A RECURSIVE ALGORITHM FOR DIGITAL IMAGE PROCESSING USING LOCAL STATISTICS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An algorithm is presented for digital image processing based on local statistics. The algorithm constitutes a recursive implementation of an approach proposed and implemented nonrecursively by J. S. Lee (Naval Research Laboratory Report 8192, March 1978). Calculations show that the proposed recursion introduces considerable improvement in efficiency.

1. INTRODUCTION

Recently, Jong-Sen Lee [1] proposed an algorithm for contrast enhancement and white noise filtering of a digital image based on local statistics. Lee's algorithm is nonrecursive and requires large computational effort in the calculation of local means and variances for individual pixels.

In what follows, we present a recursive algorithm for the calculation of the above means and variances, which reduces considerably the number of computations required to process given sets of pixels.

2. BASIC ASSUMPTIONS

Let $x(i,j)$ denote the intensity (grey level) of the pixel (i,j). Lee [1] makes the following assumptions:

(a) $x(i,j)$ is a realization of a real-valued random variable $X(i,j)$ whose mean and variance we denote by $\mu_X(i,j)$ and $\sigma_X^2(i,j)$;

(b) the observation $z(i,j)$ of $x(i,j)$ consists of $x(i,j)$ corrupted by white additive or/and multiplicative noise, the mean and variance of the noise being assumed known;

(c) observations $z(k,\ell)$ of pixels (k,ℓ) located in a $(2n + 1) \times (2m + 1)$ rectangular window centered around a given pixel (i,j) are independent and all come from the same scalar distribution. Hence estimates $\hat{\mu}_Z(i,j)$ and $\hat{\sigma}_Z^2(i,j)$ of $\mu_Z(i,j)$ and $\sigma_Z^2(i,j)$ are obtained by local averaging over those pixels, i.e.

$$\hat{\mu}_Z(i,j) = \frac{1}{(2n+1)(2m+1)} \sum_{k=i-n}^{i+n} \sum_{\ell=j-m}^{j+m} z(k,\ell), \tag{1}$$

$$\hat{\sigma}_Z^2(i,J) = \frac{1}{(2n+1)(2m+1)} \sum_{k=i-n}^{i+n} \sum_{\ell=j-m}^{j+m} (z(k,\ell) - \hat{\mu}_Z(i,j))^2. \tag{2}$$

For each pixel, Lee computes $\hat{\mu}_Z(i,j)$ and $\hat{\sigma}_Z^2(i,j)$ by means of (1) and (2) and then uses these values in obtaining a minimum mean square estimate $\hat{x}(i,j)$ of $x(i,j)$, as explained (for the additive noise case) in section 4.

Actually, the observation $z(i,j)$, should not be included in the above estimates, since it is used later on (see section 4) in the estimation of $x(i,j)$. For this reason, in the present paper, we replace (1) and (2) by (1a) and (2a) below:

$$\hat{\mu}_Z(i,j) = \frac{1}{(2n+1)(2m+1)-1}\left(\sum_{k=i-n}^{i+n}\sum_{\ell=j-m}^{j+m} z(k,\ell) - z(i,j)\right) \qquad (1a)$$

$$\hat{\sigma}_Z^2(i,j) = \frac{1}{(2n+1)(2m+1)-1}\left(\sum_{k=i-n}^{i+n}\sum_{\ell=j-m}^{j+m} (z(k,\ell) - \hat{\mu}_Z(i,j))^2 - (z(i,j)-\hat{\mu}_Z(i,j))^2\right) \qquad (2$$

There are some other issues that Lee's approach raises which we will not discuss here. The only objective of the present paper is to develop a recursive algorithm for the calculation of $\hat{\mu}_Z(i,j)$ and $\hat{\sigma}_Z^2(i,j)$ defined by (1a) and (2a).

3. RECURSIVE ESTIMATION OF LOCAL MEAN AND VARIANCE

Let the domain of the image be N X M. Our recursion applies to a concentric (N-2n) x (M-2m) rectangular subset S of the image (see Fig. 1). Thus the recursion begins at the pixel (n+1, m+1) from where it moves horizontally rightward up to the pixel (n+1, M-m). Then it shifts to the next horizontal line moving rightward from the pixel (n+2, m+1) to pixel (n+2, M-m). This procedure is repeated up to the $(N-n)^{th}$ horizontal line.

3.1. Recursion for the Local Mean

Let us define the following entities

$$\zeta(i,j) = \frac{1}{(2n+1)(2m+1)-1} z(i,j) \qquad (3)$$

$$\rho(i,j) = \sum_{k=i-n}^{i+n} \zeta(k,j), \quad i \geq n+1, \tag{4}$$

$$\tilde{\rho}(i,j) = \rho(i,j) - \zeta(i-n,j), \quad i \geq n+1, \tag{5}$$

$$\tilde{\mu}_Z(i,j) = \hat{\mu}_Z(i,j) + \zeta(i,j) - \rho(i,j-m), \quad i \geq n+1, \quad j \geq m+1 . \tag{6}$$

Substituting (3) and (4) in (1a), we obtain

$$\hat{\mu}_Z(i,j) = \sum_{\ell=j-m}^{j+m} \rho(i,\ell) - \zeta(i,j) . \tag{7}$$

A detailed consideration of the above variables vis-á-vis formula (7) leads to our proposing the following recursion for the generation of $\hat{\mu}_Z(i,j)$.

A. <u>$(n+1)^{th}$ row</u>

(a) <u>Initialization</u>

(I) For $j=1,\ldots,2m+1$, calculate:

Step 1: $\zeta(i,j)$, $i=1,\ldots,2n+1$,

Step 2: $\rho(n+1,j)$.

(II) Calculate:

Step 3: $\hat{\mu}_Z(n+1, m+1)$

(b) Remaining elements of $(n+1)^{th}$ row

For $j = m+2,\ldots,M-m$, calculate

Step 4: $\zeta(i,j+m),\ i=1,\ldots,2n+1$;

Step 5: $\rho\ (n+1,j+m)$

Step 6: $\tilde{\mu}_Z\ (n+1,j-1)$

Step 7: $\hat{\mu}_Z(n+1,j) = \tilde{\mu}_Z\ (n+1,j-1) + \rho(n+1,j+m) - \zeta(n+1,j)$

B. q^{th} row, $q = n+2,\ldots,N-n$

(a) Initialization

I. For $j=1,\ldots,2m+1$, calculate:

Step 8: $\zeta(q+n,j)$

Step 9: $\tilde{\rho}\ (q-1,j)$

Step 10: $\rho\ (q,j) = \tilde{\rho}\ (q-1,j) + \zeta(q+n,j)$

II. Calculate:

Step 11: $\hat{\mu}_Z(q,\ m+1)$

(b) Remaining elements of q^{th} row

For $j = m+2,\ldots,M-m$, calculate:

Step 12: $\zeta(q+n,j+m)$

Step 13: $\tilde{\rho}\ (q-1,j+m)$

Step 14: $\rho(q,j+m) = \tilde{\rho}\ (q-1,j+m) + \zeta(q+n,j+m)$

Step 15: $\tilde{\mu}_Z\ (q,j-1)$

Step 16: $\hat{\mu}_Z\ (q,j) = \tilde{\mu}_Z\ (q,j-1) + \rho(q,j+m) - \zeta(q,j)$.

3.2. Recursion for the Local Variance

Equation (2a) can be rewritten in the form

$$\hat{\sigma}_Z^2(i,j) = \Big[\frac{1}{(2n+1)(2m+1)-1} \Big(\sum_{k=i-n}^{i+n} \sum_{\ell=j-m}^{j+m} z^2(k,\ell) - z^2(i,j)\Big) - \hat{\mu}_Z^2(i,j)\Big] . \tag{8}$$

This shows that a recursion for $\hat{\sigma}_Z^2(i,j)$ can be formulated which is almost identical to that for the local mean with $z(k,\ell)$ replaced by $z^2(k,\ell)$. In fact, let

$$\eta(i,j) = \frac{1}{(2n+1)(2m+1)-1} z^2(i,j) \tag{9}$$

$$\gamma(i,j) = \sum_{k=i-n}^{i+n} \eta(k,j) , \quad i \geq n+1 \tag{10}$$

$$\tilde{\gamma}(i,j) = \gamma(i,j) - \eta(i-n,j), \quad i \geq n+1 \tag{11}$$

$$\tilde{\sigma}_Z^2(i,j) = \hat{\sigma}_Z^2(i,j) + \eta(i,j) - \gamma(i,j-m) + \hat{\mu}_Z^2(i,j), \quad i \geq n+1,\ j \geq m+1 \tag{12}$$

Note that (8) may be rewritten as

$$\hat{\sigma}_Z^2(i,j) = \sum_{\ell=j-m}^{j+m} \gamma(i,\ell) - \eta(i,j) - \hat{\mu}_Z^2(i,j) . \tag{13}$$

We may now proceed to state the algorithm for the recursive generation of $\hat{\sigma}_Z(i,j)$ as follows.

A. <u>$n + 1^{th}$ row</u>

(a) <u>Initialization</u>

I. For $j = 1,\ldots,2m+1$, calculate:

Step 1: $\eta(i,j)$, $i = 1,\ldots,2n+1$;

Step 2: $\gamma(n+1,j)$

II. Calculate:

Step 3: $\hat{\sigma}_Z^2(n+1,\ m+1)$ (by means of formula (13), where in this formula $\hat{\mu}_Z(n+1,\ m+1)$ is obtained from Step 3 in the algorithm of the preceding subsection).

(b) <u>Remaining elements of $(n+1)^{th}$ row</u>

For $j = m+2,\ldots,M-m$, calculate:

Step 4: $\eta(i,j+m)$, $i = 1,\ldots,2n+1$;

Step 5: $\gamma(n+1,\ j+m)$

Step 6: $\tilde{\sigma}_Z^2(n+1,\ j-1)$

Step 7:
$$\hat{\sigma}_Z^2(n+1,j) = \tilde{\sigma}_Z^2(n+1,j-1) + \gamma(n+1,j+m) - \eta(n+1,j) - \hat{\mu}_Z^2(n+1,j)$$

(where $\hat{\mu}_Z(n+1,j)$ is obtained from Step 7 in the algorithm of the preceding subsection).

B. q^{th} row, $q = n + 2, \ldots, N-n$

(a) Initialization

I. For $j = 1, \ldots, 2m + 1$, calculate

Step 8: $\eta\,(q + n, j)$

Step 9: $\tilde{\gamma}\,(q - 1, j)$

Step 10: $\gamma\,(q,j) = \tilde{\gamma}(q-1,j) + \eta\,(q + n, j)$

II. Calculate

Step 11: $\hat{\sigma}_Z^{\,2}\,(q, m+1)$ (by means of formula (13) where, in this formula, $\hat{\mu}_Z\,(q, m+1)$ is obtained from Step 11 in the algorithm of the preceding section).

(b) Remaining elements of q^{th} row

For $j = m+2, \ldots, M-m$, calculate:

Step 12: $\eta\,(q+n, j+m)$

Step 13: $\tilde{\gamma}\,(q-1, j+m)$

Step 14: $\gamma\,(q,j+m) = \tilde{\gamma}\,(q-1, j+m) + \eta\,(q+n, j+m)$

Step 15: $\tilde{\sigma}_Z^{\,2}\,(q,j-1)$

Step 16: $\hat{\sigma}_Z^{\,2}\,(q,j) = \tilde{\sigma}_Z^{\,2}\,(q,j-1) + \gamma(q,j+m) - \eta(q,j) - \hat{\mu}_Z^{\,2}\,(q,j)$

(where $\hat{\mu}_Z^{\,2}\,(q,j)$ is obtained from Step 16 in the algorithm of the preceding section).

4. CONCLUSION OF THE ALGORITHM AND FINAL REMARKS

The local mean and variance obtained as above can now be used in Lee's [1] procedure. To show this, we briefly discuss the case of filtering in the presence of white additive noise. For the cases of multiplicative noise and joint additive and multiplicative noise, the reader is referred to [1].

In the additive noise case then, we write

$$z(i,j) = x(i,j) + w(i,j) \quad , \tag{14}$$

where $w(i,j)$ is a realization of $W(i,j)$, which is independent of $X(i,j)$, with zero mean and variance σ_W^2 (Note also assumption (c) in section 2).

Assuming that the "a-priori" mean and variance of $X(i,j)$ are respectively

$$\hat{\mu}_X(i,j) = \hat{\mu}_Z(i,j) \tag{15}$$

and

$$\hat{\sigma}_X^2(i,j) = \hat{\sigma}_Z^2(i,j) - \sigma_W^2 \ , \tag{16}$$

Lee deduces the a-posteriori estimate $\hat{x}(i,j)$ of $x(i,j)$ to be

$$\hat{x}(i,j) = \hat{\mu}_X(i,j) + K(i,j)(z(i,j) - \hat{\mu}_X(i,j)) \tag{17}$$

where

$$K(i,j) = \frac{\hat{\sigma}_X^2(i,j)}{\hat{\sigma}_X^2(i,j) + \sigma_W^2} \ . \tag{18}$$

So, in this case, we would simply substitute $\hat{\mu}_Z(i,j)$ and $\hat{\sigma}_Z^2(i,j)$, obtained in the preceding sub-sections, into (15) and (16), and then proceed to (17) and (18).

At this point, it is worthwhile to compare the computational effort in executing the recursive (given above) and non-recursive schemes for the estimation of local means and variances. For this purpose, the number of scaling (multiplication by a constant), addition, and multiplication operations for the $(n+1)^{th}$ line and for the q^{th} line, where $q > n + 1$, are listed in Table 1 for each case. Typically, $m = n = 3$ and $M = N = 256$. Table 2 shows the numbers of operations listed in Table 1 for these values of m, n, M and N.

Typically, in the calculation of all the local means and variances associated with the qth line, where $5 \leq q \leq 253$, the number of scaling operations as well as the number of multiplications for the nonrecursive and recursive algorithms are nearly the same. However, (for the qth line) the recursive algorithm requires only about 1/8 of the number of additions required by the nonrecursive one in the calculation of the local means and about 1/6 of the number of additions needed by the nonrecursive algorithm in the evaluation of local variances.

From the above we conclude that in processing a picture based on local statistics, one could combine recursion with parallel processing either to simplify the parallel processor without sacrifice in speed, or to speed up the overall processing.

REFERENCE

[1] Lee, Jong-Sen, "Digital Image Processing by Use of Local Statistics", NRL Report 8192, Washington, D.C., March 1, 1978. To appear in the IEEE Transactions on Pattern Analysis and Machine Intelligence.

TABLE 1

NUMBERS OF ELEMENTARY OPERATIONS
FOR PROCESSING A SINGLE LINE

(Estimation applies to pixels (i,m+1) to (i,M-m) for the ith line)

Local Means

Type of Operation	Line	No. of Elementary Operations: Non-Recursive Algorithm (J.S. Lee) (We use (1a) and (8) instead of (1) and (2))	No. of Elementary Operations: Recursive Algorithm (Present Paper)
Scaling	$(n+1)^{th}$ line	M - 2m	(2n+1)M
	q^{th} line ($n+2 \leq q \leq M-m$)	M - 2m	M
Addition	$(n+1)^{th}$ line	((2n+1)(2m+1)-2)·(M-2m)	2(n+2)M-3(2m+1)
	q^{th} line ($n+2 \leq q \leq M-m$)	((2n+1)(2m+1)-2).(M-2m)	6M-3(2m+1)

Local Variances

Type of Operation	Line	Non-Recursive Algorithm	Recursive Algorithm
Scaling	$(n+1)^{th}$ line	M-2m	(2n+1)M
	q^{th} line ($n+2 \leq q \leq M-m$)	M-2m	M
Addition	$(n+1)^{th}$ line	[(2n+1)(2m+1)-1].(M-2m)	(2n+6)M-5(2m+1)+1
	q^{th} line ($n+2 \leq q \leq M-m$)	[(2n+1)(2m+1)-1].(M-2m)	8M-5(2m+1) + 1
Multiplication	$(n+1)^{th}$ line	(2n+1)M+M-2m	2(n+1)M-2m
	q^{th} line ($n+2 \leq q \leq M-m$)	M+M-2m	2(M-m)

TABLE 2

VALUES OF NUMBERS OF ELEMENTARY OPERATIONS
Indicated in Table 1, with m=n=3, M=N=256

Type of Operation	Line	No. of Elementary Operations: Non-Recursive Algorithm (J.S. Lee)	Recursive Algorithm (Present Paper)
Locals Means			
Scaling	4^{th} line	250	1,792
	q^{th} line ($5 \leq q \leq 253$)	250	256
Addition	4^{th} line	11,750	2,539
	q^{th} line ($5 \leq q \leq 253$)	11,750	1,515
Local Variances			
Scaling	4^{th} line	250	1,792
	q^{th} line ($5 \leq q \leq 253$)	250	256
Addition	4^{th} line	12,000	3,038
	q^{th} line ($5 \leq q \leq 253$)	12,000	2,014
Multiplication	4^{th} line	2,042	2,042
	q^{th} line ($5 \leq q \leq 253$)	506	506

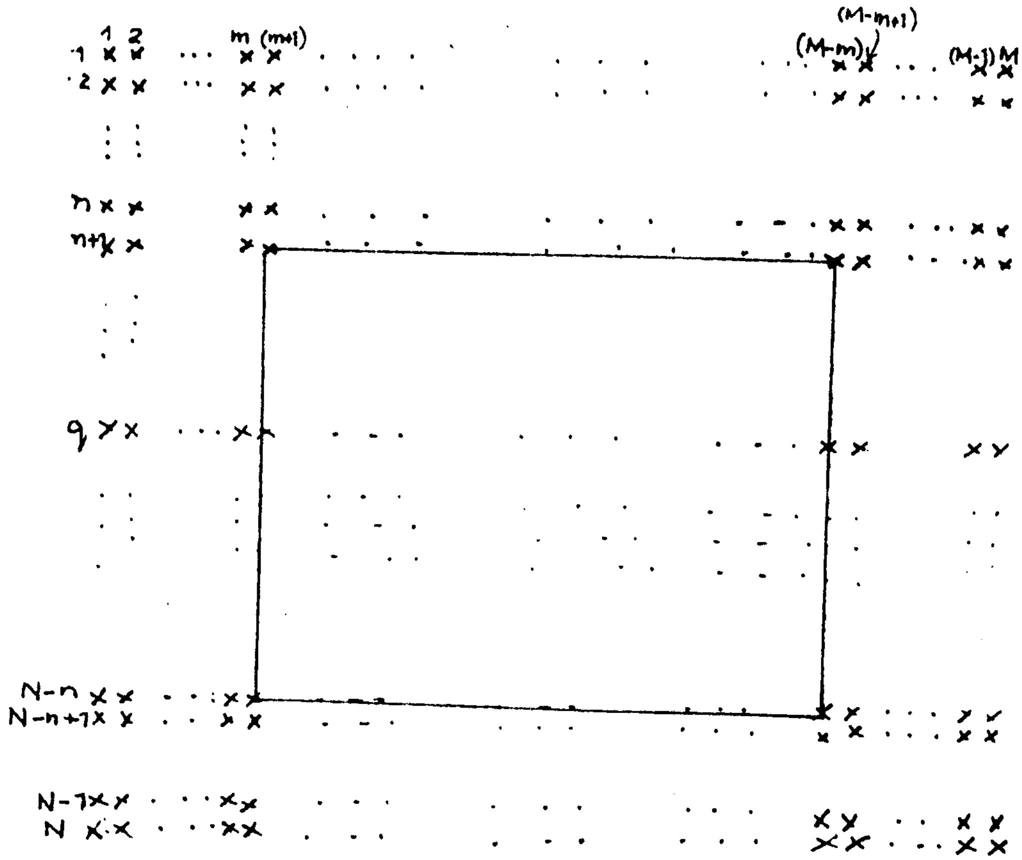

Fig. 1