Estimation-Quantization Geometry Coding Using Normal Meshes

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Abstract

We propose a new algorithm for compressing three-dimensional triangular mesh data used for representing surfaces. We apply the Estimation-Quantization (EQ) algorithm originally designed for still image compression to the normal mesh wavelet coefficients. The EQ algorithm models the wavelet coefficients as a Gaussian random field with slowly varying standard deviation. By designing the quantizers in a rate-distortion optimal fashion, we improve upon the recently proposed zerotree normal mesh compression algorithm by 0.5 to 1 dB in distortion.

1 Introduction

Three-dimensional (3-D) surface applications such as animation, object modeling, and visualization require modeling complex 3-D surface geometry [1]. The model describes the geometry, color, texture, and other information regarding the surface. A common method to model the geometry is a mesh of polygons such as triangles or quadrilaterals. Typically, a large number of polygons are required to approximate a complex 3-D surface. This can result in an enormous amount of data; for example, the 3-D surface representing Michelangelo's statue of David contains about a billion triangles [1]. The large data size makes storing and transmitting the mesh data in many applications difficult. For efficient storage and fast transmission of mesh data, the development of an efficient mesh compression algorithm is essential.

The data representing a 3-D polygon mesh consists of two parts: the coordinates of the polygon vertices and the mesh connectivity. In the naive representation of vertices, we need three numbers to represent the coordinates of each vertex in 3-D space. The mesh connectivity represents how the vertices are connected to form polygons. In this paper, we focus on triangular meshes with geometry information.

Subdivision is a procedure to obtain a multiresolution mesh representation [2]. Depending on the structure of the mesh, meshes can be classified as regular, semi-regular, or irregular. Regular and semi-regular meshes enable simple schemes to

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obtain a multiresolution mesh representation, and so they are commonly used in 3-D modeling. In particular, semi-regular meshes are of particular interest because of their flexibility. Starting with a simple irregular mesh at the coarsest scale, we can successively regularly subdivide each triangle to obtain meshes at multiple resolutions. *Lifting* combined with subdivision scheme naturally defines a *wavelet transform* on the mesh [3]. Furthermore, by restricting new vertices to lie only in the normal direction in the local coordinate system, we can represent each vertex with just one parameter to obtain a *normal mesh* [4] representation.

Normal meshes are particularly attractive for mesh compression, because the number of parameters representing the vertex coordinates is greatly reduced. A number of efficient multiresolution mesh compression algorithms have been proposed, of which progressive zerotree compression gives the best results [5]. The zerotree algorithm arranges the wavelet coefficients in a quadtree and applies the zerotree compression algorithm originally developed for images [6]. Another recent mesh compression scheme has been proposed for irregular meshes [7], which uses vertex-quantization to compress the vertices of the mesh. This algorithm assumes that the connectivity of the irregular mesh algorithm is compressed separately.

In this paper, we develop a new compression algorithm for the normal mesh wavelet coefficients that exploits their very high spatial correlations. In smooth regions, the wavelet coefficients tend to be small, while in rough regions they are large. Thus, we can model the normal mesh wavelet coefficients as an outcome of a random field with slowly varying standard deviation. To compress the normal mesh wavelet coefficients using the local correlations within each scale, we adopt the *Estimation-Quantization* (EQ) framework that has been successfully applied in still image compression [8]. By modeling the statistics of each wavelet coefficient through local standard deviation estimation, the rate-distortion (R-D) optimal quantizer is chosen followed by an entropy coder for coding the quantization symbols. Compared to the state-of-the-art zerotree mesh compression algorithm [5], our EQ mesh coder provides 0.5-1 dB improvement in coding performance.

2 Normal Triangular Meshes

In this paper, we will focus on triangular semi-regular normal meshes [4]. In a normal mesh, new vertices are added at each scale; each new vertex lies in the direction normal to a surface fit through the neighboring vertices as shown in Figure 1. Any subdivision scheme, such as the modified butterfly subdivision scheme [9], can be used to predict the normal direction and the base point for each new vertex using the coarse scale vertices in the local neighborhood of the new vertex. The intersection of the normal line with the original surface gives the new vertex at the current scale. In this fashion, each triangle is regularly subdivided to obtain the mesh at the next finer resolution as shown in Figure 2.

A wavelet transform, such as the butterfly wavelet transform [9] based on the modified butterfly subdivision scheme, can be used to compute the wavelet trans-

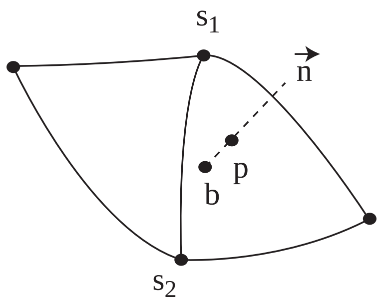

Figure 1: *The new vertex p associated with edge (s_1, s_2) is given by the intersection of the normal line $\vec{n}$ passing through base point b with the original surface.*

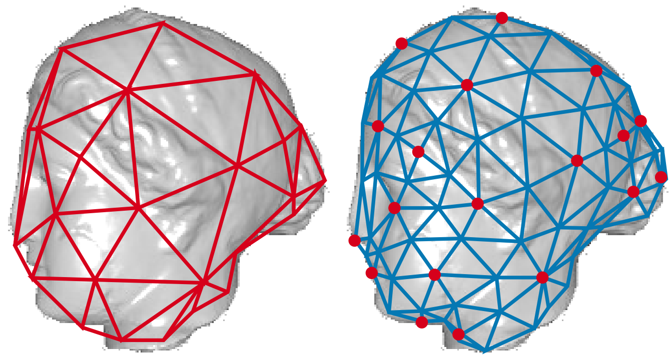

Figure 2: *The Venus head normal mesh on the left is subdivided once to obtain the mesh at the next scale. The thick dots on the right show the vertices of the mesh at the previous coarser scale.*

form of a normal mesh. The wavelet coefficients typically require three numbers for each new vertex, and can be expressed in the local coordinates using one normal and two tangential components. However, since each new vertex of a normal mesh lies in a direction normal to the local surface defined using the coarse scale vertices, the wavelet coefficients of a majority of the vertices can be expressed in the local coordinates using the normal component of the wavelet coefficient for each vertex.

For smooth surfaces, the tangential components of the normal mesh wavelet coefficients are either small or zero. The geometry of a smooth surface can therefore be described using a single normal component of the wavelet coefficient for each vertex. Thus, normal meshes offer a significant redundancy reduction over other mesh representations.

3 Wavelet Coefficient Statistics

The wavelet coefficient vector for each vertex consists of one normal and two tangential components. The distributions of the normal and the tangential wavelet coefficients at an intermediate scale of the Venus head normal mesh are shown in Figures 3 and 4.

We can see from Figure 4 that the tangential wavelet coefficients are small and that a majority of them are zero. Therefore, we model the tangential wavelet coefficients as *iid* zero-mean generalized Gaussian distribution (GGD) with fixed unknown standard deviation and shape parameters. The GGD function is given by

$$f_{(\nu,\mu,\sigma)}(X = x) = \frac{\nu\eta(\nu,\sigma)}{2\Gamma(1/\nu)}\, e^{-[\eta(\nu,\sigma)|x-\mu|]^{\nu}} \tag{1}$$

where $\eta(\nu,\sigma) = \frac{1}{\sigma}\sqrt{\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}}$, $0 < \nu \leq 2$, ν is the shape, μ is the mean, and σ is the

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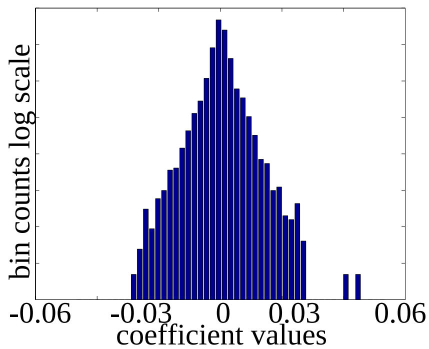


Figure 3: *Distribution of the normal wavelet coefficients of the Venus head normal mesh.*

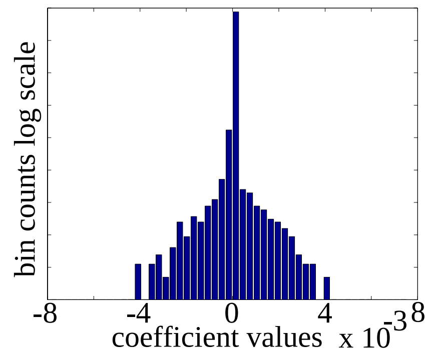


Figure 4: *Distribution of the tangential wavelet coefficients of the Venus head normal mesh, with small range on the horizontal axis.*

standard deviation.

The distribution of the normal wavelet coefficients normalized by the standard deviation of the local neighborhood of same-scale vertices is shown in Figure 5. A Gaussian density function can be used to approximate the distribution as shown in the figure. Encoding the *normalized* normal wavelet coefficients gives a better R-D performance compared to encoding the original normal wavelet coefficients as shown by the R-D curves in Figure 6. The rate and distortion measures used here are discussed in Section 6. Thus, we are motivated to model the normal wavelet coefficients using a Gaussian random field with slowly spatially varying standard deviation.

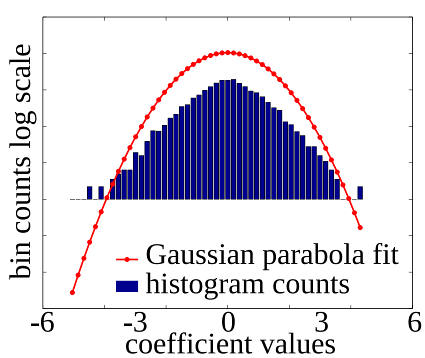


Figure 5: *Normal wavelet coefficients normalized by the standard deviation of the local neighborhood for the Venus head normal mesh.*

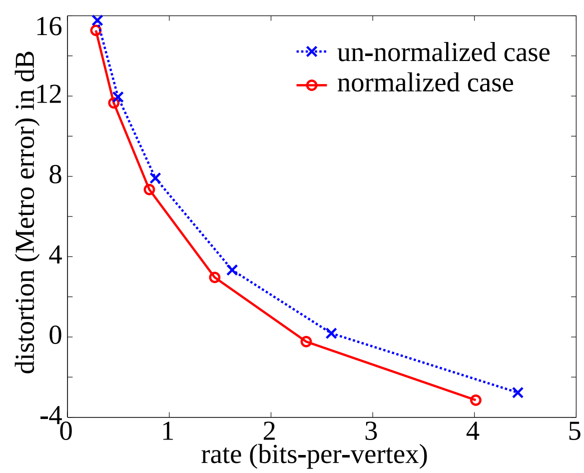


Figure 6: *R-D curve comparing the encoding of the original and normalized normal wavelet coefficients for the Venus head normal mesh.*

The standard deviation for each normal wavelet coefficient is obtained using the quantized coefficients in the causal neighborhood, instead of the original values.

This allows the decoder to use the same algorithm to obtain the same standard deviation estimate. However, since we are using the quantized neighborhood to obtain an estimate for the standard deviation, the resulting probability density is more peaky compared to a Gaussian distribution [10]. We therefore approximate the distribution with a GGD given by Equation (1), where the shape ν controls the peakiness of the distribution at each scale.

The EQ algorithm [8] is used to estimate the shape and standard deviation parameters and then encode the normal wavelet coefficients using a R-D optimization. In the next section, we discuss the EQ coder for images and then introduce our EQ coder for meshes in the following section.

4 EQ Image Coding

The EQ coder was originally proposed for still image compression [8]. The coefficients from each wavelet subband are encoded using an independent GGD model with a slowly spatially varying unknown standard deviation and a fixed unknown shape parameter.

A simple raster scan order is used to traverse through the list of the coefficients in each wavelet subband. In the simplest form of EQ, the statistics for the GGD model of each pixel are obtained from the quantized causal subset of the local 3×3 pixel neighborhood. The maximum likelihood estimate of the standard deviation is obtained from this neighborhood [8].

Once we obtain an estimate for the standard deviation, the coefficient is quantized using a scalar dead-zone quantizer. The choice of the quantizer is made based on a R-D optimization. The optimization is based on the Lagrangian cost function for each normal wavelet coefficient with vertex i

$$J_i = r_i + \lambda d_i \tag{2}$$

where λ is the Lagrangian parameter or the R-D slope, and r_i and d_i are the corresponding rate and distortion terms respectively. The rate r_i, is expressed in terms of the entropy associated with the corresponding bin probabilities of the quantizer, and the distortion d_i, is expressed in terms of the squared quantization error associated with each quantizer. The corresponding bin probabilities are used to encode the quantization symbol with an entropy coder such as an arithmetic coder [11]. The bin probabilities for each quantizer from the R-D table are computed off-line to speed up the algorithm.

With the simple EQ algorithm, a problem occurs when all the causal neighbors of a coefficient are quantized to zero. In this case, the standard deviation estimate is zero and the coefficient is automatically quantized to zero. This phenomenon can propagate through the rest of the coefficients at the same level, resulting in a stability problem for the EQ algorithm. A simple solution to this problem is to place the coefficients with all causal neighbors quantized to zero into a separate set called the *unpredictable set*, and place the remaining coefficients in the *predictable* *set*. The coefficients in the unpredictable set are modeled as *iid* zero-mean GGD with fixed shape and standard deviation. These parameters are estimated from the normal wavelet coefficients in the unpredictable set and sent to the decoder.

The initial estimates for the shape parameters of the predictable and the unpredictable sets and the initial estimate for the standard deviation parameter of the unpredictable set are obtained based on the initial partitioning of the coefficients into the predictable and the unpredictable sets at each scale. The initial partitioning of the normal wavelet coefficients into the predictable and the unpredictable sets is obtained by a simple thresholding technique [8].

5 EQ Normal Mesh Coding

Our EQ mesh coder uses the local statistics of the normal wavelet coefficients to estimate the shape ν and the standard deviation σ for the each vertex.

The local neighborhood of the current vertex is used to estimate the standard deviation for the normal wavelet coefficient of the current vertex. We construct the local neighborhood of a normal wavelet coefficient based on the local neighborhood of its vertex. Figure 7 shows a vertex v and its local neighbors at different scales. As we see, there is no obvious choice for the vertex neighborhood; in particular, the simple neighborhood consisting of the four adjacent same-scale neighbors is too small to obtain a reasonable estimate for the standard deviation. Therefore, we use the neighborhood that includes the first three layers or rings of same-scale neighbors as shown in Figure 8.

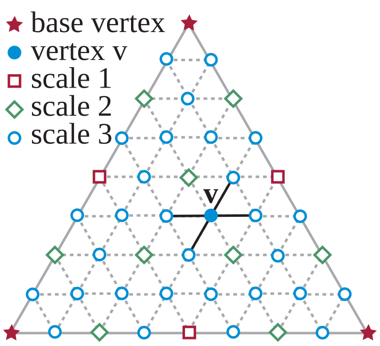


Figure 7: *A base triangle showing vertices at different scales. The four same-scale neighbors of vertex v are connected to v with solid edges.*

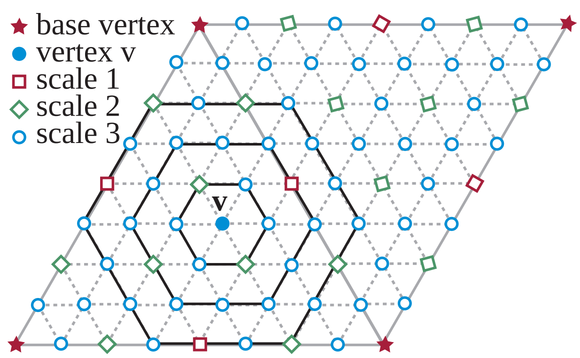


Figure 8: *Two base triangles showing the neighborhood of a vertex v. We use three layers of vertices at the same scale as v for the neighborhood.*

A structured ordering for the vertices is necessary in order to improve the number of causal neighbors in the local vertex neighborhood and to preserve the semi-regular connectivity of the normal mesh. We propose an ordering for the vertices starting with the vertices at the coarse level and then proceeding to finer levels, where at each level, each base triangle is scanned separately. Therefore, the global ordering of the vertices is reduced to an ordering of the base triangles followed by an ordering of the vertices inside each base triangle.

Figure 9 shows the ordering of vertices inside each base triangle, where the ordering starts from the bottom-left and then moves right and up. The orientation of the base triangle is selected such that the number of causal same-scale neighbors is maximized for all the vertices inside the base triangle. The vertices from the neighboring base triangles are also considered while constructing the causal vertex neighborhoods for the vertices of the current base triangle.

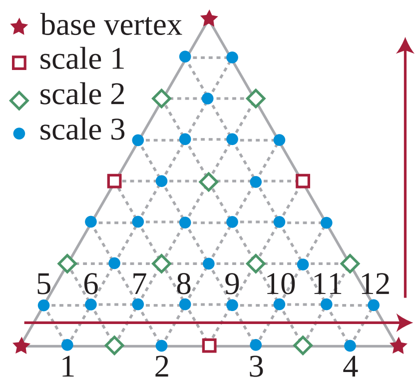


Figure 9: *Scale 3 vertex ordering inside a base triangle.*

Using this local neighborhood definition and scanning order, the standard deviation and the shape estimates are obtained for each vertex from its causal quantized neighborhood. A scalar dead-zone quantizer is selected using a R-D optimization. The R-D optimization is given by Equation (2), where the rate r_i is expressed in terms of the entropy of the bin probabilities, and the distortion d_i is measured using the mean-squared quantization error. The quantized symbols are then encoded using the corresponding bin probabilities and an entropy coder such as an arithmetic coder.

The EQ mesh coder has the same problem as the EQ image coder when all quantized coefficients in the causal neighborhood are zero. Therefore, we treat such coefficients as *unpredictable* in the same manner as discussed in Section 4.

The tangential wavelet coefficients are modeled as *iid* zero-mean GGD with fixed shape and standard deviation parameters for each scale. A single scalar dead-zone quantizer is chosen for each scale based on a R-D optimization. For a smooth surface, a small displacement in a direction normal to the surface changes the original surface much more than the same displacement in a direction tangential to the surface. Therefore, the tangential wavelet coefficients are encoded using a higher R-D slope λ compared to the normal components.

The coordinates of the vertices of the base mesh form the scaling coefficients. The scaling coefficients are uniformly quantized using variable bit rate. Note that the scaling coefficients are expressed in terms of absolute coordinates while the wavelet coefficients are expressed as local offsets in a local coordinate system. Therefore, the scaling coefficients have a significantly higher global error contribution compared to the wavelet coefficients.

The proposed vertex ordering preserves the semi-regular connectivity of the normal mesh. Therefore, we only need to encode the connectivity of the base mesh. We encode the base mesh connectivity using the TG coder [12].

6 Results

We use the Venus head, the horse, and the rabbit normal meshes [5] for analyzing the performance of our EQ mesh coder. We fix the value of λ for all normal mesh coefficients in the mesh. We use a higher λ for the tangential wavelet coefficients, as their error contribution is much smaller than the normal wavelet coefficients. In our simulations, we used 100λ for the tangential wavelet coefficients. The normal wavelet coefficients are encoded using the EQ algorithm. The scaling coefficients are uniformly quantized and then encoded using an entropy coder. The base mesh connectivity is encoded using a TG coder [12].

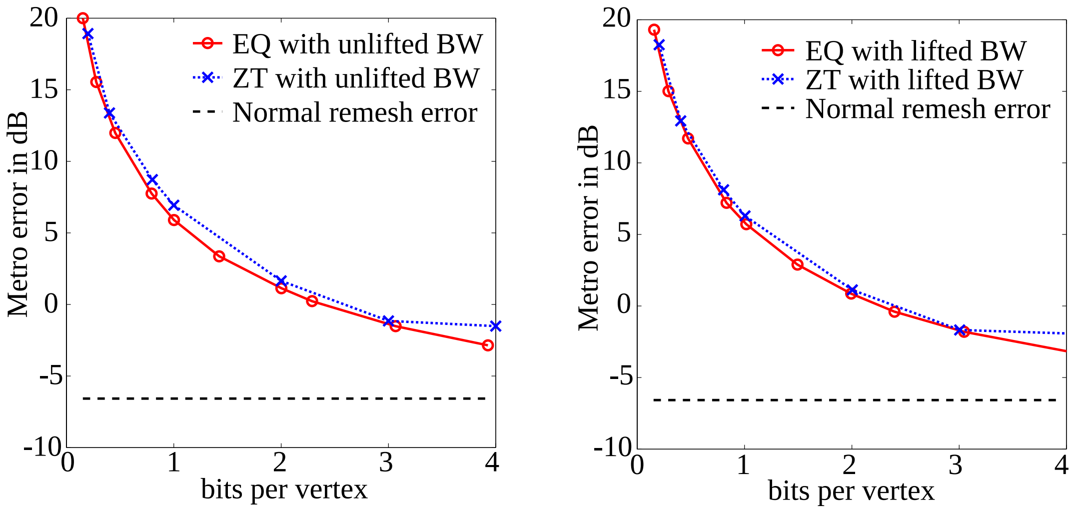
The Metro tool [13] is used to measure the final error. The Metro tool measures the squared symmetric distance between two surfaces averaged over the first surface. We refer to this error as the RMS Metro error. PSNR [1] values are expressed as a ratio of the bounding box diagonal over the RMS Metro error, where the bounding box diagonal is the longest diagonal of the box that bounds the original surface. The rate is expressed in terms of the bits-per-vertex values, given by the ratio of total number of bits over the number of vertices in the original irregular mesh.

We apply the unlifted and the lifted versions of the butterfly wavelet transforms [9] to the normal mesh, since the butterfly subdivision scheme was used to construct the normal mesh from the original irregular mesh. The Metro error is measured between the original irregular and the reconstructed normal meshes. The performance of the EQ mesh coder for the Venus head normal mesh is compared with that of the state-of-the-art zerotree mesh coder [5]. Figure 10 shows the R-D curves for the EQ mesh coder and the zerotree mesh coder and compares the distortion with the normal remeshing error, which is the error between the original irregular mesh and the original normal mesh. PSNR values as a function of the bits-per-vertex are given in Table 1. We obtained similar results for the horse and rabbit normal mesh datasets.

7 Conclusions

In this work, we have presented a context-based EQ coder for normal meshes and compared it with the state-of-the-art zerotree coder for normal meshes. We observe

[1] PSNR in $dB = 20 \log \left(\frac{\text{bounding box diagonal}}{\text{Metro error}} \right)$.



(a) *Unlifted case.*　　(b) *Lifted case.*

Figure 10: *R-D curves comparing the EQ mesh coder, the zerotree (ZT) mesh coder, and the normal remeshing error for the Venus head normal mesh using the unlifted and lifted versions of the butterfly wavelet transform (BW). Note that the error cannot go below the normal remeshing error.*

Table 1: *Venus head mesh PSNR in dB* [1] *comparing the EQ mesh coder and the zerotree (ZT) mesh coder using the lifted and unlifted versions of the butterfly wavelet (BW) transform.*

bits-per-vertex	0.25	0.5	1.0	2.0	3.0	4.0
EQ lifted BW	63.69	68.63	74.16	79.16	81.70	83.16
ZT lifted BW	62.95	68.21	73.66	78.85	81.66	81.92
EQ unlifted BW	63.48	68.59	74.08	78.85	81.37	82.96
ZT unlifted BW	62.40	67.78	73.00	78.40	81.20	81.50

performance gains of approximately 0.5-1 dB. Although the EQ mesh coder is not strictly progressive like the zerotree mesh coder, it is scale-wise progressive.

In the future, we hope to improve the results by replacing the mean squared error (MSE) with a vertex-based error metric while performing the R-D optimization, as a vertex based error metric would better correspond with the final global error measurement compared to the MSE. We hope to study the R-D trade-offs between the scaling coefficients, the normal wavelet coefficients, and the tangential wavelet coefficients.

Acknowledgments

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