Partially Coherent Constellations for Multiple-Antenna Systems

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***Abstract*— We consider the problem of digital communication in a Rayleigh flat fading environment using a multiple antenna system. We assume that the transmitter doesn't know the channel coefficients, and that the receiver has only an estimate of them. We further assume that the transmitter and receiver know the statistics of the estimation error. We will refer to this system as a partially coherent system. In an earlier work, we had derived a design criterion for the partially coherent constellations based on maximizing the minimum KL distance between conditional distributions. We had also designed single transmit antenna constellations using this criterion, which showed substantial improvement in the performance over existing and widely-used constellations. In this work, using the KL-based design criterion, we design partially coherent constellations for multiple antenna systems, and evaluate their performance through simulation. We show that, even with only a few percent channel estimation error, the new constellations achieve significant performance gains over the conventional constellations and existing multiple antenna techniques. The proposed constellations are multi-level, with multi-dimensional spherical constellations at each level. We also propose a recursive construction for the constituent spherical subsets of the multiple-antenna partially coherent constellations.**

I. Introduction

Exploiting propagation diversity by using multiple antennas at the transmitter and receiver in wireless communication systems has been proposed and studied using different approaches [1]–[7]. In [1], [2], it has been shown that in a Rayleigh flat-fading environment, the capacity of a multiple antenna system increases linearly with the smaller of the number of the transmit and receive antennas, provided that the fading coefficients are known at the receiver. In a slowly fading channel, where the fading coefficients remain approximately constant for many symbol intervals, the transmitter can send training signals that allow the receiver to accurately estimate the fading coefficients. In this case the results of [1], [2] are applicable.

In practice, due to the finite length of the training sequence, there will always be some errors in the channel estimates. In order to maintain a given data rate, one has to use shorter training sequences for more rapidly fading channels, resulting in even less reliable channel estimates. Having multiple transmit antennas adds up to this problem by requiring longer training sequences for the same estimation performance. Therefore, the usual assumption of known channel parameters at the receiver in designing optimal codes/constellations is not exactly valid in practice. In the presence of channel estimation errors (partially coherent systems), constellations which are designed using the statistics of the estimation error are more desirable than the ones designed for perfect channel state information at the receiver. In [8], a design criterion for partially coherent constellations has been derived based on the Kullback-Leibler (KL) distance [9] between conditional distributions. Several single transmit antenna constellations have also been designed in [8] and shown to provide significant performance improvement as compared to the conventional constellations.

In this paper, we consider the problem of designing vector constellation for multiple-antenna partially coherent systems. Simplifying the KL-based design criterion of [8] for the vector constellations and imposing a multi-level spherical structure on the constellation, we decouple the design problem into two simpler optimization problems of

a) designing spherical constellations based on maximizing the minimum Euclidean distance between the constellations points, and
b) constructing multi-level constellations of spherical subsets based on maximizing the minimum intra-subset and inter-subset KL distances.

Furthermore, we propose a recursive construction method for the spherical constellations which simplifies the design problem and provides close-to-optimal packings on the surface of n-dimensional complex spheres. Using the recursive constellations in the levels of the multi-level spherical constellations, we solve the decoupled optimization problem and design partially coherent vector constellations for multiple-antenna systems. We show, through simulation, that the new multiple-antenna constellations can achieve significant performance gains over the conventional multiple-antenna approach of V-BLAST [3] and the orthogonal transmit diversity scheme of [4] with comparable computational complexity.

In Section II, we introduce the system model and review the design criterion. In Section III, we explain the proposed design method and structure of the multiple-antenna constellations. In Section IV, we introduce a recursive technique to construct spherical signal sets which are used as subsets of the partially coherent constellations. In Section V, we present some simulation results that show significant improvement in the performance by using the new constellations instead of the conventional constellations and multiple-antenna techniques. Finally, we draw some conclusions in Section VI.

II. System Model and Design Criterion

A communication system with M transmit and N receive antennas in a block Rayleigh flat fading channel with coherence interval of T symbol periods is modelled using the following complex baseband notation

$$X = SH + W, \tag{1}$$

where S is the $T \times M$ matrix of transmitted signals with power constraint $\sum_{t=1}^{T}\sum_{m=1}^{M}\mathbb{E}\left\{|s_{tm}|^2\right\} = TP$, where s_{tm}'s are the elements of the signal matrix S, X is the $T \times N$ matrix of received signals, H is the $M \times N$ matrix of fading coefficients, and W is the $T \times N$ matrix of the additive received noise. Elements of H and W are assumed to be statistically independent, identically distributed circular complex Gaussian random variables from the distribution $\mathcal{CN}(0,1)$. We also assume that $H = \widehat{H} + \widetilde{H}$, where $\widehat{H}$ is known to the receiver but $\widetilde{H}$ is not. Furthermore, we assume that $\widetilde{H}$ has i.i.d. elements from $\mathcal{CN}(0,\sigma_E^2)$, and is statistically independent from $\widehat{H}$ (this can be obtained, e.g., by using an LMMSE estimator).

With the above assumptions, the conditional probability density of the received signal can be written as

$$\begin{aligned} p(X|S,\widehat{H}) &= \mathbb{E}_{\widetilde{H}}\left\{p(X|S,\widehat{H},\widetilde{H})\right\} \\ &= \frac{\exp\{-\mathrm{tr}[(I_T+\sigma_E^2 SS^H)^{-1}(X-S\widehat{H})(X-S\widehat{H})^H]\}}{\pi^{TN}\det^N(I_T+\sigma_E^2 SS^H)}. \end{aligned} \tag{2}$$

Assuming a signal set of size L, $\{S_i\}_{i=1}^{L}$, and defining $p_l(X) = p(X|S_l,\widehat{H})$, the Maximum Likelihood (ML) detector for this system will have the following form

$$\widehat{S}_{ML} = S_{\widehat{l}_{ML}}, \quad \text{where} \quad \widehat{l}_{ML} = \arg\max_{l\in\{1,\cdots,L\}} p_l(X). \tag{3}$$

In [10], it has been shown that the performance of the ML detector is related to the KL distance between conditional distributions, and that a design criterion based on maximizing the minimum KL distance between constellation points can result in an improved performance for the ML detector. Based on this result, in [10] multiple-antenna constellations have been designed for non-coherent systems, and significant performance improvements have been demonstrated over some of the existing unitary constellations. In [8], the KL-based design criterion has been derived for partially coherent systems, and several scalar constellations have been designed for single transmit antenna systems. These constellations have also shown significant performance improvement over the conventional PSK and QAM constellations. In this work, we use the design criterion of [8] and propose a construction method for multiple-antenna vector constellations for partially coherent systems.

In [8], the *expected KL distance* between conditional distributions corresponding to the signal points S_i and S_j has been derived as

$$\begin{aligned} \overline{\mathcal{D}}(S_i\|S_j) &= N\mathrm{tr}\left\{K_iK_j^{-1}\right\} - N\ln\det\left\{K_iK_j^{-1}\right\} - NT \\ &+ N\ln\det\left\{I_M + \left(1-\sigma_E^2\right)(S_i-S_j)^H K_j^{-1}(S_i-S_j)\right\}, \end{aligned} \tag{4}$$

where

$$K_i = I_T + \sigma_E^2 S_i S_i^H, \tag{5}$$

and

$$K_j = I_T + \sigma_E^2 S_j S_j^H, \tag{6}$$

Adopting the above KL distance as the performance criterion, the signal set design can be formulated as the following optimization problem

$$\underset{\frac{1}{L}\sum_{l=1}^{L}\|S_l\|^2 = TP}{\text{maximize}} \quad \min_{i\neq j} \quad \overline{\mathcal{D}}(S_i\|S_j), \tag{7}$$

where $\|S_l\|^2 = \sum_{t=1}^{T}\sum_{m=1}^{M}|(S_l)_{tm}|^2$ is the total power used to transmit S_l. Since the actual value of N does not affect the maximization in (7), in designing the optimal signal sets we can always assume $N = 1$.

III. Multiple-Antenna Constellation Design

In this section, we design partially coherent constellations for the multiple-antenna systems. For a fixed spectral efficiency, the constellation size grows exponentially with T, e.g., to achieve a spectral efficiency of 4 bits per symbol with $T = 5$, one needs to design a constellation of $2^{20} = 1,048,576$ points. With multiple antennas at the transmitter and receiver, even larger spectral efficiencies are expected, making the constellation design over multiple symbol intervals even more difficult, and their decoding complexity prohibitively large. For these reasons, in this paper we only consider constellations for the case of $T = 1$. Notice that the actual coherence interval of the channel can be larger than this value. It is also important to notice that, with $T = 1$, each transmitted matrix will have a unit rank, and thus will not be able to provide any transmit diversity gain. Therefore, if the actual coherence interval of the channel is larger than one, and in the absence of channel estimation errors, any transmit diversity scheme is expected to show a better performance at high SNR. However, as we will see later, in the presence of channel estimation errors, the performance of the proposed constellations can be significantly better than transmit diversity schemes with comparable computational complexity.

Assuming $T = 1$, each S_l will be a complex row vector. The expression for the expected KL distance in (4) reduces to

$$\begin{aligned} \overline{\mathcal{D}}_1(S_i\|S_j) &= \frac{1+\sigma_E^2\|S_i\|^2}{1+\sigma_E^2\|S_j\|^2} - \ln\left(\frac{1+\sigma_E^2\|S_i\|^2}{1+\sigma_E^2\|S_j\|^2}\right) - 1 \\ &+ \ln\det\left\{I_2 + \frac{1-\sigma_E^2}{1+\sigma_E^2\|S_j\|^2}\left(S_i-S_j\right)^H\left(S_i-S_j\right)\right\}. \end{aligned} \tag{8}$$

Using the identity

$$\det\left(I_M + A_{M\times N}B_{N\times M}\right) = \det\left(I_N + B_{N\times M}A_{M\times N}\right), \tag{9}$$

We will have

$$\overline{\mathcal{D}}_1(S_i\|S_j) = \frac{1+\sigma_E^2\|S_i\|^2}{1+\sigma_E^2\|S_j\|^2} - \ln\left(\frac{1+\sigma_E^2\|S_i\|^2}{1+\sigma_E^2\|S_j\|^2}\right) - 1 + \ln\left\{1+\frac{1-\sigma_E^2}{1+\sigma_E^2\|S_j\|^2}\|S_i-S_j\|^2\right\}. \quad (10)$$

From (10), one can see that if two constellation points (vectors) have the same norm (i.e., lie on the same M-dimensional complex sphere centered at the origin), the first three terms will cancel out, and the KL distance will be a monotonic function of the Euclidean distance between them. Therefore, if one considers only constant power constellations (i.e., constellations for which all of the points lie on the same sphere centered at the origin), then the design criterion becomes maximizing the minimum Euclidean distance between constellation points, similar to the case of perfect CSI at the receiver. The design problem in this case reduces to the problem of packing points on the surface of an M-dimensional complex sphere (or $2M$-dimensional real sphere), which is a well studied problem (see, e.g., [11] and references therein).

On the other hand, if two points lie on different spheres, the minimum KL distance between them will happen when they lie on a line which passes through the origin, and will be determined by the radiuses of the two spheres. This means that if one partitions the constellation into subsets of concentric M-dimensional complex spheres $C_1,\cdots,C_K$, with radiuses $r_1,\cdots,r_K$, containing $l_1,\cdots,l_K$ points, respectively, and defines the intra-subset and inter-subset distances as

$$\mathcal{D}_{intra}(k) = \min_{S_i,S_j\in C_k} \ln\left\{1+\frac{1-\sigma_E^2}{1+\sigma_E^2 r_k^2}\cdot 4r_k^2\sin^2\left(\angle\overline{S_i},\overline{S_j}\right)\right\}, \quad (11)$$

and

$$\mathcal{D}_{inter}(k,k') = \frac{1+\sigma_E^2 r_k^2}{1+\sigma_E^2 r_{k'}^2} - \ln\left(\frac{1+\sigma_E^2 r_k^2}{1+\sigma_E^2 r_{k'}^2}\right) - 1 + \ln\left\{1+\frac{1-\sigma_E^2}{1+\sigma_E^2 r_{k'}^2}|r_k-r_{k'}|^2\right\}, \quad (12)$$

then the minimum KL distance between the constellation points will be greater than or equal to the minimum of the inter-subset and intra-subset KL distances. In (11), $\overline{S_i}$ and $\overline{S_j}$ are real vectors constructed by concatenating the real and imaginary parts of S_i and S_j, respectively:

$$\overline{S_i} = \left[\Re(S_i), \Im(S_i)\right]^T, \quad \overline{S_j} = \left[\Re(S_j), \Im(S_j)\right]^T, \quad (13)$$

and $\angle\overline{S_i},\overline{S_j}$ denotes the angle between the two $2M$-dimensional real vectors.

Therefore, instead of solving the (computationally complex) original optimization in (7), we can solve the following simplified maximin problem to find a close-to-optimal L-point multilevel constellation of $1\times M$ vectors with average power P:

$$\underset{\substack{1\le K\le L,\ \frac{1}{L}\sum_{k=1}^{K} l_k r_k^2 = P,\ \sum_{k=1}^{K} l_k = L \\ 0\le r_1<r_2<\cdots<r_K}}{\text{maximize}} \quad \min A, \quad (14)$$

where

$$A = \left\{\{\mathcal{D}_{intra}(k)\}_{k=1}^{K}, \{\mathcal{D}_{inter}(k,k+1)\}_{k=1}^{K-1}\right\}, \quad (15)$$

and, without loss of generality, we have assumed that $r_1 < r_2 < \cdots < r_K$.

In (14), K and $l_1,\cdots,l_K$ are discrete variables, while $r_1,\cdots,r_K$ are continuous variables. This optimization problem can be solved using the approach explained in [10]. For any fixed value of K and $l_1,\cdots,l_K$ satisfying the specified constraints, (14) reduces to a continuous optimization over $r_1,\cdots,r_K$, which can be solved numerically. Moreover, since the intra-subset distance is an increasing function of r_k, it can be shown that the optimum constellation also satisfies the extra constraint $l_1\le l_2\le\cdots\le l_{K-1}$, which can be used to further restrict the domain of search.

As mentioned above, the design problem for each subset is equivalent to a packing problem on the surface of an M-dimensional complex ($2M$-dimensional real) sphere. This problem is a well studied problem (see, e.g., [11] and references therein). However, since the design and decoding complexities of the optimal packings are usually high, in the next section we propose a recursive construction for the spherical constellations, which results in a systematic design and, in some cases, low complexity decoding algorithm.

IV. A Recursive Construction for Spherical Constellations

We denote, by $S_n(L)$, the L-point recursively constructed n-dimensional real spherical constellation. We start from $n = 2$, and define

$$S_2(L) = \left\{\begin{bmatrix}\cos((l-1)2\pi/L)\\ \sin((l-1)2\pi/L)\end{bmatrix}\right\}_{l=1}^{L}. \quad (16)$$

For $n>2$, we construct the constellation by using a number of $(n-1)$-dimensional recursive constellations as latitudes of the n-dimensional constellation. In the following, we explain this procedure with more detail for the case of $n=3$.

Fig. 1 shows a 32-point 3-dimensional constellation ($S_3(32)$) constructed from two $S_2(9)$ and one $S_2(14)$ constellations. The minimum angle between points in the $S_2(9)$ constellations is denoted by β. However, when this constellation is used as a subset of an S_3 constellation, the effective angle between the constellation points is no longer β. The following equation gives the effective minimum intra-subset angle, α,

$$\sin(\frac{\alpha}{2}) = \cos(\theta)\sin(\frac{\beta}{2}), \quad (17)$$

where θ is the latitude of the subset.

Assuming a constellation of P levels (latitudes), we define the intra-subset and inter-subset distances by

$$d_{intra}(p) = \sin^2(\alpha_p), \quad \text{for } p=1,\cdots,P, \quad (18)$$

and

$$d_{inter}(p,p') = \sin^2(\theta_p-\theta_{p'}), \quad \text{for } p,p'=1,\cdots,P, \quad (19)$$

where α_p is the effective minimum intra-subset angle of the pth subset, and θ_p and $\theta_{p'}$ are the latitudes of the subsets p and p', respectively.

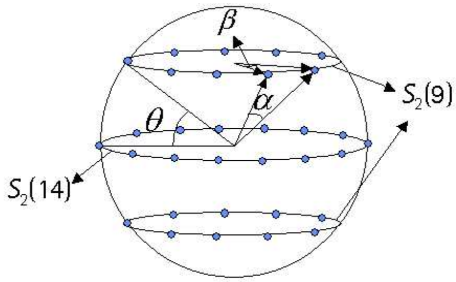

Fig. 1. A three-level three-dimensional real spherical constellation

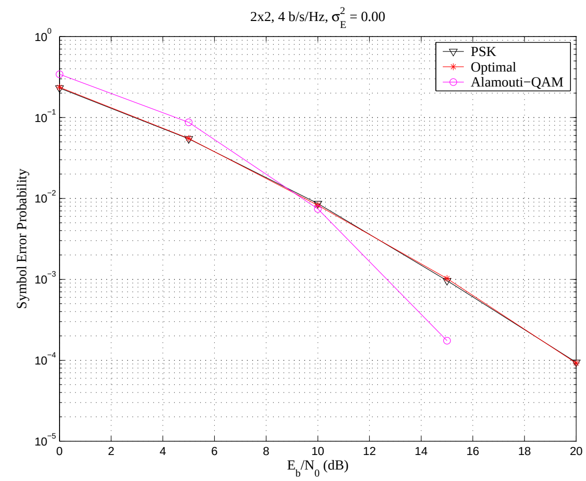

Fig. 2. Symbol error rate for 4 b/s/Hz with $M = N = 2$, and $\sigma_E^2 = 0.0$

Next, similar to the approach in the previous section, we simplify the optimization problem by only maximizing the minimum of the intra-subset and inter-subset distances, instead of maximizing the minimum distance between all pairs of constellation points. For this, we solve the following optimization problem:

$$\underset{1\le P\le L,\ \sum_{p=1}^{P} l_p=L,\ -\frac{\pi}{2}\le\theta_1<\cdots<\theta_P\le\frac{\pi}{2}}{\text{maximize}} \quad \min B \quad , \tag{20}$$

where l_p denotes the number of the points in the pth subset, and

$$B = \left\{ \{d_{intra}(p)\}_{p=1}^{P}, \{d_{inter}(p, p+1)\}_{p=1}^{P-1} \right\}. \tag{21}$$

Similar to the optimization in the previous section, here P and $l_1, \cdots, l_P$ are discrete variables, whereas $\theta_1, \cdots, \theta_P$ are continuous variables. For a given choice of P and $l_1, \cdots, l_P$, satisfying the specified constraints, we have a continuous optimization problem which can be solved numerically to find the optimal values for $\theta_1, \cdots, \theta_P$. We do not usually need to try all of the possible values for P. Starting from $P = 1$, and increasing the value of P by one at each time, the search can be stopped once the optimum minimum distance obtained from the above optimizations stops increasing. Also it can be shown that the optimal values for $l_1, \cdots, l_P$ will satisfy the following extra constraint

$$l_p \ge l_{p-1} \text{ or } l_p \ge l_{p+1}, \quad \text{for } p = 2, \cdots, P-1, \tag{22}$$

which can be used to further restrict the search domain.

For the case of $n > 3$, we use the same procedure as explained above, with the difference that instead of S_2 constellations, we use S_{n-1} constellations as the subsets, and construct the constellations recursively.

V. Simulation Results

We considered two spectral efficiencies of 4 and 8 bits/symbol (b/s/Hz, assuming a symbol interval equal to the reciprocal of the bandwidth), and designed partially coherent constellations for a 2×2 (two-transmit and two-receive antenna) system, for different values of channel estimation error (0%, 1%, and 5%). We evaluated the performance of the these constellations through simulation. As our reference curves, we also simulated the case in which two independent QPSK or 16QAM constellations were used at the two transmit antennas (resulting in spectral efficiencies of 4 and 8 b/s/Hz, respectively). These cases are similar to the V-BLAST scheme of [3], with the difference that, in order to have a fair comparison, we have used the optimal (joint ML) detector, and not a linear or successive interference cancellation algorithm as suggested in the V-BLAST scheme.

In order to compare the proposed constellations with a transmit diversity scheme, we considered the orthogonal transmit diversity approach of [4], which has a decoding complexity similar to our scheme, and evaluated its performance for a 2×2 system with 16QAM and 256QAM constellations (4 and 8 b/s/Hz) through simulation. These orthogonal transmit diversity schemes are designed for $T = 2$. Therefore, even though our constellations are designed for $T = 1$, in our simulations, we used a channel with coherence time of two symbol intervals.

Figs. 2 and 3 show the symbol error rate curves for the case of 4 b/s/Hz and estimation variances of 0 and 0.05, respectively. As we see, in the absence of estimation error (Fig. 2), the QPSK and the optimal two-antenna constellations have almost the same performance. We also notice that, because of higher order of transmit diversity, at high SNR the Alamouti scheme in this case shows better performance compared to both QPSK and the optimal two-antenna constellations.

With 5% estimation error (Fig. 3), the new constellations start showing better performance for SNR values larger than 10dB, and the performance improvement becomes substantial at higher SNR values. We also notice that the Alamouti scheme suffers from a severe performance degradation due to channel estimation errors, and its performance becomes even worse than the conventional QPSK constellations without any transmit diversity.

Figs. 4 and 5 show the symbol error rate curves for the case of 8 b/s/Hz and estimation variances of 0 and 0.01, respectively. As we see, in the absence of estimation error (Fig. 4), the 16QAM and the optimal two-antenna constellations have almost the same performance. We also notice that, even

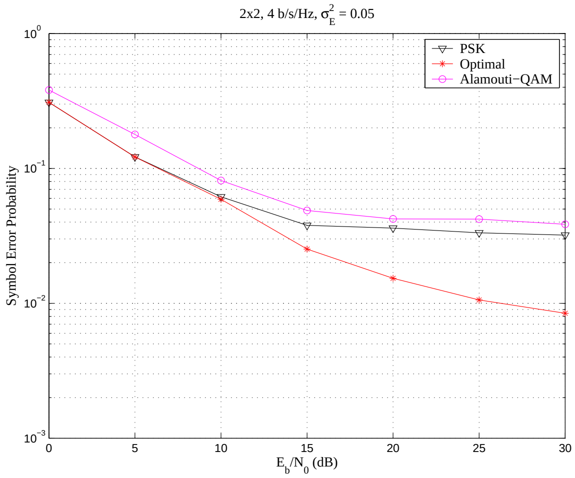

Fig. 3. Symbol error rate for 4 b/s/Hz with $M = N = 2$, and $\sigma_E^2 = 0.05$

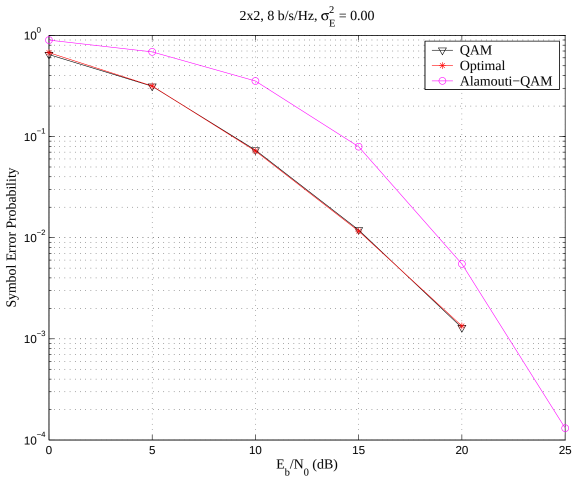

Fig. 4. Symbol error rate for 8 b/s/Hz with $M = N = 2$, and $\sigma_E^2 = 0.0$

though the Alamouti scheme has a larger transmit diversity advantage (larger slope for the error rate curve at high SNR), because of smaller coding advantage performs worse than the conventional 16QAM constellations for SNR values of up to around 23dB.

With 1% estimation error (Fig. 5), the new constellations show significant performance improvement compared to the 16QAM constellations. We also notice that the Alamouti scheme suffers from a severe performance degradation and reaches an error floor of about 3×10^{-1}.

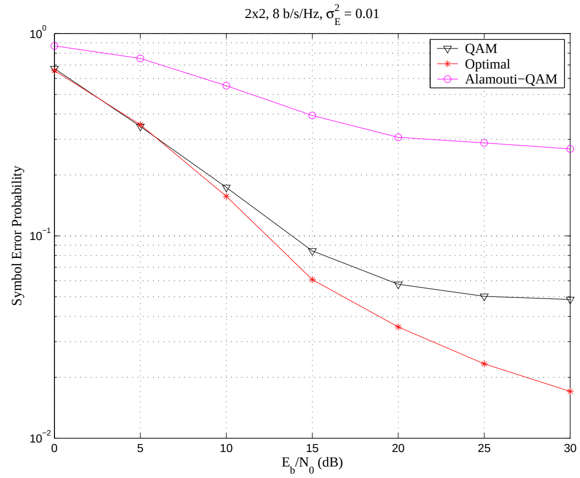

Fig. 5. Symbol error rate for 8 b/s/Hz with $M = N = 2$, and $\sigma_E^2 = 0.01$

VI. Conclusions

We considered the problem of digital communication in a Rayleigh flat fading environment using a multiple antenna system, when only partial (imperfect) channel state information is available at the receiver. Using a previously derived design criterion based on the Kullback-Leibler distance between distributions, we designed partially coherent constellations for multiple-antenna systems. We evaluated the performance of the new constellations through simulation, and showed that even with only a few percent channel estimation error, they achieve significant performance gains over conventional constellations and existing multiple-antenna techniques. The proposed constellations are multi-level, with multi-dimensional spherical constellations at each level. We also proposed a recursive construction for the constituent spherical subsets of the multiple-antenna partially coherent constellations.

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