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On Power Control with Finite Rate Feedback for Cooperative Relay Networks

Nasir Ahmed, Mohammad Khojastepour, Ashutosh Sabharwal and Behnaam Aazhang

Department of Electrical and Computer Engineering
Rice University,
6100 Main St., 77005 Houston, TX, USA
E-mail: {nasir,amir,ashu,aaz}@rice.edu

Abstract

Power control strategies with finite rate feedback are studied for the cooperative channel. The main contribution of this paper is to show that quantized feedback information can lead to significant reduction in outage probability for the cooperative relay network. To obtain an increase in diversity order and significant reductions in outage probability over constant power cooperative signaling, we develop algorithms that exploit the channel states of all network links. With one feedback bit, the proposed power control algorithm is shown to double the diversity order of constant power transmission. To quantify the performance increase of using power control in the cooperative network, we derive a lower bound on the diversity order. Based on these results, it is evident that future network protocols should utilize feedback in order to exploit the potential gains of network coding.

1. INTRODUCTION

In a distributed network of nodes, cooperation can be exploited to achieve diversity. This type of cooperation diversity was first studied for the case of two transmission nodes and one destination in [1], and was shown to provide gains in achievable rate over multiple access transmission. However, in order to fully realize the benefits of network coding, feedback information must be exploited when it is available. In this paper, we investigate the performance improvements of feedback information on outage performance in a network setting.

Our main contribution demonstrates that transmitter power control in cooperative communication networks can lead to significant improvements in outage performance *if* the entire network state is used to determine the instantaneous transmitter power. For the case of amplify and forward (AF) protocols in 'cheap' relay networks [2], we show that only one bit of feedback information suffices to double the diversity order of the system compared to the non-feedback method proposed in [3]. The proposed power control policy is simple to compute since the power control levels can be obtained in a recursive manner, whereas the optimal power control policy requires the solution to a complex optimization problem with a nonlinear constraint. Finally, we hint to the possibility that using all the channel states may be essential to extract the large gains, by considering power control policies which use only direct link information.

The results in this paper motivate the need to study the role of feedback in network information theory. Our work in this paper focuses on the simplest possible network: one transmitter-receiver pair being assisted by one relay. Although the capacity of the relay or 'cheap' relay channel is still not known, even the known suboptimal methods can be used to obtain the performance gains from feedback as shown in this paper. Our work also motivates the fact that network protocols managing contention in cooperative networks should collect some form of channel states from all participating links.

The organization of the paper is as follows. In Section 2, the network and channel model are described. In Section 3, a general network power control procedure is described, and based on this procedure, an algorithm utilizing one bit of feedback is developed and analyzed. Section 4 looks at the scenario of power control using reduced channel state information at the receiver. Section 5 analyzes the performance of the proposed power control algorithms and Section 6 concludes the paper.

2. PROBLEM FORMULATION

2.1. Network Model

The network model is shown in Figure 1. Node R acts as a relay for node S, in order to send data to destination D. The transmission is assumed to occur in a time division manner. In the first half of the time slot, the source transmits to both the relay and

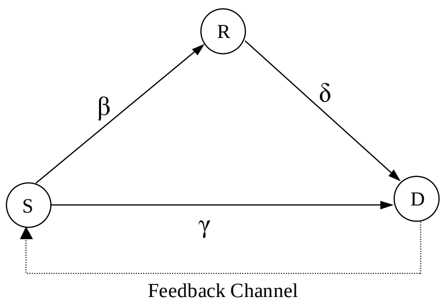

Figure 1: Model showing layout of relay channel

destination. In the second half of the time slot, the relay transmits the same information to the destination, while the source remains idle. At the relay and destination, the received signal is corrupted by additive white Gaussian noise with unit variance.

The fading values for the links in the relay channel are denoted as $a_{i,j}$, where $i \in (S, R)$ and $j \in (R, D)$. It is assumed that the gains, $a_{i,j}$, for each channel are independent, circularly symmetric Gaussian random variables with zero mean. The variance of the fading distributions are $\sigma_{i,j}^2$, where $i \in (S, R)$ and $j \in (R, D)$. For the remainder of this work, we will denote $\gamma = |a_{S,D}|^2$, $\beta = |a_{S,R}|^2$ and $\delta = |a_{R,D}|^2$.

In [3], an amplify and forward (AF) protocol was developed and shown to achieve full diversity. Its simplicity and the fact it achieves full diversity are the reasons we chose the AF protocol as the relaying method. The amplifying at the relay node is performed such that the relay experiences no more than P_{rel} power on average. For this protocol, the performance limits are characterized by the following achievable rate expression

$$R_{AF}(\gamma, \beta, \delta, P, P_{rel}) = \frac{1}{2} \log \left(1 + P\gamma + \frac{P\beta P_{rel}\delta}{1 + P\beta + P_{rel}\delta} \right) \quad (1)$$

In (1), P is the transmit power for the source, and P_{rel} is the relaying node's average power.

2.2. Power Control with Finite Rate Feedback

In this section, the power control procedure with finite rate feedback for the relay network is described. Power control with finite rate feedback for MIMO systems was analyzed in [4], and it was shown that with a long term power constraint, the probability of outage could be significantly reduced compared to constant power transmission. We assume that the receiver quantizes the power control information and transmits this quantized information through a noiseless feedback link to both the source and relay.

Consider the case where destination D can perfectly measure the relay *network channel state* (γ, β, δ). Given that the receiver uses Q bits for feedback, the power control algorithm selects a power-tuple $\mathcal{P}_q = (P_q, P_{rel,q})$ from a power control codebook $\mathcal{C}$ of cardinality 2^Q, where $q \in \{1, ..., 2^Q\}$. The index of the selected power-tuple is transmitted to both the source and relay. The source and relay also have copies of $\mathcal{C}$. Given that index q is sent on the feedback link, the source will then transmit with power P_q and the relay will use power $P_{rel,q}$.

The elements of $\mathcal{C}$ are chosen to maintain the power constraints of the source and relay. Consider a power control function $\mathcal{P}(\gamma, \beta, \delta)$ which maps the network channel state to a codebook element. To maintain the long term power constraint of the source and relay, we need to ensure that $E[\mathcal{P}(\gamma, \beta, \delta)] = (P, P_{rel})$. The objective of the power control algorithm is to find a $\mathcal{P}(\gamma, \beta, \delta)$ that minimizes the outage probability while meeting the power constraint.

3. POWER CONTROL USING THE ENTIRE NETWORK CHANNEL STATE

In this section, a power control algorithm is developed that takes into account the entire network channel state in the outage minimization process. The main focus of this section is to develop a power control algorithm in which the source performs power control and the relay simply transmits with constant power. Along with the algorithm, we analyze the outage probability and it will be seen how power control with even one bit of feedback can double the diversity order of constant power transmission. After the outage analysis, the case where the relay also transmits with a long term power constraint is analyzed.

3.1. Fixed Relay Power

In this section we consider a power control algorithm in which the relay is restricted to use a constant power in each time slot, but the source has the ability to vary its power to meet a long term average power constraint. In other words, power-tuple q from $\mathcal{C}$ has the form $\mathcal{P}_q = (P_q, P_{rel})$. The algorithm takes into account the entire network channel in an effort to minimize the outage probability. We limit our attention to one feedback bit in the algorithm, but the results can be easily extended to multiple bits of feedback.

Consider a receiver that has a perfect estimate of the network channel states (γ, β, δ). For ease of explanation, assume that $\beta = 1$, and later we will discuss how to extend the algorithm to the case of random β. Given one bit of feedback, the transmitter can select one out of two possible power levels. Referring to Figure 2, it can be seen that the space defined by all pairs (γ, δ) can be divided into two regions $\mathcal{R}_1$ and $\mathcal{R}_2$, corresponding to the power levels P_1 and P_2, respectively. To use the power control algorithm, the power levels must be determined along with the curve defining the boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$. Given power levels P_1 and P_2, the long term average power constraint of the source can be written as

$$P = \int_{\mathcal{R}_1} P_1 f(\gamma, \delta) d\gamma d\delta + \int_{\mathcal{R}_2} P_2 f(\gamma, \delta) d\gamma d\delta, \quad (2)$$

where $f(\gamma, \delta)$ is the joint probability distribution of the channel attenuations for the cooperative channel.

One key feature of the power control regions is that in region $\mathcal{R}_2$, the assigned power P_2 is the minimum required to guarantee zero outage for any point in the region. This is a fundamental property of all finite rate feedback power control algorithms [4]. With this in mind, given a transmission rate R and a constant relay power P_{rel}, power level P_2 is the solution to

$$\mathcal{R}_{AF}(\gamma, 1, \delta, P_2, P_{rel}) = R. \quad (3)$$

From Figure 2 it can be seen that the boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$ is separated by a curve $G(\gamma, P_2)$. This curve is found by solving for δ in (1), and has the following form

$$\delta = G(\gamma, P_2) = \frac{(1 + P_2)(K - \gamma P_2)}{P_{rel}(P_2 - K) + P_{rel} P_2 \gamma}, \quad (4)$$

where $K = e^{2R} - 1$. Any (γ, δ) along this curve requires exactly power P_2 for zero outage, while any other points in $\mathcal{R}_2$ require less than P_2 for zero outage. In this way, the entire region $\mathcal{R}_2$ is in zero outage. Therefore, calculating the outage probability for this power control method requires analysis of region $\mathcal{R}_1$.

As was discussed in [4], two possibilities exist for P_1. If $P_1 < P_2$, then it suffices to set $P_1 = 0$ and save power because doing so will not change the outage probability since channel states closer to the origin require more power to invert the effects of the channel. Therefore, the two cases of interest are when $P_1 = 0$, and $P_1 > P_2$. The outage probability is calculated for both cases, and the minimum is taken for the particular power constraint.

First, consider policies where $P_1 = 0$. The outage probability is simply the likelihood of being in $\mathcal{R}_1$, and can be expressed as

$$\Pi_{out}^a = \int_{\mathcal{R}_1} f(\gamma, \delta) d\gamma d\delta. \quad (5)$$

For $P_1 = 0$, the boundary of region $\mathcal{R}_1$ is determined by the curve $G(\gamma, P_2)$. The power level P_2 is found as the solution to

$$P = P_2 \int_{\mathcal{R}_2} f(\gamma, \delta) d\gamma d\delta. \quad (6)$$

Next, consider the case where $P_1 > P_2$. In general the optimal solution in this scenario is difficult to calculate [4], and instead we resort to a more tractable solution. The main idea of our scheme is to allocate equal power to the subregions $\mathcal{R}_1$ and $\mathcal{R}_2$. This technique was shown to be close to optimal for the single link channel even with one feedback bit [5]. Referring to Figure 2, in $\mathcal{R}_1$ power P_1 is sufficient to guarantee zero outage for all (γ, δ) to the right of $G(\gamma, P_1)$. It can be easily verified that $G(\gamma, P_1)$ intersects the γ axis at $\gamma_{out} = K/P_1$ and $G(\gamma, P_2)$ has a γ-intercept at $\gamma_B = K/P_2$. As a result of the simplifying assumption regarding the total power in each region, the power levels P_1 and P_2 can be solved in a recursive manner, as follows. First, power level P_2 is solved as the solution of $P_2 \Delta_2 = \frac{P}{2}$, where Δ_2 is the probability of the network channel state being in region $\mathcal{R}_2$, i.e,

$$\Delta_2 = \int_{\gamma_B}^{\infty} \int_0^{\infty} f(\gamma, \delta) d\delta d\gamma + \int_{\gamma_A}^{\gamma_B} \int_{G(\gamma, P_2)}^{\infty} f(\gamma, \delta) d\delta d\gamma,$$

where $\gamma_A = K/P_2 - 1$. Once P_2 is known, P_1 can be easily solved since we know that $P_1(1 - \Delta_2) = P/2$.

This recursive procedure is useful in that it can be easily extended to multiple feedback bits, which is not the case for the optimal power control scheme. To calculate the outage probability of this scheme, we simply find the probability that the network channel state (γ, δ) lies below the curve $G(\gamma, P_1)$. In order to do this, if we consider P^* as the minimal power required for zero outage, then P^* can be found as the solution to

$$\mathcal{R}_{AF}(\gamma, 1, \delta, P^*, P_{rel}) = R. \quad (7)$$

With this solution in hand, the outage probability using equal power subregions can be expressed as

$$\Pi_{out}^b = \int_{(\gamma, \delta): P^* \geq P_1} f_{\gamma, \delta}(\gamma, \delta) d\gamma d\delta. \quad (8)$$

The overall outage probability is the minimum of the outage probabilities obtained using the two possible scenarios. In other words, $\Pi_{out} = \min\{\Pi_{out}^a, \Pi_{out}^b\}$.

When β is also a random quantity, the regions $\mathcal{R}_1$ and $\mathcal{R}_2$ are volumes in the space defined by all positive (γ, β, δ). For a given β, the plane defined by all positive (γ, δ) is identical to Figure 2, except now $\gamma_A = \gamma_B - \beta$. By consider different values of β , the 3-dimensional volumes for $\mathcal{R}_1$ and $\mathcal{R}_2$ can be visualized. The recursive power control algorithm operates in a similar manner.

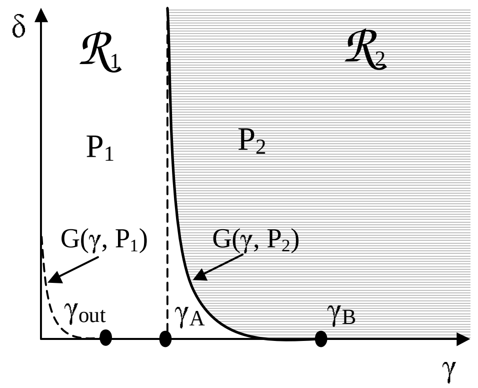

Figure 2: Structure of power control regions when $\beta = 1$ and γ and δ are random

First, power level P_2 is found by integrating over region $\mathcal{R}_2$ and assuming that the total power in this region is $P/2$. Once P_2 is found, P_1 is determined through direct substitution. In Section 5, results will be presented for cases where β is a constant and also where it can be a random quantity.

3.2. Lower Bound on Power Control Gain

It can be shown that one bit of power control on the single link channel can be shown to double the diversity over constant power transmission [5]. We next show a similar trend through bounds on the diversity order obtained by using the proposed network power control strategy with the amplify and forward transmission protocol. The main result can be summarized in the following theorem.

Theorem 1 *For the amplify and forward protocol, as $P_{rel} = P$ increases, the optimal one bit network power controls offers at least a fourth order diversity gain. The outage probability can be upper bounded by*

$$\Pi_{out} \approx \frac{2K^4}{\sigma_{r,d}^2 P^3 g(P, K, \sigma_{r,d}) - 4\sigma_{r,d}^2 P^3 K}, \tag{9}$$

where

$$g(P, K, \sigma_{r,d}) = 2K + P + \sqrt{4K^2 + \mathcal{M} + 2K^2/\sigma_{r,d}},$$

$K = e^{2R} - 1$ *and* $\mathcal{M} = P(P - 4K)$.

Proof: Omitted due to space considerations. ■

It can be seen that the effect of $\sigma_{r,d}^2$ should provide a shift in outage curve. Recall that constant power cooperative transmission provides a diversity order of two when the amplify and forward protocol is used. Using one bit for power control has doubled the slope of the outage versus power curve to four.

3.3. Effect of Varying Relay Power

In the power control algorithm discussed previously, the relay node has transmitted with constant power P_{rel} in each time slot. Constant power transmission is always inferior to power control in fading channels. Consider a simple example where the power control algorithm uses on-off signaling. When the receiver tells the source to transmit nothing, it makes no sense for the relay to simply amplify the noise, and in fact the relay could save power by not transmitting. In portions of time where the source transmits at maximum power, the relay could also send at a power higher than its average and help reduce the outage probability further. Using the above logic, it is apparent that controlling the power at the relay can provide further reductions in outage probability.

An example of such a scheme will be described next. The destination, upon obtaining the network channel state, decides a global power level which both the relay and the source are to transmit at concurrently. Based on the notation of Section 2.2, this corresponds to power control policies where element q from $\mathcal{C}$ has the form $\mathcal{P}_q = (P_q, P_q)$. The achievable rate for such a transmission scheme is simply $\mathcal{R}_{AF}(\gamma, \beta, \delta, P, P)$. The curve defining the boundary between $\mathcal{R}_1$ and $\mathcal{R}_2$ can be found by solving for δ in $\mathcal{R}_{AF}(\gamma, \beta, \delta, P_2, P_2) = R$. This is a similar to Equation 3, except now P_{rel} is replaced by P_2. Aside from this new curve, the algorithm operates identically to that in Section 3.1, and the details will be omitted. In Section 5, it will be seen how performing such a technique offers gains over simply setting P_{rel} to a constant value over all network states.

4. REDUCED CHANNEL KNOWLEDGE

Up to this point, the power control strategies relied on the entire network state (γ, β, δ). In this section, we explore the importance of using the entire network state in the power control process. More specifically, a power control algorithm is presented which relies solely on the source-destination fading state γ in the power control process, and the outage probability obtained will be compared to the network power control strategies derived earlier. In order to do this, the following two lemmas will be necessary to analyze the outage probability.

Lemma 1 *Consider the amplify and forward protocol transmitting at a rate R and average power P. For a fixed β and δ, assuming that $P_1 \leq P_2$, the outage probability for 1-bit power control can be written as*

$$\Pi_{out}^{a}(\gamma_0, \alpha_2|\delta, \beta) = 1 - e^{-\gamma_0} + e^{-\gamma_0}(1 - e^{-z_{out}(\alpha_2, \beta, \delta)})\mathcal{I}(z_{out}(\alpha_2, \delta, \beta) > \gamma_0),$$

where $\alpha_2 = P_1/P$, $\mathcal{I}(\cdot)$ is the indicator function and z_{out} is given by

$$z_{out}(x,\delta,\beta) = \frac{e^{2R}-1}{Px} - \frac{\delta P_{rel}\beta}{1+P_{rel}\delta+P\beta x},$$

and P_{rel} is the average relay transmit power.

Lemma 2 *Consider the amplify and forward protocol transmitting at a rate R and average power P. For a fixed β and δ, and assuming that $P_1 > P_2$, the outage probability for 1-bit power control can be written as*

$$\Pi^b_{out}(\gamma_0,\alpha_1,\alpha_2|\delta,\beta) = (1-e^{-\gamma_0})(\mathcal{I}(z_1>\gamma_0) + \mathcal{I}(z_1<\gamma_0)(1-e^{-z_1})) + e^{-\gamma_0}\mathcal{I}(z_2>\gamma_0)(e^{-\gamma_0}-e^{-z_2}),$$

where, $\alpha_1 = P_1/P$,$\alpha_2 = P_2/P$, $z_1 = z_{out}(\alpha_1,\delta,\beta)$ and $z_2 = z_{out}(\alpha_2,\delta,\beta)$.

With these lemmas in hand, the outage probability for a network power control algorithm using reduced channel state information can be derived. The result can be summarized in the following theorem.

Theorem 2 *For the amplify and forward protocol transmitting at a rate R, and average power P, 1-bit power control based only on the direct link fading state γ leads to an outage probability of*

$$\Pi_{out} = \min\{\int_\beta\int_\delta \Pi^a_{out}(\gamma^a_0,\alpha^a_2|\delta,\beta)f_{\beta,\delta}(\beta,\delta)d\beta d\delta \quad \int_\beta\int_\delta \Pi^b_{out}(\gamma^b_0,\alpha^b_1,\alpha^b_2|\delta,\beta)f_{\beta,\delta}(\beta,\delta)d\beta d\delta\},$$

where $f(\beta,\delta)$ is the joint probability distribution for β and δ, $\alpha^a_2 = e^{\gamma^a_0}$, and γ^a_0 is the solution to $\gamma^a_0 e^{\gamma^a_0} = \frac{e^R-1}{P}$. Additionally, $\gamma^b_0 = \frac{e^R-1}{P\alpha^b_2}$ and α^b_2 and α^b_1 can be solved through the following set of equations

$$(\alpha^b_2-1)\frac{e^R-1}{(\alpha^b_2)^2} + 1 - e^{-\frac{e^R-1}{P\alpha^b_2}} = 0,$$

$$\alpha^b_1 = \frac{1-\alpha^b_2}{1-e^{-\frac{e^R-1}{P\alpha^b_2}}} + \alpha^b_2.$$

Proof: Omitted due to space considerations. ■

Here it is assumed that the destination only uses the direct link channel state γ in its power control algorithm. In some situations where γ is large, poor channels on the relay links may corrupt the transmission, yet the algorithm ignores this point. In the results section, it will be seen that simply relying on δ results in poor performance compared to the scenario where the entire network state is accounted for.

5. ANALYSIS

In this section, numerical results showing the performance of our proposed power control schemes for the cooperative channel will be shown. Observing Figure 3, the second order diversity for constant power transmission using the amplify and forward protocol can be seen, as was discussed in [3].

Next, the outage probability curve for the network power control strategy is shown using the technique described in Section 3.1 for the case where $\beta = 1$. In this strategy, the total power in each subregion is equal. It can be seen that the outage performance with this method is far superior to constant power allocation. In fact, with one bit of feedback, the slope of the outage curve for the network power control the slope is four, as predicted by the lower bound analysis. However, for constant power transmission, the slope is only two. In this power allocation scheme, the relay simply transmits with a constant power in each time slot. This restriction was removed in the discussion of Section 3.3. The results for variable source and relay powers is also shown in Figure 3. This joint method of power control can be seen to provide gains on the order of 1dB at high powers over constant relay power allocation. In this technique, the destination transmits a single bit of feedback corresponding to a global power control level to both the source and relay. From the results of this figure, it is evident that power control using the entire state of the network provides significant gains over constant power allocation.

Additionally, on this same figure, we show the outage probability results discussed in Section 4, where the receiver uses only the direct link channel state γ in deciding a power control level. Surprisingly, it is seen that ignoring the relay links is asymptotically worse than constant power transmission. The reason for this is that for some portions of time, the direct link scheme allocates high power when the measured γ is really small, but in these cases high values for the relay channel states β and δ may occur, and less power should actually be used in order to transmit with higher power at a later time and maintain the same average power. As a result, in this network setting, it is better to do no power control at all then to use only the direct link channel state. As a final point of comparison, we plot the optimal one bit power control results for a network with no relay, from the results of [4]. It can be seen that this curve has a slope of two, whereas the network power control schemes were shown to have double this slope. This justifies the utility of using power control in conjunction with a network code.

In Figure 4, results are shown for $\sigma_{r,d} = 1$ and $\sigma_{r,d} = 2$, where the latter case corresponds to the sce-

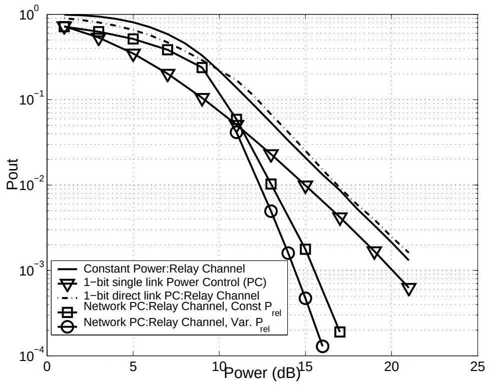

Figure 3: Simulation results comparing direct transmission power control and various network power control strategies

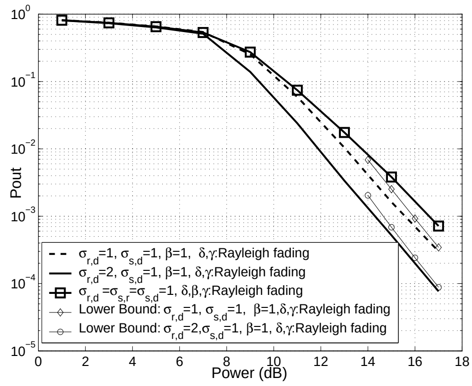

Figure 4: Outage probability results for the relay node being at a closer distance to the destination, and also for random β. In all curves, $\sigma_{s,d} = 1$ and both γ and δ undergo Rayleigh fading.

nario where the relay is located at a closer distance to the destination than the source. It can be seen that the closer distance does not increase the diversity order, but it provides a shift in the outage curves and better performance. This is expected as the benefits of cooperation are especially evident for relays experiencing good channel conditions.

Up to this point, all the results have assumed $\beta = 1$ and is deterministic. The scenario when β is random was also discussed in Section 3.1, with the control regions now being volumes in a space corresponding to the 3-tuple (γ, β, δ). Power control under such a scenario is also shown in Figure 4. Under this channel model, the diversity order is the same, however there is a shift to the right of the outage probability curve. This is expected because the extra fading does not give an independent look at the same data, but further corrupts the data sent on the relay link. Fading in such scenarios is never beneficial. On this same figure, the bounds to the outage probability are shown and it can be seen that the bounds closely follow the simulated results and confirm the fourth order diversity behavior for one bit of network power control.

6. CONCLUSIONS

In this work, the problem of outage minimization through network power control has been considered. It has been observed that using the entire state of the network to perform power control is needed to obtain sizable reductions in outage probability and specifically to provide diversity gains over constant power transmission. Additionally, we have presented a lower bound to demonstrate the increased diversity order obtained by using our network power control algorithm. We note that there are two assumptions in our analysis which limit it from being the optimal network power control strategy. The first fact is that the optimal coding strategy for the relay channel is not yet known. However, we have used the amplify and forward protocol which is optimal in terms of diversity order [3]. The second point is that the work in [4] has not completely addressed the problem of optimal power control with finite rate feedback even for the simple case of one bit of feedback.

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