POST-COMPENSATION OF RF NON-LINEARITY IN MOBILE OFDM SYSTEMS BY ESTIMATION OF MEMORY-LESS POLYNOMIAL

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Abstract—**The high peak-to-average-power-ratio (PAPR) of OFDM systems introduces inevitable non-linear distortion in the transmitter and causes both in-band distortion and out-of-band spectrum re-growth. In this paper we propose a novel scheme to compensate for the non-linearity of RF transmitters with solid-state-power-amplifiers (SSPA) in OFDM by Least Square (LS) estimation of the nonlinear parameters modeled by memory-less polynomials. The compensation is easily achieved by subtracting the higher-order polynomials of the input from the RF output. Simulation results for spectrum masking as well as the BER curve in 16-QAM OFDM show very stable and promising performance as well as simple implementation.**

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has attracted considerable interest recently because of several significant advantages such as robustness to multi-path fading and great simplification of channel equalization which typically dominates the complexity of implementation in a communication system. The application of FFT/IFFT makes the implementation at very high data rates possible. Therefore, OFDM has been adopted in several wireless standards such as IEEE 802.11a WLAN and ETSI terrestrial video broadcasting.

However, OFDM systems suffer from very high Peak-to-Average Power Ratio (PAPR)[1]. This will cause serious problems with in-band non-linear distortion as well as out-band interference for adjacent channels with spectrum re-growth. To make the power amplifier (PA) work in linear range, PAPR reduction schemes [1] as well as backoff schemes [7] were studied recently. However, the backoff scheme dramatically reduces the power efficiency of RF transmitters. This may have deleterious effects on battery lifetime in mobile systems and out-weigh all potential benefits of OFDM. In order to maximize the power efficiency, low-cost nonlinear power amplifiers, such as Class C, are preferable and the power amplifier is even forced to operate near saturation range. This will bring undesirable but inevitable non-linearity into the whole system performance.

Several schemes were proposed in different systems using high power amplifier (HPA) in order to compensate for the non-linearity in RF transmitters [4,5,6]. One complicated technique is pre-distortion which distorts the input signals to the HPA with an inverse function of the HPA non-linearity and generates linearly amplified signals at the output [4,5]. Most previous schemes applied the memory-less model of *Saleh* [3] for traveling wave tube (TWT) amplifiers [3]. But this model is not very suitable for mobile systems where SSPA's are generally used instead. Unfortunately not much work has been reported yet for mobile SSPA in the context of OFDM. Moreover most of the implementations of a pre-distorter applied look-up-tables (LUT) [5,6] to provide numerical solution of the inverse function. LUT methods have inherent quantization error limited by the size of the table and long time update of the LUT [4], so it is not very suitable for dynamic compensation.

In this paper we propose a polynomial model for memory-less SSPA. We show that this model accurately captures the AM/AM characteristics of SSPA and provides efficiencies for post-compensation. The polynomial coefficients are estimated using a least square (LS) method based on a linear equation with *Vandermonde* matrix. By simply subtracting the higher order polynomials of the input signal from the distorted SSPA output we obtain the linearly amplified signal with high precision. The analysis of response curve, RF spectrum masking, received constellation and BER in a 16QAM-OFDM system for different backoff power levels shows very stable and promising performance compared to previous schemes and enhances the power efficiency.

II. OFDM SYSTEM MODEL & PAPR

A. OFDM System Model

The system model of an OFDM transmitter with RF is shown in Fig.1. The receiver is a counterpart of the transmitter and is not shown here. First a set of information bits $[b_1\ b_2\ b_3 \ldots b_M]$ are mapped into the I/Q channel baseband symbols $\{S_n^{(i,r)}\}$ using a modulation scheme such as phase-shift-keying (PSK) or quadrature-amplitude-modulation (QAM). Then each group of N symbols are packed into a parallel block $[S_1^{r,i}\quad S_2^{r,i}\quad \cdots\quad S_N^{r,i}]^T$ at the input to the IFFT. OFDM symbols in the time domain over time interval $t \in [0, T_s]$ are generated by the IFFT operation as,

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} S_n e^{j2\pi(k-1)(n-1)/N} \qquad (1)$$

for $k=[1,2,\ldots,N]$. Then in Cyclic Prefix (CP) insertion, the first G coefficients are repeated after the original N coefficients and made serial for quadrature modulation. The analog signal at DAC output after quadrature modulation in a software radio type modulator is,

$$\tilde{s}(t) = \Re\{x(t)e^{j(\omega_c t+\phi(t))}\} \qquad (2)$$

with amplitude of the input signal to the RF transmitter

$$x(t) = \sum_{i=0}^{+\infty} \sum_{k=0}^{N_{\tilde{s}}-1} \sqrt{s_I^2(k)+s_Q^2(k)}\, p(t-k\frac{T_{\tilde{s}}}{N_{\tilde{s}}}-iT_{\tilde{s}}) \qquad (3)$$

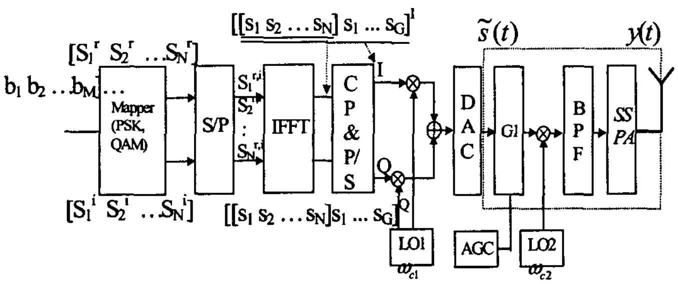

Fig1. OFDM Transmitter with RF

and the phase

$$\phi(t) = \tan^{-1}\left(\frac{\tilde{s}_Q(t)}{\tilde{s}_I(t)}\right), \quad (4)$$

where $T_{\tilde{s}}$ is the period of one time-domain OFDM symbol, $N_{\tilde{s}}$ is the number of samples in one symbol after CP and $p(t)$ is the pulse function of the symbols.

In an RF transmitter with super-heterodyne architecture, there are several cascaded analog components such as passive band-pass filter, passive mixer and power amplifier. Each of these RF components has a certain level of nonlinear effects and can be modeled as a non-linear system. The power amplifier is of particular interest because its specification will determine the maximum transmitted power.

B. Probability-based PAPR

The PAPR of OFDM signals are defined previously as [1]

$$\Gamma_{old} \equiv \frac{1}{P_{av}} \max_{0 \le t \le T_s} |\tilde{s}(t)|^2 . \quad (5)$$

Here P_{av} is the average power in one time domain symbol. Theoretically, if N sub-carriers are modulated with QPSK, then the upper bound of the *PAPR* in (5) will be N. However, when N increases, the probability of the occurrence of this upper bound is negligible. For M-ary PSK OFDM system, there are at most M^2 patterns to yield the upper bound. The probability of observing such a PAPR is $M^2 / M^N = M^{(2-N)}$. Even with QPSK ($M=4$) and $N=64$ sub-carriers as in 802.11a, this probability is only $4.7e^{-38}$ [2].

Therefore, the upper bound definition in (5) is not very meaningful and people are more interested in effective peak factor ζ_{EFT} defined as [9]

$$\Pr(\Gamma_{old} > \zeta_{EFT}) < \varepsilon_P , \quad (6)$$

where ε_P is a small number probability that can be considered negligible in practice. Actually this is the starting point of several recent performance analysis papers [2,8]. The interesting implication here is that the maximum input signal power $P_{\max}$ is inevitably larger than the input limit of RF. This is a strong motivation for the compensation of amplifier non-linearity in OFDM systems.

III. NON-LINEARITY MODEL

Two types of amplifiers are used in communication: TWT and SSPA. TWT is mostly used for high power satellite transmitters while SSPA is used in many other applications because of its small size, including mobile transmitters.

In general, Volterra series can be used to model nonlinear systems with memory. However this model involves the solution of high order filtering coefficients fitted by an adaptive learning with extremely high complexity. Several previous papers used *Saleh*'s model to analyze the HPA [4,5]. The AM/AM and AM/PM response in *Saleh*'s model is

$$\begin{cases} A(x) = \alpha_A x / (1 + \beta_A x^2) \\ \Phi(x) = \alpha_\Phi x^2 / (1 + \beta_\Phi x^2) \end{cases}, \quad (7)$$

where $\{\alpha_A, \beta_A, \alpha_\phi, \beta_\phi\}$ is a set of parameters. Most of the previous work applied LUT implementation because the mathematical solution of the inverse response is difficult. Finding a simple model with sufficient precision is of great importance.

We will consider the SSPA for mobile transmitter and assume no AM/PM effects in this paper. This is reasonable not only because SSPA has much lower AM/PM than TWT, but also because the AM/AM effects on an OFDM system are more significant than those of AM/PM. We use a polynomial model for the memory-less non-linearity as,

$$y(t) = \sum_{i=0}^{P} \alpha_i x(t)^i = \alpha_0 + \alpha_1 x(t) + \sum_{k=2}^{P} \alpha_k x(t)^k . \quad (8)$$

Here $x(t)$ and $y(t)$ are the input and output of the nonlinear RF respectively. α_0 represents the DC offset, α_1 is the linear scalar, $\{\alpha_2, \ldots, \alpha_P\}$ contributes to non-linearity in the system where P is the highest order of non-linearity.

IV. ESTIMATION & POST-COMPENSATION

A. Least Square (LS) Estimation of Polynomial

Assume we can get samples of $x(t)$ and $y(t)$ in (8) using training data with sufficient dynamic range as in $Y_t = [y_1 \ y_2 \ \cdots \ y_N]^T$, and $X_t = [x_1 \ x_2 \ \cdots \ x_N]^T$ (subscript t denotes training). By considering the combinational thermal noise $\underline{w}$ from the RF circuits, we will get a linear equation

$$Y_t = \overline{X}_t * \underline{\alpha} + \underline{w} \quad (9).$$

Here $\overline{X}_t = [\underline{1}_{N\times 1} \ X_t \ X_t^{\bullet 2} \ \cdots \ X_t^{\bullet P}]$ is a *NXP Vandermonde* matrix and $\underline{\alpha} = [\alpha_0 \ \alpha_1 \ \alpha_2 \ \cdots \ \alpha_P]^T$ are coefficients for the overall non-linearity of the RF transmitter system. The least square (LS) estimation of (9) is

$$\underline{\hat{\alpha}} = [\overline{X}_t^H \overline{X}_t]^{-1} \overline{X}_t^H * Y_t . \quad (10)$$

B. Post-Compensation

The compensation of non-linearity from the polynomial model is straight-forward. Since α_1 is the linear term in (8), a linearly amplified signal can be obtained by substracting those nonlinear terms when sending actual data as,

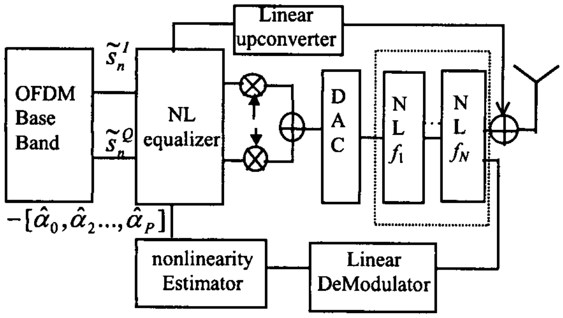

Fig.2. Post-compensation by estimation of nonlinear polynomial.

$$\begin{aligned}\tilde{y}(n) &= y(n) - \sum_{\substack{i=0\\ i\neq 1}}^{P} \hat{\alpha}_i \tilde{x}(n)^P \\ &= \alpha_1 \tilde{x}(n) + \sum_{\substack{i=0\\ i\neq 1}}^{P} (\alpha_i - \hat{\alpha}_i) \tilde{x}(n)^P = \alpha_1 \tilde{x}(n) + \varepsilon(n)\end{aligned} \tag{11}$$

The error of this compensation is caused by the estimation error $\varepsilon(n)$ and is considerably small. The system architecture for the proposed non-linear estimation and compensation is shown as in Fig.2. A linear demodulator samples the SSPA output of the training data and estimates the polynomial coefficients. Then the baseband signal formulated by (2) is multiplied by the α coefficients to generate the compensation signal. Because the compensation signal is in general much smaller than the amplified signal, it can be linearly up-converted to the RF without using another power amplifier, similar to the previous feed-forward method [9]. Then it is subtracted from the SSPA output of the transmitted signal. The final output signal is effectively linearly amplified with scalar α_1. Although here we use a discrete formulation to explain the idea here, the corresponding analog signal for compensation at RF is straightforward.

V. SIMULATION RESULTS

We used 16QAM and 64 sub-carriers in simulation. We also used a SSPA with typical RF specifications for a mobile system (Gain=20dB, P1dB=24dBm, IP2=40dBm, IP3=40dBm, IP4=100dBm) to build the whole chain of carrier modulation. We used P=10 which yields a very good fit of the model to the actual non-linearity. In Fig. 3, we compared the received signal after the low-pass filter at the quadrature demodulator. In this time frame we see the obviously high PAPR and the compensated signal has almost the same shape as linear-gained signal, while the non-linearity amplified signal is distorted severely at high amplitude. We then measured the actual response at the input/output of the RF with more than 40000 samples as in Fig. 4. It is found that even below the saturation point the response curve is not ideally linear. But the compensated response is perfectly overlapped with linear gain. The spectrum masking at RF output is shown in Fig. 5 with output backoff(OBO)=2dB. OBO is defined as the ratio of maximum possible amplifier

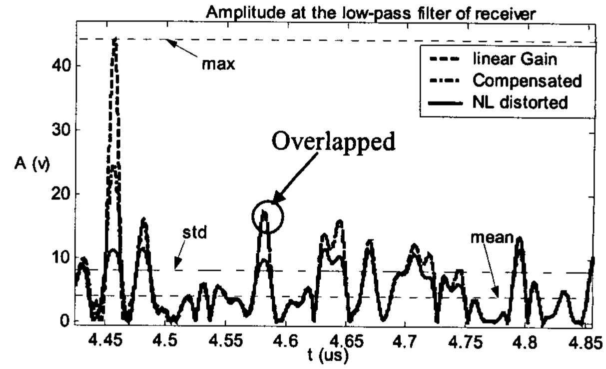

Fig.3. Amplitude after the low-pass filter of receiver I/Q-demodulator.

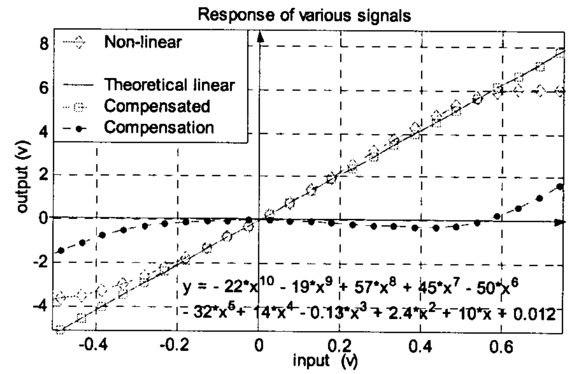

Fig.4. Response curve for the non-linear system with SSPA specifications: Gain=20dB, P1dB=24dBm, IP2=40dBm, IP3=40dBm , P=10.

output power to the average amplifier output power as in OBO (dB)=$10\log_{10}(P_{max}/P_{av})$. Note that OBO implicates power efficiency. The higher OBO is less efficient because the operating point is far below the saturation point. It can be seen that the non-linearity caused about 20dB spectrum re-growth, while the compensated signal almost has the same spectrum as the linear gain. Even though there is still some interference around the higher harmonic of the carrier frequency, it can be easily filtered by a wide band analog filter. The constellation of the received signal is shown in Fig. 6. It is found that for the non-linear case, the lower the OBO, the more dispersed the constellation. But in each OBO level, the compensated constellation is almost the same as the linear transmission. This indicates the stability of the scheme at different OBO levels.

The BER performance is shown in Fig.7, also with different OBO. It can be seen that only at OBO=14dB is the non-linear BER curve close to the linear case. This is almost useless because the transmitting power will be too weak to have a high SNR. In a more practical case, when OBO=8dB, a significant loss appears. When OBO=3,4,5dB, there will be a flat-top on the BER curve, which means the operating range of the system is dramatically limited. However, with the compensation, even at OBO=3dB, the BER performance is almost the same as ideally linear, with slight loss at BER 10^{-4}. This is better than the reported result in [5] with pre-distorter.

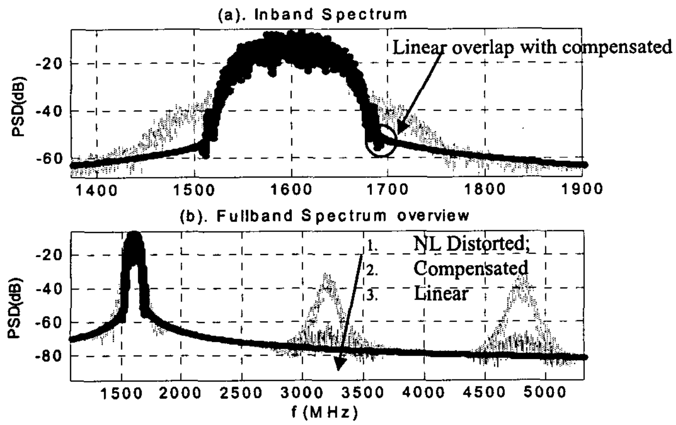

Fig.5. Spectrum re-growth at the transmitter RF output, 16QAM.

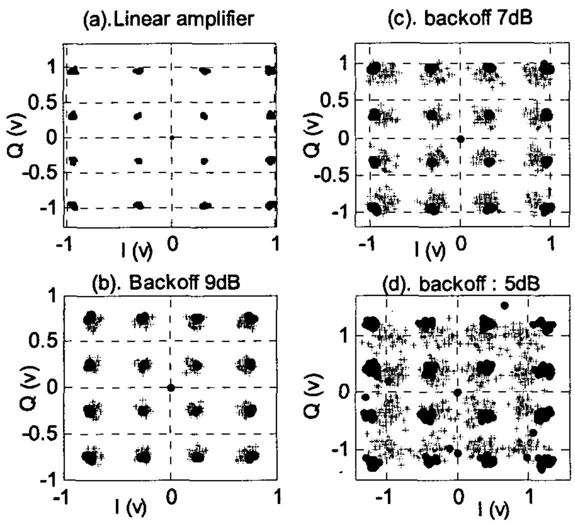

Fig. 6. Received Constellation with different OBO level.
(+: Non-linear; .: Post-compensated)

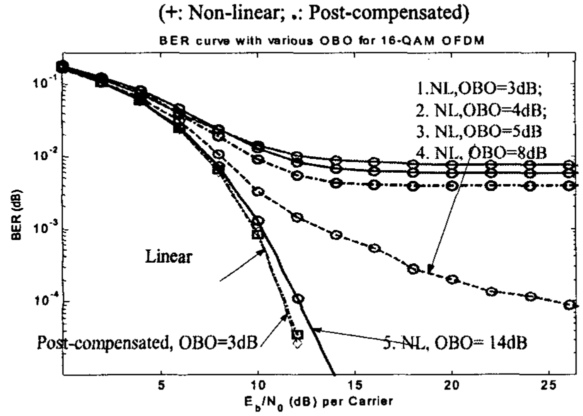

Fig. 7. BER curve for different OBO.

Order P will determine the tradeoff between estimation error and computational complexity of estimation. Fig.8 shows the residual norm vs. polynomial order P. At P=5, the error becomes considerably small. This means we can use a small order to model non-linearity accurately. This guarantees

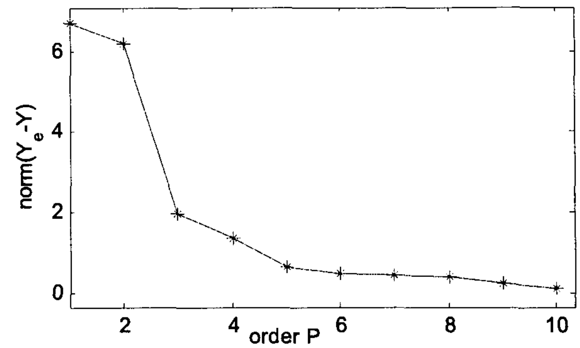

Fig. 8. Residual norm vs. polynomial order P.

a simple implementaion of the estimator. Actually we can even apply the structure in *Vandermonde* matrix to reduce the computation complexity further.

VI. CONCLUSION

In this paper we proposed a polynomial model for the non-linearity in OFDM transmitters with SSPA. The model parameters are estimated with Least Square criterion and applied for post-compensation. The simulation displays very promising performance in terms of spectrum masking and BER.

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