Empirical mode decomposition based time-frequency attributes

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Summary

This paper describes a new technique, called the *empirical mode decomposition (EMD)*, that allows the decomposition of one-dimensional signals into intrinsic oscillatory modes. The components, called *intrinsic mode functions (IMFs)*, allow the calculation of a meaningful multicomponent instantaneous frequency. Applied to a seismic trace, the EMD allows us to study the different intrinsic oscillatory modes and instantaneous frequencies of the trace. Applied to a seismic section, it provides new time-frequency attributes.

Introduction

Seismic signals, as with many real-world signals, are non-stationary, making Fourier analysis unsatisfying since the frequency content changes across the time. Exploration of the frequency content of such signals dictates a local study of the frequency content. In time-frequency analysis, we analyze the frequency content across a small span of time and then move to another time position (Flandrin, 1993; Cohen, 1995). The major drawback of most time-frequency *transforms* is that the rectangular tiling of the time-frequency plane does not match the shape of many signals.

On the other hand, basis *decomposition* techniques such as the Fourier decomposition or the Wavelet decomposition (Daubechies, 1992) have also been used to analyze real-world signals. The main drawback of these approaches is that the basis functions are fixed, and do not necessarily match varying nature of signals.

In this paper, we use a new technique, called the *empirical mode decomposition (EMD)*, first introduced by Huang et al. (1998). This technique adaptively decomposes a signal into oscillating components. The different components match the signal itself very well, and the time-frequency representation extracted from the decomposition, called the *Hilbert amplitude spectrum*, does not have any constraint on the way to tile the time-frequency plane. Because the approach is algorithmic, it does not allow to express the different components in closed form.

The EMD is in fact type of adaptive wavelet decomposition whose subbands are built up as needed to separate the different components of the signal.

Empirical Mode Decomposition

The empirical mode decomposition was first introduced by Huang et al. (1998). The principle of this technique is to decompose a signal $x(t)$ into a sum of functions that: (1) have the same numbers of zero crossings and extrema; and (2) are symmetric with respect to the local mean. The first condition is similar to the narrow-band requirement for a stationary Gaussian process. The second condition modifies a global requirement to a local one, and is necessary to ensure that the instantaneous frequency will not have unwanted fluctuations as induced by asymmetric waveforms. These functions are called *intrinsic mode functions (IMFs)*, and denoted $imf_i(t)$. The intrinsic mode functions are obtained using the algorithm of Table 1.

1. Initialize: $r_0(t) = x(t)$, $i = 1$
2. Extract the i-th IMF:
   (a) Initialize: $h_0(t) = r_i(t)$, $j = 1$
   (b) Extract the local minima and maxima of $h_{j-1}(t)$
   (c) Interpolate the local maxima and the local minima by a cubic spline to form upper and lower envelopes of $h_{j-1}(t)$
   (d) Calculate the mean $m_{j-1}(t)$ of the upper and lower envelopes
   (e) $h_j(t) = h_{j-1}(t) - m_{j-1}(t)$
   (f) if stopping criterion is satisfied
   then set $imf_i(t) = h_j(t)$
   else go to (b) with $j = j + 1$
3. $r_i(t) = r_{i-1}(t) - imf_i(t)$
4. if $r_i(t)$ still has at least 2 extrema
   then go to 2 with $i = i + 1$
   else the decomposition is finished
   and $r_i(t)$ is the residue.

Table 1: *Algorithm for the EMD.*

At the end of the algorithm, we have:

$$x(t) = \sum_{i=1}^{n} imf_i(t) + r_n(t)$$

where $r_n(t)$ is the residue of the decomposition.

Another way to explain how the empirical mode decomposition works is that it picks out the highest frequency oscillation that remains in the signal. Thus, locally, each IMF contains lower frequency oscillations than the one extracted just before. This property can be very useful to pick up frequency changes, since a change will appear even more clearly at the level of an IMF.

To illustrate the EMD, we plot in Fig. 1 a seismic trace, the first three IMFs, and the residue. We can see easily

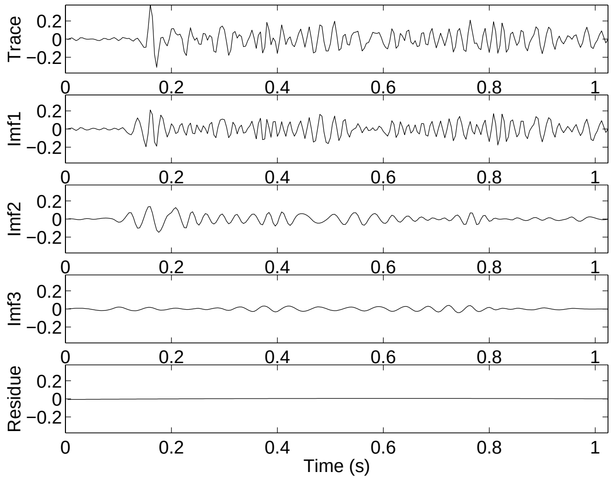

Fig. 1: *Illustration of the EMD: seismic trace, first three IMFs, and the residue of the decomposition (the IMFs 4 to 9 are not shown).*

that each component has the same number of zero crossings as extrema and is symmetric with respect to the zero line. Hence, we can calculate for each component a meaningful instantaneous frequency. The residue, in this case, has almost a zero value which means that the original signal had a zero mean, as it is the case for most seismic traces. More generally, the residue captures the trend of the original signal.

Complex Traces and Instantaneous Frequency

The notions of complex signals associated with real signals and of instantaneous frequency (Flandrin, 1993; Cohen, 1995) have been first used in seismic analysis by Taner et al. (1979).

Given a real signal $x(t)$, we can build the corresponding analytic signal (or complex trace):

$$X(t) = x(t) + j\mathcal{H}\{x(t)\}$$

where $\mathcal{H}\{x(t)\}$ is the quadrature signal corresponding to $x(t)$ obtained using the Hilbert transform

$$\mathcal{H}\{x(t)\} = \frac{1}{\pi}\, pv. \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau$$

with $pv. \int_{-\infty}^{+\infty}$ being the Cauchy principle value of the integral.

The analytic signal can also be obtained by: (1) taking the Fourier transform of $x(t)$; (2) zeroing the amplitude for negative frequencies and doubling the amplitude for positive frequencies; and (3) taking the inverse Fourier transform.

The complex trace can be expressed as:

$$X(t) = A(t)\, e^{j\,\theta(t)}$$

where $A(t)$ is the instantaneous amplitude (or reflection strength) and $\theta(t)$ is the instantaneous phase. We can then define the instantaneous frequency $\omega(t)$ as

$$\omega(t) = \frac{d\theta(t)}{dt}.$$

Instead of calculating the complex trace associated with the seismic trace $x(t)$ itself, we can calculate the complex signal associated with each of the intrinsic mode functions, and thus the instantaneous frequency of each of them, in order to obtain a multicomponent instantaneous frequency.

Fig. 2 represents the instantaneous frequency (normalized between 0 and 0.5) of the original trace and of the first three IMFs.

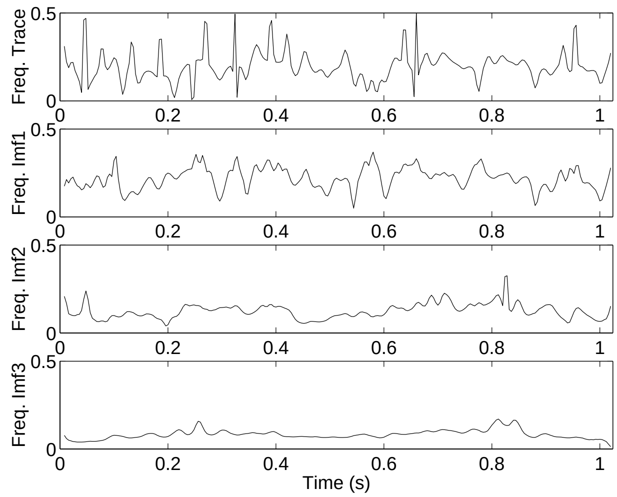

Fig. 2: *Normalized instantaneous frequencies of the original trace and of the first three IMFs.*

We observe that the instantaneous frequency of the first IMF is very similar to the instantaneous frequency of the original trace, but with much fewer peaks due to noise blow-up, which is a great advantage. The instantaneous frequencies of the other components give additional information about the different oscillatory modes of the original trace.

The fact that the instantaneous frequency of the first IMF possesses so few noise peaks suggests we use it as a time-frequency attribute instead of the instantaneous frequency of the original trace itself.

Hilbert Amplitude Spectrum

Once we have calculated the instantaneous frequency of each IMF, we can represent the Hilbert amplitude spectrum of the initial seismic trace as in Huang et al. (1998), which is obtained in the following way: for each intrinsic mode function $imf_i(t)$, if $\omega_i(t)$ is the corresponding instantaneous function, we represent in the time-frequency plane the triplet $\{t, \omega_i(t), A_i(t)\}$ where $A_i(t)$ is the amplitude of the complex trace associated to $imf_i(t)$.

Fig. 3 represents the Hilbert amplitude spectrum of the previous seismic trace, using 512 frequency bins.

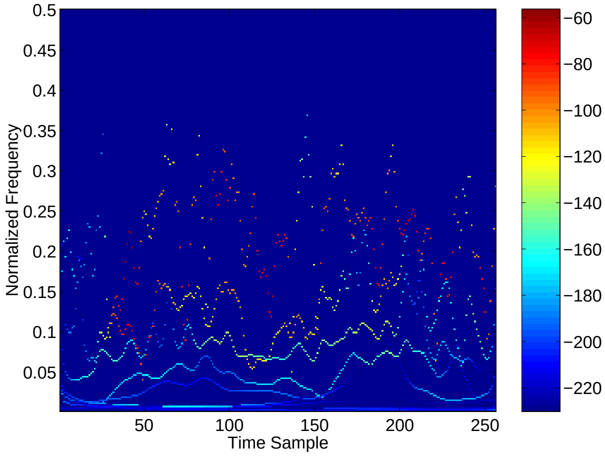

Fig. 3: *Hilbert amplitude spectrum using 512 frequency bins. The amplitude is represented in dB.*

EMD-Based Time-Frequency Attributes

The next step of this analysis is to define new time-frequency attributes based on the EMD. Fig. 4 represents a migrated seismic section from the Alboran sea composed of 100 traces of 256 samples each.

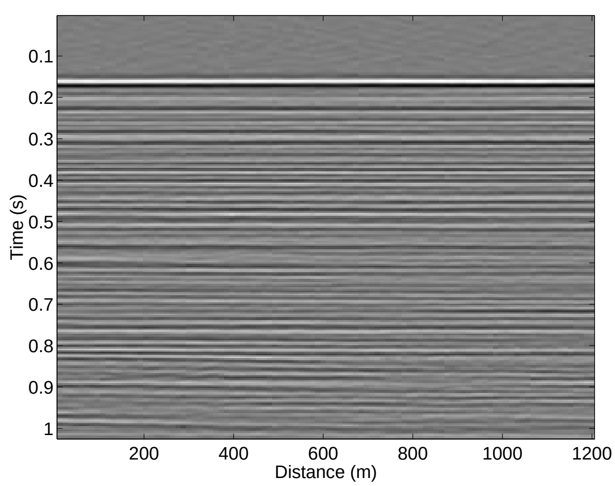

Fig. 4: *Migrated seismic section from the Alboran sea.*

On each trace, we compute the instantaneous frequency as defined in Taner *et al.* (1979). Fig. 5 represents the instantaneous frequency attribute for the Alboran sea section.

We also compute for each trace the instantaneous frequency of the IMFs obtained by the EMD. The time-frequency attributes represented Fig. 6, Fig. 7, and Fig. 8 were obtained respectively by computing for each trace the instantaneous frequency of the first, second, and third IMFs.

The new time-frequency attributes do not have as many saturated lines as the instantaneous frequency attribute. These saturated lines, corresponding to the peaks which are due to noise blow-up, tend to disappear on the new time-frequency attributes.

The second time-frequency attribute makes apparent a change around 0.42 seconds that may correspond to a change in the stratigraphy of the section.

Conclusion and Future Directions

We have proposed a new methodology to decompose a seismic trace into different oscillatory modes and to extract the instantaneous frequencies of these modes. If we represent all these frequency functions in the time-frequency plane, we have a complete description of the frequency behavior of the trace. We have also extracted new time-frequency attributes from this decomposition, based on an instantaneous frequency calculation of each component of the decomposition.

The next step in this study will be to extract other time-frequency attributes based on different calculations in the time-frequency plane, such as the mean frequency, the local bandwidth, or more generally some higher moments. We can also use other methods for calculating the instantaneous frequencies of the IMFs, for example by using a pseudo-Wigner-Ville transform.

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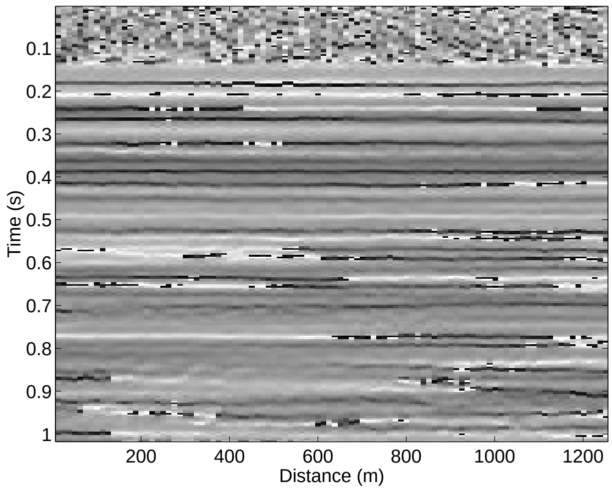

Fig. 5: *Frequency attribute based on the calculation of the instantaneous frequency as proposed in Taner et al. (1979).*

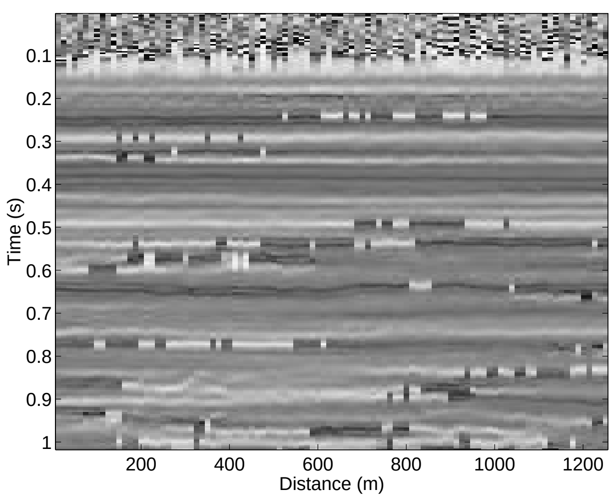

Fig. 6: *Time-frequency attribute based on the calculation of the instantaneous frequency of the first IMF.*

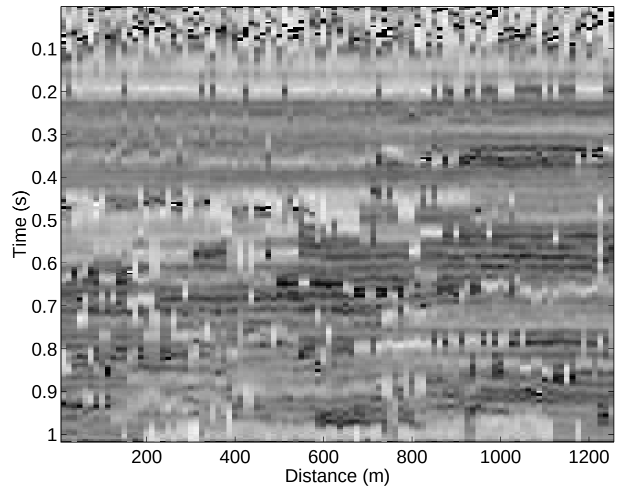

Fig. 7: *Time-frequency attribute based on the calculation of the instantaneous frequency of the second IMF.*

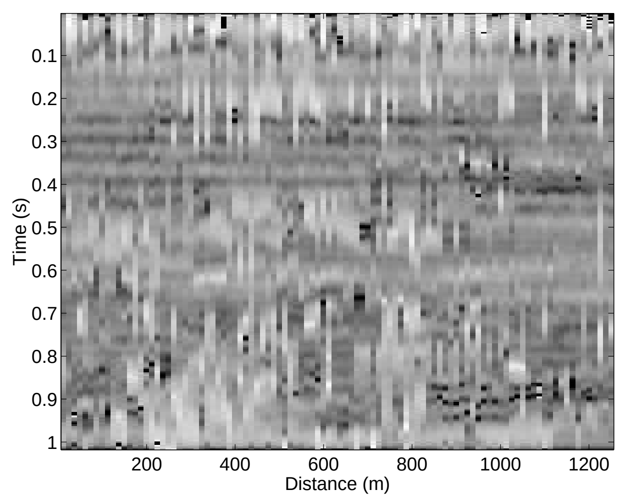

Fig. 8: *Time-frequency attribute based on the calculation of the instantaneous frequency of the third IMF.*