OPTIMAL DIGITAL COMMUNICATION OF ANALOG SIGNALS

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ABSTRACT

In this paper, the problem of optimally communicating analog sources using a bandwidth and power limited digital system is considered. We propose and analyze optimal combined source-channel coding schemes that jointly optimize the compression of the source while controlling the individual bit error probabilities to minimize the mean-squared distortion in reconstructing the signal. For a spectrally shaped channel, the proposed schemes are most efficiently implemented using multicarrier modulation. We optimize the power allocation across subchannels and the constellation within each subchannel. The results indicate that the optimal power allocation is not achieved by a water-filling solution and significant gains (several dB) can be obtained by our optimal design.

1. INTRODUCTION

In a resource limited system with complexity constraints, joint source-channel coding is known to achieve superior performance as compared to designs based on the separation theorem of Shannon [1]. The common approach to digitally communicating an analog signal is PCM: sample and quantize the signal amplitude into bits that are sent over a noisy channel. These bits are not equally important since they contribute differently to the mean-squared distortion. For example, an error in the most significant bit (MSB) is far more destructive than an error in the least significant bit (LSB). Therefore, bits need to be unequally protected against communication errors. Additionally, the number of quantization bits or in other words, the compression level has a direct impact on the achievable distortion. Therefore, an optimal joint source-channel coding has to simultaneously address both issues.

The idea of providing unequal bit error probabilities for optimal communication of analog signals was introduced by Bedrosian [2] and pursued in later related work [3, 4]. Bedrosian and others assumed an AWGN channel and their schemes try solely to find the optimal share of each bit's power in an effort to only minimize the channel-induced distortion. However, for an intersymbol interference (ISI) channel where the spectrum is spectrally shaped, the optimal strategy, even for minimizing the channel-induced distortion only, is not achieved by a simple power allocation scheme.

Multicarrier modulation (MCM) [5] is best known for its simplicity and efficiency in combating ISI. In this method, the transmission band is decomposed into orthogonal frequency bins (subchannels) over each, the channel transfer function is almost constant. Since the transmission scheme of each subchannel can be independently designed, MCM enables efficient implementation of joint source-channel coding without introducing additional system complexity. Ho et al. were first to suggest multicarrier modulation for unequal bit error protection [4]. Because they assume AWGN channel, their proposed scheme is again a power allocation problem among the frequency bins carrying bits of different importance. Zheng et al. consider transmitting layered coded multimedia data over a spectrally shaped channel using MCM [6]. However, their approach is suboptimal as source compression is not considered. Moreover as we will show, encoding bits of different importance into the same subchannel might result in superior performance, an idea not considered in [6]. Therefore to achieve optimality, we developed new design techniques and here we analyze the performance improvements.

2. OPTIMAL JOINT SOURCE-CHANNEL CODING AND THE DISTORTION CRITERION

Without loss of generality, we consider analog to digital (A/D) conversion as the quantization technique and the analog signal amplitude to be normalized to the range $(0, 1)$. Therefore, a sampled amplitude s is represented by a K bit sequence as

$$s = \sum_{k=1}^{K} b_k 2^{-k}, \tag{1}$$

where b_k is the k^{th} bit in the sequence. The mean-squared distortion in transmitting the analog signal through the noisy

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channel is expressed as

$$D = E[(s - \widehat{s})^2], \tag{2}$$

where $\widehat{s}$ denotes the recovered amplitude corresponding to s using the received bits $\widehat{b}_k$. Replacing (1) in (2), we will have

$$D = E\left[\sum_{k=1}^{K}\sum_{l=1}^{K}(b_k - \widehat{b}_k)(b_l - \widehat{b}_l)2^{-(k+l)}\right] \tag{3}$$

Denoting the correlation coefficient between the k^{th} and l^{th} bit errors as $\rho_{k,l}$, the general expression for the distortion is obtained as

$$D = \sum_{k=1}^{K}\sum_{l=1}^{K}\rho_{k,l}2^{-(k+l)}\sqrt{p_e^{(k)}p_e^{(l)}}, \tag{4}$$

where $p_e^{(k)}$ and $p_e^{(l)}$ are the bit error probabilities of the k^{th} and l^{th} bits, respectively.

To simplify the analysis and design, let us consider independent bit error probabilities where the correlation coefficients between different bit errors are zero.

$$D = \sum_{k=1}^{K} 2^{-2k} p_e^{(k)} \tag{5}$$

We note that an optimal design based on minimizing the distortion in (5) involves unequal protection for bits of different importance. Additionally, increasing the number of quantization bits has an exponentially decreasing effect on reducing the mean-squared error. At the same time, the power consumed to send the additional low significant bits of a less compressed signal might be better invested by higher protection of a more compressed signal's bits against channel errors. It is this effect that calls for considering source compression and channel bit error rates together to optimize system's performance.

Let us consider an MCM system that decomposes the ISI channel into N orthogonal frequency bins. We also assume that the source sampling period equals the MCM symbol interval. We assume a stationary channel that filters the signal, transfer function $H(f)$, and adds white Gaussian noise at the receiver with variance σ^2. The overall channel and noise effect is characterized by each bin's channel gain to noise ratio (CGNR), G_n, defined as

$$G_n = \frac{|H(f_n)|^2}{\sigma^2}. \tag{6}$$

where σ^2 denotes the noise variance and f_n equals the frequency of the n^{th} subchannel. The optimal joint-source channel coding is integrated into the MCM transmitter and involves determining the source compression level, apportionment of bits to frequency bins (bit loading) and therefore signal set design as well as power distribution among subchannel.

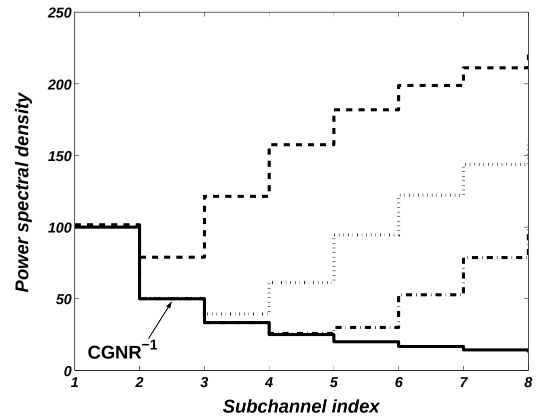

Fig. 1. The dotted lines depict the optimal power allocation for different power constraints. The amount of power poured into the frequency bins increases as the total power increases.

3. MULTICARRIER MODULATION WITH BPSK SIGNALING

The simplest transmission scheme based on the criterion of (5) employs BPSK modulation on each subchannel. Denoting P_n and G_n as the power assigned to and the CGNR of the n^{th} bin, the bit error probability of the corresponding BPSK symbol is the tail of a Gaussian distribution $Q(\sqrt{P_n G_n})$. Interestingly, it can be proven that for a distortion criterion, implementing a diversity scheme in frequency by transmitting a symbol over more than one frequency bin is not optimal when their CGNRs differ. Therefore, a total of N quantization bits at the output of the A/D converter can be optimally encoded into each MCM symbol.

To implement unequal bit error protection, we have shown that the assignment of bits to subchannels can be optimally performed by the following algorithm: Decreasingly sort both bits and bins based on their weights and gains, respectively. The optimal scheme assigns bits to bins based on this ordering; that is, assign the highest weight bit to the best channel and so on until all bits are assigned. The optimal design is achieved by solving the following convex optimization problem

$$\min_{P_n \geq 0} \quad D = \sum_{n=1}^{N}\omega_n Q\left(\sqrt{P_n G_n}\right), \tag{7}$$
$$\text{subject to} \quad \sum_{n=1}^{N} P_n = P_t,$$

where P_t is the system's power constraint. As an example, Fig. 1 shows the optimal power allocation for an analog signal quantized to 8 bits and modulated as BPSK symbols that are transmitted over eight frequency bins. Different dotted lines correspond to different power constraints.

The optimizer allocates higher power to MSBs. In the example, the MSB was always assigned to subchannel 8 and the LSB to subchannel 1. At smaller power constraint values the LSB is not allocated any power meaning that it is not sent. Note that the optimal design is not achieved by a water-filling solution. It can also be seen that especially for small power constraints, some of the least important bits are not assigned any power, which means we are compressing the source more to reduce distortion.

To study the achievable gains, we have plotted the mean-squared distortion gain of our approach along with water-filling, equal error and equal power allocation schemes compared to the worst case when all bit error probabilities are $1/2$. Equally protecting bits, $P_n G_n$ = constant, results in the lowest gain. It can be seen that the optimal approach achieves the highest gain and performs better than water-filling by approximately 5 dB.

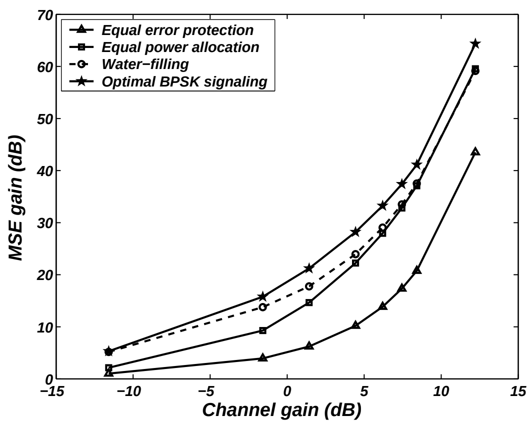

Fig. 2. The mean-squared distortion gain of different schemes as compared to the worst case scenario when all bit error probabilities are 0.5.

4. QAM SIGNALING AND SUBCHANNEL CONSTELLATION DESIGN

We analyze increasing the constellation size of each subchannel for two reasons. First of all by sending more bits over each frequency bin, less compressed signals can be encoded into each MCM symbol without the need for increasing the bandwidth. Secondly, this approach provides the possibility of transmitting more bits over the higher gain subchannels which could reduce distortion. While the limits of this approach has to be further studied, in this section we present how QAM signaling can be used to achieve this goal.

The constellation of the rectangular QAM signal set of the n^{th} bin is depicted in Fig. 3, where Gray coding has been used to map the bit pairs into symbols. Considering x and y axes as the boundaries of the decision region, Gray coding on a QAM signal set will result in independent bit error probabilities for the two modulated bits, regardless of the channel gain. Therefore, the design criterion is determined by (5). In a traditional QAM, $\theta_n = \pi/4$ and there-

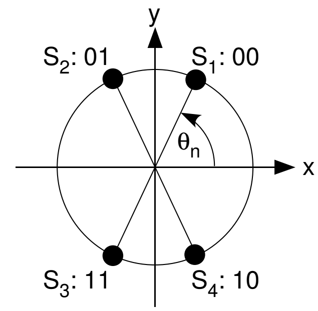

Fig. 3. QAM constellation of the n^{th} bin with Gray coding. It can be seen that the bit error events of the modulated bits are independent of each other.

fore the two generating bits will be assigned equal powers, resulting in equal bit error probabilities. To provide unequal error protection for individual bits, we optimally design the QAM signal set of different bins by letting the angle, θ_n, of different frequency bins be design variables to be optimized. The joint source-channel coding problem for QAM becomes

$$\min_{\substack{P_n \geq 0 \\ \theta_n \in [0, \pi/2]}} \quad D = \tag{8}$$

$$\sum_{n=1}^{N} \omega_{2n-1} Q(\sqrt{P_n G_n} \cos\theta_n) + \omega_{2n} Q(\sqrt{P_n G_n} \sin\theta_n)$$

$$\text{subject to} \quad \sum_{n=1}^{N} P_n = P_t,$$

where ω_{2n-1} and ω_{2n} are the weighting coefficients of the first and second bits modulated to the QAM symbol over the n^{th} bin, and $P_n \cos^2\theta_n$ and $P_n \sin^2\theta_n$ are the corresponding power allocations.

The bit loading algorithm is optimally performed in a similar fashion to BPSK except that two bits are assigned to each frequency bin. We have proved that only by optimally designing the QAM constellation angle in each bin, does the scheme provide lower distortion. Note that QAM receivers of the different subchannels need not know what value of θ_n the transmitter used. In other words, optimal design can be implemented at no additional receiver complexity.

Fig. 4 shows the distortion gains that resulted. The analog source is sampled with a 16-bit A/D converter. The bits are to be transmitted over 8 frequency bins using QAM and BPSK signaling. The BPSK approach retains at most the 8 most significant bits. Significant gains accrue using QAM

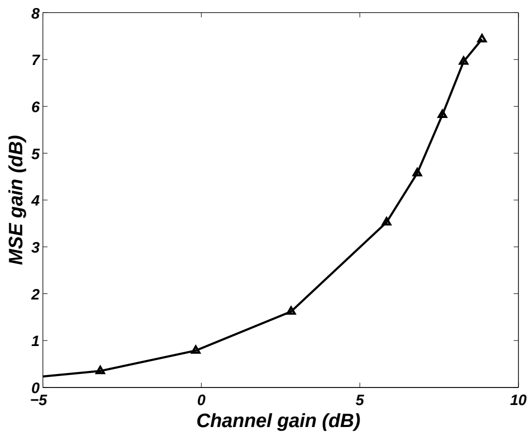

Fig. 4. The distortion gain obtained for an analog source quantized to 16 bits and transmitted over 8 subchannels by optimal QAM design as compared to BPSK which drops the 8 LSBs.

without any increase in the transmission bandwidth by better protecting the MSBs and transmitting less compressed signal (16 instead of 8 bit quantizer).

5. CONCLUSIONS

Optimal digital communication of analog signals requires considering the source and channel coding designs jointly. The goal is to optimize the amount of source compression and, at the same time, control the individual bit error probabilities to minimize the overall signal reconstruction error. Especially for ISI channels, which is the focus of this paper, multicarrier modulation provides an easy and efficient way of implementing joint source-channel coding. In addition to determining the optimum compression level of the source, the design involves optimal allocation of bits to subchannels with different gains (bit loading) and power allocation among them. Moreover, optimality can only be achieved if the constellation of each subchannel supports unequal protection of different bits.

The optimal power distribution is not the one that optimizes the capacity, i.e. water-filling. Unfortunately, characterizing the optimal solution is not simple due to the nonlinearity of the Q-function and the exponential weights. Our examples show that even for the simplest design approach (BPSK signaling), the gain over water-filling is about 5 dB. However, this gain depends strongly on the CGNR profile and is thus dependent on the spectral characteristics of the channel and the noise.

Employing optimal QAM signal sets over the subchannels improves the performance by allowing unequal error protection. The gains over BPSK are mainly due to better protection of the most important bits and providing the possibility of transmitting less compressed signals. We have explored the rate-distortion function for this problem and have not reached the rate-distortion limit. We feel that higher order constellations may provide even more gains.

We have generalized this approach to the simultaneous transmission of multiple analog signals. Here the optimal transmission scheme has to determine the compression of each signal based on the channel's characteristics. We use a weighted sum of mean-squared errors as the distortion criterion. This problem easily fits into this framework. However, since different signal's bits are transmitted over same subchannels, the receiver's complexity increases, but substantial distortion gains over simpler, suboptimal schemes result.

6. REFERENCES

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