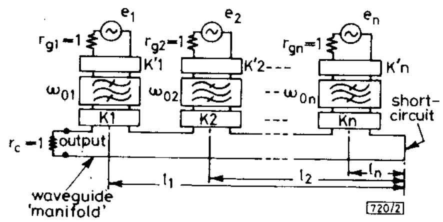

Fig. 2 Prototype manifold multiplexer

(corresponding to nonlinear functions) are the sampling frequencies in the frequency range $[\omega_0(i) - \Delta\omega_0(i)/2 ; \omega_0(i) + \Delta\omega_0(i)/2]$ (i = 1, ..., n. n is the number of channels, $\omega_0(i)$ the resonance frequency and Δ the bandwidth of the ith channel). Fig. 4a and b illustrate the nonlinear functions. For each multiplexer, we use approximately 300 nonlinear functions to optimise 16 parameters per channel for six-pole dual-mode filters.

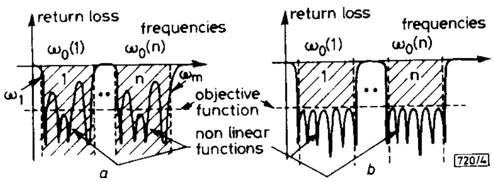

Fig. 4 Nonlinear functions before and after optimisation

a Before optimisation
b After optimisation

When F(x) is less than a prespecified acceptable return loss value (objective function), then all return losses (nonlinear functions) are acceptable. We can therefore stop the optimisation (Fig. 4b).

This procedure does not require dummy matching elements because all parameters are optimised. An example of a 12 channel, 12GHz contiguous band multiplexer with optimisation (Fig. 3) is presented. Each channel has a useful bandwidth of 27MHz with a channel separation of 30 MHz and return losses prespecified equal to 26dB.

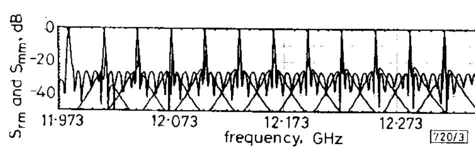

Fig. 3 Multiplexer transmission and return loss responses with optimisation

Bandwidth : 27MHz, distance between two resonance frequencies : 30MHz, return loss : 26dB

Conclusion: An optimisation procedure for the computer-aided design of a waveguide multiplexer has been described. This method is based on an analysis algorithm for the equivalent circuit of the multiplexer. The filters used in the multiplexer can be either direct-coupled or multiple-coupled cavity filters. An example of a 12 channel, 12GHz waveguide output multiplexer is included. To our knowledge, such a structure constitutes the state of the art in output muitiplexers for satellite transponders.

Acknowledgment: This work is supported by the Centre National d'Etudes Spatiales (CNES), Toulouse, France.

12 July 1993
Electronics Letters Online No: 19931069

G. Tanne and S. Toutain (Laboratoire d'Electronique et Systèmes de Télécommunications (LEST), URA CNRS n° 1329, UBO 6, av. Le Gorgeu, BP 452, 29275 Brest Cedex, France)

J. F. Favennec (ENIB Laboratoire RESO, Case Postale N°15, 29608 Brest Cedex, France)

P. Jarry (ENSERB - 351, Cour de la Libération, 33405 Talence, France) C. Boschet (CNES - 18, av Ed. Elin, 31055 Toulouse Cedex, France)

References

1 RHODES, J.D., and LEVY, R.: 'A generalized multiplexer theory', IEEE Trans., 1979, **MTT-27**, (2) pp. 99–111
2 RHODES, J.D., and LEVY, R.: 'Design of general manifold multiplexers', IEEE Trans., 1979, **MTT-27**, (2) pp. 111–123
3 RHODES, J.D.: 'A low-pass prototype network for microwave linear phase filters', IEEE Trans., 1970 **MTT-18**, (6) pp. 290–301
4 CAMERON, R.J., and RHODES, J.D.: 'Assymetric realizations for dual-mode bandpass filters', IEEE Trans., 1981 **MTT-29**, (1) pp. 51–58
5 HALD, J., and MADSEN, K.: 'Combined LP and quasi-Newton methods for minimax optimization', Mathematical Programming, 1981, **20**, (1) pp. 49–62

Signal transform covariant to scale changes

. R.G. Baraniuk

Indexing terms: Signal processing, Transforms

A unitary signal transformation that is covariant by translation to scale changes (dilations and compressions) in the signal is formulated and justified. Unlike the Mellin transform, which is invariant to scale changes, this new transform is a true indicator of the scale content of a signal.

Introduction: The concept of scale has been a subject of considerable interest recently, due to the introduction of the wavelet transform and classes of bilinear timescale distributions [1–3]. Because these tools analyse signals only in terms of their joint timescale content, it is relevant to consider also transforms indicating solely the scale content of signals. Traditional concepts such as the time content and frequency content of a signal (x)t can be expressed in terms of x(t) and its Fourier transform (Fx)(f)[Note 1] . However, the situation with scale is more confused, with a variety of transforms proposed to indicate scale content.

Most approaches to date have been based on or have resulted in the Mellin transform [2]

$$(Mx)(c) = \int_0^\infty x(u)e^{-j2\pi c \log u}\frac{du}{\sqrt{u}} \qquad (1)$$

It is natural to insist that a transform indicating the scale content of a signal must change as the scale of the signal is changed. This requirement immediately eliminates the Mellin transform from contention as a scale indicating transform, because it is invariant to scale changes. That is, defining the scale change or dilation operator as $(D_d x)(t) = x(e^{-d}t)\, e^{-d/2}$, we have

$$|(MD_d x)(c)| = |(Mx)(c)| \qquad (2)$$

The aim of this Letter is to derive and justify the signal transform

$$(Sx)(\sigma) = e^{\sigma/2}x(e^{\sigma}t_0) \qquad (3)$$

that is covariant by translation to scale changes

$$(SD_d x)(\sigma) = (Sx)(\sigma - d) \qquad (4)$$

The derivation of this scale transform will use some simple concepts from unitary operator theory and eigenanalysis.

Mathematical background: Define the time-shift and frequency-shift operators as $(T_t g)(u) = g(u-t)$ and $(F_f g)(u) = e^{j2\pi fu}g(u)$, respectively. The time, frequency, and dilation operators are unitary. Because a unitary operator V maps the signal space of square-inte-

[Note 1]. The bracket notation (Ug) (v) represents the result of operating on the function (signal) g with the operator U and then evaluating at the point v.

grable functions $L^2(R)$ back onto itself in a way that preserves its structure exactly, it can be interpreted as simply a relabelling operator that takes every function $x \in L^2(R)$ and gives it a new name Vx. This relabelling is equivalent to changing the frame of reference or to changing bases. Two operators, A and B, are defined as unitarily equivalent if they are equivalent modulo a change of basis, that is, if $B = U^{-1}AU$ for some unitary transformation U [4,5].

Given a linear operator Q on $L^2(R)$, solution of the eigenequation

$$\left(Qe_q^Q\right)(\tau) = \lambda_q^Q e_q^Q(\tau) \qquad (5)$$

yields the eigenfunctions $\{e_q^Q(\tau)\}$ and the eigenvalues $\{\lambda_q^Q\}$ of Q, both of which are indexed by the parameter q. If Q is unitary, then the eigenfunctions form an orthonormal basis for $L^2(R)$. In this case, the basis expansion onto these eigenfunctions yields another unitary operator, which we will refer to as the Q-Fourier transform F_Q. The forward transform of a signal $x \in L^2(R)$ is given by

$$(\boldsymbol{F}_Q x)(q) = \int x(u) e_q^{Q*}(u)\, du \qquad (6)$$

Note that F_Q is invariant (up to a phase shift) to the operator Q; that is, $|(F_QQx)(q)| = |(F_Qx)(q)|$.

The eigenfunctions of the time operator $e_f^T(u) = e^{j2pfu}$ yield the usual Fourier transform $F_T = F$, while the eigenfunctions of the frequency operator, $e_t^F(u) = \delta(u-t)$, yield the (trivial) signal transform $(F_Fx)(t) = x(t)$. The eigenfunctions of the dilation operator on $L^2(R_+)$ are the 'hyperbolic chirp' functions

$$e_c^D(u) = u^{-1/2} e^{j2\pi c \log u}, \quad u > 0 \qquad (7)$$

Because these functions are by definition invariant to dilations, the Mellin transform F_D in eqn. 1 is likewise invariant.

Scale transform: The derivation of the scale transform follows directly from the unitary equivalence of the time, frequency, and dilation operators. The key observation is as follows. The transform F_T based on the time operator eigenfunctions indicates the frequency content of a signal x, because it is covariant to frequency shifts Fx of the signal. Similarly, the transform F_F based on the frequency operator eigenfunctions indicates the time content of a signal, because it is covariant to time shifts Tx of the signal. Therefore, the transform F_H covariant to the scale operator will be based on the eigenfunctions of an operator H that is dual to the scale operator in the same sense that the time and frequency operators are dual.

Fortunately, the time and dilation operators are unitary equivalent, allowing the direct computation of H. The time operator (on $L^2(R)$) and the dilation operator (on $L^2(R_+)$) are related by

$$D_d = \varepsilon^{-1} T_d \varepsilon \qquad (8)$$

where $\varepsilon : L^2(R_+) \rightarrow L^2(R)$ is the unitary exponential axis warping

$$(\varepsilon g)(u) = e^{u/2} g(e^u t_0) \qquad (9)$$

and t_0 is an arbitrary positive reference time. Because the dual operator to time shift is frequency shift, the dual operator to dilation is therefore $H_h = \varepsilon^{-1}F_h\varepsilon$. The eigenfunctions of H are given by

$$e_\sigma^H(u) = \left(\varepsilon^{-1} e_\sigma^F\right)(u) = e^{\sigma/2}\delta(u - e^\sigma t_0), \quad u > 0 \qquad (10)$$

The expansion of eqn. 6 onto these eigenfunctions yields the scale transform $F_H = S$ given in eqn. 3 for signals in $L^2(R_+)$. This transform clearly possesses the dilation covariance property eqn. 4.

Interpretation: Although almost disappointingly simple, the transform of eqn. 3 has the attributes of a scale indicating transform. Just as the value x(t) of the time function at the point t indicates the signal content at the reference point t = 0 when the signal is translated by -t seconds, the value $(Sx)(\sigma)$ of the scale transform at the point σ indicates the signal content at the reference point $t = t_0$ when the signal is dilated by the factor $-\sigma$. Furthermore, by analogy to the pure frequency functions $e_f^T(u) = e^{j2pfu}$ of the F_T Fourier transform and the pure time functions $e_t^F(u) = \delta(u - t)$ of the F_F transform, the functions $e_s^H(u)$ in eqn. 10 play the role of pure scale functions having no spread about their inherent scale σ.

The scale transform can be applied to measure scale content in any domain, so long as the variable t of the function x(t) is interpreted appropriately. One particularly enlightening interpretation considers t as a spatial variable in an imaging system, with x(t) the distribution of an object in the direction perpendicular to the image plane; then σ represents a zoom parameter and the spread of Sx indicates the amount of focus change required to successively bring all of the object into focus at the image plane.

While S is defined only for one-sided signals in $L^2(R_+)$, signals in $L^2(R)$ can be handled by computing separate scale transforms along the positive and negative time axes. Alternatively, we can consider two values $\pm t_0$ of the reference time parameter.

Conclusions: Scale is a relative quantity ('this thing is twice as large as that thing'), and so we should anticipate difficulties with a transform indicating purely the scale content of a signal. The scale transform derived here is simply a warped version of the signal being analysed, and thus it is instantaneous in the sense that its value $(Sx)(\sigma)$ at point σ depends only on the signal value $x(e^\sigma t_0)$ at point $e^\sigma t_0$. This is contrary to most other popular signal transforms, including the Fourier and Mellin transforms, which use integral averages over the totality of the signal time axis to compute a single value of the transform. However, the instantaneity of the scale transform is not completely heretical, because the concept of scale is bound closely to the physical domain where it is applied.

Finally, we note that the concept of duality introduced in the scale transform section can be related to the concept of orthogonality of operators developed in detail in [5]. This approach is used to generalise the results of this Letter to other operators in [6].

Acknowledgments: This work was supported by an NSERC-NATO postdoctoral fellowship and was completed while the author was with the Laboratoire de Physique, Ecole Normale Supérieure de Lyon, France.

20 July 1993
Electronics Letters Online No: 19931073

R. G. Baraniuk (Department of Electrical and Computer Engineering, Rice University, PO Box 1892, Houston, TX 77251-1892, USA)

References

1 RIOUL, O., and FLANDRIN, P.: 'A general class extending wavelet transforms', IEEE Trans. Signal Process., 1992, **40**, (7), pp. 1746–1757
2 COHEN, L.: 'The scale representation', to be published in IEEE Trans. Signal Process., December 1993, **41**
3 PAPANDREAOU, A., HLAWATSCH, F., and BOUDREAUX-BARTELS, G.F.: 'The hyperbolic class of quadratic time-frequency representations. Part I: Constant-Q warping, the hyperbolic paradigm, properties and members', IEEE Trans. Signal Process. December 1993, **41**
4 BARANIUK, R.G., and JONES, D.L.: 'Warped wavelet bases: Unitary equivalence and signal processing', Proc. IEEE Int. Conf. Acoustics, Speech and Signal Processing - ICASSP '93, 1993
5 BARANIUK, R.G., and JONES, D.L.: 'Unitary equivalence: A new twist on signal processing', submitted to IEEE Trans. Signal Process., 1993
6 BARANIUK, R. G.: 'Integral transforms covariant to operators'. Preprint, 1993

Interfacial polarisation in Al-Y_2O_3-SiO_2-Si capacitor

C.H. Ling

Indexing terms: Capacitors, Dielectric materials

The variation by a factor of 2 in the observed permittivity of yttrium oxide film is explained in terms of interfacial polarisation