

Data Collection by a Mobile Observer in a Single-hop Sensor Network

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In this paper, we demonstrate significant reduction in communication power by using a mobile observer to collect data from a wireless single-hop network of static sensors. The key challenge in using a mobile observer is that it remains within range of any sensor for a limited period, and inability to transfer data in this period causes data loss. This prompts us to choose the fraction of nodes successful in communicating data as our performance constraint as we try to minimize network power. First, we establish that the process of data collection by a mobile observer is analogous to a queue. The queuing-based data collection model is then used as a design tool to identify the combination of system parameters that satisfies the performance metric with minimum power. Second, we derive a sufficient condition on node placement for zero data loss in a mobile observer network. As our third contribution, we demonstrate that in a dense network, the data collection analysis is much simpler, which allows us to obtain closed expressions for the network power. Power comparisons with static observer networks show that power reduction by multiple orders of magnitude is typical. The scenarios chosen for power comparisons also provide guidelines on the choice of path, if such a choice is available. As an example, a disc shaped network is analyzed and among the set of concentric circular paths, the best path is identified.

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1. INTRODUCTION

This paper demonstrates significant reduction of communication power by using a mobile observer (data collector) in a single hop WSN (wireless sensor network). One of the biggest challenges of WSNs is to provide realistic throughput over a period of several years using battery-operated sensors. To make things harder, recently discovered scaling laws for energy-limited WSNs [El Gamal 2005; Chakrabarti et al. 2004] have indicated that per-node throughput scales with the number of nodes N

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as $\Theta(\frac{\log N}{N})$, which rapidly vanishes to zero as N increases. The scaling behavior of throughput with N remains unchanged even when sophisticated beamforming techniques are used. Therefore, the success of energy-limited wide-area WSNs calls for non-conventional energy saving solutions. In this direction, we thoroughly investigate the practicality, and quantify the benefits of having a mobile observer, which reduces communication power by travelling to different parts of the network for data collection. Our problem formulation is inspired by ubiquitous public transport vehicles such as buses, trains and elevators.

Our main contributions are as follows. First, we develop a queuing-based model to analyze and evaluate the process of data collection by a mobile observer. Using this model, we quantify the success in data collection in terms of the communication range and data rate. Since the range and the data rate also determine communication power, our model is used as an optimization tool prior to sensor deployment to discover the set of system parameters that will let us collect the desired amount of data with minimum network power.

In a mobile observer WSN, data loss occurs when the observer passes through a region of high local node density, so that not all sensors succeed in sending data to the observer while within range. This data loss is inevitable if sensors are independently distributed over the network. However, not all WSN applications can tolerate data loss. Our second contribution is to find a constraint on minimum sensor separation that guarantees zero data loss with a mobile observer. The condition we derive is a sufficient condition for zero data loss, and although it is not necessary, we show that it is tight.

Our third contribution is an asymptotic analysis of data collection in dense networks. We show that as the number of sensors grows large, the tradeoff between power and data loss can be expressed by closed form expressions, and without the need for queuing-based numerical analysis. Our conclusion from the asymptotic analysis is that in a dense network, almost all sensors that enter within range are able to transfer data successfully, provided that the communication rate equals or exceeds the rate of data collection by the desired fraction of sensors. As a consequence, if we require that the data loss fraction should not exceed F_{max} , then the range need only be large enough for a fraction $(1 - F_{max})$ of sensors to enter within range.

To quantify the gains due to mobility, we study two cases of interest, and in each case we compare a static observer WSN with a mobile observer WSN. In the first case, our calculations indicate that the communication power can be reduced by two orders of magnitude or more in a typical neighborhood WSN. The second case deals with a disk-shaped WSN with the observer moving on a concentric circular path. This case throws light on several practical issues, such as edge effects, and the consequence of a model mismatch between the actual mobility pattern and our mobility model. We derive the optimal path radius for minimizing network power. This example serves as a design guideline in practical situations where there are multiple paths over which the observer may move, and the problem is to choose the one that minimizes network power.

The above idea of saving communication power through the use of predictable mobility in a WSN was originally proposed by the authors in [Chakrabarti et al.

2003]. Independently, the same idea was explored in [Shah et al. 2003]. Although [Chakrabarti et al. 2003] and [Shah et al. 2003] have the same motivation, the emphasis in the former is on analysis of data collection, whereas the latter focusses on network architecture. Related ideas have also been explored in [Rao and Kesidis 2004; Tong et al. 2003; Gandham et al. 2003; Wu et al. 2004]. An important contribution on the use of mobility in ad-hoc networks came in [Grossglauser and Tse 2001], wherein it was shown that mobility can be used to improve transport capacity significantly. The solution in [Grossglauser and Tse 2001] was based on a random mobility model, as contrasted to the deterministic mobility that we assume in this paper. Formulations based on random mobility often suffer from large, and sometimes unbounded latency. In this direction, the feasibility of mobility based solutions that provide improved capacity with low delay guarantees was researched in [Bansal and Liu 2003]. The mobility based solution proposed in this paper is intended for applications that can tolerate large but bounded delays.

The remainder of this paper is organized as follows. In Section 2, we describe our WSN model. We formulate our problem, and pose the important questions to be answered in Section 3. Section 4 studies the tradeoff between power and data loss in our network. Conditions for zero data loss are derived in Section 4.4. Section 5 is devoted to the asymptotic data collection analysis for dense networks. A power comparison between static and mobile observer WSNs is presented in Section 6. Section 7 summarizes our conclusions and discusses future directions.

2. SYSTEM MODEL

Here, we first provide a general description of a WSN, and the roles played by sensors and the observer. This is followed by a description of the observer mobility pattern. Last, we outline the model for single-hop communication between static sensors and the mobile observer.

2.1 Model of a WSN

The WSN consists of N static sensor nodes that are independently and uniformly distributed over a planar surface with area A (see Fig. 1). Sensors collect information and send it to a common mobile observer O . The network is homogeneous, i.e., all sensors are identical. Thus, each node has the same amount of energy (battery), uses the same communication range, and the same data rate D to transmit. Nodes do not perform power control. The above assumptions are usually valid for networks with cheap unsophisticated sensors that can be rapidly deployed.

Each sensor collects useful information at a fixed rate $D_{sens} = \frac{D_{net}}{N}$, where D_{net} is the rate at which the entire network collects information. This assumption is motivated by a bit-conservation principle - data of the same quality can be obtained by a dense network of low-resolution sensors as by a sparse network of high-resolution sensors [Ishwar et al. 2003]. Since sensors collect as well as communicate data at identical rates, it can be concluded that the time T taken to communicate information collected by a sensor in one cycle (this will be defined in Section 2.2) is also fixed. Finally, sensors are energy constrained but the observer is not, since it derives its energy from a mobile vehicle.

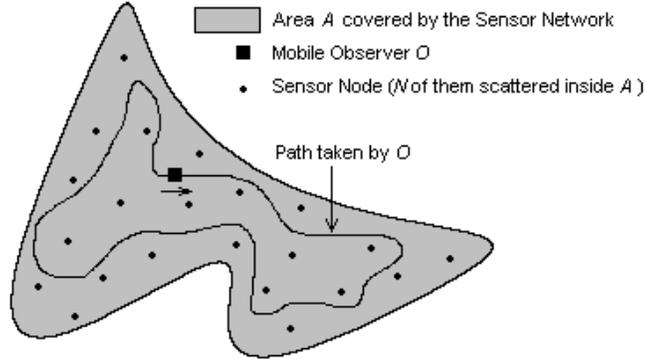


Fig. 1. Diagram of the WSN

2.2 Observer Mobility Model

The observer O repeatedly traverses a deterministic path inside A . This path is fixed and cannot be chosen at will. However, we do assume that the path has the following properties:

- It is continuous.
- It is smooth, meaning that it can be approximated by a straight line over distances of the order of the communication range of a sensor. Stated more precisely, the radius of curvature at each point on the path is large compared to the communication range.
- As the observer moves on its path, it covers a *strip* of width $2R$ within which sensors can communicate with it.¹ This strip contains all points that are within distance R from at least one point on the path of the observer. We require the path of the observer to be such that this strip covers all points in the network, but does not overlap with itself. As a consequence, the total area covered by this strip equals the area of the network.

When the observer, starting from some point and moving on its path, has been within distance R of each sensor once, we say that it has completed a *cycle*. If the path is closed, as in Fig. 2a., the observer returns to its starting point X at the end of each cycle. Otherwise, the observer completes one cycle from X to Y , as shown in Fig. 2b., and in the next cycle, the observer retraces its path from Y to X . The speed of the observer is fixed at v and the time taken to complete one cycle is denoted as T_{cycle} .

The following relation can be derived from the path properties.

$$A = 2RvT_{cycle} \quad (1)$$

The equality

$$TD = T_{cycle}D_{sens}, \quad (2)$$

captures the fact that all the information collected by a sensor at a rate D_{sens} in a period T_{cycle} is transmitted at a rate D in a period T to the observer.

¹Here, R may simply be thought of as the communication range. A precise definition of R is given in Section 2.3.

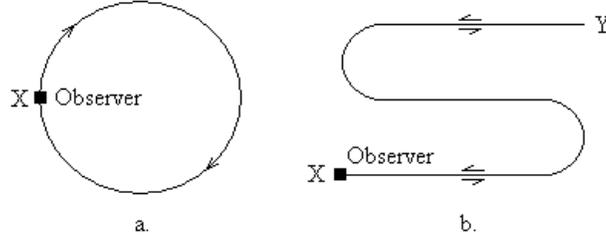


Fig. 2. Motion of the Observer on a. A Closed Path and b. A Path that is not Closed

2.3 Communication Model

We assume single-user single-hop communication over an AWGN channel with bandwidth and noise power spectral density normalized to unity. The channel attenuation exponent is γ . We also assume that communication takes place over continuous time slots that are long enough for the asymptotic capacity theorem for Gaussian channels to be invoked. All channels are assumed to be time-invariant. Thus, from a physical layer perspective, our channel model is

$$Y = \frac{X}{(d)^{\gamma/2}} + N, \quad (3)$$

where Y is the channel output, X is the input, d is the distance between transmitter and receiver, and N is additive Gaussian noise.

Network topology dictates, to an extent, the minimum communication range for each node. For successful data transfer, the sensor and the observer should be within range, and must communicate continuously over a period T . This condition is both necessary and sufficient for data transfer. The above requirement imposes constraints on the communication range. Let $\mathbf{S} = \{S_1, S_2, \dots, S_N\}$ denote the set of sensors as well as their positions. For the observer to be within range of each sensor in the network at some point on its path, the range should be at least

$$R = \sup_i \left(\inf_t \|S_i - O(t)\|_2 \right). \quad (4)$$

where $O(t)$ is the time-varying position of the observer, and $\|\cdot\|_2$ is the Euclidean norm. In other words, if we were to draw a circle of radius R at every point on the path of the observer, then the union of the areas of all these circles would be just sufficient to contain all the sensors. Additionally, the observer must *remain* within range for at least a period T . Taking this into account, the communication range for sensors must be chosen to satisfy²

$$R_m \geq R' = \sqrt{R^2 + (vT/2)^2}. \quad (5)$$

Choosing the communication range to be R' ensures that a sensor will remain in range of the observer long enough to transfer its data. Therefore, (5) is a necessary condition for *all* sensors in the network to be successful in transferring data to the observer. Conversely, (5) is not necessary if data loss can be tolerated.

Interestingly, (5) is not a sufficient condition for successful data collection from all sensors. Due to the independent sensor distribution, at times there may be

²The straight line approximation for the path is implicit here.

too many sensors in range, contending to send data to the observer. If the rate of communication is not sufficiently high, there may not be enough time for all sensors to send their data to the observer, leading to data loss.

Our characterization of data loss is motivated by packet-based communication, where a data packet is either received or dropped. No credit is given for partially received data. Sensors are assumed to have knowledge (acquired through a suitable protocol [Chakrabarti et al. 2003]) that enables them to predict whether the observer will remain in range for the entire data transfer. Therefore, we will assume that all communications are successful, and that data loss is due to sensors that never transmitted because of predictable communication failure.

3. PROBLEM FORMULATION

The average communication power per sensor $P(R_m, D)$ and the average fraction of nodes that fail to send information to the observer $F(R_m, D)$ are both functions of the communication range R_m and the data rate D . The average network power P_{net} is a summation of the average node powers, and can be expressed in terms of P and F as

$$P_{net} = (1 - F)NP. \quad (6)$$

We are faced with the task of designing the communication system for a WSN i.e. of choosing the parameters R_m and D appropriately. The area of the network, the path over which the observer is constrained to move, and the number of sensors are parameters that are specified. We require that the fraction of sensors losing data should not exceed a threshold F_{max} . Our main goal is to minimize P_{net} subject to the above constraints and the design limitations that are a part of our modelling assumptions. This optimization problem can be stated as

$$\min_{F(R_m, D) \leq F_{max}} P_{net}. \quad (7)$$

The following are vital questions that must be answered to meet the above goal, and to evaluate the effectiveness of a mobile observer relative to a static one in a WSN.

- (1) How can we characterize the tradeoff between the data loss fraction and the communication power? The problem is to find the combination of range and data rate that achieves minimum power for a specified data loss fraction.
- (2) Certain WSN applications may not be able to afford data loss. In such cases, how can we deterministically guarantee zero data loss?
- (3) How does the power consumption of mobile observer WSNs compare with that of static observer WSNs?

The rest of the paper will address the above questions.

4. THE POWER - DATA LOSS TRADEOFF

In this section, we derive the functions for power $P(R_m, D)$ and loss fraction $F(R_m, D)$. The dependence of P on R_m and D is well known. To describe F however, we will first need to establish the analogy between the data collection

process and a queuing system, and use this queuing formulation to get our answers. The procedure for determining F will be explained with an example.

The function P is derived as follows. In a cycle of period T_{cycle} , a sensor collects $T_{cycle}D_{sens}$ bits, which are transferred to the observer at a rate D in a period $T_{cycle}D_{sens}/D$. Therefore, a sensor transmits only for a fraction D_{sens}/D of the total cycle time, and is idle at all other times. Hence, P_t , the power during actual transmission, is related to P , the average transmission power over one cycle as

$$P_t = \frac{PD}{D_{sens}}. \quad (8)$$

D , the capacity of the AWGN channel over which sensors communicate with the observer, is given by

$$D = \log_2 \left(1 + \frac{P_t}{R_m^\gamma} \right), \quad (9)$$

we derive that

$$P = \frac{D_{sens}R_m^\gamma(2^D - 1)}{D}. \quad (10)$$

Note that P increases monotonically with both R_m and D . Although we have not shown it yet, F decreases monotonically with R_m and D , and as a consequence, P_{net} is also monotonically increasing in R_m and D .

The dependence of F on R_m and D is hard to model for the following reasons. First, the fraction of unsuccessful nodes F depends on the actual node locations, which are an outcome of a process of random scattering. Averaging over all possible outcomes is challenging. Second, the data-loss fraction depends on the sequence in which the observer communicates with sensors. It seems natural that the value of F should correspond to the sequence that minimizes data loss. However, finding this optimal sequence in our framework is very hard. Third, it is difficult to quantify the influence of R_m . The choice of R_m influences F in two different ways. When the chosen R_m satisfies (5), data loss is only due to insufficiently high data rate and contention among sensors in areas of high local node density. On the other hand, if R_m does not satisfy (5), then there is another component of the data loss in the form of sensors that never enter within range of the observer with the potential to transfer data. To discover how F varies with R_m and D , we will establish that the process of data collection can be mathematically modelled as a single-server queue with deadlines. The queuing analogy will also provide valuable insight into the process.

An important parameter related to F is the fraction of time the observer is busy communicating, which we call f_b . The relation between f_b and F is

$$f_b = \frac{(1 - F)NT}{T_{cycle}}. \quad (11)$$

It follows from (2), (6), (10), and (11) that the average network power can be written in terms of f_b as

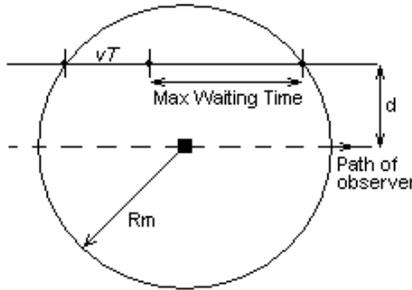
$$P_{net} = f_b R_m^\gamma (2^D - 1). \quad (12)$$

Equations (11) and (12) will be invaluable for analyzing the power-data loss tradeoff in a dense network.

4.1 Data Collection as a Queue

As the observer moves, new sensors come in range, while ones that were in range go out of range. Since sensors are independently distributed, there may be imbalances in local node density leading to contention among sensors in regions of high local node density. If the data rate is insufficient, then there is not enough time for all sensors to communicate with the observer while in range, leading to data loss. Therefore, the process of data collection from a field of independently distributed sensors bears resemblance to a single-server queue with deadlines. Here, we will establish the queuing analogy mathematically.

The event that a new node comes in range is termed an *arrival*. The observer may be busy when a new arrival occurs, in which case, the node must wait in order to send its data. This corresponds to *queuing* of arrivals. When the observer finishes communicating with a node, it immediately starts communicating with another (if there is another node waiting for its turn). If there are multiple arrivals waiting in the queue, the observer chooses one based on some rule, which we call the *queue discipline*. In our framework, it is not useful to start communicating with a sensor node that will not stay within range long enough to transfer all its data, since partial data transfer is treated as failure. For each sensor there is a maximum waiting time or *laxity*, which is a function of its distance from the path of the observer. If the observer does not start communicating with the node before this time, it will be impossible for the sensor to send all its data. This is depicted in Fig. 3. We assume that the observer has accurate knowledge of node deadlines, as if guided by an oracle. In [Chakrabarti et al. 2003], the authors outlined a communication protocol explaining how knowledge of node deadlines can be acquired in practice.



Maximum waiting time for the sensor node shown is

$$\frac{2 \cdot (\sqrt{R_m^2 - d^2}) - vT}{v}$$

Fig. 3. Relation between Waiting Time and Data Loss: The sensor is at a distance d from the path of the observer. If the sensor enters within range at $t = 0$, then the observer must start communicating with the node before $t = \text{Maximum waiting time (laxity)}$, failing which the node will go out of range before it can transfer all its data.

To describe the queue, we need to specify its *arrival process*, the *distribution of deadlines*, the *service mechanism* which provides a statistical description of service times, and the *queue discipline* used to schedule arrivals. We start by introducing our notation. Let t_0 be the time when the observer starts moving. As the observer

moves, sensors enter within range of communication. By convention, we will call the i^{th} sensor to enter within range S_i . Let t_i be the time when S_i enters within range. The interarrival times are denoted by $T_i = t_i - t_{i-1}$. S_i remains within range of the observer for d_i seconds, by which time communication has to be completed to avoid data loss. We will refer to d_i as the *deadline* of S_i . The time taken by S_i to communicate all its information to the observer is denoted by c_i , which is analogous to the *service time* of the queue. For fixed data rate, c_i is directly proportional to the amount of information S_i has, and by our assumption that all sensors have the same amount of data, $c_i = T$ for all sensors. Another quantity of interest to us is the *laxity* $l_i = d_i - c_i$, which is the time by which communication must have *begun*.³

The characteristics of our queue are described as follows.

4.1.1 Arrival Process. In a time interval of length t , the observer moves a distance vt . If R_m satisfies (5), then nodes within an area $2Rvt$ enter within range with the potential to transfer data.⁴ The arrival process of the queue is derived through the following lemma.

LEMMA 4.1. *If the communication range R_m satisfies (5), then sensors enter within range of the observer with the following interarrival PDF,*

$$p_{T1}(t) = \frac{2RvN(A - 2Rvt)^{N-1}}{A^N}; \quad 0 \leq t \leq \frac{A}{2Rv}. \quad (13)$$

Proof: In a time period t , the observer travels a distance vt . Nodes within an area $2Rvt$ that were previously out of range now come within range. Let $p_{T1}(t)$ be the PDF of interarrival times. Then ⁵,

$$\int_0^t p_{T1}(x)dx = \text{Probability that at least one node arrives in time } t \quad (14)$$

$$= 1 - (\text{Probability that no node arrives in time } t) \quad (15)$$

$$= 1 - \left(\frac{A - 2Rvt}{A}\right)^N. \quad (16)$$

Taking the derivative on both sides with respect to t yields (13), which is the PDF of interarrival times. \square

COROLLARY 4.2. *If the communication range R_m does not satisfy (5), then sensors enter within range of the observer with the following interarrival PDF,*

$$p_{T2}(t) = \frac{2vN\sqrt{R_m^2 - (\frac{vT}{2})^2}\left(A - 2vt\sqrt{R_m^2 - (\frac{vT}{2})^2}\right)^{N-1}}{A^N}; \quad 0 \leq t \leq \frac{A}{2Rv}. \quad (17)$$

Proof: Same as in Lemma 4.1, with R replaced by $\sqrt{R_m^2 - (vT/2)^2}$. \square

³The terms *laxity* and *deadline* have been borrowed from the processor scheduling literature.

⁴If (5) is satisfied with strict inequality, then some nodes may be encountered more than once in a cycle. In that case, the observer will only acknowledge the arrival with the longest deadline.

⁵This follows from the fact that nodes are independently and uniformly distributed and the probability for any single node to be outside the area $2Rvt$ equals $(A - 2Rvt)/A$. Note that the time t cannot exceed $(A/2Rv)$ because $(A/2Rv) = T_{cycle}$ from (1).

Obtaining the arrival process is necessary for describing the queue. However, the PDF of interarrival times, as given by Lemma 4.1, is rarely encountered in queuing theory, which limits its usefulness. Interestingly, the PDF of interarrival times can be approximated by an exponential PDF. Such an approximation is motivated by the observation that the PDF of interarrival times is memoryless for small t , and it is known that the exponential PDF is the only continuous PDF with the memoryless property. We approximate the interarrival PDF $p_{T_1}(t)$ with

$$p_{T_1}^*(t) = \frac{2RvN}{A} \exp\left(-\frac{2RvN}{A}t\right); \quad 0 \leq t \quad (18)$$

and we approximate $p_{T_2}(t)$ with

$$p_{T_2}^*(t) = \frac{2vN\sqrt{R_m^2 - (\frac{vT}{2})^2}}{A} \exp\left(-\frac{2vN\sqrt{R_m^2 - (\frac{vT}{2})^2}}{A}t\right); \quad 0 \leq t. \quad (19)$$

The above approximation is accurate for small t and large N , which becomes evident when we compare the Taylor series expansions of the exact and approximate distributions. Note that the approximate interarrival distribution has an infinite support region, whereas the exact distribution has finite support. The difference, however, is negligible unless the number of nodes is very small. As a consequence of (18) and (19), the arrival process can be treated as a Poisson process.

4.1.2 Distribution of Deadlines. The distribution of deadlines d_i is derived in the following lemma.

LEMMA 4.3. *If R_m satisfies (5), the PDF of service deadlines d_i is*

$$p_{D1}(d) = \frac{dv^2}{4R\sqrt{R_m^2 - (vd/2)^2}}; \quad \frac{2\sqrt{R_m^2 - R^2}}{v} \leq d \leq \frac{2R_m}{v}. \quad (20)$$

Proof: The PDF of deadlines may be derived as follows. The deadline for a sensor node at a distance r from the path of the observer is

$$d(r) = \frac{2\sqrt{R_m^2 - r^2}}{v}. \quad (21)$$

Note that r is uniformly distributed from 0 to R as a consequence of the independent and uniform spatial distribution of sensors, and the assumption that the path can be approximated by a straight line over short distances. Hence, the PDF of waiting times may be obtained by transforming this uniform PDF using (21). \square

COROLLARY 4.4. *If R_m satisfies (5), the PDF of sensor laxity l_i is*

$$p_{L1}(d) = \frac{(d+T)v^2}{4R\sqrt{R_m^2 - (v(d+T)/2)^2}}; \quad \frac{2\sqrt{R_m^2 - R^2}}{v} - T \leq d \leq \frac{2R_m}{v} - T. \quad (22)$$

Proof: $l_i = d_i - T$. \square

COROLLARY 4.5. *If R_m does not satisfy (5), the PDF of service deadlines d_i is*

$$p_{D2}(d) = \frac{dv^2}{4\sqrt{R_m^2 - (vT/2)^2}\sqrt{R_m^2 - (vd/2)^2}}; \quad T \leq d \leq \frac{2R_m}{v}. \quad (23)$$

and the PDF of sensor laxity l_i is

$$p_{L2}(d) = \frac{(d+T)v^2}{4\sqrt{R_m^2 - (vT/2)^2}\sqrt{R_m^2 - (v(d+T)/2)^2}}; \quad 0 \leq d \leq \frac{2R_m}{v} - T. \quad (24)$$

Proof: The proof is as earlier with R replaced by $\sqrt{R_m^2 - (vT/2)^2}$. \square

4.1.3 Service Mechanism. The distribution of service times depends on the distribution of data among sensors. Since sensors collect equal amounts of data, service times are also equal.

We will consider only *non-preemptive* and *non-idling* service mechanisms in the context of our queue. In a non-preemptive service mechanism, a task that has begun must reach completion before another task can start. In situations where task-switching has little or no associated cost, pre-emption can sometimes improve performance. In a wireless network, preemption would carry a protocol overhead, making it undesirable.

An idling policy allows the server to remain idle in anticipation of arrivals with short deadlines even when the queue is not empty. We will consider only non-idling policies in this paper because, as we will see later, our queuing system will be working under overloaded conditions, where idling will carry no advantage.

4.1.4 Queue Discipline. Unlike the arrival process and the distribution of deadlines, the queue discipline is not fixed by the network parameters. The observer can freely choose the rule by which it will serve arrivals, and must make the best choice to minimize data loss. In our framework, the observer traces the same path repeatedly in a field of static sensors. Consequently, after the first cycle, the sequence of arrivals is known. Therefore, the observer can act as a clairvoyant scheduler and find the schedule that minimizes data loss.

A clarification may be necessary at this point. The determinism described above should not be misinterpreted to mean that the queuing formulation is unnecessary. At the design stage, node locations are not known. Sensors must be designed to meet performance criteria in the absence of such knowledge. The queuing formulation acts as a useful tool that predicts network behavior on average without knowledge of node locations.

In summary, we see that the process of data collection can be modelled as an M/D/1 queue with deadlines. We are not aware of a general expression characterizing the data loss fraction for this queue. However, such an expression can be found if the network is dense ($N \rightarrow \infty$). As a consequence of this queuing formulation, data collection can be analyzed independent of quantities that are hard to characterize, such as the shape of the network and the path of the observer. It is interesting to note that all the information that we need about these quantities is contained in the parameter R defined in (4). Therefore, the queuing formulation reduces a complex network power optimization problem to a simpler, tractable, and more familiar one. We will also see in Section 5 that asymptotic analysis using the queuing formulation gives us analytical expressions for data loss in a dense WSN.

We conclude our discussion of the queuing analogy here. We are now in a position to discuss power minimization using the queue.

4.2 Power Minimization

Our goal is to find the pair (R_m^*, D^*) that achieves data loss below a specified fraction F_{max} with the minimum P .⁶ In other words, we are trying to solve a constrained minimization problem with R_m and D as our variables, an upper bound on $F(R_m, D)$ as the constraint, and $P(R_m, D)$ as our objective function. The value of F can be estimated from the average fraction of arrivals dropped by the queue. In the absence of an analytical expression for $F(R_m, D)$, the power optimization procedure will involve identifying pairs (R_m, D) for which the data loss fraction is below F_{max} , followed by choosing the pair among these that minimizes P . A procedure to find the minimum power solution is provided below, and it is used in an example in Section 4.3.

- (1) $D \leftarrow (1 - F_{max})D_{net}$
- (2) $R_m \leftarrow \sqrt{((1 - F_{max})R)^2 + (vT_{cycle}D_{sens}/2D)^2}$
- (3) **until** $(F(R_m, D))^7 < F_{max}$
 increment R_m **in small steps**.
- (4) **calculate** $P_{net}(R_m, D)$.
 if (a power minimum has been reached⁸)
 then (output the current values of D , R_m , and $P_{net}(R_m, D)$)
 else (increment D by a small amount δ and go back to step 2).

The initial values of D and R_m in the above procedure correspond to the following lower bounds

$$D \geq (1 - F_{max})D_{net}, \quad (25)$$

$$R_m \geq \sqrt{((1 - F_{max})R)^2 + (vT/2)^2} \geq (1 - F_{max})R, \quad (26)$$

which are proved in Section 5.

4.3 Power Minimization - A Numeric Example

We now present an example of network power minimization.⁹ Table 1 lists the parameters used in this example.

The queue is simulated for the parameters in Table 4.3. Fig. 4 is a contour plot showing how data loss depends on R_m and T (equivalently D) in this network. The tradeoff between range R_m and data rate D is evident from this plot. A long range compensates for low data rate (large T), and similarly high data rate (small T) compensates for short range, to reach the same value of F in each case. Also note that this tradeoff is effective only within the limits specified by (25) and (26). No range, however large, can attain a given value of F_{max} , if D does not

⁶It can be seen from (6) that minimizing P is the same as minimizing P_{net} when F is fixed.

⁷A queuing simulation is necessary to determine $F(R_m, D)$ for each value of R_m .

⁸At minimum, the power corresponding to D is less than those corresponding to $D + \delta$ and $D - \delta$.

⁹Our purpose here is not only to explain how the power minimization procedure works, but to give a comprehensive idea of the queuing performance. Therefore, we also simulate the queue for parameters that may be ignored when the sole purpose is power minimization.

Table I. List of System Parameters

Parameter	Value
F_{max} = Maximum allowable data loss fraction	0.2
γ = Path attenuation of wireless channel	2,3,4,5,6
R = Network Parameter related to range	100 m
A = Area covered by the WSN	4 sq km
v = Observer speed	10 m/sec
N = Number of nodes	100
B = Bandwidth of the system	100 kHz
N_0 = Noise power spectral density	10^{-19} Watt/Hz
D_{sens} = Rate at which a sensor collects data	1 kbps
D_{net} = Rate at which the network collects data	ND_{sens}
T_{cycle} = Time needed to complete one cycle	$A/(2Rv)$
D_{cycle} = Data collected by a node in one cycle	$D_{sens} * T_{cycle}$

satisfy (25). Similarly, to attain a given F_{max} , R_m must satisfy (26). These bounds corresponding to $F_{max} = 0.2$ are marked in Fig. 4.

A point worth mentioning is that for accurate estimates of F , the simulation should be carried on over a period corresponding to several cycles. In this example, the simulation covered a period of 10 cycles.

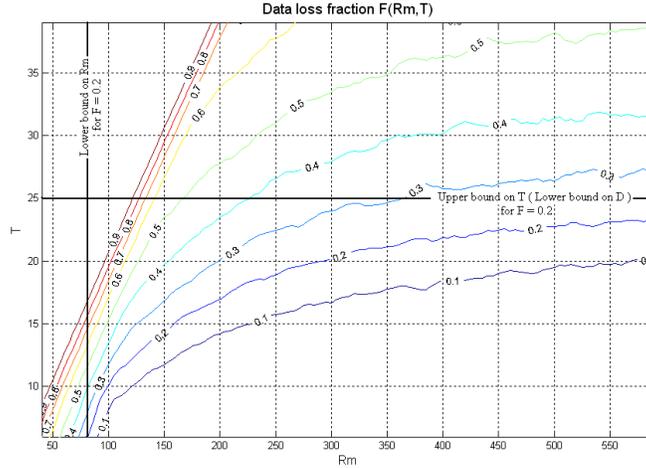


Fig. 4. A contour plot showing the data loss pattern for different choices of R_m and T

Next, we calculate P_{net} for pairs (R_m, D) that achieve $F \leq 0.2$. This corresponds to points on or below the contour for $F = 0.2$ in Fig. 4. Fig. 5 shows P_{net} as a function of R_m for different values of the channel attenuation γ . It is implicit that for each R_m , the smallest value of D necessary to ensure $F \leq 0.2$ is chosen. It can be seen that the network power attains its minimum at a value of R_m which is higher than the asymptotic lower bound of 80m calculated from (26). Finally, note that γ does not seem to affect the power minimum significantly.

We will prove in Section 5, that in a dense network, the power minimum is attained when D and R_m are both equal to their respective lower bounds. But this is clearly not true in a sparse network, as seen from Fig. 6 and Fig. 7. The plots

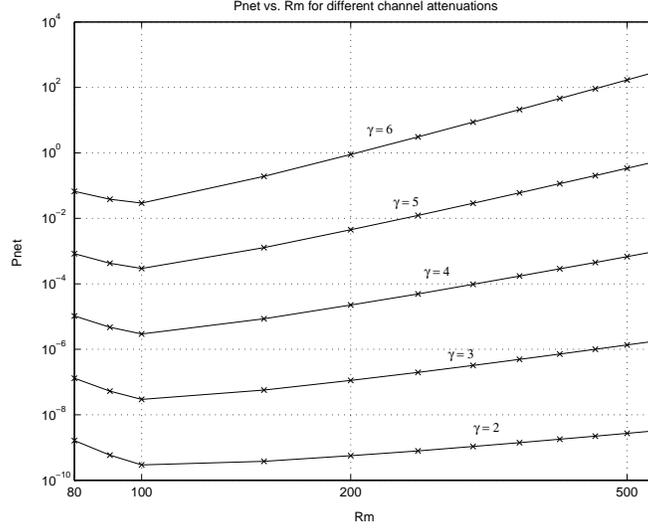


Fig. 5. Variation of network power with R_m for different channel attenuations.

show the values of R_m and D which correspond to minimum P_{net} for each of four different values of N . The network parameters (except N) are same as in Table 4.3. In a sparse network with 30 nodes in a 4 sq. km. area, the minimum power range is 150m, which is nearly double the asymptotic lower bound of $(1 - F_{max})R = 80$ m, and the corresponding data rate is nearly four times its asymptotic lower bound of $(1 - F_{max})D_{net} = 80$ kbps. We also see that the minimum power range tends to the asymptotic lower bound of 80m, and the corresponding data rate becomes close to 80 kbps as N becomes large. As $N \rightarrow \infty$, the minimum power range and data rate would actually equal the respective lower bounds, but the fact that we are not far from the lower bound even with $N = 300$ is of empirical significance. It indicates that asymptotic results can give fair estimates of the optimum range and data rate at realistic node densities. Also note from Fig. 6 that for large communication range, the network power remains same irrespective of N . The fact that data is being received from nodes in discrete bursts is no longer significant with the smoothing effect of a large communication range.

4.4 Communication without Data Loss

Our discussion until now has been concentrated on the tradeoff between data loss and power. But, certain applications may not tolerate data loss at all. For such applications, it is not sufficient to provide probabilistic guarantees on data loss. Rather, we are interested in deriving conditions that can deterministically ensure that there will be no data loss. It is possible to ensure zero data loss by increasing the communication range or the data rate to large values. However, such solutions have little or no advantage over stationary observer sensor networks in terms of power, which limits their usefulness.

As we have discussed, the independent distribution of sensors can lead to high local node density, and cause data loss. This suggests that if a minimum separation between sensors can be ensured (not an independent distribution any more), then zero data loss is a possibility. This is indeed true, and the result is quantified in

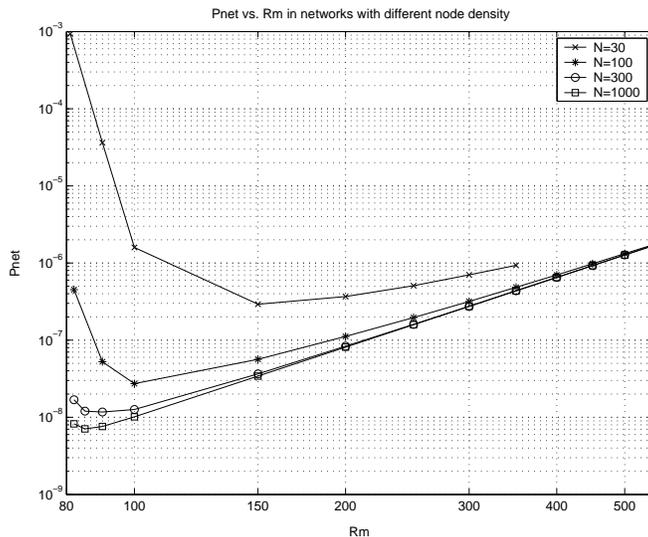


Fig. 6. Network power vs. R_m for different node densities.

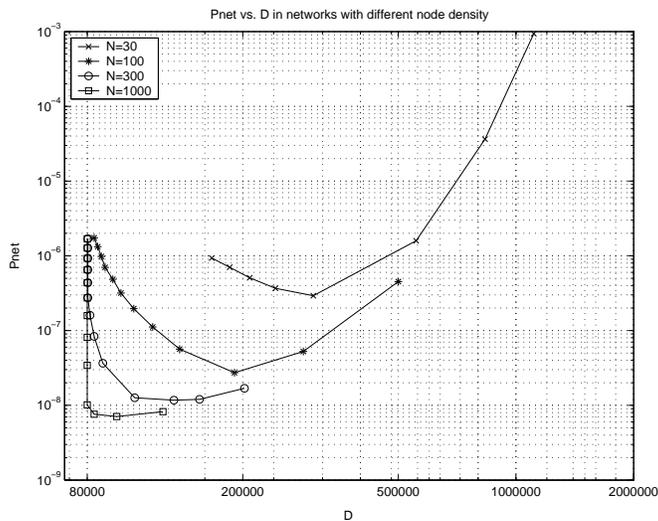


Fig. 7. Network power vs. D for different node densities.

the following theorem.

THEOREM 4.6. *No data is lost on account of observer mobility if the communication range R_m satisfies (5), and the minimum distance of separation d between all pairs of nodes obeys the condition,*

$$d \geq \sqrt{(2R)^2 + (vT)^2} . \tag{27}$$

Proof: See Appendix A. □

The above theorem applies to situations where some amount of control can be exercised on the positioning of sensors, or alternatively, if data from sensors that

are too close to each other is redundant enough to be ignored.

Recall from Section 2.3 that (5) is a necessary but not sufficient condition for ensuring zero data loss. Condition (27) is sufficient for preventing data loss, but unlike (5), it is not necessary. It is possible that (27) is violated but the path of the observer is such that no data is lost. However, (27) is tight in the sense that if (5) is a strict equality, then for every $d' < d$, it is possible to arrange sensors and to choose a path such that data loss is inevitable. An example of this is provided in Appendix A.

5. ASYMPTOTIC ANALYSIS OF POWER - DATA LOSS TRADEOFF

In this section, we will derive expressions characterizing the tradeoff between power and data loss in the case of a dense network ($N \rightarrow \infty$). As $N \rightarrow \infty$, individual sensors collect information at a rate $D_{sens} = D_{net}/N$, which goes to 0, as does the time T taken by a sensor to transfer its information to the observer. Our goal, as before, is to find the pair (R_m^*, D^*) that minimizes P while limiting data loss below a specified fraction $F_{max} > 0$. In this direction, we present the following theorem.

THEOREM 5.1. *As $N \rightarrow \infty$, for any given $F_{max} > 0$, the pair (R_m^*, D^*) that minimizes $P(R_m, D)$ is,*

$$\lim_{N \rightarrow \infty} R_m^* = \sqrt{((1 - F_{max})R)^2 + (vT/2)^2} \simeq (1 - F_{max})R, \quad (28)$$

$$\lim_{N \rightarrow \infty} D^* = (1 - F_{max})D_{net}. \quad (29)$$

Proof: See Appendix B. □

COROLLARY 5.2.

$$\lim_{N \rightarrow \infty} (1 - F_{max})NT = T_{cycle}. \quad (30)$$

Proof: Using (11), and Lemma B.1. □

□

COROLLARY 5.3. *The average network power in a dense network is*

$$P_{net} = ((1 - F_{max})R)^\gamma (2^{(1 - F_{max})D_{net}} - 1). \quad (31)$$

Proof: Using (10), (6), and Theorem 5.1. □

Thus, we see that in a dense network, the network power can be expressed analytically in terms of the system parameters R and D_{net} and the specified data loss fraction F_{max} .

Theorem 5.1 is a consequence of the law of large numbers. In a dense network, even minor variations in local node density become extremely improbable. As a result, sensor arrivals at the observer become almost uniform, and the observer can serve all sensors in range with high probability if the sensors have non-zero laxity. Note from the proof that the exact nature of the distribution of deadlines is irrelevant in a dense network, so long as nodes have non-zero laxity with high probability.

6. POWER COMPARISON WITH STATIC WSNS

The power consumption in a mobile observer WSN depends on the area and the shape of the WSN, as well as the path of the observer. The influence of these quantities manifests through the parameters A and R . Power savings due to mobility depend on these parameters. In this section, we illustrate the impact of mobility in two representative scenarios, by comparing a static WSN with a mobile observer WSN in each case. Analyzing the first case gives us an appreciation of the amount of savings in communication power that one might expect in a typical neighborhood WSN. The second case demonstrates the role played by the parameter R in power savings, and also throws light on issues of practical importance such as the influence of network edges, and factors that would govern the choice of path, if a choice existed.

The networks are assumed to be dense in each case. We will consider networks shaped like planar discs. In the case of a static observer, we will assume that the observer is positioned at the center of the disc. For fair comparison, we will require that in both the static and mobile observer networks, only a fraction $(1 - F)$ of the sensors send data to the observer, and that each sensor uses the same range and data rate.¹⁰

6.1 Case 1: A Neighborhood WSN

The expression for network power in a dense mobile observer network is given by (31), which becomes,

$$P_{mobile} = BN_0((1 - F_{max})R)^\gamma(2^{(1-F_{max})ND_{sens}/B} - 1), \quad (32)$$

on taking into account the bandwidth B and the noise power spectral density N_0 .

Now, suppose that the same WSN has a centrally placed static observer. In order for a fraction $(1 - F_{max})$ of nodes to be within range, the range must equal $\sqrt{1 - F_{max}}$ times the radius of the network. The rate of communication equals the rate of data collection by $(1 - F_{max})N$ sensors, which is $(1 - F_{max})ND_{sens}$. From these observations, we conclude that each sensor uses the same power, which can be calculated from the range and data rate in the usual manner. The network power, which is a summation of individual node powers, is therefore,

$$P_{static} = BN_0 \left(\sqrt{\frac{(1 - F_{max})A}{\pi}} \right)^\gamma (2^{(1-F_{max})ND_{sens}/B} - 1) \quad (33)$$

for the static observer scenario.

From (32) and (33), we see that,

$$\frac{P_{mobile}}{P_{static}} = \left(\frac{\sqrt{\pi(1 - F_{max})} R}{\sqrt{A}} \right)^\gamma. \quad (34)$$

¹⁰This requirement may seem unfair because the network power can be reduced if sensors that are nearer to the observer use less power. However, note that the network power is dominated by the power consumption of the farthest sensors, therefore the minimum network power achievable will differ from our calculations only by a small constant factor.

Thus, for a fixed network area A , and a specified data loss fraction F_{max} , the ratio of power consumptions depends only on R . The distance R is a measure of the extent to which the path reaches different parts of the network. A dense path (such as a space filling curve) will lead to a small value of R , and therefore significant power savings.

If a typical neighborhood WSN has an area $A = \pi(1)^2$ sq km, and the observer makes use of a local vehicle that carries it to within $R = 0.1$ km of each sensor, then from (34) we can see that the network communication power is reduced by at least two orders of magnitude (since $\gamma \geq 2$). This comes as no surprise since the required communication range is reduced by a factor of ten. Even from this rough estimate, it is clear that in wide area WSNs, where the communication power constitutes a large percentage of the total power, mobility can aid in reducing communication power considerably.

6.2 Case 2: Disk Network with Circular Closed Path

This example evaluates power savings in a disk shaped network with a circular path that is concentric with the network. We will study how power savings change with the radius of the path. Apart for that, this example will also throw light on edge effects and the consequences of a mobility model mismatch.

We begin by examining the path itself. A circular path in a circular network may not fit our mobility model in several situations. First, if the radius is small, it will be inaccurate to treat the path as a straight line over distances of the order of the communication range. Second, equation (1) will be valid only if the path is of exactly half the radius as the network. Third, a path with large radius (nearly equal to the network radius) will require a large range to collect data from nodes near the center of the network. This range will be large enough to collect data from points outside the network, or in other words, the network edge will come into picture. But, as we will see, even though the path may not fit our mobility model, it can still save power irrespective of its radius. First, note that the arrival rate will be uniform due to the symmetry of the path, and because the network is dense. Second, since the network is dense, $T \rightarrow 0$, and therefore almost all nodes in range will remain in range long enough to receive service. In a dense network, the data rate is given by (29). The choice of communication range is dictated by the requirement that a fraction $(1 - F_{max})$ of sensors remain within range. This boils down to ensuring that the set of points that are within range R_m from the path of radius R_{path} constitute an area $(1 - F_{max})A$ (see Fig. 8). From the above criterion, the minimum R_m necessary for $(1 - F_{max})N$ nodes to be in range is found to be,

$$R_m = \begin{cases} \sqrt{1 - F_{max}} R_{net} - R_{path} & 0 \leq R_{path} \leq \frac{\sqrt{1 - F_{max}} R_{net}}{2} \\ \frac{(1 - F_{max})R_{net}^2}{4R_{path}} & \frac{\sqrt{1 - F_{max}} R_{net}}{2} < R_{path} < \frac{(1 + \sqrt{F_{max}}) R_{net}}{2} \\ R_{path} - \sqrt{F_{max}} R_{net} & \frac{(1 + \sqrt{F_{max}}) R_{net}}{2} \leq R_{path} \leq R_{net} \end{cases}$$

Using the above, we plot the mobile observer network power (normalized with the static observer network power) corresponding to $F_{max} = 0.2$ for different R_{path} and γ in Fig. 9. The plot reiterates that power savings depend on R_{path} . As R_{path} increases from zero, the range R_m necessary to cover the same area decreases, thereby saving power. Power savings are maximized when $R_{path} = \frac{(1 + \sqrt{F_{max}}) R_{net}}{2}$.

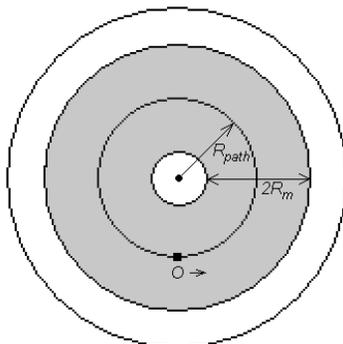


Fig. 8. A disk network with a circular path of radius R_{path} . Only sensors in the shaded region are able to communicate with the observer O .

For larger values of R_{path} , edge effects degrade performance. By edge effect, it is implied that the edge of the network is inside communication range of the observer. When this happens, the observer stops receiving data from some areas that are within range (but outside the network). In other words, a longer range is now necessary to receive data from the same number of sensors, leading to increased power consumption.

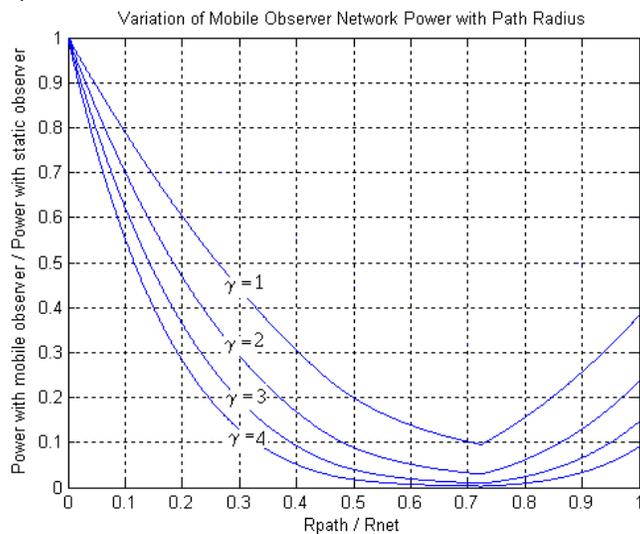


Fig. 9. Role of R in determining power savings

7. CONCLUSIONS AND FUTURE WORK

We have demonstrated the feasibility of using a mobile observer to save communication power in a WSN. Not only does the use of a mobile observer save overall network power, it also equates the power consumption at different nodes, thereby ensuring that the network does not become inefficient through localized node failures.

There are certain challenges in using a mobile observer that need to be studied further. We have explored the case of a mobile observer that moves on a predictable route. Extensions to partially predictable mobility patterns need to be explored.

Other extensions include: having nodes with correlated data, or which collect information at different rates; having multiple mobile observers; using heterogeneous networks where not all nodes are power-limited; and using a combination of mobility and multi-hop communication. Work remains to be done in exploring these and other related extensions of this problem.

APPENDIX

A. CONDITION FOR GUARANTEEING ZERO DATA LOSS

The sufficient condition for ensuring zero data loss with minimum separation of nodes d is derived as follows (see Fig. A(a)). In this proof, we will assume that (5) is satisfied with equality. It is obvious that the proof will also be valid for larger communication ranges (if (5) is a strict inequality). Data transfer from a node to the observer takes time T . If we can ensure that the entry of two nodes into the range of the observer is spaced in time by T , then data loss will not occur. Thus, if a sensor node is placed anywhere on the arc \widehat{AB} (see Fig. A(a)), then no other node can lie within the shaded region $ABCD$. This implies that the minimum separation d must be greater than or equal to the distance from any point on arc \widehat{AB} to any other point on the boundary of $ABCD$ i.e.,

$$d \geq \max(|x - y|); \quad x \in \widehat{AB}, y \in ABCD . \quad (35)$$

Let $x = U$ and $y = V$ be the pair for which $|x - y|$ is maximized.

THEOREM A.1. *$U = A$ and $V = C$ (or $U = B$ and $V = D$) are the pair that maximizes $|x - y|$.*

Proof: The result follows from a series of claims. The claims themselves are almost trivial to prove, therefore for brevity we will omit their proofs here.

CLAIM A.2. *Choose and fix an arbitrary point $x = P \in \widehat{AB}$. Then, the point $y = Q \in ABCD$, for which $|P - y|$ is maximized does not lie on the line segments BC or AD except possibly that Q is one of the end points A, B, C or D .*

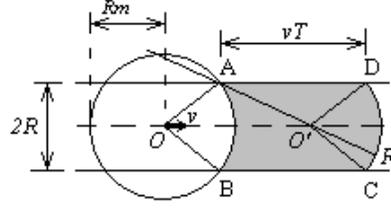
COROLLARY A.3. *Since this claim holds for arbitrary x , it must hold for $x = U$, the point which achieves the overall maximum. Therefore, $V \in \{\widehat{AB}, \widehat{CD}\}$.*

CLAIM A.4. *Choose and fix an arbitrary point $x = P \in \widehat{AB}$. Suppose that the point $y = Q \in ABCD$ is the point for which $|P - y|$ is maximized. Then, unless P is one of the end points of \widehat{AB} , $|P - Q| < |U - V|$. In other words, $U \in \{A, B\}$.*

COROLLARY A.5. *V lies on \widehat{CD} for if V were to lie on \widehat{AB} , by moving horizontally to the corresponding point on \widehat{CD} , one could show that this point is farther from U than V is.*

The problem is therefore reduced to that of finding V from the set of points on \widehat{CD} . U has been ascertained to be either A or B (which one we choose makes no difference). Consider the line passing through A and O' . This line cuts the arc \widehat{CD} if and only if,

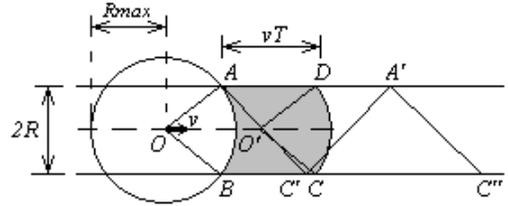
$$(2R)^2 + (vT)^2 > (2R_m)^2, \quad (36)$$



$AOO'D$ and $BOO'C$ are parallelograms

The observer moves along the line OO' with a velocity v . The circle denotes its range.

(a)



$$d < \sqrt{(2R)^2 + (vT)^2}$$

$$d = |A-C| = |C'-A'| = |A'-C''| \dots$$

Nodes are positioned at A , C' , A' , C'' and so on. Nodes enter the range of the observer at a rate such that the time in between arrivals is less than T and outage is inevitable after some time.

(b)

which contradicts (5). The case of interest is when the line passing through A and O' does not cut \widehat{CD} . This happens when A lies within the circle centered at O' and having radius R_{\max} . We prove the following claim for this situation.

CLAIM A.6. *When the line passing through A and O' does not cut \widehat{CD} ,*

$$|A - C| = \max(|x - y|); \quad x \in AB, y \in ABCD. \quad (37)$$

This also proves our original claim that $U = A$ and $V = C$ (or $U = B$ and $V = D$) are the pair that maximizes $|x - y|$. \square

This is the result using which we obtain a meaningful relationship between the minimum separation d and our system parameters. Since,

$$|A - C| = |B - D| = \sqrt{(2R)^2 + (vT)^2}, \quad (38)$$

we conclude that

$$d \geq \sqrt{(2R)^2 + (vT)^2} \quad (39)$$

is a sufficient condition to guarantee zero data loss.

The above condition is not necessary to avoid data loss, meaning that it may be violated but data may not be lost. However, an interesting point to note is that if this condition is not satisfied, then there exist bad arrangements of sensors that lead to data loss. Fig. A(b) shows one such arrangement of sensors where data loss is unavoidable with

$$d < \sqrt{(2R)^2 + (vT)^2}. \quad (40)$$

B. PROOF OF THEOREM 5.1

The following is a sketch of the proof. We know that power is monotonically increasing with R_m as well as D . We will prove this theorem by showing that

- (1) Data loss exceeds F_{max} if either $R_m < R_m^*$ or $D < D^*$.
- (2) Data loss does not exceed F_{max} if $(R_m, D) = (R_{m\epsilon}^*, D^*)$, where¹¹

$$R_{m\epsilon}^* = \sqrt{((1 - F_{max})R(1 + \epsilon))^2 + (vT/2)^2}, \quad (41)$$

for any given $\epsilon > 0$ and for sufficiently large N .

It is easy to see that the data rate D must equal or exceed D^* for data loss to remain below F_{max} . From (2) and (11), we have,

$$D = \frac{(1 - F)D_{net}}{f_b} \geq (1 - F)D_{net} \geq (1 - F_{max})D_{net} = D^*. \quad (42)$$

Intuitively, (42) means that the rate D at which data is transferred must equal or exceed the rate at which it is collected by the requisite fraction $1 - F_{max}$ of sensors.

It is also easy to see that R_m cannot be less than R_m^* for the data loss fraction to be less than F_{max} . Assume for a contradiction that $R_m < R_m^*$. Then, since R_m does not satisfy (5), from (19) and (1) the arrivals at the observer are Poisson distributed with a mean arrival rate of $2vN\sqrt{R_m^2 - (vT/2)^2}/A = N\sqrt{R_m^2 - (vT/2)^2}/RT_{cycle}$. Therefore,

$$\text{Average number of arrivals per cycle} = \frac{N\sqrt{R_m^2 - (vT/2)^2}}{RT_{cycle}}T_{cycle} \quad (43)$$

$$= \frac{N\sqrt{R_m^2 - (vT/2)^2}}{R} \quad (44)$$

$$< \frac{N\sqrt{R_m^{*2} - (vT/2)^2}}{R} \quad (45)$$

$$= (1 - F_{max})N, \quad (46)$$

where the inequality follows from our assumption that $R_m < R_m^*$, and the last step follows from the definition of R_m^* .

Hence, the average number of arrivals falls short of $(1 - F_{max})N$. Even if all arrivals are successfully serviced, the data loss fraction will still be in excess of F_{max} , contradicting our assumption that $R_m < R_m^*$ suffices to keep the data loss fraction below F_{max} .

¹¹We wish to define a quantity that is infinitesimally larger than R_m^* . Using $R_m^* + \epsilon$ makes subsequent steps cumbersome, therefore we define $R_{m\epsilon}^*$.

It remains to be shown that for any $\epsilon > 0$, $(R_m, D) = (R_{m\epsilon}^*, D^*)$ limits data loss to F_{max} . The following result will be necessary for this.

For any $\epsilon > 0$, if $(R_m, D) = (R_{m\epsilon}^*, D^*)$, then,

$$\lim_{N \rightarrow \infty} f_b = 1. \quad (47)$$

The above result will be the most crucial link in substantiating our claim in (28). For the sake of continuity, let us assume that (47) holds. Then, observe that,

$$D = \frac{(1 - F(R_m, D))D_{net}}{f_b} \Rightarrow F(R_m, D) = 1 - \frac{Df_b}{D_{net}} \quad (48)$$

$$\Rightarrow F(R_m^*, D^*) = 1 - (1 - F_{max})f_b \quad (49)$$

$$\Rightarrow \lim_{N \rightarrow \infty} F(R_{m\epsilon}^*, D^*) = F_{max}, \quad (50)$$

which completes our proof. Here, (48) follows from (42), (49) from (29), and (50) from (47).

It is now time to prove claim (47).

LEMMA B.1. $\lim_{N \rightarrow \infty} f_b = 1$.

Proof: We define the following events:

E_1 . The observer is idle at a randomly chosen instant t .

E_2 . All sensor arrivals in $(-\infty, t]$ have either been serviced completely before t , or they are past their laxity.

E_3 . All sensor arrivals in $(t - c/\sqrt{N}, t]$ have either been serviced completely before t , or they are past their laxity. Here, c is a constant.

It should be clear from the definitions, and from the fact that the scheduling policy is non-idling, that $E_1 = E_2$, and $E_2 \subseteq E_3$. Thus,

$$P(E_3) \geq P(E_2) = P(E_1) = 1 - f_b. \quad (51)$$

Now, consider the event E_3 . It is easy to see from (22) that the probability that an arrival in $(t - c/\sqrt{N}, t]$ will be past its laxity at t is negligibly small, being at most $O(1/\sqrt{N})$. Therefore, for E_3 to be true, (nearly) all arrivals in $(t - c/\sqrt{N}, t]$ must be serviced completely by t . From (2) and (42), the service time per arrival is

$$T = \frac{T_{cycle}D_{net}}{ND} = \frac{f_b T_{cycle}}{(1 - F)N}. \quad (52)$$

Therefore, the maximum number of arrivals that can be serviced in a period $\frac{c}{\sqrt{N}}$ is

$$\frac{c}{\sqrt{N}T} = \frac{(1 - F)c\sqrt{N}}{f_b T_{cycle}}. \quad (53)$$

For E_3 to be true, the number of arrivals in $(t - c/\sqrt{N}, t]$ must not exceed the number that can be serviced in that period. Hence,

$$P(E_3) \leq P\left(\text{Number of arrivals in } \frac{c}{\sqrt{N}} \leq \frac{c}{\sqrt{N}T}\right). \quad (54)$$

Since the value of R_m we are considering does not satisfy (5), the arrivals will be Poisson with a mean arrival rate of $\frac{2vN\sqrt{R_m^2 - (vT/2)^2}}{A} = \frac{N\sqrt{R_m^2 - (vT/2)^2}}{RT_{cycle}} \rightarrow \infty$ as

$N \rightarrow \infty$. In the limit, as the arrival rate becomes large, the Poisson distribution becomes a Normal distribution with the same mean and variance as the Poisson. Thus,

$$\frac{e^{-\mu} \mu^x}{x!} \approx \frac{1}{\sqrt{2\pi\mu}} \exp\left(-\frac{(x-\mu)^2}{2\mu}\right) \quad (55)$$

$$\text{where } \mu = \left(\frac{\sqrt{R_m^2 - (vT/2)^2} N}{RT_{cycle}}\right) \left(\frac{c}{\sqrt{N}}\right) = \frac{c\sqrt{N(R_m^2 - (vT/2)^2)}}{RT_{cycle}}.$$

As a consequence of this,

$$\begin{aligned} P(E_3) &\leq P\left(\text{Number of arrivals in } \frac{c}{\sqrt{N}} \leq \frac{c}{\sqrt{NT}}\right) \\ &= \sum_{x=0}^{\lfloor \frac{c}{\sqrt{NT}} \rfloor} \frac{e^{-\mu} \mu^x}{x!} \\ &\approx \int_{-\infty}^{\frac{c}{\sqrt{NT}}} \frac{1}{\sqrt{2\pi\mu}} \exp\left(-\frac{(x-\mu)^2}{2\mu}\right) dx \\ &= \frac{1}{2} + \text{erf}\left(\frac{\frac{c}{\sqrt{NT}} - \mu}{\sqrt{\mu}}\right) \\ &= \frac{1}{2} + \text{erf}\left(\frac{\frac{(1-F)c\sqrt{N}}{f_b T_{cycle}} - \frac{c\sqrt{N(R_m^2 - (vT/2)^2)}}{RT_{cycle}}}{\sqrt{\frac{c\sqrt{N(R_m^2 - (vT/2)^2)}}{RT_{cycle}}}}\right), \end{aligned} \quad (56)$$

where $\text{erf}(x)$ is the function

$$\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt, \quad (57)$$

so that $\text{erf}(\infty) = 1/2$ and $\text{erf}(-\infty) = -1/2$. It is easy to see that for $R_m = R_{m\epsilon}$,

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{\frac{(1-F)c\sqrt{N}}{f_b T_{cycle}} - \frac{c\sqrt{N(R_m^2 - (vT/2)^2)}}{RT_{cycle}}}{\sqrt{\frac{c\sqrt{N(R_m^2 - (vT/2)^2)}}{RT_{cycle}}}} &= \infty \quad \frac{1}{f_b} > 1 + \epsilon \\ &= -\infty \quad \frac{1}{f_b} < 1 + \epsilon. \end{aligned} \quad (58)$$

Now, if $f_b > 1/(1 + \epsilon)$, then all is well and we find that (56) equals 0 so that,

$$\begin{aligned} 0 &\geq P(E_3) \geq P(E_2) = P(E_1) = 1 - f_b \\ &\Rightarrow f_b = 1. \end{aligned} \quad (59)$$

If, on the other hand, we assume that $f_b < 1/(1 + \epsilon)$, then we can arrive at a contradiction by showing that $F = 0$ contrary to our assumption that $F_{max} > 0$. Physically, this would mean that the system is working at higher power than necessary to ensure the specified F_{max} , hence this is not the minimum.

For obtaining such a contradiction, consider an arrival at some arbitrary time $t - c/\sqrt{N}$. As argued before, the probability that it will be past its laxity at t

is negligibly small. We will show that this arrival will have received service with probability 1 if $f_b < (1 - F)$, showing that there is no data loss ($F = 0$). Since the observer acts as a clairvoyant scheduler with perfect knowledge of node arrivals and deadlines, it will outperform any schedule in terms of data loss F . We will show that even with a non-preemptive non-idling LCFS (Last Come First Served) schedule, $F = 0$ can be achieved. For convenience, we define the following event:

E_4 . The arrival at $t - c/\sqrt{N}$ does not begin to receive service by t .

Since the schedule is non-idling, the number of arrivals in $(t - c/\sqrt{N}, t]$ must exceed the number of sensors that can be serviced in that period for E_4 to be true. Hence,

$$\begin{aligned} P(E_4) &\leq P\left(\text{Number of arrivals in } \frac{c}{\sqrt{N}} > \frac{c}{\sqrt{NT}}\right) \\ &= 1 - P\left(\text{Number of arrivals in } \frac{c}{\sqrt{N}} \leq \frac{c}{\sqrt{NT}}\right) \\ &\approx \frac{1}{2} - \text{erf}\left(\frac{\frac{(1-F)c\sqrt{N}}{f_b T_{\text{cycle}}} - \frac{c\sqrt{N}}{T_{\text{cycle}}}}{\sqrt{\frac{c\sqrt{N}}{T_{\text{cycle}}}}}\right). \end{aligned} \quad (60)$$

If $f_b < (1 - F)$, then (60) evaluates to 0 as $N \rightarrow \infty$, so that we have $P(E_4) = 0$. Thus, an arbitrary arrival will receive service with probability 1, implying that the data loss fraction $F = 0$ which disagrees with our assumption that $F_{\text{max}} > 0$. \square

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