AN IDENTIFICATION SCHEME FOR DETERMINATION OF SYSTEMIC ARTERIAL LOAD PARAMETERS

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AN IDENTIFICATION SCHEME FOR THE DETERMINATION OF SYSTEMIC ARTERIAL LOAD PARAMETERS

R.Y.S. Ling*, J.W. Clark*, R. Srinivasan†, J.S. Cole‡, R.C. Pruett*

ABSTRACT

An identification scheme is developed for the determination of several parameters of a modified "Windkessel" model of the systemic arterial system for an individual patient undergoing cardiac catheterization. The scheme utilizes a modification of the well-known Prony method [10,11] as a "starter method" to determine good nominal values for the model parameters being varied. These values then serve as input to an iterative nonlinear least squares identification method (Marquardt method [14]) which then converges to final values of the parameters. Solution of the model equations with these parameter values yields the best fit of model generated and observed aortic and brachial artery pressures. This technique, coupled with a method for determining the mechanics of the left ventricle (e.g., the elastance concept [1-5]), permits the functional characterization of the hemodynamic properties of the left ventricle and its systemic load for an individual patient.

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I. INTRODUCTION

The general objectives of this paper are (1) the development of a clinically oriented, lumped-parameter model of the systemic load of the left heart and (2) the employment of conventional parameter estimation techniques to identify circulatory parameters for an individual patient undergoing cardiac catheterization. In this study a two-stage parameter identification scheme is used to identify the systemic load parameters, while the patient's ventricular mechanics are specified by his ventricular elastance curve [i.e., instantaneous pressure-volume ratio (Suga [1-3], Greene and Clark [4,5])]. The parameter estimation scheme utilizes averaged pressure data (obtained from a single solid state transducer) at three anatomical sites (right proximal brachial artery, ascending aorta, and left ventricle); the parameter values obtained using this estimation scheme are reasonable in a physical sense and provide a good fit to aortic and brachial arterial pressures. The method is simple, utilizes only pressure data (which is recorded with relatively little measurement error), and involves the use of measurement techniques that are routinely employed in cardiac catheterization laboratories. Furthermore, it provides an estimate of several hemodynamic parameters of clinical interest [e.g., lumped proximal and distal arterial load parameters, aortic valve resistance, ascending aortic flow, ventricular mechanics, stroke volume, and ejection fraction] for an individual patient.

II. THE MODEL

In order to adequately characterize the circulatory effects of left-ventricular pumping, models of the left heart and systemic arterial system must be chosen. The model considered in this study is shown in Fig. 1 and consists
of two parts, a model of the left ventricle and a systemic arterial load model. The heart model is based on the elastance concept and is identical to that considered in the other studies [5]. The systemic load model is a linear third-order "modified Windkessel" model similar in many respects to those of Spencer and Denison [6], and Goldwyn and Watt [7]. These models lump the arterial tree into two major compartments, proximal and distal. In the model utilized in this study (Fig. 1), the proximal compartment characterizes the lumped viscoelastic properties of the aorta and large arteries (parameters $C_L$ and $R_T$), while the resistive and compliant properties of the distal aspects of the systemic circulation are characterized by the compliance $C_R$ and the peripheral resistance $R_R$. The ineritance of the long fluid columns connecting the compartments is modeled by the parameter $L$ in Fig. 1. This model differs from the Goldwyn-Watt model [7] in that the proximal segment is considered viscoelastic and contains transverse resistance $R_T$ in series with the compliance $C_L$ (See Fig. 2). The addition of this resistor tends to damp the upstream pressure $P_1$ (which in the Goldwyn-Watt model tends to be quite oscillatory for poor choices of parameter values) and further provides a more realistic input driving point impedance to flow at the aortic root at higher driving frequencies.* The Spencer-Denison model (Fig. 2) contains two additional resistance parameters $R_L$ and $R_2$ which were found to be relatively insensitive.

* The magnitude of the input impedance $|Z_{IN}(\omega)|$ approaches zero as $\omega$ approaches larger values in the physiological range. The model of Fig. 1 provides a better fit to experimental impedance data obtained in experiments on experimental animals and human subjects (e.g., Noordergraaf [8]).
and therefore were eliminated from consideration. This provided a model that was capable of accurately mimicking pressure data, yet contained as few parameters as possible.

The system equations associated with the model of Fig. 1 are derived on the basis of Kirchhoff's current law applied to nodes 1 and 2 of the model. They are as follows:

\[
\dot{P}_1 = -W_1 P_1 + W_2 P_2 - W_3 f + S \left( \frac{1}{A R C_L} P_{LV} + W_4 P_{LV} \right) \quad (1)
\]

\[
\dot{P}_2 = -\frac{1}{R} C_R P_2 + \frac{1}{C_R} f \quad (2)
\]

\[
\dot{f} = \frac{1}{L} P_1 - \frac{1}{L} P_2 \quad (3)
\]

where

\[W_0 = (1 + S A R_L / R_A)\]

\[W_1 = (R_T / L + S A R_A L C_L) / W_0 \quad (4)\]

\[W_2 = \frac{R_T}{L W_0} \quad (5)\]

\[W_3 = \frac{1}{C_L W_0} \quad (6)\]

\[W_4 = \frac{R_T}{R_A W_0} \quad (7)\]

Here \(P_{LV}\) is left ventricular pressure and \(P_1\) is aortic root pressure (mmHg), \(P_2\) is peripheral pressure (measured at the brachial artery), \(f\) is peripheral flow through the inerstance \(L\), and \(S_A\) is a switching function defined as
\[ S_A = \begin{cases} 1 & \text{if } P_{LV} > P_1 \\ 0 & \text{if } P_{LV} \leq P_1 \end{cases} \]  

which represents the pressure-operated nature of the aortic valve. Notice that the term involving \( P_{LV} \) and its time derivative \( \dot{P}_{LV} \) in equation (1) represents a forcing function for the systemic load equations and is not present when the aortic valve closes \((S_A = 0; P_{LV} \leq P_1)\).

III. DATA ACQUISITION

Pressure waveforms at three anatomical sites (brachial artery, ascending aorta and the left ventricle), left ventricular volume and electrocardiographic data were obtained from patients undergoing cardiac catheterization at the Methodist Hospital, Houston, Texas. Pressure recordings were obtained using a solid state catheter-tipped transducer (Millar Instrument Co., Houston, Texas) and an FM tape recorder (Hewlett-Packard, Model 3960). The derivative of the electrocardiogram was simultaneously recorded on tape for use as a timing signal. With the pressure transducer in the left ventricle, a radiopaque dye (hypaque) was injected through a second hollow catheter and an enhanced X-ray fluoroscope image of the heart was filmed for at least three cardiac cycles. The left ventricle was filmed in the right anterior oblique (RAO) position (in which the long axis of the ventricle is generally perpendicular to the X-ray beam) using a standard X-ray source, image intensifier, and a 35-mm camera run at a speed of 64 frames per sec. The resulting single-plane cineangiograms were analyzed using a servoplotting table adapted for projection of cinefilms onto the plotting surface. The plotting table was connected
directly to a small computer programmed to calculate actual dimensions from the cinefilms. The ventricular outline was projected onto the plotting surface and a grid was placed over the projected image at an angle so that it was aligned with the long axis of the ventricle. An operator then scanned the ventricular outline with an electromagnetic cursor which transferred to the point coordinates of the ventricular outline to the computer.

The formula used for volume calculation is:

\[ V_{LV} = (0.848 \cdot A^2 \cdot C^3/L) \cdot 0.787 + 7.8 \text{ ml} \]  \hspace{1cm} (9)

where \( A \) is calculated area of planar projection, \( L \) is length from aortic valve to apex, and \( C \) is a correction factor which accounts for the distortion and magnification errors resulting from nonparallel X-ray beams and from the projection system. Ventricular stroke volume is obtained from the difference between end-diastolic and end-systolic volumes and cardiac output is determined by multiplying stroke volume by heart rate. The recorded arterial pressure data \((P_1, P_2)\) was digitized using a Digital Equipment Corporation PDP-12 computer and a sampling interval of 2 msec. The pressure curves were averaged over ten cardiac cycles. Smoothing cubic spline fits were then obtained for the left ventricular pressure tracing associated with the cardiac cycle chosen for ventricular volume analysis, the volume data and the averaged arterial pressure data using a standard spline subroutine.† The cubic spline fit to

* This formula was adapted from Kasser and Kennedy [9].
† IBM System/360 IMSL Scientific Subroutine ICSSMU.
the data has desirable smoothness properties, is continuous and has continuous
first and second derivatives. One will note, for example, that equation (1)
contains a term involving \( \frac{dP_{LV}}{dt} \). The parameter estimation methods employed
in this study are also somewhat sensitive to measurement noise and this "pre-
filtering" step also aids the accuracy of these methods.

IV. IDENTIFICATION METHOD

Referring to Fig. 1 and the model equations (1-7), the actual parameters
to be identified are the model parameters \( C_R, L, R_T, C_L \) and the initial flow
f(0). The values of these parameters are to be estimated, given known values
for the parameters \( R_A \) and \( R_R \) and the pressures in the left ventricle,
ascending aorta, and proximal brachial artery over an entire cardiac cycle.

The identification scheme employed here consists of two stages as indi-
cated in Fig. 3. The systemic load parameters including the unknown initial
flow and pressures at the beginning of diastole are first identified via a
modified Prony method using only the diastolic portion of the pressures in the
ascending aorta (\( P_1 \)) and proximal brachial artery (\( P_2 \)). The Prony method is
a simple, one-step procedure in which the parameter values are determined so
as to minimize the so-called model or equation errors [10]. Starting values
for the parameters are not required. The method is based on the original work
done by Prony in 1795 [11] for exponential curve fitting and has been success-
fully employed by Burrus, Parks and Watt [10] for the identifying the parameters
of the systemic load model of Goldwyn and Watt (Fig. 2b). They used only one

* Methods of determining values for \( R_A \) and \( R_R \) for a particular patient
will be discussed later.
pressure curve, namely the pressure in the brachial artery and assumed $R_R$ to be known.

Our identification scheme using Prony method follows exactly the same lines as Burrus, Parks and Watt insofar as the identification of $L$, $C_L$ and $C_R$ is concerned as shown in Fig. 3. That is, these parameter values are estimated for a guessed value of the parameter $R_T$. The load resistance $R_R$ is assumed to be known; it is taken to be the ratio of the mean pressure in the brachial artery and cardiac output. The optimal value of $R_T$ is determined via an iterative Fibonacci search [12] so as to minimize the observational errors in the diastolic portion of both $P_1$ and $P_2$ (see Appendix for details).

In applying the modified Prony method, the coefficients associated with the Z-transform of the system's output (brachial artery pressure, $P_2$) are first determined. The coefficients associated with the Laplace transform of the same are then determined by the impulse invariant method [13]; that is, by requiring that the responses of the continuous and discrete system coincide at the sampling instants. The coefficients associated with the Laplace transform are nonlinear functions of the model parameters. These nonlinear algebraic equations, it turns out, can be solved without resorting to iterative procedures for the problem at hand. The details are given in the Appendix.

The systemic load parameters determined by the modified Prony method are used as starting values for parameter identification by the Marquardt method [14]. This method is an iterative nonlinear least-squares algorithm that is a modification of the classical Gauss-Newton method [15]. Pressures $P_{LV}(t)$,
$P_1(t)$ and $P_2(t)$ over the entire cardiac cycle, together with calculated values of $R_A$ and $R_R$ are used as input to the model; the Marquardt algorithm adjusted the parameter vector $\alpha$ (consisting of four model parameters $C_L$, $R_T$, $L$, and $C_R$) to minimize the square of the difference between observed proximal and distal pressure ($P_{1,\text{obs}}$ and $P_{2,\text{obs}}$) and values computed from the model ($P_{1,\text{mod}}$ and $P_{2,\text{mod}}$)

$$E(\alpha) = \frac{1}{2} e^T e$$

(10)

where

$$e(\alpha) =
\begin{bmatrix}
e_{11}(\alpha, t_1) \\
\vdots \\
e_{1n}(\alpha, t_n) \\
e_{21}(\alpha, t_1) \\
\vdots \\
e_{2n}(\alpha, t_n)
\end{bmatrix}$$

(11)

$$\alpha =
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_m
\end{bmatrix}$$

(12)

and

$$e_1(\alpha, t) = P_{1,\text{mod}}(\alpha, t) - P_{1,\text{obs}}(t)$$

(13)

$$e_2(\alpha, t) = P_{2,\text{mod}}(\alpha, t) - P_{2,\text{obs}}(t)$$

(14)

The parameter adjustment algorithm associated with this method is given by

$$\Delta \alpha = [(J J^T)^{-1} J + \lambda I] e$$

(15)

where $J$ is the $m \times n$ Jacobian matrix

$$J = \left[\frac{\partial e}{\partial \alpha}\right]^T$$

(16)

$I$ is the $m \times n$ identity matrix.

* The time derivative of $P_L(t)$ is also required as input to the model equations and this derivative is obtained numerically from the pressure data by employing conventional Lagrangian differentiation formulas [16].
and $\lambda$ is the Levenberg adjustment parameter \cite{14,15}. This parameter limits the step size ($\Delta \alpha$) of each iteration, and the appearance of $\lambda$ in the equation (15) results from the minimization of the performance index $E(\alpha)$ subject to the constraint

$$A\Delta \alpha = r^2$$

where $r$ is the radius of a hypothetical hypersphere about the nominal point $\alpha^0$ in the parameter space. Observing equation (15), one notes that for large values of $\lambda$, the algorithm behaves as a gradient where

$$\Delta \alpha = \lambda \Delta e$$

As $\lambda \to 0$, the algorithm is identical to that of the classical Gauss-Newton method which is a quasilinearization method. Thus use of appropriate values for the Levenberg parameter allows one to take advantage of the good starting properties of the gradient method and the rapid, quadratic-convergence properties of the quasilinearization method in the vicinity of the extremum.

With regard to the initial conditions, although they can also be treated as unknown parameters in the Marquardt method, this was not done in order to reduce the number of adjustable parameters and thus simplify the problem. Time $t = 0$ is chosen to be the beginning of diastole. For each iteration, $P_1(0)$ and $P_2(0)$ are taken to be the measured pressures in the ascending aorta and brachial artery at the beginning of diastole. The initial flow for each iteration is computed as follows:

$$f(0) = \frac{P_1(0)}{R} + C \frac{dP_2}{R dt} \bigg|_{t=0}$$

This follows from the model equation (2).
The two stage identification scheme was first used on model-generated data and it produced convergence (for several different sets of initial parameter values) to the proper known model parameter values. The scheme was then employed on measured pressure from human subjects undergoing routine cardiac catheterization.

V. RESULTS

All computations were done on the IBM 370/155-11 system at Rice University. The programs were written in FORTRAN. The numerical integration of the model equations were carried out by the Runge-Kutta method using a step size of 16 msec.

For the Fibonacci search, $R_1$ was assumed to lie between 0 and 10 initially. The iterations were stopped when this initial interval was reduced to 1/1000 of its size.

Two representative sets of results are shown in figures 5 and 6 (patients G.M. and G.S.). Although deviations up to 10 mmHg between the patient and model data are present, the overall fit is good considering the lumped parameter model used in this study.

Figs. 6 and 7 show the comparison of the measured left ventricular volume curve during systole (by cineangiography) with that produced by the model. The time course of ventricular volume (during systole) computed from model results was determined from the formula

$$V_{LV}(t) = V_{ed} - \int_0^T S f_A(t) dt$$

(20)

where the interval $(0, T_s)$ is the ejection time.
and $f_A(t)$ is the instantaneous flow through the aortic valve during systole, $V_{ed}$ is end-diastolic volume determined via cineangiography and $V_{LV}$ is left ventricular volume. The overall agreement is reasonably good in both patients.

The parameter values obtained using the Prony method are listed in Table 1. The RMS error between measured and model data for both proximal and distal diastolic pressure is also given. Similar data for the Marquardt method operating on $P_1(t)$ and $P_2(t)$ over the entire cardiac cycle are given in Table 2. The identified parameter values for both patients can be seen to be of the same order of magnitude.

If one utilized the parameters identified in stage 1 to obtain a model solution over the entire cycle, the resultant fits obtained are found to be much worse than those obtained using the Marquardt method as a second identification stage. Thus the Prony method serves as a one-step "starter" method that provides reasonably good nominal values for the model parameters to be used in the more accurate Marquardt method; this latter iterative method is somewhat sensitive to starting values for the model parameters. These methods have been compared in previous studies by our group [17].

Using the identified parameter values for each patient, derived expressions for the driving point impedance at the input to the systemic load (point $P_1$ in Fig. 1) could be evaluated and the magnitude and phase of the input impedance plotted as a function of frequency. This has been done and the results are shown in Figs. 8 and 9; these results agree generally with other low-order model approximations to impedance data obtained experimentally from human subjects (see reference [8]).

* Fibonacci iterations for values of $R_T$ are contained within this single step.
Fig. 10 shows the ventricular elastance characterization* during the systolic portion of the cardiac cycle for the two representative patients. These patients were diagnosed by a cardiologist as having normal ventricular mechanics and the general appearance of these curves is consistent with this diagnosis. Both elastance curves have the same basic shape; differences between waveforms may be generally attributed to differences in end-diastolic volume \( V_{ed} \), heart-rate, and positive inotropic state [18,5]. Since patient G.M. has a higher resting heart rate than patient G.S. (see Table 3), this may also be indicative of an increased basal level of positive inotropic state (i.e., phasic activity on sympathetic augmentor fibers to the heart) and would account for increased peak magnitude of the elastance \( E_{LV,\text{max}} \), the decrease in the time to peak elastance \( t_{\text{max}} \) and the slight increase in the slope of the rising phase of the elastance curve \( \frac{dE_{LV}}{dt} \) [see reference [18]].

Given the basal heart rate and inotropic state of each patient, the elastance curves of Fig. 10 serve as a relatively simple functional characterization of the ventricular mechanics of that patient under quiescent conditions.

* See references [1-5 and 18].
VI. DISCUSSION

The foregoing results indicate the feasibility of using conventional identification techniques to identify the systemic load parameters of an individual patient undergoing cardiac catheterization. The parameter values identified for patients G.M. and G.S. were of the same order of magnitude and appeared reasonable in a physical sense. These model parameters yielded very good fits to proximal and distal arterial pressure data as well as reasonably good predicted values for ventricular volume changes during systole.

The elastance curves calculated for these patients characterize the mechanics, or "pump" properties, of the left ventricle and, coupled with the systemic load identification scheme, allow the assessment of the hemodynamic properties of the left heart and its systemic load for an individual patient. The method uses less than two minutes of CPU time on an IBM 370/155 digital computer and is therefore a reasonably fast off-line identification technique.

As experience is obtained with the use of this identification scheme, it may prove useful in both a clinical and an experimental sense. For example, Goldwyn and Watt [7] indicate that such a technique may be useful in the clinical diagnosis of peripheral arterial disease. This technique may also be useful in the periodic assessment of the load parameters of a patient receiving mechanical circulatory assistance in an intensive care unit. These examples remain quite speculative, however, until such time as a larger clinical and experimental data base is accumulated with this technique allowing more quantitative comparisons.
APPENDIX

Let \( y(t) \) denote the output of the system, the brachial artery pressure. An appropriate representation for \( y(t) \) is:

\[
y(t) = q_1 e^{-q_2 t} + q_3 e^{-q_4 t} \cos(q_5 t + q_6)
\]

(A1)

This takes into account the oscillatory nature of the output. Now the Z-transform of \( y(t) \) is

\[
Y_Z = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}
\]

(A2)

The corresponding Laplace transform of the output \( y(t) \) is

\[
Y_S = \frac{N_2 s^2 + N_1 s + N_0}{s^3 + D_2 s^2 + D_1 s + D_0}
\]

(A3)

Here \( a_0, a_1, a_2, b_1, b_2, \) and \( b_3 \) are first determined by the Prony method. The coefficients \( q_1 \) through \( q_6 \) are obtained by equating the Z-transform of the RHS of Eq. (A1) to the RHS of Eq. (A2). \( N_2, N_1, N_0, D_2, D_1, \) and \( D_0 \) are then found by equating the Laplace transform of the RHS of Eq. (A1) to the RHS of Eq. (A3).

Now, in terms of the model parameters, \( N_0, N_1, N_2, \) etc. are given by

\[
N_2 = P_2(0)
\]

(A4)

\[
N_1 = \frac{R_T P_2(0)}{L} + \frac{f(0)}{C_R}
\]

(A5)

\[
N_0 = \frac{P_1(0)}{LC_R} + \frac{P_2(0)}{LC_L} + \frac{R_T f(0)}{LC_R}
\]

(A6)
\[ D_2 = \frac{R_T}{L} + \frac{1}{R_L C_R} \]  
\[ (A7) \]

\[ D_1 = \frac{R_T}{LR_L C_R} + \frac{1}{LC_L} + \frac{1}{LC_R} \]  
\[ (A8) \]

\[ D_0 = \frac{1}{LC_L C_R R_R} \]  
\[ (A9) \]

Observe that the initial conditions are associated with only the numerator coefficients. The parameters \( L, C_L, \) and \( C_R \) are to be calculated using eqs. (A7), (A8), and (A9) given the values of \( R_T \) and \( R_R \). Eliminating \( L \) and \( C_R \) from equations (A7-9) we obtain

\[ C_L^3 + 2k_2 C_L^2 + k_1 C_L + k_o = 0 \]  
\[ (A10) \]

where

\[ k_2 = \frac{(D_2/D_3)}{R_T + R_R} \]  
\[ (A11) \]

\[ k_1 = \frac{D_1/D_3}{R_T(R_T+R_R)} + k_2^2 \]  
\[ (A12) \]

\[ k_o = \frac{(D_3 - D_1 D_2)/D_3^2}{R_T(R_T + R)^2} \]  
\[ (A13) \]

Knowing the value of \( C_L \), the values of the unknown parameters and initial conditions are readily computed. If eq. (A10) yields three real values for \( C_L \), then three sets of parameter values are computed and the one that produces minimum observational errors in \( P_1 \) and \( P_2 \) (the criterion function for the Fibonacci search) is chosen.
REFERENCES


FIGURE CAPTIONS

FIG. 1  Lumped parameter model of the left heart and its systemic load.

FIG. 2  Modified Windkessel models of various forms (a) the model used in this study; (b) the Goldwyn-Watt model [7]; and (c) the Spencer-Denison model [6].

FIG. 3  Parameter identification scheme.

FIG. 4  (a) Comparison of the ascending aorta pressure obtained from the model and that measured in patient G.M. (b) Comparison of the proximal brachial artery pressure obtained from the model and that measured in patient G.M.

FIG. 5  (a) Comparison of the ascending aorta pressure obtained from the model and that measured in patient G.S. (b) Comparison of the proximal brachial artery pressure obtained from the model and that measured in patient G.S.

FIG. 6  Comparison of the left ventricular volume obtained from single plane cineangiogram and that obtained from the model during systole for patient G.M.

FIG. 7  Comparison of the left ventricular volume obtained from single plane cineangiogram and that obtained from the model during systole for patient G.S.

FIG. 8  Systemic load impedance as a function of frequency for patient G.M.

FIG. 9  Systemic load impedance as a function of frequency for patient G.S.
FIG. 10 A plot of the ventricular elastance function \( E_{LV}(t) = P_{LV}(t)/V_{LV}(t) \) as a function of time for each patient. Only the systolic portion of the cardiac cycle is shown.
FIG 1

SYSTEMIC ARTERIAL SYSTEM MODEL

LEFT HEART MODEL

PULMONARY CIRCUIT

CATHETER

P1

P2

PLV

E(t)
FIG 2
FIG 3
PATIENT C.S.
DATA ——
MODEL ——

Ascending Aorta Pressure - mmHg

Time - msec

Fig 5a
Proximal Brachial Artery Pressure

PATIENT O.S.
DATA •••
MODEL ——•

Time — MSEC

FIG 56
Fig 6

LEFT VENTRICULAR VOLUME DURING SYSTOLE - C.C.

PATIENT G.M.
DATA •••
MODEL FIT •••
LEFT VENTRICULAR VOLUME DURING SYSTOLE -- C.C.

PATIENT G.S.
DATA ○○○
MODEL FIT ○○○

TIME - MSEC

Fig 7
<table>
<thead>
<tr>
<th>PATIENT'S NAME</th>
<th>G.M.</th>
<th>G.S.</th>
</tr>
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<tbody>
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<td>PARAMETER VALUES</td>
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<td>RT = 0.285</td>
<td>RT = 0.330</td>
<td></td>
</tr>
<tr>
<td>L = 0.00385</td>
<td>L = 0.00336</td>
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</tr>
<tr>
<td>CL = 0.00136</td>
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<tr>
<td>CR = 0.0000438</td>
<td>CR = 0.0000595</td>
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<tr>
<td>FLOW(0) = 7.751</td>
<td>FLOW(0) = 8.765</td>
<td></td>
</tr>
<tr>
<td>R = 14.164</td>
<td>R = 10.65</td>
<td></td>
</tr>
</tbody>
</table>

| RMS ERROR | 3.423 | 4.467 |

**TABLE 1.** Resistance: mmHg·min/liter

Inductance: mmHg·min·min/liter

Capacitance: liter/mmHg

RMS error is the root mean square error of both curves during diastole. Parameter Values obtained at the end of stage 1. The number of Fibonacci iterations was 15 in each case.
<table>
<thead>
<tr>
<th>PATIENT'S NAME</th>
<th>G.M.</th>
<th>G.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV</td>
<td>0.6410</td>
<td>RV = 0.4202</td>
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<td>RT = 2.127</td>
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<tr>
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<td>L = 0.003474</td>
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<tr>
<td>CL</td>
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<td>CL = 0.001201</td>
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<td>CR = 0.00008457</td>
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<td>RR</td>
<td>14.16</td>
<td>RR = 10.65</td>
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<td>RMS ERRORS</td>
<td>RMS error of A.A. pressure</td>
<td>RMS error of A.A. pressure</td>
</tr>
<tr>
<td></td>
<td>= 2.522</td>
<td>= 3.187</td>
</tr>
<tr>
<td></td>
<td>RMS error of P.B.A. pressure</td>
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<tr>
<td></td>
<td>= 2.663</td>
<td>RMS error of both pressures</td>
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<td></td>
<td></td>
<td>= 3.486</td>
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<tr>
<td>NO. OF ITERATIONS</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>STOPPING CRITERIA</td>
<td>$E_1 = 0.03427$</td>
<td>$E_1 = 0.04469$</td>
</tr>
<tr>
<td></td>
<td>$E_2 = 0.001987$</td>
<td>$E_2 = 0.01988$</td>
</tr>
</tbody>
</table>

**TABLE 2.** Resistance: mmHg·min/liter  
Inductance: mmHg·min·min/liter  
Capacitance: liter/mmHg  
$E_1$: the maximum relative change of parameter value  
$E_2$: the relative change of total error (defined as the sum of the squares of the error at each point of both A.A. and P.B.A. pressure)
<table>
<thead>
<tr>
<th>PATIENT NAME</th>
<th>END DIASTOLIC VOLUME</th>
<th>HEART RATE</th>
<th>EJECTION FRACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.M.</td>
<td>151 ml</td>
<td>74 bpm</td>
<td>0.649</td>
</tr>
<tr>
<td>G.S.</td>
<td>153 ml</td>
<td>95 bpm</td>
<td>0.601</td>
</tr>
</tbody>
</table>

TABLE 3. Miscellaneous Patient Data.