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Code Design for Multiple-Antenna Systems

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Abstract

We propose a systematic method for the design of space-time codes for AWGN slowly and fast Rayleigh fading channels. This can be accomplished by adopting a concatenated space-time code structure, where an orthogonal transmit diversity system constitutes the inner encoder. We will show that this will cause in decoupling of the problems of spatial and temporal diversity gains maximization, involved in the design of space-time codes. This decoupling significantly simplifies the code design procedure and presents a systematic code construction technique. In the case of slowly fading channels, where no temporal diversity gain is available, the concatenated structure of the space-time code, will help to decouple the problems of spatial diversity and coding gains maximization. However in a fast fading channel, the proposed system will decouple the problems of spatial and temporal diversity gains maximization. At the end, some issues involved in designing codes for downlink broadcast channels will be discussed.

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Contents

Abstract	ii
Acknowledgments	iii
List of Illustrations	vi
1 Introduction	1
2 Background	5
2.1 Code Design Criteria in Rayleigh Fading Channels	5
2.1.1 Single Transmit and Receive Antenna Systems	5
2.1.2 Multiple-Antenna Systems and Space-Time Codes	7
2.1.3 Orthogonal Transmit Diversity (OTD) System	9
2.2 Design Criteria of Space-Time Codes in Block Fading Channels	10
3 Code Design Criteria For Orthogonal Transmit Diversity	
System	13
3.1 The System Model	13
3.2 Design Criteria for Slowly Fading Channels	14
3.3 Design Criteria for Fast Fading Channels	16
3.4 Optimal Decoding Algorithm	19
4 Design of Trellis Coded Modulation Schemes for the OTD	
System	23

4.1	Code Design for Slowly Fading Channels	23
4.2	Code Design for Fast Fading Channels	25
4.2.1	Constellation Expansion in Size: Design of MTCM for OTD	26
5	Design of Concatenated Space-Time Codes with Generalized Orthogonal Transmit Diversity	30
5.1	Design Criteria for Slowly Fading Channels	32
5.2	Design Criteria for Fast Fading Channels	36
6	Broadcast Coding for Multiple-Antenna Systems	39
6.1	Overview	39
6.2	System Model	41
6.3	Achievable Rates Region	43
6.3.1	Perfect CSIT	43
6.3.2	Noisy CSIT	44
6.4	Numerical Results and Future Work	45
7	Conclusions	48
	Bibliography	50

Illustrations

2.1	Orthogonal Transmit Diversity (OTD) system	9
3.1	Concatenated Orthogonal Space-Time Code	13
4.1	Performance comparison of rate 2 b/s/Hz 4-state space-time trellis codes, with two transmit and one receive antennas, designed to achieve diversity gain of 2 in slowly fading channels.	24
4.2	Performance comparison of rate 2 b/s/Hz 8-state space-time trellis codes, with two transmit and one receive antennas, designed to achieve diversity gain of 2 in slowly fading channels.	25
4.3	Performance comparison of MTCM scheme designed for the concatenated orthogonal system of Figure 3.1, and the single transmit and receive antenna system. Codes are designed to achieve a total diversity gain of 4 in fast fading channels ($R = 1.5$ b/s/Hz). . .	27
4.4	Performance comparison of AT&T smart-greedy space-time trellis code with concatenated orthogonal space-time MTCM code with two transmit and one receive antennas in a slowly fading channel ($R = 1$ b/s/Hz).	28

4.5	Performance comparison of AT&T smart-greedy space-time trellis code with concatenated orthogonal space-time MTCM code with two transmit and one receive antennas in a fast fading channel ($R = 1$ b/s/Hz).	29
5.1	Error performance of codes designed for generalized orthogonal transmit diversity systems with more than two transmit or one receive antennas. Codes are designed for slowly fading channels ($R = 1.5$ b/s/Hz).	33
5.2	Comparison of the performance of rate 1.5 b/s/Hz codes designed for generalized orthogonal transmit diversity systems with 1 receive antenna, with the outage probability in a slowly fading channel. . . .	34
5.3	Comparison of the performance of rate 1.5 b/s/Hz codes designed for generalized orthogonal transmit diversity systems with 2 receive antennas, with the outage probability in a slowly fading channel. . . .	35
5.4	Error performance of codes designed for generalized orthogonal transmit diversity system with three and four transmit and single receive antennas. Codes are designed for fast fading channels ($R = 1$ b/s/Hz).	37
6.1	Broadcast Beamforming Scheme	41
6.2	Achievable rates region for perfect and noisy ($\sigma_z^2 = 0.1$) CSIT ($N = 2$ and $n_T = 2$).	42
6.3	Per user capacity for different values of σ_z^2 ($N = 2$ and $n_T = 2$)	44
6.4	Per user capacity vs number of transmit antenna elements for fixed number of users ($N = 2$).	45

6.5	Total user capacity vs number of transmit antenna elements for variable number of users ($\frac{N}{n_T} = \alpha$).	47
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Chapter 1

Introduction

Wireless channels are characterized as random and unpredictable. The major inherent impairments of these channels are the time varying fading, multipath, noise and interference. Because of the unavailability of the line of sight path between the transmitter and the receiver in most cases, the wireless channel is statistically modeled to be Rayleigh fading. As a result, recovering the transmitted data bits at the receiver is extremely difficult and this is the main challenge of designing a reliable wireless communication system.

A well studied method to combat the destructive effects of a wireless channel is the use of diversity techniques. These techniques fall into three major categories: *time*, *frequency* and *space*. The main idea of diversity is to provide the receiver with multiple replicas of the same transmitted signal which are faded independently.

In a time diversity system, the same information bearing signal is transmitted in different time intervals. If the separation between these intervals is large enough to guarantee independent fades, there is a good chance that not all the copies of the transmitted signal are severely attenuated. In a similar fashion, in a frequency diversity system, the same signal is being transmitted over different frequency bands. Again, if the separation between these bands are chosen to be large, we will have independent fades.

Not both of the above mentioned diversity techniques are appropriate for different types of fading conditions. For example, in a slowly fading channel, where there are

very slight time variations in the channel, the assumption of independent fading coefficients between the time intervals corresponding to the transmission of the same information bearing signal is very unrealistic. Thus, in these channels, diversity gain due to the time variations of the channel (referred to as *temporal diversity gain*), is not available. A similar argument holds for the case of frequency diversity techniques.

To achieve temporal diversity gains in a mobile radio wireless channel, we need to have very high velocity mobile terminals. This is not usually achievable unless interleaving/de-interleaving is applied. However, the use of interleaving/de-interleaving techniques does not provide a good performance for delay sensitive applications (like voice or video). Thus, the necessity of using spatial diversity techniques becomes evident.

The spatial diversity techniques suggest the use of multiple-element transmit and/or receive antennas. Providing sufficient spacing between the elements of both the transmit and the receive antennas guarantees independent fading channels between transmit and receive antenna pairs. The gain provided by the use of multiple-antenna systems is called *spatial diversity gain*. This source of diversity, if exploited efficiently, provides significant performance improvements over the single-element transmit and receive antenna systems, irrespective of the slowly or fast fading nature of the channel.

The information theory aspects and the capacity of multiple transmit and receive antenna systems have been well studied in the literature [21, 9, 16]. It has been shown that employing multiple antennas, results in a considerable increase in the system capacity. Thus, in addition to overcoming the fading effect of the wireless channel, the multiple transmit and receive antenna systems are considered as a solution to the increasing demand for higher data rates in today's mobile wireless communications .

Receive diversity has received more attention than transmit diversity in today's practical systems. For example, most of the current mobile radio wireless communication systems, use multiple antennas at the base station to exploit receive diversity in the uplink channel (mobile terminal to base station). However, because of the difficulty of having multiple antennas at the mobile terminals (for the size limitations and the cost of multiple RF down conversions), the use of receive diversity in the downlink channel has not yet been deployed.

To summarize some previous work on the transmit diversity systems, they can be categorized as follows: systems with feedback [10, 25, 17], systems with feedforward and no feedback first proposed in [26], and blind systems (with neither feedforward and nor feedback information). Recently, an effective coding scheme for multiple transmit antennas was proposed in [20]. These codes, which are capable of providing both spatial and temporal diversity gains without sacrificing the bandwidth, are referred to as *space-time codes* in the literature. Space-time coding schemes assume perfect channel information at the receiver and no channel information at the transmitter.

In [20], the criteria for the design of optimum space-time codes to achieve maximum diversity and coding gains in slow and fast fading channels were derived. In a slowly fading channel, where no temporal diversity gain can be exploited, the code design rule is based on the maximization of spatial diversity and coding gain. However, in a fast fading environment, the space-time coding scheme can provide significant amounts of temporal diversity in addition to spatial diversity gain.

Based on the design criteria of [20], most of the space-time codes designed so far, try to maximize the spatial and temporal diversity, and the coding gains simultaneously. As a result, no systematic way of constructing space-time codes for transmission over fading channels has been introduced so far. Most of the proposed

codes are hand made and ad hoc and mostly the result of extensive searches over all possible codes.

In this work, we will introduce a systematic method for the design of space-time codes over AWGN Rayleigh slowly and fast fading channels. It will be shown that applying *Orthogonal Transmit Diversity (OTD)* systems [1], the problems of spatial and temporal diversity gains maximization involved in the design of space-time codes, can be decoupled. As will be shown in Chapter 3, this will significantly simplify the code design procedure and will present a systematic code design technique.

In Chapter 2, some background information for the performance criteria and design of coding schemes in fading channels will be provided. Chapter 3, will present the design criteria for concatenated space-time codes using orthogonal transmit diversity systems as the inner code. It will be shown that depending on the type of the fading channel, the criteria will result either in decoupling the problems of spatial diversity and coding gain maximization (slowly fading channel), or spatial and temporal diversity gains maximization (fast fading channel). Simulation results for a bandwidth efficient outer coded modulation scheme will be demonstrated in Chapter 4. Generalization of the design criteria scheme to the case of orthogonal systems with more than two transmit and one receive antennas will be discussed in Chapter 5. In Chapter 6, some issues of designing codes for downlink broadcast channels will be discussed, and Chapter 7 will present the conclusions.

Chapter 2

Background

In the first section of this chapter, we will review some existing results from the literature on the code design criteria in Rayleigh fading channels. We will first discuss the design criteria and techniques for single transmit and receive antenna systems. Later, the use of multiple transmit and receive antenna systems will be motivated and the performance criteria of space-time codes in slowly and fast fading channels will be discussed. we will then talk about the orthogonal transmit diversity systems and their characteristics.

In the second section, the Chernoff upper bound and the design criteria for space-time codes in block fading channels derived in [3], will be presented. The background information provided in this chapter, will be used as a basis for the rest of the discussions in this thesis.

2.1 Code Design Criteria in Rayleigh Fading Channels

2.1.1 Single Transmit and Receive Antenna Systems

It has been well established in the literature that the performance of optimum codes for transmission over single transmit and receive antenna systems, are guided by different factors in Additive White Gaussian Noise (AWGN) and Rayleigh fading channels. In [24], it has been shown that the criterion for the design of optimum codes in AWGN channels is based on the maximization of the code free Euclidean distance.

The code design criteria in fading channels with complete channel state information at the receiver, was first studied by Divsalar and Simon in [7]. They proved that in fast fading channels, unlike the additive white Gaussian noise channels, the performance of the code is not ruled by the code free Euclidean distance. Instead the performance in this case is mainly determined by two other factors: *the length of the shortest error event path* and *the product of the squared Euclidean distances of the code symbols along the shortest error even path*.

The first factor, which is primarily the code minimum Hamming distance, is measured by the number of code symbols with nonzero Euclidean distance along the error path with the shortest length. This determines the rate of decay (slope) of the error rate of the code vs. Signal to Noise Ratio (SNR) and is usually referred to as *diversity gain*. The intercept point of the error rate curve with the error rate axis is determined by the second factor which is referred to as *coding gain*.

Motivated by the code performance criteria in fast fading channels, Divsalar, *et al*, proposed Multiple Trellis Coded Modulation (MTCM), as a new coded modulation technique with better performance compared to the conventional Trellis Coded Modulation (TCM) schemes. The superior performance of the MTCM scheme is due to its better Hamming distance properties which in turn results in higher achievable diversity gains.

In a conventional TCM scheme, just one code symbol is assigned to each transition of the trellis. Thus, the diversity gain is limited to the number of the branches of the shortest error event path. In the case of a trellis with parallel paths, the diversity gain is limited to one, which means that the asymptotic error rate of such code in a fast fading channel varies inverse linearly with SNR. On the contrary, in an MTCM scheme, more than one code symbol is assigned to each trellis transition. Thus, even

in the case of the existence of parallel paths, the error probability performance varies with the inverse of the SNR at a faster rate.

In [8], a systematic set partitioning method for designing optimum MTCM schemes using MPSK modulation for single transmit and receive antenna systems is presented. The code construction technique is based on the Hamming and product distance criteria for the design of optimum codes in fading channels with interleaving/de-interleaving. The design method of [8] provides codes with higher diversity gains as compared to the conventional TCM schemes having the same number of trellis states.

One of the contributions of this project is to develop a systematic code design method for multiple transmit and receive antenna systems in fast fading channels based on the set partitioning scheme of [8]. This work will be motivated in the next sections.

2.1.2 Multiple-Antenna Systems and Space-Time Codes

It is well known that the application of multiple transmit and receive antenna systems results in remarkable performance improvements and higher data rates in mobile wireless communications. By ensuring enough spacing between the antenna elements of the transmitter and the receiver, considerable amounts of spatial diversity gain can be provided by these systems. This is due to the fact that the signals transmitted over different antennas undergo independent fades. So, there is a high chance that some of the replicas of the transmitted signal will be less faded and so the transmitted information can be recovered at the receiver with less error probability.

Inspired by the design criteria of optimum codes in fading channels introduced in [7], Tarokh, *et al*, initially proposed the concept of space-time coding. Space-time codes are bandwidth efficient coding techniques for multiple transmit and receive antenna systems. They provide both spatial and temporal diversity gains and have

the potential of significantly improving the error performance and system capacity as compared to single transmit and receive antenna coding schemes.

Based on the Chernoff upper bounds for the pairwise error probability, the design criteria of space-time codes in slowly and fast fading channels, were presented in [20]. The maximum achievable diversity gain of a space-time code with n_T transmit and n_R receive antennas can be shown to be $n_T n_R$. In a slowly Rayleigh fading channel, where the fading coefficients are assumed to be constant during the transmission of a whole block of data and independently varying from one block to another, the design rule has been shown in [20] to be based on the following criteria:

- *The Rank Criterion:* In order to achieve the maximum diversity gain of $n_T n_R$, the code difference matrices have to be full rank for all pairs of codewords. If the minimum rank of the code difference matrices over all codewords is $r < n_T$, then the achievable diversity gain would be $r n_R$.
- *The Determinant Criterion:* To achieve maximum coding gain, the minimum of the determinants of the matrices $\mathbf{D}(\mathbf{c}, \mathbf{e})\mathbf{D}^H(\mathbf{c}, \mathbf{e})$ taken over all pairs of codewords should be maximized ($\mathbf{D}(\mathbf{c}, \mathbf{e})$ is the code difference matrix).

In a fast fading channel, it is assumed that the fading coefficients of the channel vary independently from one symbol interval to another. This can be achieved by using interleaving/de-interleaving techniques. Unlike the slowly fading case, the time varying nature of this kind of channels results in considerable temporal diversity gains. The code design criteria for Rayleigh fast fading channels are derived in [20] as the following:

- *The Distance Criterion:* In order to achieve the diversity gain of νn_R , any two codeword matrices should differ in at least ν columns.
- *The Product Criterion:* For a given diversity gain, in order to achieve the maximum coding gain, the minimum of the products of the Euclidean distances of the code symbols in those columns of the codeword matrices in which they differ, taken over all pairs of codewords, should be maximized.

The above design criteria has later been applied in [20] to construct space-time trellis coding schemes for transmission over multiple-antenna systems.

2.1.3 Orthogonal Transmit Diversity (OTD) System

For two transmit antennas, a simple transmit diversity technique which is capable of providing full spatial diversity gain is the *Orthogonal Transmit Diversity (OTD)* and was first discovered by Alamouti [1]. The system is shown in Figure 2.1 and works as follows: suppose that the symbol sequence c_1, c_2, \dots, c_L is to be transmitted. During the first symbol interval, the two symbols c_1 and c_2 are simultaneously transmitted from antennas one and two, respectively. At the second interval, antenna one transmits $-c_2^*$ and the second antenna transmits c_1^* . This procedure continues to transmit the rest of the symbols, similarly.

It can be seen that the system is a full rate system (one symbol per transmission) and can be shown to provide a full spatial diversity gain of 2. Moreover it has been proven in [15] that this system preserves the capacity of multiple transmit and single receive antenna systems.

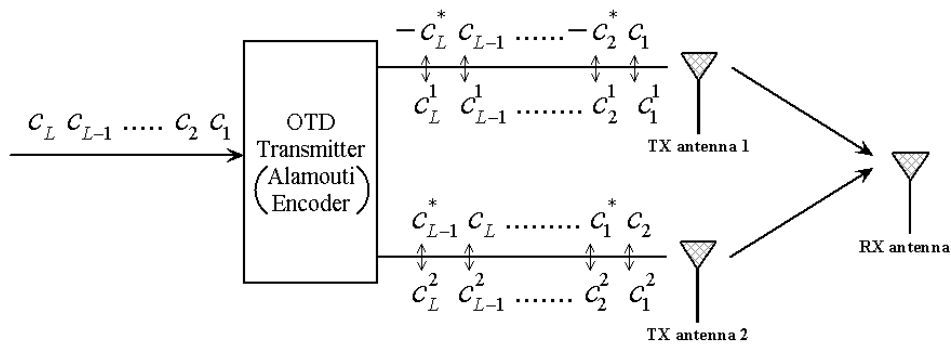


Figure 2.1 Orthogonal Transmit Diversity (OTD) system

However, the most appealing characteristic of the OTD system is the simplicity of its decoder structure. The maximum likelihood decoding can be achieved by the use of a linear combiner [1] to decouple the symbols transmitted from each transmit antenna, as opposed to joint detection techniques.

The simplicity and good performance of Alamouti's OTD system has been used as a motivation in some recent research works [23, 22] to find similar orthogonal designs for more than 2 transmit antennas. These designs which we will refer to as *Generalized Orthogonal Transmit Diversity (GOTD)* systems, use the theory of orthogonal matrices studied by several mathematicians like Radon-Hurwitz. While the GOTD systems provide full diversity and use a simple linear combiner as the maximum likelihood decoder, they are not full rate. This will be discussed in more detail in Chapter 5.

The so far discussed characteristics of the OTD and GOTD systems are motivations to think of a concatenated space-time code structure. This concatenated structure can use the OTD or the GOTD system as the inner code to provide full spatial diversity. The outer code which can be a bandwidth efficient coded modulation scheme, will provide both temporal diversity and coding gain. The idea of concatenated space-time codes and their design criteria in slowly and fast fading channels is the topic of Chapter 3.

2.2 Design Criteria of Space-Time Codes in Block Fading Channels

In [3], the performance criterion of space-time codes in block fading channels has been studied and the Chernoff upper bound for the pairwise error probability has been derived. The approach in the analysis is very similar to the one taken in [20]

for the design criteria of space-time codes in slowly and fast fading channels. In this section, those results are presented from [3].

Assume that

$$\mathbf{c} = \begin{pmatrix} c_1^1 & c_2^1 & \cdots & c_L^1 \\ c_1^2 & c_2^2 & \cdots & c_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{n_T} & c_2^{n_T} & \cdots & c_L^{n_T} \end{pmatrix}$$

and

$$\mathbf{e} = \begin{pmatrix} e_1^1 & e_2^1 & \cdots & e_L^1 \\ e_1^2 & e_2^2 & \cdots & e_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^{n_T} & e_2^{n_T} & \cdots & e_L^{n_T} \end{pmatrix}$$

represent the matrices of transmitted and erroneously decoded symbols, respectively, where n_T is the number of transmit antennas and L is the length of the code block. It is assumed that the channel is block fading with block length M , i.e. fading coefficients are constant across blocks of length M and are independently varying from one block to another, and that there are K such blocks in a frame, i.e. $L = K \times M$. So, the code difference matrix in the k th block $\mathbf{D}_k(\mathbf{e}, \mathbf{e})$ would be

$$\mathbf{D}_k(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} c_{(k-1)M+1}^1 - e_{(k-1)M+1}^1 & \cdots & c_{kM}^1 - e_{kM}^1 \\ c_{(k-1)M+1}^2 - e_{(k-1)M+1}^2 & \cdots & c_{kM}^2 - e_{kM}^2 \\ \vdots & \ddots & \vdots \\ c_{(k-1)M+1}^{n_T} - e_{(k-1)M+1}^{n_T} & \cdots & c_{kM}^{n_T} - e_{kM}^{n_T} \end{pmatrix}.$$

Assuming that complete channel state information is available at the receiver, the Chernoff upper bound for the pairwise error probability is shown in [3] to be

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^K \frac{1}{\det(I_{n_T} + \mathbf{D}_k(\mathbf{c}, \mathbf{e})\mathbf{D}_k^H(\mathbf{c}, \mathbf{e})E_s/4N_0)^{n_R}}, \quad (2.1)$$

where n_R is the number of receive antennas, I_{n_T} is the identity matrix of size n_T , and $Es/4N_0$ is a measure of signal to noise ratio.

Later in [3], it has been shown that the above upper bound results in the same rank and determinant criteria for slowly fading channels and the distance and product criteria for the fast fading channels both introduced in [20]. These criteria were described in Section 2.1.2.

The fact is that these criteria do not provide a methodical procedure of designing space-time codes in fading channels. That is because maximizing the spatial and temporal diversity gains simultaneously, and optimizing the coding gain at the same time, does not result in a systematic code design technique. So far, this has been the only approach taken in this direction. Thus, most of the codes proposed so far [20], are manual and hand made and the result of broad searches over all possible codewords. In the next chapter, we will present a systematic method for the design of space-time codes in both slowly and fast fading channels.

Chapter 3

Code Design Criteria For Orthogonal Transmit Diversity System

In this chapter, we consider the design of space-time codes in Rayleigh fading channels, using the orthogonal transmit diversity system of Figure 2.1. We will also develop guidelines for the design of bandwidth efficient coded modulation schemes for this system in slowly and fast fading channel conditions.

3.1 The System Model

The model of our proposed system is shown in Figure 3.1. The input information bits are first encoded using a coded modulation block. These encoded symbols are later passed through the OTD transmitter, acting as an inner encoder in this scenario. The two symbol streams resulting from the Alamouti's orthogonal transformation (Figure 2.1) are then transmitted through the two transmit antennas. The two symbol streams resulting from the Alamouti's orthogonal transformation (Figure 2.1) are then transmitted through the two transmit antennas.

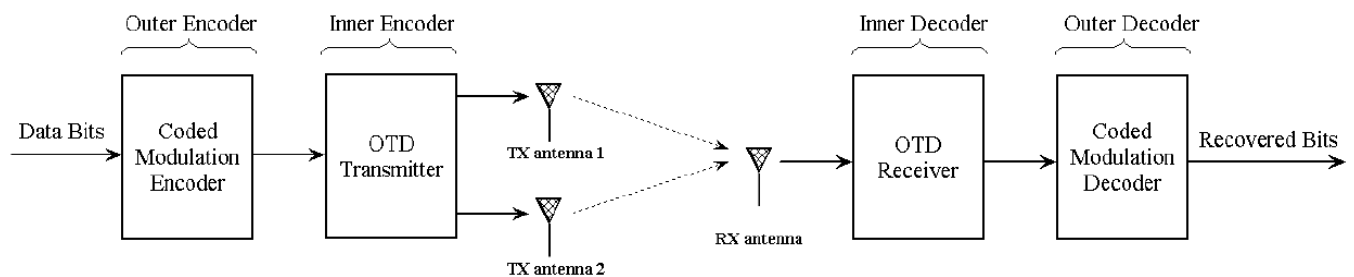


Figure 3.1 Concatenated Orthogonal Space-Time Code

The channel is modeled to be Rayleigh fading AWGN, with statistically independent fading coefficients between each pair of transmit and receive antennas. The additive noise terms at different symbol intervals are assumed to be independent samples of a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension.

The received symbol at the receiver during each symbol interval is the sum of the two faded symbols of the two transmitters affected by the additive white Gaussian noise. Later in this section, we will show that the optimum decoder for this scheme is the concatenation of Alamouti's linear combiner (standard OTD receiver) and a maximum likelihood decoder for the outer coded modulation scheme. The OTD receiver performs a combining operation on the received symbols and finally, the combined symbols are sent to the outer coded modulation decoder to recover the data bits.

3.2 Design Criteria for Slowly Fading Channels

As explained in Chapter 2, in a slowly fading environment, it is assumed that the fading coefficients of the channel remain constant during the transmission of the whole block of length L . To conform with (2.1), this suggests considering a single block ($K = 1$) of length $M = L$. Thus, for the system of Figure 3.1, (2.1) reduces to

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\det(I_2 + \mathbf{D}(\mathbf{c}, \mathbf{e})\mathbf{D}^H(\mathbf{c}, \mathbf{e})\frac{E_s}{4N_0})}, \quad (3.1)$$

where $\mathbf{D}(\mathbf{c}, \mathbf{e})$ is the difference matrix between the code block \mathbf{c} and the erroneously decoded block \mathbf{e} . Because of the orthogonal transmission structure of the OTD system shown in Figure 2.1 and assuming that L is even, we will have

$$\begin{aligned} c_{l-1}^1 &= c_{l-1} & c_l^1 &= -c_l^* \\ c_{l-1}^2 &= c_l & c_l^2 &= c_{l-1}^* \end{aligned}, \text{ for } l = 2, 4, \dots, L.$$

Thus, the difference matrix $\mathbf{D}(\mathbf{c}, \mathbf{e})$ can be expressed as

$$\mathbf{D}(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} c_1 - e_1 & -(c_2 - e_2)^* & \cdots & c_{L-1} - e_{L-1} & -(c_L - e_L)^* \\ c_2 - e_2 & (c_1 - e_1)^* & \cdots & c_L - e_L & (c_{L-1} - e_{L-1})^* \end{pmatrix}.$$

Substituting the above matrix in (3.1), it follows that

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\left[1 + \sum_{l=1}^L |c_l - e_l|^2 \left(\frac{E_s}{4N_0}\right)\right]^2}. \quad (3.2)$$

An upper bound for the pairwise error probability of the system of Figure 3.1 in slowly fading channel conditions follows from the Chernoff bound of (3.2) as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \left[\left(\sum_{l=1}^L |c_l - e_l|^2 \right) \left(\frac{E_s}{4N_0} \right) \right]^{-2}. \quad (3.3)$$

The upper bound of (3.3), shows a full spatial diversity gain of 2 resulting from the OTD system. It should also be noticed that because of the slowly fading nature of the channel during the transmission of the whole block of symbols, no temporal diversity gain can be provided by the coded modulation scheme. Instead, the code design criterion in this case would involve the maximization of the coding gain, which is expressed as

$$d_e(\mathbf{c}, \mathbf{e}) = \sum_{l=1}^L |c_l - e_l|^2. \quad (3.4)$$

The above expression is the definition of the Euclidean distance of the code. Thus, the criterion for the design of optimum coded modulation schemes for the system of Figure 3.1 is exactly equivalent to the code design rule for single transmit and receive antenna systems in AWGN channels. The important conclusion drawn from this argument is that any coded modulation scheme, already designed for optimum performance in an AWGN channel with single transmit and receive antennas, would also be optimum for the system of Figure 3.1.

Moreover, it is evident that designing a code based on just maximization of the free Euclidean distance and benefiting the inherent spatial diversity gain of the OTD system, is much simpler than designing space-time codes according to the rank and determinant criteria in [20]. Interestingly, the error performance simulation results reported in Chapter 4, show that the system of Figure 3.1 provides a higher coding gain as compared to the space-time trellis codes in [20] with the same complexities, both designed for optimum performance in a slowly fading environment.

3.3 Design Criteria for Fast Fading Channels

In this section, it is considered that the block length of the fast fading channel of interest for the system of Figure 3.1 is equal to two symbol intervals and if needed, sufficient interleaving is applied. This means that the fading coefficients of the channel remain constant during the transmission of a block of two symbols and are statistically independent between different blocks. To derive the Chernoff upper bound as in (2.1), the whole block of length L can be partitioned into $K = L/2$ blocks of length 2 each (assuming that L is an even number). Thus, (2.1) can be expressed as

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^{L/2} \frac{1}{\det(I_2 + \mathbf{D}_k(\mathbf{c}, \mathbf{e})\mathbf{D}_k^H(\mathbf{c}, \mathbf{e})\frac{E_s}{4N_0})} , \quad (3.5)$$

where $\mathbf{D}_k(\mathbf{c}, \mathbf{e})$ is the difference matrix between \mathbf{c} and \mathbf{e} for the k^{th} block of length 2. Because of the orthogonal transmission structure of the OTD system, we will have

$$\begin{aligned} c_{2k-1}^1 &= c_{2k-1} & c_{2k}^1 &= -c_{2k}^* \\ c_{2k-1}^2 &= c_{2k} & c_{2k}^2 &= c_{2k-1}^* \end{aligned} , \text{ for } k = 1, 2, \dots, L/2.$$

So, the matrix $\mathbf{D}_k(\mathbf{c}, \mathbf{e})$ can be written as

$$\mathbf{D}_k(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} c_{2k-1} - e_{2k-1} & -(c_{2k} - e_{2k})^* \\ c_{2k} - e_{2k} & (c_{2k-1} - e_{2k-1})^* \end{pmatrix}.$$

Substituting the matrix $\mathbf{D}_k(\mathbf{c}, \mathbf{e})$ in (3.5) results in

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^{L/2} \frac{1}{\left[\left(1 + (|c_{2k-1} - e_{2k-1}|^2 + |c_{2k} - e_{2k}|^2) \frac{E_s}{4N_0} \right) \right]^2} \quad (3.6)$$

An upper bound for the pairwise error probability of the system of Figure 3.1 in a fast fading channel with block length of two, follows from (3.6) as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \prod_{\substack{(c_{2k-1}, c_{2k}) \neq (e_{2k-1}, e_{2k}) \\ k = 1, \dots, L/2}} \left[(|c_{2k-1} - e_{2k-1}|^2 + |c_{2k} - e_{2k}|^2) \left(\frac{E_s}{4N_0} \right) \right]^{-2} \quad (3.7)$$

As can be seen from (3.7), a full spatial diversity gain of 2 results from the orthogonal transmission system. Hence, applying the OTD system, we have maximized the spatial diversity gain achievable by two transmit antennas. It should also be noticed that unlike the slowly fading scenario, in a fast fading channel, a considerable temporal diversity gain can be provided by the coding scheme. Hence, the optimum coded modulation scheme in this case is the one which maximizes the temporal diversity gain as well as the coding gain.

In [7], it has been shown that the performance of codes in fast fading channels with single transmit and receive antennas is controlled by two factors: the code minimum Hamming distance (length of the shortest error event path), and the product of squared symbol distances along the shortest error event path. These two factors determine the temporal diversity gain and the coding gain, respectively. However, from (3.7) it follows that the optimum code for transmission over the OTD system is not simply obtained by the criteria in [7], but instead it involves maximization of new distance quantities defined in terms of pairs of consecutive symbols. These new *pairwise Hamming* and *pairwise product* distances, denoted by $d_{PW,H}$ and $d_{PW,P}$

respectively, are defined as:

$$d_{PW,H}(\mathbf{c}, \mathbf{e}) = \sum_{\substack{(c_{2k-1}, c_{2k}) \neq (e_{2k-1}, e_{2k}) \\ k=1, \dots, L/2}} 1, \quad (3.8)$$

and

$$d_{PW,P}(\mathbf{c}, \mathbf{e}) = \prod_{\substack{(c_{2k-1}, c_{2k}) \neq (e_{2k-1}, e_{2k}) \\ k=1, \dots, L/2}} (|c_{2k-1} - e_{2k-1}|^2 + |c_{2k} - e_{2k}|^2). \quad (3.9)$$

The pairwise distances motivate us to introduce a new code design technique for the OTD system in a fast fading channel, based on expanding the signal constellation. The expansion can be performed in either dimension or size of the constellation (going to higher orders of modulation). Each point in the new constellation can be considered as the concatenation of two consecutive signal points from the original signal set. Denoting the sequences of transmitted and erroneously decoded symbols in the new constellation by $\mathbf{C} = \{C_1, C_2, \dots, C_{L/2}\}$ and $\mathbf{E} = \{E_1, E_2, \dots, E_{L/2}\}$, respectively, we have:

$$\begin{aligned} C_k &= (c_{2k-1}, c_{2k}) \quad k = 1, 2, \dots, L/2, \\ E_k &= (e_{2k-1}, e_{2k}) \quad k = 1, 2, \dots, L/2. \end{aligned}$$

Thus, (3.8) and (3.9) can be rewritten as

$$d_{PW,H}(\mathbf{c}, \mathbf{e}) = \sum_{\substack{C_k \neq E_k \\ k=1, \dots, L/2}} 1 = d_H(\mathbf{C}, \mathbf{E}), \quad (3.10)$$

and

$$d_{PW,P}(\mathbf{c}, \mathbf{e}) = \prod_{\substack{C_k \neq E_k \\ k=1, \dots, L/2}} |C_k - E_k|^2 = d_P(\mathbf{C}, \mathbf{E}). \quad (3.11)$$

Hence, the code design criteria will be based on maximizing the minimum symbol Hamming and product distances in the expanded constellation.

Thus, to achieve a temporal diversity gain of L equal to the length of the code block, only a minimum symbol Hamming distance of $L/2$ in the expanded constellation is needed. This does not necessarily require a minimum Hamming distance of L in the old signal set. In fact, it is clear from the upper bound of (3.7) that adopting the orthogonal transmit diversity, the Hamming distance requirement of the codewords halves. This is the consequence of the inherent spatial diversity gain of two resulting from the orthogonal transmission system. The reduction in the Hamming distance requirement allows us to have larger subsets in the signal set partitioning, which in turn results in higher achievable code rates with less code complexities. This will be explained with more detail in Chapter 4.

3.4 Optimal Decoding Algorithm

In this section, we will prove that the optimum decoding algorithm for the system of Figure 3.1 can be obtained by concatenating Alamouti's linear combiner (standard OTD receiver as the inner decoder) and a maximum likelihood decoder for the outer coded modulation scheme.

Assuming perfect channel state information at the receiver, the decision metric of the optimum (maximum likelihood) decoder for a space-time coding scheme with block length of L , can be expressed as

$$\sum_{j=1}^{n_R} \sum_{l=1}^L \left| r_l^j - \sum_{i=1}^{n_T} \alpha_l^{i,j} c_l^i \right|^2, \quad (3.12)$$

where r_l^j denotes the received signal at the j^{th} receiver during the l^{th} symbol interval, and $\alpha_l^{i,j}$ is the complex Gaussian fading coefficient of the channel between the i^{th} transmitter and the j^{th} receiver in the l^{th} symbol interval. So, the maximum likelihood receiver decides in favor of the codeword which maximizes the decision metric of (3.12).

For the orthogonal transmission system of Figure 3.1 with two transmit and single receive antennas, (3.12) reduces to

$$\sum_{p=1}^{L/2} \left(|r_{2p-1} - \alpha_{2p}^1 c_{2p-1} - \alpha_{2p}^2 c_{2p}|^2 + |r_{2p} + \alpha_{2p}^1 c_{2p}^* - \alpha_{2p}^2 c_{2p-1}^*|^2 \right), \quad (3.13)$$

where the index j in the expression $\alpha_{2p}^{i,j}$ is dropped as the number of receive antennas is assumed to be one. The received signal at each symbol interval is the sum of the two faded transmitted symbols affected by the additive white complex Gaussian noise with variance $N_0/2$ per dimension

$$\begin{aligned} r_{2p-1} &= \alpha_{2p}^1 c_{2p-1} + \alpha_{2p}^2 c_{2p} + n_{2p-1}, \\ r_{2p} &= \alpha_{2p}^2 c_{2p-1}^* - \alpha_{2p}^1 c_{2p}^* + n_{2p} \end{aligned}, \quad \text{for } p = 1, 2, \dots, L/2. \quad (3.14)$$

Expanding (3.13), the maximum likelihood decision rule can also be written as

$$\begin{aligned} \sum_{p=1}^{L/2} \{ & (|r_{2p-1}|^2 + |r_{2p}|^2) + (|\alpha_{2p}^1|^2 + |\alpha_{2p}^2|^2) (|c_{2p-1}|^2 + |c_{2p}|^2) \\ & - 2\Re [(r_{2p-1} \alpha_{2p}^{1*} + r_{2p}^* \alpha_{2p}^2) c_{2p-1}^*] + 2\Re [(r_{2p-1} \alpha_{2p}^{2*} - r_{2p}^* \alpha_{2p}^1) c_{2p}^*] \}, \end{aligned} \quad (3.15)$$

where $\Re[x]$ denotes the real part of x .

Now, let's define z_{2p-1} and z_{2p} as

$$\begin{aligned} z_{2p-1} &= r_{2p-1} \alpha_{2p}^{1*} + r_{2p}^* \alpha_{2p}^2, \\ z_{2p} &= r_{2p-1} \alpha_{2p}^{2*} - r_{2p}^* \alpha_{2p}^1. \end{aligned}, \quad \text{for } p = 1, 2, \dots, L/2. \quad (3.16)$$

These quantities are exactly the outputs of the standard OTD receiver/combiner proposed by Alamouti in [1]. The important point of the discussion so far is that based on the Neyman-Fisher factorization theorem [12], z_{2p-1} and z_{2p} defined as above are sufficient statistics for the maximum likelihood decoding of (3.13). This means that using the standard OTD combiner at the front end of the receiver causes no information loss in estimating the symbol sequence c_1, c_2, \dots, c_L .

Considering that the first term of the summation in (3.15) is independent of the symbol sequence to be estimated, it can be replaced by any other expression

which is also independent of c_1, c_2, \dots, c_L . Thus, we replace $|r_l|^2$ with $|z_l|^2/\beta_l$ for $l = 1, 2, \dots, L$, where

$$\beta_l = |\alpha_l^1|^2 + |\alpha_l^2|^2, \text{ for } l = 1, 2, \dots, L. \quad (3.17)$$

It can be easily seen that minimizing the decision metric of (3.13) is equivalent to minimizing the new maximum likelihood metric

$$\sum_{p=1}^{L/2} \left\{ \left(\frac{|z_{2p-1}|^2}{\beta_{2p}} + \frac{|z_{2p}|^2}{\beta_{2p}} \right) + \beta_{2p} (|c_{2p-1}|^2 + |c_{2p}|^2) + 2\Re(z_{2p-1}c_{2p-1}^* + z_{2p}c_{2p}^*) \right\}. \quad (3.18)$$

Equation (3.18) can also be expressed as

$$\sum_{l=1}^L \left(\frac{|z_l|^2}{\beta_l} + \beta_l |c_l|^2 - 2\Re(z_l c_l^*) \right) = \sum_{l=1}^L \frac{|z_l - \beta_l c_l|^2}{\beta_l} \quad (3.19)$$

On the other hand, substituting (3.14) in (3.16), the linear combiner outputs at two consecutive symbol intervals would be

$$\begin{aligned} z_{2p-1} &= (|\alpha_{2p}^1|^2 + |\alpha_{2p}^2|^2)c_{2p-1} + \alpha_{2p}^{1*}n_{2p-1} + \alpha_{2p}^2 n_{2p}^* \\ z_{2p} &= (|\alpha_{2p}^1|^2 + |\alpha_{2p}^2|^2)c_{2p} + \alpha_{2p}^{2*}n_{2p-1} - \alpha_{2p}^1 n_{2p}^* \end{aligned}, \text{ for } p = 1, 2, \dots, L/2. \quad (3.20)$$

Defining N_{2p-1} and N_{2p} the noise terms at the output of the OTD receiver during two consecutive symbol intervals, as

$$\begin{aligned} N_{2p-1} &= \alpha_{2p}^{1*}n_{2p-1} + \alpha_{2p}^2 n_{2p}^* \\ N_{2p} &= \alpha_{2p}^{2*}n_{2p-1} - \alpha_{2p}^1 n_{2p}^* \end{aligned}, \text{ for } p = 1, 2, \dots, L/2, \quad (3.21)$$

It can be easily shown that N_l 's for $l = 1, \dots, L$, are zero mean, iid complex Gaussian random variables with variance $\beta_l N_0/2$ per dimension.

Therefore, the decision metric of (3.19), is in fact the metric for the maximum likelihood decoding algorithm performed on the output symbols of the linear combiner (OTD receiver). Thus, it is concluded that the optimum decoder for the orthogonal system of Figure 3.1 is achieved by concatenating the linear combining scheme

of (3.16) with a standard maximum likelihood decoder operating on the combined symbols. Moreover, the complexity of the decoder remains unchanged compared to the existing space-time trellis decoders.

Chapter 4

Design of Trellis Coded Modulation Schemes for the OTD System

In Chapter 3, it was explained that the system in Figure 3.1 can considerably simplify the design procedure of maximum diversity gain coding schemes for multiple-antenna communication systems in fading channels. We later derived upper bound expressions for the pairwise error probability of this system in slow and fast fading channel conditions. Based on these bounds, guidelines for the design of concatenated space-time coded modulation schemes were proposed and discussed in detail. In this chapter, we will put these criteria into practice by designing bandwidth efficient TCM schemes as the outer code for the system in Figure 3.1. The simulation results reported in the next sections show that codes designed for this system have better performance compared to the space-time trellis codes of [20] with similar complexities.

4.1 Code Design for Slowly Fading Channels

It was shown in Section 3.2 that the criterion for designing optimum outer codes for the system in Figure 3.1 in slowly fading channels, is based only on the maximization of the free Euclidean distance. To analyze the error performance of the codes designed as such, consider designing rate 2 b/s/Hz codes for the system in Figure 3.1. We will use Ungerboeck's 4 and 8-state TCM codes with 8PSK modulation, designed based on the maximization of the free Euclidean distance criterion, for optimum performance in AWGN channels [24]. In Figures 4.1 and 4.2, the simulation results for the error

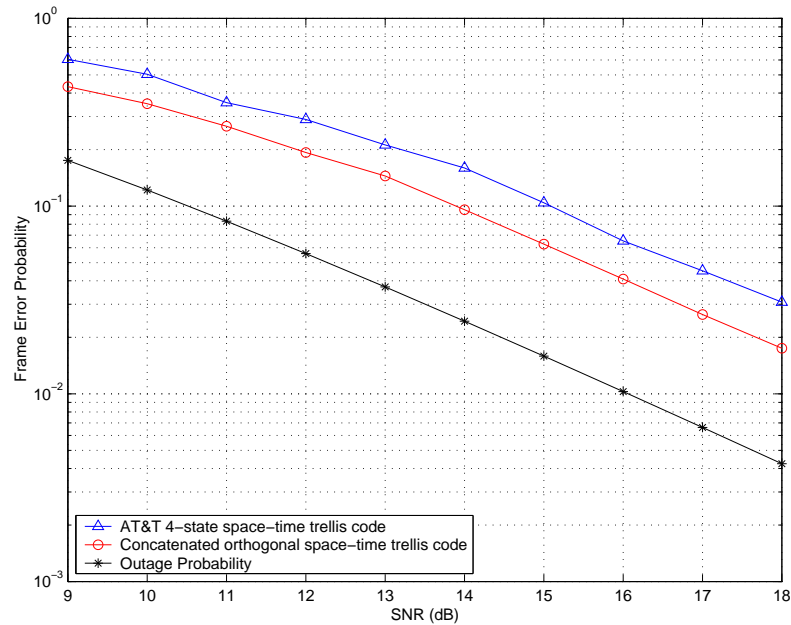


Figure 4.1 Performance comparison of rate 2 b/s/Hz 4-state space-time trellis codes, with two transmit and one receive antennas, designed to achieve diversity gain of 2 in slowly fading channels.

performance of these codes for different levels of SNR are shown and compared to the outage probability given in [21].

For comparison purpose, we have also plotted the error performance of two space-time trellis codes proposed in [20] with two transmit and single receive antennas and the same trellis complexities and code rates, designed based on the rank and determinant rules of [20] for slowly fading channels. While both codes demonstrate a diversity gain of two, it can be seen that the system of Figure 3.1 shows more than 1 and 2 db gain over the space-time trellis codes of [20] for the two cases of 4 and 8-state trellises, respectively. Besides, it is observed that the concatenated orthogonal space-time system performs closer to the outage probability.

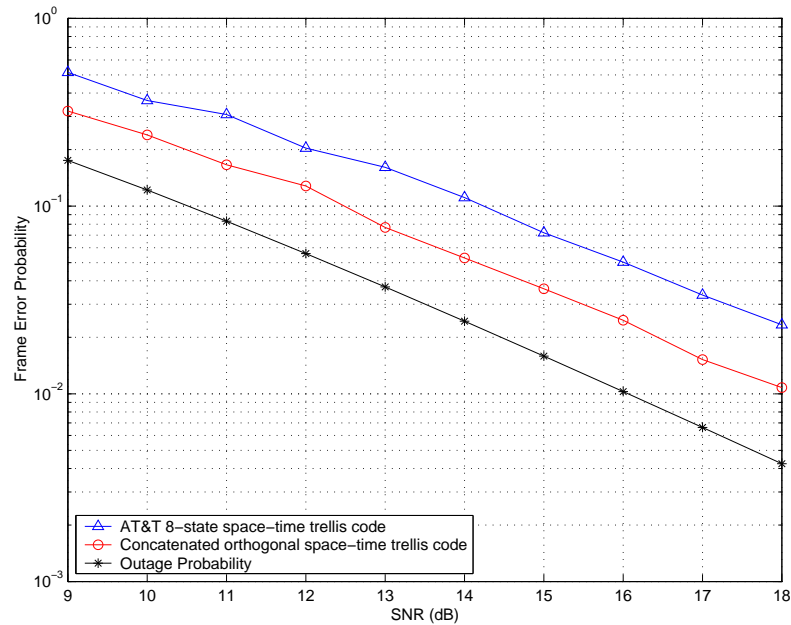


Figure 4.2 Performance comparison of rate 2 b/s/Hz 8-state space-time trellis codes, with two transmit and one receive antennas, designed to achieve diversity gain of 2 in slowly fading channels.

4.2 Code Design for Fast Fading Channels

In Section 3.3, it was described that the outer code design criteria for the system in Figure 3.1 in fast fading channels, is based on the maximization of the Hamming and product distances in the expanded constellation, where each point is the concatenation of two points from the original signal set. Assuming that the original signal set is two dimensional (2D), one way of constructing the new constellation is to consider the four dimensional (4D) Cartesian product of the original 2D signal set by itself (signal set expansion in dimension). Another way is to construct a new 2D constellation of size M^2 , where M is the size of the original signal set (signal set expansion in size). The code design will later be performed for the new constellation trying to maximize its Hamming and product distances. At the output of the coded modulation block,

each encoded symbol from the new constellation will be considered as the concatenation of two signal points from the original constellation, and will be transmitted in two consecutive symbol intervals through the OTD system as in Figure 2.1. Multiple Trellis Coded Modulation (MTCM) and MultiLevel Coding (MLC) design techniques satisfying the Hamming and product criteria of Section 3.3, have already been proposed in the literature ([8, 18]). These are appropriate coded modulation schemes for fast fading channels, as they can be designed to achieve good distance properties required by the criteria derived in [7]. Thus, the use of MTCM and MLC schemes as the outer code for the system of Figure 3.1 is recommended.

In the next subsection, the idea of signal set expansion in size and the above design procedure is demonstrated through an example, where an MTCM scheme is designed for the system in Figure 3.1. The idea of signal set expansion in dimension is very similar and is shown in [3].

4.2.1 Constellation Expansion in Size: Design of MTCM for OTD

Consider designing a code with an overall diversity gain of 4 and rate 1.5 bits/s/Hz using QPSK modulation. In order to achieve a minimum Hamming distance of 2 resulting in a total diversity gain of 4 using the OTD system, it suffices to consider an MTCM code with multiplicity of 4 and perform the set partitioning task for a 2-fold Cartesian product of a 16PSK signal set. Each point in the 16PSK signal set is considered as the concatenation of two consecutive QPSK symbols. If the set partitioning scheme of [8] is adopted for a 2-fold Cartesian product of 16PSK symbols, a maximum of 16 code sets can be assigned to each subset. This means that a maximum of 16 parallel paths can be considered for the trellis. So, if a 4-state fully connected trellis is considered, the encoder would be capable of encoding 6 input bits. Together with the 4 QPSK symbols assigned to each transition of the trellis,

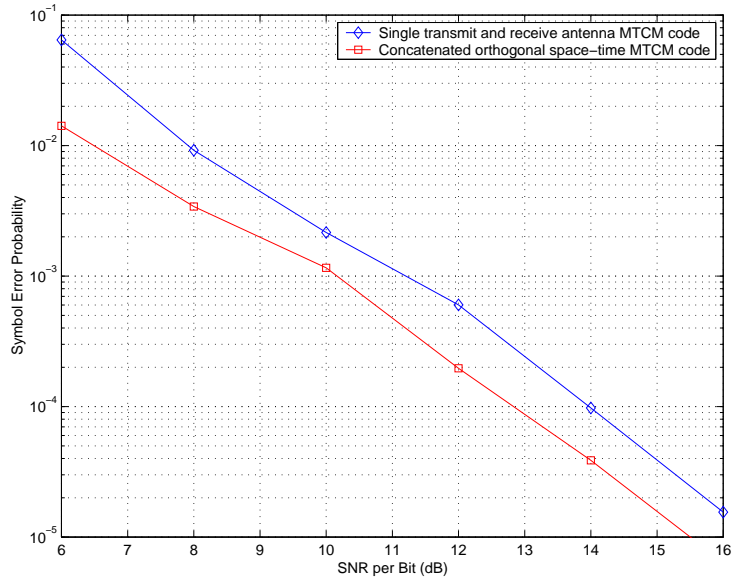


Figure 4.3 Performance comparison of MTCM scheme designed for the concatenated orthogonal system of Figure 3.1, and the single transmit and receive antenna system. Codes are designed to achieve a total diversity gain of 4 in fast fading channels ($R = 1.5$ b/s/Hz).

this results in the desired rate of $6/4 = 1.5$ bits/sec/Hz. Note that if we wanted to use the same trellis to design a code with diversity gain of 4 for single transmit and receive antenna system, the maximum achievable rate would be 1 bit/s/Hz. That is because the set partitioning of the 4-fold Cartesian product of QPSK symbols, would result in subsets with a maximum size of four [8]. Thus, for comparison purpose, an 8 state fully connected trellis with 8PSK modulation has been used to design an MTCM code with rate 1.5 bits/s/Hz and diversity gain of 4 for single transmission scheme.

The error performance comparison of these two transmission schemes is shown in Figure 4.3. It can be seen that while both codes provide a diversity gain of 4, the MTCM code designed for the orthogonal system in Figure 3.1, outperforms the single transmit and receive antenna transmission scheme by 1 dB.

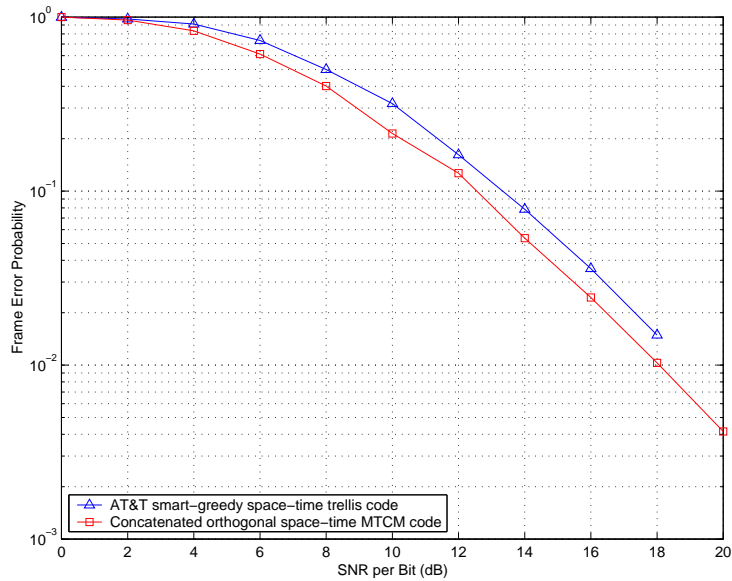


Figure 4.4 Performance comparison of AT&T smart-greedy space-time trellis code with concatenated orthogonal space-time MTCM code with two transmit and one receive antennas in a slowly fading channel ($R = 1$ b/s/Hz).

To demonstrate the robustness of the system in Figure 3.1, its performance has also been compared to the smart-greedy space-time code of [20] in Figures 4.4 and 4.5. Smart-greedy codes are designed to provide a good performance in both slowly and fast fading channels. Thus, even if the transmitter doesn't know the channel, the code is constructed to take advantage of both the space diversity provided by the use of multiple antennas, and the possible time variations of the channel. As such, the smart-greedy codes guarantee a diversity gain of r_1 in slowly and $r_2 \geq r_1$ in fast fading channel conditions [20].

For comparison purpose, we picked up the 2-state smart-greedy space-time code of [20] with QPSK modulation which is designed to achieve a diversity gain of 2 and 3 in slowly and fast fading channels, respectively, using two transmit and one receive antennas. The error performance has been simulated in both cases and compared to

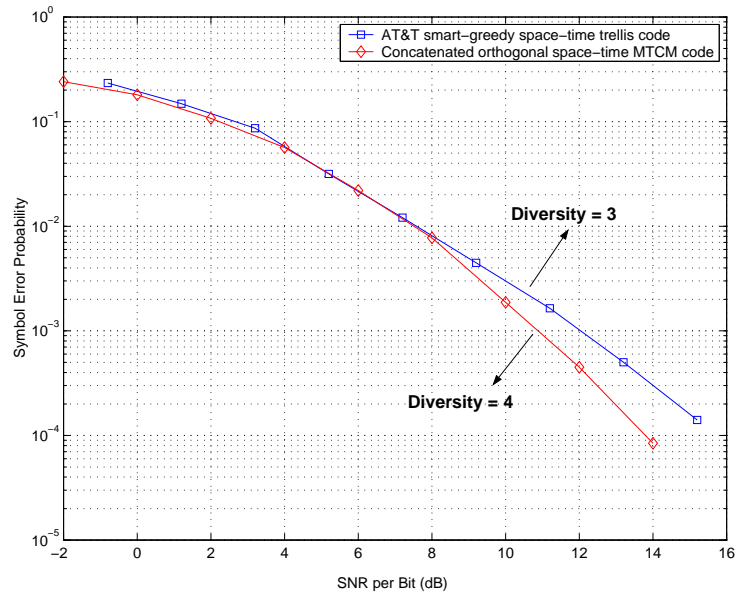


Figure 4.5 Performance comparison of AT&T smart-greedy space-time trellis code with concatenated orthogonal space-time MTCM code with two transmit and one receive antennas in a fast fading channel ($R = 1$ b/s/Hz).

the concatenated orthogonal space-time code in Figure 3.1 applying an MTCM outer code of multiplicity 4. The complexities of the two trellises are similar (both have 2 states), and the code rate is 1 b/s/Hz in both cases.

As can be seen from the simulation results of Figures 4.4 and 4.5, the concatenated space-time code shows a better performance in both slowly and fast fading channels. In slowly fading case, the concatenated system provides just a spatial diversity gain of 2 and no additional temporal diversity is gained from the outer MTCM scheme. That is why the two error curves are parallel with slope of almost -2 . However, the concatenated system presents a higher coding gain. In the case of fast fading channel, the concatenated space-time code demonstrates an asymptotic diversity gain of 4 (2 spatial and 2 temporal from the inner code), as compared to the gain of 3 resulting from the smart-greedy code.

Chapter 5

Design of Concatenated Space-Time Codes with Generalized Orthogonal Transmit Diversity

In this chapter, we will extend the design criteria of Chapter 3 to orthogonal systems with more than two transmit and one receive antennas. We refer to these as *Generalized Orthogonal Transmit Diversity (GOTD)* systems.

In [19], it was proven that for complex signal constellations, full rate, full diversity orthogonal designs of size n_T exist if and only if $n_T = 2$ (n_T is the number of transmit antennas). For more than two transmit antennas, full diversity generalized orthogonal designs with rates less than one, have been introduced in the literature [19, 22]. These designs fall into two main categories: rate halving codes and square matrix embeddable codes. The rate halving codes [19] are basically built by concatenating real orthogonal code block matrices and their complex conjugates together, halving the overall rate of the resulting complex orthogonal design. For example the rate 1/2 code for 4 transmit antennas built from the real orthogonal design has the following code block matrix

$$\mathbf{c} = \begin{pmatrix} c_1 & -c_2 & -c_3 & -c_4 & c_1^* & -c_2^* & -c_3^* & -c_4^* \\ c_2 & c_1 & c_4 & -c_3 & c_2^* & c_1^* & c_4^* & -c_3^* \\ c_3 & -c_4 & c_1 & c_2 & c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & c_3 & -c_2 & c_1 & c_4^* & c_3^* & -c_2^* & c_1^* \end{pmatrix}. \quad (5.1)$$

The square matrix embeddable codes are based on orthogonal square code matrices with dimension n_T (assuming to be a power of 2). For number of transmit antennas

that are not a power of two, the code block matrix dimension is assumed to be $n_T \times Q$ ($Q \geq n_T$ for the code to be linearly decodable) which is obtained by deleting a row from an orthogonal design of higher dimension. For example, for the case of three antennas, the code is constructed by deleting one row from the 4×4 square code. Codes for 5, 6 and 7 antennas are built similarly from the 8×8 square orthogonal design and so on.

Some examples of the square matrix embeddable codes are the sporadic codes of [19] for $n_T = 3$ and 4 (with rate $3/4$), and the unitary designs of [22]. It has been proven in [22] that the maximum achievable rate of these codes with n_T transmit antennas is $\frac{\lceil \log_2 n_T \rceil + 1}{2 \lceil \log_2 n_T \rceil}$ ($\lceil x \rceil$ is the integer greater or equal to x). Unitary designs and their construction have been introduced and fully explained in [22]. The code matrix for a unitary design with four transmit antennas and the maximum rate of $3/4$, constructed in [22] has the code block matrix

$$\mathbf{c} = \begin{pmatrix} c_1 & -c_2^* & -c_3^* & 0 \\ c_2 & c_1^* & 0 & c_3^* \\ c_3 & 0 & c_1^* & -c_2^* \\ 0 & -c_3 & c_2 & c_1 \end{pmatrix}, \quad (5.2)$$

while the code matrix of the sporadic code of [19] having the same rate and number of transmit antennas is expressed as

$$\mathbf{c} = \begin{pmatrix} c_1 & -c_2^* & \frac{c_3^*}{\sqrt{2}} & \frac{c_3^*}{\sqrt{2}} \\ c_2 & c_1^* & \frac{c_3^*}{\sqrt{2}} & \frac{c_3^*}{\sqrt{2}} \\ \frac{c_3}{\sqrt{2}} & \frac{c_3}{\sqrt{2}} & \frac{(-c_1 - c_1^* + c_2 - c_2^*)}{\sqrt{2}} & \frac{(c_2 + c_2^* + c_1 - c_1^*)}{\sqrt{2}} \\ \frac{c_3}{\sqrt{2}} & -\frac{c_3}{\sqrt{2}} & \frac{(-c_2 - c_2^* + c_1 - c_1^*)}{\sqrt{2}} & -\frac{(c_1 + c_1^* + c_2 - c_2^*)}{\sqrt{2}} \end{pmatrix}. \quad (5.3)$$

It can be seen that the unitary design of (5.2) has simpler structure and is more power balanced compared to the sporadic code of (5.3).

For a generalized orthogonal design of rate R and code matrix block size of $n_T \times Q$, we assume that RQ symbols are transmitted during Q consecutive symbol intervals, through the GOTD transmitter. As an example, for the orthogonal design of (5.2) where $Q = 4$ and $R = 3/4$, 3 coded symbols are transmitted during the transmission period of the code block.

The important characteristic of all the above designs is the orthogonality of their code block matrix, \mathbf{c}

$$\mathbf{c}\mathbf{c}^H = \sum_{q=1}^{RQ} |c_q|^2 I_{n_T}. \quad (5.4)$$

This causes the orthogonal systems to be capable of providing a full spatial diversity gain of $n_T n_R$ (n_R is the number of receive antennas) as will later be shown in this chapter. Moreover, in a similar way to the discussion of Section 3.4, it can be shown that the optimum decoder for the concatenated space-time codes with GOTD systems, is also obtained by concatenation of a linear combiner with a maximum likelihood decoder for the outer coded modulation scheme.

In the next two sections, we will study the criteria of designing concatenated space-time codes with the GOTD system in slow and fast fading channels.

5.1 Design Criteria for Slowly Fading Channels

As in Section 3.2, in a slowly fading environment, we assume constant fading coefficients for the channel during the transmission of the whole block of length L . Considering a single channel block ($K = 1$) of length $M = L$, (2.1) results in

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\det(I_{n_T} + \mathbf{D}(\mathbf{c}, \mathbf{e})\mathbf{D}^H(\mathbf{c}, \mathbf{e})\frac{E_s}{4N_0})^{n_R}}. \quad (5.5)$$

On the other hand, because of the linearity of all the orthogonal designs discussed so far, the code difference matrix $\mathbf{D}(\mathbf{c}, \mathbf{e})$, will also inherit the orthogonality property

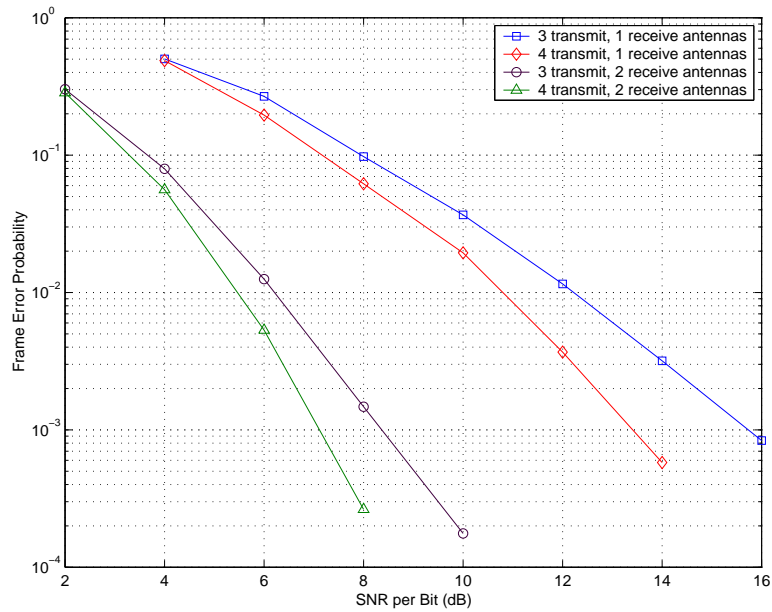


Figure 5.1 Error performance of codes designed for generalized orthogonal transmit diversity systems with more than two transmit or one receive antennas. Codes are designed for slowly fading channels ($R = 1.5$ b/s/Hz).

(5.4) of the code block matrix [22]. Thus for the code difference matrix $\mathbf{D}(\mathbf{c}, \mathbf{e})$ expressed as

$$\mathbf{D}(\mathbf{c}, \mathbf{e}) = (\mathbf{D}_1(\mathbf{c}, \mathbf{e}), \mathbf{D}_2(\mathbf{c}, \mathbf{e}), \dots, \mathbf{D}_{L/Q}(\mathbf{c}, \mathbf{e})), \quad (5.6)$$

we will have

$$\mathbf{D}_p(\mathbf{c}, \mathbf{e})\mathbf{D}_p^H(\mathbf{c}, \mathbf{e}) = \sum_{q=1}^{RQ} |c_{(p-1)RQ+q} - e_{(p-1)RQ+q}|^2 I_{n_T}, \quad \text{for } p = 1, 2, \dots, L/Q. \quad (5.7)$$

So, (5.5) reduces to

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\det(I_{n_T} + \sum_{p=1}^{L/Q} \sum_{q=1}^{RQ} |c_{(p-1)RQ+q} - e_{(p-1)RQ+q}|^2 \frac{E_s}{4N_0} I_{n_T})^{n_R}}, \quad (5.8)$$

or equivalently

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{(1 + \sum_{l=1}^{RL} |c_l - e_l|^2 \frac{E_s}{4N_0})^{n_T n_R}}. \quad (5.9)$$

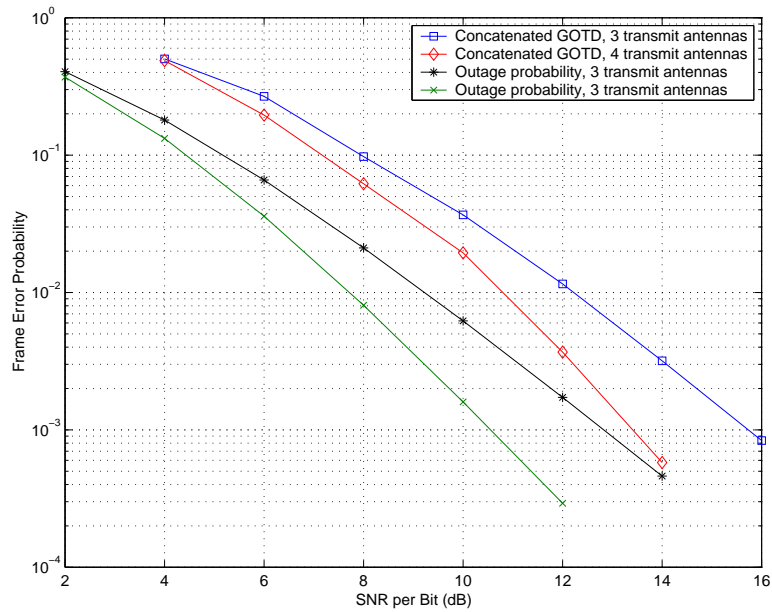


Figure 5.2 Comparison of the performance of rate 1.5 b/s/Hz codes designed for generalized orthogonal transmit diversity systems with 1 receive antenna, with the outage probability in a slowly fading channel.

An upper bound for the pairwise error probability of concatenated space-time codes employing GOTD systems in slowly fading channel conditions, follows from the Chernoff bound of (5.9) as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \left[\left(\sum_{l=1}^{RL} |c_l - e_l|^2 \right) \left(\frac{E_s}{4N_0} \right) \right]^{-n_T n_R}. \quad (5.10)$$

The above upper bound shows a full spatial diversity gain of $n_T n_R$ provided by the GOTD system. It can also be seen that due to the slowly fading characteristic of the channel, the coding scheme cannot provide any temporal diversity gain. Thus, the criterion for the design of optimum outer coded modulation schemes in this case, is also based on the maximization of the code free Euclidean distance

$$d_e(\mathbf{c}, \mathbf{e}) = \sum_{l=1}^{RL} |c_l - e_l|^2. \quad (5.11)$$

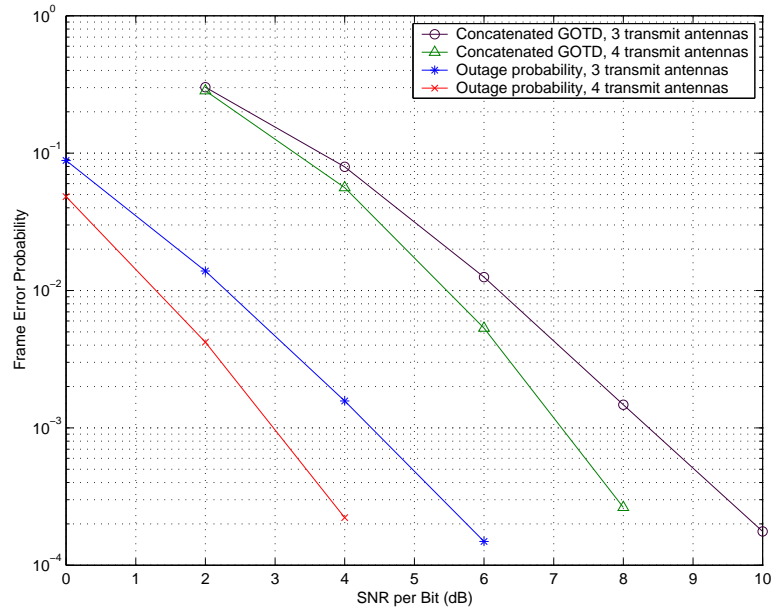


Figure 5.3 Comparison of the performance of rate 1.5 b/s/Hz codes designed for generalized orthogonal transmit diversity systems with 2 receive antennas, with the outage probability in a slowly fading channel.

Thus, it is concluded that for slowly fading channels, the design criterion of concatenated space-time codes with GOTD systems is the same as the code design rule for the system of Figure 3.1 described in Section 3.2.

In Figure 5.1, the simulation results for the frame error probability of codes designed for the unitary design of (5.2) are provided. As the outer code, we have used the 8-state Ungerboeck's TCM code [24] designed based on the maximization of the free Euclidean distance, for optimum performance in AWGN channels. The inner code is the GOTD unitary design of (5.2) with $n_T = 3$ and 4 antennas. The unitary design for 3 transmit antennas is obtained by deleting one arbitrary row of (5.2). Ungerboeck's TCM code has a rate of 2 b/s/Hz, which together with the rate 3/4 of the unitary design, results in an overall rate of 1.5 b/s/Hz.

It can be noticed that there is an increase in the slope of the frame error probability curves as the number of transmit and receive antennas increase. This increase in the diversity gain is expected according to (5.10). Moreover, the codes with 2 receive antennas provide a higher coding gain with respect to the single receive antenna codes. It is seen that at frame error rates of 10^{-3} and lower, the code with 4 transmit and 2 receive antennas gives more than 6 dB gain over the case of 4 transmit and single receive antennas.

The performance comparison with the outage probability is demonstrated in Figures 5.2 and 5.3. It can be seen that at frame error rate of 0.1 (in these simulations, each frame consists of 176 coded symbols transmitted from each transmit antenna), the code for 4 transmit and single receive antennas performs within approximately 2.5 dB of the outage probability.

5.2 Design Criteria for Fast Fading Channels

In this section, we assume that the block length of the fast fading channel equals to the length of the code block matrix ($M = Q$). Thus, for a total of $K = L/Q$ blocks, it follows from (2.1) that

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^{L/Q} \frac{1}{\det(I_{n_T} + \mathbf{D}_k(\mathbf{c}, \mathbf{e})\mathbf{D}_k^H(\mathbf{c}, \mathbf{e})\frac{E_s}{4N_0})^{n_R}}. \quad (5.12)$$

Substituting (5.7) into (5.12), an upper bound for the pairwise error probability of concatenated space-time codes with the GOTD systems in fast fading channels, can be derived as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \prod_{\substack{(c_{(k-1)RQ+1}, \dots, c_{kRQ}) \neq \\ (e_{(k-1)RQ+1}, \dots, e_{kRQ}) \\ k = 1, \dots, L/Q}} \left[\left(\sum_{q=1}^{RQ} |c_{(k-1)RQ+q} - e_{(k-1)RQ+q}|^2 \right) \left(\frac{E_s}{4N_0} \right) \right]^{-n_T n_R}. \quad (5.13)$$

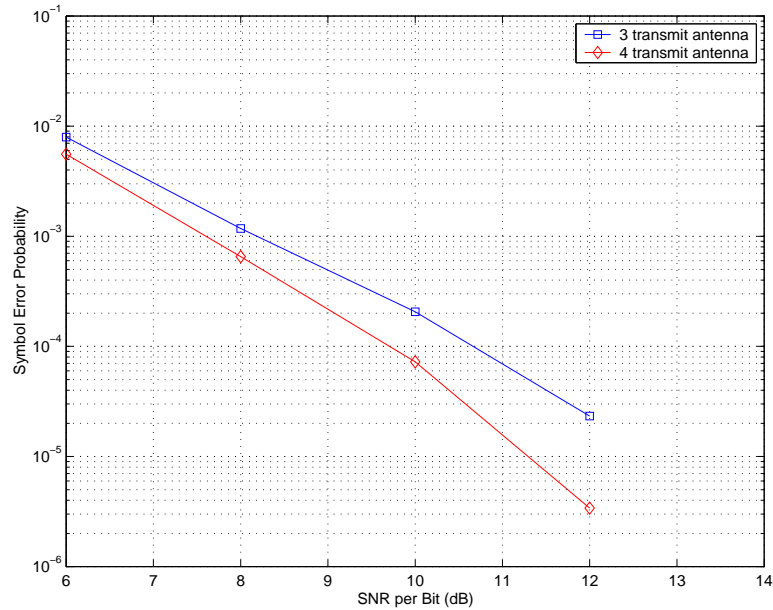


Figure 5.4 Error performance of codes designed for generalized orthogonal transmit diversity system with three and four transmit and single receive antennas. Codes are designed for fast fading channels ($R = 1$ b/s/Hz).

It should be noted from (5.13) that in the case of fast fading channels, in addition to the full spatial diversity gain of $n_T n_R$ resulting from the GOTD system, the coding scheme is also capable of providing some temporal diversity gain. Similar to the discussion of Section 3.3, it can be concluded that the code design criteria in this case is also based on the maximization of minimum product and Hamming distances in the expanded signal set. This expansion can be performed in both dimension or size, however here for the generalized orthogonal designs, the constellation points in the new signal set are the concatenation of RQ signal points (2 for the system of Figure 3.1) in the original signal set. Thus for an original 2D signal set of size M , the signal set expansion in size is equivalent to constructing a new 2D constellation of size M^{RQ} and performing the set partitioning in the new signal set.

To demonstrate the above design procedure, we have simulated the performance of codes designed for the unitary design of (5.2) with 3 and 4 transmit and single receive antennas in fast fading channels. The simulation results are reported in Figure 5.4. The outer encoder is a 4-state fully connected MTCM code with 64 parallel paths, multiplicity of 6 and uses QPSK modulation. The code design would be based on the maximization of Hamming and product distances in the expanded constellation, where each point is the concatenation of $RQ = 3$ signal points from the original QPSK signal set. The aim is to achieve diversity gains of 6 and 8 for three and four transmit antennas respectively. The set partitioning technique of [8] for the 2-fold Cartesian product of $4^3 = 64$ PSK signal set would result in subsets of size 64 (that is 64 parallel paths). Thus the 4-state fully connected trellis would be capable of encoding 8 bits. Together with 6 QPSK output symbols per each transition of the trellis, the rate of the MTCM encoder would be $4/3$ b/s/Hz. Since the rate of the unitary design of (5.2) is $3/4$, the overall rate of the concatenated code will be 1 b/s/Hz.

It can be seen from Figure 5.4 that asymptotically, the codes are achieving their expected diversity gains of 6 and 8 for three and four transmit antennas, respectively. The code for 4 transmit antennas shows over 1 dB gain over the 3 antenna code at symbol error rates of 10^{-4} and lower.

Chapter 6

Broadcast Coding for Multiple-Antenna Systems

This chapter is mainly intended to motivate our ongoing research on the problem of designing codes for fading broadcast channels. We start by overviewing the problem and will later provide a system model description. The achievable mutual information rates for different cases of channel state information at the transmitter will be discussed in Section 6.3. In the last section of this chapter, some preliminary numerical results on the achievable user capacities, adopting multiple-antenna systems in broadcast channels, will be presented. These results will motivate our future research plans which will be outlined in the same section.

6.1 Overview

Next generation of wireless networks are expected to support high rate data applications in addition to voice. In fact it is anticipated that packet data will dominate voice data in future's communication systems. Wireless networks supporting data applications can range from cellular networks with central controllers in the form of base stations to ad hoc networks where no central control units are available.

High data rate applications are characterized by their required throughput and maximum sustainable delay and error rates. In order to meet these constraints and provide quality of service to all simultaneous data users, the issue of downlink resource allocation and scheduling in wireless networks has recently been addressed by some researchers [2, 4]. Some of the proposed scheduling schemes take advantage of the

feed back channel information at the transmitter to change the transmission rates so as to satisfy the delay and throughput constraints of most of the users [4].

In this research, we study a downlink broadcast scenario where a single transmitter is simultaneously transmitting data to multiple data users and no time sharing scheme is adopted. The transmitter is a multiple-element antenna system. The use of multiple antennas at the receivers is optional. Each user is interested only in part of the transmitted data and for complexity reasons, we assume that each receiver is equipped with a single-user optimal decoder.

The issue of finding the set of simultaneously achievable rates by different users in a broadcast channel was first introduced by Cover in [5]. He showed that by superimposing high-rate and low-rate information in a Gaussian broadcast channel, it is possible to design codes to achieve rates greater than those achievable by simple time-sharing schemes [5, 6].

For the case of fading broadcast channels, the capacity region has been studied in [13, 14], under the assumption that both the transmitter and the receiver have perfect channel information. In [13, 14], the capacity regions of code division (CD), time division (TD) and frequency division (FD) schemes have been calculated. It is also shown that the CD schemes can achieve higher simultaneous rates as compared to TD and FD schemes.

In this work, we evaluate the effect of using multiple-antenna systems in a downlink broadcast fading scenario. We will show that using a beamforming scheme, it would be possible to simultaneously transmit to more than one user. The beamforming vectors at the transmitter are computed such that to cancel out the interference of other users. Thus, it suffices for the users to be equipped with single-user detectors as opposed to multi-user detectors, which are more complex. We will quantify the gains in achievable user capacities resulting from the use of multiple-antenna systems in a

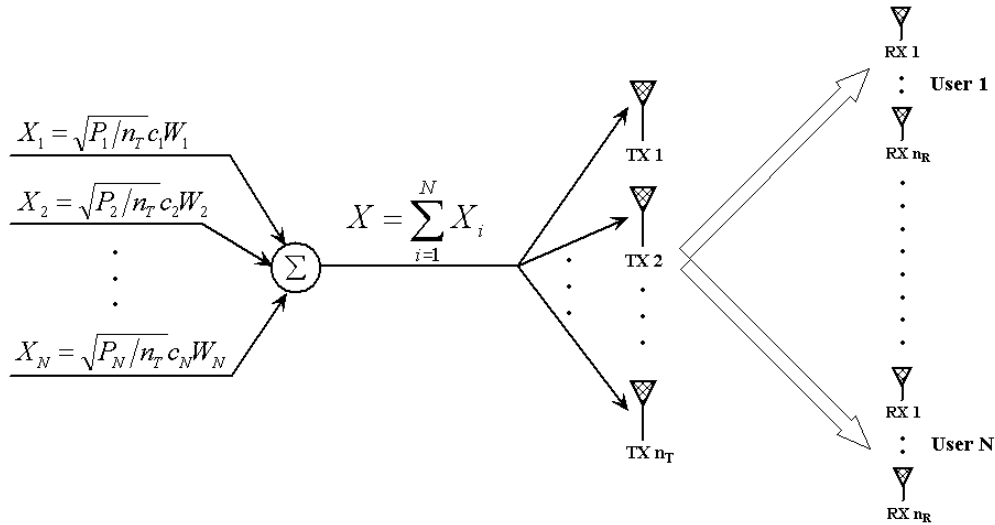


Figure 6.1 Broadcast Beamforming Scheme

broadcast channel with different number of simultaneous data users. But first, let's present the system model.

6.2 System Model

The system model is shown in Figure 6.1. We assume multiple-element antennas at the transmitter and the receiver of each user. The number of transmit and receive antenna elements are denoted by n_T and n_R , respectively. It is known that using a beamforming scheme, the system can support a maximum number of $N = n_T$ simultaneous users [11].

The system works as follows: at a certain symbol interval, the code symbol of the i^{th} user, c_i , chosen from a unit energy constellation distributed as $\mathcal{CN}(0, 1)$, is multiplied by its corresponding $n_T \times 1$ normalized beamforming vector, \mathbf{W}_i , summed over all users and then transmitted over the n_T antennas. The power assigned to user

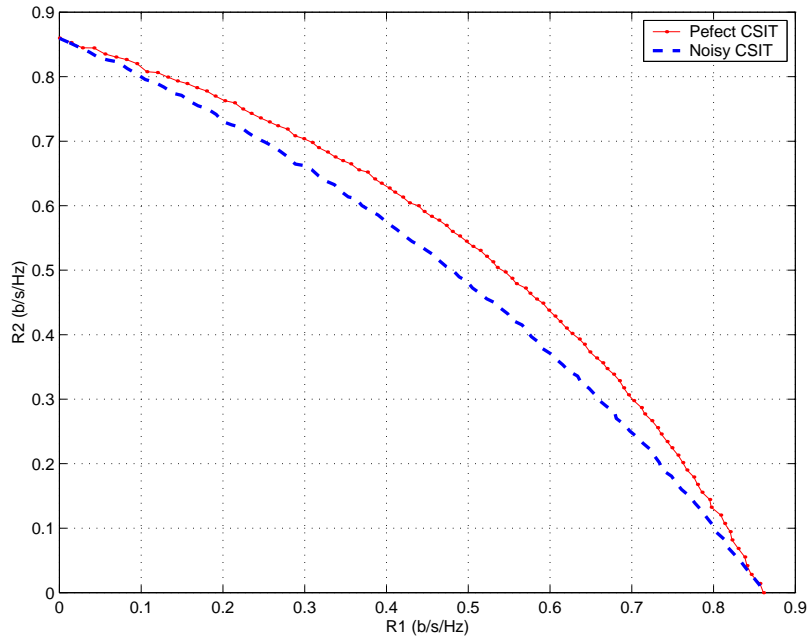


Figure 6.2 Achievable rates region for perfect and noisy ($\sigma_z^2 = 0.1$) CSIT ($N = 2$ and $n_T = 2$).

i is denoted by P_i , and the average power constraint of the system is expressed as

$$\text{trace}(\mathbb{E}[\mathbf{X}\mathbf{X}^H]) = \sum_{i=1}^N \mathbf{P}_i \leq \mathbf{P}. \quad (6.1)$$

The channel is modeled as slowly Rayleigh fading AWGN, with independent and identically distributed (i.i.d.) fading coefficients between each pair of transmit and receive antennas of each user. The fading is also considered to be i.i.d. between different users. The baseband representation for the received signal of the i^{th} user is

$$\mathbf{Y}_i = \mathbf{H}_i^T \mathbf{X} + \mathbf{N}_i, \text{ for } i = 1, 2, \dots, N, \quad (6.2)$$

where \mathbf{X} is the $n_T \times 1$ vector of the sum of the simultaneous transmitted information data of all users as in Figure 6.1, \mathbf{H}_i is the $n_T \times n_R$ channel matrix of user i , \mathbf{N}_i is the complex circularly symmetric additive white Gaussian noise vector, and \mathbf{Y}_i is

the $n_R \times 1$ received signal. Both the channel matrix \mathbf{H}_i and the noise vector \mathbf{N}_i are distributed as $\mathcal{CN}(0, I)$.

6.3 Achievable Rates Region

In this section, the achievable rates region of the broadcast beamforming scheme of Figure 6.1 will be analyzed for two cases of perfect and noisy channel state information at the transmitter (CSIT). It is considered that perfect channel state information is always available at the receiver. For simplicity, we assume that the receivers of all users consist of a single-element antenna.

6.3.1 Perfect CSIT

When perfect channel state information is available at the transmitter, the beamforming vectors can be chosen such that the power of the undesired signals at each user is made equal to zero at any instant. In order to do this and also to maximize the average mutual information, it can be easily shown that the beamforming vector of the i^{th} user, \mathbf{W}_i , should be computed as the projection of its own channel vector \mathbf{H}_i onto the intersection of the null spaces of the channel vectors of all other users. Having chosen the beamforming vectors as such, the baseband representation of the received signal in (6.2) reduces to

$$\mathbf{Y}_i = \mathbf{H}_i^T \mathbf{X}_i + \mathbf{N}_i, \text{ for } i = 1, 2, \dots, N. \quad (6.3)$$

This means that each user will receive only that part of the information data which is primarily intended for him. It can be further shown that the achievable rates region of the users can be expressed as

$$R_i \leq \mathbb{E}_{\mathbf{H}_i} \left[\log \left(1 + \frac{P_i}{n_T} |\mathbf{H}_i^T \mathbf{W}_i|^2 \right) \right], \text{ for } i = 1, 2, \dots, N. \quad (6.4)$$

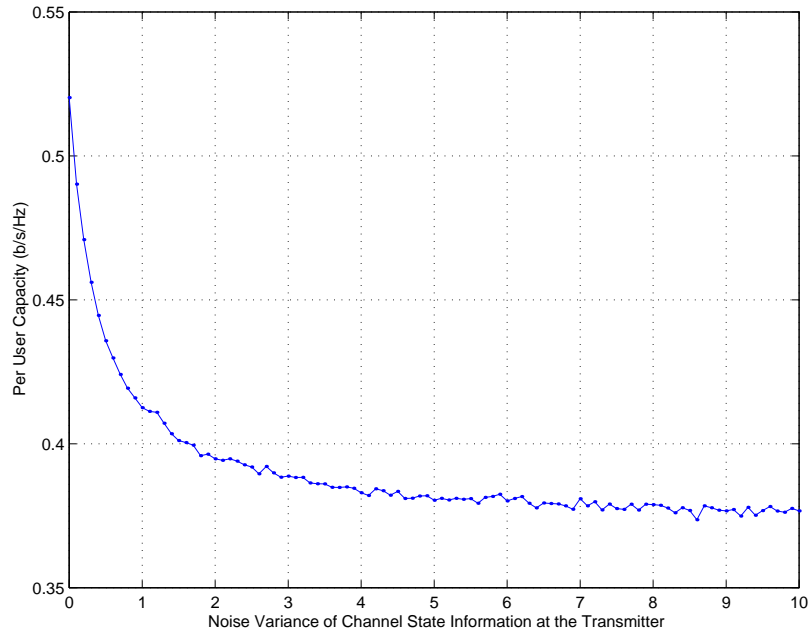


Figure 6.3 Per user capacity for different values of σ_z^2 ($N = 2$ and $n_T = 2$)

6.3.2 Noisy CSIT

Here, it is assumed that a noisy version of the channel state information is available at the transmitter. Denoting the noisy channel state information of the i^{th} user as $\hat{\mathbf{H}}_i$ and its corresponding beamforming vector as $\hat{\mathbf{W}}_i$, we will have

$$\hat{\mathbf{H}}_i = \mathbf{H}_i + \mathbf{Z}_i, \text{ for } i = 1, 2, \dots, N, \quad (6.5)$$

where \mathbf{Z}_i is a complex additive white Gaussian noise distributed as $\mathcal{CN}(0, \sigma_z^2 I)$.

Using the beamforming scheme explained in Section 6.3.1, it is evident that in this case the undesired signal energy at the users cannot be completely neutralized. The achievable rates region of the users can be shown to be smaller than the perfect

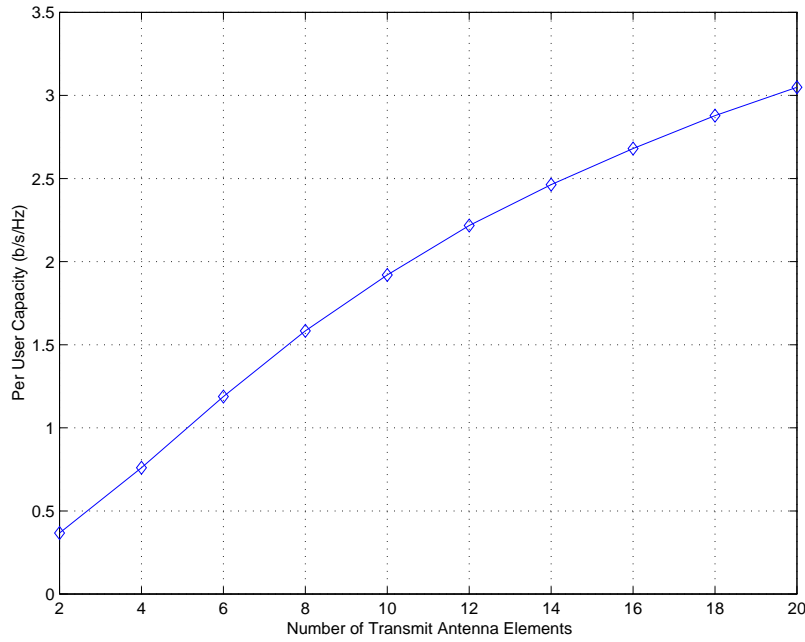


Figure 6.4 Per user capacity vs number of transmit antenna elements for fixed number of users ($N = 2$).

CSIT case as

$$R_i \leq \mathbb{E}_{\mathbf{H}_i, \hat{\mathbf{H}}_j} \left[\log \left(1 + \frac{P_i/n_T |\mathbf{H}_i^T \hat{\mathbf{W}}_i|^2}{1 + \sum_{\substack{j=1 \\ j \neq i}}^N P_j/n_T |\mathbf{H}_i^T \hat{\mathbf{W}}_j|^2} \right) \right], \text{ for } i = 1, 2, \dots, N. \quad (6.6)$$

6.4 Numerical Results and Future Work

In this section, some preliminary numerical results on the achievable rates of the broadcast beamforming scheme of Figure 6.1, using Monte-Carlo integration technique, are presented. These results motivate further research work on related issues.

In Figure 6.2, the achievable rates regions of the broadcast beamforming scheme with two users are plotted for the two cases of perfect and noisy CSIT. In the second case, the variance of the additive noise of the channel state information at the trans-

mitter (σ_z^2) is considered to be 0.1. The number of transmit antenna elements (n_T) are 2 for both cases, and the total power constraint of the transmitter (P) is assumed to be 0 dB.

Similarly, the per user capacity of the noisy CSIT case for different values of noise variance (σ_z^2) is plotted in Figure 6.3. Here, we have considered equal powers assigned to each user. No power control scheme is adopted in any of the above cases. These results show the importance of channel state information at the transmitter for the broadcast beamforming scheme of Figure 6.1. It can be seen that as the variance of the noise increases the scheme becomes less and less efficient.

In Figure 6.4 and 6.5, the effect of the number of transmit antenna elements as well as the number of simultaneous users in the network, have been analyzed. In Figure 6.4, the per user capacity is plotted for different numbers of transmit antennas. Here, it is assumed that the number of users are fixed and equal to 2. It can be seen that as the number of transmit antennas increase, higher rates can be achieved. This is in fact expected as we know that the application of multiple-antenna systems results in higher data rates in wireless networks [21].

The case of variable number of users has been studied in Figure 6.5, where the total user capacity is plotted versus the number of transmit antennas for fixed $\frac{N}{n_T} = \alpha$. It is noticed that there is still an increase in the total achievable capacity as the number of transmit antennas increase.

Inspired by the gains obtained in achievable rates using a multiple-element antenna structure in a broadcast channel, it would be interesting to do further research on the design of codes which can actually achieve those gains. We will consider the design and error performance analysis of space-time codes for broadcast channels as future work.

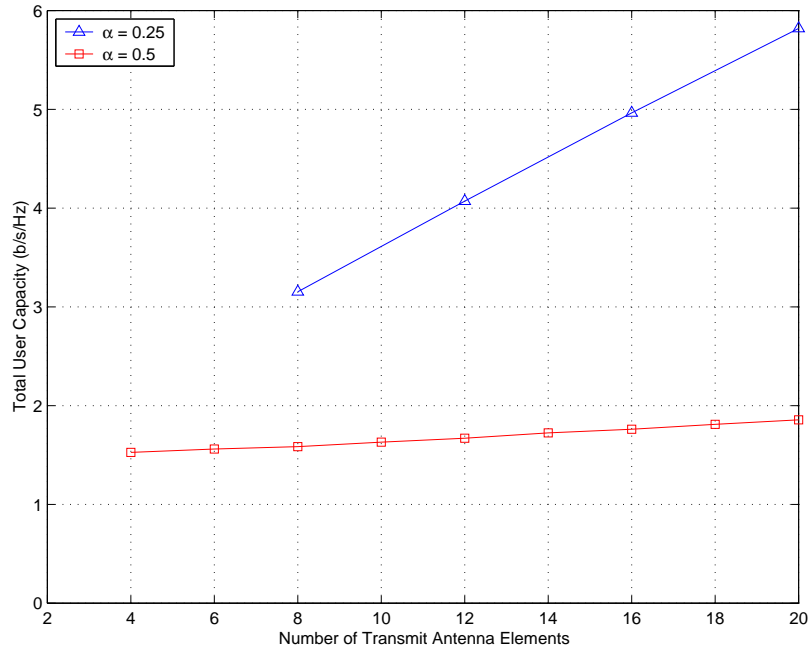


Figure 6.5 Total user capacity vs number of transmit antenna elements for variable number of users ($\frac{N}{n_T} = \alpha$).

So far, we have been assuming that in addition to channel state information, the information regarding the beamforming vectors are also available at the receiver of all users. This implies a network where all the users (nodes) share information with each other (global channel knowledge). We are also planning to analyze a distributed network where no information is shared between different nodes (local channel knowledge). And finally the design of power control schemes for the broadcast beamforming scheme of Figure 6.1 is another important issue to be considered in this research.

Chapter 7

Conclusions

In this thesis, we propose a systematic technique in order to design space-time codes for slowly and fast AWGN Rayleigh fading channels. It is shown that this can be done by constructing a concatenated space-time code structure, with an orthogonal transmit diversity system as the inner code. Applying this system will result in decoupling the problems of temporal and spatial diversity gains maximization, involved in the design of space-time codes and hence will considerably simplify the code design procedure. We also derive the code design criteria of the outer encoder for both slowly and fast fading channel conditions.

In a slowly fading channel, no temporal diversity gain can be provided by the outer coding scheme. Thus, the concatenated space-time code structure will help in decoupling the problems of spatial diversity gain and coding gain maximization. The inner orthogonal transmit diversity system, will provide full spatial diversity gain. In order to maximize the coding gain, it is shown that the criterion for the design of the outer encoder, is just based on the maximization of the free Euclidean distance. This is exactly equivalent to the design criterion of optimum codes for AWGN channels with single transmit and receive antennas. So, the codes designed for optimum performance in an AWGN channel are well suited as optimum outer codes in the concatenated space-time code structure.

For the case of a fast fading channel, significant temporal diversity gains can be provided by the outer coding scheme. Here, the concatenated structure of the space-

time code will help in decoupling the problems of spatial and temporal diversity gains maximization. The inner orthogonal transmit diversity system will again provide full spatial diversity gain. In order to design the outer code, we introduce the idea of constellation expansion, and show that the outer code design criteria in this case is based on the maximization of the Hamming and product distances in the expanded signal set.

To analyze the performance of our proposed design technique, we construct codes for both slowly and fast fading channels and compare them to some existing codes in the literature. It is observed from the error performance simulations that the codes showed better performance compared to some other codes having the same complexities. For the case of slowly fading channels, the codes are shown to perform close to the outage probabilities.

We also evaluate the effect of using multiple-antenna systems in downlink broadcast channels for different cases of channel state information at the transmitter. Inspired by the gains of multiple-antenna systems observed by some preliminary numerical analysis, it would be interesting to design codes which can actually achieve those gains. The design of power control schemes and the analysis of distributed networks, where the nodes do not share information among each other, are also among our future research plans.

Bibliography

- [1] S. M. Alamouti. A simple transmit diversity technique for wireless communications. *IEEE Journal on Selected Areas in Communications*, 16(8):1451–1458, Oct. 1998.
- [2] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, and P. Whiting. Providing quality of service over a shared wireless link. *IEEE Communications Magazine*, pages 150–154, February 2001.
- [3] M. J. Borran, M. Memarzadeh, and B. Aazhang. Design of coded modulation schemes for orthogoanal transmit divesity. *to be submitted to IEEE Transactions on Communications*.
- [4] S. Borst and P. Whiting. Dynamic rate control algorithms for hdr throughput optimization. In *Proceedings of IEEE INFOCOM '01*, April 2001.
- [5] T. M. Cover. Broadcast channels. *IEEE Transactions on Information Theory*, IT-18(1):2–14, January 1972.
- [6] T. M. Cover. Comments on broadcast channels. *IEEE Transactions on Information Theory*, 44(6):2524–2530, October 1998.
- [7] D. Divsalar and M. K. Simon. The design of trellis coded MPSK for fading channels: Performance criteria. *IEEE Transactions on Communications*, 36(9):1004–1012, Sept. 1988.

- [8] D. Divsalar and M. K. Simon. The design of trellis coded MPSK for fading channels: Set partitioning for optimum code design. *IEEE Transactions on Communications*, 36(9):1013–1021, Sept. 1988.
- [9] G. J. Foschini and M. J. Gans. On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Personal Communications*, 6(3):311–335, March 1998.
- [10] P. S. Henry and B. S. Glance. A new approach to high capacity digital mobile radio. *Bell Syst. Tech. J.*, 51:1611–1630, Sept. 1972.
- [11] D. H. Johnson and D. E. Dudgeon. *Array Signal Processing, Concepts and Techniques*. Prentice Hall, 1993.
- [12] S. M. Kay. *Fundamentals of Signal Processing: Estimation Theory*. Prentice Hall International Editions, 1993.
- [13] L. Lifang and A. Goldsmith. Capacity and optimal resource allocation for fading broadcast channels: Part I: Ergodic capacity. *To appear: IEEE Transactions on Information Theory*, March 2001.
- [14] L. Lifang and A. Goldsmith. Capacity and optimal resource allocation for fading broadcast channels: Part II: Outage capacity. *To appear: IEEE Transactions on Information Theory*, March 2001.
- [15] T. Muharemovic and B. Aazhang. Information theoretic optimality of orthogonal space-time transmit schemes and concatenated code construction. In *Proceedings of ICT*, June 2000.

- [16] A. Narula, M. Trott, and G. Wornell. Information theoretic analysis of multiple-antenna transmission diversity. *Submitted to IEEE Transactions on Information Theory*.
- [17] G. Raleigh and J. M. Cioffi. Spatio-temporal coding for wireless communications. In *Proceedings of IEEE GLOBECOM'96*, pages 1809–1814.
- [18] N. Seshadri and CE. W. Sundberg. Multilevel trellis coded modulations for the rayleigh fading channel. *IEEE Transactions on Communications*, 41(9):1300–1310, Sept. 1993.
- [19] V. Tarokh, H. Jafarkhani, and A. R. Calderbank. Space-time block codes from orthogonal designs. *IEEE Transactions on Information Theory*, 45(5):1456–1467, July 1999.
- [20] V. Tarokh, N. Seshadri, and A. R. Calderbank. Space-time codes for high data rate wireless communication: Performance criterion and code construction. *IEEE Transactions on Information Theory*, 44(2):744–765, March 1998.
- [21] I. E. Telatar. Capacity of multi-antenna gaussian channels. *AT&T-Bell Laboratories Internal Tech. Memo.*, June 1995.
- [22] O. Tirkkonen and A. Hottinen. Square-matrix embeddable space-time block codes for complex signal constellations. *submitted to IEEE Transactions on Information Theory*.
- [23] O. Tirkkonen and A. Hottinen. Complex space-time block codes for four tx antennas. In *Proceedings of Globecom*, pages 1005–1009, 2000.
- [24] G. Ungerboeck. Channel coding with multilevel/phase signals. *IEEE Transactions on Information Theory*, IT-28:55–67, Jan. 1982.

- [25] J. H. Winters. Switched diversity with feedback for DPSK mobile radio systems. *IEEE Trans. Veh. Technol.*, VT-32:134–150, Feb. 1983.
- [26] A. Wittneben. Base station modulation diversity for digital SIMULCAST. In *Proceedings of IEEE'VTC*, pages 505–511, May 1993.