RICE UNIVERSITY

Performance Improvements with Feedback in Cooperative Relay Networks

by

Nasir Ahmed

A Thesis Submitted
in Partial Fulfillment of the
Requirements for the Degree
Doctor of Philosophy

Approved, Thesis Committee:

Behnaam Aazhang, Chair
J.S. Abercrombie Professor
Electrical and Computer Engineering

Joe Cavallaro, Professor
Electrical and Computer Engineering

Ashutosh Sabharwal, Faculty Fellow
Electrical and Computer Engineering

Rudolf Riedi, Associate Professor
Statistics

Houston, Texas
August, 2005

Abstract

Performance Improvements with Feedback in Cooperative Relay Networks

by

Nasir Ahmed

Recent results on multiple antenna transmission techniques have shown great potential in their ability to improve the overall performance in fading channels. Despite the promise shown by employing multiple antenna's, practical implementations may not be feasible due to size and hardware limitations of mobile nodes. *Cooperative Coding* is a new transmission paradigm that overcomes these limitations by pooling together the resources of neighboring nodes in a network to create a distributed antenna array.

The power of node collaboration can be seen by considering the relay channel, the simplest cooperative network. Recently, protocols have been developed for the wireless relay channel that allow the network to behave as a virtual multiple antenna system. In this thesis we show that in addition to efficient network protocols, exploiting channel state information can yield even more performance in the relay setting by allowing for temporal power and rate control.

When power control is used for a given transmission rate, minimizing the *outage probability* is the appropriate method to maximize performance in the block fading channel. In a relay setting, we derive the optimal power control strategy when the transmitters in the network have perfect knowledge of the network channel state. In practice having perfect channel state knowledge at the transmitters is not possible. In this direction, we derive a power control policy that minimizes the outage probability based on the rate of the feedback link. Interestingly, we observe that only a few bits of feedback are needed to extract much of the gains of the perfect feedback power control policy.

For applications that can support a variable rate of transmission, such as data transfers, the feedback can be used to vary both the transmission rate and power. The appropriate performance metric in this case is throughput. We derive throughput maximizing policies for various cooperative transmission protocols. Once again, we show that with a limited rate of feedback, significant throughput gains are possible in relay networks. Interestingly, we show that simultaneous power and rate adaptation is usually not needed. For small average power constraints, power control is imperative, while for large average powers, rate control is sufficient to achieve a large throughput.

Our results reveal that power and rate adaptation can lead to significant performance improvements. Even a few bits of feedback can lead to large power savings and throughput gains, and as a result, channel state feedback can be readily implemented with minimal communication overhead in next generation protocols.

Acknowledgments

First, I would like to thank my advisor Dr. Behnaam Aazhang whose enthusiasm, encouragement and guidance helped make this work possible. Second, I would like to thank Dr. Ashutosh Sabharwal for the interesting discussions and collaboration that we had over the years. Third, I would also like to thank my other two committee members Dr. Joe Cavallaro and Dr. Rudolf Riedi for their useful comments and suggestions. I also want to thank past and present members of the Systems group, specifically Amir Khojastepour, Tarik Muharemovic, Muharem Jacubovic, Jaber Borran, Chris Steger, Nadeem Ahmed, and Alex de Baynast. Most importantly, I would like to thank my family, my wife, parents and brothers, for their support and encouragement during the course of my studies.

Contents

Illustrations

Chapter 1
Introduction

The increasing prevalence of mobile devices and need for wireless information access has led to more demands on system designers to provide higher throughput and improved battery longevity. Wireless channels differ from their wired counterparts due to a phenomenon known as *fading*. As a result, techniques and algorithms from wired systems cannot always be directly applied to wireless scenarios.

Increasing the *diversity* of the transmission is a technique used to exploit the random fading effect in wireless systems. Diversity gains are possible when an information sequence is passed through multiple, independent realizations of the channel. *Spatial diversity* diversity gains can be achieved by using multiple transmit antennas. Performance is improved due to the increased likelihood of one of the data streams experiencing a good channel condition. Despite the promise shown by multiple antennas in mitigating the effects of fading, increasing the number of transmit antennas on small mobile devices is often impractical as a result of size and hardware complexity constraints.

To meet the demands of increased spectral and power efficiency without increasing the size of mobile devices, fundamentally new paradigms are needed to improve performance. User cooperation, in which nodes pool their resources together and cooperatively transmit their data, is a transmission technique that has recently emerged for the network setting. Cooperation provides a method of achieving spatial diversity without the need for multiple antenna's at the mobile nodes. Furthermore, utilizing cooperative techniques leads to a higher throughput than direct communication between nodes [1]. However, due to the relative infancy of network information theory, especially in wireless applications, none of the current proposed network coding schemes have fully exploited the true benefits of node cooperation.

The cooperation paradigm is certainly useful in the communication between handsets and base stations. Cooperation can lead to improved battery life and a higher throughput, thereby enabling high data rate multimedia applications. Although useful in the cellular context, conceptually cooperation can be applied in more general settings. Two immediate applications which can have improved performance from collaborating nodes are ad-hoc and sensor networks [2]. For example, in sensor networks, where power conservation is of paramount importance, low complexity cooperation protocols can be used to reduce the likelihood of a decoding error, which allows the nodes to operate at a lower power and still meet target data and error rate requirements.

1.1 Relay Channel

The simplest example of a cooperative network is the relay channel, which was first introduced in [3]. Relaying occurs when a helper node assists the source-destination nodes in communicating. An example of a relay system is shown in Figure 1.1. Although the concept of relaying is more than 30 years old, there are still many open problems for this channel. For example, in general the capacity of the relay channel is unknown even for the case of Gaussian channels. As a result, most of the research efforts have focused on finding efficient protocols that lead to lower bounds on the capacity. As a second example, consider channel coding. In direct transmission, techniques such as Turbo[4] and Low-Density Parity Check[5] coding exist that are nearly capacity achieving. In the relay setting, code design principles from direct transmission cannot be directly applied, and research efforts are ongoing to find powerful channel codes that approach the lower bounds on capacity for the relay channel [6, 7].

Fading channels are an area where relaying has shown great potential. Research efforts in the fast fading environment have generally been in the form of protocols that lead to lower bounds on the capacity. In the slow fading channel, results are typically expressed in terms of outage probability, which provides a lower bound to the frame error rate for coded systems [8]. Recent results have shown that low complexity relay protocols can achieve full diversity* and exhibit an outage performance similar to multiple antenna systems [9]. The relay node, through its spatial separation with

*A diversity order of d is obtained if for some constant C and power P, the outage behaves as $\frac{C}{P^d}$. Increased diversity can be obtained by adding antenna's and/or relays to the system. As shown later, diversity can also be increased through feedback.

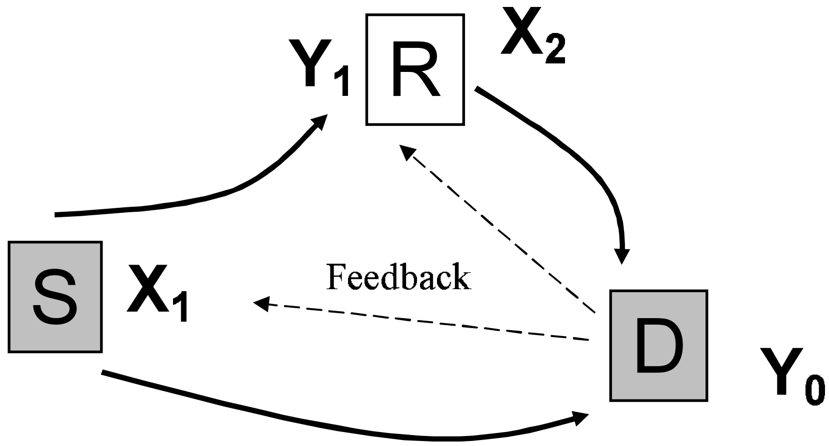


Figure 1.1 : Layout of the relay network with 3 nodes. The source transmits to the destination, and the relay node assists in the communication process. Communication along the links are corrupted by pathloss along the links in the network and noise at the receivers.

---

the source, transmits its signal through a channel that is independent of the source-destination channel, creating a distributed antenna array.

1.2 Performance Improvements through Feedback

Relaying, like multiple antenna systems, provides a powerful way to exploit the effects of fading in wireless systems. Another such method, which is the focus of this thesis, is to further improve performance in fading relay channels by exploiting feedback of the channel state information when it is available. Figure 1.1 demonstrates the feedback process in relay networks. The destination is assumed to have a perfect measure of the quality of all the links in the network. The destination can then use this perfect estimate to decide the power and rate for the source node (S) and the relay node (R).

The source and relay nodes, upon reception of the feedback information, transmit with the appropriate parameters to maximize the performance.

The feedback can be used to improve performance in various ways. In this work, we use the feedback information to adapt the transmission rate and power of the source and relay. When the source has applications that transmit data at a constant rate, the feedback is used to minimize the outage probability by performing temporal power control. On the other hand, when the source application allows for variable rate transmission, such as a file transfer, the feedback is used to maximize the effective data rate, or throughput.

In both constant rate and variable rate applications, the performance improvement with feedback is dependent on the degree of channel knowledge at the source and relay. Limits of performance can be derived when the source and relay have perfect knowledge of the channel state information in the network. However, in practice the feedback link to the transmitters will be limited. As a result, the perfect feedback scenario represents a bound on the performance of rate and power control algorithms with a limited rate feedback link.

1.3 Related Work

Our work is concerned with maximizing performance in a wireless system based on the amount of feedback available. The use of feedback in cooperative systems is a relatively new area, however, related studies have been performed for direct transmission.

We highlight some of the most similar work below.

For direct transmission, in the limit of perfect feedback, the work of [10] deals with the problem of minimizing the information outage probability in the limit of perfect channel knowledge at the transmitter and receiver. Similar results have also been derived [11] for the case of multiple antenna systems. The basic idea behind [10, 11] is to use power control to minimize the outage probability. The solution involves guaranteeing zero-outages for a subset of the channel states, and declaring outages and transmitting with no power for channel states that are too prohibitive in terms of the required power to invert the channel effects [11].

For multiple-antenna systems, the practical problem of outage minimization with a finite capacity feedback link has been explored recently. When a long-term power constraint is available, [12] constructs a power control algorithm based on the rate of the feedback link. The work of [13] discusses the concept of feedback diversity, and shows how increasing feedback bits leads to a diversity gain similar to the gains achieved by adding more spatial dimensions. When a short term power constraint is used, the work of [14] constructs optimal beamformers to reduce the outage probability based on the amount of feedback available. The authors in [15] looked at increasing the data rate through power and rate adaptation with limited feedback for a single antenna direct transmission system.

In the fading relay channel, as discussed in Section 1.1, the majority of the proposed work has dealt with searching for efficient protocols and generally assumes a constant transmit power. The results of [9] show how in the relay channel, efficient full-diversity achieving protocols are available that use simple repetition coding. Furthermore, [16, 17] discuss methods to calculate the diversity order and provides large power asymptotic results regarding the outage probability of the proposed relaying methods.

Recently, [18] addressed power allocation for some specific relay codes in the fast fading environment when perfect feedback is available at the transmitters. However, the authors did not consider outage minimization for the slow fading channel. The authors in [19] derived the delay-limited capacity for some time division relay protocols when perfect channel knowledge is available to the transmitters, and showed that this quantity is non-zero. We show in Chapter 5 that delay-limited capacity is not a very useful measure of performance in the slow fading channel and that throughput should be considered.

1.4 Contributions

Our contributions can be categorized based on the constraints of the source in the network. We first consider the case where the source transmits with a constant rate over all time. This type of model is valid in applications such as voice, where the desired system metric is a low probability of error for a given transmission rate. On the other hand, for applications such as file transfers, the transmission rate may be adapted over time based on the channel conditions. The adaptation can lead to a larger throughput.

1.4.1 Constant Rate Transmission

Assuming transmission at a fixed rate, we consider the use of power control to minimize the outage probability in cooperative networks. The gains from power control can also be understood by considering a fixed performance level, and observing the power savings achieved through temporal power adaptation. Our contributions in this regard are outlined next.

1.4.1.1 Perfect Channel Knowledge at the Transmitters

- For a given rate of transmission, we derive the optimal outage minimizing power control policy assuming a sum power constraint on the source and relay for any relay protocol when perfect channel knowledge is available to the transmitters. The procedure is general, and we show how to apply the results to various relay coding protocols.

- When perfect feedback is available, but power control not possible, we show that gains can still be achieved using phase control. By having the source and relay adjust their phases to allow for coherent combining at the destination, significant improvements are achieved over direct transmission. Furthermore, a hybrid between the decode and forward (DF) [20] and estimate and forward (EF) [21] protocols nearly achieves the universal lower bound on outage probability.

- We derive the optimal power control policy with perfect feedback and a peak power constraint on the source and relay. This constraint is very practical, as power amplifier non-linearities and FCC regulations can limit the instantaneous peak transmit power. For small average power constraints, the peak limitation does not degrade the outage performance. However, significant performance degradation occurs for large average power constraints.

1.4.1.2 Limited Feedback at the Transmitters

- Having perfect channel state knowledge at the transmitters may not be feasible in practice. In this direction, we derive a power control policy based on the rate of the feedback link. We construct a low complexity, sub-optimal procedure that performs very well for large average power constraints. Our analysis shows that with only a few bits of feedback, most of the gap to the perfect feedback power control policy can be achieved.

- We show that with the use of one bit of feedback, the diversity order of the amplify and forward (AF) [9] transmission scheme doubles to four. The increased diversity order leads to a performance comparable to a system with 4 transmit antenna's and constant power transmission on each antenna.

- When no feedback is available, we show the outage minimizing spatial power allocation policy for the AF protocol. Interestingly, we demonstrate that the source power must always exceed the relay power to minimize the outage. Furthermore, the performance loss from using equal source and relay powers is minimal for small source-relay distances.

1.4.2 Variable Rate Transmission

For applications that can support a variable rate of transmission, the use of feedback from the destination can be used to temporally adapt both the transmission rate and power. The rate of transmission is increased for good channel conditions, whereas the rate is decreased in poor channels to reduce the likelihood of a decoding error. Our contributions are outlined next.

- We derive a rate control procedure based on the limited capacity of the feedback link. The algorithm is general, and can be applied to any network code. We explicitly demonstrate results for the DF protocol, multi-hopping, selection relaying (SR) [9] and direct transmission.

- For a finite rate of feedback, we demonstrate that it is best to select transmission rates and powers such that the outage probability is relatively high. Small transmission rates lead to a minimal probability of a frame error, but a small throughput. On the other hand, large transmission rates lead to larger frame error rates. Throughput maximization requires a balancing of the transmission rate to guarantee the maximum spectral efficiency.

- We show that both power and rate control are usually unnecessary for throughput maximization. When the average power constraint is large, rate control with constant power is nearly optimal, especially for small rates of the feedback link. On the other hand, for small average power constraints, rate control is unnecessary, and power control with rate selection suffices.

- We show the power of relay coding over direct transmission. Decode and forward is shown to have tremendous throughput gains over direct transmission, with the largest percentage gains occurring at small average power constraints. Furthermore, DF has large gains over multi-hopping, suggesting the importance collaboration between the source and relay transmissions. We also show that for small to moderate average power constraints, having a half-duplex constraint on the relay node does not lead to a significant loss in performance compared to full-duplex relaying.

1.5 Outline

The thesis is organized as follows. Chapter 2 overviews relevant background information and notation. Next in Chapter 3 the optimal power control policy for a relay code is outlined for the case of perfect channel knowledge at the transmitters. Chapter 4 considers the case of finite rate feedback and power control. Chapter 5 looks at variable rate applications, and methods to maximize the throughput based on the rate of the feedback link. Chapter 6 provides concluding remarks and avenues for future research.

Chapter 2
Background

2.1 Relay Network

Consider the relay network in Figure 2.1, with one relay node and one source-destination pair. The relay assists in the communication of data between the source and the destination, and it does not produce its own data. It is assumed that link i in the network is attenuated by fading coefficient h_i, where $i \in \{0, 1, 2\}$. At both the source and relay, the received signal is corrupted by additive white Gaussian noise with zero mean and unit variance. The received signal at the relay is $y_1 = h_0 x_1 + z_1$, where x_1 is the relay input and z_1 is the noise at the relay. At the destination, the received signal is $y = h_1 x_1 + h_2 x_2 + z$, where x_2 is the input signal at the relay, and z is the noise at the destination. Direct transmission can be considered a special case where the relay is not present, or does not participate in the communication process. In this case, the transmit signal is x_1 and the received signal at the destination is $y = h_1 x_1 + z$.

2.2 Fading Channel Model

As implied by the receive signal equations for the relay channel, the fading is represented by a multiplicative gain on the transmit signal. This type of fading is referred to as block fading. The fading is characterized by the fact that the time scale of the channel changing is larger than a time slot. In a time slot, a packet or frame of length N symbols is transmitted, and each of the transmitted symbols undergo the same fading value. The length of time over which the channel is assumed constant is referred to as the *coherence time*. In our model, the coherence time is N symbols.

We assume communication where line of sight is not present. Under this assumption, the magnitudes of the fading coefficients are assumed to follow a Rayleigh distribution [22]. In the sequel, we will denote $\gamma_0 = |h_0|^2$, $\gamma_1 = |h_1|^2$ and $\gamma_2 = |h_2|^2$. The *network channel state* is defined by the 3-tuple $\underline{\gamma} = (\gamma_0, \gamma_1, \gamma_2)$, where γ_i follows an exponential distribution with mean λ_i, $i \in \{0, 1, 2\}$. The distribution for link i in the network is expressed as

$$f(\gamma_i) = \frac{1}{\lambda_i} e^{-\gamma_i/\lambda_i}, \tag{2.1}$$

and

$$F(\gamma_i) = 1 - e^{-\gamma/\lambda_i} \tag{2.2}$$

the PDF and CDF, respectively. The parameter λ_i captures the pathloss across link i in the network, which is a function of the length of the link, and the pathloss exponent α; typically, α lies in the range $(2, 5)$.

To consider the effect of the relay nodes positioning, we use the model shown in Figure 2.2. We assume that the distance between the source and relay is one unit, and the relay is located in a line between the source and destination. The parameter

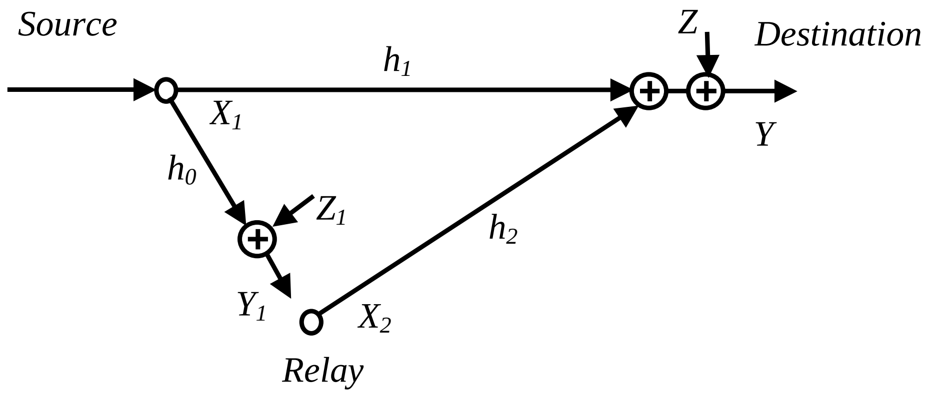


Figure 2.1 : Layout of the relay network with 3 nodes. The source transmits to the destination, and the relay node assists in the communication process. Communication along the links are corrupted by pathloss along the links in the network and Gaussian noise at the receivers.

---

d represents the distance from the source to the relay, and $1 - d$ is the distance from the relay to the destination. The mean value of the fading distribution for the source-relay link is consequently $\lambda_0 = \frac{1}{d^\alpha}$ and for the relay-destination link we have $\lambda_2 = \frac{1}{(1-d)^\alpha}$.

2.3 Performance Metric

2.3.1 Outage and Throughput

A practical analysis tool for the block fading environment is the outage probability [8], which for large blocklengths, serves as a lower bound to the frame error rate, making it useful for the analysis of coded systems. Outage probability is the probability that the instantaneous achievable rate of the channel is less than the transmission rate,

$$P_{out}(R, \mathbf{P_{avg}}) = \text{Prob}[R > R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))], \quad (2.3)$$

where R_{gen} is the instantaneous achievable rate of the transmission protocol used. In (2.3), $P_s(\underline{\gamma})$ is the transmit power of the source, $P_r(\underline{\gamma})$ is the transmit power of the relay and R is the attempted rate of transmission. Note that in (2.3) the source and relay powers have been written as functions of the instantaneous network channel state $\underline{\gamma}$ to show that power control is possible when information regarding the network channel state is available to the transmitters. In (2.3), the average power constraint is $\mathbf{P_{avg}}$, which is dependent on the type of power constraint in the network (see Section 2.3.2).

Due to outage events, the effective data rate is less than the attempted rate of transmission. Given source and relay power allocation policies of $P_s(\underline{\gamma})$ and $P_r(\underline{\gamma})$, respectively, the throughput can be expressed as

$$\begin{aligned} T = \int_{\underline{\gamma}} & R(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma})) \cdot \\ & \mathcal{I}_F\{R(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma})) < R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))\} f(\underline{\gamma}) d\underline{\gamma}, \end{aligned} \tag{2.4}$$

where $R_{gen}(\cdot)$ is the achievable rate of the transmission scheme, $f(\underline{\gamma})$ is the distribution function of the network channel state, $\mathcal{I}_F\{\cdot\}$ is the indicator function, and $R(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))$ denotes the fact that the in general, the transmit power and the attempted transmission rate can be a function of the channel state.

2.3.2 Power Constraints

The outage and throughput metrics are calculated for a given average power constraint in the network. The transmit power need not be constant, and can be adapted over time to improve performance. The degree to which power adaptation takes place depends on the amount of feedback available at the transmitters. If the source and relay have individual power constraints of $\overline{P}_s$ and $\overline{P}_r$, respectively, a power control policy satisfies a long term power constraints if $E[P_s(\underline{\gamma})] \leq \overline{P}_s$ and $E[P_r(\underline{\gamma})] \leq \overline{P}_r$. If channel state information is unavailable, then power adaptation is not possible. In this case the source transmits with a constant power $\overline{P}_s$ and the relay with a constant power $\overline{P}_r$.

If the source and relay are allowed to pool their power resources together, then the power adaptation can be performed using a sum power constraint. The sum power constraint problem has a long term average power constraint of Ψ, and when comparing against the individual power constraint problem, we consider the case where $\Psi = \overline{P}_s + \overline{P}_r$. The transmission policy satisfies the sum power constraint if $E[P_s(\underline{\gamma}) + P_r(\underline{\gamma})] \leq \Psi$. By using this constraint, more feasible power control policies are available and as a result the performance obtained with a sum power constraint will always exceed that of individual power constraints.

Referring back to the definition of outage probability in (2.3), if individual power constraints are utilized, then $\mathbf{P_{avg}} = (\overline{P}_s, \overline{P}_r)$. This means that the outage probability

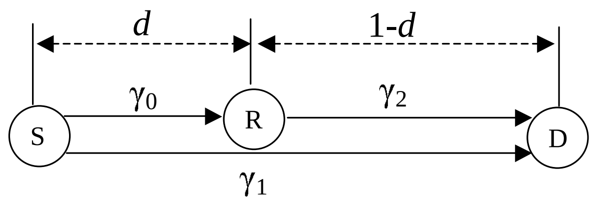


Figure 2.2 : Layout of the relay network with the relay node located along a straight line from the source to the destination. Assuming the fading value is inversely proportional to the distance, then $E[\gamma_0] = 1$, $E[\gamma_1] = \frac{1}{d^\alpha}$ and $E[\gamma_2] = \frac{1}{(1-d)^\alpha}$.

is defined subject to individual average power constraints on the source and relay. On the other hand, if the source and relay pool their resources and a sum power constraint is used, then $\mathbf{P_{avg}} = \Psi$. This means that the mean value of the sum of the source and relay powers must satisfy $\mathbf{P_{avg}}$.

2.4 Relaying Protocols

The *achievable rate* [10], the highest reliable data rate for a codeword, is determined by maximizing the mutual information of a given relay transmission protocol for a given channel state. The achievable rate is an asymptotic quantity that is achieved as the blocklength $N \to \infty$. Assuming a Gaussian noise process and perfect channel state knowledge at the receivers, we next highlight some important network coding protocols.

2.4.1 Half-Duplex Relays

If the nodes are 'cheap'[23], then transmission and reception simultaneously in the same frequency band is not possible. In this case, a practical transmission protocol is the amplify and forward (AF) technique, developed in [9]. Given a source with average power P_s and a relay with average power P_r, the achievable rate of the AF transmission protocol is [9]

$$R_{AF}(\underline{\gamma}, P_s, P_r) = \frac{1}{2}\log\left(1 + 2\gamma_1 P_s + \frac{4\gamma_2 P_s \gamma_0 P_r}{1 + 2P_s\gamma_0 + 2P_r\gamma_2}\right). \tag{2.5}$$

Amplify and forward operates in two phases. A time slot of N symbols is divided into two equal portions, each of length $N/2$ symbols. For the first half of the time slot, the source broadcasts its transmit signal to both the destination and the relay. In the second half of the slot, the source remains silent, and the relay forwards the signal it received from the source to the destination. Prior to forwarding, the relay scales the signal to meet its power constraint. The scaling assumes knowledge of the source-relay link parameter γ_0, which is a valid assumption for the relay.

Note that in (2.5), since each transmitter sends data for half the time slot, the source uses power $2P_s$ and the relay uses power $2P_r$ to guarantee an average power of $P_s + P_r$. The AF protocol was shown to achieve a diversity order of two in the fading channel with constant power transmission. In (2.5) the transmit powers in general can be functions of the channel state $\underline{\gamma}$. However, for clarity of exposition, we have removed references to the power being a function of the channel state in the description of the relay protocols.

Another recently proposed 'cheap' protocol is selection relaying (SR). Given a source with average power P_s and a relay with average power P_r, the achievable rate of SR is [9]

$$R_{SR}(\underline{\gamma}, P_s, P_r) = \begin{cases} R_{SR,1}, \ \text{if } \frac{1}{2}\log(1 + P_s\gamma_0) > R \\ R_{SR,2}, \ \text{if } \frac{1}{2}\log(1 + P_s\gamma_0) \leq R, \end{cases}$$

where $R_{SR,1} = \frac{1}{2}\log(1+P_s\gamma_1+P_r\gamma_2)$, $R_{SR,2} = \frac{1}{2}\log(1+2P_s\gamma_1)$ and R is the attempted rate of transmission. Additionally, P_s is the source power, and P_r is the power of the relay. For a given rate of transmission, if the source-relay link results in an outage, then the relay remains silent in the second half of the time slot, and the source repeats its transmission to the destination. However, if the relay is able to decode the source's transmission, then in the second time slot, the relay re-transmits the packet to the destination and the source remains silent.

2.4.2 Full-Duplex Relays

For nodes that can transmit and receive simultaneously, protocols with higher achievable rates are available. The limits of communication on the relay channel are defined by the cut-set upper bound(UB)[24]. When no channel side information is available at the transmitters, the upper bound on the achievable rate is

$$R_{UB}(\underline{\gamma}, P_s, P_r) = \max_{0\leq|\rho|\leq 1} \min\{\log(1 + (1 - |\rho|^2)(\gamma_0 + \gamma_1)P_s), \quad (2.6)$$
$$\log(1 + \gamma_1 P_s + \gamma_2 P_r + 2\rho\cos(\measuredangle h_1 - \measuredangle h_2)\sqrt{\gamma_1\gamma_2 P_s P_r})\}.$$

The parameter ρ controls the correlation between the input of the source and relay channels. Note that in general, when channel state is not available at the transmitters, then ρ is a constant and cannot be adjusted based on channel state. However, the outage minimizing constant can be obtained based on the statistics of the network channel states. The achievable rate of (2.6) is an upper bound, and no coding schemes have been found that have an achievable rate of R_{UB}.

If the channel state $\underline{h} = (h_0, h_1, h_2)$ is known to all nodes, then transmitters can perfectly align their signals at the receiver of the destination node. More specifically, we can completely ignore all the phases values of the constants h_1, h_2, h_0 by multiplying the transmitted signals X_1, X_2 by the values $e^{-j\measuredangle h_1}$, $e^{-j\measuredangle h_2}$ and the relay received signal Y_1 by the value $e^{j\measuredangle h_1 - j\measuredangle h_0}$. Also, $\rho = \rho(\underline{h})$ can also be optimally found, therefore the upper bound in the case of transmitter channel knowledge is given by

$$\begin{aligned} R_{UB}(\underline{\gamma}, P_s, P_r) = \max_{0 \leq \rho \leq 1} \min\{&\log(1 + (1 - \rho^2)(\gamma_0 + \gamma_1)P_s), \\ &\log(1 + \gamma_1 P_s + \gamma_2 P_r + 2\rho\sqrt{\gamma_1\gamma_2 P_s P_r})\}. \end{aligned} \tag{2.7}$$

In terms of achievable schemes, under the assumption that the relay fully decodes the transmission from the source, an achievable rate was derived in [20] and shown to be

$$\begin{aligned} R_{DF}(\underline{\gamma}, P_s, P_r) = \max_{0 \leq |\rho| \leq 1} \min\{&\log(1 + (1 - |\rho|^2)\gamma_0 P_s), \\ \log(1 + \gamma_1 P_s + \gamma_2 P_r + 2\rho\cos(\measuredangle h_1 - \measuredangle h_2)&\sqrt{\gamma_1\gamma_2 P_s P_r})\}. \end{aligned} \tag{2.8}$$

Once gain, in (2.8), if channel state is not available at the transmitters, then the outage minimizing ρ is chosen based on the channel statistics and used for all time. Similar to the case of the upper bound on the capacity, when channel knowledge is available to the transmitters, then the decode and forward (DF) achievable rate is reduced to

$$R_{DF}(\underline{\gamma}, P_s, P_r) = \max_{0 \leq \rho \leq 1} \min\{\log(1 + (1 - \rho^2)\gamma_0 P_s), \\ \log(1 + \gamma_1 P_s + \gamma_2 P_r + 2\rho\sqrt{\gamma_1 \gamma_2 P_s P_r})\}. \quad (2.9)$$

The Markovian decode and forward scheme is limited by the fact that the relay fully decodes, which would result in poor performance when the source-relay link is in a deep fade. Recently, a new transmission protocol called estimate and forward [21] was proposed, and it relies on the principle that the relay uses a statistical estimate of the signal from the source instead of fully decoding. The achievable rate of this scheme is

$$R_{EF}(\underline{\gamma}, P_s, P_r) = \log\left(1 + P_s\gamma_1 + \frac{P_s\gamma_0 P_r \gamma_2}{1 + P_s\gamma_0 + P_s\gamma_1 + P_r\gamma_2}\right). \quad (2.10)$$

The interesting point regarding this transmission scheme is that using estimate and forward allows for an achievable rate that always exceeds direct transmission.

It was shown in [21], that both EF and DF have their utilities in certain instances of the network channel state. When the relay is positioned near the source, then the DF protocol performs well, while the EF protocol has a high achievable rate for relays that are close to the destination. With this in mind, we propose an adaptive protocol which chooses the protocol with the larger achievable rate in each time slot. The proposed transmission scheme, which is a hybrid between the estimate and forward and the decode and forward transmission schemes, has an achievable rate as follows

$$R_{HB}(\underline{\gamma}, P_s, P_r) = \max\{R_{EF}(\underline{\gamma}, P_s, P_r), R_{DF}(\underline{\gamma}, P_s, P_r)\}. \tag{2.11}$$

Interestingly, it will be seen that the hybrid performs well in terms of outage performance in cases where EF and DF perform poorly.

2.4.3 Other Transmission Protocols

To assess the performance of the described relay codes, two non-cooperative benchmarks will be used. First is the case of direct transmission, which has an achievable rate of,

$$R_{DT}(\gamma_1, P_s) = \log(1 + P_s\gamma_1). \tag{2.12}$$

In comparing direct transmission with cooperative schemes, the total power consumption of each protocol must be considered. Cooperative techniques have two power supplies, one at the source and the other at the relay. To fairly compare against direct transmission, for most of this work we consider the case where the total network power (i.e. source plus relay power) is equated with the power used in direct transmission.

Another transmission protocol worth comparing against relaying is data forwarding or multi-hopping. In this scheme a relay node is present that receives information from the source and decodes the message. The relay then transmits the decoded information using an optimal channel code towards the destination. The destination only utilizes the message from the relay. A decoding error at the relay will clearly lead to an error at the destination. The achievable rate of this cascaded system is simply the minimum of the capacity of the individual links in the network, i.e.

$$R_{MH}(\underline{\gamma}, P_s, P_r) = \min\{\log(1 + P_s\gamma_0), \log(1 + P_r\gamma_2)\}. \tag{2.13}$$

2.5 Direct Transmission Analysis

In Chapter 3 we consider outage minimization in a relay network when perfect channel state knowledge is available at the source and relay, and Chapter 4 addresses the practical constraint of outage minimization with a limited rate feedback link. In Chapter 5, we derive a throughput maximization policy for relay networks based on the rate of the feedback link and on constraints imposed by the source. To better understand these solutions for relay networks, it is instructive to review some well-known results for direct transmission. We next consider the outage minimization and throughput maximization process for a direct transmission system in the limit of perfect feedback.

2.5.1 Outage Minimization for Direct Transmission

Given a source transmitting at a rate R nats/sec/Hz, the objective is to minimize the outage probability subject to a long term constraint on the average power,

$$E_{\gamma_1}[P_s(\gamma_1)] \leq P_{avg}.$$

Since perfect feedback of γ_1 is available at the source, the power can be adapted for each channel state to minimize the outage probability while satisfying the long term constraint on the average power. The optimal solution for $P_s(\gamma_1)$ turns out to be [10],

$$P_s(\gamma_1) = \begin{cases} P_s^*(\gamma_1), & \text{if } P_s^*(\gamma_1) \leq p^* \\ 0, & \text{if } P_{st}^*(\gamma_1) > p^*, \end{cases} \tag{2.14}$$

where

$$P_s^*(\gamma_1) = \frac{e^R - 1}{\gamma_1}.$$

$P_s^*(\cdot)$ is the power required to invert the instantaneous effects of the channel. The optimal outage minimization solution is to guarantee zero outages for a subset of channel states, and to declare outage and not transmit with any power for channel states below some threshold. The power threshold p^* is chosen to satisfy the long term power constraint,

$$\int_{\zeta}^{\infty} \frac{e^R - 1}{\gamma_1} f(\gamma_1) d\gamma_1 = P_{avg},$$

where $\zeta = \frac{e^R - 1}{p^*}$ and $f(\gamma_1)$ is the probability distribution function of the source-destination channel gain γ_1. The resulting outage probability is then $P_{out} = Prob[\gamma_1 \leq \zeta]$.

When constant power transmission is used, plotting the outage probability as a function of the SNR on a log scale results in a constant decay rate of the outage curve, as can be seen in Figure 2.3. When power control is used with perfect feedback, the decay rate becomes exponential. This leads to significant power savings over constant

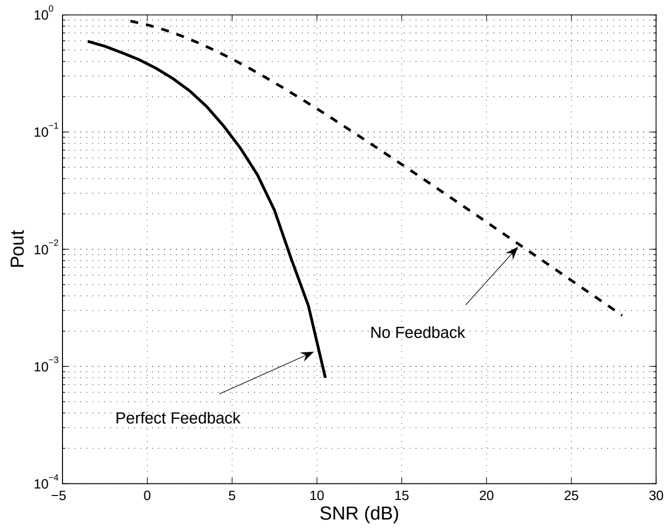


Figure 2.3 : Probability of outage vs. SNR for direct transmission with constant power transmission (no feedback) and with perfect feedback. The transmission rate is $R = 1$ nats/sec/Hz.

---

power transmission at a target outage probability [10]. For example, at an outage probability of 10^{-2}, having a perfect estimate of the feedback link leads to an SNR saving of more than $14dB$.

2.5.2 Throughput Maximization for Direct Transmission

Maximizing the throughput for direct transmission depends on the amount of feedback available to the transmitter. We next consider the case with perfect feedback.

If the transmission rate is adapted for each channel state, for constant power transmission, the throughput can be expressed as

$$T = \int_0^\infty \log(1 + P_s\gamma_1) f(\gamma_1) d\gamma_1.$$

Since constant power transmission is used, the adaptation can be done without any

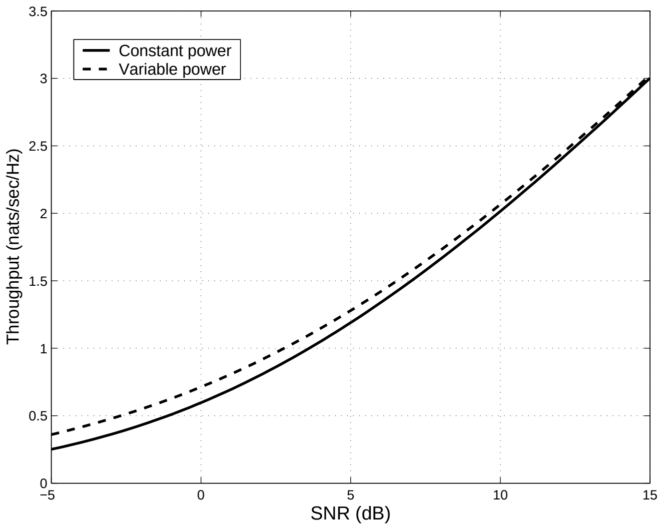


Figure 2.4 : Throughput vs. SNR for direct transmission with both constant and variable power transmission.

---

outages. When power control is performed in addition to rate control, a higher throughput is possible. If the power is adapted along with the rate for each channel state, then the optimal solution to maximize the throughput is to perform water-filling [25], which leads to a power adaptation of

$$P_s(\gamma_1) = \begin{cases} \frac{1}{\lambda} - \frac{1}{\gamma_1}, & \text{if } \gamma_1 \geq \lambda \\ 0, & \text{if } \gamma_1 < \lambda. \end{cases} \quad (2.15)$$

The cut-off parameter λ is chosen to satisfy the long term power constraint

$$\int_{\lambda}^{\infty} \left(\frac{1}{\lambda} - \frac{1}{\gamma_1} \right) f(\gamma_1) d\gamma_1 = P_{avg}.$$

We see that for poor channel conditions, transmission is shut off and an outage is declared, whereas in better channel conditions the rate and power are increasing functions of γ_1. The throughput of such a scheme is then

$$T = \int_0^{\infty} \log(1 + P_s(\gamma_1) \cdot \gamma_1) f(\gamma_1) d\gamma_1 = \int_{\lambda}^{\infty} \log\left(\frac{\gamma_1}{\lambda}\right) f(\gamma_1) d\gamma_1.$$

Figure 2.4 shows the throughput results for direct transmission when perfect feedback is available. We see from this figure that at large SNR's, constant power transmission is nearly optimal. On the other hand, the combined power and rate control policy can lead to modest gains over constant power transmission in the low power regime.

Any finite rate of feedback will lead to a throughput that is upper-bounded by the perfect feedback limit. The objective is to try to achieve significant gains with as few bits as possible.

Chapter 3

Outage Minimization with Perfect Feedback

3.1 Introduction

In this chapter we consider the block fading environment and derive the performance limits in the relay channel when the transmitters have perfect channel state knowledge. First, if the source and relay must transmit with a constant power, then with the use of channel state information, they can allocate for phase offsets to ensure that the signals at the receiver add coherently. Additionally, for different channel states, the source and relay can modify the correlation between their transmitted signals to further reduce the outage. Under these assumptions, the hybrid protocol of (2.11) is shown to be sufficient to approach the lower bound on outage probability.

Second, we derive the optimal power control strategy for any relay code, and compare the performance against the lower bound on outage probability. It is shown that the decode-forward protocol, which exhibits optimal performance for special cases of the relay nodes position in the ergodic environment, is nearly optimal when the relay is positioned close to the source. The estimate-forward protocol is shown to be robust to the position of the relay node, and outperforms decode-forward when the relay moves toward the destination. Our results also outline the power of network coding, as we show that relaying can significantly outperform multiple antenna transmission, with the gains being most evident when the relay is midway between the source and destination.

Third, we consider a practical constraint on the power policies of the source and relay. Due to the effects of amplifier nonlinearities, the peak power would often be limited at the source and relay to avoid excessive distortion in the transmit signal. Furthermore, peak limitations imposed by FCC regulations will limit the range of powers available in a given power control policy. In this direction, we derive the optimal power control policy when the source and relay have peak power limitations.

The rest of the chapter is organized as follows. Section 3.2 investigates the outage performance of the relay protocols under the assumption of constant power transmission. Section 3.3 describes the optimal power allocation when the perfect network channel state is available to the transmitters and power control is performed. Section 3.4 considers the effect of a peak power limitation on the outage minimization process, and Section 3.5 concludes the chapter.

3.2 Outage Minimization with Constant Power Transmission

In a relay system, even when the source and relay are restricted to transmit with a constant power in each time slot, methods exist to reduce the outage probability, which is not the case for a direct transmission system. When channel state information is available to the source and relay, two optimizations can be performed. First, the phase at the source and relay can be corrected, such that the signals at the destination

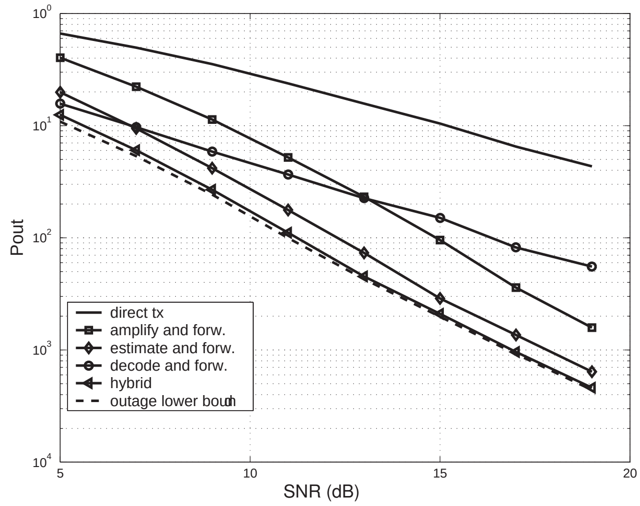


Figure 3.1 : Probability of outage vs. SNR for various relaying protocols using constant power and a rate $R = 1$ nats/sec/Hz, $d = 0.5$, and $\alpha = 3$. It is assumed that the source and relay have equal power constraints.

---

node combine coherently. Second, for some network coding protocols, the correlation between the signals from the source and relay can be adjusted to maximize the rate in each channel state. Both of these optimizations can be performed while performing constant power transmission.

Figure 3.1 shows the outage probability results for the case of $d = 0.5$, where d is the distance parameter from Figure 2.2, with the source and relay able to use phase correction and select the optimal ρ for each transmission. It is assumed that the source and relay both have the same average power constraint, and the plots are versus the network SNR (source and relay power) for comparison. The results for the direct link system using the same power as the total power in the relaying system is shown as a baseline for comparison. Since the decode and forward protocol suffers from decoding errors at the relay, it also has the same diversity order as direct link transmission, although a better coding gain. The amplify and forward protocol has a second order diversity, yet it has poor performance at low powers. This protocol suffers from the fact that the source and relay remain idle for half of each transmission slot. The estimate and forward is also seen in the figure, and it has a 2.5dB advantage over amplify and forward. Amazingly, the hybrid protocol is shown in the figure to closely follow the outage lower bound for a relay channel with constant power. This confirms that from an outage perspective with constant power transmission, the hybrid protocol is sufficient to approach the fundamental limits.

Using a distance of $d = 0.5$ indicates that the relay is midway between the source and relay. However, it is interesting how robust the transmission schemes are to the relay node's position. In Figure 3.2, for a a fixed SNR of 10dB, the outage probability is shown as a function of d. The amplify and forward protocol performs well when the relay is located midway between the source and destination, but performance degrades when d is small or large. For a good source-relay link, which occurs when d is small, the decode and forward protocol performs well, as the chance of outage on this link is low. As the relay moves towards the destination, the performance degrades substantially. The estimate and forward protocol exhibits almost the opposite behavior. For relays that are closer to the source, it performs poorly, while as the relay-destination link reduces, the performance approaches the optimal solution. Finally, the hybrid protocol is also shown in Figure 3.2, and it can be seen that

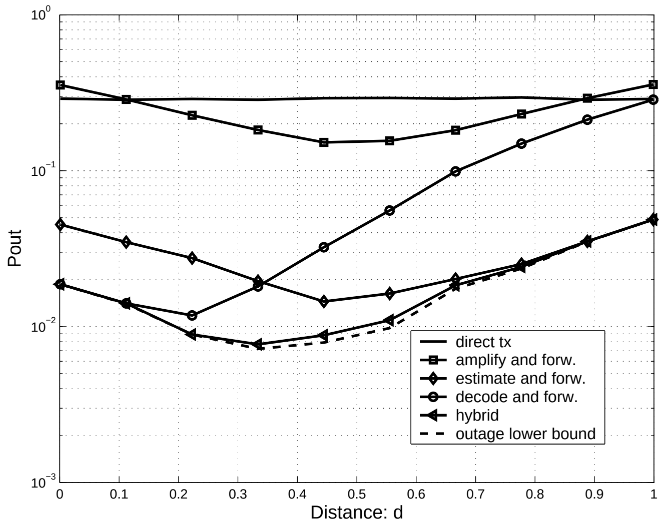


Figure 3.2 : Probability of outage vs. relay distance to source for various relaying protocols using constant power and a rate $R = 1$ nats/sec/Hz, and $\alpha = 3$ with an SNR of 10dB. Source and Relay have equal power constraints.

---

throughout nearly the entire range of d, the outage performance follows closely to the lower bound on outage probability. This indicates that the hybrid protocol is robust to node positioning. The power of the HB protocol becomes clearer by looking at distance d=0.33, where both the EF and DF protocols are far from the lower bound, but the HB still closely follows the bound. By using a combination of the DF and EF protocols, the HB protocol can compensate for their mutual weakness.

3.3 Optimal Power Control with Perfect Feedback

In this section, we derive the optimal power control policy for any relay code when perfect feedback is available to both the source and relay. The solution has similarities to that of a two-block fading channel[10], with one important difference. The source and relay power are not independent, since when the source power is zero, the relay power must also be zero. On the other hand, in the two-block fading channel, each sub-block is independent.

3.3.1 General Procedure

When the network channel state is available at the source and relay, outage minimization with power control can provide significant performance improvements, as will be seen next. Given a network channel state of $\underline{\gamma}$ that is perfectly measured at the destination, based on the power control, the source and relay are instructed to transmit with powers $P_s(\underline{\gamma})$ and $P_r(\underline{\gamma})$, respectively. Assuming a generic transmission protocol with an achievable rate of $R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))$, the outage probability becomes

$$\text{Prob}(R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma})) < R) = E_{\underline{\gamma}}[\mathcal{I}_F\{R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma})) < R\}], \tag{3.1}$$

where $\mathcal{I}_F(\cdot)$ is the indicator function. To obtain significant reductions in outage probability, the minimization is done with respect to an average long term sum power constraint, meaning that

$$E_{\underline{\gamma}}[P_s(\underline{\gamma}) + P_r(\underline{\gamma})] \leq 2P_{avg}. \tag{3.2}$$

The network power optimization problem involves the minimization of outage subject to a sum power constraint with two variables P_s and P_r, which seems intractable. However, we next show that this problem can be turned into a single variable optimization problem which allows us to use the same idea of outage minimization used for the single link fading channel [10].

Theorem 3.3.1. *The optimal power allocation that minimizes the outage for a relaying protocol with achievable rate* $R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))$ *under a long term power constraint is*

$$P_{LT}(\underline{\gamma}) = \begin{cases} P^*_{st}(\underline{\gamma}), & \text{with probability 1, if } P^*_{st}(\underline{\gamma}) < p^* \\ P^*_{st}(\underline{\gamma}), & \text{with probability } w_0, \text{ if } P^*_{st}(\underline{\gamma}) = p^* \\ 0, & \text{with probability } 1 - w_0, \text{ if } P^*_{st}(\underline{\gamma}) = p^* \\ 0, & \text{with probability 1, if } P^*_{st}(\underline{\gamma}) > p^*. \end{cases} \tag{3.3}$$

Here P^*_{st} *is the solution to* $T(\underline{\gamma}, P_{st}(\underline{\gamma})) = R$*,* $w_0 \in (0,1)$*, and*

$$T(\underline{\gamma}, P_{st}(\underline{\gamma})) = \max_{P_s(\underline{\gamma}), P_r(\underline{\gamma})} \{R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma})) : P_s(\underline{\gamma}) + P_r(\underline{\gamma}) \leq 2P_{st}(\underline{\gamma})\}, \tag{3.4}$$

where $P_s(\underline{\gamma})$ *is the instantaneous source power and* $P_r(\underline{\gamma})$ *is the instantaneous relay power. Furthermore,* p^* *is chosen such that the average power constraint is satisfied.*

Proof: In general, the procedure for minimizing the outage probability under a long term constraint involves two steps, as was shown in [10]. The first step requires the solution to a short term power allocation, which minimizes the network power and guarantees zero outage in each network channel state while transmitting at the target spectral efficiency. However, as will be seen, the key difference with network power control and traditional power control is in the solution of the short term power, where an additional optimization must be performed to maximize the achievable rate by finding the optimal values of P_s and P_r given a constraint on their sum. The second step involves finding a cutoff region that modifies the short term power allocation to shut off transmission in poor channel conditions. The cutoff region is determined to satisfy the average sum power constraint. Next, we describe the algorithm in more detail.

The minimization of (3.1) with a long-term sum power constraint requires the solution of a short term power constraint $P_s(\underline{\gamma}) + P_r(\underline{\gamma}) = 2P_{st}^*(\underline{\gamma})$ which is a function of the current network state $\underline{\gamma}$. Given the solution of the short term power allocation, the optimal power allocation with a long term constraint has the following structure [10]

$$P_{LT}(\underline{\gamma}) = \begin{cases} P_{st}^*(\underline{\gamma}), \text{with probability } w(\underline{\gamma}) \\ 0, \text{with probability } 1 - w(\underline{\gamma}). \end{cases} \tag{3.5}$$

The minimized outage probability is then $P_{out}(\underline{\gamma}, R) = E_{\underline{\gamma}}[1 - w(\underline{\gamma})]$. To obtain $P_{st}^*(\underline{\gamma})$, we find a power control policy with minimum power that guarantees zero outage at transmission rate R. Note that for a given $P_{st}(\underline{\gamma})$, infinitely many choices exist such that $P_s(\underline{\gamma}) + P_r(\underline{\gamma}) = 2P_{st}(\underline{\gamma})$. However, based on Proposition 3 in [10], under a short-term power allocation, the optimal policy is to maximize $R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))$ for a given P_{st}. If we let

$$T(\underline{\gamma}, P_{st}(\underline{\gamma})) = \max_{P_s(\underline{\gamma}), P_r(\underline{\gamma})} \{R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma})) : P_s(\underline{\gamma}) + P_r(\underline{\gamma}) \leq 2P_{st}(\underline{\gamma})\}, \tag{3.6}$$

then for a fixed $P_{st}(\underline{\gamma})$, (3.6) can be written as

$$T(\underline{\gamma}, P_s(\underline{\gamma})) = \max_{P_s(\underline{\gamma})}\{R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), 2P_{st}(\underline{\gamma}) - P_s(\underline{\gamma})) : 0 \leq P_s(\underline{\gamma}) \leq 2P_{st}(\underline{\gamma})\}. \quad (3.7)$$

The maximization can be done over the single variable $P_s(\underline{\gamma})$. Note that with the use of (3.7), the original optimization over two variables $P_s(\underline{\gamma})$ and $P_r(\underline{\gamma})$ is now turned into a single variable maximization over $P_s(\underline{\gamma})$ given the sum power constraint $2P_{st}(\underline{\gamma})$. Now, for any $P_{st}(\underline{\gamma})$, the optimal short term power allocation between the source and relay is known that maximizes the achievable rate. Since the short term allocation is simply a function of P_{st}, we can use the same procedure as [10] in finding the optimal solution. The next step is to determine the optimal value of $P^*_{st}(\underline{\gamma})$.

The optimal short term power, $P^*_{st}(\underline{\gamma})$, can be obtained as the solution to

$$P^*_{st}(\underline{\gamma}) = \min_{P_{st}(\underline{\gamma})} T(\underline{\gamma}, P_{st}(\underline{\gamma})) \geq R. \quad (3.8)$$

Note that since $T(\underline{\gamma}, P_{st}(\underline{\gamma}))$ is monotonically increasing in P_{st}, then P^*_{st} exists. This corresponds to the minimum power for zero outage, given that the source and relay power are optimally calculated to maximize the achievable rate.

The policy $P^*_{st}(\underline{\gamma})$ guarantees zero outage for each channel state $\underline{\gamma}$. To ensure that the long term power constraint is satisfied, a weighting function $w(\underline{\gamma})$ is found as the solution to

$$\max_{w}\{E[P^*_{st}(\underline{\gamma})w(\underline{\gamma})] \leq P_{avg}, 0 \leq w(\underline{\gamma}) \leq 1\}. \quad (3.9)$$

Following the discussion of [10], the optimal value of $w(\underline{\gamma})$ has the form

$$w(\underline{\gamma}) = \begin{cases} 1, \text{ if } P_{st}^*(\underline{\gamma}) < p^* \\ w_0, \text{if } P_{st}^*(\underline{\gamma}) = p^* \\ 0, \text{ if } P_{st}^*(\underline{\gamma}) > p^* \end{cases} \tag{3.10}$$

with $w_0 \in (0, 1)$ and

$$p^* = \sup \left\{ p : \int_{\mathcal{R}(p)} P_{st}^*(\underline{\gamma}) dF(\underline{\gamma}) < P_{avg} \right\}, \tag{3.11}$$

and

$$\mathcal{R}(p) = \{\underline{\gamma} : P_{st}^*(\underline{\gamma}) < p\}. \tag{3.12}$$

Note that for a continuous distribution of $\underline{\gamma}$, $F(\underline{\gamma})$, $Prob(P_{st}^* = p^*)$ is a set of measure zero [10], so any $w_0 \in (0, 1)$ suffices. The result then follows. ■

From the proof of Theorem 3.3.1, it can be seen that the region $\mathcal{R}(p)$ is the set of all network channel states $\underline{\gamma}$ that require $P_{st}^*(\underline{\gamma}) < p$. The outage region can be interpreted as a volume in the 3-D space of all $(\gamma_0, \gamma_1, \gamma_2)$ that requires more power than p^* to invert the channel effects.

To summarize, minimizing the outage and satisfying the average sum power constraint involves first solving a short term power allocation problem that completely inverts the effects of the channel, and this is followed by a cutoff power that guarantees the sum average power constraint. By using (3.7), the best allocation of power between the source and relay is determined for any given network channel state. The power allocation procedure for specific protocols is similar, except the solution of $T(\underline{\gamma}, P_{st}(\underline{\gamma}))$ varies depending on the form of the achievable rate. In the next section, an analytical expression $T(\underline{\gamma}, P_{st}(\underline{\gamma}))$ will be found for the protocols of interest in this work.

3.3.2 Power Control for Specific Relaying Protocols

We next focus the results of Section 3.3.1 to specific relay protocols. We will study the form of $T(\underline{\gamma}, P_{st}(\underline{\gamma}))$ for the EF, AF, and Hybrid Protocols. We limit our study to $T(\underline{\gamma}, P_{st}(\underline{\gamma}))$ since the solution to the optimal power allocation for each of these protocols is similar, except for the form of $T(\underline{\gamma}, P_{st}(\underline{\gamma}))$. In general the power functions P_s, P_r, P_{st}, which were defined in Section 3.3.1, are functions of $\underline{\gamma}$. However, for simplicity of presentation, we drop the reference to $\underline{\gamma}$ in this section.

3.3.2.1 Estimate/Amplify and Forward

To solve for the optimal power allocation using the estimate and forward protocol, two steps need to be performed. First, the function $T_{EF}(\underline{\gamma}, P_{st})$ must be obtained, by using (3.6). For the estimate and forward technique, using $R_{EF}(\underline{\gamma}, P_s, P_r)$, the function $T_{EF}(\underline{\gamma}, P_{st})$ can be written as

$$T_{EF}(\underline{\gamma}, P_{st}) = \max_{P_s} \left\{ \log \left(1 + \gamma_1 P_s + \frac{P_s \gamma_0 (2P_{st} - P_s))\gamma_2}{1 + P_s \gamma_0 + (2P_{st} - P_s)\gamma_2} \right) : 0 \leq P_s \leq 2P_{st} \right\}, \tag{3.13}$$

where we have replaced P_r with $2P_{st} - P_s$. The maximizing source power can be computed by solving for P_s in the following

$$\frac{\partial R_{EF}(\underline{\gamma}, P_s, 2P_{st} - P_s)}{\partial P_s} = 0,$$

where $P_s \in (0, 2P_{st})$. After solving for P_s^*, the form of $T_{EF}(\underline{\gamma}, P_{st})$ is then known. This gives the maximum achievable rate given that the constraint on the power is $P_s + P_r = 2P_{st}$.

Next, assuming a transmission rate R, we need to solve for the minimum value of P_{st} which satisfies the satisfies the rate requirement, which is shown in (3.8). However, since the achievable rate is monotonically increasing in P_{st}, the minimum power occurs when the R is met with equality. That is,

$$T_{EF}(\underline{\gamma}, P_{st}^*) = R. \tag{3.14}$$

Once P_{st}^* is known, then the cutoff power from (3.11) is determined that ensures the sum power constraint is met. This cutoff point, in conjunction with P_{st}^* determines the optimal power allocation. For the amplify and forward protocol, the same procedure is performed, except now $R_{AF}(\underline{\gamma}, P_s, P_r)$ is used.

3.3.2.2 Hybrid Protocol and Outage Lower Bound

For the hybrid protocol, a simplification can be done to solve for $T_{HB}(\underline{\gamma}, P_{st})$, which can be expressed as

$$T_{HB}(\underline{\gamma}, P_{st}) = \max_{P_s, P_r} \left\{ \max\{R_{EF}(\underline{\gamma}, P_s, P_r), R_{DF}(\underline{\gamma}, P_s, P_r)\} : P_s + P_r \leq 2P_{st} \right\}.$$

Note that in the above, the outer 'max' operation maximizes over all power control policies with average source power $P_s \in (0, 2P_{st})$. The inner max operation selects the largest value between the estimate and forward and the decode and forward protocols for a given P_s. Because the inner max is simply selecting between two choices, the two operations can be interchanged without affecting the solution. As a result, we have

$$\begin{aligned} T_{HB}(\underline{\gamma}, P_{st}) = \max \Big\{ & \max_{P_s} \{R_{EF}(\underline{\gamma}, P_s, 2P_{st} - P_s) : 0 \le P_s \le 2P_{st}\}, \\ & \max_{P_s} \{R_{DF}(\underline{\gamma}, P_s, 2P_{st} - P_s) : 0 \le P_s \le 2P_{st}\} \Big\} \\ & = \max\{T_{EF}(\underline{\gamma}, P_{st}), T_{DF}(\underline{\gamma}, P_{st})\}. \end{aligned} \tag{3.15}$$

Note that in (3.15), all references to P_r have been replaced with $2P_{st} - P_s$, allowing for the maximization to be done over the single variable P_s. Maximizing $T_{EF}(\underline{\gamma}, P_{st}(\underline{\gamma}))$ has already been discussed, so we focus on the decode and forward protocol.

Recall that when channel state is available at the transmitters, with phase cancellation, we have that

$$\begin{aligned} & R_{DF}(\underline{\gamma}, P_s, P_r) = \\ & \quad \max_{0 \le \rho \le 1} \min\{\log(1 + (1 - \rho^2)\gamma_0 P_s), \log(1 + \gamma_1 P_s + \gamma_2 P_r + 2\rho\sqrt{\gamma_1 \gamma_2 P_s P_r})\}. \end{aligned} \tag{3.16}$$

The solution for $T_{DF}(\underline{\gamma}, P_{st})$ depends on the values of $\underline{\gamma}$, P_s and P_r. We discuss the solution based on three intervals.

First, by equating the two terms inside the minimum, it is apparent that if $\gamma_0 < \gamma_1 + \gamma_2 \frac{P_r}{P_s}$, then $\log(1 + (1 - \rho^2)\gamma_0 P_s) \le \log(1 + \gamma_1 P_s + \gamma_2 P_r + 2\rho\sqrt{\gamma_1 \gamma_2 P_s P_r})$ for any

ρ, and as a result

$$R_{DF}(\underline{\gamma}, P_s, P_r) = \log(1 + \gamma_0(1 - \rho^2)P_s).$$

However, since $\rho \in (0, 1)$, it is sufficient to write the maximizing value of R_{DF} as

$$R_{DF}(\underline{\gamma}, P_s, P_r) = \log(1 + \gamma_0 P_s). \tag{3.17}$$

Since $\gamma_0 < \gamma_1 + \gamma_2 \frac{P_r}{P_s}$ and $P_r = 2P_{st} - P_s$, we have that $P_s \frac{\gamma_2}{\gamma_2+\gamma_0-\gamma_1} < P_s < 2P_{st} \frac{\gamma_2}{\gamma_2+\gamma_0-\gamma_1}$. Maximizing R_{DF} is equivalent to setting $P_s^* = \frac{2P_{st}\gamma_2}{\gamma_2+\gamma_0-\gamma_1}$. This leads to

$$T_{DF}(\underline{\gamma}, P_{st}) = \log\left(1 + \frac{2P_{st}\gamma_0\gamma_2}{\gamma_2 + \gamma_0 - \gamma_1}\right). \tag{3.18}$$

Second, if $\gamma_0 < \gamma_1$, then no additional restriction is placed on P_s, since in this case direct transmission is superior to the decode and forward scheme, it is sufficient to set $P_s^* = 2P_{st}$. This leads a maximum rate of

$$T_{DF}(\underline{\gamma}, P_{st}) = \log(1 + 2P_{st}\gamma_1). \tag{3.19}$$

The final interval occurs if $\gamma_0 > \gamma_1 + \gamma_2 \frac{P_r}{P_s}$, then (3.16) is simplified by noting that the minimum occurs when $\log(1+(1-\rho^2)\gamma_0 P_s) = \log(1+\gamma_1 P_s + \gamma_2 P_r + 2\rho\sqrt{\gamma_1\gamma_2 P_s P_r})$, which corresponds to the optimal value of ρ of

$$\rho^* = \frac{\sqrt{\gamma_2\gamma_1 P_s P_r}}{P_s\gamma_0} + \frac{\sqrt{\gamma_2\gamma_1 P_s P_r - P_s\gamma_0(P_s\gamma_1 + P_r\gamma_2 - P_s\gamma_0)}}{P_s\gamma_0}. \tag{3.20}$$

This leads to an achievable rate of

$$R_{DF}(\underline{\gamma}, P_s, P_r) = \log(1 + \gamma_0(1 - \rho^{*2})P_s). \tag{3.21}$$

The maximizing P_s for this achievable rate must now be found. Equivalently, it is sufficient to maximize $\gamma_0(1-\rho^{*2})P_s$, since (3.21) is monotonically increasing with P_s. We then must solve for P_s^* in,

$$\gamma_0(1-\rho^{*2}) - 2\gamma_0 P_s \rho^* \frac{\partial \rho^*}{\partial P_s} = 0. \tag{3.22}$$

This leads to a quadratic solution for P_s with solutions

$$P_s^* \in \left\{ \frac{2\gamma_2 P_{st}}{\gamma_1+\gamma_2}, \frac{2P_{st}(\gamma_0\gamma_1 + 2\gamma_1\gamma_2 + \gamma_2^2)}{\gamma_2^2 + \gamma_1\gamma_2 + \gamma_0\gamma_1 + \gamma_0\gamma_2} \right\}. \tag{3.23}$$

From, (3.23), the solution which lies in the range $(0, 2P_{st})$ and gives the largest achievable rate is taken. If none of the solutions lie within this range, then the maximum rate occurs at the endpoint, when $P_s^* = 2P_{st}$. Given the value of P_s^*, then the maximum rate of R_{DF} is expressed as

$$T_{DF}(\underline{\gamma}, P_{st}) = \log(1 + \gamma_0(1-\rho^{*2})P_s^*). \tag{3.24}$$

Based on the discussion for the hybrid protocol, the solution of $T_{UB}(\underline{\gamma}, P_{st})$ for the cut-set bound on outage probability is now trivial, as it has a similar form to the decode and forward protocol, with the term γ_0 replaced by $\gamma_0 + \gamma_1$. Otherwise, the procedure to solve for P_s^* follows directly from that of the decode and forward protocol.

3.3.3 Analysis and Discussion

In Figure 3.3, the outage probability with power control at the source and relay is shown. The relay is assumed to be at a distance of $d = 0.2$, which allows for a

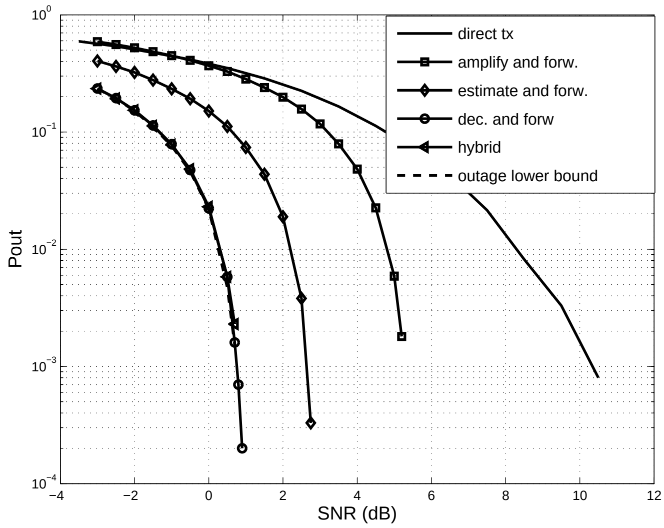


Figure 3.3 : Probability of outage vs. SNR for $\alpha = 3$, $R = 1$ nats/sec/Hz, and d=0.2. Decode and forward is near optimal at small source-relay distances.

good source-relay link. The pathloss exponent is $\alpha = 3$. The tremendous gains of performing optimal power allocation over the fading channel are seen. The lower bound on outage probability is shown, and it can be seen that the hybrid protocol and the decode and forward protocol closely follow the lower bound.

The estimate and forward protocol, on the other hand, is approximately 2dB away from the lower bound. This result makes sense with what was seen for the scenario of phase cancellation, where for small values of d, decode and forward and the hybrid protocol closely followed the lower bound. For large distances, however, the rate of the DF protocol degrades substantially compared to the outage lower bound. On this same figure, the outage results for the amplify and forward protocol are also seen, and it is seen that there is a tremendous loss in performance by performing such a

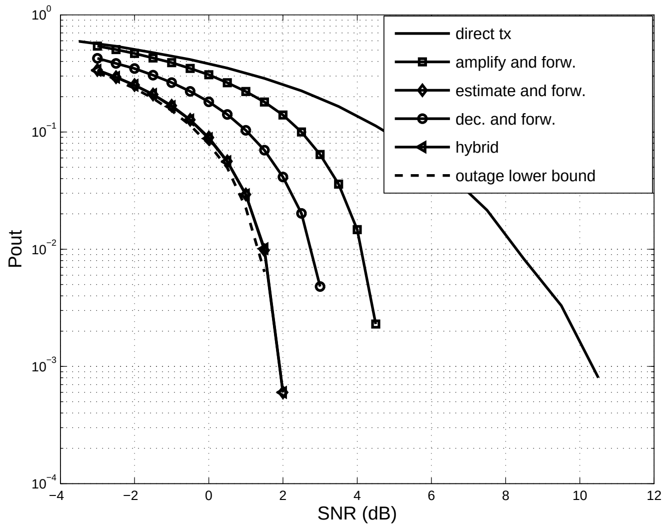


Figure 3.4 : Probability of outage vs. SNR for $\alpha = 3$, $R = 1$ nats/sec/Hz, and d=0.95. Estimate and forward has near optimal behavior at this value of d, the source-relay distance.

'cheap' transmission protocol.

Figure 3.4 shows the outage probability results when $d = 0.95$, which implies that the relay is closer to the destination than it is to the source. In this case, it can be seen that once again, the hybrid protocols performance follows closely that of the outage lower bound. In [26], the estimate and forward protocol was shown to be almost capacity achieving for large values of d, while the performance of the DF protocol dramatically declined for increasing d. This trend is also seen from in the case of optimal power control. Here, the performance of the DF scheme is approximately 1.5dB away from the outage lower bound, while the EF protocol closely follows the lower bound.

In Figure 3.5, for a fixed sum power, the outage is shown as a function of the relay's positioning with respect to the source and destination. Amplify-forward (AF) and estimate-forward (EF) perform better as the relay moves towards the destination, and behave similar to a system with multiple receive antenna's. AF suffers from a rate loss due to the time-orthogonality constraint, which is reflected in the gain of EF over AF. Decode-forward performs close to the lower bound on outage probability when the source-relay distance is small. Interestingly, the hybrid between EF and DF performs better than both, which is most evident at $d = 0.8$. Also shown in this figure is a 2x1 multiple antenna system, which performs the same as DF at the extremes (i.e. $d = 0$ and $d = 1$). On the other hand, DF significantly outperforms the 2x1 system when the relay is positioned between the source and relay. which justifies the use of relaying in favor of multiple transmit antenna's.

An interesting point regarding Figure 3.5 is the performance of a multi-hop system, in which the source and relay do not perform collaboration in their communication to the destination. Clearly, this protocol has its best performance when the relay is midway between the source and destination, which allows for strong source-relay and relay-destination links. However, it is apparent that using 'expensive' relay techniques such as EF and DF clearly outperform multi-hopping. This fact justifies the use of collaborative communication between the source and relay.

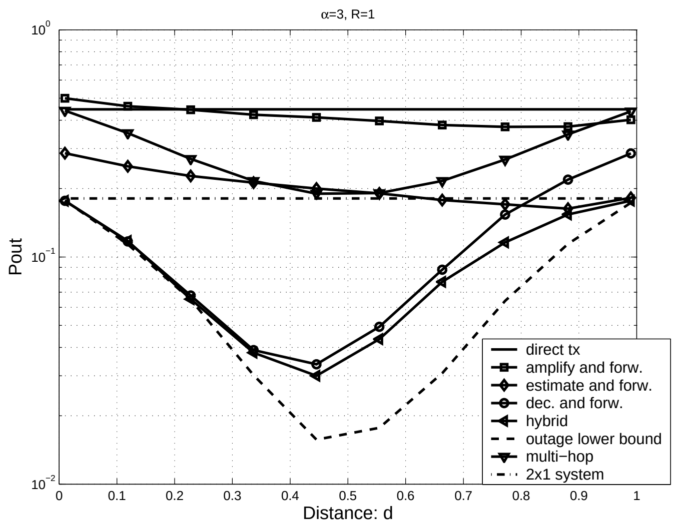


Figure 3.5 : Probability of outage vs. relay distance to source for various relaying protocols using constant power and a rate $R = 1$ nats/sec/Hz. The SNR is -1dB.

3.4 Effect of Practical Constraints on Outage Minimization

In our formulation of the optimal power control policy, we make a critical assumption that in practical scenarios may not exist. In real systems, the source and relay cannot transmit an unbounded amount of energy in a particular time slot, even if the power control policy meets the average power constraint. To take this point into account, we next describe the optimal power control policy with peak power constraints.

3.4.1 Outage Minimization with Peak Power Constraints

The optimal power control policy described earlier assumes that for channel states lying within $\mathcal{R}(p^*)$, both the source and relay can use an unbounded amount of power to invert the effects of the channel. In practice, however, due to limitations imposed by the power amplifier, transmission would be peak power limited at both the source and relay. We next consider the optimal power control policy under both individual peak constraints, and a sum power constraint.

Theorem 3.4.1. *The optimal power allocation that minimizes the outage for a relaying protocol with achievable rate $R_{gen}(\underline{\gamma}, P_s(\underline{\gamma}), P_r(\underline{\gamma}))$ under long term average and peak power constraints is*

$$P_{LT}(\underline{\gamma}) = \begin{cases} P^*_{st}(\underline{\gamma}), & \textit{with probability 1, if } \underline{\gamma} \notin \mathcal{G}(P_s^{max}, P_r^{max}) \textit{ and } P^*_{st}(\underline{\gamma}) < p^* \\ P^*_{st}(\underline{\gamma}), & \textit{with probability } w_0, \textit{ if } \underline{\gamma} \notin \mathcal{G}(P_s^{max}, P_r^{max}) \textit{ and } P^*_{st}(\underline{\gamma}) = p^* \\ 0, & \textit{with probability } 1 - w_0, \textit{ if } \underline{\gamma} \notin \mathcal{G}(P_s^{max}, P_r^{max}) \textit{ and } P^*_{st}(\underline{\gamma}) = p^* \\ 0, & \textit{with probability 1, if } \underline{\gamma} \notin \mathcal{G}(P_s^{max}, P_r^{max}) \textit{ and } P^*_{st}(\underline{\gamma}) > p^* \\ 0, & \textit{with probability 1, if } \underline{\gamma} \in \mathcal{G}(P_s^{max}, P_r^{max}), \end{cases}$$

for some subset $\mathcal{G}(P_s^{max}, P_r^{max}) \subset R^3_+$ and $w_0 \in (0, 1)$. Here $P^*_{st} = \frac{1}{2}(P^*_s + P^*_r)$, where $P^*_k = \min\{\hat{P}_k(\eta^*), P_k^{max}\}$, $k \in \{s, r\}$, and $\hat{P}_k(\eta^*)$ is found from the solution of

$$R_{gen}(\underline{\gamma}, \min\{\hat{P}_s(\eta^*), P_s^{max}\}, \min\{\hat{P}_r(\eta^*), P_r^{max}\}) = R \tag{3.25}$$

Furthermore, P_s^{max} is the source peak power, P_r^{max} is the relay peak power, and p^* is chosen such that the peak and average power constraints are satisfied, and η^* is chosen to satisfy the rate constraint.

Proof: Let the subset $\mathcal{G}(P_s^{max}, P_r^{max})$ define all possible channel states in R^3_+ for which transmitting at both peak powers does not allow zero outage at a transmission rate R,

$$\mathcal{G}(P_s^{max}, P_r^{max}) = \{\underline{\gamma} : R_{gen}(\underline{\gamma}, P_s^{max}, P_r^{max}) < R\}.$$

Next, define

$$\mathcal{R}(p) = \{\underline{\gamma} \notin \mathcal{G}(P_s^{max}, P_r^{max}) : P_{st}^* < p\}.$$

as the set of channel states not in $\mathcal{G}(P_s^{max}, P_r^{max})$ that use the optimal power allocation strategy P_{st}^*.

Based on the above definitions, when $\underline{\gamma} \notin \mathcal{G}(P_s^{max}, P_r^{max})$, the optimal power allocation policy under peak and average power constraints for

$$P_{LT}(\underline{\gamma}) = \begin{cases} P_{st}^*(\underline{\gamma}), & \text{with probability } 1, \text{ if } P_{st}^*(\underline{\gamma}) < p^* \\ P_{st}^*(\underline{\gamma}), & \text{with probability } w_0, \text{ if } P_{st}^*(\underline{\gamma}) = p^* \\ 0, & \text{with probability } 1 - w_0, \text{ if } P_{st}^*(\underline{\gamma}) = p^* \\ 0, & \text{with probability } 1, \text{ if } P_{st}^*(\underline{\gamma}) > p^*, \end{cases} \tag{3.26}$$

where $w_0 \in (0, 1)$ and

$$p^* = \sup\left\{p : \int_{\mathcal{R}(p)} P_{st}^*(\underline{\gamma}) dF(\underline{\gamma}) < P_{avg}\right\}. \tag{3.27}$$

Next, we need to find the form of P_{st}^*, which minimizes the power while transmitting at rate R and with peak power constraints P_s^{max} and P_r^{max}. Based on the discussion outlined in the proof of Theorem 3.3.1, we seek to solve

$$\min_{P_s, P_r} \{P_s + P_r : R_{gen}(\underline{\gamma}, P_s, P_r) = R, 0 \le P_s \le P_s^{max}, 0 \le P_r \le P_r^{max}\}. \tag{3.28}$$

The above has similarities with a 2-block fading channel with peak and average power constraints, and can be solved in a similar manner as was done in [27]. The following Lagrangian equation is setup

$$J = \sum_{k\in\{s,r\}} P_k - \sum_{k\in\{s,r\}} \psi_k P_k + \sum_{k\in\{s,r\}} \mu_k (P_k - P_k^{max}) + \eta(R_{gen}(\underline{\gamma}, P_s, P_r) - R). \quad (3.29)$$

The objective function is clearly convex, as well as the set of feasible points, and as a result the optimal power allocation strategy (P_s^*, P_r^*), and the corresponding $(\psi_k^*, \mu_k^*, \eta^*)$ satisfy the KKT conditions [28]

$$\begin{aligned} P_k^* &\geq 0 \\ P_k^* &\leq P_k^{max} \\ R_{gen}(\underline{\gamma}, P_s^*, P_r^*) &= R \\ \psi_k^* &\geq 0 \\ \mu_k^* &\geq 0 \\ \psi_k^* P_k^* &= 0 \\ \mu_k^*(P_k^* - P_k^{max}) &= 0 \\ \frac{\partial J}{\partial P_k^*} = 1 - \psi_k^* + \mu_k^* + \eta^* \frac{\partial R_{gen}(\underline{\gamma}, P_s^*, P_r^*)}{\partial P_k^*} &= 0 \end{aligned}$$

For the relay protocols discussed in this work, it can be verified that solutions of the form

$$P_k^* = \min\{\hat{P}_k(\eta^*), P_k^{max}\}, \quad (3.30)$$

satisfy the KKT conditions [27] , where η^*, $\hat{P}_s(\eta^*)$ and $\hat{P}_r(\eta^*)$ are found from the solution of

$$R_{gen}(\underline{\gamma}, \min\{\hat{P}_s(\eta^*), P_s^{max}\}, \min\{\hat{P}_r(\eta^*), P_r^{max}\}) = R \tag{3.31}$$

and η^* is the Lagrange multiplier chosen to meet the rate constraint. This is in fact quite similar to the power control policy of Theorem 3.3.1. The only difference in the short term power inversion process is in the derivation of $P_{st}^*(\underline{\gamma})$. Rather than allowing for any (P_s^*, P_r^*), there is now a maximum value on these quantities. At most one of these values may reach a peak, and then the problem reduces to searching for one power level that meets the rate constraint.

■

Functionally, the optimization procedure is similar to that of the case where no peak power constraints exists. The major difference is the region $\mathcal{G}(P_s^{max}, P_r^{max})$, which increases the size of the outage region (where transmission is shut off). In terms of the short term power allocation process, to incorporate the peak power constraint, first the unbounded zero outage power is found, which is identical to that operations performed in Section 3.3.2. However, it is possible that at most one of the powers, $\hat{P}_k(\eta^*)$, $k = s$ or r, could violate the peak power constraint. In such a case, then power $\hat{P}_k(\eta^*)$ would be set to the maximum P_k^{max}, and then $\hat{P}_i(\eta^*)$, $i \neq k$, can be easily solved for. The individual power allocations for the source and relay are not uniform, however, which is the case for K-block transmission on the single link fading

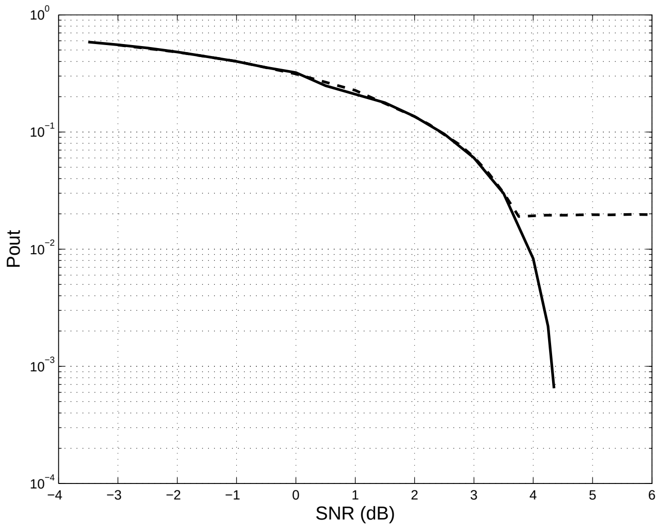


Figure 3.6 : Comparison of outage performance for amplify and forward with a global network power constraint, and also one with the addition of a peak SNR constraint of $8dB$, with $\alpha = 3$, $R = 1$ nats/sec/Hz, and d=0.5.

---

channel. The source power is always non-zero in the solution to the short-term power constraint, whereas the relay power can become zero.

3.4.2 Analysis and Discussion

Figure 3.6 shows the effects of a peak power constraint on system performance. For small SNR's, the peak limited system performs the same as the peak unconstrained system. However, as the SNR becomes large, the peak limitation becomes prominent, and in fact after a certain average power the outage remains constant. This is expected as the optimal power control solution with higher sum power allows for a larger region of states for which the channel inversion process is possible. However, due to the

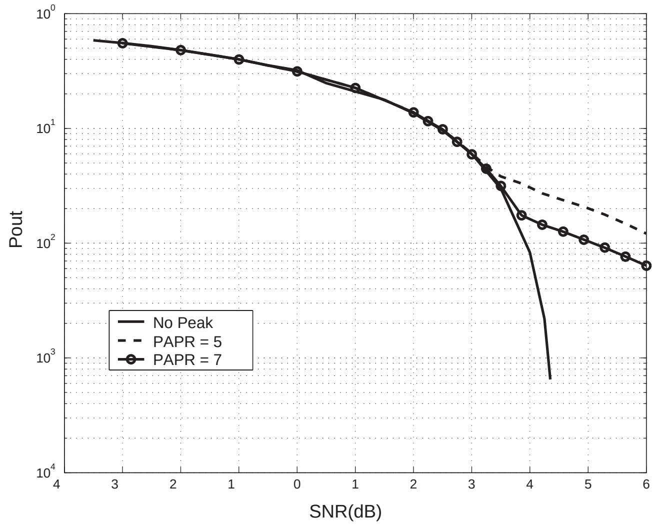


Figure 3.7 : Outage probability results for amplify and forward for different values of the PAPR ratio. Increasing ratios lead to better outage performance. For a given PAPR, the outage curve follows the no peak curve up to a point, then diverges from this curve. In this figure, $\alpha = 3$, R=1 nats/sec/Hz, and d=0.5.

---

peak power limitation, these states will not be useable, and as a result the outage will remain constant. Figure 3.7 shows effect of a peak to average power ratio on the outage performance. When the PAPR=∞, then this corresponds to the case with no peak power constraint. When the peak to average power ratio is finite, then for a small average power constraint the peak constraint does not effect the outage probability. However, as the average power constraint increases, the peak constraint effects the outage probability and leads to a finite diversity order. The diversity of the power control policy with a finite PAPR is equal to the diversity of the same protocol with constant power transmission.

The PAPR results can be easily understood by considering direct transmission. In direct transmission, without a peak constraint, the channel inversion process leads to a power allocation of $P^* = \frac{e^R-1}{\gamma_1}$ for a given channel state γ_1. A cutoff value s^* is chosen such that if $\gamma_1 < s^*$, then an outage is declared. Recall that the cutoff value is chosen to meet the long-term power constraint. The outage probability then becomes $1-e^{-s^*}$. The cutoff value $p^* = \frac{e^R-1}{s^*}$ represents a peak value on P^*. For a peak power constraint of κ, the outage curve follows the no-peak power case as long as $p^* \geq \kappa$. When the average power constraint increases, p^* decreases and becomes less than κ. In this case, κ will determine the outage probability, which will remain constant at $P_{out} = 1 - e^{-\frac{e^R-1}{\kappa}}$. When a peak to average power constraint is used, when p^* is less than κ, the outage will once again be a function solely of the peak value κ. However, defining the PAPR as $\zeta = \frac{\kappa}{P_{avg}}$, the outage will decrease as a function of $\zeta \cdot P_{avg}$. For large average power constraints, the outage probability can be expressed as

$$P_{out} = 1 - e^{-\frac{e^R-1}{\zeta P_{avg}}}.$$

For a fixed ζ, in the regime of large P_{avg}, the first order approximation to the exponential Taylor series leads to the following

$$P_{out} \approx \frac{e^R-1}{\zeta P_{avg}}.$$

Clearly, for the case of direct transmission the outage probability decreases with a diversity order of one. The coding gain over constant power transmission is proportional to ζ, which is the peak to average power ratio. A similar argument holds for relaying.

3.5 Conclusions

In this chapter, we have analyzed the outage performance of different relaying protocols for the fading channel with side information both at the transmitters and the receiver. The main contributions of this chapter are twofold. First, having side information at the source and the relay provides a tremendous gain which can be exploited by having feedback. Moreover, even a finite rate of feedback can significantly improve the performance and lower the outage probability [29]. Second, by using a hybrid of the coding protocols discussed in [21] the universal lower bound on the outage probability can be almost achieved.

This work reveals that side information at the transmitters for the relay channel is more crucial than that of the single link channel. The side information at the transmitter can be used to devise coding schemes which perform better even if power variation is not allowed at the transmitter. However, in a single link side information at the transmitter is useless if transmitting power is constant. Additionally, in this chapter, we derived the optimal power allocation for different relaying protocols when power control is allowed. It is imperative to note that by exploiting power control the outage probability decays much faster.

A major assumption in this chapter is the presence of perfect feedback at both the source and relay. In reality, having a perfect estimate is not feasible, due to the finite capacity of the feedback links. In Chapter 4, a power control policy is devised for the scenario where a finite rate feedback link exists.

Chapter 4
Power Control with Limited Feedback

4.1 Introduction

Chapter 3 discussed the fundamental limits in the block fading relay channel under the assumption of perfect channel knowledge at the transmitter. Adapting the power over time leads to significant power savings for a target error performance level. Furthermore, significant gains over direct transmission are possible with the optimal power control policy. However, in practice having a perfect measure of the instantaneous channel state is not practical, and methods must be developed to reduce the outage probability based on the degree of channel state knowledge at the transmitters.

The objective of this chapter is to investigate methods to approach the performance limits in the fading relay channel under different assumptions of network channel state information at the transmitters (CSIT). The first performance limit considered is the one defined by the optimal power control policy when perfect network channel state information is available at the source and relay. However, in practice, having a perfect channel estimate at the transmitters is impractical, especially in network scenarios. Hence, we consider the effect of finite rate feedback links. We derive a power control policy based on the rate of the feedback link, and we show how it can be used to approach the perfect feedback power control limit. Second, when channel state information is unavailable to the transmitters, we find the optimal performance limit for a given protocol and provide a simple method to approach this limit.

To approach the performance of the optimal CSIT power control algorithm, we describe a power control procedure based on a limited feedback channel that is extendable to any number of feedback bits. Interestingly, we see that with just one or two bits of power control information, the finite rate feedback algorithm can overcome most of the performance gains that the optimal CSIT power control policy achieves over constant power transmission. Furthermore, we show a simple power control policy, where equal average power is given to each power control subregion. This practical policy allows for efficient computation of the power control regions, and is easily extendable to any rate of the feedback link. Our results are general and can be extended to many relay coding protocols. However, we show results based on the amplify and forward protocol. Amplify and forward is an attractive network code due to its simplicity and ability to achieve full diversity. For the AF technique, through an analysis of the outage probability, we are able to show that the use of a feedback bit doubles the diversity order over constant power transmission. The effect of the increased diversity order is a significant savings in power over constant power transmission for a target frame error rate. Such power savings are of particular importance in systems requiring energy efficiency, such as ad-hoc and sensor networks [30]. It is therefore imperative that next-generation network protocols utilize feedback to enable power control, as it will result in significant battery life improvements.

The second performance limit considered occurs when no channel state knowledge is available to the transmitters. When no CSIT is available, then temporal power control is not possible. However, based on the statistics of the links in the network, the source and relay are able to determine the fraction of the total available power with which to transmit. For the AF protocol, we derive the optimal spatial power allocation between the source and relay. Interestingly, it is seen that for relays positioned close to the source, which is a scenario where relaying becomes feasible, equal power allocation between the source and relay is close to optimal. As a result, in the absence of CSIT, there is minimal power savings from using spatial power allocation at the transmitters. Our work suggests that to obtain large performance improvements over constant power transmission, it is imperative to have feedback for each realization of the channel state to allow for temporal power control.

The rest of the chapter is organized as follows. Section 4.2 investigates the outage performance of the relay protocol under the assumption that limited channel state information is available to the transmitters. Section 4.3 looks at the case of no transmitter channel state information, and Section 4.4 provides concluding remarks.

4.2 Power Control with Finite Rate Feedback

In this section, we derive a power control algorithm for the relay channel that uses limited feedback. First, we outline the general procedure, and then we present a

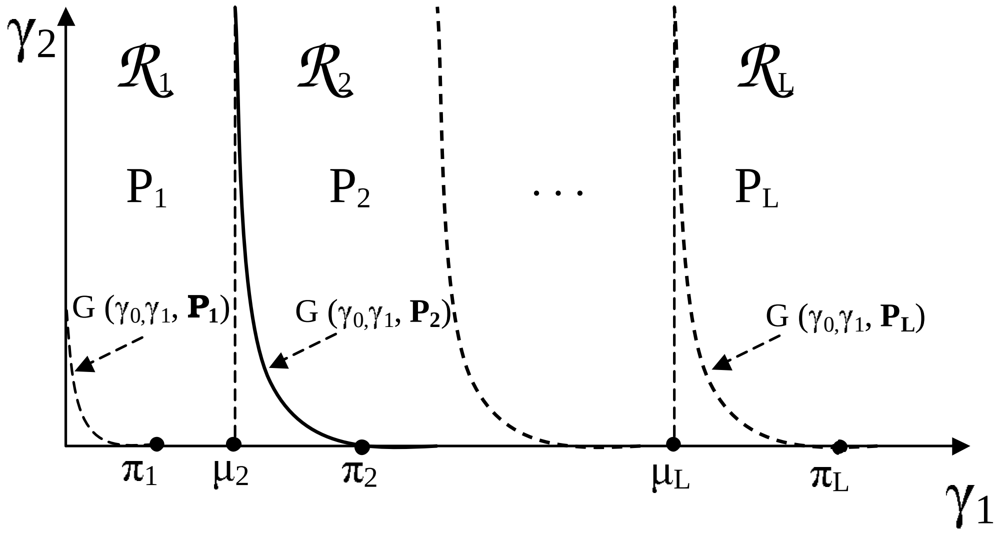


Figure 4.1 : Structure of power control regions for a fixed γ_0. Using $\log_2 L$ bits of feedback, the space of all (γ_1,γ_2) is divided into L subregions. In region $\mathcal{R}_i$, $i \in \{1, ..., L\}$, power level P_i is used.

---

low complexity suboptimal solution. The low complexity solution has the property that it can be easily extended to an arbitrary number of feedback bits. For the case of one feedback bit, an approximation to the outage probability is developed, and the diversity gain for the AF protocol is shown to double over constant power transmission.

4.2.1 General Procedure

Consider the destination, which has a perfect estimate of the network channel state $\underline{\gamma}$. Given M bits of feedback, the space defined by all possible sets of $\underline{\gamma}$ is quantized into $L = 2^M$ regions. For the network channel state $\underline{\gamma}$, the region is a volume in the space defined by all positive $(\gamma_0, \gamma_1, \gamma_2)$. The destination, upon measuring the channel state, selects a power-tuple $\mathbf{P_q} = (P_{s,q}, P_{r,q})$, from a power control codebook $\mathcal{C}$, of size L, where $q \in \{1, 2, ..., L\}$. The index q to the selected power-tuple is transmitted to both the source and relay through a noiseless feedback link. It is assumed that both the source and relay have copies of $\mathcal{C}$. Upon reception of the index q, the source transmits with power $P_{s,q}$ and the relay with power $P_{r,q}$.

The elements of $\mathcal{C}$ are chosen to maintain the average power constraints of both the source and relay. We consider the case where both the source and relay have individual average power constraints. The power control policy described by Theorem 3.3.1 involves outage minimization with a sum power constraint, and serves as a lower bound to the outage for the case of individual power constraints on the source and relay. As a result, even as $L \to \infty$, the power control policy of Theorem 3.3.1 will provide a lower bound on the outage probability of the developed power control algorithm.

Consider the power control function $S : \mathbb{R}^3_+ \to \mathbb{R}^2_+$, which maps the current channel state $\underline{\gamma} \in \mathbb{R}^3_+$ to a codebook element $\mathbf{P_q} \in \mathbb{R}^2_+$. To satisfy the average power constraint, $E_{\underline{\gamma}}[S(\underline{\gamma})] \leq (P_s, P_r)$ must hold on a per element basis. The objective of the power control algorithm is to find a $S(\underline{\gamma})$ that minimizes the outage probability while meeting the power constraint. In general, the elements of $\mathbf{P_q}$ can differ, chosen to meet individual power constraints of the source and relay. To simplify the analysis, we impose one of two possible restrictions on $P_{r,q}$. The first restriction is where the

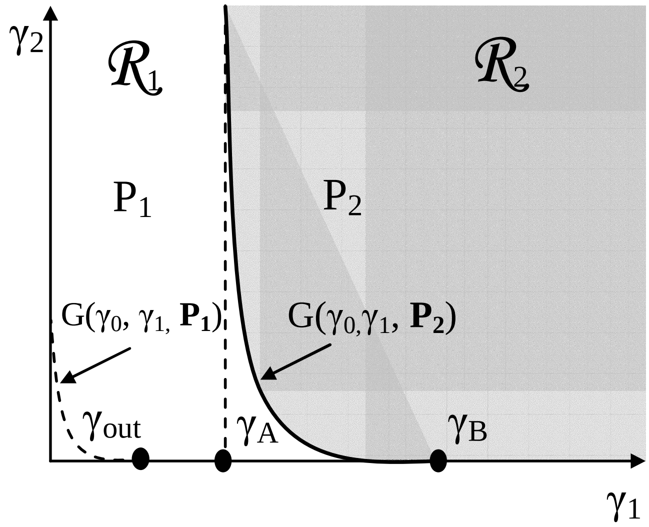


Figure 4.2 : Structure of power control regions for a fixed γ_0 and 2 subregions. The function $G(\gamma_0, \gamma_1, P_1)$ defines the outage region such that all points lying below this curve require more than power P_1 to guarantee zero outage.

---

relay transmits with a constant power P_r in each time slot. This leads to a power-tuple of $\mathbf{P_q} = (P_{s,q}, P_r)$. The second restriction is where the relay takes a similar action as the source, depending on its power constraint. That is, if $P_r = \eta P_s$, then we impose a constraint on power-tuple q as $\mathbf{P_q} = (P_{s,q}, \eta P_{s,q})$. We will show later that the second form of the power-tuple allows for an increase in performance over using a constant relay power. The results presented next are applicable to both scenarios.

Given M bits of feedback, the space defined by all $(\gamma_0, \gamma_1, \gamma_2)$ will be divided into $L = 2^M$ subregions $\mathcal{R}_q$, $q \in \{1, 2, ..., L\}$. If the instantaneous value of $\underline{\gamma}$ falls into region $\mathcal{R}_q$, the destination indicates to the source and relay to use power-tuple $\mathbf{P_q}$. The power levels $(P_{s,q}, P_{r,q})$, $q \in \{1, 2, ..., L\}$ are chosen to satisfy the long term power constraint, i.e.,

$$(P_s, P_r) = \left(\sum_{q=1}^{L} P_{s,q} \int_{\mathcal{R}_q} f(\underline{\gamma}) d\underline{\gamma}, \sum_{q=1}^{L} P_{r,q} \int_{\mathcal{R}_q} f(\underline{\gamma}) d\underline{\gamma} \right), \tag{4.1}$$

where $f(\underline{\gamma})$ is the joint probability distribution of the network channel state $\underline{\gamma}$.

In Figure 4.1, for the amplifyand forward protocol and for a given γ_0, the power control regions $\mathcal{R}_q$ are shown. Although we have shown the space of (γ_1, γ_2) for a given γ_0, changing γ_0 changes the position of μ_q. The power control regions are in fact volumes in the space defined by $\underline{\gamma}$, where for any particular γ_0, a cross-section of the 3-D space is similar to that shown in Figure 4.1.

One key feature of the power control regions is that in region $\mathcal{R}_q$, $q \geq 2$, the assigned power $\mathbf{P_q}$ is the minimum required to guarantee zero outage for any point in the region. This is a fundamental property of all optimal finite rate feedback power control algorithms [12]. With this property in mind, assuming a relaying protocol with achievable rate R_{gen} and transmitting at a constant rate R, power level $P_{s,q}$ is the solution to

$$\mathcal{R}_{gen}(\underline{\gamma}, P_{s,q}, x) = R. \tag{4.2}$$

Note that in (4.2), $x = P_r$ when the relay power is always constant, and $x = \eta P_{s,q}$ when the relay also adapts its power.

From Figure 4.1 observe that for a fixed γ_0, the boundary between $\mathcal{R}_q$ and $\mathcal{R}_{q+1}$ is separated by a curve $G(\gamma_0, \gamma_1, \mathbf{P_{q+1}})$. This curve is found by solving for γ_2 in (4.2). Any (γ_1, γ_2) along this curve requires exactly powers $\mathbf{P_{q+1}}$ for zero outage, while any other points in $\mathcal{R}_{q+1}$ require instantaneous source and relay powers less than $P_{s,q}$ and $P_{r,q}$, respectively, for zero outage. We state this formally in the following theorem.

Theorem 4.2.1. *For the amplify and forward protocol, any points lying below the curve $G(\gamma_0, \gamma_1, \mathbf{P_q})$ require source and relay powers greater than $P_{s,q}$ and $P_{r,q}$, respectively, to guarantee zero outage. Furthermore, any points lying above this curve require source and relay powers less than $P_{s,q}$ and $P_{r,q}$, respectively, to guarantee zero outage.*

Proof: See Appendix A.

As a result of Theorem 4.2.1, the entire region $\mathcal{R}_{q+1}$ has no outages. This property holds for all $\mathcal{R}_q$, $q \in \{2, ..., L\}$. Based on the power constraint, however, a portion of $\mathcal{R}_1$ would be in outage. Therefore, calculating the outage probability reduces to an analysis of region $\mathcal{R}_1$.

Region $\mathcal{R}_1$ uses power-tuple $\mathbf{P_1}$ corresponding to a source power of $P_{s,1}$ and a relay power of $P_{r,1}$. The outage probability is the probability that the source power required to invert the channel is greater than $P_{s,1}$. Note that our analysis stems from the properties of the source power, as the relay power is either constant or a scaled version of the source power. Defining P^* as the minimum power to guarantee zero outage for network channel state $\underline{\gamma}$, then P^* can be written as the solution of

$$R_{gen}(\underline{\gamma}, P^*, x) = R,$$

where $x = P_r$ when the relay transmits with constant power, or else $x = \eta P^*$ when the

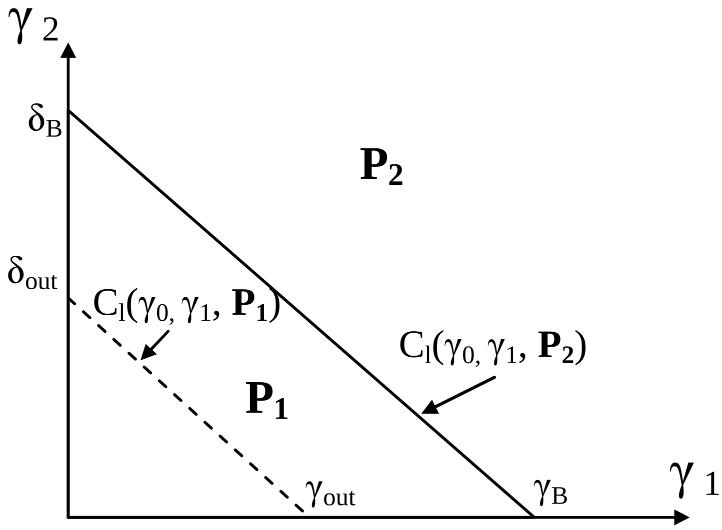


Figure 4.3 : Structure of power control regions for a fixed γ_0 and 2 subregions, using large power approximation. Regions $\mathcal{R}_1$ and $\mathcal{R}_2$ are separated by a line, $C_l(\gamma_0, \gamma_1, \mathbf{P_2})$. Below the dotted line $C_l(\gamma_0, \gamma_1, \mathbf{P_1})$, the power required to invert the channel is greater than $\mathbf{P_1}$, so the area below the dotted curve defines the outage probability.

---

relay also adapts its power. With the solution to P^* in hand, the outage probability can be expressed as

$$P_{out} = \int_{\underline{\gamma}:P^* \geq P_{s,1}} f(\underline{\gamma}) d\underline{\gamma}. \tag{4.3}$$

Different relaying protocols will have different solutions of $G(\gamma_0, \gamma_1, \mathbf{P_q})$. Considering the amplify and forward protocol, solving for γ_2 leads to the following

$$G_{AF}(\gamma_0, \gamma_1, \mathbf{P_q}) = \frac{K(1 + P_{s,q}\gamma_0) - P_{s,q}\gamma_1(1 + P_{s,q}\gamma_0)}{P_{r,q}(-K + P_{s,q}\gamma_1 + P_{s,q}\gamma_0)}, \tag{4.4}$$

where $K = e^{2R} - 1$. It can be easily verified that $\mu_q = K/P_{s,q} - \gamma_0$ for $q \in \{2, ..., L\}$, and $\pi_q = K/P_{s,q}$ for $q \in \{1, ..., L\}$, where π_q and μ_q are defined in Figure 4.1. The power control regions for variable γ_0 can be visualized by considering the effect of γ_0 on μ_q and also in the form of $G_{AF}(\cdot)$.

4.2.2 Suboptimal Power Control Method

In general, solving the regions $\mathcal{R}_q$ and the associated power levels is computationally complex. However, for a more efficient approach, we consider a method similar to [13], where equal total power is allocated to each subregion. For the case of multiple antenna systems with finite rate feedback, this technique was shown to be a good solution and close to optimal for large powers and for increasing bits of feedback. The power of this method is that, instead of jointly solving for the power control levels, they can be found in a successive fashion, which makes this algorithm amenable to a large number of feedback bits. The procedure is described next.

First, the power levels $(P_{s,L}, P_{r,L})$ are solved by noting that

$$\frac{P_s}{L} = P_{s,L} \int_{\mathcal{R}_L} f(\underline{\gamma}) d\underline{\gamma}. \tag{4.5}$$

The solution determines the power levels $(P_{s,L}, P_{r,L})$ and the region boundary $G(\gamma_0, \gamma_1, \mathbf{P_L})$. Once region L has been solved, then region $L-1$ can be found. This process is continued until power level $\mathbf{P_1}$ has been found. Solving power level $\mathbf{P_q}$ requires knowledge of $\mathbf{P_{q+1}}$, and by the simplifying assumption that $\frac{P_s}{L} = P_{s,q} \int_{\mathcal{R}_q} f(\underline{\gamma}) d\underline{\gamma}$. Note that we have used the total power in each region as P_s/L, since the power level for each region $\mathcal{R}_q$ corresponds to the transmit power of the source, and the relay can either transmit with a constant P_r or a variable power related to the source power. In either case, the relay's action is reflected in the algorithm by the form of $G(\gamma_0, \gamma_1, \mathbf{P_q})$ and hence in the solution of regions $\mathcal{R}_q$. In Section 4.2.4, we will see how using this suboptimal technique with just a few power levels leads to tremendous savings in power at a target outage probability over constant power transmission.

4.2.3 Lower Bound on Diversity Order

It was seen in [9] that the amplify and forward protocol transmitting at constant power has a diversity order of two compared to a first order diversity for the single antenna direct transmission system. We next show that for the case of one bit of feedback, the diversity gain doubles from two to four for the AF protocol.

To show the behavior of 1-bit of feedback for the amplify and forward protocol, we first consider the effect of the source-relay fading value, γ_0. It should be noted that even in the case of a Gaussian source-relay link with a fixed γ_0, amplify and forward still exhibits a second order diversity. This can be shown rigorously by analyzing the asymptotic behavior of the exponential distribution.

Theorem 4.2.2. *For the amplify and forward protocol with a fixed γ_0, and random γ_1 and γ_2, transmitting at a constant power leads to a second order diversity.*

Proof: See Appendix B.

Aside from Theorem 4.2.2, the fact that a fixed value of γ_0 does not effect the diversity can also be understood by observing that the destination node still sees two independent copies of the information, through the random source-destination and the relay-destination links. With this in mind, we state the following theorem.

Theorem 4.2.3. *For the amplify and forward protocol, as $P_r = P_{avg}$ increases, the optimal one bit network power control offers at least a diversity order of four.*

Proof: Assume that the source-relay link is Gaussian, with a fixed source-relay link gain of γ_0. As was proven in Theorem 4.2.2, this assumption does not affect the diversity order analysis. Additionally, it is assumed that the relay simply transmits with power P_r in each time slot, as we are seeking a lower bound for diversity order. The analysis for outage probability that follows assumes large values of SNR. Under such a scenario, the hyperbola shown in Figure 4.2 intersects the γ_2 axis. To compute a lower bound to the outage probability under such a scenario, we approximate the hyperbola as a triangle, as seen in Figure 4.3. The concavity of the hyperbola guarantees that the approximate outage analysis will be a lower bound, since $P_{s,1} \geq P_{s,2}$ for large power constraints. Looking in R_1, a line defined as $C_l(\gamma_0, \gamma_1, \mathbf{P_1}) = \delta_{\mathbf{out}} - \gamma_{\mathbf{1}}\delta_{\mathbf{out}}/\gamma_{\mathbf{out}}$ defines the outage region. Also, in this figure the line $C_l(\gamma_0, \gamma_1, \mathbf{P_2}) = \delta_{\mathbf{B}} - \gamma_{\mathbf{1}}\delta_{\mathbf{B}}/\gamma_{\mathbf{B}}$ is the boundary between R_1 and R_2. Note that $\gamma_{out} = K/P_{s,1}$ and points below this curve are assumed to be in outage. Also, δ_{out} is found by setting $\gamma_1 = 0$ in $G_{AF}(\gamma_0, \gamma_1, (P_{s,1}, P_r))$, and δ_B is found by solving for γ_2 in $G_{AF}(\gamma_0, 0, (P_{s,2}, P_r))$. The outage probability can be written as

$$P_{out} = \int_{\gamma_1=0}^{\gamma_{out}} \int_{\gamma_2=0}^{C_l(\gamma_0,0,P_{s,1})} f_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2) d\gamma_1 d\gamma_2. \tag{4.6}$$

Denoting the probability that the network state (γ_1, γ_2) is in region R_2 as Δ_2, we can then write

$$\Delta_2 = \frac{1}{\lambda_1\lambda_2}\int_{\gamma_1=\gamma_B}^{\infty} e^{-\frac{\gamma_1}{\lambda_1}} \int_{\gamma_2=0}^{\infty} e^{-\frac{\gamma_2}{\lambda_2}} d\gamma_2\gamma_1 + \frac{1}{\lambda_1\lambda_2}\int_{\gamma_1=0}^{\gamma_B} e^{-\frac{\gamma_1}{\lambda_1}} \int_{\gamma_2=C_l(\gamma_0,\gamma_1,P_{s,2})}^{\infty} e^{-\frac{\gamma_2}{\lambda_2}} d\gamma_2 d\gamma_1 \quad (4.7)$$

Using the second order Taylor approximation to the exponential function, $e^{-x} \approx 1 - x + \frac{x^2}{2}$, it can be shown that $\Delta_2 \approx 1 - \frac{\delta_B\gamma_B}{2\lambda_1\lambda_2}$. Using similar arguments, it can be shown that $P_{out} = \frac{\delta_{out}\gamma_{out}}{2\lambda_1\lambda_2}$, where $\gamma_{out} = K/P_{s,1}$. Therefore, we have the following approximation for the outage probability for large $P_{s,1}$

$$P_{out} = \frac{\delta_{out}\gamma_{out}}{2\lambda_1\lambda_2} = \frac{K^2(1 + P_{s,1}\gamma_0)}{2P_rP_{s,1}(P_{s,1}\gamma_0 - K)\lambda_1\lambda_2} \approx \frac{K^2}{2P_rP_{s,1}\lambda_1\lambda_2}. \quad (4.8)$$

To complete the analysis, we need to find $P_{s,1}$ as a function of P_{avg}. Using the power constraint on region R_1, we can show that $P_{s,1}(1 - \Delta_2) = P_{avg}/2$, which leads to $P_{s,1} = \frac{P_{avg}P_rP_{s,2}\lambda_1\lambda_2(P_{s,2}\gamma_0 - K)}{K^2(P_{s,2}\gamma_0+1)}$. With this in mind, the outage probability is now rewritten as a function of $P_{s,2}$ as

$$P_{out} \approx \frac{K^4(P_{s,2}\gamma_0 + 1)}{2\lambda_1^2\lambda_2^2P_r^2P_{avg}P_{s,2}(P_{s,2}\gamma_0 - K)}. \quad (4.9)$$

Clearly, (4.9) has a fourth order decay with respect to power P_{avg} as long as $P_{s,2}$ is a linear function of P and when $P_r = P_{avg}$. Next, $P_{s,2}$ is found as a function of P. Using the fact that we are considering an algorithm where each power control region has the same total power, i.e., $P_{s,2} \cdot \Delta_2 = \frac{P_{avg}}{2}$, $P_{s,2}$ is the solution to the following

$$P_{s,2} \approx \frac{P_{avg}}{4} + \frac{K}{2\gamma_0} + \frac{\sqrt{(P_{avg}/\gamma_0)^2 + 2K^2/(\lambda_1\lambda_2) - 4P_{avg}K/\gamma_0 + 4K^2/\gamma_0^2}}{4}.$$

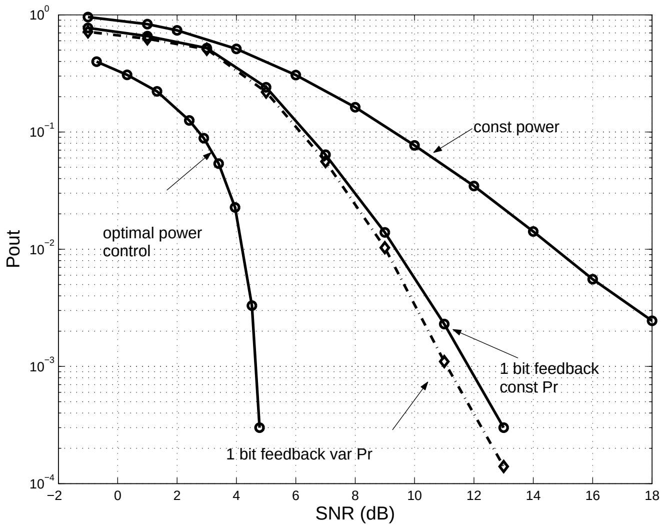


Figure 4.4 : Outage performance vs. SNR for the amplify and forward scheme, with d = 0.5, $R = 1$ nats/sec/Hz and $\alpha = 3$. For the case of 1 feedback bit, the solid line indicates a constant P_r, and a dashed line indicates a variable P_r. The source and relay are given equal average power constraints.

As a result, since $P_{s,2}$ depends on P_{avg}, substitution into (4.9) leads to a fourth order decay of the outage probability as a function of the power. ■

From this result, it is clear that, with the use of just one feedback bit, the diversity order has doubled from two to four. A similar effect was seen in [13], for the case of direct transmission, where the decay in outage probability was proportional to the number of elements in the power control codebook. An additional point of interest is the effect of the mean values of the fading links λ_1 and λ_2. Increased values of these parameters lead to a decrease in the outage probability. However, the diversity order is still four. For the case of constant power transmission using the AF protocol, changes in λ_1 and λ_2 also do not effect the diversity order [9].

In [9], the authors explored the use of one feedback bit to improve system performance and proposed a technique known as *incremental relaying*. This relay protocol makes more efficient use of the available degrees of freedom by using a feedback bit to indicate the success/failure of the source transmission to the destination and only relaying when the source transmission leads to a decoding failure. This results in gains over traditional AF by increasing the rate for good source-relay conditions. However, it is shown in [9] that for a fixed transmission rate this technique provides a diversity order of 2. Our work reveals that for a fixed rate of transmission, increased power savings can be obtained by using the feedback information for power control.

4.2.4 Analysis and Discussion

In Figure 4.4, power control with one bit of feedback for the AF protocol is shown. With just one bit of feedback, fourth order diversity is obtained, compared to a second order diversity for constant power transmission. At an outage probability of 10^{-2}, there is approximately 5dB of SNR savings with just one feedback bit. Furthermore, we observe that, at this same outage probability, one bit of feedback substantially reduces the gap to the optimal power control strategy. This motivates the need for future network protocols to allocate a few bits in feedback packets to allow for power control.

Recall that two possibilities were described for the action of the relay. First, the

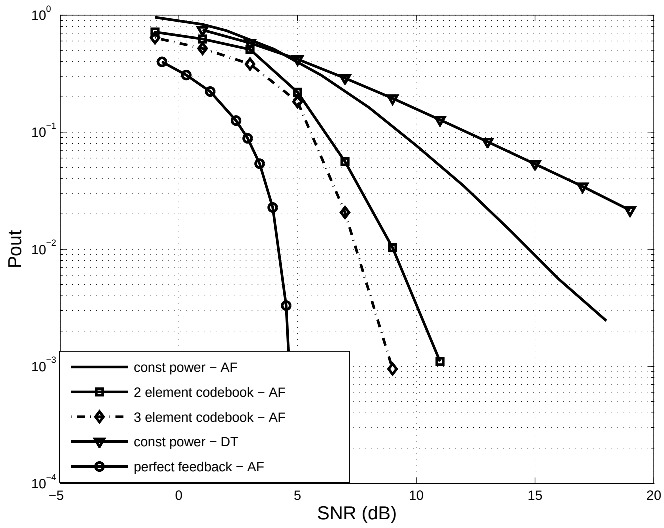


Figure 4.5 : Effect of more feedback bits on outage performance, for d=0.5, $\alpha = 3$, $R = 1$ nats/sec/Hz using the AF protocol. The relay in this case transmits with variable power in each time slot, and $P_s = P_r$. For comparison, the case of constant power transmission is shown, and also the optimal power control policy when perfect CSIT is available. Additionally the performance of a direct transmission system using constant power is shown.

---

relay transmits with a constant power in each time slot. Second, the relay takes the same action as the source (when they have the same average power constraints). In Figure 4.4, for the case of 1-feedback bit, the gains of using a variable relay power are also shown. We see that there is a small gain from performing this type of power adaptation at the relay.

In Figure 4.5, for the amplify and forward protocol, the effect of increasing feedback bits is shown. Constant power transmission is compared to the proposed power control strategy with 2 power levels (1 bit feedback). Additionally, the gain from

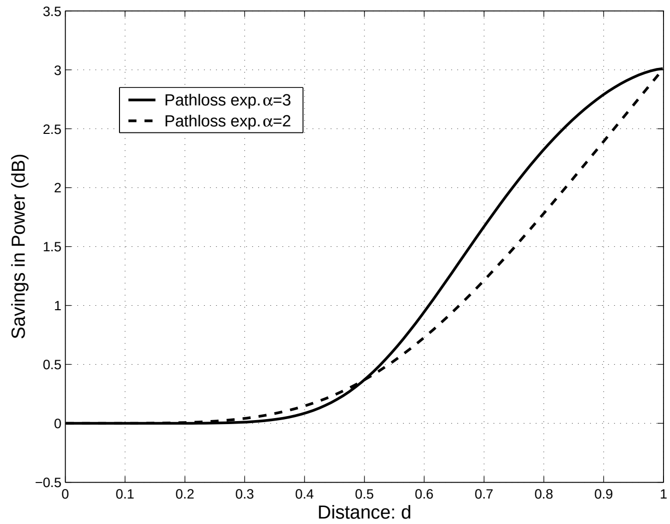


Figure 4.6 : Savings in power by using the optimal source-relay power ratio vs. equal power among source and relay assuming a rate $R = 1$ nats/sec/Hz and $P_{out} = 10^{-2}$. The d-axis represents the relay's fractional distance between the source and destination. The savings in power corresponds to the reduction in average power that is achieved by using the optimal power ratio versus equal power allocation between the source and relay.

---

adding one more power control level is shown, and we see that, for small outage probabilities, much of the gap to the optimal power control strategy has been bridged. This suggests that only a few bits of feedback are necessary to extract large savings in power, and further increases in the feedback rate offer diminishing returns. Also shown in the figure is the performance of a direct transmission system using the same total power as AF and transmitting at the same rate. Clearly, direct transmission offers only a first order diversity, whereas AF has double this diversity, which translates into large power savings.

4.3 Outage Minimization with No CSIT

In the previous sections, the potential gains of using the optimal power control strategy were seen, and also the effects of limited feedback on outage minimization. We observed that only a few bits of feedback are needed to bridge much of the gap to the optimal CSIT power control algorithm. We next consider the case where the transmitters have no channel state information(CSIT) and thus cannot perform temporal power control.

Even though the transmit powers for the source and relay are fixed, the outage probability can be minimized by determining the optimal fraction of the total power to be allocated to the source and relay. In each time slot, we have that $P_s + P_r = 2P_{avg}$. The objective is to find a $\kappa \in (0, 1)$ such that the outage probability is minimized given that $P_s = 2P_{avg}\kappa$ and $P_r = 2P_{avg}(1-\kappa)$. In addition to the derivation of the optimal source-relay power ratio κ^*, we will see how the practical choice of using equal power at the source and relay performs close to optimal for many cases of interest. Next, we consider the performance of the amplify and forward protocol for the case of constant power transmission.

Consider the amplify and forward protocol and an optimal source-relay power ratio κ^*. The achievable rate is

$$R_{AF}(\underline{\gamma}, P_s, P_r) = \frac{1}{2}\log\left(1 + 2\gamma_1 P_{avg}\kappa^* + \frac{4\gamma_2 P_{avg}^2 \kappa^* \gamma_0 (1-\kappa^*)}{1 + 2P_{avg}\kappa^*\gamma_0 + 2P_{avg}(1-\kappa^*)\gamma_2}\right).$$

We next characterize the outage probability for the amplify and forward protocol in the limit for large powers and for a given κ.

Lemma 4.3.1. *As the average power $2P_{avg}$ becomes large, the outage probability of the amplify and forward protocol can be approximated as*

$$P_{out} \approx \left\{ \frac{e^{2R}-1}{2\sqrt{2}P_{avg}} \right\}^2 \left[\frac{1}{\kappa\lambda_1} \cdot \frac{(1-\kappa)\lambda_2 + \kappa\lambda_0}{\kappa(1-\kappa)\lambda_0\lambda_2} \right], \tag{4.10}$$

where λ_i, $i \in \{0,1,2\}$ is the mean value of the fading for link i in the relay network and $0 \leq \kappa \leq 1$ allocates power between the source and relay.

Proof: The proof is based on asymptotic analysis of the exponential distribution, which was described in [9]. The total network power is $2P_{avg}$. Based on [9], it can be shown that

$$\lim_{s\to\infty} s \cdot \text{Prob}[s\kappa\gamma_1 < t] = \frac{t}{\lambda_1\kappa} = f(t),$$

and

$$\lim_{s\to\infty} s \cdot \text{Prob}[f(s\kappa\gamma_0, s(1-\kappa)\gamma_2) < t] = \frac{t}{\lambda_0\kappa} + \frac{t}{\lambda_2(1-\kappa)} = g(t),$$

where $f(x,y) = \frac{x \cdot y}{x+y+1}$. In the above formulations, $s\kappa$ is the total power used by the source and $s(1-\kappa)$ is the power used by the relay. Using Theorem 1 from [17],

$$\begin{aligned} \lim_{s\to\infty} s^2 \cdot \text{Prob}[s\kappa\gamma_1 + f(s\kappa\gamma_0, s(1-\kappa)\gamma_2) < t] = \\ \int_0^t g(t-x) \cdot f^{'}(x)dx = \\ \frac{t^2}{2\kappa\lambda_1}\left[\frac{1}{\kappa\lambda_0} + \frac{1}{(1-\kappa)\lambda_2}\right]. \end{aligned}$$

Substituting $s^2 = 4P_{avg}^2$ and $t = e^{2R} - 1$, the result follows. ■

We next investigate how the optimal source power $2P_{avg}\kappa^*$ is a function of the position of the relay. To do this, we consider again the scenario where the source and destination are one unit apart, and the relay is a distance $0 \leq d \leq 1$ from the source. Given a pathloss exponent α, this leads to $\lambda_1 = 1$, $\lambda_0 = \frac{1}{d^\alpha}$ and $\lambda_2 = \frac{1}{(1-d)^\alpha}$. To find the optimal source-relay power ratio, it suffices to minimize the outage probability of (4.10) over all κ. Performing the minimization, the optimal value of κ is

$$\kappa^* = \frac{(1-d)^\alpha - 4d^\alpha + \sqrt{(1-d)^{2\alpha} + 8(1-d)^\alpha d^\alpha}}{4(1-d)^\alpha - 4d^\alpha}. \tag{4.11}$$

An interesting property of the power ratio is that the solution is independent of the network power constraint $2P_{avg}$. Another interesting point is that the solution $\kappa^* \geq 0.5$, meaning that the relay should never transmit with more power than the source. In Figure 4.6, the savings in power by using the optimal power ratio of (4.11) are seen. By using the optimal ratio, up to 3dB is saved over equal power allocation between the source and relay. However, we see that for small distances ($d < 0.5$), the gains of using the optimal ratio are minimal. As a result, when the source is close to the destination, using equal power for the source and relay is a good strategy.

4.4 Conclusions

In this chapter, we have analyzed power control methods to approach the fundamental limits in the fading relay channel for varying degrees of side information at the source and relay. When perfect side information is available at the transmitters, significant energy savings over constant power transmission can be obtained through optimal power control. However, we showed that only a few bits of feedback are sufficient to achieve most of the gains of the optimal CSIT power control policy. This result suggests the importance of designing protocols that incorporate feedback in future wireless networks, as even limited amounts of feedback will translate to significant increases in battery life for mobile nodes.

We also analyzed the case where no side information was available at the transmitters. Interestingly, transmitting with equal power at the source and relay is close to optimal, especially for relays positioned close to the source. This hints at the importance of the relay's contribution in improving system performance, as the power of the relay needs to be similar to the source's power in order to minimize the outage.

Chapter 5

Throughput Gains with Limited Feedback

5.1 Introduction

The work described in the previous chapters has focused on outage minimization subject to a power constraint on the transmitters. We have looked at both peak and average power constraints, and also considered the practical problem of having a limited feedback channel. The derived algorithms are appropriate for a source which transmits at a fixed rate over time. However, for applications that can support a variable rate of transmission such as data transfer, one is more concerned with maximizing the overall throughput to allow as many correctly decoded data packets as possible at the destination.

Typically, the performance of systems in the block fading network environment can be characterized by the *outage probability* [8] and the *delay-limited capacity* [31]. However, due to outage events, the effective data rate, or *throughput*, is less than the attempted transmission rate. Higher transmission rates leads to a higher frame error rate (FER), and low transmission rates translate to low FER's. The FER, which serves as a tight lower bound to the outage probability [32], needs to be balanced with the transmission rate to maximize the throughput. We consider the practical problem of throughput maximization, which reflects end-to-end performance more accurately than the rate of transmission.

In the network setting, we consider different possible methods of transmission: relay coding, data forwarding, and direct transmission. We derive throughput maximization algorithms based on properties of the transmit buffer. First, assuming that the source always has enough information bits to send, we outline the optimal rate control procedure which maximizes the system throughput for the above mentioned transmission techniques. The analysis assumes that the rate control process is solely a function of the network channel state with no finite backlog effects. Interestingly, we observe that for a finite rate of feedback there is an optimal outage operating point that maximizes the throughput which, for a small rate of feedback, can be greater than 10%. This suggests that minimizing the outage probability is unnecessary to maximize the throughput. In fact, for a finite rate of feedback, it is better to increase the coding rate and allow for some frame errors. Furthermore, we show that for a limited rate of feedback, it is unnecessary to temporally vary both the rate and power. For small average power constraints, power control suffices to maintain a high throughput. On the other hand, for large average power constraints, adapting the transmission rate while using a constant power is nearly as useful as a transmission employing a variable rate and variable power.

Second, we consider the scenario where the buffer size is limited, and supplied by packets at a constant rate. Outages can occur in a network from both decoding errors at the receiver and dropped packets at the source resulting from buffer overflows. We show the gains in throughput achievable by increasing the buffer size and by having channel state information available at the transmitters. Our results demonstrate that transmission with the aid of a relay node leads to a large increase in throughput over direct transmission. Also, the effect of power control in conjunction with rate control is shown to provide significant gains in throughput over constant power transmission. Much like the case of ergodic capacity, temporal power adaptation provides most of its gains over constant power transmission for small average power constraints.

Our results suggest the power of the relay transmission paradigm. Decode and forward, one of several cooperative protocols considered in this chapter, is shown to have significant gains in throughput over both data forwarding and direct transmission. We show that even a low complexity relay protocol such as selection relaying, which was defined in (2.6), can achieve large throughput gains over direct transmission for many practical cases of interest. Our analysis reveals that the true benefits of relaying become evident in wireless networks when feedback regarding channel state information is available at the transmitters. Even limited amounts of feedback lead to large gains in throughput. As a result, next generation relay protocols should incorporate channel state feedback for throughput maximization in order to realize the true benefits of node cooperation.

The remainder of this chapter is as follows. Section 5.2 outlines the rate control procedure assuming an infinite supply of source data. Section 5.3 analyzes the throughput when the source rate is limited, and the queue size is finite. Section 5.4

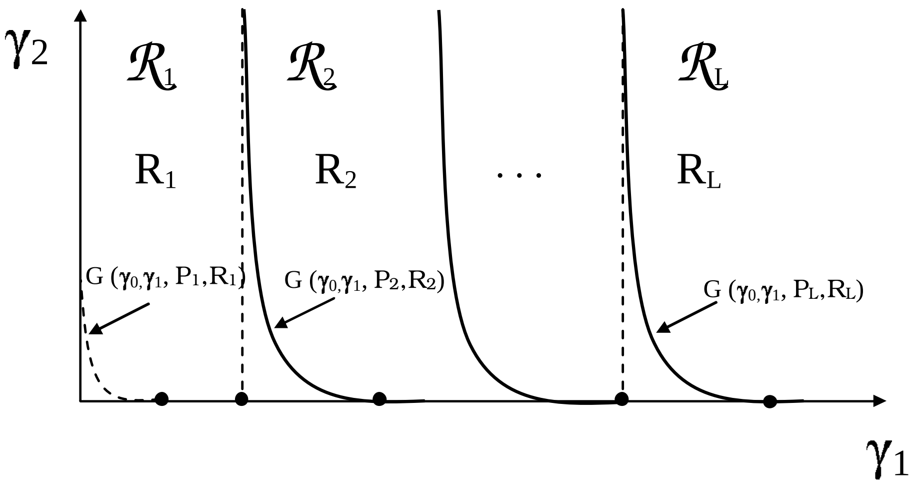


Figure 5.1 : For a fixed γ_0, a typical set of rate control regions is shown for $\log_2 L$ bits of feedback in the relay channel. Below the dotted line in $\mathcal{R}_1$ indicates the outage region. The contour separating $\mathcal{R}_{q-1}$ and $\mathcal{R}_q$ denotes all points which can guarantee zero outage while transmitting at a rate R_q and a power P_q.

---

provides concluding remarks.

5.2 Throughput Maximization based on the Network Channel State

In this section, we describe a rate control algorithm for cooperative relay networks which maximizes the throughput for a limited feedback channel. First, the general problem setup is described, and then the analysis is refined for relay codes, multi-hopping and direct transmission. The transmission policy assumes packet fragmentation is possible up to arbitrary precision [33]. Furthermore, we assume that there is an infinite backlog of data at the source. These assumptions together imply that the transmission policy is a function solely of the network channel state.

5.2.1 Problem Setup

5.2.1.1 Rate Control Regions

Consider a network of 3 nodes with a source communicating to a destination with the possible assistance of a relay node. We assume that the destination has a perfect estimate of the network channel state $\underline{\gamma}$. Given M bits of feedback, the space defined by all possible $\underline{\gamma}$ is quantized into $L = 2^M$ regions $\mathcal{R}_q, q \in \{1, 2, \ldots, L\}$. Upon measuring the channel state, if $\underline{\gamma} \in \mathcal{R}_q$, the destination selects a rate level R_q from a rate control codebook $\mathcal{C}$ and a power P_q from a power control codebook $\mathcal{P}$, where $q \in \{1, 2, ..., L\}$. The index q is transmitted to both the source and relay through a noiseless feedback link. It is assumed that both the source and relay have copies of $\mathcal{C}$ and $\mathcal{P}$. Upon reception of the index q, the source and relay then transmit with rate $R_q = \mathcal{C}(q)$ and power $P_q = \mathcal{P}(q)$.

An example of the L rate control subregions is shown in Figure 5.1 for a given value of γ_0. The rate control regions can be visualized as three dimensional volumes, where a cross section of the regions at γ_0 is represented in Figure 5.1. One key feature of the rate control regions is that in region $\mathcal{R}_q$, $q \geq 2$, for a given power, the rate assigned is the maximum that guarantees zero outage for any point in the region. This is a fundamental property of all optimal finite rate feedback algorithms [12]. In $\mathcal{R}_1$, a subset of the channel states cannot support a rate of R_1 for a finite average power constraint, and channel states belonging to this subset would result in an outage event. The objective of the throughput maximization algorithm is to select the rate control regions to maximize the throughput. Throughput maximization does not necessarily require a minimization of the outage region. As we show later, it is often beneficial to increase the probability of an outage in $\mathcal{R}_1$ to increase the throughput.

Given that all regions except $\mathcal{R}_1$ have a zero outage condition, the boundary between regions $\mathcal{R}_{q-1}$ and $\mathcal{R}_q$ is separated by a contour, $\gamma_2 = G(\gamma_0, \gamma_1, P_q, R_q)$ [29], which represents all $(\gamma_0, \gamma_1, \gamma_2)$ such that

$$R_{gen}(\underline{\gamma}, P_q, P_q) = R_q, \tag{5.1}$$

where $R_{gen}(\cdot)$ is the achievable rate of a generic transmission protocol. Any points lying along the contour can support rate R_q and guarantee zero outage while transmitting with power P_q. Any points lying above the contour can support a higher rate than R_q while maintaining zero outage, while points below the contour require a smaller rate than R_q to invert the effect of the channel. The outage region is a subset of $\mathcal{R}_1$. For a given γ_0, the outage region is denoted by any points lying below the dotted curve in Figure 5.1. Any points in this region require a rate less than R_1 to invert the effect of the channel. The outage probability in the general setting of time-varying $(\gamma_0, \gamma_1, \gamma_2)$ is expressed as

$$\epsilon = \int_{\underline{\gamma}:R_{gen}(\underline{\gamma}, P_1, P_1) \leq R_1} f(\underline{\gamma}) d\underline{\gamma}, \tag{5.2}$$

where $f(\underline{\gamma})$ is the joint distribution of the network channel state $\underline{\gamma}$.

5.2.1.2 Throughput Analysis

Due to the effect of outages, the effective data rate will be less than the attempted rate of transmission. Given L rate control regions, with outages occurring in $\mathcal{R}_1$, the throughput can be expressed as

$$T = R_1(\Delta_1 - \epsilon) + \sum_{q=2}^{L} R_q \Delta_q,$$

where ϵ is the outage probability defined in (5.2) and

$$\Delta_q = \int_{\mathcal{R}_q} f(\underline{\gamma}) d\underline{\gamma}.$$

The objective is to maximize the throughput subject to an average power constraint of

$$\epsilon \cdot \min\{P_1, P_2, \cdots, P_L\} + (\Delta_1 - \epsilon)P_1 + \sum_{q=2}^{L} P_q \Delta_q \leq P. \tag{5.3}$$

The first and second terms on the left hand side of (5.3) imply that in $\mathcal{R}_1$, whenever an outage event occurs, the destination should indicate to the source and relay to use the least amount of power possible rather than sending with power P_1 for any point in R_1. This technique saves power without a reduction in performance.

For a given outage probability, the throughput can be maximized by finding appropriate rates and powers for the codebooks $\mathcal{C}$ and $\mathcal{P}$, which involves searching for a set of L contour functions. For variable rate transmission with a constant transmit power (VRCP), as $L \rightarrow \infty$, the outage probability can be made arbitrarily small. In the limit with perfect feedback, the throughput can be expressed as

$$T = \int_{\underline{\gamma}} R_{gen}(\underline{\gamma}, P, P) f(\underline{\gamma}) d\underline{\gamma}. \tag{5.4}$$

Note that (5.4) is achieved without outages, as the rate is adapted for each and every channel state. It also serves as an upper bound in throughput to the rate control finite rate feedback procedure whenever constant power transmission is assumed. For VRCP, maximizing the throughput involves increasing the rate for better channel conditions, and as a result we can say that $R_1 \leq R_2 \cdots \leq R_L$ when there are $\log_2 L$ bits of feedback.

For constant rate transmission with variable power (CRVP), throughput maximization involves two steps. First, given the rate of the feedback channel and a fixed transmission rate, the outage probability is minimized using the power control strategies in [29]. Second, rate selection is performed to find the transmission rate that leads to the largest throughput. When perfect feedback is available, the power allocation procedure resembles channel inversion for all but a subset of the channel states. In poor channel conditions, transmission is shut off and an outage is declared to meet the long-term power constraint [29]. When the transmission rate is fixed and the power is variable, outage minimization leads to a power allocation policy with decreasing transmit powers for better channel conditions, meaning that $P_1 \geq P_2 \geq \cdots P_L$.

Variable rate transmission with power control (VRVP) leads to the largest possible throughput given a long-term power constraint. In the limit of perfect feedback, the optimal solution involves power control based on the concept of 'water-filling' [25]. In this case, transmission is shut off in poor channel conditions, and, in better channel conditions, increased rate and power are used to maximize the throughput. For a finite rate of feedback, this leads to $P_1 \leq P_2 \leq \cdots P_L$ and $R_1 \leq R_2 \leq \cdots R_L$. The power allocation strategies for constant rate and variable rate transmission are quite different. In constant rate transmission, more power is allocated for poor channel states to reduce the risk of an outage. In variable rate transmission, increased power is used for better channels to maximize the throughput.

It will be seen that, even with a few bits of feedback, using VRVP to maximize the throughput leads to tremendous gains over the case of constant rate/constant power transmission (CRCP). Interestingly, we will show that in some domains of operation, it suffices to use CRVP and in other cases using VRCP is sufficient to achieve large throughput gains over CRCP transmission.

5.2.2 Relay Codes

To apply the throughput analysis to relay codes, the rate control regions must be derived for a given protocol. For the decode and forward protocol, the rate control regions are shown in Figure 5.2 for the case of 1 bit of feedback. With 1 bit of feedback, there are two rate control regions. In $\mathcal{R}_1$, a rate of R_1 and a power of P_1 is used, and in $\mathcal{R}_2$, a rate of R_2 and a power of P_2 is used. The boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$ is separated by $G(\gamma_0, \gamma_1, P_2, R_2)$, which is the shaded surface in the figure, and

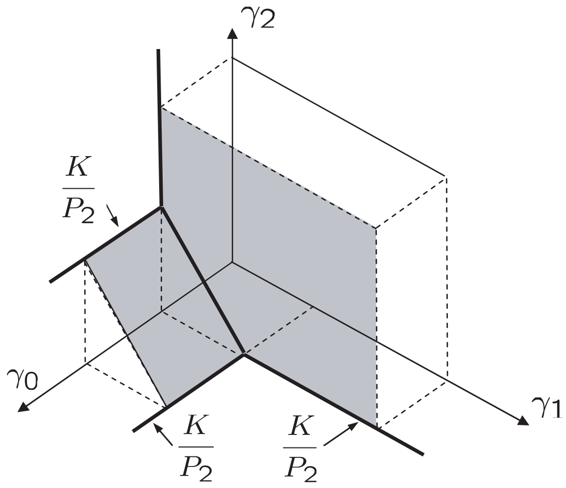


Figure 5.2 : Rate control region for decode and forward for 1-bit of feedback. The parameter K/P_2 determines the contour which separates regions 1 and 2. Rate level R_1 is used in region 1 and R_2 is used in region 2.

is obtained by solving (5.1) with $R_{gen}(\cdot)$ substituted by the achievable rate expression for decode and forward. The contour $G(\gamma_0, \gamma_1, P_2, R_2)$ is characterized by the term K/P_2 in Figure 5.2, where $K = e^{R_2} - 1$.

The rate control region of Figure 5.2 is general and can be applied to VRVP, VRCP and CRVP. If we impose the restriction that the transmit power is constant, then the contour is characterized by the term $(e^{R_2} - 1)/P$, where P is the constant power value. On the other hand, if the transmission rate is constant and we are using CRVP, then the contour is characterized by $(e^{R} - 1)/P_2$, where R is the selected rate of transmission.

In Figure 5.2, we see that as long as $\gamma_0 < K/P_2$, then the channel state lies in $\mathcal{R}_1$, and as a result the possibility of an outage exists, irrespective of the value of γ_1 or γ_2.

This emphasizes that the performance of decode and forward is strongly influenced by the source-relay link. The outage region for decode and forward is a subset of $\mathcal{R}_1$, and its boundary is a contour, similar to the boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$ shown in Figure 5.2, except the outage region contour is parameterized by $\frac{e^{R_1-1}}{P_1}$. The following theorem formally states that the boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$ shown in the figure is a contour based on the definition in (5.1).

Theorem 5.2.1. *For the rate control contour of decode and forward separating $\mathcal{R}_1$ and $\mathcal{R}_2$ with 1 bit of feedback, any points lying above this contour can support a rate larger than R_2 and still guarantee zero outage, while points below this contour require a rate less than R_2 to guarantee zero outage.*

Proof: See Appendix C ■

In the derivation of the rate control region for decode and forward, we have assumed that $\rho = 0$, implying that the source and relay transmit signals are uncorrelated. Adapting ρ along with the rate and power would lead to a gain over assuming a constant ρ over all time. However, we next show that for the case of constant rate and power transmission, selecting a value of $\rho = 0$ is optimal.

Theorem 5.2.2. *For decode and forward relaying in the absence of channel state information, selection a source-relay correlation value of $\rho = 0$ maximizes the throughput.*

Proof: See Appendix D ■

The rate control region for selection relaying resembles that of decode and forward, and is described next. Consider the case of 1 bit of feedback. A key difference between SR and DF is that the contour $G(\gamma_0, \gamma_1, P_2, R_2)$ is now defined by $K = e^{2R_2} - 1$. The factor of two in the exponent is to compensate for the loss associated with half-duplex relaying. In SR, when $\gamma_0 \geq \frac{K}{P_2}$, the contour $G(\gamma_0, \gamma_1, P_2, R_2)$ is identical to that of DF, except that $K = e^{2R_2} - 1$. When $\gamma_0 < \frac{K}{P_2}$, the contour $G(\gamma_0, \gamma_1, P_2, R_2)$ is a cube defined by all $\underline{\gamma}$ such that $\gamma_0 = \frac{K}{P_2}$ and $\gamma_1 = \frac{K}{2P_2}$. The structure of the contour can be verified by substituting the achievable rate of selection relaying for $R_{gen}(\cdot)$, and solving (5.1). Note that in selection relaying, the acknowledgement feedback from the relay to the source allows for a reduction in the size of rate control region 1. The feedback from the relay to the source reduces the strong influence on γ_0 seen in Decode and Forward. As a result, the performance of SR does not significantly degrade with poor source-relay channel conditions as long as the source-destination channel condition is strong.

5.2.3 Multi-hopping

For multi-hopping, the throughput maximization procedure is similar to that of decode and forward, except now the rate control contours are defined by the space of all (γ_0, γ_2) rather than $(\gamma_0, \gamma_1, \gamma_2)$. The rate control contour for multi-hopping is shown in Figure 5.3 for 1 bit of feedback. In region $\mathcal{R}_2$, rate R_2 and power P_2 are used, and in region $\mathcal{R}_1$, rate R_1 and power P_1 are used. Outages only occur in $\mathcal{R}_1$, and they correspond to all channel states that require a smaller rate than R_1 to invert the effects of the channel. The contour $G(\gamma_0, \gamma_1, P_2, R_2)$ is characterized by the parameter K/P_2, where $K = e^{R_2} - 1$. Although the rate control region is shown for the case of VRVP, by setting $K = e^R - 1$ we have CRVP transmission. Similarly, by changing P_2 to the average power constraint P, the contour defining parameter now becomes $(e^{R_2} - 1)/P$ and this leads to the case of VRCP. For CRCP transmission, closed form results are possible for multi-hopping. For constant rate transmission, closed form results are possible for multi-hopping.

Theorem 5.2.3. *For multi-hopping with the source and relay transmitting with a constant power P and at a constant rate, the maximum throughput is*

$$T = (1 - \epsilon^*) \log \left(1 - P \log(1 - \epsilon^*) \frac{\lambda_2 \lambda_0}{\lambda_2 + \lambda_0} \right), \tag{5.5}$$

where

$$\epsilon^* = 1 - e^{\left(\frac{1}{\mathcal{W}(PC)} - \frac{1}{PC} \right)}, \tag{5.6}$$

and $C = \frac{\lambda_2 \lambda_0}{\lambda_2 + \lambda_0}$. The function $\mathcal{W}(\cdot)$ is known as the Lambert-W function, which satisfies $\mathcal{W}(x) e^{\mathcal{W}(x)} = x$.

Proof: For the case of constant rate transmission, only one rate R_1 and one power P is used for all channel states. The outage region is characterized by a contour, which subdivides $\mathcal{R}_1$ into two subregions. Channel states lying below the contour are always in outage, while points lying above the contour are never in outage.

The structure of the outage contour can be understood by considering the rate control region for 1 bit of feedback and VRVP shown in Figure 5.3. The boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$ is a contour, which represents all channel states that require a rate R_2 to guarantee zero outage. In much the same way, the outage region contour is a curve that represents all channel states that require a rate R_1 to guarantee zero outage. Any points lying below this contour require a smaller transmission rate than R_1 to invert the effects of the channel, and there is an outage for those states. Considering Figure 5.3 with $P_2 = P$ and by interpreting the shaded region as the outage region contour, we have the case of CRCP. The probability of a channel state lying within the outage region can be expressed as

$$\epsilon = e^{-\frac{K}{P\lambda_2}}(1 - e^{-\frac{K}{P\lambda_0}}) + 1 - e^{-\frac{K}{P\lambda_2}},$$

where $K = e^{R_1} - 1$. Solving for the rate leads to

$$R_1 = \log\left(1 - P\log(1-\epsilon)\frac{\lambda_2\lambda_0}{\lambda_2 + \lambda_0}\right).$$

Maximizing the throughput $(1-\epsilon)R_1$ with respect to ϵ leads to (5.6). ■

As the source-relay distance decreases, then $\lambda_0 \to \infty$, and $\lambda_2 \to 1$. This scenario corresponds to the throughput of direct transmission, which is less than multi-hopping for any given ϵ, under the relay model shown Figure 2.2.

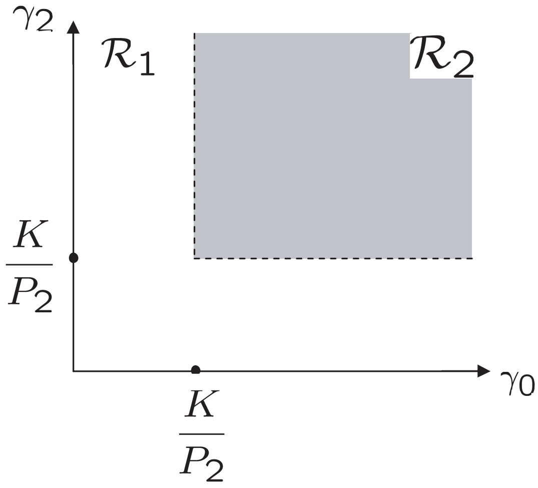


Figure 5.3 : For the multi-hopping system, rate control regions are shown for 1-bit of feedback. Rate level R_2 is used in region 2, and rate level R_1 is used in region 1.

5.2.4 Direct Transmission

The analysis for direct transmission is simplified in that instead of a network channel state, only the fading parameter γ_1 is used. Furthermore, rather than a rate control contour, a threshold is used to separate two subregions. A similar approach for direct transmission was taken in [34], however in this section we consider throughput as a function of the outage probability, which in the results section leads to interesting insight with regards to optimal outage operating points and the importance of rate selection.

We next proceed to define the throughput maximization problem for direct transmission using VRVP, and consider the solution for some special cases. The space defined by all γ_1 is divided into L subregions, with each subregion corresponding to a rate of transmission. Rate control value R_q is chosen if $\beta_{q-1} \leq \gamma_1 \leq \beta_q$. As noted earlier, the outage region is always a subset of $\mathcal{R}_1$. The throughput with limited feedback at an outage probability of ϵ is expressed as,

$$T = \sum_{q=1}^{L} R_q \int_{\beta_{q-1}}^{\beta_q} f(\gamma_1) d\gamma_1, \tag{5.7}$$

where R_q is the rate of transmission in region q, $\beta_q = \infty$, and β_0 is the cutoff value chosen such that all channel gains lying below this cutoff will be in outage.

The throughput maximization of (5.7) is done over all power allocations which satisfy

$$\epsilon \cdot \min\{P_1, P_2, \cdots, P_L\} + P_1(\Delta_1 - \epsilon) + \sum_{q=2}^{L} \Delta_q P_q \leq P, \tag{5.8}$$

where $\Delta_q = \int_{\beta_{q-1}}^{\beta_q} f(\gamma_1) d\gamma_1$ denotes the probability of the channel state lying in region $\mathcal{R}_q$ and the outage probability is $\epsilon = \int_0^{\beta_0} f(\gamma_1) d\gamma_1$. Looking at (5.8) we observe that in $\mathcal{R}_1$, whenever the channel is in outage power can be saved by using the minimum power in the set $\{P_1, P_2, \cdots, P_L\}$.

For Rayleigh fading this probability can be expressed as $\Delta_q = e^{-\beta_{q-1}} - e^{-\beta_q}$. If we rewrite the throughput as $T = \sum_{q=1}^{L} [e^{-\beta_{q-1}} - e^{-\beta_q}] R_q$, then to find the optimal value of β_q, we need to solve $\frac{\partial T}{\partial \beta_q} = 0$, which leads to

$$e^{-\beta_q} \log(1 + P_q \beta_{q-1}) + \frac{(e^{-\beta_q} - e^{-\beta_{q+1}}) P_{q+1}}{1 + P_{q+1} \beta_q} - e^{-\beta_q} \log(1 + P_{q+1} \beta_q). \tag{5.9}$$

Therefore, threshold β_q will be a function of thresholds β_{q-1} and β_{q+1}. We know that β_0 is fixed for a given outage probability ϵ, and that $\beta_L = \infty$. As a result, for a few special cases of the rate of the feedback link, closed form results are possible.

For the case of 1 bit of feedback, its not hard to verify that the objective function T is convex and as such a unique maximum exists. Additionally, we know that $\beta_2 = \infty$, and threshold level β_1 can be solved for explicitly as

$$\beta_1 = \frac{1}{\mathcal{W}\left(\frac{P_2}{1-P_1\log(1-\epsilon)}\right)} - \frac{1}{P_2}, \tag{5.10}$$

where $\mathcal{W}$ is the Lambert-W function. The throughput using VRVP is

$$T = (1-\epsilon-e^{-\beta_1})\log(1-P_1\log(1-\epsilon)) + e^{-\beta_1}\log(1+P_2\beta_1). \tag{5.11}$$

As another special case, consider the scenario when $M = 0$, i.e., when there is no feedback. In this case, a constant rate and power (CRCP) is used for each time instant. The solution can be obtained by setting $\lambda_2 = 1$ and $\lambda_0 = \infty$ in (5.5) for multi-hopping. As a result, we obtain the following throughput expression

$$T = \log(1-P\log(1-\epsilon)) \cdot (1-\epsilon). \tag{5.12}$$

Interestingly, for any outage probability that does not equal 0 or 1, the throughput is greater than zero. The same cannot be said about the delay-limited capacity, which is zero for the block Rayleigh fading channel.

5.2.5 Analysis and Results

Figure 5.4 shows the throughput versus outage results for decode and forward and direct transmission. In this figure, we use VRCP. We assume that the total network

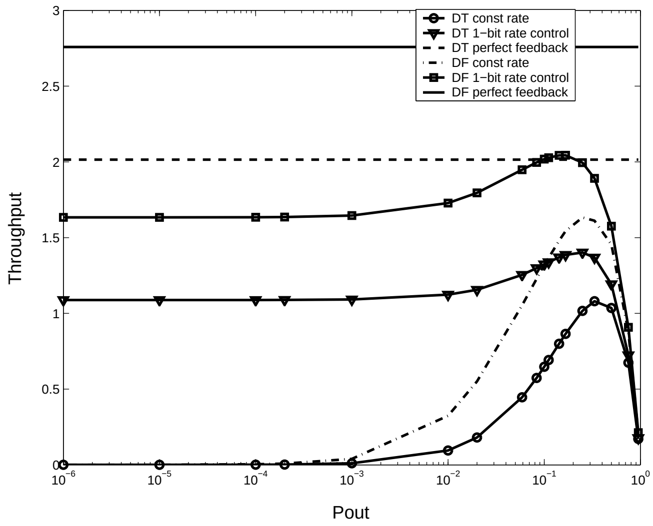


Figure 5.4 : Throughput versus outage probability for system with a constant SNR = 10dB and with a distance parameter of d=0.5. Only rate control is used and the transmit power is a constant (VRCP). The performance of the decode and forward (DF) protocol is compared to that of direct transmission (DT) for different sizes of the rate control codebook.

---

signal to noise ratio (SNR) is $10dB$ for the relay coding case, which includes power from both the source and relay. For direct transmission, to fairly compare against relaying, we use a source power that equals the total network power used in the relay case. Additionally, we assume that the source-relay distance is $d = 0.5$. This implies that the relay is midway between the source and destination. We see that for decreasing outage probabilities, both decode and forward and direct transmission have a decreasing throughput. This result confirms that for Rayleigh block fading the delay-limited capacity is zero, meaning that a non-zero throughput is not possible when error free communications is desired. However, we observe that a large throughput is in fact possible, which suggests that delay-limited capacity is not a good measure of performance in block fading channels.

Observing Figure 5.4, it is apparent that for small outage probabilities, the throughput is always non-zero for both decode and forward and direct transmission when feedback is incorporated. With VRCP, the maximum throughput for decode and forward increases by more than 33% with the use of 2 rate control levels. Furthermore, the gap to the perfect feedback rate control limit is substantially reduced with the addition of just one bit of feedback. Interestingly, from Figure 5.4, we see that with one feedback bit, decode and forward has a larger maximum throughput than direct transmission with perfect feedback.

Power control in conjunction with finite rate feedback has shown great promise in the relay channel for outage reduction with fixed rate transmission [29]. When the rate and power are adapted over time, in the limit of perfect feedback, performance is upper-bounded by the ergodic capacity of the channel, which can be achieved with a water-filling based power control algorithm [25]. Figure 5.5 demonstrates the effect of a combined power/rate control (VRVP) procedure in comparison with variable rate/constant power (VRCP) and constant rate/variable power (CRVP) throughput maximization at an SNR of $10dB$. Adapting both the rate and power leads to a larger throughput than VRCP. However, we see that the gains over VRCP are not tremendous for large average power constraints. For example, using 1-bit of feedback, the throughput gains over 0 feedback bits are approximately 28% at an SNR of $10dB$ for both VRCP and VRVP. Furthermore, both VRCP and VRVP achieve diminishing returns with increasing bits of feedback.

Also shown in Figure 5.5 is the throughput when a constant rate is transmitted and the feedback bits are used solely for power control (CRVP). In this case, rather than using multiple transmission rates, one rate is selected that maximizes the throughput. We see that CRVP leads to a reduced throughput over techniques which use a variable rate. Rate adaptation is essential at large average powers. On the other hand, for small power constraints, power adaptation is imperative. This is seen by comparing Figure 5.5 and Figure 5.6. When the SNR is $10dB$, VRCP is nearly as useful as VRVP. However, for smaller average power constraints, power control is essential to maximize the throughput. In fact, as shown in Figure 5.6, transmitting at a constant rate and using power control (CRVP) significantly outperforms VRCP at an SNR of $-5dB$. At an SNR of $2dB$, the performance of VRCP and CRVP are almost similar, with CRVP having a modest throughput gain over VRCP, as can be seen from Figure 5.7.

Another interesting conclusion that can be drawn is that for a low rate feedback link, simultaneous power and rate adaptation is unnecessary. For small SNR's, power control and rate selection (CRVP) is nearly optimal for 1 bit of feedback. On the other hand, for large SNR's, rate control with constant power transmission (VRCP) is nearly optimal when 1 bit of feedback is used. Our results show that although VRVP leads to the highest possible throughput, the extra complexity required to adapt both the rate and power is not warranted. CRVP is a powerful technique for small SNR's,

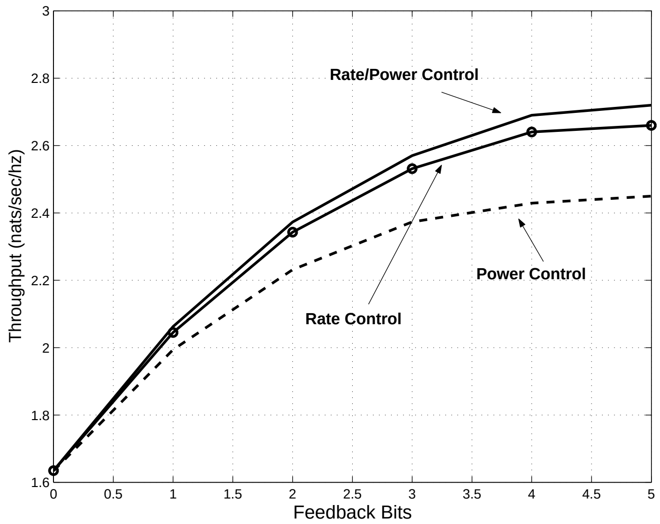


Figure 5.5 : For an SNR of $10dB$, the throughput of decode and forward is shown as a function of the number of feedback bits for a source-relay distance of $d = 0.5$. Adapting both the rate and power leads to increases over constant power transmission, with the gains most evident as the feedback rate increases. Power control alone leads to good performance for small average powers.

---

and by using one rate codebook at the transmitters, it leads to a lower complexity than VRVP. For large power constraints, amplifier nonlinearities and FCC regulations may prohibit the use of a large dynamic power range. However, our results show that the use of power control in this regime is not necessary for a high throughput.

5.2.5.1 Optimal Outage Operating Point

Observing Figure 5.4 once again for the case of decode and forward with constant rate transmission, we see that the throughput is maximized at a large outage probability, greater than 10%. For increasing amounts of feedback, the optimal outage probability decreases, and only in the limit of perfect feedback does it equal zero. The result

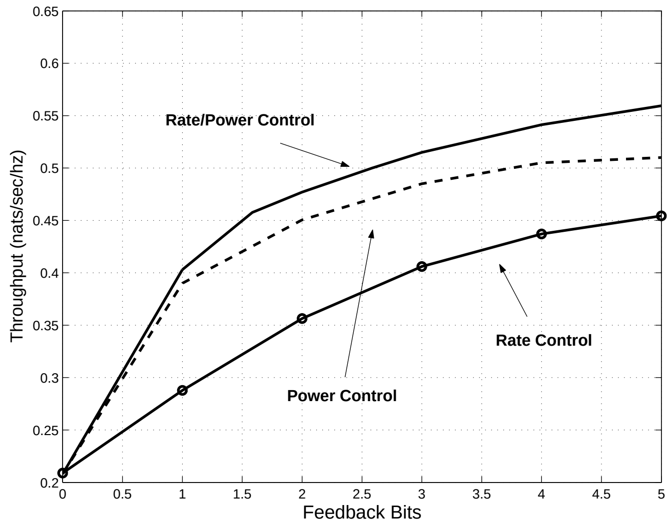


Figure 5.6 : For an SNR of $-5dB$, the throughput of decode and forward is shown as a function of the number of feedback bits for a source-relay distance of $d = 0.5$. Adapting both the rate and power leads to increases over constant power transmission, with the gains most evident as the feedback rate increases. The performance is further from the combined rate/power control limited at high average powers.

---

suggests that, for a finite rate of feedback, it is unnecessary to minimize the outage probability when constant power transmission is used. Rather, the coding rates should be appropriately chosen to meet the throughput maximizing outage probability, which can be quite high.

When power control is utilized with rate selection to maximize the throughput, the outage probability can still be quite large. For example, for direct transmission with perfect feedback and power control, the maximum throughput at $SNR = 10dB$ is $1.84nat/sec/Hz$, at an outage probability of approximately 25%. This outage can be calculated by using the water-filling power control technique described in [25]. This emphasizes that high outage operating points are necessary whenever power control

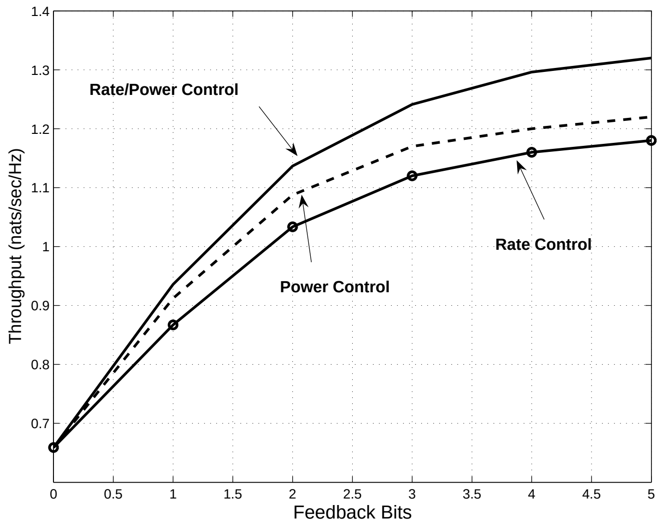


Figure 5.7 : Throughput of decode and forward for SNR=2dB. At this SNR, power and rate control have similar performance, with the power control algorithm having a slightly higher throughput than rate control.

---

is employed. It is better to increase the coding rate, and allow for frame errors to maximize the long-term throughput. Recall that with perfect feedback and constant power transmission, there are no outages since the rate is adapted for every channel state. This implies that VRCP leads to a smaller optimal outage operating point than VRVP.

For the case of direct transmission with constant power transmission, explicit results can be found for the case of constant rate transmission. Maximizing the throughput of (5.12) over all ϵ leads to the following optimal operating point

$$\epsilon^* = 1 - e^{-\left(\frac{1}{\mathcal{W}(P)} - \frac{1}{P}\right)}, \tag{5.13}$$

where $\mathcal{W}$ is the Lambert-W function. For example, at an SNR of 10dB, the through-

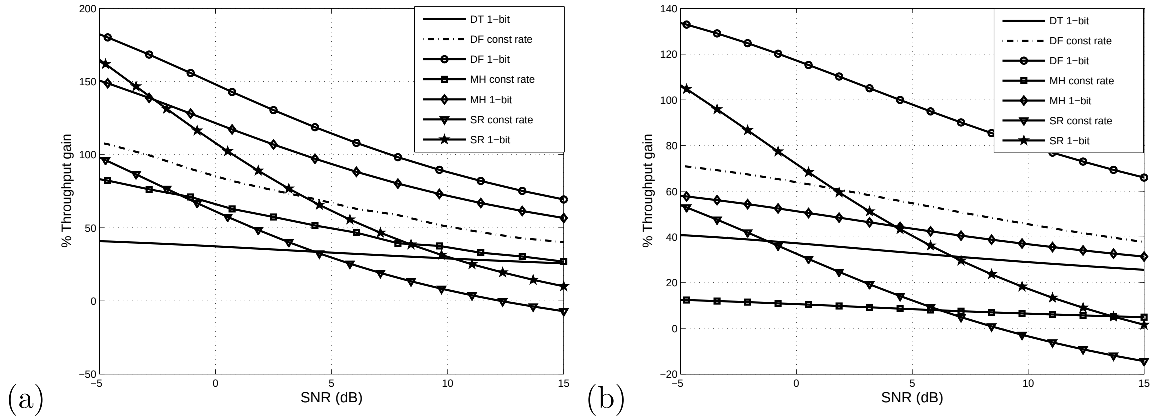


Figure 5.8 : For a source-relay distance of (a) $d = 0.5$, and (b) $d = 0.25$, the % gain in throughput of various protocols over constant rate direct transmission is shown as a function of SNR. The protocols shown are direct transmission (DT), decode and forward (DF), multi-hopping (MH) and selection relaying (SR). In all cases, the transmit power does not change over time (VRCP).

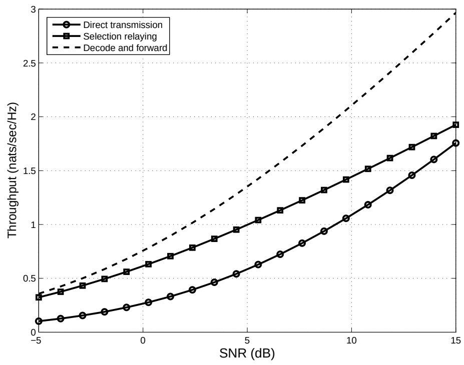
---

put is maximized when the outage probability is approximately 20%.

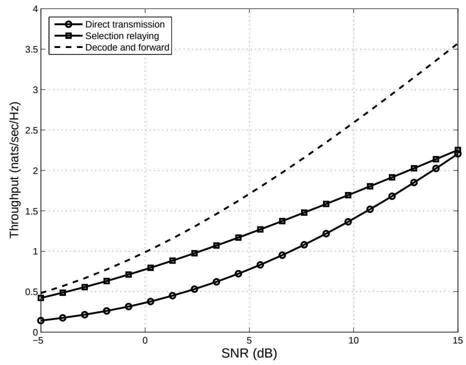
5.2.5.2 Comparison of Cooperative Techniques

We next compare results for various collaborative communication techniques. Figure 5.8(a) shows the percentage gain of various protocols over constant rate direct transmission when the source-relay distance $d = 0.5$ for VRCP. Multi-hopping has the smallest gains over direct transmissions. Furthermore, the gains over direct transmission are diminishing with respect to power. Decode and Forward has large gains over both direct transmission and multi-hopping. Adding feedback increases the gains over constant rate direct transmission for all the protocols. The gains of cooperative communication are quite apparent when feedback is considered. For an SNR of $5dB$, Decode and Forward with 1 bit of feedback has a 115% gain over constant rate direct transmission, whereas direct transmission with 1 bit of feedback has a 33% gain over constant rate direct transmission. As the SNR increases, the gains over direct transmission are diminishing. However, for a practical range of SNR's, tremendous gains in throughput are seen with the rate control procedure in combination with relay coding. Even the low complexity selection relaying (SR) protocol offers large gains over direct transmission for small SNR's. We observe that at an SNR of $-5dB$, SR has nearly a 100% gain in throughput over direct transmission. With 1-bit of feedback, SR has gains over direct transmission up to an SNR of $10dB$. For large SNR's, SR performs worse than direct transmission due to the loss from repetition coding and the half-duplex constraint.

When considering a smaller source relay distance, such as $d = 0.25$ depicted in Figure 5.8(b), we see that the gap in performance between multi-hopping and Decode and Forward increases. For small source-relay distances, it has been observed that Decode and Forward is nearly optimal in the case of outage minimization with power control [35] and for the case of ergodic capacity [26]. For a value of $d = 0.25$, multi-hopping has a small gain over direct transmission, but cooperative coding clearly surpasses multi-hopping. Even selection relaying has large throughput gains over multi-hopping in this figure for a large range of SNR's. Multi-hopping gains over direct transmission from strong source-relay and relay-destination links. As the source-relay distance decreases, there is an almost noiseless source-relay link. However, the relay-



(a)



(b)

Figure 5.9 : The effect of individual energy sources for the source and relay with decode and forward (DF) and selection relaying (SR) are shown in comparison to direct transmission (DT) for a source-relay distance of $d = 0.5$ and constant power transmission for (a) no feedback and (b) 1 bit of feedback for rate control.

---

destination average channel gain approaches that of the source-destination, making multi-hopping only marginally superior to direct transmission.

Our results show that Decode and Forward is a powerful technique in the fading channel, with large throughput gains over direct transmission. However, we see that selection relaying also performs well in the power limited regime, with gains over multi-hopping and direct transmission. In scenario's where low complexity is required at the relay the use of this simple protocol is attractive.

5.2.5.3 Individual Power Constraints on the Source and Relay

In comparing relaying to direct transmission, the results to this point have used a sum power constraint on the source and relay and equated this with the power used in direct transmission. Next, we consider another practical scenario in which we compare a system where the source always has the same power for both direct transmission and relaying. In relaying schemes, the relay has its own individual energy source, and we assume $P_r = P_s$. The results shown in Figure 5.9(a) and Figure 5.9(b) demonstrate the gains of relaying in this scenario. The performance of direct transmission is shown for constant rate transmission in comparison with selection relaying and decode and forward in in Figure 5.9(a). We see that, despite the rate loss from the source not transmitting data over the entire time slot, selection relaying has a gain of $4dB$, and decode and forward a gain of $7dB$ over direct transmission at a spectral efficiency of 1 nats/sec/Hz. As the SNR increases, selection relaying eventually performs worse than direct transmission. However, for a practical range of SNR's, it is superior to direct transmission. In the low power regime, the spectral efficiency is proportional to the average received power. Due to the better channels in the source-relay and relay-destination links, the use of selection relaying leads to a large gain over direct transmission in the power limited regime. Figure 5.9(b) shows a similar trend for the case of 1 bit of feedback used for rate control.

5.3 Impact of Finite Queue Backlogs on Throughput

Controlling the transmission rate over time can lead to significant improvements in throughput. In practice, the transmission rate is not only a function of the channel state, but also of the amount of data available in the transmission buffer. In periods of poor channel conditions, transmitting at low rates leads to an increased backlog in the transmit buffer. When the backlog exceeds the buffer size, dropped packets result and this leads to another source of outage in a network. Transmitting at a higher rate is possible only if the data backlog is sufficient to allow for large transmit spectral efficiencies. In this section, we study the interplay between finite backlogs, maximum buffer sizes and throughput.

5.3.1 Transmission and Queueing Model

The network channel state $\underline{\gamma}$ is assumed constant for T_c seconds, and varies independently the next T_c seconds. Given a symbol period of T_s seconds, for large values of $\kappa = T_c/T_s$, the achievable rates of Section 2.4 serves as a good indicator of performance for coded systems [8]. Hence, given a source power of P_s and a relay power of P_r, an outage event occurs if the attempted rate of transmission is larger than $R_{gen}(\underline{\gamma}, P_s, P_r)$, where $R_{gen}(\cdot)$ is the conditional mutual information of a generic network transmission protocol. This implies that an outage occurs if the number of information bits in a packet exceeds $\kappa \times R_{gen}(\cdot)$. For the remainder of this section, we assume that $P = P_s = P_r$ for simplicity of exposition and that κ is large enough so that the achievable rate serves as a good indicator of performance.

A constant rate source provides a single packet of size $R \cdot \kappa$ information bits in each time slot into a first come first serve queue of size V packets located at the source node. The buffer state at time t is $q_t \leq V$, which is the number of buffered packets. In state q_t, the source may transmit any of $r_t = \{0, 1, \ldots, q_t\}$ packets. The transmission policy implies a packet integrity constraint [33], meaning that multiple packets may be transmitted in a particular time slot, but packets may not be fragmented over multiple time slots. In this work, the cooperative protocols considered have both the source and relay transmit at a rate of r_t. Although cooperative techniques exist without this constraint, DF and SR are examples of protocols that exhibit excellent performance when the source and relay transmit with codebooks of equal size. The transmission rate and power are determined by the amount of feedback available from the destination, the current buffer state, and the chosen network coding protocol. In poor channel states, no transmission occurs and the data is buffered. This is a similar model to that used in [36].

The source and relay have V power codebooks $\mathcal{P}_1, ..., \mathcal{P}_V$, where $\mathcal{P}_k = \{P_{k,0}, ..., P_{k,k}\}$ and $P_{k,0} = 0$. Power level $P_{k,i}$ is used when the buffer size $q_t = k$ and the transmission rate is iR, $i \in \{0, 1, \ldots, k\}$. This has similarities with the VRVP policy of the previous section, except in this case the transmission rate is limited by both the channel state *and* the buffer size. The decision as to which rate and power to use is determined by the destination, which has a perfect measure of the network channel state $\underline{\gamma}$. If the destination feeds back index i to the transmitters when $q_t = k$, then the source and relay are to transmit with rate iR, $i \leq k$, and power $\mathcal{P}_k(i) = P_{k,i}$ in the current time slot.

It is assumed that the feedback link is noiseless, and as a result the source and relay are always able to correctly infer the current transmission rate and power to be used. In order to calculate the appropriate rate and power allocation, the relay and destination must be aware of the current source buffer state. The relay and destination only need the starting buffer state, which can be discovered in the initialization phase of the communication process. The protocol overhead to communicate the initial buffer size is $\log_2 V$ bits. Since the destination calculates the transmission rate for the source and relay to use, and since the arrival rate to the source is constant, the destination will be able to keep track of the state of the source buffer without any additional protocol overhead. Similarly, the relay will have the initial buffer state, and throughout the communication process it can maintain an update of the state of the source buffer when it receives feedback from the destination.

5.3.2 Throughput Maximization

Given the probability of being in buffer state k is s_k, $k \in \{1, \ldots, V\}$, the long-term throughput can be expressed as

$$T = \sum_{k=1}^{V} s_k \sum_{i=0}^{k} i \cdot \Delta_{k,i} \cdot R, \tag{5.14}$$

where for a given power control policy, $\Delta_{k,i}$ is the probability of transmitting i packets error-free while in buffer state k. Note that i varies from 0 to k because when the buffer has k packets, at most k can be transmitted at a time. Let $\mathcal{R}_{k,i}$ denote all channel states that lead to the transmission of i packets in buffer state k. In the transmission scenario described above, outages occur at the source if a packet is dropped due to buffer overflow. When zero packets are transmitted due to a poor channel state and one packet arrives in a time slot, then the arriving packet must be dropped when $q_t = V$ since the buffer is already full. This is the only scenario in which packets will be dropped due to buffer overflow. The blocking probability is expressed as

$$\Pi = s_V \cdot \Delta_{V,0}. \tag{5.15}$$

When feedback is unavailable, then the source will always transmit one packet in each time slot. Consequently, there will be no dropped packets, but outages will occur at the destination due to decoding errors. In summary, there are two types of outage events. When no feedback is available to the transmitters, outages occur at the destination as a result of decoding errors. When feedback is available, decoding errors can be prevented by controlling the transmission rate, however buffer overflows result in dropped packets, which can be characterized as another source of outage.

It is not hard to verify that the buffer state q_t forms a stationary Markov chain with V possible states [36]. Defining the state transition probability as $p_{ij} = Pr[q_{t+1} = j|q_t = i]$, and $\mathbf{C} = [p_{ij}]$, then the stationary buffer state probabilities can be found as the solution to

$$\mathbf{C}\mathbf{s} = \mathbf{s}, \tag{5.16}$$

where $\mathbf{s} = [s_1 s_2 \ldots s_V]^T$. We seek to maximize the throughput in (5.14) for a given average power constraint

$$\sum_{k=1}^{V} s_k \sum_{i=0}^{k} P_{k,i}\Delta_{k,i} \leq P, \tag{5.17}$$

where $P_{k,i}$ is the power used when the buffer state $q_t = k$ and the transmission rate $r_t = i$. The value of $P_{k,i}$ can be optimized to maximize the throughput. Alternately, the power can also be constant whenever it is greater than zero. This implies that $P_{k,i} = P_{j,m}, \forall i, m \neq 0$. We will show results for both variable and constant $P_{k,i}$ in Section 5.3.4.

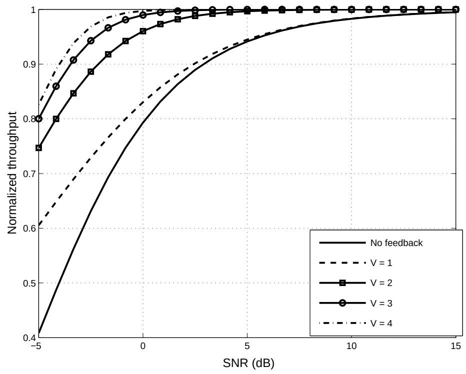
The rate probabilities $\Delta_{k,i}$ are dependent on the fading distribution and the transmission protocol. For Decode and Forward, $\Delta_{k,i}$ is derived by considering the rate control contour of Figure 5.2. Given a rate iR and power $P_{k,i}$, a contour exists with defining parameter $\psi_{k,i} = (e^{iR} - 1)/P_{k,i}$. The rate probability $\Delta_{k,i}$ is the probability of a channel state lying between the contours with parameters $\psi_{k,i}$ and $\psi_{k,i+1}$. Let

$$\Psi_{k,i} =$$

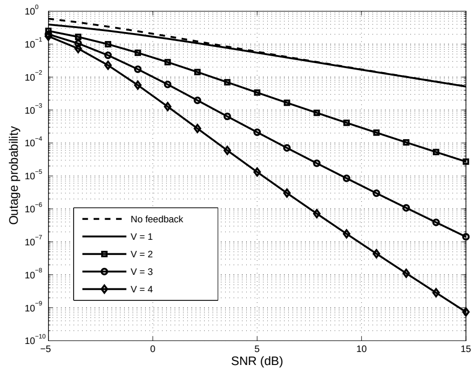
$$1 - \left[\left(1 - e^{-\psi_{k,i}/\lambda_1} - e^{-\psi_{k,i}/\lambda_2} \left(1 - e^{-\psi_{k,i}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)} \right) \frac{\lambda_2}{\lambda_2 - \lambda_1} \right) e^{-\psi_{k,i}/\lambda_0} + 1 - e^{-\psi_{k,i}/\lambda_0} \right],$$

where $\psi_{k,i} = (e^{iR} - 1)/P_{k,i}$. The expression for $\Psi_{k,i}$ can be found by calculating the probability that the channel state lies above the contour of Figure 5.2 (using a defining parameter of $\psi_{k,i}$) under the assumption of Rayleigh fading. The rate probability can then be written as

$$\Delta_{k,i} = \Psi_{k,i} - \Psi_{k,i+1}$$



(a)



(b)

Figure 5.10 : For a constant arrival rate of $R = 0.5$ nats/sec/Hz, the (a) normalized throughput and (b) outage probability is shown for decode and forward various protocols over constant rate as a function of SNR. The results are shown for the case of no feedback, and also for delay bounds of $V = 1, 2, 3, 4$. The power is constant whenever the rate is non-zero (VRCP).

---

for $i = \{0, 1, \ldots, k\}$, $\Psi_{k,0} = 1$ and $\Psi_{k,k+1} = 0$. Similar expressions can be obtained for direct transmission, multi-hop and selection relaying. Maximizing the throughput of (5.14) is analytically intractable for the relay channel. We will therefore resort to numerical optimization techniques.

5.3.3 Increasing Feedback Bits

As described previously, all nodes in the network have the ability to track the buffer state of the source. The feedback bits are used to determine which power and rate are to be used by the source and relay. The power codebook chosen is determined by the current buffer state. The source and relay then use the index received from the destination in the selected power control codebook to decide which power to send with. As a result, the rate of feedback required is $\log_2(V+1)$ bits, which corresponds to the size of power codebook $\mathcal{P}_V$. All other power codebooks have less than $V+1$ elements, and therefore this feedback rate is sufficient to index into any power codebook. The transmission rate to use is easily determined by noting that the index into the power codebook also corresponds to the number of packets to be transmitted. In other words, received index $i \in \{0, 1, \ldots, k\}$ in buffer state k corresponds to i packets transmitted in the current time slot.

It was shown in [36] that further performance improvements are possible through the use of additional feedback bits for power control. Given a buffer state of $q_t = k$, then the source and relay use power control codebook $\mathcal{P}_k$ which has elements $\{P_{k,0}, P_{k,1}, P_{k,2}, ..., P_{k,k}\}$. Power $P_{k,i}$ corresponds to a transmission rate of iR. It is possible to include additional power control levels for any particular rate of transmission. For example, if additional power levels of P_a and P_b are added to codebook k in the following manner

$$\mathcal{P}_k^* = \{P_{k,0}, P_{k,1}, P_a, P_{k,2}, ..., P_{k,k}, P_b\},$$

then the transmission rate of R now has two potential power levels of $P_{k,1}$ or P_a, and similarly transmission rate kR also has two power levels. The index for this codebook must now range from 0 to $k+2$. In general the rate of the feedback link will be $\log_2 X$, where X is the size of the largest power codebook in the set $\mathcal{P}_1, \mathcal{P}_1, \ldots, \mathcal{P}_V$.

Near the end of Section 5.3.4 we look at simultaneous rate and power adaptation (VRVP) based on the rate of the feedback link. Specifically, we compare the outage

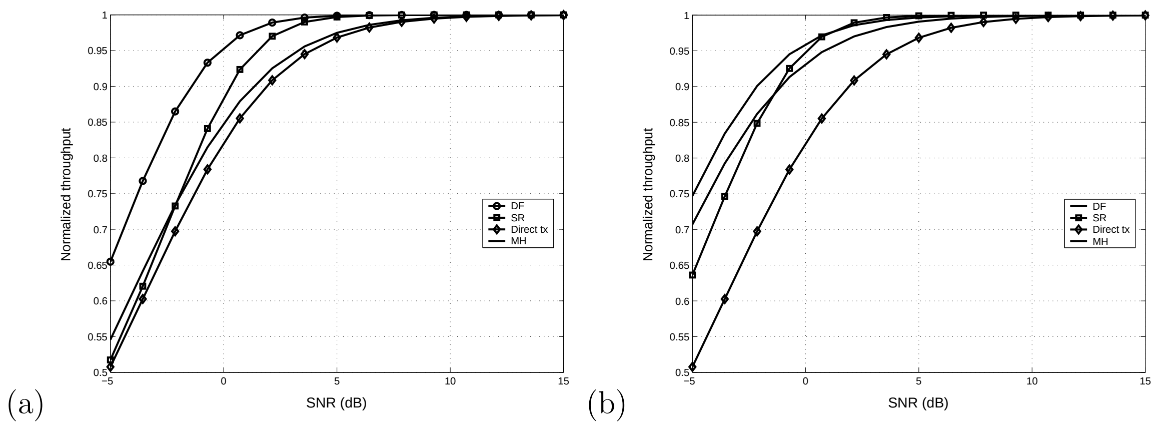


Figure 5.11 : For an arrival rate of $R = 0.5$ nats/sec/Hz, and a source-relay distance of (a) d = 0.2 (b) d = 0.5, the normalized throughput as a function of SNR is shown for various cooperative transmission schemes. Results are shown for $V = 2$ and with constant power whenever the rate is non-zero (VRCP).

---

performance for two buffer sizes $V = 1$ and $V = 2$ with 2-bits of feedback. Having two bits of feedback implies that there are 4 power control levels for each possible buffer state. The results will demonstrate that for a given rate of the feedback link, having a larger buffer size leads to a smaller outage probability for a given SNR, or equivalently a larger throughput.

5.3.4 Analysis and Results

We next consider the throughput performance of the proposed rate and power control strategy. At any given time, for a queue size of q_t, the transmission rate is iR, $i \in \{0, 1, \ldots, q_t\}$. Although the transmission rate varies over time, the average throughput is upper bounded by R, the arrival rate into the source buffer. The throughput is dependent on many factors such as the size of the source buffer, the number of feedback bits, and whether or not power control is utilized. We will consider each of these factors.

Figure 5.10(a) shows the normalized throughput for decode and forward as a function of SNR for different buffer sizes. In this figure, constant power transmission is used whenever data is transmitted (VRCP). This implies that $P_{k,i}$ is a constant for all $i > 0$ and k in (5.17). As a reference, the performance of a system with no feedback or queuing is shown. In this transmission scheme, one packet is always transmitted, and outage events only occur at the destination since the source is always transmitting at a fixed rate R. Next, when $V = 1$, rate control is performed with the use of feedback and no data is sent for poor channels. When $V = 1$, the transmission rate is either 0 or 1 packet, and the source can buffer at most one packet. There is a large gain in throughput over constant rate transmission from the additional power savings achieved by not transmitting during poor channel states. However, for large SNR's, both the throughput and the outage probability (shown in Figure 5.10(b)) converge for $V = 1$ and the no-feedback case. Using an increased buffer size allows for more variation in the transmission rates and an increased throughput. This is evident from the gains in throughput with $V = 2$. Increasing V leads to diminishing returns, but it is clear that the addition of delay through the use of the feedback allows for a reduced outage probability and a significantly higher throughput. In the limit of $V = \infty$ packets are never dropped and the normalized throughput will be 1 in the range of SNR's shown in Figure 5.10(a).

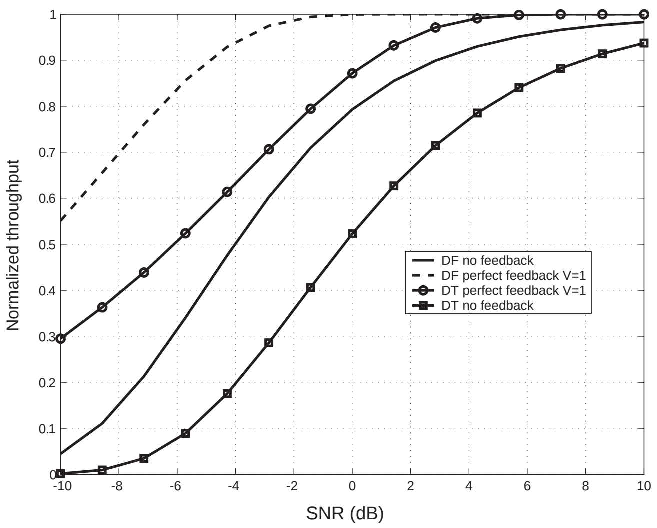


Figure 5.12 : For an arrival rate of R=0.5 nats/sec/Hz, the normalized throughput is shown for $V = 1$ as a function of SNR for decode and forward (DF) and direct transmission (DT) with perfect feedback (VRVP), and with no power control (CRCP).

The outage probability is seen in Figure 5.10(b), and shows an interesting result. When feedback is unavailable, then the outage probability is solely a function of the channel as a transmission rate of R is used in each time slot. When feedback is available, then channel outages can be avoided, and the outage probability results depict the blocking probability defined in (5.15). For $V = 1$ and for no-feedback, a first order diversity behavior is experienced with decode and forward. Furthermore, for constant power transmission, additional levels of delay increase the diversity. From this figure, it is apparent that for a buffer size of V, the diversity order is also V. This suggests significant power savings with increasing delay for a target probability of buffer overflow.

Figures 5.11(a) and 5.11(b) shows throughput results for decode and forward in comparison with other network transmission schemes. Figure 5.11(a) considers a source-relay distance of $d = 0.2$, which implies a strong source-relay channel gain. In this case, decode and forward is seen to have a large throughput gain over multi-hop and selection relaying. Multi-hopping performs about the same as direct transmission in this case. For this source-relay distance, there are hardly any decoding errors at the relay. Moreover, the relay-destination channel gain has a mean value that is very close to that of the source-destination link. As a result, multi-hopping and direct transmission have comparable performance. Selection relaying gains over direct transmission at large average powers, and for small power constraints it has a similar performance as direct transmission. When the source-relay distance is $d = 0.5$, as in Figure 5.11(b), we see that decode and forward once again outperforms selection relaying and multi-hop, but multi-hop performs quite well in this scenario. When $d = 0.5$, multi-hop performs at its best as it relies heavily on strong source-relay and relay-destination links. Despite the fact that selection relaying is half-duplex and relies on a repetition code at the relay node, it achieves large throughput gains over direct transmission for $d = 0.5$.

Recall the results from Section 5.2.5 where the protocols had an infinite queue backlog and the transmission was a function solely of the channel state. Selection relaying had a loss in performance compared to direct transmission for large SNR's. We showed that the throughput maximizing transmission rate led to a relatively high outage operating point. For small SNR's, the power of SR can be attributed to performance in the power-limited regime, where the average received power determines the transmission rate. For large SNR's, the repetition coded nature of SR leads to a loss in throughput compared to DT. In Figure 5.11 we see that for a constant arrival rate and a finite buffer size, SR outperforms DT even at higher SNR's. When the arrival rate to the source buffer is fixed, then the throughput maximizing policy is to minimize the probability of outage. As the outage probability approaches zero, the throughput approaches the arrival rate. For large SNR's, SR performs very well in terms of outage probability [9], which explains the large throughput gains over DT.

The addition of power control in addition to rate control can lead to significant reductions in outage probability, and gains in throughput. The use of power control is shown in Figure 5.12 for $V = 1$ and for both direct transmission and decode and forward. In this case, we assume that there is a perfect amount of feedback so that the optimal power control strategy can be performed, which amounts to channel inversion for channel states in which the transmission rate is non-zero [10, 35]. We see that, for both direct transmission and decode and forward, tremendous gains are possible with power control [25]. When a limited feedback channel is available, the gains will be reduced accordingly.

Figure 5.12 also shows the gains of decode and forward over direct transmission for $V = 1$. When no feedback is used, decode and forward has as much as a 30% gain in throughput over direct transmission for the same SNR. Similarly, decode and forward is quite powerful when feedback is introduced for power control, as can be

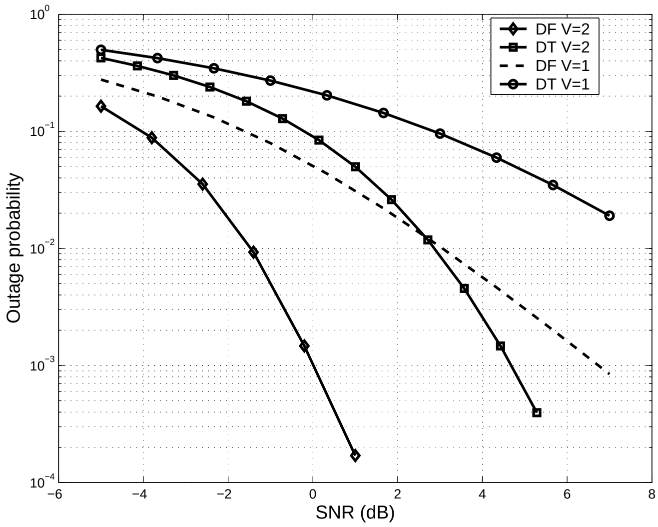


Figure 5.13 : The effect of having a fixed number of feedback bits for power/rate control is more useful for higher delays. Outage results are shown for $V = 1$, $V = 2$ for 2-bits of feedback using DF and DT with VRVP.

---

seen in this figure. The gains of decode forward can also be seen in Figure 5.13 in terms of outage probability. We see that when $V = 1$, or $V = 2$, for a fixed outage probability, decode and forward has a large power savings over direct transmission. For example, when $V = 1$, at an outage probability of 10%, Decode and forward has a $4dB$ reduction in SNR. Figure 5.13 shows outage results for the case of 2-bits of feedback, which implies that additional feedback bits are used for power control (see Section 5.3.3). Interestingly, at any given outage probability, the additional feedback bits provide larger power savings for larger buffer sizes (V=2).

5.4 Conclusions

In this chapter, we have analyzed the throughput performance of different network transmission protocols in the fading channel when a finite rate of feedback is provided to the transmitters. We derived the optimal rate control algorithm that maximizes the throughput. Our work leads to some interesting design guidelines for next generation network protocols. First, only a few coding rates are necessary to maximize the throughput, which is useful as it shows that throughput gains are achievable with minimal additional hardware complexity at the transmitter. Second, the selected transmission rates should be large enough to allow for a moderate amount of frame errors to maximize the long-term throughput. Third, cooperative techniques such as decode and forward can give significant performance improvements over traditional point-to-point communications, and should therefore be implemented in next generation protocols.

We also looked at the use of simultaneous rate and power adaptation. An interesting conclusion is that simultaneous rate and power adaptation is not needed. When the average power constraint is small, using one rate control codebook is sufficient as long as power control is employed. On the other hand, for systems operating in the high power regime, power control is unnecessary.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

The intense research focus by many groups on cooperative communications is with merit, as node collaboration has the potential to provide significant performance improvements in networks. Relaying, a simple example of the cooperation paradigm, has shown the potential for tremendous gains over direct transmission, especially in the context of fading channels.

By utilizing a relay, a virtual multiple antenna system is created. Relaying leads to similar performance as a multiple transmit/single receive antenna system in the block fading channel in terms of outage and diversity. In this thesis, we showed that the true potential of relaying becomes evident with the use of feedback. Feedback can be used for different purposes based on requirements at the source. If the source transmits at the same rate irrespective of the channel conditions, then the optimal strategy is to minimize the outage probability. On the other hand, if adaptive rate transmission is allowed, then throughput maximization should be the goal.

Our analysis has resulted in several interesting conclusions. With constant rate transmission, the outage probability is minimized by adapting the transmit power both spatially and temporally. We saw that:

- The optimal power control policy with perfect feedback involves a 'channel-inversion' type procedure where transmission is shut off for a subset of channel states which require large amounts of power to invert the channel effects.

- Much like in the case of ergodic-capacity, decode and forward is nearly optimal for strong source-relay links in the block fading environment. In fact, a hybrid between decode and forward and estimate and forward nearly approaches the universal lower bound on outage probability for a wide array of channel conditions.

- Power control with finite rate feedback quickly approaches the limit defined by the perfect feedback power control policy. Only a few bits of feedback are really needed to achieve significant gains over constant power transmission.

- Feedback can be used to achieve diversity gains similar to spatial diversity. For example, with 1 feedback bit, the diversity order of amplify and forward doubles to four, which leads to similar outage behavior as a 4x1 multiple antenna system. For direct transmission, the use of 1 feedback bits results in only a second order diversity.

When feedback is available and the transmission rate is not constrained to be constant, we showed results for throughput maximization. Here we saw that:

- For a limited rate of feedback, the throughput is maximized for a relatively high outage operating point (i.e. greater than 10%). This suggests using higher rate codes and allowing for frame errors to maximize the effective data rate. As the rate of the feedback link increases, the optimal outage operating point decreases.

- It is unnecessary to temporary adapt both the rate and power, especially for a small amount of feedback. For small average power constraints, power control with rate selection is sufficient. On the other hand, for large average power constraints, adapting the transmission rate is imperative, and the addition of power control does not provide any significant throughput gain.

- Cooperation provides large throughput gains over direct transmission. Even simple protocols such as selection relaying can achieve gains over direct transmission in the low power regime.

The results motivate the need for next generation network protocols to incorporate feedback regarding channel state information. Even limited amounts of feedback can lead to significant gains. These gains can translate into significant improvements in battery life for a target performance level. Our results suggest that although relaying has shown excellent potential in the fading channel in the case of no transmitter channel knowledge, its true power becomes evident when feedback is available and temporal adaptation of rate and power are performed.

6.2 Future Work

Our analysis focuses on a network with three nodes: one source-destination pair, and a relay node. In a more general setting, the use of additional relays in the cooperation process leads to a larger achievable rate. An interesting protocol for a multi-relay network was proposed in [16]. A set of relay nodes (the 'decoding set') comprises all relays that were able to successfully decode the source transmission. In the second phase of transmission, the relays from the decoding set use a space-time structure to transmit to the destination. This is one example of a multi-relay network. The potential throughput gains of such a system needs to be quantified, and also appropriate rate and power control strategies need to be derived. There is clearly a tradeoff in using more relay nodes to increase the throughput at the expense of increasing the interference with neighboring communications.

The half-duplex constraint is a practical restriction on the relay terminals. The results presented in this work for half-duplex relays involved primarily the amplify and forward and selection relaying protocols. These protocols, while simple to implement, impose an additional time orthogonality constraint. The time-orthogonality reduces the achievable rate. Furthermore, the above mentioned protocols utilize repetition coding at the relay nodes, which also leads to a throughput loss. The half-duplex protocol described in [23] does not impose a time orthogonality constraint. As a result, it is expected that for throughput maximization, the 'cheap' decode-forward [23] protocol will have a throughput behavior that more closely follows that of full-duplex relaying. Power and rate control analysis is an area of future work for more advanced half-duplex protocols.

In the work described in this thesis, feedback is used to improve performance either by outage minimization or throughput maximization. A crucial assumption in this work relates to the nature of the feedback link. First, we assume that the feedback transmission is error-free. This may be practical for limited rates of feedback, but for large feedback rates, errors would be expected. One area of future work is to construct power and rate control algorithms that maximize throughput with errors in the feedback link. Second, we assume the feedback information sent from the destination is instantaneously received at the transmitters without delay. However, in a practical system, the transmission delay may lead to 'outdated' quantized channel information at the transmitters. The addition of feedback delay leads to an interesting problem of determining the optimal throughput maximization algorithm for a given delay.

Appendix A

Proof of Theorem 4.2.1

Note that $\mathcal{R}_{AF}(\underline{\gamma}, P_{s,q}, x)$ is a monotonically increasing function of $P_{s,q}$, for $x \in \{P_r, \eta P_{s,q}\}$. This can be verified by confirming that $\frac{\partial \mathcal{R}_{AF}(\underline{\gamma}, P_{s,q}, x)}{P_{s,q}} > 0$. Recall that $G(\cdot)$ is found by solving for γ_2 in $\mathcal{R}_{gen}(\underline{\gamma}, P_{s,q}, x) = R$. For any $\underline{\gamma}_a = (\gamma_0, \gamma_1, \gamma_2)$ lying on $G(\cdot)$, we will next show that for any $\underline{\gamma}_b = (\gamma_0 - \epsilon_0, \gamma_1 - \epsilon_1, \gamma_2 - \epsilon_2)$, $\mathcal{R}_{AF}(\underline{\gamma}_b, P_{s,q}, x) < \mathcal{R}_{AF}(\underline{\gamma}_a, P_{s,q}, x)$. As a result, since $\mathcal{R}_{AF}(\cdot)$ is monotonically increasing in the source power, to transmit at rate R with new power $P_{s,q}^b$, an increase in source power ($P_{s,q}^b > P_{s,q}$) is necessary to guarantee $\mathcal{R}_{AF}(\underline{\gamma}_b, P_{s,q}^b, x^b) = R$, where $x^b \in \{P_r, \eta P_{s,q}^b\}$.

For AF, consider $y(\underline{\gamma}) = P_{s,q}\gamma_1 + \frac{P_{s,q}\gamma_0 x \gamma_2}{1+P_{s,q}\gamma_0 + x\gamma_2}$. Let $y_1(\underline{\gamma}) = 1 + P_{s,q}\gamma_1$, and $y_2(\underline{\gamma}) = \frac{P_{s,q}\gamma_0 x \gamma_2}{1+P_{s,q}\gamma_0 + x\gamma_2}$. For $y_1(\underline{\gamma})$, clearly $1 + P_{s,q}\gamma_1 > 1 + P_{s,q}(\gamma_1 - \epsilon_1)$. For $y_2(\underline{\gamma})$, it needs to be shown that

$$\frac{P_{s,q}\gamma_0 x(\gamma_2 - \epsilon_2)}{1 + P_{s,q}(\gamma_0 - \epsilon_0) + x(\gamma_2 - \epsilon_2)} < \frac{P_{s,q}\gamma_0 x \gamma_2}{1 + P_{s,q}\gamma_0 + x\gamma_2}. \tag{A.1}$$

After some manipulation, this can be rewritten as

$$(1 + P_{s,q})\left(\frac{\gamma_0}{\epsilon_0} - 1\right) + (1 + x)\left(\frac{\gamma_2}{\epsilon_2} - 1\right) > -1. \tag{A.2}$$

Since all fading elements are positive, then $\epsilon_i \leq \gamma_i$. As a result, (A.2) is always satisfied. We have then shown that $\frac{1}{2}\log(y(\underline{\gamma} - (\epsilon_0, \epsilon_1, \epsilon_2))) < \frac{1}{2}\log(y(\underline{\gamma}))$ which corresponds to $\mathcal{R}_{AF}(\underline{\gamma}_b, P_{s,q}, x) < \mathcal{R}_{AF}(\underline{\gamma}_a, P_{s,q}, x)$. Consequently, an increase in source power is required to guarantee zero outage for any channel state lying below the curve $G(\cdot)$.

Proving the second part of the theorem, that points lying above $G(\gamma_0, \gamma_1, \mathbf{P_q})$ requires a source power less than $P_{s,q}$ and a relay power less than $P_{r,q}$, is straightforward. By following similar steps as above, except now setting $\underline{\gamma}_b = (\gamma_0 + \epsilon_0, \gamma_1 + \epsilon_1, \gamma_2 + \epsilon_2)$, the result follows. ■

Appendix B

Proof of Theorem 4.2.2

Consider the function $g(P_{avg}, \gamma_0, \gamma_2) = \frac{P_{avg}^2 \gamma_0 \gamma_2}{1+P_{avg}\gamma_0+P_{avg}\gamma_2}$. We first will find

$$Pr[g(P_{avg}, \gamma_0, \gamma_2) < t].$$

This can be re-written as $Pr[1/g(P_{avg}, \gamma_0, \gamma_2) > 1/t]$. Next, note that

$$\begin{aligned} Pr[1/g(P_{avg}, \gamma_0, \gamma_2) > 1/t] = Pr\left[\frac{1 + P_{avg}\gamma_0 + P_{avg}\gamma_2}{P_{avg}^2\gamma_0\gamma_2} > \frac{1}{t}\right] = \\ Pr\left[\gamma_2 < \frac{t(1 + P_{avg}\gamma_0)}{P_{avg}(P_{avg}\gamma_0 - t)}\right] = \\ 1 - \exp\left(-\frac{t(1 + P_{avg}\gamma_0)}{\lambda_2 P_{avg}(P_{avg}\gamma_0 - t)}\right). \end{aligned} \tag{B.1}$$

Next, using the fact that $e^{-x} = 1 - x + x^2/2 + ...$, we can approximate $\lim_{P_{avg}\to\infty}(P_{avg} \cdot Pr[g(P_{avg}, \gamma_0, \gamma_2) < t])$ as

$$\begin{aligned} \lim_{P_{avg}\to\infty}(P_{avg} \cdot Pr[g(P_{avg}, \gamma_0, \gamma_2) < t]) = \lim_{P_{avg}\to\infty} \frac{P_{avg} t(1 + P_{avg}\gamma_0)}{\lambda_2 P_{avg}(P_{avg}\gamma_0 - t)} = \\ \frac{tP_{avg}\gamma_0}{\lambda_2(P_{avg}\gamma_0 - t)} = \frac{t}{\lambda_2}. \end{aligned} \tag{B.2}$$

Next, consider $Pr[P_{avg}\gamma_1 < t]$. It can be verified that

$$\lim_{P_{avg}\to\infty}(P_{avg} \cdot Pr[P_{avg}\gamma_1 < t]) = \lim_{P_{avg}\to\infty}(P_{avg} \cdot (1 - e^{-\frac{t}{P_{avg}\lambda_1}})) = \frac{t}{\lambda_1}. \tag{B.3}$$

Let $a(t) = t/\lambda_1$, and $b(t) = t/\lambda_2$. Using Theorem 1 from [17],

$$\begin{aligned} \lim_{P_{avg}\to\infty} P_{avg}^2 \cdot Pr\left(P_{avg}\gamma_1 + \frac{P_{avg}^2\gamma_0\gamma_2}{1 + P_{avg}\gamma_0 + P_{avg}\gamma_2} < t\right) = \int_0^t a(t-x)b'(x)dx \\ = \frac{t^2}{2\lambda_1\lambda_2} \end{aligned} \tag{B.4}$$

Using the fact that $t = e^{2R} - 1$, for large P, we have that

$$Pr\left(P_{avg}\gamma_1 + \frac{P_{avg}^2\gamma_0\gamma_2}{1 + P_{avg}\gamma_0 + P\gamma_2} < e^{2R} - 1\right) \approx \frac{\left(e^{2R} - 1\right)^2}{2P_{avg}^2\lambda_1\lambda_2}. \tag{B.5}$$

Clearly, there is a second order decay of the outage with respect to power for this case. As a result, a deterministic γ_0 does not affect the diversity order.

Appendix C

Proof of Theorem 5.2.1

Consider the rate control region for decode and forward, shown in Figure 5.2. This figure contains a surface which separates regions 1 and 2, for the case of 1-bit of feedback. The achievable rate for decode and forward is given by

$$R = \min\{R_a, R_b\}, \tag{C.1}$$

where $R_a = \log(1 + P_2\gamma_0)$, and $R_b = \log(1 + P_2\gamma_1 + P_2\gamma_2)$. Along the contour, since $K = e^{R_2} - 1$.

Consider the case where $\gamma_0 = K/P_2$, where $K = e^{R_2} - 1$, and $\gamma_1 > K/P_2$. Then the achievable rate can be expressed as

$$R = \min\{R_2, R_b\} = R_2,$$

for any value of γ_2. Furthermore, any ψ decrease in γ_0 leads to an achievable rate of $\log(1 + P_r(\gamma_0 - \psi)) < R_2$, since in this case R_a is active in the expression for the achievable rate of decode and forward in (C.1).

Consider the case where $\gamma_0 > K/P_2$, and $\gamma_1 + \gamma_2 = K/P_2$, which also lies along the surface. In this case, the achievable rate is expressed as

$$R = \min\{R_a, R_2\} = R_2.$$

Any ψ decrease in γ_2 leads to an achievable rate of $\log(1 + P_r(\gamma_2 - \psi) + P\gamma_1) < R_2$. The same trend holds for a decrease in γ_1.

Finally, consider where $\gamma_0 = K/P_2$ and $\gamma_1 + \gamma_2 = K/P_2$. Here we have $R = R_a = R_b = R_2$. Any decrease in γ_0 will lead to an achievable rate of $R = \log(1 + P_r(\gamma_0 - \psi)) < R_a = R_2$. As a result, the rate required for zero outage is less than R_2. Similarly, when $\gamma_0 = K/P_2$ and either γ_1 or γ_2 is decreased by ψ, then $R = \log(1 + P_r(\gamma_1 - \psi) + P_2\gamma_2) < R_b = R_2$. This implies than any point lying below the triangular portion of the contour requires a rate less R_2 to invert the effects of the channel.

As a result, any points lying on the surface require a rate of R_2 to invert the channel effects, while points lying below the contour require a rate less than R_2 to maintain zero outage. Similarly, it can be shown that points above the contour can support a higher rate than R_2 and still maintain zero outage. The surface is then a contour with rate R_2, and separates $\mathcal{R}_1$ and $\mathcal{R}_2$.

Appendix D

Proof of Theorem 5.2.2

Consider (2.8), and to indicate the dependence on the correlation ρ, denote it by $R_{DF}(\rho)$. Since channel state information is unavailable at the transmitters, the outer maximization is not present, and the achievable rate is

$$R_{DF}(\rho) = \min\{\log(1 + (1 - |\rho|^2)\gamma_1 P_s),$$
$$\log(1 + \gamma_1 P_s + \gamma_2 P_r + 2Re\{\rho e^{j\measuredangle h_1 - j\measuredangle h_2}\}\sqrt{\gamma_1\gamma_2 P_s P_r})\Big\}, \tag{D.1}$$

At a transmission rate R, the outage probability as a function of ρ can be expressed as

$$P_{out}(\rho) = 1 - E[\mathcal{I}_F(R_{DF}(\rho))], \tag{D.2}$$

where $\mathcal{I}_F(R_{DF}(\rho))$ is the indicator function defined as

$$\mathcal{I}_F(R_{DF}(\rho)) = \begin{cases} 1, & R_{DF}(\rho) > R \\ 0, & R_{DF}(\rho) \leq R. \end{cases} \tag{D.3}$$

When no channel state information is available at the transmitters, the throughput is simply $R \cdot (1 - P_{out}(\rho))$. Therefore for any given R, maximizing the throughput requires a minimization of the outage probability.

The indicator function (D.3) operates on a convex set, and as a result, $g(\rho) = \mathcal{I}_F(R_{DF}(\rho))$ is log-concave [28]. Note that $P_{out}(\rho) = 1 - E[g(\rho)]$, and is then log-convex. Considering $R_{DF}(\rho)$, it should be noted that the first log term is independent of the phase of ρ. Moreover, since $\measuredangle h_1$ and $\measuredangle h_2$ are uniformly distributed on $[0, 2\pi)$, in the calculation of P_{out} the second log term of $R_{DF}(\rho)$ would also be independent of the phase of ρ. Consequently, we have that $P_{out}(\rho) = P_{out}(-\rho)$. The log-convexity of P_{out} implies that

$$\frac{\log P_{out}(\rho) + \log P_{out}(-\rho)}{2} \geq \log P_{out}(0),$$

which leads to the fact that

$$\sqrt{P_{out}(\rho) \cdot P_{out}(-\rho)} \geq P_{out}(0).$$

Also, based on the arithmetic-geometric mean inequality

$$\frac{P_{out}(\rho) + P_{out}(-\rho)}{2} \geq \sqrt{P_{out}(\rho) \cdot P_{out}(-\rho)},$$

and as a result

$$P_{out}(\rho) = \frac{P_{out}(\rho) + P_{out}(-\rho)}{2} \geq P_{out}(0).$$

Therefore, having $\rho = 0$ would minimize the outage probability and hence maximize the throughput.

Bibliography

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–part i: System description," *IEEE Trans. on Communications*, pp. 1927–1938, Nov. 2003.

[2] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," vol. 42, October 2004.

[3] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Prob.*, vol. 3, 1971.

[4] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near shannon limit error-correcting coding and decoding: turbo-codes," *IEEE ICC 1993*, pp. 1064–1070, May 1993.

[5] D. MacKay and R. Neal, "Near shannon limit performance of low density parity check codes," *IEE Electronic Letters*, vol. 32, pp. 1645–1655, August 1996.

[6] M. Khojastepour, N. Ahmed, and B. Aazhang, "Code design for the relay channel and factor graph decoding," in *Asilomar Conf. on Signals, Systems and Computers*, (Pacific Grove, CA), Nov. 7-10 2004.

[7] B. Zhao and M. Valenti, "Cooperative diversity using distributed turbo codes," in *in Proc. Virginia Tech Symp. on Wireless Personal Commun.*, (Blacksburg,

VA), June 2003.

[8] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. on Vehicular Technology*, vol. 43, pp. 359–378, May 1994.

[9] N. Laneman, D. Tse, and G. Wornell, "Coooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. on Info. Theory*, vol. 50, Dec. 2004.

[10] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans Info. Theory*, vol. 45, pp. 1468–1489, Jul. 1999.

[11] G. Taricco, E. Biglieri, and G. Caire, "Limiting performance of block-fading channels with multiple antennas," in *Info. Theory and Comm. Workshop*, (Kruger National Park, South Africa), pp. 27–29, 1999.

[12] S. Bhashyam, A. Sabharwal, and B. Aazhang, "Feedback gain in multiple antenna systems," *IEEE Trans. on Communications*, vol. 50, pp. 785–798, May 2002.

[13] A. Khoshnevis and A. Sabharwal, "Performance of quantized power control in multiple antenna systems," in *IEEE ICC 2004*, (Paris, France), June 20-24 2004.

[14] K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On Beamforming with Finite Rate Feedback in Multiple Antenna Systems," *IEEE Transactions*

on Information Theory, vol. 49, pp. 2562–2579, October 2003.

[15] L. Lin, R. Yates, and P. Spasojevic, "Adaptive transmission with discrete code rates and power levels," *IEEE Trans. on Communications*, vol. 51, pp. 2115–2125, December 2003.

[16] N. Laneman and G. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. on Info. Theory*, vol. 49, pp. 2415–2525, Oct. 2003.

[17] N. Laneman, "Limiting analysis of outage probabilities for diversity schemes in fading channels," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, (San Francisco, USA), Dec. 2003.

[18] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channel," *IEEE Trans. on Info. Theory*, vol. 51, pp. 2020–2040, June 2005.

[19] D. Gunduz and E. Erkip, "Opportunistic cooperation and power control strategies for delay-limited capacity," in 39^{th} *Annual Conference on Information Sciences and Systems*, (Baltimore, MD), March 16-18 2005.

[20] T. M. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. on Info. Theory*, vol. 25, pp. 572–584, Sept. 1979.

[21] M. Khojastepour, A. Sabharwal, and B. Aazhang, "Lower bounds on the capacity of gaussian relay channel," in 38^{th} *Annual Conference on Information Sciences and Systems*, (Princeton, NJ.), March 17-19 2004.

[22] J. G. Proakis and D. Manolakis, *Digital Signal Processing. Principles, Algorithms, and Applications.* Prentice Hall, 1996.

[23] M. Khojastepour, A. Sabharwal, and B. Aazhang, "On the capacity of 'cheap' relay networks," in *In Proc. 37th Annual Conf. on Info. Sciences and Systems*, (Baltimore, MD.), March 12-14 2003.

[24] T. Cover and J. Thomas, *Elements of Information Theory.* Wiley, 1991.

[25] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Info. Theory*, vol. 43, pp. 1986–1992, Nov. 1997.

[26] M. Khojastepour, N. Ahmed, and B. Aazhang, "Performance limits of the fading relay channel - part i: Ergodic capacity," *To be submitted to IEEE Transactions on Information Theory*, 2004.

[27] N. Ahmed, M. Khojastepour, and R. Baraniuk, "Delay-limited throughput maximization for fading channels - part ii: Optimal rate selection and power control," *Submitted to IEEE Trans. on Info. Theory*, 2004.

[28] S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.

[29] N. Ahmed, M. Khojastepour, A. Sabharwal, and B. Aazhang, “Power control with finite rate feedback for cooperative relay networks,” in *Intern. Symposium on Inform. Theory and its Applications(ISITA)*, Oct. 10-13 2004.

[30] Y. Xu, S. Bien, Y. Mori, J. Heidmann, and D. Estrin, “Topology control protocols to conserve energy in wireless ad hoc networks,” *Under Submission to CENS Technical Report 0006*, Jan. 2003.

[31] L. Zheng and D. Tse, “Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels,” *IEEE Trans. on Info. Theory*, vol. 49, pp. 1073–1096, May 2003.

[32] R. Knopp and P. Humblet, “On coding for the block fading channel,” *IEEE Transactions on Information Theory*, vol. 46, pp. 189–205, January 2000.

[33] A. Sabharwal, D. Rajan, and B. Aazhang, “Dual problems in power control,” in *Annual Allerton Conf. on Commun., Control and Comp.*, (Monticello, IL), Oct. 2001.

[34] I. Maric and R. D. Yates, “Forwarding strategies for parallel-relay networks,” in *Proc. IEEE International Symposium on Information Theory (ISIT),*, (Chicago, IL), June 2004.

[35] N. Ahmed, M. Khojastepour, and B. Aazhang, “Outage minimization and optimal power control for the fading relay channel,” in *IEEE Information Theory*

Workshop (ITW), (San Antonio, TX), pp. 458–462, Oct. 24-29 2004.

[36] D. Rajan, A. Sabharwal, and B. Aazhang, "Outage behaviour with delay and csit," in *Proc. of International Conference on Communications (ICC)*, (Paris, France), pp. 578–582, June 2004.

[37] R. Gallager, *Information Theory and Reliable Communication.* Wiley, 1968.

[38] N. Ahmed, M. Khojastepour, and B. Aazhang, "Performance limits of the fading relay channel - part ii: Efficient protocols and the optimal power control," *To be submitted to IEEE Transactions on Information Theory*, 2004.

[39] N. Ahmed, M. Khojastepour, A. Sabharwal, and B. Aazhang, "Power control with limited feedback for relay networks," *Submitted to IEEE Transactions on Communications*, December 2004.

[40] N. Ahmed and B. Aazhang, "Throughput gains with limited feedback in cooperative relay networks," in *Intern. Conf. on Wireless Networks, Communications, and Mobile Computing*, (Maui, Hawaii), June 13-16 2005.

[41] N. Ahmed and B. Aazhang, "Cooperative Communications in the Fading Channel," in *SPIE Conference on Noise in Communication Systems*, March 2005. Accepted.

[42] S. Hanly and D. Tse, "Multiaccess fading channels: part ii: Delay limited capacities," *IEEE Transactions on Information Theory*, vol. 44, pp. 2816–2831, Nov.

1998.

[43] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushayana, and A. Viterbi, “Cdma/hdr: A bandwidth efficient high-speed wireless data service for nomadic users,” *IEEE Communications Magazine*, vol. 38, pp. 70–77, Jul. 2000.

[44] Y. Liang and V. V. Veeravalli, “Gaussian orthogonal relay channel: Optimal resource allocation and capacity,” *Submitted to IEEE Trans. on Info. Theory*, Aug. 2004.

[45] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity–part ii: Implementation aspects and performance analysis,” *IEEE Trans. on Communications*, pp. 1939–1948, Nov. 2003.

[46] R. Nabar, H. Bolceski, and F. Kneubuhler, “Fading relay channels: performance limits and space-time signal design,” *IEEE Journal on Selected Areas in Comm.*, vol. 22, pp. 1099–1109, Aug. 2004.

[47] R. Nabar, F. Kneubuhler, and H. Bolcskei, “Performance limits of amplify-and-forward based fading relay channels,” in *IEEE Conf. on Acoustics, Speech and Sig. Prc. (ICASSP)*, (Montreal,Canada), pp. 565–568, May 17-21 2004.

[48] J. N. Laneman, “Network coding gain of cooperative diversity,” in *IEEE Military Comm. Conf.*, (Monterey, CA), Nov. 2004.

[49] G. Kramer, M. Gastpar, and P. Gupta, “Capacity theorems for wireless relay channels,” in *41st Annual Allerton Conf. on Commun., Control and Comp.*, (Monticello, IL), Oct. 2003.

[50] T. E. Hunter and A. Nosratinia, “Cooperation diversity through coding,” in *Proc. IEEE International Symposium on Information Theory (ISIT),*, p. 220, 2002.

[51] M. Janani, A. Hedayat, T. Hunter, and A. Nosratinia, “Coded coopration in wireless communications: Space-time transmission and iterative decoding,” *IEEE Transactions on Signal Processing*, vol. 52, pp. 362–371, February 2004.

[52] T. Hunter and A. Nosratinia, “Coded cooperation under slow fading, fast fading, and power control,” in *36th Annual Allerton Conf. on Signals, Systems and Computers*, (Monticello, IL), November 2002.

[53] T. E. Hunter, S. Sanayei, and A. Nosratinia, “The outage behavior of coded cooperation,” in *Proc. IEEE International Symposium on Information Theory (ISIT),*, p. 220, 2004.

[54] Y. Yao, X. Cai, and G. Giannakis, “On energy efficiency and optimum resource allocation of relay transmissions,” *Submitted to IEEE Trans. on Wireless Comm.*, 2004.

[55] A. E. Gamal and S. Zahedi, "Minimum energy communication over a relay channel," (Yokohama, Japan), p. 344, June-July 2003.

[56] O. Oyman and A. J. Paulraj, "Spectral efficiency of relay networks in the power-limited regime," in *42nd Annual Allerton Conf. on Commun., Control and Comp.*, (Monticello, IL), Oct. 2004.

[57] A. Host-Madsen, "Cooperation in the low power regime," in *42nd Annual Allerton Conf. on Commun., Control and Comp.*, (Monticello, IL), Oct. 2004.

[58] A. Host-Madsen and J. Zhang, "Ergodic capacity and power allocation in wireless relay channels," in *Proc. IEEE Globecom*, (Dallas, TX), 2004.

[59] M. Haleem and R. Chandramouli, "Adaptive stochastic iterative rate selection for wireless channels," *IEEE Communication Letters*, vol. 12, pp. 292–294, May 2004.

[60] B. Zhao and M. Valenti, "Some new adaptive protocols for the wireless relay channel," in *Allerton Conf. on Commun., Control and Comp.*, (Monticello, IL), October 2003.

[61] B. Zhao and M. Valenti, "Practical relay networks: A generalization of hybrid-arq," *IEEE Journal on Selected Areas in Comm.*, vol. 23, pp. 7–18, Jan. 2005.

[62] B. Zhao and M. Valenti, "Distributed turbo coded diversity for the relay channel," *IEEE Electronics Letters*, vol. 39, pp. 7786–787, May 2003.

[63] M. Valenti and B. Zhao, "Capacity approaching distributed turbo codes for the relay channel," in *Proceedings of the 57th IEEE Vehicular Technology Conference (VTC)*, April 2003.

[64] M. Yuksel and E. Erkip, "Broadcast strategies for the fading relay channel," in *Military Communications Conference*, (Monterey, CA), 2004. Invited.

[65] A. Stefanov and E. Erkip, "Cooperative coding for wireless networks," *IEEE Transactions on Communications*, vol. 52, pp. 1470–1476, September 2004.

[66] A. Stefanov and E. Erkip, "Cooperative space-time coding for wireless networks," in *IEEE Information Theory Workshop (ITW)*, pp. 50–53, 31 March - 4 April 2003.

[67] I. Hammerstroem, M. Kuhn, B. Rankov, and A. Wittneben, "Space-time processing for cooperative relay networks," in *IEEE Vehicular tech. conf.*, (Zurich, Switzerland), Oct. 6-9 2003.

[68] P. Anghel, G. Leus, and M. Kaveh, "Multi-user space-time coding in cooperative networks," in *IEEE Conf. on Acoustics, Speech and Sig. Prc. (ICASSP)*, (Hong Kong), April 6-10 2003.

[69] P. Anghel and M. Kaveh, "Relay assisted uplink communication over frequency-selective channels," in *4th IEEE Workshop on Signal Processing Advances in*

Wireless Communications (SPAWC), (Hong Kong), pp. 125–129, June 15-18 2003.

[70] P. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a rayleigh-fading environment," *IEEE Transactions on Wireless Communications*, vol. 3, pp. 1416–1421, September 2004.

[71] E. Zimmermann, P. Herhold, and G. Fettweis, "On the performance of cooperative relaying protocols in wireless networks," *European Transactions on Telecommunications (ETT)*, vol. 16, pp. 17–35, January-February 2005.

[72] E. Zimmermann, P. Herhold, and G. Fettweis, "The impact of cooperation on diversity-exploiting protocols," in *Proceedings of the 59th IEEE Vehicular Technology Conference (VTC)*, May 2004.

[73] P. Herhold, E. Zimmermann, and G. Fettweis, "A simple cooperative extension to wireless relaying," in *Proceedings of the 2004 International Zurich Seminar on Communications*, (Zurich, Switzerland), February 2004.

[74] P. Herhold, E. Zimmermann, and G. Fettweis, "On the performance of cooperative amplify-and-forward relay networks," in *Proceedings of the 5th International ITG Conference on Source and Channel Coding (SCC)*, (Erlangen, Germany), January 2004.

[75] E. Zimmermann, P. Herhold, and G. Fettweis, "On the performance of coopera-

tive diversity protocols in practical wireless systems," in *Proceedings of the 58th IEEE Vehicular Technology Conference (VTC Fall 2003)*, (Orlando, Florida), October 2003.

[76] K. Azarian, H. E. Gamal, and P. Schniter, "Achievable diversity-vs-multiplexing tradeoffs in half-duplex cooperative channels," in *IEEE Information Theory Workshop (ITW)*, (San Antonio, TX), pp. 292–297, Oct. 24-29 2004.

[77] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *Submitted to IEEE Trans. Info. Theory*, July 2004.