An Approach to Capacity Analysis of Coarsely Managed Wideband Multiple Access Systems

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Abstract

We consider the problem of coarsely managed multiple access wideband systems, where each user can control its own transmission power and rate, independent of other users' actions, according to the *policy* specified by central controlling agent: base station. Even with such coarse coordination, multiuser detection enables a system superior to any orthogonal division system like TDMA. We fully characterise the set of such available coarse management policies for wideband systems: robust spectral efficiency slope region. Finally, we show that slopes on the boundary of robust slope region lead to an elegant interpretation of robust slope region in terms of awarding protected receiver dimensions to each user.

1 Introduction

Capacity of multiple access systems is commonly studied with an inherent assumption that the central agent (e.g., base-station) exercises tight control over transmissions of all users. This control typically includes rate and power allocation for each user, with the goal of providing reliable reception. We refer to such systems as *tightly managed.* In multiple access data networks, which are characterized by bursty data and delay-bounded traffic, tight management implies that users must re-negotiate their rates and powers with the base-station frequently throughout the call duration. Any change in transmission parameters (rate or power) of one user, affects the whole system, necessitating a rate and power adaptation for all other users. In this paper, we study the capacity limits of wideband multiple access systems where such tight control may not be feasible or desirable.[1]

We approach the problem by assuming that only a coarse control is exercised by the base-station. In effect, each user is *only* given a *policy* according to which it (user) locally varies its own data rate, and power to meet the data rate requirements. Users are assumed oblivios to each others data rate or transmit power. Each policy is carefully chosen by the base station, such that multiple access capacity constraints are not violated under any circumstances. We label such systems as *coarsely managed.* It is apparent that orthogonal multiple access schemes, like TDMA or FDMA, trivially enable coarse control; base-station allocates time or frequency slots and users can vary their rates/power based on single-user capacity limits. The question we ask is if there is a multiple access scheme which achieves higher spectral efficiencies than orthogonal systems, while allowing local data rate adaptation by each user. We answer the question in affirmative, and in fact we provide an entire family of solutions, in regime of low spectral efficiencies. This regime is defined in [1], where the capacity theorem is captured by minimum energy per bit $\mathcal{E}_{min}$, and the slope of spectral efficiency at $\mathcal{E}_{min}$.

Our main result is a new concise characterization of the above mentioned family of solutions for coarse management policies, namely *robust slope region*, first defined by the authors in [4]. Robust slope region includes all those single user wideband slopes which are achieved independent of other users' rates or powers. In many cases of interest, like multiple users in multiple antenna Rayleigh fading channels, the robust slope region is strictly larger than the slope region achieved by any of the orthogonal multiple access systems. Based on robust slope region, the base-station assigns appropriate single-user slopes, which define a policy according to which users control their individual rates and powers. Both rate-dependent slope region [2, 3] and robust slope region are effectively characterised using the *slope matrix* that we introduce in the next section. This matrix depends only on physical characteristics of the multiuser channel and is the multi-user analogue of achievable slope [1].

For the special case of Rayleigh channels with additive white Gaussian noise, the robust slope region admits a very intuitive explanation via *virtual receiver dimension partitioning.* We show that the each point on the boundary of the robust slope region can be represented as a K-tuple of single-user slopes, where each user is communicating using all its antennas to a subset of base-station's antennas *in isolation* from other users'. Thus, the receiver dimensions can be partitioned and virtually dedicated to different users.

[1] Note that our formulation differs from a totally uncoordinated multiple access system, where receivers for each user are different and perform no form of coordination; see work by Peter Gober, ISIT 2000.

Our focus on wideband systems is mostly motivated by increased interest in ultrawideband systems, where users will operate with very low powers and designers will opt for coarse control to keep system cost low. The rest of the paper is organized as follows. Section 2 provides full characterization of multiaccess slope region for tightly managed systems, given first and second partials of aggregate capacity function. Analogously, Section 3 provides such characterisation for coarsely managed systems, demonstrating relationship between receiver dimensions and coarse control policy, and provides several design guidelines. Section 4 is the conclusion. Proofs are only sketched due to space limitations.

2 Wideband Multiple Access Slope Region - Tightly Managed Systems

In this section, we first give a new characterization of multiuser slope regions and define the multiuser generalization of achievable slope, labeled as *slope matrix.* Results of this section generalize single user discussion [1], and provide a concise parameterization of known results for two-user systems [3]. Consider the following generic class of additive noise channels

$$Y = f_1(X_1) + \cdots + f_K(X_K) + \eta, \tag{1}$$

where f_k denotes a random or deterministic transformation of user k signal X_k. We assume that each user employs codebooks derived from optimal single user input distributions, which happens to be optimal for linear channels with additive gaussian noise. For cases where the optimal multiuser codebooks are not same as single-user optimal codebooks, our results give an inner bound to actual slope region.

Achievable rate region of K-user channel is a polymatroid with $2^K - 1$ faces [5], defined as follows. Let $\bar{A}$ denote the complement of set $A \subseteq \{1,..,K\}$. Aggregate date rate of users in set A must be less than

$$C_A(P_A) \stackrel{\triangle}{=} I\left(X_A; Y|X_{\bar{A}}\right) = I\left(X_A; Y|X_{\bar{A}} = 0\right), \tag{2}$$

where, $P_A = [P_k]_{k\in A}$ represents the vector of power constraints for users in the set A, and $C_A(P_A)$ denotes the family of maximal mutual informations indexed by A. For simplicity of notation, indexing set A is suppressed when $A = \{1, 2, ..., K\}$, and we write $C(P)$.

Our analysis is performed for wideband low power systems, where capacity relationship between data rate and power is effectively described by the minimum energy per bit $\mathcal{E}_{min,k}$ and slope $\mathcal{S}_k$ of spectral efficiency R_k [1] for each user k. Minimum energy per bit, $\mathcal{E}_{min,k}$ is simply the reciprocal of first derivative of capacity function at zero transmit power [1], and it is the same in single and multiuser systems. Besides this, the slope $\mathcal{S}_k$ of spectral efficiency R_k at $\mathcal{E}_{min,k}$, captures bandwidth requirements of the system and completes the theorem. This slope is defined in decibel scale as

$$\mathcal{S}_k \stackrel{\triangle}{=} \lim_{\mathcal{E}_k \downarrow \mathcal{E}_{min,k}} \frac{R_k}{\log_2 \mathcal{E}_k - \log_2 \mathcal{E}_{min,k}} \quad \text{[bits]/[symbol} \times 3\text{ dB]}. \tag{3}$$

For single user systems slope $\mathcal{S}$ depends on the channel via derivatives of capacity function $C(P)$, evaluated at $P = 0$, and not on actual transmit power or rates [1]. For multiple users, the slope region depends on the ratio of rates between the two users [2]. Though the slope region depends on ratio of rates (or direction of rate vector in the two-dimensional rate space), we show that the following matrix, labeled as *slope matrix*, is the multiuser generalization of single-user slopes.

Definition 1. Slope Matrix $\mathfrak{C}$ *for a* $K-$*user multiaccess channel is a* $K \times K$ *matrix defined as follows*

$$\mathfrak{C}^{-1} \stackrel{\triangle}{=} \frac{1}{2}\,\mathrm{diag}\big[\nabla C(0)\big]^{-1}\,\big[-\nabla^2 C(0)\big]\;\mathrm{diag}\big[\nabla C(0)\big]^{-1} \tag{4}$$

$\nabla^2 C(0)$ *is the Hessian matrix of second partials, and* $\nabla C(0)$ *is the gradient of* C, *both evaluated at* 0. *Operator* diag *produces a diagonal matrix from the gradient vector* $\nabla C(0)$. *Finally,* $\mathfrak{C}_A$ *denotes the Slope Matrix with both rows and columns only from the set* A.

Definition (4) simply states that entires of $\mathfrak{C}^{-1}$ are second order partials of $C(P)$ divided by the product of corresponding first partials of $C(P)$. Following properties of $\mathfrak{C}$ can be immediately noted: $\mathfrak{C}$ is symmetric, and non-negative definite because $-\nabla^2 C(0) \geq 0$, which is multiplied from left and right by the same diagonal matrix in (4). More importantly, $\mathfrak{C}$ depends only *physical channel characteristics*, e.g., fading distribution and number of antennas. All dependency of $\mathfrak{C}$ on transmit power P has been removed by evaluating the partials at $P = 0$. Optional constraints, such as discrete alphabet or a set of spreading codes, are implicitly incorporated in $\mathfrak{C}$, as part of physical channel characteristics. Furthermore, $\mathfrak{C}_A$ equals the Slope Matrix for users within A, when all other users are silent. Put $A = \{k\}$ to see that diagonal entries of $\mathfrak{C}$ are relevant for each individual single user channel. Off-diagonal entries capture the multiple access interference. We now state the main result for this section.

Theorem 1. *Let Σ denote a diagonal matrix of single-users slopes $\Sigma \triangleq \mathrm{diag}\left(\mathcal{S}_1 \ldots \mathcal{S}_K\right)$. Let Σ_A denote the submatrix of Σ, maintaining only rows and columns within A. Let $R \triangleq \left[R_1 \ldots R_K\right]^T$ such that $\|R\| = 1$, is the direction vector in the K dimensional space of actual transmission rates, and similarly denote by R_A the sub-vector of R with indices from subset A. The slope region* CDMA(R) *is given as*

$$\mathrm{CDMA}(R) = \left\{\Sigma: \; R_A^T \left[\Sigma_A^{-1} - \mathfrak{C}_A^{-1}\right] R_A \geq 0, \;\; \forall A \subseteq \{1,..,K\}\right\}. \tag{5}$$

Proof. Follows from (3) and multi-dimensional Taylor series expansion of generic capacity constraint (2). □

Notation CDMA(R) comes from achievability by superposition, and not necessarily orthogonal codes. The key point to note is that the slope region (5) does not depend on actual rates but only the ratio of rates or a $K-1$ direction vector in a K dimensional space. In [3], the slope for two-user multiple access channel is characterized by a single parameter θ which is the ratio of actual rates transmitted by the two users.

Following example clarifies the structure of slope region and slope matrix. Consider K users in a correlated multiple antenna each using a spreading code. User k has N_k transmit antennas, receiver is made up of M antennas, and symbols of each user are modulated by row spreading sequence s_k^T of length L and unit energy. Let columns of the received $M \times L$ matrix Y represent consecutive chip intervals, and let matrix η represent complex additive white Gaussian noise

$$Y = \sum_{k=1}^{K} \left[\Gamma^{1/2} H_k \Upsilon_k^{1/2}\right] X_k s_k^T + \eta. \tag{6}$$

In channel model (6) we implicitly assume that coherence period lasts for at least L chips. Transmit antenna correlation for each user is labeled with Υ_k, and the receive correlation Γ is the same for across all users. Also, channel randomness is captured by matrix H_k, which is independent, identically distributed and rotationally invariant. Generic entry h of H_k has kurtosis κ, which is defined as $\kappa(h) = E[|h|^4]/E^2[|h|^2]$.

Example 1. *Let $\rho(k,l) = s_k^\dagger s_l$ denote correlation between spreading sequences of users k and l. If users allocate powers evenly across all transmitters, then $\mathcal{E}_{min,k} = (\ln 2)/M$, and slopes in single user mode are*

$$2\mathcal{S}_{k,su}^{-1} = \frac{tr\left(\Gamma^2\right) tr\left(\Upsilon_k^2\right)}{M^2 N_k^2}(\kappa - 2) + \frac{tr\left(\Gamma^2\right)}{M^2} + \frac{tr\left(\Upsilon_k^2\right)}{N_k^2}. \tag{7}$$

On the other hand, if $\Upsilon_k \neq \mathrm{I}$, and users choose to exploit antenna correlation by directing their transmissions along the maximum eigenvalue of transmit correlation matrix Υ_k , then $\mathcal{E}_{min,k} = \lambda_{max}^{-1}(\Upsilon_k)(\ln 2)/M$, and

$$2\mathcal{S}_{k,su}^{-1} = \frac{tr\left(\Gamma^2\right)}{M^2}(\kappa - 1) + 1. \tag{8}$$

Further define $\omega = M^2/tr\left(\Gamma^2\right)$, then the slope matrix is given as

$$\mathfrak{C}^{-1} = \left(2\omega\right)^{-1} \begin{bmatrix} 2\omega\mathcal{S}_{1,su}^{-1} & |\rho(1,2)|^2 & \ldots & |\rho(1,K)|^2 \\ |\rho(2,1)|^2 & 2\omega\mathcal{S}_{2,su}^{-1} & \ldots & |\rho(2,K)|^2 \\ & & \ddots & \\ |\rho(K,1)|^2 & |\rho(K,2)|^2 & \ldots & 2\omega\mathcal{S}_{K,su}^{-1} \end{bmatrix} \tag{9}$$

The slope matrix (9) is found by evaluating first and second partials of the capacity function for the channel model (6), and special cases may be derived from the above example. One special case is given by assuming random long spreading codes, and further expectation must be taken to arrive at $E|\rho(k,l)|^2 = L^{-1}$, except if $k = l$. Notice that off-diagonal entries in (9) reflect two brands of user separation: multi-antenna by ω, and spreading sequences by $\rho(k,l)$. If receiver antennas are uncorrelated, then $\Gamma = \mathrm{I}$ and $\omega = M$. For a more general case, ω measures the equivalent of number of effective uncorrelated receivers and can thus be thought in units of [uncorrelated receivers]. As ω increases, off-diagonal entries of $\mathfrak{C}^{-1}$ vanish, as all K users become increasingly uncorrelated.

Tight control of transmission parameters by the base station may be implemented as follows using Theorem 1, and channel-specific slope matrix $\mathfrak{C}$. A particular user alters data rate based on request from higher communication layer, and a brand new set of slopes must be re-computed for *all users* from (5), provided that spectral efficiencies are small. Region (5) allows for many solutions, but a simple strategy which solves $\left[\Sigma^{-1} - \mathfrak{C}^{-1}\right] R = 0$ for Σ would suffice to satisfy all constraints in (5). The following section will actually describe as to why this strategy would be desirable, out of all possible solutions. With new slope $\mathcal{S}_k$, and data rate R_k, each user must re-adjust his energy per information bit $\mathcal{E}$ according to (3), thereby completing the tight control policy. We now address the case where such tight control is not available, and transmission parameters are controlled locally by the mobiles.

3 Robust Slope Region - Coarsely Managed Systems

In this section, we present the main result of the paper, which is the characterization of all single-user slope which are simultaneously achievable *such that* each user can adapt its power and rate individually without affecting other users. Since the admissible slopes in CDMA(R) are dependent on the rate proportions, users cannot use these slopes for their individual power and rate control. Instead their slopes have to be chosen from a set which is *independent* of R, which leads us to the following definition of robust slope region.

Definition 2 ([4]). Robust Slope Region *is defined to be the intersection of all slope regions* CDMA(R), *taken over all possible rate proportions* $R = \left[R_1 \dots R_K\right]$, *such that* $\|R\| = 1$,

$$\mathrm{CDMA}_\star = \bigcap_R \mathrm{CDMA}(R). \tag{10}$$

The above definition of a rate-invariant slope region is in the same spirit as that of *compound channel capacity* [6], where channel itself has a parameter θ, which is unknown to the transmitter, and capacity in most cases turns out to be $\min_\theta \max I(X, Y)$. In our case, the unknown parameters are other users' transmission rate and power. While min-max formulation is appropriate for single user, multiple access requires a generalization to higher-dimensional objects (sets), and $\cap\cup$ operation generalizes min-max. For us, $\cup$ is already evaluated by assuming optimality of single user distributions, and $\cap$ is inside the definition of the robust slope region. Figure 1 provides an interpretation for two users, where inside envelope of all regions CDMA(R) defines $\mathrm{CDMA}_\star$. In coarsely managed systems, $\mathcal{S}_k$ defines policy of user k, according to which rate and energy per information bit are to be varied locally, and defining relationship (3) serves this purpose. For example, given $\mathcal{S}_k$ along with another parameter, say R_k, user may solve for the missing variable $\mathcal{E}_k$ locally, and adjust transmission power accordingly. Transmission reliability is assured, provided that all users adhere to their policy $\mathcal{S}_k$, and provided that receiver utilizes multiuser detection. We now provide a closed-form characterization of the robust slope region CDMA_*, again in terms of the slope matrix $\mathfrak{C}$.

Theorem 2 (Robust Region Characterization). *For any* $\mathfrak{C}$, *the robust slope region is described by*

$$\mathrm{CDMA}_\star = \left\{\Sigma : \quad \lambda_{max}\left(\Sigma^{1/2}\mathfrak{C}^{-1}\,\Sigma^{1/2}\right) \leq 1\right\}. \tag{11}$$

Proof. Based on Rayleigh quotient and positive semi-definiteness of $\mathfrak{C}$. □

Above theorem provides additional intuition about the slope matrix: if $\mathfrak{C}^{-1} = \Sigma_{su}^{-1}$ were diagonal, then users would be entirely orthogonal with $\Sigma \leq \Sigma_{su}$, which holds for each diagonal element. Intuitively, when $\mathfrak{C}^{-1}$ is close to diagonal, users are separable and close to orthogonal. By "close" we mean that in coarsely controlled systems, almost full single user slopes may be allocated to each user, as if no other users were present. We conclude that in such fortunate cases, coarse control is sufficient, even if tight control were available. For a specific example of $\mathfrak{C}^{-1}$ in (9) we conclude that $\mathfrak{C}^{-1}$ approaches a diagonal matrix as receiver multi-antenna dimensionality ω grows with respect to number of users. In fact, relationship between coarsely managed systems and receiver dimensionality is even stronger, and is given by the following corollary.

Corollary 1 (Virtual Antenna Partition). *Assume K users with N_k transmit antennas each, and M receive antennas, where coefficients are Rayleigh distributed, and with* $\Gamma = \Upsilon_k = \mathrm{I}$ *in (6). Let*

$$M = M_1 + M_2 + ... + M_K \tag{12}$$

be arbitrary (virtual) division of M receivers into K (not necessarily integer) partitions. The rectangle specified by

$$2\mathcal{S}_k^{-1} \geq N_k^{-1} + M_k^{-1} \quad \text{for} \quad k = \{1, ..., K\}, \tag{13}$$

belongs to the robust slope region $\mathrm{CDMA}_\star$. *Conversly, the region itself is union of all such rectangles. Furthermore, (13) are slopes as if user k was communicating with N_k transmit antennas to M_k receivers, shielded away from others. To see this put* $\kappa = 2$, $\Gamma = \mathrm{I}$, *and* $\Upsilon_k = \mathrm{I}$ *in (7).*

The above corollary is portrayed in Figures 1-4. Point A is equivalent to the extreme case where all receive antennas are allocated to to User 1, and User 2 can have no transmission whatsoever, i.e., $\mathcal{S}_2 = 0$. If policies are divided according to Point B, User 1 is given slope $\mathcal{S}_1$ equivalent to three receive antennas, whereas User 2 is granted slope corresponding to one receive antenna. Point C is the symmetric, fair case, of equal slopes. In reality, all signals are received on all antennas, and multiple access interference is unavoidable, but it is still possible to perform a

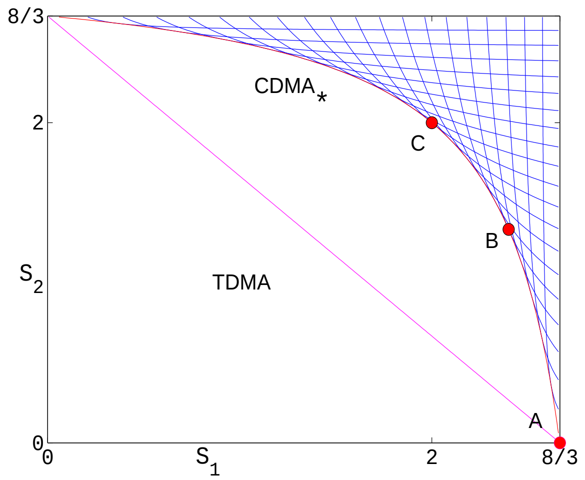

Figure 1: Slope regions for 2 users. Each User has 2 transmit antennas. There are 4 receive antennas.

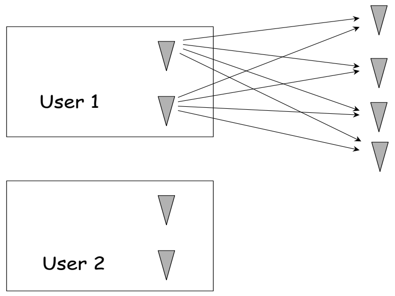

Figure 2: Equivalent to Point A on CDMA$_\star$

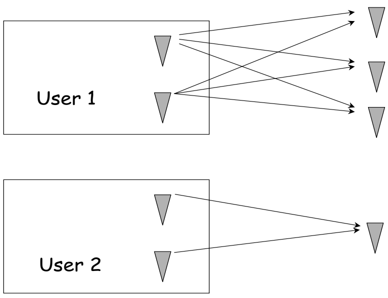

Figure 3: Equivalent to Point B on CDMA$_\star$

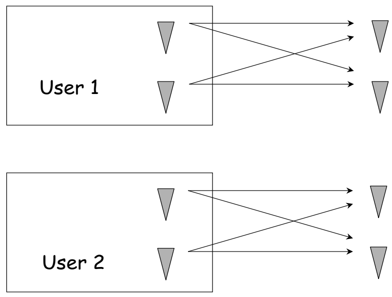

Figure 4: Equivalent to Point C on CDMA$_\star$

virtual partition of the receive antenna spectrum for coarsely managed systems. Above corollary concludes that spatial dimensionality is amenable to divisions in coarsely managed systems, much like dimensionality of time or frequency. Practical difference between the two is that dividing spatial dimensions requires multiuser detection, while dividing temporal dimensions does not (like in TDMA). Finally, channel gain is not portrayed in Figures 1-4, but it is always equivalent to four receive antennas, since $\mathcal{E}_{min}$ is not affected by multiple access [1]. Furthermore, virtual antenna partition need not be an integer partition (M_k can be any non-negative real number), and antenna correlation can be incorporated by partitioning $\omega = M^2/tr(\Gamma^2)$, "effective width" of receive antenna spectrum. Following corollary is the more general version of Corollary 1.

Corollary 2 (Dividing Receiver Dimensions). *Suppose connectivity matrix is provided in (9) with no temporal spreading $\rho(k,l) = 1$. The robust slope region can be parametrized as follows*

$$\text{CDMA}_\star = \bigcup_{\sum_k \omega_k = \omega} \Big\{(\mathcal{S}_1, .., \mathcal{S}_K): \quad 2\mathcal{S}_k^{-1} - 2\mathcal{S}_{k,su}^{-1} \geq \omega_k^{-1} - \omega^{-1}\Big\}. \tag{14}$$

As is apparent from (14), a particular partition of the receive antenna spectrum ω, namely the choice of ω_k, results in a specific set of unconditionally achievable slopes, which belong to the robust slope region CDMA$_\star$. Non-integer partition should not be surprising, since ω measures spatial dimensionality and when multiplied with time-frequency product of transmission, the resulting number of dimensions is an integer. Finally, we revisit the slope allocation policy suggested at the end of Section 2, which works at the operating point satisfying $\left[\Sigma^{-1} - \mathfrak{C}^{-1}\right] R = 0$ for Σ on the region boundary of $CDMA(R)$. Incidentally, this operating point also lies on CDMA$_\star$ as well, and hence is not only optimal for a given R but also provides insensitivity to variable transmission parameters. In other words, the above operating is insensitive to rate and power perturbations by other users.

3.1 Further Consequences of CDMA$_*$

Orthogonal Systems: Consider an orthogonal system, like TDMA or FDMA. The following theorem characterizes their performance, and shows that they are inferior even for coarse management policies.

Theorem 3 (Robust Slope Region of TDMA). *Slope region achieved by TDMA is contained in* CDMA$_\star$

$$\text{TDMA} = \left\{(\mathcal{S}_1,..,\mathcal{S}_K):\quad \sum_{k=1}^{K}\mathcal{S}_k\mathcal{S}_{k,su}^{-1} \le 1\right\} = \left\{\Sigma:\ tr\left(\Sigma^{1/2}\,\mathfrak{C}^{-1}\,\Sigma^{1/2}\right)\le 1\right\} \subseteq \left\{\Sigma:\ \lambda_{max}\left(\Sigma^{1/2}\,\mathfrak{C}^{-1}\,\Sigma^{1/2}\right)\le 1\right\} \tag{15}$$

Proof. To derive slope region of TDMA, divide time unit into slots of length α_k, given to user k. Then $\mathcal{S}_k = \alpha_k\ \mathcal{S}_{k,su}$, and the left equality in (15) follows. Chain follows since inverse single user slopes are on diagonal of $\mathfrak{C}^{-1}$. □

Necessary and sufficient condition for equality of TDMA and CDMA$_\star$ is that the matrix $\Sigma^{1/2}\mathfrak{C}^{-1}\Sigma^{1/2}$ is rank one and in this case, the connectivity matrix $\mathfrak{C}^{-1}$ has only one eigenvalue. In single antenna systems, $\mathfrak{C}^{-1}$ has only one eigenvalue if there is no fading in the system to separate the users statistically (or equivalently, the spatial channels are perfectly correlated).

Optimal sequences for Symmetric Policies: Consider a system when the set of spreading sequences, of fixed length L, are not allowed to vary with data rates. Here, sequences may be designed so that every user is allocated with an equal slope policy. We will find the set of spreading codes such that CDMA$_\star$ is achieved for the case of $\mathcal{S} = \mathcal{S}_k$. Here, $\Sigma = \mathcal{S}\,\mathrm{I}$ and the admissibility condition (11) states

$$\mathcal{S} \le \lambda_{max}^{-1}\left(\mathfrak{C}^{-1}\right). \tag{16}$$

Theorem 4 (Saddle Point of WBE Sequences). *When $\mathcal{S}_{k,su}$ are all equal, then WBE sequences minimize the maximum eigenvalue of the $\mathfrak{C}^{-1}$. Corresponding eigenvector is the vector of all ones* $\mathbf{1} = [1..1]$.

Proof. Express maximum eigenvalue using the Rayleigh quotient. First, $\mathbf{1}^T\mathfrak{C}^{-1}\mathbf{1}$ is minimized when $\mathfrak{C}^{-1}$ consists of WBE sequences, because they minimize total square correlation [1, 7] and others. From the other side, if $\mathfrak{C}^{-1}$ consists of WBE sequences, then $\mathbf{1}$ is eigenvector with the maximum eigenvalue. □

4 Conclusion

In this paper, we defined and characterized the notion of robust slope region, a key concept to study the performance of coarsely controlled systems. Fundamental to our characterizations was the slope matrix, which is a function of the multiuser channel between the transmitters and base-station. For Rayleigh fading channels, the concept of robust rate region is equivalent to virtual partitioning of receive dimensions and allocating them to each user such their received signals are isolated from other users.

The concept of dimension defined using the parameters ω appears to be more generally applicable, and could also impact design of suboptimal low-complexity receivers and resource management.

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