis, we have analyzed the error probability for transmissions over a multiple transmitter multiple receiver channel in a wireless communication scenario. Our work has mainly focused on the case where the receiver is assumed to have complete channel information with no information at the transmitter. We argue that this assumption is not unrealistic and fits particularly well in an indoor environment where the process of fading (and hence the changes in the channel) is found to be slow, which facilitates good estimation of channel at the receiver.

The main contribution of the thesis is the construction of codes based on coding criterion referred to as the equal eigenvalues criterion. We first defined Euclidean distance between codeword matrices in a meaningful fashion for a multiple transmitter system. We have shown that equal eigenvalues condition ensures maximum coding gain for a given Euclidean distance. The coding gain is to be understood as the product of the non-zero eigen values of the code difference matrix. We also argued that the Euclidean distance is a function of the complexity of the code employed (viz., the number of states in the trellis for a trellis code) and the constellation used for signaling on each antenna. Hence, given the constel-
lation and the complexity of the code, the minimum Euclidean distance is upper-bounded and we show that the equal eigenvalues lead to maximum coding gain. Thus, our claim that the optimal coding gain is achieved by our criterion.

We have shown that the new coding criterion leads to robustness. By this, we mean that the codes designed offer a minimum performance guarantee in varying channel conditions. To prove this, we considered a reference case where the transmitter also has complete information about the channel in addition to the receiver. We obtained an upper bound for the error performance of this system. We showed that the so called beam-forming achieves the best performance for this system. We then showed that the equal eigenvalues criterion, with no information at the transmitter, performs within $10 \log(M) \text{dB}$ of this case, $M$ being the number of transmit antennas. We argue that this difference is fundamental in nature (i.e., for schemes with channel information at the transmitter and no channel information at the transmitter). Hence, our claim of optimality is proved.

We have designed a trellis code for two antennas with QPSK constellation containing 8 states, based on the coding criteria presented in this thesis. The rate of the code is 1.5 bits/sec/Hz. We have also designed another code with QPSK constellation at a higher rate of 2 bits/sec/Hz which is a sub-optimal implementation of the equal eigenvalues criterion. We have compared the performance of this code with those occurring in the literature with the same complexity and transmission rate. We observe that the codes presented here perform significantly better in different kinds of channel conditions. We have also discussed the construction of sub-
optimal codes where the most likely error events ensure that the equal
eigen value criterion is satisfied and the condition is relaxed for less prob-
able error events.

5.2 Scope for future work

We achieved the equal eigenvalues condition, which was used in the con-
struction of codes, through a sufficient condition. We showed that the
Radon Hurwitz transformation 2.1, first observed by Alamouti [2] ensures
an orthogonal structure over sub codeword lengths. We then showed that
this is a sufficient condition for equal eigen values over the entire codeword
length. It is yet to be proved (or disproved) that this condition is also a
necessary condition for the design criterion presented in this work.

We have constructed codes for two antennas with QPSK constellation
with 8 and 16 states in the trellis. It is to be noted that the complexity
of the code increases exponentially with the number of antennas and also
the number of points in the constellation. Hence, it would be necessary to
look into lower complexity sub-optimal coding schemes with comparable
performance. Further, it is possible to design codes with constellations of
different sizes on different antennas so that data rates can be increased
with controlled increase in complexity.

It would be interesting to consider the role of feedback at the transmitter
in designing codes for multiple antennas. It is not totally unrealistic to
assume (partial) channel state information at the transmitter, especially
in systems like time division duplex signaling.
The field of multiple transmitter systems is beginning to attract the attention of researchers all over, offering a wide range of open problems. This certainly appears to be among the most viable solutions in bridging the performance gap between the wired communications and the wireless communications.
Bibliography


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