

# Analysis of Decision-Feedback Based Broadband OFDM Systems

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## Technical Areas:

- A. Communications Systems and Networking: 8. OFDM / Multicarrier;
- B. Adaptive Systems and Processing: 1. Adaptive Filtering;
- E. Signal Processing Algorithms and Applications: 1. DSP in Wireless Communications.

**Abstract**—In wireless communications, about 25% of the bandwidth is dedicated to training symbols for channel estimation. By using a semi-blind approach, the training sequence length can be reduced while improving performance. The principle is as follows: the detected symbols (hard decision) are fed back to the channel estimator in order to re-estimate the channel more accurately. However, semi-blind approach can significantly deteriorate the performance if the bit error rate is high. In this paper, we propose to determine analytically the minimum Signal to Noise Ratio (SNR) from which a semi-blind method starts to outperform a training sequence based only system.

## I. INTRODUCTION

<sup>1</sup>Spectral efficiency is a crucial issue in the wireless communications systems such as 4G, IEEE802.11n, 802.16. Approximately 25% of the bandwidth is dedicated to training symbols for channel estimation. Semi-blind (SB) approaches have been proposed in order to reduce the training sequence length ([1],[2],[6]). The principle is as follows: the detected symbols (hard decision) are fed back to the channel estimator in order to re-estimate the channel more accurately. However, semi-blind approach can deteriorate the performance if the bit error rate is high. Therefore it is essential to determine the minimum Signal-to-Noise Ratio (SNR\*) such that the semi-blind approach outperforms a Training Sequence Based Only (TSBO) system for any SNR > SNR\*.

The main difficulty to evaluate SNR\* is the detection stage which is a non linear function. Some authors have approximated the detector function by  $\frac{2}{\pi} \arctan(x)$  [4]. However, this function remains highly non linear and the calculations for the closed-loop are barely intractable.

In this paper, we propose an elegant solution to determine analytically SNR\*: we express the channel re-estimate as a function of the probability of error of the symbols  $P_e$ . Since  $P_e$  can easily be written as a function of the SNR of the system, we are able to determine analytically SNR\*.

## II. SYSTEM DESCRIPTION

We consider a OFDM transmission for quasi-static fading channel. Assuming proper cyclic insertion and sampling, the OFDM system with  $N_c$  subcarriers decouples the frequency-selective channel into  $N_c$  flat-fading channels with the following input-output relation:

$$r_i = h_i s_i + n_i, \quad i = 1, \dots, N_c, \quad (1)$$

where  $h_i$  is a complex channel spectrum coefficient.  $s_i$  and  $r_i$  are, respectively, the  $M$ -QAM transmitted signal and the received signal at the  $i$ -th subcarrier;  $n_i$  is the i.i.d. zero-mean additive noise with a variance  $\sigma^2$ . For each transmitted block of  $N$  OFDM symbols,  $N_1$  symbols are dedicated to the training whereas  $N_2 = N - N_1$  symbols form the payload.

<sup>1</sup>This work was supported in part by NSF under grants ANI-9979465, EIA-0224458, and EIA-0321266.

## III. LINEAR EQUALIZATION BASED ON TRAINING SEQUENCES ONLY

In this section, we recall briefly main results [5] for zero-forcing linear equalization applied to a broadband (MIMO) OFDM system when TSBO approach is used.

The MMSE solution  $\hat{h}_i$  for the channel estimation for each subcarrier  $i$ ,  $i = 1, \dots, N_c$  using a preamble of  $N_1$  OFDM symbols is:

$$\hat{h}_i = \frac{1}{N_1} \sum_{l=1}^{N_1} \frac{1}{\sigma_{s_i}^2} r_i[l] s_i^*[l], \quad (2)$$

and the corresponding Zero-Forcing solution  $\tilde{s}_i[l]$  for the estimated symbol  $l$  for each subcarrier  $i$ ,  $i = 1, \dots, N_c$  is given by:

$$\tilde{s}_i[l] = r_i[l] / \hat{h}_i \quad (3)$$

*Proposition 1:* The probability of error per symbol for each subcarrier is bounded by ([5]):

$$P_e^{TSBO} \leq 2 \operatorname{erfc} \left( \sqrt{\frac{3|h_i|^2}{4\sigma^2(1 + D_M/N_1)(M-1)}} \right) \quad (4)$$

where  $M$  is the modulation order and  $D_M$  is the dispersion of the modulation.

## IV. SEMI-BLIND LINEAR EQUALIZATION

In this section, we determine analytically the performance of the semi-blind channel estimation and equalization. The principle of a SB approach for channel estimation is as follows:

- 1) A short training sequence composed by  $N_1$  symbols, is used at the beginning in order to get a rough channel estimation,
- 2) The first symbol of the payload data is estimated by using the channel estimation of the first step.
- 3) In order to improve the performance, we update the channel estimation by treating the previously detected symbols as training.

$N$  and  $N_2$  denote the total length of a transmitted frame and the length of the payload, respectively:  $N - N_1 = N_2$ . The semi-blind channel estimation  $\hat{h}_i$  per subcarrier  $i$ ,  $i = 1, \dots, N_c$  is:

$$\hat{h}_i = \frac{1}{N_1 + N_2} \left( \sum_{l=1}^{N_1} \frac{1}{\sigma_{s_i}^2} r_i[l] s_i^*[l] + \sum_{l=N_1+1}^{N_1+N_2} \frac{1}{\sigma_{\tilde{s}_i}^2} r_i[l] \tilde{s}_i^*[l] \right) \quad (5)$$

*Proposition 2:* The probability of error per symbol and per subcarrier is bounded by:

$$P_e^{SB} \leq 2 \operatorname{erfc} \left( \sqrt{\frac{3|h_i|^2}{4(\mu_{sb}^2 + \sigma_{sb}^2)(M-1)}} \right) \quad (6)$$

where  $\mu_{sb}$  and  $\sigma_{sb}^2$  are the bias and the variance of the estimation error, respectively. Their analytical expressions are given in Eqs. 7 and 8, respectively. Here,  $P_e^{TSBO}(N_1)$  denotes the probability of symbol error by using TSBO approach with  $N_1$  training samples.  $C_1(M)$  is a constant

$$\mu_{sb} = h \cdot \frac{N_2}{N_1 + N_2} \left[ P_e^{TSBO}(N_1)(-1 + C_1 + C_{21} - C_{22}) + C_{22} \right] \quad (7)$$

$$\sigma_{sb}^2 = \frac{\sigma^2 D_M}{N_1 + N_2} + \frac{|h|^2 N_2}{(N_1 + N_2)^2} \left\{ \left[ P_e^{TSBO}(N_1)(-1 + C_1 + C_{21} - C_{22}) + C_{22} \right] \left[ P_e^{TSBO}(N_1)(-2 + C_1 + C_{21} - C_{22} + D'_M) - 2 \right] \right\} \quad (8)$$

$$C_{21}(M, P_e^{TSBO}(N_1), |h|) = \frac{2D_M \exp(-5d^2|h|^2/4\sigma^2) \sinh(d^2|h|^2/\sigma^2)}{\sqrt{\pi}\sigma^2 [\operatorname{erf}(3d|h|/2\sigma) - \operatorname{erf}(d|h|/2\sigma)]} \quad (9)$$

where  $d = \frac{\sqrt{6}}{(M-1)}$ .

$$C_{22}(M, P_e^{TSBO}(N_1), |h|) = \frac{8[(\sqrt{M} - 2) + \exp(-d^2|h|^2/8\sigma^2) \sinh(d^2|h|^2/8\sigma^2)] \exp(-d^2|h|^2/8\sigma^2) \sinh(d^2|h|^2/8\sigma^2)}{M\sqrt{\pi}\sigma^2 [1 - \operatorname{erf}(d|h|/2\sigma)]} \quad (10)$$

depending only of the modulation order.  $C_{21}(M, P_e^{TSBO}, |h_i|)$  and  $C_{22}(M, P_e^{TSBO}, |h_i|)$  are defined in Eqs. 9 and 10. Proofs will be provided in the full paper.

## V. COMPARISON BETWEEN THE TSBO AND THE SB APPROACHES

Clearly, if the bias in the semi-blind approach is large, then the TSBO approach performs better. However, since the bias and the variance of the semi-blind estimator decreases linearly with the total frame length, better performance is expected for high SNRs.

*Theorem 1:* Assuming that the channel estimation error is Gaussian in both TSBO and SB schemes, SB outperforms TSBO if and only if:

$$\log \sigma_{sb}^2 + \frac{\mu_{sb}}{\sigma_{sb}^2} \geq \log \left( \frac{\sigma^2 D_M}{N_1} \right) \quad (11)$$

Therefore, we are able to determine the threshold  $SNR^* = 1/N_c \sum_i |h_i|^2 / (\sigma^2)^*$  such that below this point, the semi-blind approach outperforms the approach based on training sequence only. The threshold  $(\sigma^2)^*$  is solution of Eq. 11 when equality holds. It can be determined numerically with a fixed-point method.

## VI. SIMULATION RESULTS

To illustrate the performance gain of SB over a TSBO approach, we simulate an OFDM transmission according to the IEEE802.16 standard [3]. Results are shown in Figs. 1 and 2 for 16-QAM and 64-QAM constellations, respectively. Analytical thresholds from Eq. 11 are: For 16-QAM,  $SNR^* = 8.76, 8.29$  and  $8.37$  in dB for  $N_1 = 1, 2, 4$ , respectively; For 64-QAM,  $SNR^* = 14.03dB, 13.21$  and  $12.63$  in dB for  $N_1 = 1, 2, 4$ , respectively. The gap between the theoretical and simulated thresholds (0.5-1 dB) is essentially due to the upper bound of the probability of symbol error which is not tight at low SNR. Clearly, SB outperforms TSBO as soon as SER is small enough for any training sequence length and modulation order.

## VII. CONCLUSION

In this paper, we determined analytically the threshold  $SNR^*$  from which Semi-Blind Approach outperforms Training Sequence Based Only method. Simulations validate our

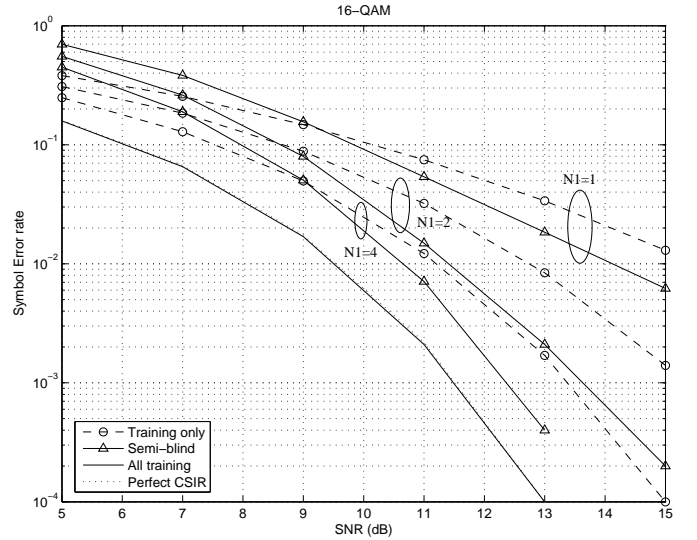


Fig. 1. Comparison between TBO and SB approaches for several training sequences length ( $N_1 = 1, 2, 4$ ) - 16-QAM.

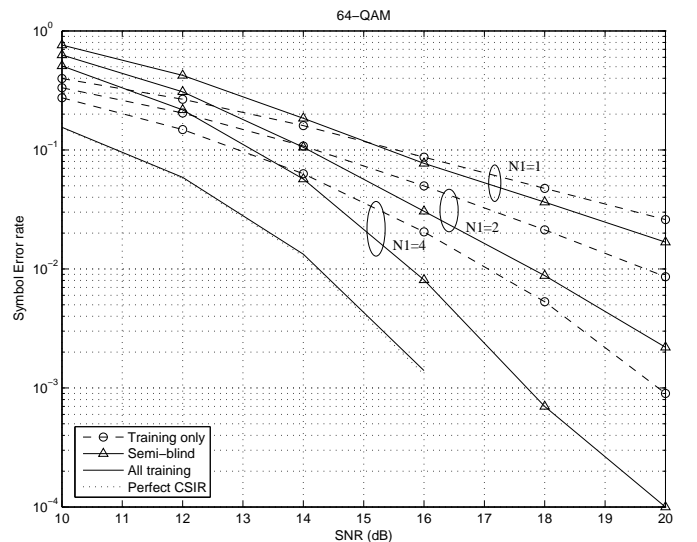


Fig. 2. Comparison between TBO and SB approaches for several training sequences length ( $N_1 = 1, 2, 4$ ) - 64-QAM.

approach. Interestingly, SB outperforms TSBO even for  $P_e$  as large as  $10^{-1}$ . Moreover, this analysis can be extended to any linear processing (synchronization, ...).

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