LDPC Code Design for Relay Channel in Time-Division mode

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***Abstract*— We address the problem of LDPC code design for relay channel in time-division mode based on a distributed strategy: In the first time slot, the source transmits part of the codeword. The relay and the destination receive it but only the relay can decode it. In the second time slot, the relay transmits the additional redundant bits to the destination. The destination is able to decode the transmitted codeword based on the data received in both time slots. The asymptotic performance of the LDPC codes that we designed are as close as 0.6 decibel from the theoretical limit.**

I. Introduction

Consider the relay channel in Fig. 1. We assume that the relay operates in a Time-Division (TD) manner [3], *i.e.* for a given time window, the relay listens for a first phase (say t), and transmits for the rest $(1-t)$. The source sends a message encoded at rate R_1 with power P_1 during the first phase. The relay decodes the message, re-encodes it, then transmits the remaining redundant bits with power P_2' to the destination during the second phase. The source may transmit during the second phase an additional message with power P_2. This scheme is called 'Decode-Forward' ([2]).

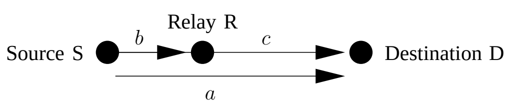

Fig. 1. The relay channel. The three different nodes are: the source node (S), the relay node (R) and the destination node (D). In Time-Division transmission mode, the source transmits with power P_1 in the first time slot and with power P_2 in the second time slot. The relay transmits only during the second time slot with power P_2'. The coefficients a, b and c are the amplitude of the channel gains.

II. Practical strategies for TD Relay Channel

We consider optimum power allocation among the nodes under network power and bandwidth constraints:

$$tP_1 + (1-t)\left(P_2 + P_2'\right) \leq 1 \quad (1)$$

In order to avoid a joint optimization over the power allocation, the time-sharing and the correlation factor between source and relay signals in the second phase, several constraints can be considered:

1) source off in the second phase ($P_2 = 0$),
2) equal time-sharing between both phases ($t = 1/2$), ([4], [8], [7] with $t = 2/3$).
3) equal power allocation,
4) uncorrelated source and relay signals in the second phase ($\rho = 0$),
5) the source and the relay transmit the same signal in the second phase ($\rho = 1$) [1].

In Figure 2, we evaluate the relative loss in term of signal-to-noise ratio for several combinations compare to the Shannon limit (p. 2033 in [3]). According to these results, we consider in this paper: optimal time-sharing and power allocation and fully correlated source and relay signals in the second phase ($\rho = 1$). This strategy is almost optimal and requires optimization of only a single LDPC code as shown next.

III. LDPC Code Design for Relay Channel in TD mode

LDPC codes have recently been proposed for Relay Channel in TD mode. In [4], the overall performance is close to the achievable rate. However, they assume equal time-sharing ($t = 1/2$) which is largely suboptimal. In [1], we proposed a capacity-approaching scheme based on LDPC codes but we observe significant performance loss due to the error propagation.

We now describe our distributed coding strategy: Assume that a codeword of length N has been generated by a LDPC encoder defined by the parity-check matrix H. In the first phase, the source transmits all RN information bits, R being the coding rate, and $N_1 - RN$ redundant bits such that the ratio RN/N_1 corresponds exactly to the transmission rate between the source and the relay. The relay decodes this partial codeword and re-encodes it (instantaneously) whereas the destination stores the received data. In the second phase, the relay and the source transmit the remaining $N - N_1$ redundant bits. The destination gathers the N bits received in both phases as a single packet and is able to decode it.

The first N_1 bits of the codeword have to be (perfectly) decoded at the relay, therefore they have to contribute at least in $N_1 - RN$ parity-check equations where none of the remaining redundant bits is involved. This condition of "separation" gives to the parity-check matrix H the following structure:

$$H = \begin{bmatrix} H_{11_{(N_1-RN)\times N_1}} & 0_{(N_1-RN)\times(N-N_1)} \\ H_{21_{(2RN-N_1)\times N_1}} & H_{22_{(2RN-N_1)\times(N-N_1)}} \end{bmatrix} \quad (2)$$

As shown in [6], the degree distributions $\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1}$ and $\rho(x) = \sum_{i=2}^{d_c} \rho_i x^{i-1}$ fully describe the ensemble of random LDPC codes for Gaussian channel. In our case, we define the ensemble of random codes by separating the degree distributions $\lambda(x)$ and $\rho(x)$ into $\lambda_1(x) = \sum_{i=2}^{d_v} \lambda_{i,1} x^{i-1}$, $\lambda_2(x) = \sum_{i=2}^{d_v} \lambda_{i,2} x^{i-1}$ and $\rho_1(x) = \sum_{i=2}^{d_c} \rho_{i,1} x^{i-1}$, $\rho_2(x) = \sum_{i=2}^{d_c} \rho_{i,2} x^{i-1}$ in order to take into account the structure of the parity-check matrix in Equation 2. Then, density evolution analysis [6] is used to optimize the four polynomials $\lambda_1(x)$, $\lambda_2(x)$, $\rho_1(x)$ and $\rho_2(x)$.

Symmetry is an important property associated with the message distribution in the density evolution of belief propagation algorithm to ensure its convergence. The symmetry condition was found in [5] for frequency-selective channel and can similarly be applied to the relay channel. The symmetry property is then used to prove another important property of density evolution, the stability condition:

Theorem 1 (Stability condition): Assume that the noise variance is below a threshold related to the average of the channel log-likelihood ratios. If $RN\lambda_{21}/N_1 + (1 - RN)\lambda_{21}/N_1 < \lambda_2^*$, then the fraction of incorrect messages will converge to zero under density evolution as the number of decoding iterations tends to infinity where λ_2^* is upper bounded

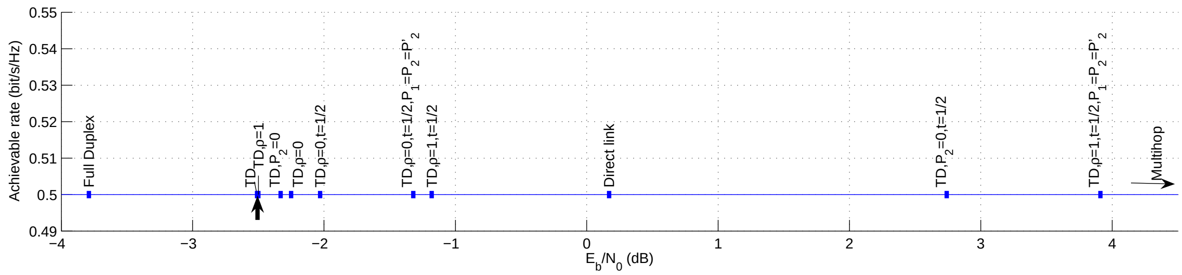

Fig. 2. Minimal E_b/N_0 in dB required for an error-free transmission with binary rate-1/2 coded inputs with several constraints. The arrow indicates the near-optimal scheme that we select in this paper: optimal time-sharing and power allocation and fully correlated source and relay signals in the second phase ($\rho = 1$).

TABLE I

GAP BETWEEN THE THRESHOLD VALUES OBTAINED THROUGH THE DENSITY EVOLUTION ANALYSIS AND THE THEORETICAL MINIMAL SIGNAL-TO-NOISE RATIO $(E_b/N_0)^*$ GIVEN BY THE SHANNON LIMIT.

	Relay node		Destination node	
R	$\frac{E_b}{N_0}$ dB	Gap (dB)	$\frac{E_b}{N_0}$ dB	Gap (dB)
0.42	1.30	0.16	-1.81	0.91
0.47	1.87	0.25	-1.63	0.59
0.53	2.44	0.21	-1.33	0.89
0.59	3.55	0.35	-1.16	0.52

as:

$$\lambda_2^* < \frac{1/\sum_{i=2}^{d_c}\left(\rho_{i,1}(i-1)+\rho_{i,2}(i-1)\right)}{\frac{R}{R_1}\mathrm{e}^{\frac{a^2P_1}{2\sigma^2}}+\left(1-\frac{R}{R_1}\right)\mathrm{e}^{\frac{(a\sqrt{P_1}+c\sqrt{P_2'})^2}{2\sigma^2}}} \qquad (3)$$

In the distributed coding scheme that we propose, we have an additional constraint: the weight of the i-th column of the sub-parity-check equation H_{11} cannot exceed the weight of the i-th column of the parity-check equation H, or equivalently:

$$\begin{cases} \lambda'_{2,1} & \leq & \lambda'_{2,11} \\ \lambda'_{3,1} & \leq & \lambda'_{3,11}+\lambda'_{2,11}-\lambda'_{2,1} \\ \lambda'_{4,1} & \leq & \lambda'_{4,11}+\lambda'_{3,11}-\lambda'_{3,1}+\lambda'_{2,11}-\lambda'_{2,1} \\ & \vdots & \\ \lambda'_{d_v,1} & \leq & \sum_{i=1}^{d_v}\lambda'_{i,11}-\sum_{i=1}^{d_v}\lambda'_{i,1} \end{cases}$$

where the coefficients $\lambda'_{i,j}$ are the coefficients of the degree distribution of the bit nodes from the node perspective [6]. The coefficients $\lambda'_{i,11}$ are the coefficients of the degree distribution related to the submatrix H_{11}.

IV. THRESHOLD CALCULATION AND BIT ERROR RATE RESULTS

In Table I, we show for several rates, the gap between the threshold values of the designed LDPC codes and the theoretical minimal signal-to-noise ratios E_b/N_0 required to achieve the corresponding rates.

In order to reduce the propagation of the errors, it is crucial to have a very low packet error rate at the relay. Therefore, we optimize first the small parity-check matrix H_{11} and then we design the global parity-check matrix H subject to H_{11}. The gap to $(E_b/N_0)^*$ (smaller than 1 dB in all simulated cases) is mainly due the choice of H_{11}. It is possible to test a larger set of matrices H_{11} in order to reduce this gap.

We also simulated the performance of a designed rate-0.53 LDPC code with a block size of 10^5 over relay channel. The relay listens the source during 63% of the time (it maximizes throughput) and transmits simultaneously with the source during the reminded time. The relay node is located at half-distance between the source and the destination nodes. Power allocation is optimal; the average source-destination SNR is equal to -2 dB.

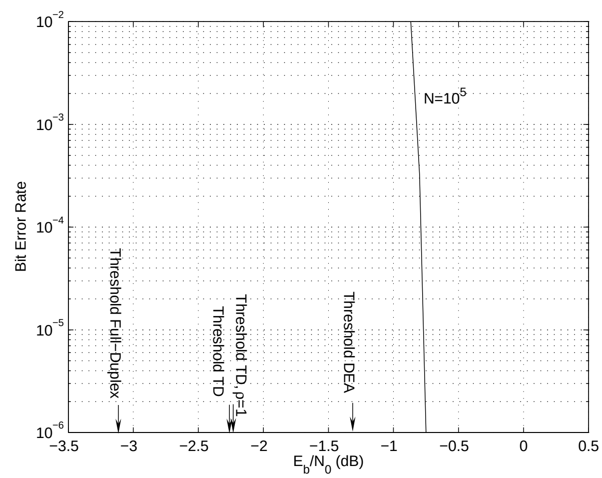

Fig. 3. Bit Error Rate Performance over relay channel in TD mode for the 0.53-rate LDPC code that has been optimized through density evolution analysis. The codeword size is equal to 10^5. The thresholds for full duplex relay channel, half duplex relay channel and half duplex relay channel with full correlation between the source and relay signals in the second phase are also shown for comparison. For a bit error rate of 10^{-6}, our code performance is approximatively 0.5 dB below the threshold and 1.5 dB from the theoretical achievable rate.

V. CONCLUSION

In this paper, we designed LDPC Code for relay channel in time-division mode based on Density Evolution Analysis. The gap between the asymptotic performance of the designed codes and the theoretical achievable rate is less than 1 decibel for a large range of coding rates. For finite length (10^5), the gap between the performance of the LDPC code that we generated and the theoretical limit is smaller than 1.5 decibels.

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