Performance of Quantized Power Control in Multiple Antenna Systems

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***Abstract*— In this paper, we analyze the outage probability of a single user system with multiple antennas at the transmitter, single antenna at the receiver, and finite rate feedback power control. The optimum power control is complex and the analysis is not tractable. Hence we propose a sub-optimal power allocation scheme, with very low computational complexity, which is asymptotically optimum. Analyzing the proposed algorithm we show that the diversity order can potentially be increased unboundedly by increasing the feedback rate and *without* increasing number of transmit or receive antennas. We find a closed form approximation to this diversity-like gain at large SNRs, as a function of number of transmit antennas, number of quantization levels, and average available SNR. Simulation results confirm the validity of the analysis.**

I. Introduction

Power control is a well established method to improve communication performance in fading channels. Its efficacy has been established in single [1, 2] and multiuser [3–6] channels. Even with the extensive research, performance of power control with finite number of feedback bits remains unclear. In this paper, we study the performance of power control in block fading i.i.d. channels with quantized transmitter information.

For systems with closed loop power control, we show that the outage probability has a finite diversity order unlike the systems with complete Channel State Information at Transmitter (CSIT). Figure 1 shows the performance of perfect channel information at the transmitter versus no information for a single transmit-receive antenna system; no CSIT case has a diversity order of 1 while the system with full CSIT has an exponential decay. More importantly, the diversity order of the power-controlled system depends on both the number of transmit antennas and the number of feedback bits.

Interestingly, fast feedback for power control contributes to diversity much more than the number of antennas. For example a system using space-time codes with 2 bits of SNR feedback with 2 transmit and one receive antennas has a diversity order of 30 (derivation is given in Section IV). Thus, antenna diversity can be traded with *feedback diversity*. An important point to note is that temporal power-control achieves this diversity without coding over multiple fading blocks, and hence the diversity order is not a result of coding over multiple fading coefficients. The lower bound on the diversity order is derived using a simple suboptimal power control scheme, which allocates equal average power to each quantized bin. We show that asymptotically, in number of feedback bits, the equal power allocation achieves the same outage probability as the optimal quantizer.

The results in this paper extend our prior analysis on the utility of finite rate feedback in multiple antennas presented in [7], where only 1-bit optimal quantizers were presented and higher rate quantizers were found numerically. Furthermore, it complements our results on finite rate feedback based beamforming in multiple antenna systems [8], and will allow a unified design of feedback channels in multiple antenna systems.

The work in [9] is closest to our analysis. Using BER as the metric, the authors derive optimal power control methods for closed loop power control. The quantizer design procedure in [9] is a modified Lloyd-Max algorithm. In contrast, we opt for suboptimal procedures with an emphasis on characterizing the performance of power control with finite rate information about channel conditions.

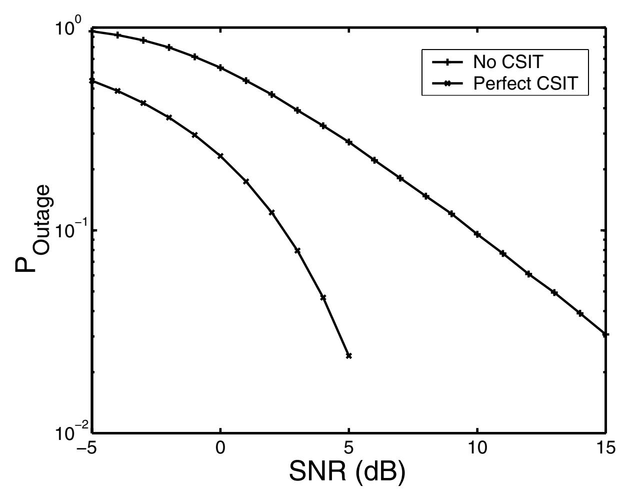

Fig. 1. Outage probability as a function of SNR with full and no channel state information at the transmitter (CSIT) for a single transmit-receive antenna system; receiver is assumed to have perfect channel information.

The rest of the paper is organized as follows. The problem is formulated in Section II. The suboptimal solution is derived in Section III, which is used to derive lower bounds on the diversity order in Section IV.

II. Problem Formulation

We consider the independent and identically distributed (i.i.d.) block fading channel model of [10]. For a multiple antenna system with M transmit antennas and one receive antenna, this model leads to the following complex baseband representation of the received signal

$$y = hx + n,$$

where x is the $M \times 1$ vector of transmitted signals, h is $1 \times M$ channel vector, and n is the additive noise. The channel coefficient, h, is assumed to vary independently from block to block and is assumed to have an arbitrary distribution unless it is mentioned specifically. Furthermore, the block fading assumption implies that h is constant for the whole codeword. Additive noise, n, is assumed to be circularly symmetric complex Gaussian distribution with zero mean and unit variance. For simplicity, we use γ to represent the magnitude squared of the channel vector, *i.e.*, $\gamma = \|h\|_2^2$.

The receiver is assumed to have perfect knowledge of the channel. The receiver quantizes channel SNR, γ, using a quantizer with L bins, resulting in a feedback codeword of $B = \log_2(L)$ bits. The receiver sends the generated feedback codeword back to the transmitter using a noiseless and zero-delay feedback channel. The transmitter then uses space-time coding and performs power control, based on the received feedback information, to transmit data. Our objective is to design the quantizer used by the receiver, $\mathcal{Q}^*$, and the power allocation policy used by the transmitter, P_t^*, such that the outage probability is minimized for a given average transmit power constraint. Outage probability [4, 11] is defined as the probability with which the achievable mutual information falls below a pre-specified threshold R,

$$\Pi(R, P_t) = \Pr\left(I(X;Y|h) < R\right), \tag{1}$$

where $I(X;Y|h)$ is the instantaneous mutual information between the transmitted and received signals. If Gaussian codebooks are used, the outage probability, $\Pi(R, P_t)$, can be rewritten as

$$\Pi(R, P_t) = \Pr\left(\log\left(1 + \frac{P_t \gamma}{M\sigma^2}\right) < R\right), \tag{2}$$

where P_t is the transmitted power for the given feedback γ_t at time t, and σ^2 is the noise variance. Note that if $\gamma \neq 0$ having $P_t = M(2^R - 1)\sigma^2/\gamma$ is enough for having zero outage [12]. For simplicity throughout the paper we use k for quantity $M(2^R - 1)\sigma^2$, and define $SNR_n = MP_{av}/k$, where P_{av} is the average available power.

The objective is to minimize outage probability $\Pi(R, P_t)$ subject to an average power constraint, *i.e.*,

$$(P_t^*, \mathcal{Q}^*) = \arg \min_{(P_t, \mathcal{Q}):\mathbb{E}(P_t) \leq P_{av}} \Pi(R, P_t). \tag{3}$$

Note that the expectation in (3) is a time average. Later we will change this time average with expectation with respect to the channel distribution. One needs a first order ergodicity in order to make such a transition.

Caire *et. al.* in [2] showed that the solution to (3) is temporal water-filling. Also it is shown in [2] that channel inversion power allocation is a practical sub-optimal scheme, which asymptotically converges to optimum solution at high SNRs. In channel inversion power $p = k/\gamma$ is allocated to each channel state $\gamma > \gamma_{th}$. Threshold γ_{th} is set such that average power constraint is met [12]. In our case we would like to approximate this power allocation strategy with finite number of power levels such that (3) is satisfied.

Recall that the quantizer is a set of thresholds, $\{\gamma_i\}_{i=1}^{L-1}$, determining L quantization bins, $[0, \gamma_1), [\gamma_1, \gamma_2), \ldots, [\gamma_{L-1}, \infty)$. Let $b_i, 1 \leq i \leq L$, denote the i^{th} quantization bin. For each quantization bin b_i, there is a power level $P_t(i)$ such that it guaranties reliable communication. With channel inversion strategy we have $P_t(i) = k/\gamma_{i-1}, 2 \leq i \leq L$. If we allocate power level $P_t(1)$ to the first bin, it only guarantees reliable channel for $\gamma > \gamma_0$, $\gamma_0 = k/P_t(1)$, and for $\gamma < \gamma_0$ the power is wasted. Therefore we solve the problem for two different approaches, namely $P_t(1) = 0$ and $P_t(1) \neq 0$, and choose the scheme that yields a smaller probability of outage. Although not trivial, simulation results show that for high SNRs, choosing $P_t(1) \neq 0$ always yields lower outage.

Let $\gamma_0 = \gamma_1$ when $P_t(1) = 0$ and $\gamma_0 = k/P_t(1)$ otherwise, then system is in outage for $\gamma < \gamma_0$, and probability of outage in (3) can be expressed as

$$\Pi(R, P_t) = \Pr(\gamma < \gamma_0). \tag{4}$$

In the subsequent section, we will derive a suboptimal quantizer based on the asymptotic form of the optimal quantizer using the dual problem of (3). The suboptimal solution is computationally tractable, has an intuitive explanation and will be used to derive lower bound on the diversity order of quantized power control (equivalently it provides an upper bound on the outage probability).

III. Suboptimal Power Control with Equi-power Allocation

In this section, we will derive a sub-optimal power control scheme, based on the asymptotic properties of the optimal power control. The derivation of the sub-optimal power control follows naturally from the dual[1] of optimization problem (3) given by

$$(P_t^*, \mathcal{Q}^*) = \arg \min_{\Pi(R, P_t) \leq \alpha} \mathbb{E}(P_t). \tag{5}$$

The expected value of power in (5) can be written as

$$\mathbb{E}(P_t) = P_t(1)F_\gamma(0, \gamma_1) + \cdots + P_t(L)F_\gamma(\gamma_{L-1}, \infty), \tag{6}$$

where $f_\gamma(\cdot)$ is the probability distribution of γ and $F_\gamma(\alpha, \beta) = \int_\alpha^\beta f_\gamma(\gamma)d\gamma$ is the probability mass accumulated in interval $[\alpha, \beta]$.

[1]The duality of (3) and (5) is discussed more in detail in [13]

To describe L power levels, $P_t(1),\ldots,P_t(L)$, we need to define $L-1$ thresholds, $\gamma_1,\ldots,\gamma_{L-1}$. From (4) and (5) one can conclude that the constraint on the outage in (5) is equivalent to fixing γ_0, or equivalently the largest power level, $P_t(1)$. Thus, the dual problem in (5) can be converted into an unconstrained problem with $(L-1)$ unknown power levels and $(L-1)$ thresholds. The solution to (5) needs to satisfy the first order derivative condition, $\vec{\nabla}_{P_t}\mathbb{E}_\gamma[P_t]=0$, which leads to the following system of nonlinear equations

$$\begin{aligned}
\frac{f_\gamma(\gamma_1)}{\gamma_0}-\frac{F_\gamma(\gamma_1,\gamma_2)}{\gamma_1^2}-\frac{f_\gamma(\gamma_1)}{\gamma_1} &= 0\\
\frac{f_\gamma(\gamma_2)}{\gamma_1}-\frac{F_\gamma(\gamma_2,\gamma_3)}{\gamma_2^2}-\frac{f_\gamma(\gamma_2)}{\gamma_2} &= 0\\
&\vdots\\
\frac{f_\gamma(\gamma_{L-1})}{\gamma_{L-2}}-\frac{F_\gamma(\gamma_{L-1},\infty)}{\gamma_{L-1}^2}-\frac{f_\gamma(\gamma_{L-1})}{\gamma_{L-1}} &= 0.
\end{aligned} \tag{7}$$

Solution to the system of nonlinear equations in (7) gives the optimum power control scheme along with corresponding optimal thresholds, $\{\gamma_i\}_{i=1}^{L-1}$, through channel inversion relation. This solution provides a common language between transmitter and receiver. On one hand $\gamma_i's$ define the quantizer at the receiver, and on the other hand corresponding $P_t(i)'s$ determine the power allocation strategy at the transmitter.

Consider the i^{th} equation in (7) ($1\le i\le L-1$, with $\gamma_L=\infty$), that is,

$$\frac{1}{\gamma_{i-1}}f_\gamma(\gamma_i)-\frac{1}{\gamma_i^2}F_\gamma(\gamma_i,\gamma_{i+1})-\frac{1}{\gamma_i}f_\gamma(\gamma_i)=0. \tag{8}$$

We can rewrite (8) as,

$$\begin{aligned}
\frac{1}{\gamma_i}(\gamma_{i+1}-\gamma_i)f_\gamma(\gamma_i) &= \frac{1}{\gamma_{i+1}}F(\gamma_{i+1},\gamma_{i+2})\\
P_t(i)(\gamma_{i+1}-\gamma_i)f_\gamma(\gamma_i) &= P_t(i+1)F(\gamma_{i+1},\gamma_{i+2}).
\end{aligned} \tag{9}$$

As number of bits in feedback, $B=\log_2(L)$, approaches infinity, the length of quantization bins, (γ_i,γ_{i+1}), approaches zero, and hence by mean value theorem, we can further simplify (9) when $B\to\infty$ as

$$P_t(i)F(\gamma_i,\gamma_{i+1})\approx P_t(i+1)F(\gamma_{i+1},\gamma_{i+2}). \tag{10}$$

The term $P_t(i)F(\gamma_i,\gamma_{i+1})$ is the average power allocated to the i^{th} bin. Thus from (10), it follows that an approximation to the optimal power allocation is to allocate equal average power to each quantization bin. From the above discussion, it follows that the equal allocation power control is asymptotically optimum in the feedback rate B. The above approximate solution (10) can now be used in the primal problem (3) to find what the average power in each bin should be. Solution to (3) is on the boundary of constraint set, *i.e.*, at the optimum point, P_t^*, we have $\mathbb{E}(P_t^*)=P_{av}$. More precisely, at P_t^* we have

$$P_t^*(1)F(0,\gamma_1^*)+\cdots+P_t^*(L)F(\gamma_{L-1}^*,\infty)=P_{av}. \tag{11}$$

Combining (11) with (10) we get,

$$P_t^*(i)F(\gamma_{i-1}^*,\gamma_i^*)=\frac{P_{av}}{L},\quad \forall\, i, 1\le i\le L, \tag{12}$$

with $\gamma_L^*=\infty$ (and $\gamma_0^*=0$), then for $i=L$ in (12) we have

$$P_t^*(L)F(\gamma_{L-1}^*,\infty)=\frac{P_{av}}{L}. \tag{13}$$

Also by channel inversion power allocation we have $\gamma_{i-1}^* = k/P_t^*(i)$ for $0<i<L$. Hence equation (13) is a function of $P_t^*(L)$ (or γ_{L-1}^*) only and we can solve (13) for $P_t^*(L)$ (or γ_{L-1}^*). Replacing the value for γ_{L-1}^* in (12) for $i=L-1$, we end up with an equation with a single variable $P_t^*(L-1)$ (and corresponding threshold γ_{L-2}^*). By recursively repeating the same procedure, we can obtain all the power levels, $\{P_t^*(i)\}_{i=1}^L$.

Figure 2 compares the performances of systems with quantized feedback (optimal, and equal allocation power control), and a system with perfect CSIRT, as a function of SNR. A single transmit and single receive antenna system is considered, and the feedback rate is $B=\log_2(3)$ bits/code-block. Note that the performance of optimal and equi-power schemes are not distinguishable in Figure 2 indicating that equal power allocation performs very close to optimum for the range of simulated SNRs.

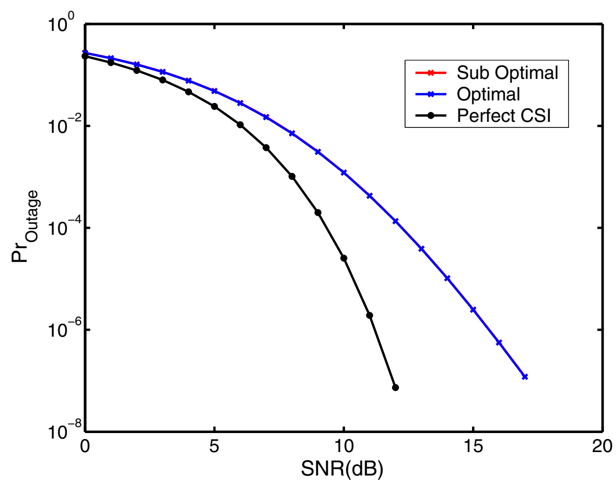

Fig. 2. Outage for $\log_2(3)$ bits of feedback for optimal, equal allocation power control, and perfect CSIRT with respect to SNR for a system with single antenna at transmitter and receiver, and transmission rate of R=2 b/s/Hz

IV. Lower Bound on Diversity Order

Figure 3 shows the performance of optimal and equal power allocation for a system with two transmit and single receive antennas with respect to SNR. Figure 3 reveals that as the number of bits in feedback increases, the slope of probability of outage with respect to SNR increases as well. This observation suggests a diversity order gain via feedback, without increasing the number of transmit antennas. In this section we capture this phenomenon by analyzing the behavior of proposed sub-optimum equal power allocation quantizer.

Throughout this section we assume that there are M transmit and single receive antennas, and equal power quantizer is used at both ends. The analysis is carried on for large values of SNR. Also channel realization, h, is assumed to

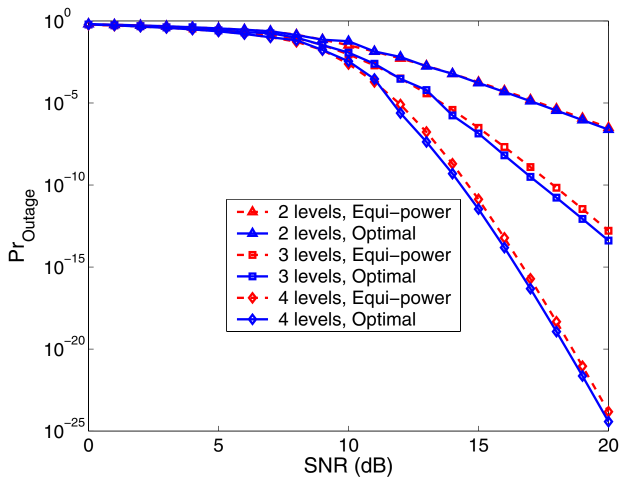

Fig. 3. Outage probability as a function of SNR for a system with 1, $\log_2(3)$, and 2 bits of feedback, 2 transmit antennas, and rate R=2 bit/s/Hz.

be a $1 \times M$ complex Gaussian random vector with identity covariance matrix, *i.e.*, $h \sim \mathcal{CN}(0, I)$. Therefore, $\gamma = |h|^2$ is chi-squared with $2M$ degrees of freedom and its probability density function is given by

$$f_\gamma(\gamma) = \frac{\gamma^{M-1} e^{-\gamma}}{\Gamma(M)}. \tag{14}$$

Replacing (14) in (4) we get

$$\begin{aligned} \Pi(R, P_t) &= \int_0^{\gamma_0} f_\gamma(\gamma) d\gamma \\ &= 1 - \sum_{i=0}^{M-1} \frac{\gamma_0^i}{i!} e^{-\gamma_0}, \end{aligned} \tag{15}$$

where $0! = 1$. As stated in (15) probability of outage is a function of γ_0, which in turn is a function of very first power level, $P_t(1)$; power levels $P_t(i)$'s, $1 \le i \le L$, are solutions to (12). Hence in order to analyze the outage probability, we need to analyze the solution of (12) as a function of received SNR, number of quantization bins, L, and number of transmit antennas, M. Lemma 4.1 shows the relation between outage probability, $\Pi(R, P_t)$, and the quantizer, Q. Theorem 4.3 demonstrates the relation between quantization thresholds, γ_i's, and average power constraint (or available SNR).

Lemma 4.1: For a system with M transmit antennas, probability of outage is proportional to γ_0^M.

Proof: [4.1] For simplicity let $F(\cdot)$ be defined as

$$\begin{aligned} F(\gamma) &= \int f_\gamma(\gamma) d\gamma \\ &= -\sum_{i=0}^{M-1} \frac{\gamma^i}{i!} e^{-\gamma}. \end{aligned} \tag{16}$$

Using (16) we can rewrite the outage probability in (15) as

$$\Pi(R, P_t) = F(\gamma_0) + 1. \tag{17}$$

At high SNR_n, or equivalently low probability of outage, $\gamma_0 \to 0$, hence we can replace $F(\gamma_0)$ by its Taylor expansion around origin, *i.e.*,

$$F(\gamma_0) \approx -1 + \gamma_0^M. \tag{18}$$

Note that $f_\gamma(\gamma)$ has a zero at origin with multiplicity of $M-1$. Replacing (18) in (17) yields the desired result. ■

Corollary 4.2 (Single antenna case): In a single antenna system $\Pi(R, P_t) \approx \gamma_0$.

Proof: Proof is immediate by setting $M = 1$ in Lemma 4.1. ■

Lemma 4.1 shows the relation between probability of outage and γ_0. In order to complete the analysis, we need to have γ_0 as a function of available SNR_n and number of quantization bins, L (or equivalently feedback rate $B = \log_2(L)$). Theorem 4.3 provides us with this relation.

Theorem 4.3 (main result): In a system with M transmit antennas and an L level quantized feedback with equal power allocation power control,

$$\gamma_0 \approx \frac{1}{SNR^{1+M+M^2+\cdots+M^{L-1}}} \tag{19}$$

for large values of SNR_n/ML.

Proof: [4.3] We use recursive argument in the proof to find γ_i's. We start from the very last bin. Total power allocated to the last bin, given by (12), only depends on γ_{L-1}. Hence it can be solved for γ_{L-1} as a function of average SNR, number of transmit antennas, and number of quantization bins. Having γ_{L-1} we replace it in the relation for total power at the $(L-1)^{st}$ bin to obtain γ_{L-2} and so on. In particular

$$\begin{aligned} P_t(L) \int_{\gamma_{L-1}}^{\infty} f_\gamma(\gamma) d\gamma &= \frac{P_{av}}{L} \quad or \\ F(\infty) - F(\gamma_{L-1}) &= \frac{SNR_n}{ML} \gamma_{L-1}. \end{aligned} \tag{20}$$

In obtaining (20) we used the fact that $P_t(L) = k/\gamma_{L-1}$ and the definition of $SNR_n = MP_{av}/k$ given in section II. When $SNR_n/ML \to \infty$, $\gamma_i \to 0$ for all $1 \le i \le L-1$. Therefore in (20) we can replace $F(\gamma_{L-1})$ by its equivalent Taylor expansion given by (18). Doing so we get

$$1 - \frac{\gamma_{L-1}^M}{M!} = \frac{SNR_n}{ML} \gamma_{L-1}. \tag{21}$$

We denote SNR_n/ML by s for simplicity. Also let $x = \gamma_{L-1}/\sqrt[M]{M!}$. Hence we can rewrite the left hand side of (21) as

$$1 - x^M \approx 1 - x. \tag{22}$$

Using the above approximation we can rewrite (21), for large values of s, as

$$\begin{aligned} 1 - \gamma_{L-1}/\sqrt[M]{M!} &\approx s\gamma_{L-1} \quad or \\ \gamma_{L-1} &\approx \frac{1}{s + \frac{1}{\sqrt[M]{M!}}} \\ &\approx \frac{1}{s}. \end{aligned} \tag{23}$$

Now for the next level using equal power allocation, we get

$$F(\gamma_{L-1}) - F(\gamma_{L-2}) = s\gamma_{L-2}. \tag{24}$$

Replacing $F(\cdot)$ with its equivalent at (18) and with algebraic manipulation of the resultant relation we get

$$\frac{\gamma_{L-1}^{M}}{M!} = \gamma_{L-2}(s - \frac{\gamma_{L-2}^{M-1}}{M!}). \quad (25)$$

Noting that $\gamma_{L-2}^{M-1}/M!$ is negligible at high SNRs, (25) can be rewritten as

$$\begin{aligned} \gamma_{L-2} &\approx \frac{1}{M!s}\gamma_{L-1}^{M} \\ &\approx \frac{1}{M!s^{1+M}}. \end{aligned} \quad (26)$$

Repeating the above procedure $L-1$ times and replacing $s = SNR_n/ML$, we get

$$\gamma_0 \approx \frac{1}{(M!)^{1+M+\cdots+M^{L-2}}(\frac{SNR_n}{ML})^{1+M+\cdots+M^{L-1}}}. \quad (27)$$

■

Note that the analysis is asymptotic on SNR_n/ML. Hence one should be careful when using the above formulation to analyze the behavior of the system with respect to number of transmit antennas, M, or the feedback rate, $B = \log_2(L)$.

Corollary 4.4 (Outage): For a system satisfying the assumptions of *theorem* 4.3,

$$\Pi(R, P_t) \approx \frac{1}{\left(SNR_n^{1+M+\cdots+M^{L-1}}\right)^{M}}. \quad (28)$$

Proof: [4.4] Combining *lemma* 4.1 with the result of *theorem* 4.3 we get

$$\begin{aligned} \Pi(R, P_t) &\approx \gamma_0^{M} \\ &\approx \left(\frac{(ML)^{1+M+\cdots+M^{L-1}}}{(M!)^{1+M+\cdots+M^{L-2}}SNR_n^{1+M+\cdots+M^{L-1}}}\right)^{M} \end{aligned} \quad (29)$$

■

Corollary 4.5 (Single antenna case): For a system with single antenna at transmitter and receiver

$$\Pi(R, P_t) \approx \frac{1}{SNR_n^{L}}. \quad (30)$$

Proof: [4.5] Proof is immediate by replacing $M = 1$ in (29). ■

Figure 4 shows the probability of outage for systems with finite rate feedback (optimum power allocation and equal power allocation strategies) along with the derived bound given in (29). As figure 4 shows, the derived bound captures the slope of outage probability for large values of SNR very accurately.

V. Conclusions

In this paper, we characterized an interesting high SNR behavior of power control schemes which relies on finite number of feedback bits. Namely, unlike the case of perfect CSIT, systems with quantized CSIT have finite diversity. This diversity-like gain is analyzed and a closed form approximation at high SNR is found for general case as a function of number of transmit antennas and feedback rate. Results were confirmed with simulation and numerical techniques.

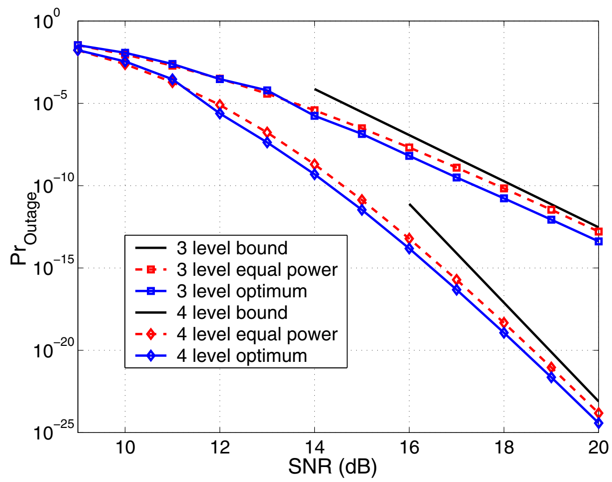

Fig. 4. Shows the bound on diversity order for L=3, and 4 with optimum and equal power allocation schemes for a system with 2 transmit antennas, and rate R=2 bit/s/Hz.

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