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RICE UNIVERSITY

Housing and Non-Housing Asset Values Under a Consumption Tax Reform: A General Equilibrium Analysis

by

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A Thesis Submitted
in Partial Fulfillment of the
Requirements for the Degree

Doctor of Philosophy

APPROVED, THESIS COMMITTEE:

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Houston, Texas

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Abstract

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This study focuses on two potential problems that often arise in the discussions of the feasibility of consumption tax reform: (1) the potential negative effect of a consumption tax reform on the value of owner-occupied housing, and (2) the tendency of such a reform to impose a one-time windfall loss on the owners of existing capital other than owner-occupied housing. The results suggest that the reform-induced one-time windfall tax on the owners of existing assets tends to be overstated in models that do not explicitly account for owner-occupied housing. The study also suggests that the potential decline in the value of owner-occupied housing could be significant, approximately 10 percent of the total value of owner-occupied housing, immediately after reform. However, the value of owner-occupied housing is likely to rebound during the transition, resulting in significant increases in the value of owner-occupied housing in the long run.
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Chapter 1

Intergenerational and Intrigenerational Redistributions Under a Consumption Tax Reform

1.1 Introduction

Interest in consumption-based tax reform has grown considerably in recent years in the academic, business and political arenas, and several alternative proposals are currently being discussed in Washington and elsewhere. Perhaps the most prominent is the Flat Tax, first proposed by Hall and Rabushka (1985, 1995), introduced in Congress by Representative Armey and Senator Shelby and supported by presidential candidate Steve Forbes. A more recent entrant in the field that has attracted considerable attention is the National Retail Sales Tax (NRST), introduced in the House by Representatives Schaefer and Tauzin and supported by Senator Lugar and House Ways and Means Committee Chairman Archer. Other proposals include the cash-flow-based USA (Unlimited Saving Allowance) Tax, proposed by Senators Nunn and Domenici, and a subtraction-method VAT, supported by Representative Gibbons.

Proponents of consumption-based reforms argue that they would generate a variety of benefits, and there is a large literature examining the relative advantages and disadvantages of income and consumption taxes in terms of efficiency (especially in terms of reducing tax disincentives against saving and investment), equity (including views of equity that take a lifetime rather than an annual perspective), and simplicity in administration and compliance.\(^1\) These issues will not be revisited in my dissertation. Rather, the focus of the study is on two potential problems that often arise in

\(^1\)For Example, see the papers in Boskin (1996) and Aaron and Gale (1996).
discussions of the feasibility of consumption tax reform and are commonly viewed as creating huge impediments to the implementation of such a reform: (1) the potential negative effect of consumption taxes on the value of existing owner-occupied housing, and (2) the tendency of such a reform to impose a one-time windfall loss on the owners of existing capital other than owner-occupied housing. The purpose of the first chapter of this dissertation will be to examine these two potential sources of reform-induced windfall losses, including their combined effects on the intergenerational and intragenerational redistributions that would accompany a consumption tax reform.

Chapter 1 is organized as follows. The following section describes briefly the two consumption tax reforms considered - the Flat Tax and the NRST - including their tax treatment of owner-occupied housing as well as the effects of their implementation on the price level. The third section considers the nature of the windfall tax on the owners of existing capital that tends to occur under a consumption tax, while the following section discusses several recent analyses that have examined the potential effect of a consumption-based tax reform on the price of owner-occupied housing. The following two sections examine the interactions between these two effects - on the price of housing and the prices of non-housing capital assets - in terms of their effects on intergenerational and intragenerational redistributions. The concluding section discusses the methodology used in the completion of this research and several possible extensions.

1.2 Two Consumption-Based Tax Proposals

The two consumption tax proposals considered in this chapter are the Flat Tax and the NRST. Since the NRST is - apart from administrative considerations - equivalent to a VAT, the analysis of the NRST generally applies to the VAT as well. These two
proposals are outlined below; the effects on the price level of implementing either the Flat Tax or NRST are discussed as well.

1.2.1 The Flat Tax

Under the Flat Tax, businesses are taxed at a flat rate on their non-financial cash flows. The Flat Tax thus allows expensing of all business-related purchases, including capital investment that would be depreciated under an income tax. Businesses are allowed to deduct wages, which are then taxed at the individual level. Individuals are allowed a standard deduction and personal exemption, with all remaining wages taxed at the same flat rate applied to business cash flows. No special transition rules are provided under the Flat Tax. In particular, depreciation deductions for existing capital assets would not be allowed. Since no special provisions for the tax treatment of housing are provided under the Flat Tax, the consumption of housing services provided by owner-occupied housing are simply taxed like all other durable consumption items. (Owner-occupiers would not be allowed a deduction for the cost of a house and would not be taxed on the future flow of housing services or the proceeds from the sale of a house.) Local property tax and home mortgage interest - like all other forms of interest - are not deductible under the Flat Tax.²

1.2.2 The National Retail Sales Tax

The NRST is a tax at a flat rate on all final sales to consumers; in principle all business-related purchases by firms are exempt from tax.³ Distributional concerns

²Providers of rental housing would be taxed liked any other business.
³In practice, a significant fraction of the tax base of retail sales taxes operated by states - estimated by Ring (1989) to be roughly 40 percent - consists of business purchases. See Gillis, Miezskowski, and Zodrow (1996) for a discussion of this problem.
at the low end of the income distribution could be addressed with either a universal or a means-tested rebate of tax paid on some minimum level of consumption. Under the NRST, sales of new owner-occupied housing would be taxed at the time of sale. However, no attempt would be made to tax the services provided by existing owner-occupied housing, so that consumption of housing services from existing owner-occupied housing would not be subject to tax. Note that such an approach is identical to the treatment of owner-occupied housing that occurs under the Flat Tax.

1.2.3 Effects of the Flat Tax or the NRST on the Price Level

An important difference between the Flat Tax and the NRST is the likely effect of their implementation on the price level. The effect on the price level of a switch from an income tax to any form of consumption tax is uncertain in that it depends on the response of the monetary authorities. However, since the Flat Tax is similar in structure to the current income tax system in that taxes on wages are collected at the individual level so that wage contracts are expressed on a pre-tax basis, there is no reason to suspect that its implementation would have much of an impact on the price level.

In marked contrast, however, the imposition of a NRST would eliminate income taxes on wage income. In order for firms to continue to earn normal economic profits, either commodity prices would have to increase or the nominal wages paid by firms would have to decrease. Wage rigidities, primarily attributable to the fact that most wage contracts are written in nominal terms, imply that the latter option would involve large transitional costs. Accordingly, most analysts assume that the monetary authorities would follow an expansionary policy that would allow for "full price ad-
justment” - that is, commodity prices would rise by the full amount of the NRST. This assumption is adopted in this paper as well.⁴

1.3 Effects of Reform on the Owners of Existing Assets Other than Owner-Occupied Housing

1.3.1 The Windfall Tax on Existing Assets Other than Owner-Occupied Housing

Much of the concern about the transition from an income tax to a consumption tax has focused on the potential one-time windfall loss imposed on the owners of existing capital assets (other than owner-occupied housing). This loss would arise under both of the consumption tax reforms discussed above, although its form would be somewhat different under the two plans.

Under the Flat Tax, the value of expensing for new investment expenditures is equivalent in present value terms to the exemption of the normal rate of return on such investments. In the absence of any special transition rules, firms are not allowed a deduction for the remaining basis of existing assets, but the returns on existing assets are included in the tax base. As a result, the rate of return on existing assets falls relative to new investments, and arbitrage across new and existing assets implies that the value of existing assets must fall proportionately by the rate of the tax. Thus, a one-time windfall loss is imposed on the owners of existing capital.⁵ (That is, a consumption tax can be described as imposing a tax on future wages and existing capital assets.) Note that since prices do not change under the Flat Tax, lenders

⁴For example, see McClure (1996), Gravelle (1995), the Joint Committee on Taxation (1993), and Bradford (1996). Opinion on this issue, however, is not unanimous. For example, Jorgenson (1996) implicitly assumes a contractionary monetary policy such that producer prices and nominal wages fall, while Auerbach (1996) assumes partial price adjustment.

⁵See Gravelle (1995) and Joint Committee on Taxation (1993).
are insulated from this loss, since the nominal value of outstanding bonds is fixed, and thus the entire reform-induced one-time windfall loss is borne by equity holders. However, as will be discussed further below, existing owner-occupied housing is not subject to the one-time tax, since existing owner-occupied housing is ignored for tax purposes. That is, owner-occupiers receive no deductions for housing purchases and are not taxed on the service flow from existing owner-occupied housing, and sales of existing housing are not included in the tax base.

By comparison, if a NRST were enacted under the full price adjustment scenario assumed in this paper, the potential windfall loss on the owners of existing assets would be reflected in the reduced purchasing power of their assets, with no change in the nominal price of assets. In this case, the one-time increase in commodity prices would also reduce the real value of outstanding bonds so that the windfall loss would be distributed equally across equity and debt holders. However, since only new construction of owner-occupied housing is included in the tax base, the price of new owner-occupied housing - and thus existing housing as well - would rise proportionally by the sales tax rate. Thus, existing owner-occupiers would be exempt from the one-time tax on existing capital (although other factors, discussed below, would also affect the price of housing).

1.3.2 Other Factors Affecting the Windfall Tax on Existing Assets Other than Owner-Occupied Housing

The discussion thus far leaves the impression that large welfare losses would be imposed on the owners of existing capital assets (other than owner-occupied housing) under the enactment of a consumption-based tax reform. However, a wide variety of other factors associated with a consumption tax reform would also affect existing
capital owners - and most of them would act to offset the one-time windfall tax on existing assets.

For example, implementing a consumption-based tax reform would lead to an increase in the rate of investment during the transition to the new equilibrium relative to the rate of investment in the income tax steady state. This increase in investment would result in a larger steady state capital stock in the long run. In addition, implementing a consumption-based tax reform would result in a reallocation of capital between the housing and non-housing industries. However, higher rates of investment during the transition to the new equilibrium, as well as a reallocation of capital between industries, would involve adjustment costs. As a result, the transition to the new equilibrium would occur only over time. During this transition period, owners of existing assets and new investments would earn inframarginal rents, and these above-normal returns would act to offset the one-time windfall loss on the owners of existing assets.

A second important factor in determining the effects on existing capital owners of implementing a consumption-based tax is the effect of such a reform on interest rates. The effect of a consumption tax on interest rates has been much debated in the literature, as the range of likely post-reform interest rates varies from a constant before-tax interest rate to a constant after-tax interest rate. As noted by Bradford (1996), a constant before-tax interest rate is consistent with a highly inelastic demand for capital and might be termed the small open economy case, while a constant after-tax interest rate is consistent with a highly elastic supply of saving and a closed economy. Most analysts assume that interest rates would fall somewhat with reform, since they would be calculated on an after-tax basis. For example, Hall and Rabushka (1995) suggest that interest rates would decline by roughly 2 percentage points after implementation of the Flat Tax.
However, Feldstein (1995) stresses that the current income tax system favors debt finance, so that a consumption tax reform would reduce effective tax rates on equity-financed investment much more than on debt-financed investment, thus increasing relative returns to equity. The fact that debt would become relatively less attractive would in turn put upward pressure on interest rates in order to maintain a financial equilibrium; that is, interest rates might rise even though the overall before-tax rate of return declines as a result of the consumption tax reform.

As long as they did not decline immediately to their values in the pre-reform equilibrium, interest rates could play an important role in offsetting the windfall tax on capital owners described above, since the owners of existing assets would earn a higher after-tax rate of return on existing assets and new investments. This effect would be more important for consumers who are able to defer consumption of existing wealth. Indeed, it is possible that the gains from earning a higher after-tax rate of return could completely offset the windfall loss caused by a decrease in the value of existing equity in business assets, especially for younger individuals.

A third factor offsetting the potential capital levy under a consumption tax arises because the current tax system allows accelerated depreciation allowances (primarily to offset the effect of inflation on nominal depreciation deductions). As a result, income is under taxed in the early years after an investment, and over taxed in its later years. This in turn implies that existing assets should sell at a discount under the current income tax regime, in order to reflect the higher effective tax rate on all remaining cash flows generated by existing assets. The switch to a NRST would eliminate this expected tax, while the move to a Flat Tax would reduce it to the extent that rates were reduced. This in turn would increase the value of existing assets, thus partially offsetting the one-time windfall tax. Auerbach (1996) estimates that the average discount on corporate fixed capital is approximately eight percent
due to this factor. However, Gravelle (1998) estimates that the average discount on capital has declined to 4 percent, since the length of asset lives, especially for structures, for depreciation purposes were increased in 1986 and 1993.

Another factor that would reduce any reform-induced windfall losses is more contentious. Under the "new view" of dividend taxation, the benefits to shareholders of deferring the individual level dividend tax on investments financed with retained earnings is equal in present value to the future tax on dividend distributions from the investment. Thus, dividend taxes at the individual level have no effect on marginal investment decisions financed with retained earnings. Moreover, under this view the market value of assets is discounted to reflect future dividend taxes, so that the enactment of a consumption-based tax would result in a one-time increase in share values. Auerbach (1996) estimates that the capitalization of future dividend tax payments would be approximately 10 percent of the value of existing corporate shares under the new view. However, the consensus of the profession seems to favor the "traditional" view of dividend taxation, under which the returns to equity-financed investments are subject to business and individual level taxes and corporate shares do not sell at a discount. Under these circumstances, the "new view effect" on share prices would not occur.

Tax-deferred assets might also play a role in mitigating any reform-induced one-time tax on the value of existing capital. These assets would have been subject to tax under the current income tax upon withdrawal. This tax would be forgiven under the NRST, generating a windfall gain that would offset the loss attributable to the reform-induced price increase. By comparison, the tax on withdrawals would be reduced under the Flat Tax to the extent that tax rates were lower (or special transition provisions provided for some forgiveness of tax on assets withdrawn from existing tax-deferred accounts). Given the large fraction of assets held in tax-deferred
accounts as well as the fact that nearly all net personal saving in the past twenty years has been in tax-deferred accounts, this factor could be very important, especially under the NRST. A related factor is that unrealized capital gains would be exempt from tax under the Flat Tax (although they would be taxed when consumed under the NRST).

Many discussions of consumption-based taxes argue that they "exempt" capital income from tax. However, as stressed by Bradford (1996) and Gentry and Hubbard (1997a, 1997b), this characterization is accurate only for the "normal" return to saving and investment (the "return to waiting") that is untaxed as a result of expensing under the Flat Tax or because investment goods are not subject to tax under the NRST. By comparison, the three other components of capital income - inframarginal returns attributable to market power, good ideas or managerial skill, the expected returns to risk taking, and returns to risk taking in excess of the risk premium (or "good luck") - are all taxed similarly under both income and consumption taxes.

Thus, as stressed by Gentry and Hubbard, the differences between the two taxes in terms of their tax treatment of capital income is much less than commonly assumed, especially in simulation analyses that ignore uncertainty and assume competitive markets. Recall that the source of the reform-induced capital levy described above is the relative tax treatment of capital income before and after reform. Since this approach indicates that the differences are smaller than under approaches that ignore imperfect competition and uncertainty, it suggests that the declines in the values of existing capital assets would be smaller than those implied by analyses that assume that all capital income taxation is eliminated with the implementation of a consumption tax.

There are also several potential sources of efficiency gains that could potentially mitigate any reform-induced windfall losses imposed on the owners of existing capital. One of the most important sources of efficiency gains in switching from a consumption-
based tax to an income-based tax is the treatment of capital income. Enactment of a consumption tax reduces the distortion on saving and investment decisions inherent in an income tax, and thus improves the allocation of resources across present and future consumption. In addition, to the extent that switching to a consumption-based tax lowers the marginal tax rate on labor supply, a reduction in the distortion of the labor-leisure decision also results in efficiency gains. A number of other distortions would also be eliminated under a consumption-based tax as well, including the tax distortions across assets and business sectors that characterize current law. Obviously, one of the most important distortions of this nature is the allocation of capital across the owner-occupied housing, rental housing, and non-housing sectors. Neglecting these gains would tend to overstate the potential reform-induced welfare losses on the owners of existing capital.

The nature of bequests and the bequest motive would also play an important role in the transmission of windfall gains and losses across generations. The most common bequest motives assumed in the literature arise either out of altruism, bequest as a form of consumption, a means of affecting the behavior of potential heirs, or as an accident due to the uncertainty of an individual's life span. Moreover, the inclusion of a simple bequest motive in most general equilibrium studies is imperative to present a more realistic pattern of saving and an explanation of the level of capital accumulation in a life-cycle model.

All of these factors suggest that the net effects on asset values and reform-induced redistributions of implementing a consumption-based tax are ambiguous from a theoretical standpoint. Although it is impossible to predict accurately changes in the value of assets and net redistribution across age and income groups, several studies have attempted to identify these effects using dynamic simulations of the life cycle
general equilibrium models. These studies suggest that with no adjustment costs, implementing a consumption-based tax reform causes an initial decline in the value of assets that is typically between 4 and 10 percent. The inclusion of moderate adjustment costs, however, mitigates the decline in the value of assets, yielding estimates of the reform-induced change in asset values that range from a 4 percent decrease to an increase of 6 percent. Without adjustment costs, the elderly generally tend to be losers from reform, while the reform-induced welfare gains to the younger and middle-aged generations are attained earlier. In the case of moderate adjustment costs, the elderly generally enjoy welfare gains from reform. However, the existence of adjustment cost delays the efficiency gains from reform, causing most young and middle-aged individuals to be losers from reform, even though significant long run gains may be achieved.

The intragenerational effects of a consumption tax reform are studied by Altig, Auerbach, Kotlikoff, Smetters and Walliser (1997) — hereafter AAKSW — who include 12 different income groups in each generation, and thus allow for the examination of the welfare effects of consumption-based tax reform across age groups and income classes. Although AAKSW perform a large number of simulations, their results for the implementation of the Flat Tax, with and without adjustment costs, are most relevant for this discussion. Without adjustment costs, asset values decline by nearly 10 percent with the implementation of the Flat Tax. By comparison, with adjustment costs, this decline is only 3.6 percent. Without adjustment costs, very old generations are generally losers from reform, while middle-aged and younger generations are generally slight gainers or slight losers. In contrast, with adjustment costs, all elderly generations are gainers from reform, while middle-income young genera-

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6For example, see Auerbach (1996), Altig, Auerbach, Kotlikoff, Smetters, and Walliser (1997), and Zodrow and Williams (1998).
tions are net losers (due primarily to higher tax rates) and high and low-income young generations are net gainers (the former due primarily to lower tax rates and the latter due primarily to increased wages attributable to greater capital intensity).

1.4 Effects of Reform on the Owners of Existing Owner-Occupied Housing

One of the more contentious issues associated with implementing a Flat Tax or a NRST is the size of the potential reform-induced decline in the price of owner-occupied housing. As noted above, this decline does not arise as part of any one-time capital levy imposed on capital owners by a consumption tax reform. Rather, this price decline is due to the reduction in housing demand attributable to the elimination of the relative tax advantage of owner-occupied housing under current law, as well as the elimination of deductions for mortgage interest and property taxes.

Owner-occupied housing is treated very generously under the current income tax. The most important factor is that the implicit capital income generated by an investment in owner-occupied housing — the imputed rents in the form of housing services, net of maintenance and depreciation expenses — is untaxed. In addition, virtually all capital gains on owner-occupied housing escape taxation, and homeowners are allowed deductions for mortgage interest (including interest on home equity loans) and property tax payments under the current income tax.

The effect of a consumption tax reform on housing is most easily analyzed by examining reform-induced changes in the user cost of owner-occupied housing, defined as the cost of consuming an additional unit of owner-occupied housing or the sum of the opportunity cost of the home owner's equity, depreciation and maintenance expenditures, and the after-tax cost of mortgage interest and (arguably) property
tax payments. The opportunity cost of the equity-financed share of owner-occupied housing is equal to the after-tax return on alternative non-housing investments. If a Flat Tax or a NRST were enacted, the after-tax return on new investments in sectors other than the owner-occupied housing sector would increase due to the elimination of the tax on the normal rate of return. Furthermore, after-tax mortgage and property tax payments would also increase for households that itemize, since tax deductions for these payments would be eliminated. All of these factors would tend to reduce the demand for owner-occupied housing, which in turn would lead to a decline in housing prices in the short run. In the long run, the cost of housing would return to an equilibrium reflecting production costs, including the cost of land.

Several analysts have commented on the impact on housing prices of consumption tax reforms, and the range of predicted effects is large. Several studies predict large declines. For example, Data Resources Incorporated (1995) predict that the aggregate price of owner-occupied housing would decline by as much as 15 percent in response to the enactment of a Flat Tax. Similarly, Capozza, Green, and Hendershott (1996) estimate that implementing a Flat Tax would reduce owner-occupied housing prices by an average of 20 percent, assuming that the interest rate falls by one percentage point. By comparison, Hall (1995) argues that implementation of the Flat Tax would result in a rather modest decline in the aggregate price of housing, assuming a two percentage point decline in the interest rate. Similarly, Gravelle (1996) and Bruce and Holtz-Eakin (1997) argue that both the short run and the long run effects of a Flat Tax on housing prices would be fairly small.

The case for large reform-induced declines in housing prices is illustrated by Capozza, Green, and Hendershott (1996). They estimate the change in the price of owner-occupied housing using an asset market equilibrium model in which the

7See Hendershott and Hu (1981) for a discussion of the appropriate treatment of property taxes.
implicit rents from owner-occupied housing equal the user cost of housing capital. Furthermore, they assume that the supply of prime residential land is perfectly inelastic, which implies that it is impossible to make quantity adjustments in the supply of owner-occupied housing, and that the marginal benefit of an additional unit of housing — the implicit rents — is constant. The implication of these assumptions is that a change in the tax treatment of owner-occupied housing must be fully capitalized into the price of owner-occupied housing.

Their model suggests that implementing the Flat Tax at a rate of 17 percent, holding constant the before-tax rate of interest, would reduce owner-occupied housing prices by an average of 29 percent. They suggest that some reduction in interest rates is likely however, and their model predicts an average price decline of 20 (9) percent if the before-tax interest rate declines by one (two) percentage points; the latter result corresponds roughly to the case of a constant after-tax interest rate. They also note that housing price effects would vary widely across geographical areas, and that the largest price declines in percentage terms would occur for expensive urban homes owned by individuals with high marginal tax rates.

Other studies cast doubt on such large price declines, and shift the focus to the dynamic factors that would mitigate the effects of a consumption tax reform on the price of housing. For example, Bruce and Holtz-Eakin (1997), hereafter BHE, employ a dynamic partial equilibrium simulation model of the owner-occupied housing market that is capable of analyzing the short-run and long-run price effects of enacting a Flat Tax. BHE assume that the Flat Tax is shifted forward, which implies that the price level — including the price of new and thus (in equilibrium) existing owner-occupied housing — will increase proportionally by the rate of the tax. Note, however, that this one-time increase in the price of housing does not reflect a change in the real value of owner-occupied housing; rather, as described above, it reflects the fact that
owner-occupied housing is not subject to the potential capital levy associated with the implementation of a consumption tax. Any decline in the real price of owner-occupied housing in the BHE model reflects the changes in consumer demand stressed by CGH.

The first version of the BHE model ignores land. In this case, reform leads to a short run decrease in the real price of owner-occupied housing since the stock of housing is initially fixed, while in the long run, the real price of owner-occupied housing reflects construction costs; that is, the long-run supply of owner-occupied housing is assumed to be perfectly elastic, as firms in the housing market are assumed to be price-takers with respect to the economy as a whole.

Their results indicate that the short-run impact of enacting a 17 percent Flat Tax, assuming the before-tax rate of interest is constant, is a 10 percent increase in the nominal price of owner-occupied housing. This implies that the real price of owner-occupied housing declines by 6 percent. In the long run, capital flows out of the owner-occupied housing sector, since the after-tax rate of return has decreased, and nominal housing prices increase by 17 percent, leaving real housing prices unchanged.

BHE then expand their model to include land in the production of housing services, thus capturing the fact that the supply response of land is more inelastic than the supply response of structures as stressed by CGH. They assume an elasticity of supply for land equal to 0.2 and an elasticity of supply for structures equal to 0.8. Somewhat surprisingly, this model yields a short-run increase in the price of owner-occupied housing that is quite similar to the version of their model that ignores land. One difference is that in the two-factor model the adjustment period is roughly twice as long as in the model that ignores land, although most of the adjustment still occurs within 7 to 10 periods after the reform.
Similar results are obtained by Gravelle (1996), who utilizes a reduced-form model of the housing market and the user cost of owner-occupied housing. She begins with the “worst case” scenario — a completely inelastic supply of housing, no savings response, and instantaneous demand adjustments. In this case, her results suggest that enactment of the Flat Tax would reduce the price of owner-occupied housing by 22 percent in the long run.

Gravelle then makes a number of plausible modifications to the model, each of which has the effect of reducing the reform-induced decline in the price of housing. Allowing the before-tax rate of return on capital to decline by a modest 0.5 percent — an estimate consistent with the findings of Engen and Gale (1996) — reduces her estimate of the long run decline in the price of owner-occupied housing to 17 percent on average. Allowing the supply of owner-occupied housing to be less than completely inelastic reduces the long run effect on housing prices of enacting a Flat Tax even further. For instance, with a housing supply elasticity of 1.0 and allowing for a one percentage point decline in the interest rate, the price (and quantity) of owner-occupied housing decline by 5 percent. In fact, if the after-tax interest rate is held constant, the model suggests that the average price of owner-occupied housing would actually increase in the long run, although housing prices would still decline somewhat for homeowners who itemize and face high marginal tax rates. Gravelle concludes that the long run impact of a consumption tax reform on the price of owner-occupied housing would be small.

She also argues that several factors would mitigate the decline in the price of owner-occupied housing in the short run. For example, she notes that the supply of owner-occupied housing is more elastic than it might seem because new home construction, which could be easily reduced in response to a reduction in demand, makes up roughly 15 percent of the market for home sales. In addition, supply could
be reduced easily if a decline in the price of owner-occupied housing — especially one that is perceived to be a short-run phenomenon — convinces owners to temporarily remove existing houses from the market or convert them to rental houses. Finally, it is unlikely that the reform-induced reallocation of capital from owner-occupied housing to other sectors of the economy could occur without significant adjustment costs. As noted above, the presence of adjustment costs would slow down the reallocation of investment and thus reduce the decline in the price of owner-occupied housing. For all of these reasons, Gravelle concludes that the implementation of a consumption tax would have a relatively limited impact on housing prices even in the short run.

To sum up, the magnitude of the effects of implementing a Flat Tax or a NRST on housing prices are uncertain. Because of its generous treatment under current law, owner-occupied housing would not experience the reform-induced decline in asset values that a consumption tax could impose on other types of assets. However, housing prices would decline due to the reduction in demand associated with higher returns on alternative investments as well as the elimination of deductions for mortgage interest and property taxes. However, given a variety of factors that provide some flexibility in the supply of housing, it appears likely that even in the short run this decline would be moderate.

1.5 Net Effects on Intergenerational and Intragenerational Redistributions

The simulation models described in Section 1.3 are single-good models that consider housing only in an ad hoc fashion — e.g., by reducing the effective tax rates applied to capital income. The analysis above, however, suggests that the effects of a consumption tax reform on the values of existing housing are quite different from those
on other assets, so an accurate picture of the effects of reform on asset values can be obtained only with an explicit analysis of housing. The same is true of the intergenerational and intragenerational redistributions attributable to reform, since ownership patterns of housing and non-housing capital as well as methods of financing housing investments differ considerably across generations and income classes.

For example, Gentry and Hubbard (1997), analyzing the 1989 Federal Reserve Survey of Consumer Finances, find that the shares of primary residences owned by young, middle-aged, and old households are 16, 45, and 39 percent respectively. In addition, Gentry and Hubbard find that older households own roughly 75 percent of all bonds and 65 percent of all stocks held directly and in mutual funds.

Differences in ownership patterns of non-housing and housing capital across income classes are even more dramatic than intergenerational differences. Gentry and Hubbard (1997) report that the top ten percent of the income distribution owns 34 percent of primary residences, 58 percent of taxable bonds, 84 percent of tax-exempt bonds, 71 percent of stock held directly, 69 percent of stock held in mutual funds, and 54 percent of assets held in retirement accounts. By comparison, individuals in the 50-90th percentiles own 45 percent of primary residences, 21 percent of taxable bonds, 12 percent of tax-exempt bonds, 24 percent of direct stock, 25 percent of stock held in mutual funds, and 40 percent of assets held in retirement accounts. Differences of these magnitudes clearly suggest that distinguishing explicitly between ownership of non-housing capital assets and owner-occupied housing could play an important role in identifying accurately the pattern of intergenerational and intragenerational welfare redistributions associated with a consumption-based tax reform.
1.5.1 General Effects of Explicitly Including Owner-Occupied Housing

The implementation of a broad-based consumption tax should result in efficiency gains due to the reduction of existing tax distortions of the allocation of investment. In particular, the current income tax seriously distorts the allocation of investment across owner-occupied and other business assets — an important effect since owner-occupied housing is the single largest component of the capital stock. However, single good models obviously cannot capture the efficiency gains from eliminating this distortion, which would tend to reduce any negative effects of reform.

The size of the inefficiency associated with the tax wedge between the owner-occupied and business sectors depends on the elasticity of substitution between housing and non-housing goods, as well as the elasticity of substitution between owner-occupied and rental housing. Poterba (1992) estimates that the tax advantage to owner-occupiers in 1990 induced households with incomes of $30,000, $50,000, and $250,000 to increase housing consumption by 12.4, 23.2, and 23.2 percent, respectively. The excess burden associated with these estimates was $53 for households with an income of $30,000, $326 for households with an income of $50,000, and $1,631 for households with an income of $250,000.

In addition, the speed with which capital would move out of owner-occupied housing and into other production sectors would be affected by the magnitude of the costs of adjusting the capital stock. Since the reallocation involved could be sizable, the adjustment costs associated with moving capital out of owner-occupied housing and into other sectors could be quite important. As described above, adjustment costs imply that the owners of existing business assets will earn inframarginal returns during the transition, but that the efficiency gains of reform would be delayed.
1.5.2 Intergenerational and Intragenerational Effects

As discussed at length above, the owners of existing capital are subject to a potential one-time windfall capital levy under a consumption-based tax reform, but owners of existing owner-occupied housing are exempt from this one-time capital levy. Therefore, explicitly including owner-occupied housing would allow for a more accurate calculation of the distribution of this reform-induced windfall loss on the owners of existing business assets across generations and income classes.

In general, the larger the share of owner-occupied housing assets in an individual’s portfolio, the smaller the capital levy under a consumption tax reform. Since middle-aged generations own a larger share of owner-occupied housing and a smaller share of the remaining assets in the economy, models that account for owner-occupied housing explicitly should, relative to single-good models, obtain smaller burdens on existing middle-aged generations and larger losses for older generations.

In addition, given the housing ownership pattern described above, the implementation of a consumption tax reform in a model that accounts for owner-occupied housing will result in smaller welfare losses for middle to high income classes within any generation, relative to the single-good case. On the other hand, the welfare of the highest income classes will be reduced, since they own a disproportionate share of existing business assets. This effect would be especially pronounced for the top two percent of the income distribution, since owner-occupied housing is such a small share of total net worth in this income group.

More generally, the effects of other factors affecting the capital levy aspect of a consumption tax would affect the intergenerational and intragenerational redistributions associated with a consumption tax reform in analogous ways. For example, in contrast to non-housing assets, owner-occupied housing does not sell at a discount due to accelerated depreciation deductions. Thus, owner-occupiers would not realize
a windfall gain on their existing housing from the elimination of business taxes under the NRST or the reduction in business tax rates under the Flat Tax, and the gain on existing non-housing assets would accrue disproportionately to older generations and individuals in the highest income classes.

Finally, under the "full price adjustment" scenario with implementation of the NRST, the one-time increase in the price level would decrease the real value of outstanding mortgages, resulting in a redistribution from lenders to borrowers. Gentry and Hubbard (1997) report the distribution of mortgage debt across young, middle-aged, and old households to be 31.7, 54.6, and 13.7 percent, respectively. In the aggregate, young and middle-aged households would receive a substantial benefit from the decline in the real value of outstanding debt if an NRST were enacted. Since the portfolios of older cohorts consist of a large share of bonds, they would be subject to a one-time windfall loss on their bond holdings under the NRST.

1.5.3 Intergenerational and Intragenerational Effects of Lower Housing Demand

The owners of existing assets would also be affected by changes in the value of assets that stem from tax-induced changes in the allocation and size of the capital stock. As discussed above, this asset price effect is of particular concern in the owner-occupied housing market, since investment in that sector is especially tax advantaged under current law and because owner-occupied housing makes up such a large share of the capital stock. The differences in the patterns of ownership of owner-occupied housing described above imply that reform-induced decreases in the user cost of owner-occupied housing, and thus the corresponding price declines in owner-occupied housing, would vary widely by income and age of the household.
In particular, middle-aged and elderly generations are more likely to be owner-occupiers and tend to own larger and more expensive homes than younger generations. In addition, middle-aged and elderly generations are more likely to face higher marginal tax rates and itemize deductions. As a result, the reform-induced decline in the user cost of owner-occupied housing, and thus the related decline in the price of owner-occupied housing, would tend to be larger for households in middle-aged and elderly generations.

Furthermore, within a given generation, high-income households face higher marginal tax rates, own more expensive homes, and are more likely to itemize deductions. This implies that the largest increase in the user cost of owner-occupied housing, and thus the largest decrease in the demand for owner-occupied housing, would occur for high-income households. For middle-aged high-income households the combination of high tax rates, the increased probability of itemizing, and high loan-to-value ratios all act to increase the value of mortgage interest deductions, and thus increase the cost of owner occupying. On the other hand, older high-income households tend to hold less mortgage debt and more equity in owner-occupied housing on average. Since consumption-based taxes exempt the “normal” returns from investments, the increased cost of the equity-financed share of owner-occupied housing would also tend to increase the cost of owner occupying.

By comparison, the welfare of young low-income households would be least affected by the decline in the price of owner-occupied housing, since the rate of home ownership, marginal tax rates, the probability of itemizing, and house values typically increase with both age and income. On the other hand, since investments in owner-occupied housing by young itemizing homeowners are highly leveraged on average, consumption tax reform would increase after-tax mortgage interest and property tax payments and thus impose welfare losses on these households as well. Alternatively,
low-income homeowners that do not itemize would suffer very small welfare losses as a result of the reform-induced increase in the user cost of owner-occupied housing.

The effect of interest rate changes on owner-occupiers relative to the owners of other assets would also be quite different. A constant before-tax interest rate would be accompanied by a relatively large increase in the user cost of owner-occupied housing, since this would accentuate the increase in the opportunity cost of equity-financed investments in owner housing and the servicing cost of mortgage debt. In this case, the reduction in the demand for owner-occupied housing would be relatively large, as would the corresponding decline in its price. On the other hand, a constant after-tax interest rate — which roughly characterizes the simulation models described above in the long run — would negate the increase in the user cost of capital attributable to higher opportunity costs of equity-financed investments and after-tax mortgage interest payments. In this case, the expected reductions in the demand for owner-occupied housing would be minimized, as would the decline in its price. In fact, the user cost of owner-occupied housing could decline for some non-itemizing homeowners, since mortgage interest payments for this group would fall. In this case, the welfare losses attributable to a reform-induced decline in the price of owner-occupied housing on the middle-aged and elderly generations and high-income households would be reduced significantly. The welfare loss on young itemizing households would also decrease as interest rates declined, since mortgage interest payments would fall.

In addition, if the cost of adjusting the capital stock were particularly large for moving capital out of owner-occupied housing and into other business sectors, the reductions in the demand for, and prices of, owner-occupied housing would be mitigated. The largest reductions in welfare losses would accrue to middle-aged and elderly generations, and to high-income households.
Income and price effects on the demand for housing would also have implications for the intergenerational and intragenerational distributions of the effects of a consumption tax reform. Most studies of the distributive effects of the Flat Tax suggest that it would increase the amount of tax paid by those in the lower and middle parts of the income distribution and reduce taxes for the highest income households. As a result, the highest income households would increase their demand for owner-occupied housing due to the increase in after-tax income. Gravelle (1996) finds that if the income and price elasticities of demand for owner-occupied housing were the same, then this effect would offset roughly one quarter of the reduced demand for owner-occupied housing among the highest income households. Moreover, Rosen (1985) finds that the demand for owner-occupied housing in the highest income brackets is relatively price inelastic, implying that the demand reductions in response to changes in the user cost of housing will be relatively smaller in the highest income classes. Both of these factors tend to dampen the decrease in the demand for owner-occupied housing and the related price declines in the highest income classes.

1.6 Conclusion

The analysis above suggests that including housing explicitly in dynamic models of the intergenerational and intragenerational redistributions caused by the implementation of a Flat Tax or a NRST could potentially be quite important. This is likely to be the case for two reasons. First, it is possible that ignoring the distinctions in asset price effects that operate in the non-housing and owner-occupied housing sectors could bias the aggregate estimates of asset price changes resulting from consumption-based tax reform. Thus, including housing explicitly would provide a more detailed and reliable estimate of the potential effects of tax reform. Second, explicitly including housing would allow for more accurate estimates of the reform-induced redistributions
across age groups and income classes. One particularly interesting possibility is that including housing would reduce to some extent the welfare gains enjoyed by the highest income households. This seems likely since, as described above, (1) they would experience a potentially significant welfare loss due to the decline in house values attributable to the reform-induced reduction in demand for expensive homes, but (2) they would benefit to a relatively small extent from the exclusion of owner-occupied housing from the capital levy attributable to the introduction of a consumption tax. Since the gains from a consumption tax reform are concentrated among the wealthy (see AAKSW), the resulting pattern of intragenerational redistributions would be less regressive than that predicted by an analysis that ignores owner-occupied housing.

However, the analysis thus far is obviously only descriptive. In Chapter 2 of this dissertation I present results from an overlapping generations general equilibrium life-cycle model that explicitly includes rental housing and owner-occupied housing assets. The model allows for more detailed estimates of the effects of consumption tax reforms on housing and non-housing asset values as well as the reform-induced intergenerational redistributions that would accompany reform. Note that the model does not provide estimates of the intragenerational reform-induced redistributions since it is constructed using a representative agent framework. Although, the model can be readily extended for such purposes, and thus, provides a solid foundation for future improvements and extensions in this area of research.
Chapter 2

Housing and Non-Housing Asset Values Under a Flat Tax Reform

2.1 Introduction

In this chapter, I employ a dynamic overlapping generations general equilibrium model to examine the two potential problems associated with the implementation of a consumption tax reform: (1) the potential negative effect of consumption taxes on the value of the existing housing stock, and (2) the tendency of such a reform to impose a one-time windfall loss on the owners of existing capital (other than owner-occupied capital). There are several distinguishing features of the model used in this paper relative to existing studies that examine the effects of consumption-based tax reform. Most importantly, the value of assets in the non-housing, rental housing, and owner-occupied housing sectors are calculated explicitly. These calculations take into account the important differences in the tax treatment of capital across sectors that exist under the current U.S. income tax. For example, accelerated depreciation deductions are allowed in the non-housing and rental housing sectors, the service flow to owner-occupied housing is untaxed, owner-occupiers are allowed tax deductions for property taxes and mortgage interest payments, and landlords but not owner-occupiers are allowed deductions for maintenance expenditures. In addition, the model includes a number of the factors, described in Section 1.3, that act to offset the one-time windfall tax imposed on the owners of existing assets. The purpose of this chapter is to provide more detailed estimates of the potential sources of reform-induced windfall losses, including their combined effects on the intergenerational redistributions that
would accompany a consumption tax reform. I begin by reviewing existing estimates of asset price effects on non-housing and housing capital, and providing an overview of the results discussed in this chapter.

Several analysts have estimated the effects on housing prices of consumption tax reforms, and the range of predicted effects is large. Several studies predict large price declines. For example, Data Resources Incorporated (1995) estimate that the aggregate price of owner-occupied housing would decline by as much as 15 percent in response to the enactment of a Flat Tax. Similarly, Capozza, Green, and Hendershott (1996) estimate that implementing a Flat Tax would reduce owner-occupied housing prices by an average of 20 percent. By comparison, Hall (1995) argues that implementation of the Flat Tax would result in a modest decline in the aggregate price of housing, and Gravelle (1996) and Bruce and Holtz-Eakin (1997) argue that both the short run and the long run effects of a Flat Tax on housing prices would be fairly small. As noted by Capozza, Green, and Hendershott (1999), these results differ primarily because of differing assumptions regarding the elasticity of supply in the residential land market and thus the degree of flexibility in the supply of owner-occupied housing. The effects of declines in housing prices on the pattern of reform-induced intergenerational redistributions described below will of course increase in importance with the magnitudes of such declines.

In addition, much of the concern about the transition from an income tax to a consumption tax has focused on the potential one-time windfall loss imposed on the owners of existing capital assets (other than owner-occupied housing). Under the Flat Tax, the value of expensing for new investment expenditures is equivalent in present value to the exemption of the normal rate of return on such investments. However, in the absence of transition rules, firms are not allowed a deduction for the remaining basis of existing assets, although the returns on such assets are included in the tax
base. As a result, the rate of return on existing assets falls relative to the return on new investments; arbitrage across new and existing assets implies that the value of existing assets must fall proportionately by the rate of the tax. Thus, a one-time windfall loss is imposed on the owners of existing capital. Note that as long as prices do not change under the Flat Tax, lenders are insulated from this loss, since the nominal value of outstanding bonds is fixed; thus the entire reform-induced one-time windfall loss is borne by equity holders. However, existing owner-occupied housing is not subject to the one-time tax, since existing owner-occupied housing is effectively ignored for tax purposes under the Flat Tax, which is equivalent to the treatment of owner-occupied housing under the current income tax. That is, owner-occupiers receive no deductions for housing purchases, are not taxed on their imputed rents, and sales of existing housing are not included in the tax base.

As described in detail by Zodrow (1999), a wide variety of other factors associated with the implementation of a Flat Tax or a NRST would also affect existing capital owners, with most of them acting to offset the one-time windfall tax on existing assets. A partial list of these factors includes:

(1) the costs of adjusting the capital stock, which would allow the owners of capital to earn above-normal returns on both existing assets and new investments during the period of transition to the new equilibrium;

(2) a short run increase in the after-tax rate of interest, which would allow the owners of capital to earn a higher after-tax rate of return on existing assets and new investments;

(3) the elimination or reduction under a consumption tax of the expected tax on assets that were allowed accelerated depreciation allowances under current law; and
(4) the efficiency gains obtained from eliminating distortions of saving and investment decisions and reducing distortions of the labor-leisure choice, as well as from improvements in the allocation of capital across alternative assets and business sectors.

In addition, as stressed by Engen and Gale (1996) and Gentry and Hubbard (1997a), the differences between the taxation of capital income under the current income tax and under a consumption-based tax are smaller than assumed in many simulation studies, which implies that any one-time capital levies would be smaller than those obtained in such simulations.

Although it is impossible to predict with complete accuracy the net effect of all these factors on changes in asset values and on net redistributions across age and income groups, several studies have attempted to estimate these effects. For example, Altig, Auerbach, Kotlikoff, Smetters and Walliser (1997) - hereafter AAKSW - construct a dynamic life-cycle general equilibrium simulation model which includes 12 different income groups in each generation, and estimate reform-induced welfare changes across age groups and income classes. Their results for the implementation of the Flat Tax, with and without adjustment costs, are most relevant for this discussion. In particular, average asset values decline by nearly 10 percent with the implementation of the Flat Tax in the absence of adjustment costs; however, the introduction of adjustment costs reduces the decline in the average value of assets to only 3.6 percent. Without adjustment costs, very old generations are generally losers from reform, while middle-aged and younger generations are generally slight gainers or slight losers. In contrast, with adjustment costs, all elderly generations are gainers from reform, while middle-income young generations are net losers (due primarily to higher tax rates) and high and low-income young generations are net gainers (the former due primarily to lower tax rates and the latter due primarily to increased wages
attributable to greater capital intensity). Auerbach (1996) uses a dynamic life-cycle general equilibrium model with a single representative agent to predict the effects of implementing a variety of consumption-based tax reforms on capital allocation, efficiency, and growth. His results suggest that implementing a Flat Tax, similar to the one proposed by Hall and Rabushka, would lead to a 5.7 percent decline in asset values in the short-run and long-run in the case without adjustment costs. In the case with adjustment costs, asset values would increase initially by 3 percent and then decline to a long-run equilibrium value 3.6 percent less than the initial steady state values. The reform-induced effects on net welfare across age groups is similar to the pattern found by AAKSW. Zodrow and Williams (1998) also obtain a similar pattern of asset price changes and intergenerational redistributions with the implementation of a NRST in the presence of adjustment costs.

This study extends the basic framework that underlies existing dynamic life-cycle general equilibrium models by explicitly modelling the non-housing, rental housing and owner-occupied housing sectors in a manner similar to Goulder and Summers (1989) and Goulder (1989). The model allows for the determinants of the costs of owner-occupied and rental housing to be determined separately within the model and thus allows for the effects of implementing a consumption-based tax reform on the cost of owner-occupied or rental housing to be directly related to housing investment decisions and house values. Note that the model distinguishes between the effects of reform on the rental price of housing services and the value of housing capital.\(^8\)

An important limitation of the structure of this model is that the ratio of rental housing to owner-occupied housing services is exogenously determined. Although the issue is not currently addressed in this paper, an important aspect of the model

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\(^8\) Rental and owner-occupied housing services are assumed to be produced using the same production function: this implies that the rental price of a unit of housing services from either owner-occupied or rental housing is the same.
is its flexibility in modelling the treatment of owner-occupied housing under a NRST. In particular, the structure of the model allows new investments in owner-occupied housing to be included in the tax base explicitly.

The features of this model allow the effects of implementing a consumption-based tax reform on the value of assets in the non-housing, rental housing, and owner-occupied housing sectors to be identified separately. This is important since the reform-induced effects on assets values in the non-housing and rental housing sectors would not be the same as those in the owner-occupied housing sector. As noted above, under the Flat Tax owner-occupied housing would not be subject to the potential one-time windfall levy that affects the value of non-housing and rental housing assets since housing would essentially be ignored for tax purposes. Furthermore, the relatively large tax advantage of owner-occupied housing under the current income tax indicates that changes in asset values that result from a reallocation of capital across sectors would be the principal concern in the owner-occupied housing sector. In addition, the transition to a consumption-based tax would tend to eliminate the distortion between non-housing and rental housing assets as well, since non-housing assets are taxed at higher effective tax rates on average than rental housing under the current income tax.

The interaction of the reallocation of capital across industries is also important in determining the reform-induced effects on asset values. For example, the reform would induce a reallocation of capital from the owner-occupied housing sector to the rental housing and non-housing sectors as well as a reallocation of capital from the rental housing to the non-housing sector. Therefore, new investment in the non-housing sector would increase in the short-run as capital was reallocated from both the owner-occupied and rental housing sectors. The reform would also shift capital out of the owner-occupied sector and into the rental sector, however, this effect is
offset by the shift of capital from the rental to non-housing sector. This implies that the reform-induced effects on asset values due to the reallocation of capital across sectors would tend to be more important in the non-housing sector relative to the rental housing sector. Note that this factor tends to offset the one-time windfall loss on existing assets (other than owner-occupied housing). Dynamic general equilibrium models that ignore this reform-induced effect on asset values will tend to overstate the effects of reform on existing non-owner-occupied asset values. In the presence of adjustment costs, the importance of this offsetting reform-induced effect on values increases, since the level of adjustment costs depends on the difference between the ratio of investment to capital and the long-run steady state growth rate during the transition.

The results of the simulations presented below suggest that with a moderate level of adjustment costs ($\beta = 5$) the total real value of equity actually increases in the non-housing sector by 3.4 percent and falls by 12.3 percent in the rental housing sector in the year of reform. The value of owner-occupied housing falls by 9.5 percent in the year of reform, but returns to the value that would have occurred under the income tax steady state 9 years after the enactment of reform. The long-run equilibrium value of owner-occupied housing is 8.6 percent higher than it would have been under the income tax. The immediate decline in the value of owner-occupied housing that is predicted in this study is smaller than the estimates of Data Resources Incorporated (1995) and Capozza, Green, and Hendershott (1996), but similar to Gravelle's (1996) estimates of the decline in the value of owner-occupied housing, which range from 9 to 13 percent assuming that the elasticity of supply of owner-occupied housing is between 1 and 0.5.\footnote{These estimates are based on a fixed before-tax rate of return and thus are consistent with the short-run predictions reported in this study.} This study also finds that varying the level of adjustment costs
has virtually no overall effect on the initial decrease in the equity value of owner-occupied housing. However, the findings presented below suggest that the level of adjustment costs does tend to have a modest effect on the value of owner-occupied housing along the transition path to the new steady state. For the lowest level of adjustment costs considered in this study ($\beta = 1$) the total real value of equity in the non-housing and rental housing sector falls by 8.9 and 18.8 percent respectively, and higher levels of adjustment costs have a significant impact on the reform-induced effects on the value of the non-housing and rental housing firms. In fact, under the highest level of adjustment costs the value of the non-housing and rental housing firm increases initially by 19 percent, and the value of the rental housing firm decreases by 3.3 percent in the year of reform.

Net investment in the non-housing sector is 136 percent higher in the year of reform than it would have been in the initial income tax steady state, and remains at least 70 (50) percent higher in the first 19 (35) years after reform. In the long-run equilibrium, net investment is 41.3 percent higher than it would have been under the income tax. In the year of reform, net investment in the rental sector increases by 50 percent in comparison to its level in the initial income tax equilibrium. Net investment in the rental sector continues to increase in the first 7 years after reform, reaching a level 57 percent higher 7 years after reform than it would have been in the initial income tax steady state. In the long-run equilibrium, net investment in rental housing is 27 percent higher than it would have been under the income tax. Net investment in owner-occupied housing decreases initially by 48 percent, and then increases gradually for 18 years after the reform to a level that is 20 percent higher than under the initial income tax steady state. In the long-run equilibrium, net investment in owner-occupied housing is 8.6 percent higher than it was under the income tax. In the long run equilibrium, the share of non-housing capital increases
by 5.6 percent, the share of capital in the rental housing sector is unchanged, and the share of owner-occupied housing falls by 5.6 percent.

The reform-induced effects on intergenerational welfare shows that explicitly accounting for housing reduces the potential for a one-time windfall loss on non-housing and rental housing assets, this is especially true for higher levels of adjustment costs. As a result, older generations actually gain after the reform since their assets earn above normal returns and the after-tax interest rate increases in the short-run. Younger generations also benefit more in this case because the efficiency gains from reform are larger than in the typical single production sector models. Higher adjustment costs increase the number of young and middle-aged generations that suffer losses, since the efficiency effects in this case tend to occur more slowly over time.

The paper is organized as follows. The following two sections present the model used in this study and explain the calibration and data used in the initial income tax steady state. The next section presents the results from the simulations including estimates of the reform-induced asset price effects and the interaction of these effects on the intergenerational redistributions. The concluding section discusses directions for further research.

2.2 Model Structure

The model employed in this paper is similar to the Auerbach and Kotlikoff (1987) dynamic overlapping generations life-cycle model. It is extended to include other relevant features of the general equilibrium models constructed by Fullerton and Rogers (1993), Keuschnigg (1990), Boulder and Summers (1989), and Boulder (1989). The distinguishing feature of the model is the treatment of the housing market; the investment demands for owner-occupied and rental housing are endogenously determined, the tax advantage of owner-occupied housing relative to other assets under
the current income tax is taken into account, and adjustment costs in the housing and non-housing markets are included explicitly. Thus, this model allows for a more detailed description of the effects of implementing a consumption-based tax reform on housing and non-housing markets.

2.2.1 Housing and Non-Housing Firm Behavior

On the production side, the model allows for a detailed treatment of the industries that produce the non-housing consumption good, rental housing services, and owner-occupied housing services. Following Goulder and Summers (1989) and Goulder (1989), the owner-occupied and rental housing sectors are modelled as firms that produce housing services and are managed by owner-occupiers and landlords. The analysis assumes that firm managers act to maximize the value of the firm in a perfectly competitive environment with no uncertainty. The approach utilized in each industry is based on Tobin's "q" theory of investment, as extended to include adjustment costs by Hayashi (1982). It is most similar to the firm modeling approaches used by Goulder and Summers (1989), Goulder (1989), and Keuschnigg (1990).

The housing and non-housing industries each produce a single output using labor and capital according to a Cobb-Douglas production function. It is assumed that owner-occupied and rental housing services are produced with the same production function. Thus, the price of owner-occupied and rental housing services is the same. In all three sectors investment is financed with debt and equity such that firms maintain a constant debt-capital ratio. Following Goulder and Summers (1989), adjustment costs per unit of investment are given by

$$\Phi_s(I_s/K_s) = p_s(\frac{\beta}{2})(\frac{I_s}{K_s} - \mu)^2/(\frac{I_s}{K_s})$$, \hspace{1cm} (2.1)
where $\delta$ and $\mu$ are the adjustment cost parameters; higher values of $\delta$ and lower values of $\mu$ imply higher adjustment costs. The value of $\mu$ is set equal to the steady state ratio of gross investment to capital so that there are no adjustment costs in the steady state equilibrium. The price of the investment and consumption good, $p_s$, is the numeraire.

In each industry, firm managers choose the quantity of labor and the path of investment in order to maximize the value of the firm. The optimal quantity of labor depends only on market conditions within that period, while the optimal path of investment depends on current as well as future market conditions and the size of adjustment costs during the transition period following a tax reform.

**Owner-Occupied and Rental Housing Value**

The modelling of the owner-occupied and rental housing sectors is similar to Goulder and Summers (1989) and Goulder (1989). It extends upon their work by allowing the value of the housing services firm to be calculated separately for the owner-occupied and rental housing sectors. Earnings in the owner-occupied and rental housing sectors in period $s$, $EARN^O_s$ and $EARN^R_s$, are defined as the value of housing services less labor cost, real interest payments on total indebtedness, and property tax payments. The index for the owner-occupied and rental housing sectors is $l = O, R$. The earnings equation in each sector is represented as

$$EARN^l_s = p^l_s F(K^l_s, L^l_s) - w_s L^l_s - i_s B^l_s - cK^l_s,$$

where $p^l_s$ is the price of housing services, $F(K^l_s, L^l_s)$ is firm output, $K^l_s$ and $L^l_s$ are the amount of capital and labor used in production, $w_s$ is the wage rate, $i_s$ is the before-tax interest rate, $B^l_s$ is the total indebtedness, and $c$ is the property tax rate imposed on the value of housing capital. Following Goulder (1989), the labor in-
put in the owner-occupied housing sector represents labor used in the production or maintenance of owner-occupied housing services. Owner-occupiers do not pay taxes on their housing earnings, but they do receive income tax deductions for mortgage interest and property tax payments. The value of these deductions are calculated according to

$$TE_s^O = \vartheta \tau_{is} [-i_s B_s^O - cK_s^O]$$

(2.3)

where $\tau_{is}$ is the tax rate on individual income and $\vartheta$ is the fraction of homeowners that itemize. Assuming that adjustment costs are fully deductible, the total tax liability on the owners of rental housing in period $s$ is given by

$$TE_s^R = \tau_s^R [p_s^R F(K_s^R, L_s^R) - w_s L_s^R - f_2 i_s B_s^R - f_2 mK_s^R - \Phi_s^R I_s^R$$

$$- f_1 I_s^R + f_3 b^R I_s^R - f_4 \delta^R b^R K_s^R - cK_s^R - f_5 \delta^R K_s^R]$$

(2.4)

where the tax rate on rental housing income, $\tau_s^R$, is a weighted average of the corporate tax rate and the tax rate on landlord profits, $m$ is the percentage of maintenance expenditures to the rental capital stock, $I_s^R$ is gross investment in the rental sector in period $s$, $b^R$ equals the debt to capital ratio, $\delta^R$ is the economic rate of depreciation, $\delta^R$ is the accelerated rate of depreciation, and $K_s^R$ is the remaining basis of the rental housing capital stock for tax purposes. In the initial income tax steady state $f_1 = f_3 = f_4 = 0$ and $f_2 = f_5 = 1$. Under the Flat Tax, the “R-based” cash flow business tax is modelled by setting $f_1 = 1$ and $f_2 = f_3 = f_4 = f_5 = 0$. Note that the rental housing firm is treated as a strictly non-corporate firm, and thus is not allowed to issue stock, since only a small fraction of rental housing is corporate owned. However, the tax rate on rental housing does reflect the fact that a small fraction of the rental housing market is corporate owned. Also, note that landlords, but not owner-occupiers, are allowed a tax deduction for maintenance and repair expenditures.
Following Goulder and Summers (1989), it is assumed that neither owner-occupied or rental housing owners accumulate cash; this implies that cash inflows in period \( s \) must be equal to total disbursements in the owner-occupied and rental housing markets, or

\[
EARN_s^l + BN_s^l = S_s^l + I_s^l(1 + \Phi_s^l) + TE_s^l \tag{2.5}
\]

for \( l = O, R \). The net service flow to owner-occupants and landlords are given by \( S_s^O \) and \( S_s^R \) respectively. Solving equation (2.5) for the net service flow of owner-occupied or rental housing, \( S_s^l \), yields

\[
S_s^l = EARN_s^l + BN_s^l - I_s^l(1 + \Phi_s^l) - TE_s^l, \tag{2.6}
\]

where \( BN_s^l \) is new bonds issued in period \( s \). Note that a fraction of all marginal investments in the owner-occupied and rental housing sectors must be financed by reductions in the net service flow to owner-occupants and landlords, since all housing owners are assumed to maintain a constant debt to capital ratio and are not allowed to issue shares of stock.

Assuming individual level arbitrage implies that the after-tax nominal return to bonds, is equal to the net return of either owning and occupying or renting out a house. This condition is represented as

\[
(1 - \tau_{is})i_s = \frac{S_s^l + (1 - \tau_{gs}^l)(V_{s+1}^l - V_s^l)}{V_s^l}, \tag{2.7}
\]

where \( V_s^l \) is the value of the owner-occupied or rental firm, \( (V_{s+1}^l - V_s^l) \) represents the capital gain on owner-occupied or rental housing, and \( \tau_{gs}^l \) is the effective annual accrual tax rate on capital gains in the owner-occupied or rental housing sector.\(^{10}\)

Solving the difference equation in 2.7 subject to the following transversality condition

\[
\lim_{T \to \infty} V_{T+1}^l \prod_{u=t}^T \frac{1}{(1 + \theta_u^l)} = 0, \tag{2.8}
\]

\(^{10}\)This takes into account the benefits of deferral of capital gains.
where \( \theta_u^l = \frac{(1 - \tau_s^l)\tau_s^l}{(1 - \tau_s^l)} \), yields

\[
V_s^l = \sum_{u=s}^T \left\{ \prod_{v=s}^u \left[ \frac{1}{1 + \theta_v^l} \right] \frac{1}{(1 - \tau_s^l)} S_u^l \right\}.
\]  

(2.9)

That is, the value of owner-occupied or rental housing equals the present value of all future net service flows that owner-occupants or landlords receive. The transversality condition implies the value of the owner-occupied or rental housing firm is not allowed to become infinitely large in the future.

The model explicitly accounts for the value of tax savings from depreciation allowances on rental housing, including both those attributable to past investments (previous to period \( s \)) and those attributable to future investments made after period \( s \). Thus, it is necessary to distinguish between the present value of the remaining depreciation allowances on past investments in rental housing and the present value of depreciation allowances on future investments in rental housing, since only the present value of depreciation allowances on future investments will affect the investors optimal choice of investment. In the rental housing sector, the present value of depreciation allowances on all past and future investments is given by

\[
\sum_{u=s}^\infty \left[ \prod_{v=s}^j \frac{1}{1 + \theta_v^R} \right] f_5^R \gamma_u^R \delta^R K_t^R = X_t^R + \sum_{j=s}^\infty [I_j^R Z_{j+1}^R \prod_{v=s}^j \frac{1}{1 + \theta_v^R}]  
\]  

(2.10)

where

\[
Z_j^R = \sum_{u=j}^\infty [f_5^R \gamma_u^R \delta^R (1 - \delta^R)^{u-j} \prod_{v=j}^u \frac{1}{1 + \theta_v^R}] 
\]

and \( X_t^R = Z_t^R K_t^R \). The first term — \( X_t^R \) — is the value of the remaining depreciation deductions on "old" rental housing capital existing at the time of reform and the second term is the value of depreciation deductions on investments made after the enactment of reform.

Given this result, the maximization problem facing firm managers in the owner-occupied and rental housing sector can be written as
\[ V_s^t = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^l)} \Gamma_u^l + X_t^l, \]  

(2.11)

where

\[ \Gamma_u^O = \Omega_u^O[p_s^O F(K_s^O, L_s^O) - w_s L_s^O] - K_s^O[\Omega_u^O (1 - \tau_{is}) i_s b^O + \Omega_u^O (1 - \tau_{is}) c b^O - \Omega_u^O b \delta^O] 
+ I_s^O [\Omega_u^O b^O - \Omega_u^O (1 + \Phi_s^O)]. \]  

(2.12)

and

\[ \Gamma_u^R = \Omega_u^R (1 - \tau_u^R) [p_u^R F(K_u^R, L_u^R) - w_u L_u^R] \]
\[ - K_u^R [\Omega_u^R (1 - \tau_u^R) c + b^R \Omega_u^R \delta^R - \Omega_u^R (1 - \tau_u^R) f_2 i_u b^R - \Omega_u^R f_2 m - f_4 \tau_u^R \Omega_u^R \delta^R] \]
\[ + I_u^R [\Omega_u^R - b^R \Omega_u^R - \tau_u^R \Omega_u^R (f_1 - f_3 b^R) + \Omega_u^R (1 - \tau_u^R) \Phi_u^R - Z_u^R]. \]

(2.13)

Owner-occupants and landlords maximize (2.11) subject to the following constraints

\[ K_{s+1}^l = I_s^l + (1 - \delta^l) K_s^l \]  

(2.14)

and

\[ \lim_{T \to \infty} K_T^l \geq 0. \]  

(2.15)

The Lagrangian for the owner-occupied and rental housing market is

\[ \mathcal{L}^l = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^l)} \{ \Gamma_u^l + X_t^l + q_{u+1}^l [I_u^l + (1 - \delta^l) K_u^l - K_{u+1}^l] \}. \]  

(2.16)

where

\[ q_{u+1}^l = [\prod_{v=s}^{u} \frac{1}{(1 + \theta_v^l)}] q_{u+1}. \]  

(2.17)

The necessary conditions for the optimal quantity of labor and optimal path of investment are derived below for the owner-occupied and rental housing sector.
Owner-Occupied Housing  It is assumed that homeowners choose the amount of labor and the optimal path of investment in order to maximize the present value of all future net service flows from owner-occupied housing. The first order condition with respect to the optimal quantity of labor in the owner-occupied housing market implies

\[ w_s = p_s^O F_s^O. \]  \hspace{1cm} (2.18)

That is, the value of the marginal product of labor must equal the wage rate.

Second, the necessary condition with respect to the optimal path of the investment in the owner-occupied housing market in each period is

\[ q_{s+1}^O = \Omega_{u}^O - b^O \Omega_{u}^O + \Omega_{s}^O [ \Phi_{s}^O + (\frac{I_{s}^O}{K_{s}^O}) \Phi_{s}^O]. \]

Again, this describes the variable commonly known as Tobin's "q". It demonstrates that the shadow price of additional owner-occupied housing capital (\(q_{s+1}^O\)) must equal the after-tax marginal cost of owner-occupied housing capital (the right hand side). The first term implies that the shadow price is simply one in the absence of debt and taxes, since the investment good is the numeraire. The second term reflects the financing of a fraction \(b\) of the cost of the investment with debt. The last term reflects the costs of installing new capital goods.

In order to express investment demand as a function of the value of the firm, \(V_s^O\), rather than \(q_{s+1}^O\), the relationship between marginal \(q\) and average \(q\) (denoted as \(Q\)) as shown by Hayashi (1982) is

\[ q_s^O = \frac{[V_s^O - X_s^O]}{K_s^O}, \quad Q_s^O = \frac{V_s^O}{K_s^O}. \]  \hspace{1cm} (2.19)

Thus, the investment demand function can be written as

\[ \frac{I_s^O}{K_s^O} = \frac{[\frac{V_s^O - X_s^O}{K_{s+1}^O} - \Omega_{s}^O + \Omega_{s}^O]}{\Omega_{s}^O \beta} + \mu. \]  \hspace{1cm} (2.20)
Rental Housing Market  It is assumed that landlords choose the amount of labor and the optimal path of investment in order to maximize the present value of all future net service flows from rental housing. The first order condition with respect to the optimal quantity of labor in the rental housing market implies

\[ w_s = p_s^R F_L^R. \]  

(2.21)

That is, the value of the marginal product of labor must equal the wage rate.

Second, the necessary condition with respect to the optimal path of the investment in the rental housing sector in each period is

\[ q_{s+1}^R = \Omega_u^R - b^R \Omega_u^R - \tau^R \Omega_u^R (f_1 - f_3 b^R) - Z_{s+1}^R + (1 - \tau_s^R) \Omega_s^R [\Phi_s^R + \frac{\Phi_s^R}{K_s^R}]. \]  

(2.22)

This describes the variable commonly known as Tobin's "q". It demonstrates that the shadow price of additional rental housing capital \(q_{s+1}^R\) must equal the after-tax marginal cost of rental housing capital (the right hand side). Since the investment good is the numeraire, the shadow price is simply one in the absence of debt and taxes. The second term reflects the financing of a fraction \(b\) of the cost of the investment with debt. The third term reflects the value of immediately expensing new investment under a "R-based" cash flow tax. The forth term reflects the reduction in the shadow price of new capital goods due to tax deductions for depreciation. The last term reflects the costs of installing new capital goods with immediate expensing of such adjustment costs.

In order to express investment demand as a function of the value of the firm, \(V_s^R\), rather than \(q_{s+1}^R\), the relationship between marginal \(q\) and average \(q\) (denoted as \(Q\)) as shown by Hayashi (1982) is

\[ q_s^R = \frac{[V_s^R - X_s^R]}{K_s^R}, \quad Q_s^R = \frac{V_s^R}{K_s^R}. \]  

(2.23)
Thus, the investment demand function can be written as

\[
\frac{I^R_s}{K^R_s} = \left[ \frac{\varphi^R_{s-1} - X^R_{s-1}}{K^R_{s-1}} - \Omega^R_u + bR\Omega^R_u + \tau^R\Omega^R_u(f_1 - f_3bR) + Z^R_{s+1}}{(1 - \tau^R_s)\Omega^R_s\rho_s\beta} \right] + \mu.
\] (2.24)

**Non-Housing Firm Value**

The non-housing firm is a composite of all non-housing corporate and non-corporate firms. Total earnings in any period are defined as the value of output less labor costs and real interest payments on the stock of outstanding debt, \(B_s\), or

\[
EARN_s = p_sF(K_s, Le_s) - w_sLe_s - i_sB_s,
\] (2.25)

where \(w_s\) is the real wage and \(i_s\) is the real interest rate in period \(s\). Assuming that adjustment costs are fully deductible and a corporate business tax rate of \(\tau_{bs}\) in period \(s\), the firm’s total tax liability is given by

\[
TE_s = \tau_{bs}[p_sF(K_s, Le_s) - w_sL_s - \Phi_sI_s - f_1I_s - f_2i_sB_s + f_3bI_s - f_4\delta K_s - f_5\delta^\tau K^T_s] \tag{2.26}
\]

where \(\delta^\tau\) is the accelerated rate of depreciation and \(K^T_s\) is the remaining basis of the capital stock for tax purposes. Assuming no cash flow on the part of the firm, cash inflows in period \(s\) must equal total disbursements, or

\[
EARN_s + BN_s + VN_s = DIV_s + I_s(1 + \Phi_s) + TE_s, \tag{2.27}
\]

where \(DIV_s\) is dividends paid in period \(s\) and \(VN_s\) is new share issues in period \(s\). It is assumed that the firm pays out a constant fraction of earnings after taxes and depreciation as dividends in each period. This implies that new investments are financed with debt, retained earnings, and new share issues if the amount of retained earnings are insufficient to finance the desired level of investment.

Following Goulden and Summers (1989), the individual level arbitrage condition requires that the after-tax nominal return to bonds be equal to the net of tax return
received by the owners of the firm. This can be written as

\[(1 - \tau_{is})i_s = \frac{(1 - \tau_{ds})DIV_s + (1 - \tau_{gs})(V_{s+1} - V_s - VN_s)}{V_s}. \tag{2.28}\]

where \(\tau_{is}\) is the tax rate on the individual’s interest income, \(\tau_{ds}\) is the tax rate on dividends, \(\tau_{gs}\) is the effective annual accrual tax rate on capital gains, \(V_s\) is the value of the firm, and \((V_{s+1} - V_s - VN_s)\) is the capital gain on outstanding shares. The treatment of equity finance follows the “traditional” view of the effects of dividend taxation.

Solving the difference equation in 2.28 subject to the following transversality condition

\[\lim_{T \to \infty} V_{T+1} \prod_{u=t}^{T} \frac{1}{(1 + \theta_u)} = 0, \tag{2.29}\]

where \(\theta_s = \frac{(1 - \tau_{is})i_s}{(1 - \tau_{gs})}\), yields

\[V_s = \sum_{u=s}^{\infty} \frac{\left[\frac{(1 - \tau_{du})}{(1 - \tau_{gu})}\right] DIV_u - VN_u}{\prod_{v=s}^{u} (1 + \theta_v)}, \tag{2.30}\]

for \(s \geq t\). That is, \(V_s\) equals the present value of all future net distributions to shareholders. The transversality condition implies that the value of the firm is finite in the future. (Note that under the assumption of individual arbitrage, the firm’s discount rate \(\theta_s = \frac{(1 - \tau_{is})i_s}{(1 - \tau_{gs})}\) is increased to reflect the fact that dividend distributions avoid the capital gains tax that arises when the firm retains earnings. Note also that this is a perfect foresight condition in that the firm must predict all future values of the interest rate (in \(\theta_s\)) and the tax rate variables.)

Solving the first order conditions for the firm in a similar manner to those in the housing services industry yields

\[q_{s+1} = 1 - b - f_5Z_{s+1} - \Omega_u \tau_{bs}(f_1 - f_3b) + (1 - \tau_{bs} \Omega_s)[\Phi_s + \frac{I_s}{K_s} \Phi_s']. \tag{2.31}\]
This is the variable commonly known as Tobin's "q" — the ratio of the market value of capital to its replacement cost. It demonstrates that the shadow price of additional capital goods \( q_{s+1} \) must equal the after-tax marginal cost of capital goods (the right hand side). The shadow price is simply one in the absence of debt and taxes, since the investment good is the numeraire. The second term reflects the financing of a fraction, \( b \), of the cost of the investment with debt. The third term reflects the immediate expensing of new investments under the "R-based" and the "R+F-based" cash flow taxes. The fourth term reflects the reduction in the shadow price of new capital goods due to tax deductions for depreciation. The last term reflects the costs of installing new capital goods with immediate expensing of such adjustment costs. This equation can be solved to give the investment demand function for the firm as

\[
\frac{I_s}{K_s} = \frac{(q_{s+1} - 1 + b + f_5 Z_{s+1} + \Omega_u \tau_{bs} (f_1 - f_3 b))}{p_s \beta (1 - \tau_{bs} \Omega_s)} + \mu \tag{2.32}
\]

### 2.2.2 Individual Behavior

Individual behavior is modeled using a dynamic overlapping generations framework that consists of fifty-five cohorts. Each generation is represented by a single individual, who has an economic life span of fifty-five years, works for the first forty-five of those years, and is retired for the last ten. Individual tastes are identical so that differences in behavior across generations are due solely to differences in lifetime budget constraints.

An individual age \( a \) accumulates assets, \( A_i(a) \), from the time of "economic birth" that are eventually used to fund (1) consumption over the lifetime, including that during the retirement period, and (2) the making of bequests. Note that assets include inheritances received. The model includes a relatively primitive "target model" of bequests, as bequests are assumed to be completely insensitive to changes in economic
conditions, including changes in income. In the model, inheritances are received at age 25 and bequests are given at the time of death (age 55).

An individual age \(a\) chooses total composite consumption \(\tilde{C}_s(a)\) and leisure \(E_s(a)\) to maximize rest-of-life utility

\[
LU_s(a) = \frac{1}{(1 - \frac{1}{\sigma_1})} \sum_{s=t}^{t+54-a} \frac{U_s(a)^{(1-\frac{1}{\sigma_1})}}{(1+\rho)^{s-t}},
\]

where \(\sigma_1\) is the intertemporal elasticity of substitution, \(\rho\) is the pure rate of time preference, \(U_s(a)\) is utility of an individual age \(a\) in each period \(s\). For notational simplicity, the age index is suppressed in the following description of the model. The utility in each period \(s\) is assumed to be a CES function of total consumption \(\tilde{C}_s\) and leisure \(E_s\)

\[
U_s = \left[\alpha^{\frac{1}{\sigma_2}} \tilde{C}_s^{\frac{\sigma_2-1}{\sigma_2}} + \alpha^{\frac{1}{\sigma_2}} E_s^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}},
\]

where \(\sigma_2\) is the intertemporal elasticity of substitution between total consumption and leisure between periods, \(\alpha_E\) and \(\alpha_C\) measure the relative intensities of household preferences for consumption and leisure. Note that leisure \((E_s)\) is defined as \(E_s = H_T - L_s\), where \(H_T\) is the total number of hours available in period \(s\) for either labor supply \((L_s)\) or leisure.

The composite good is model as a CES function of non-housing goods and housing services,

\[
\tilde{C}_s = \left[\alpha^{\frac{1}{\sigma_3}} C_s^{\frac{\sigma_3-1}{\sigma_3}} + \alpha^{\frac{1}{\sigma_3}} H_s^{\frac{\sigma_3-1}{\sigma_3}}\right]^{\frac{\sigma_3}{\sigma_3-1}},
\]

where \(C_s\) is total consumption of non-housing goods in period \(s\), \(H_s\) is total consumption of housing services in period \(s\), \(\sigma_3\) is the elasticity of substitution between non-housing goods and housing services, and \(\alpha_G\) and \(\alpha_H\) measure the relative intensities of household preferences for non-housing goods and housing services.

Total consumption of non-housing goods is modelled as a composite good of the quantities of the taxed and non-taxed (non-housing) goods in excess of the minimum
required purchases using a CES function,

\[ C_s = \left[ \alpha_{C1}^\frac{1}{\sigma4} (C_{1s} - b_{1s}) \right]^{\sigma4-1}_{\sigma4} + \alpha_{C2}^\frac{1}{\sigma4} (C_{2s} - b_{2s}) \right]^{\sigma4-1}_{\sigma4-1}, \tag{2.36} \]

where \( C_{is} \) is the consumption of good \( i \) in period \( s \), \( b_{is} \) is the minimum required purchase of good \( i \) in period \( s \), \( \sigma4 \) is the elasticity of substitution between “above-minimum” quantities of the taxed and non-taxed goods, and \( \alpha_{C1} \) and \( \alpha_{C2} \) measure the relative intensities of household preferences for taxed and non-taxed non-housing goods.

Total consumption of housing services is modelled as a composite good of rental housing and owner-occupied housing services using a CES function

\[ H_s = \left[ \alpha_O^\frac{1}{\sigma5} (OH_s - b_{Os}) \right]^{\sigma5-1}_{\sigma5} + \alpha_R^\frac{1}{\sigma5} (RH_s - b_{Rs}) \right]^{\sigma5-1}_{\sigma5-1}, \tag{2.37} \]

where \( OH_s \) is the quantity of owner-occupied housing services consumed in period \( s \), \( RH_s \) is the quantity of rental housing services consumed in period \( s \), \( b_{Os} \) and \( b_{Rs} \) represent the minimum purchase requirements of owner-occupied and rental housing, \( \sigma5 \) is the elasticity of substitution between owner-occupied housing and rental housing, and \( \alpha_O \) and \( \alpha_R \) measure the relative intensities of household preferences for owner-occupied and rental housing. Note that the price owner-occupied and rental housing services is equal, since owner-occupied and rental services are produced using the same production function.

The consumer chooses \( \tilde{C}_s \) and \( E_s \) in each period to maximize (2.33) subject to the lifetime budget constraint (with present values taken at the end of period \( s \)),

\[ TW_s = \sum_{s=t}^{t+54-a} \frac{F_s \tilde{C}_s}{\Pi_{u=t+1}^{s+1}[1 + r_u]}, \tag{2.38} \]

where \( F_s \) is the price of the composite consumption good and \( TW_s \) is the total value of lifetime wealth. The solution to the consumer’s maximization problem yields the following expression for the optimal time path of consumption
\[ \bar{C}_s = \left[ \frac{1 + r_s}{1 + \rho} \right] F_{s-1} \left[ \frac{FU_s}{FU_{s-1}} \right]^{(\alpha_1 - \sigma_1) \left[ \frac{\sigma_2 - \sigma_1}{1 - \sigma_1} \right]} \bar{C}_{s-1} \]  

where

\[ FU_s = \alpha + \alpha E \left[ \frac{w_s(1 - \tau_{ws} - \tau_{ss})}{F_s} \right]^{(1 - \sigma_2)}. \]

Repeated substitution for \( \bar{C}_s \) in (2.38) using equation (2.39) yields

\[ \bar{C}_t = \frac{TW_t}{\Psi_t} \]  

where

\[ \Psi_t = \left\{ \sum_{s=t}^{t+54-a} \frac{(1 + \rho)^{(s-t)\alpha} (F_{s-2})^\sigma (F_s)^{1-\sigma} \left( \prod_{u=t+1}^{s} \left[ \frac{FU_u}{FU_{u-1}} \right]^{(\sigma_2 - \sigma_1)} \right) \right\}. \]  

and

\[ TW_s = A_s (1 + r_s) + TDA_s (1 + i_s) - \frac{BQ_{t+54-a}}{\prod_{u=t+1}^{t+54-a} (1 + r_u)} \]

\[ + \sum_{s=t}^{t+54-a} \frac{\left[ w_s (H_T - E_s) - SD_s \right] (1 - \tau_{wags} - \tau_{ss})}{\prod_{u=t+1}^{s} (1 + r_u)} \]

\[ + \sum_{s=t}^{t+54-a} \frac{SSB_s + WD_s (1 - \tau_{ws}) + TR_s + LSR_s}{\prod_{u=t+1}^{s} (1 + r_u)} \]

\[ - \sum_{s=t}^{t+54-a} \frac{p_s (1 + \tau_{cs}) b_{1s} + p_s b_{2s} + pO_s b_{Os} + pRs b_{Rs}}{\prod_{u=t+1}^{s} (1 + r_u)}. \]

where the value of taxable assets at time \( s \) is represented as \( A_s \) (which includes the inheritance received), \( TDA_s \) is the value of tax-deferred assets at time \( s \), \( BQ_{t+55-a} \) is the exogenous bequest, \( w_s \) is the wage rate in period \( s \), \( l_s \) is labor supplied in period \( s \), \( SD_s \) is the mandatory tax-deferred savings deposit in period \( s \), which is equal to \( SD_s \) during working years and is zero during the retirement years, \( WD_s \) is the withdrawal during each of the retirement years, \( t_{wags} \) is the average tax rate on wage income in
period $s$, $t_{ss}$ is the social security tax on wage income in period $s$, $SSB_s$ represents social security benefits received (if any) in period $s$, $TR_s$ is transfers received in period $s$ (which are modeled as uniformly distributed across all 55 generations and growing at the rate of technological growth in the economy ($g$)), and $p_s(1 + c_s)b_{1s} + p_s^2b_{2s} + p_0s^2b_{0s} + p_{R3}s^2b_{R3}$ is the cost of meeting the total minimum purchase requirements in period $s$. Note that, following Auerbach and Kotlikoff (1987), the wage rate varies over the life cycle in a "hump-backed" fashion to reflect changes in productivity over the life cycle.

**Tax Deferred Assets**

The availability of tax deferred assets under the income tax is explicitly taken into account in the model, using the following formulation — which is fairly simple but nevertheless rich enough to capture the most important transition effects of the reform. In each period of an individual's working life, a fixed amount $SD_s$ is assumed to be deposited in a tax deferred account. Thus, for an individual age $a$ in period $t$ (when reform is enacted) that is in the labor force, the value of tax deferred assets at time $t$ is

$$TDA_t(a) = \sum_{v=t-a}^{t-1} SD_v \prod_{u=v+1}^{t-1} (1 + i_u),$$

where $SD_v$ represents the amount of wage earnings that are deposited in the tax deferred account in period $v$ and thus earn interest at the tax-free rate of return $i_u$ until they are withdrawn from the account. Upon reaching the retirement age of 45, individuals begin making withdrawals from their tax deferred accounts, with assets remaining in the account continuing to accumulate interest earnings at the before-tax rate of return. Individuals are assumed to make ten equal withdrawals at the end of each period in the retirement years. Generalizing to the case of an individual age $a$ (ranging from negative values for unborn generations to an individual age 54) at time
When reform is enacted yields

\[ WD = \frac{TDA_{t^*+45-a} \prod_{u=t^*+45-a}^{t^*+54-a} (1 + i_u)}{1 + \sum_{t^*+46-a}^{t^*+54-a} \prod_{u=t^*+46-a}^{t^*+54-a} (1 + i_u)} \]

where

\[ TDA_{t^*+45-a} = \sum_{v=t^*+44-a}^{t^*+44-a} SD_v \prod_{u=v+1}^{t^*+44-a} (1 + i_u). \]

Note that SDs is assumed to grow at the rate of productivity growth, so that the amount of savings deposited in the tax deferred account increases at the growth rate of the economy.

**Social Security System**

The social security benefit, SSBr, is received by those who reach an economic age of 45 in year s until death at age 54. The benefit payment depends on average wage earnings over the 45 working years and an assumed rate of replacement, RSS. Average wage earnings over the first 45 working years of the individuals life equal

\[ AVE_s = \frac{\sum_{a=1}^{45} w_{s-45+a} (Le_{s-45+a})}{45} \]

where \( w_{s-45+a} \) represents the wage in year \( s+45-a \) and \( Le_{s-45+a} \) represents effective labor in year \( s - 45 + a \) of individuals that reached age 45 in year \( s \). The benefit received is thus related to average earnings by

\[ SSBr = RSS(AVE_s). \]

Assuming that social security is self financing (in each period) implies that annual social security taxes must equal annual benefit payments:

\[ \frac{\tau_{bs} \sum_{a=1}^{45} w_{s,a} (Le_{s,a})}{(1 + n)^{(a-1)}} = \frac{SSB_{s-u}}{(1 + n)^{(45+u)}}. \]
2.2.3 The Government

Federal and State Government Expenditures

The federal government collects taxes to finance government consumption ($G_s$) and transfers ($TR_s$). The level of government spending in the model is exogenously determined such that real government spending increases in each period by the steady state rate of growth in the economy. Government spending after reform is assumed to be fixed at the same real level of expenditure that would have occurred in the initial income tax steady state. These expenditures by the government are modeled as purchases of output, which do not enter the individual utility function. The government allocates a fixed share of its total consumption expenditures to housing services and the consumption good.

The government balances its budget on an annual basis. Note that although the use of an annual balanced budget constraint is common in the literature, it imposes strong restrictions on government behavior. In particular, it precludes the use of debt policy to ameliorate the transitional effects of moving to a consumption tax (McGee 1989) or to reduce the differences in the steady state welfare effects of alternative forms of consumption taxes (Seidman 1990).

The state government collects taxes to finance state government consumption. The assumptions regarding state government expenditures are identical to those governing expenditures at the federal level.

The Federal and State Tax Systems

The federal income tax is modeled as a progressive tax on wage income net of the tax deferred saving deposit, $SD_s$, coupled with flat rate taxes on capital income. The
total tax burden on taxable wage income is defined as

\[ \psi + \left( \frac{\chi}{2} \right)(w_s L_s - SD_s)(w_s L_s - SD_s), \]

where \( \chi > 0 \) and \((w_s L_s - SD_s)\) is the individual's wage tax base in period \( s \). This implies an average tax rate on labor income of

\[ \tau_{\text{avg}} = \psi + \left( \frac{\chi}{2} \right)(w_s L_s - SD_s) \]

and a marginal tax rate of

\[ \tau_{w_s} = \psi + \chi(w_s L_s - SD_s). \]

If \( \chi = 0 \), then the income tax system is proportional.

Capital income in the corporate sector is assumed to be taxed at flat rates \( \tau_{ds} \) on dividends, \( \tau_{is} \) on interest income, and \( \tau_{gs} \) on capital gains. The tax rate on capital gains is an effective annual accrual rate, taking into account the benefits of tax deferral until gains are realized and tax exemption of gains transferred at death. Rental income in the rental housing sector is taxed at the flat rate \( \tau_{s}^{R} \). Capital gains in the rental and owner-occupied sector are assumed to be untaxed.

Under the state tax system wage income is taxed at the flat rate \( \tau_{w}^{st} \), capital income in the non-housing and rental housing sector is taxed at the flat rate business tax \( \tau_{b}^{st} \), retail consumption is taxed at the state consumption tax rate \( \tau_{c}^{st} \), and the property tax rate \( \tau_{h}^{pt} \).

### 2.2.4 Market Equilibrium

The model structure requires that two types of equilibrium conditions must hold for the market to be in equilibrium. The first type of equilibrium condition is the intratemporal condition that requires quantity demanded and quantity supplied to be
equal in each market given individual's expectations. The second type of equilibrium condition is the intertemporal condition requiring individual's expectations of future economic conditions to be fulfilled.

The intratemporal conditions require that each market must clear in every period. In the labor market, this implies that labor demand must equal labor supply at the prevailing market wage rate in every period. Asset market clearing requires that the total financial wealth of all individuals (summing across all cohorts alive in period $s$) equal the sum of the total market value of equity plus the value of all outstanding debt. The market clearing condition in the non-housing consumption goods market requires that total production equals the sum of consumption, investment in the non-housing and housing capital, the cost of adjusting the capital stock in the housing and non-housing market, and federal and state government consumption of the non-housing good. In the owner-occupied and rental housing services industry total production must equal consumption plus government consumption of housing services.

The second type of equilibrium condition is the intertemporal condition that results from the assumption that individuals and firms have perfect foresight. The farsighted nature of individual expectations imply that current decisions are based on current as well as future market equilibria; that is, current and future equilibria are intertemporally connected through the forward looking behavior of consumers. An intertemporal equilibrium is reached when consumer expectations of all current and future market equilibria are correct. To solve for the intertemporal equilibrium the model uses the method devised by Fair and Taylor (1983). This method uses an initial guess to calculate actual values for all forward looking variables, and then updates the guess using a linear combination of the initial guess and the actual value. This procedure is iterated until the model converges to a fixed point (i.e. a point where the difference between the revised guess and actual values is sufficiently small).
2.3 Parameter Values and Calibration

The parameter values used in the model are chosen so that in the year of reform the initial income tax equilibrium closely resembles the prevailing features of the U.S. economy in 1997. In addition, a number of other parameter values used in the model are chosen to be consistent with either empirical estimates or other studies that are similar in nature to this study, especially the work by AAKSW, Auerbach and Kotlikoff (1987), Auerbach (1996), Fullerton and Rogers (1993).

It is assumed that the economy is growing along a balanced growth path in the initial income tax equilibrium. The steady state rate of growth in the model’s initial equilibrium is determined by an exogenously specified rate of population and technological growth. The value of each of these parameters is set equal to one percent, implying that the steady state growth rate of the economy is 2.01 percent. This is consistent with estimates of the average U.S. growth rate from the Economic Report of the President 1999 (ERP).

The intertemporal nature of the model makes it necessary to introduce several key parameters that can play an important role in the determination of the responses of savings and consumption to changes in the after-tax rate of return caused by tax reform. The first of these is the intertemporal elasticity of substitution ($\sigma_1$), which governs the willingness of consumers to substitute consumption across periods in response to changes in the relative price of consumption across periods. In particular, the larger the value of this elasticity, the larger the savings response to a given change in the rate of return. The empirical evidence offers a wide range of possible values of the intertemporal elasticity of substitution. However, other studies of this nature, such as those by Auerbach and Kotlikoff (1987), Auerbach (1996), and Fullerton and Rogers (1993), choose values that range from 0.25 to 0.5. The analysis in this paper assumes $\sigma_1 = 0.26$. 
The second key parameter in the model is the adjustment cost parameter, \( \beta \), which is positively related to the level of adjustment costs. A higher value of \( \beta \) reduces the size of the firm's desired short run adjustments to the capital stock in each period, and thus, tends to lengthen the period of transition to the new steady state. As noted above, this is particularly important when analyzing the effect of a consumption-based tax reform on the value non-housing capital, since higher adjustment costs would reduce the short run movement of capital out of the owner-occupied housing sector.\(^{11}\) In the non-housing sector, higher adjustment costs allow the owners of existing non-housing assets to earn inframarginal returns over a longer period as the economy moves to the new steady state growth path. The base case assumes a moderate adjustment costs case of \( \beta = 5 \) in the non-housing and housing sectors; or equivalently, that a one percent increase in the investment to capital ratio would cause a 5 percent increase in the cost of investment goods to each firm. The model is also solved for \( \beta = 1, \beta = 10 \) and \( \beta = 20 \) in order to determine the sensitivity of the results with respect to this adjustment cost parameter.\(^{12}\)

Several other parameters are drawn from the literature or calculated based on data from ERP (1999). Following Fullerton and Rogers (1993), the rate of time preference, \( \rho \), is set equal to 0.005.\(^{13}\) The capital share parameters in the housing and non-housing sectors is set so that the share of capital income, net of depreciation, in the production of housing and non-housing goods is equal to 0.975 and 0.24 respectively.

\(^{11}\)The convex nature of the adjustment cost function implies that an increase or decrease in the value investment to capital ratio will lead to a higher level of adjustment costs.

\(^{12}\)Empirical estimates of the size of adjustment costs in the investment literature cover a wide range. For instance, Summers (1981) estimates that \( \beta = 32 \), while a more recent study by Cummins, Hasse, and Hubbard (1994) suggest that \( \beta = 2 \) is a more reasonable estimate of the adjustment cost parameter.

\(^{13}\)The rate of time preference is indicative of the subjective view of the discounted value of future consumption. A larger value of \( \rho \) implies that individuals tend to discount the future to a relatively greater extent.
The economic rate of depreciation of non-housing capital, \( \delta = 0.0601 \), is calculated from the estimates reported in Fullerton and Rogers (1993). The economic rate of depreciation of owner-occupied and rental housing capital is assumed to be \( \delta^h = 0.018 \). Depreciation allowances are set so that existing capital carries a discount below market value of approximately 4 percent in the non-housing sector and 3 percent in the rental housing sector.\(^{14}\) Following Auerbach (1996) and Fullerton and Rogers (1993), the debt-to-capital ratio in initial income tax equilibrium is set equal to 0.35 in the owner-occupied housing, rental housing, and the non-housing sectors. The dividend payout ratio is set at 0.68 to reflect actual corporate retained earnings of 48 percent out of after-tax corporate profits and full distribution of dividends from the non-housing non-corporate sector. Household savings in tax deferred assets is set so that the ratio of tax deferred assets to total household asset holdings is approximately 20 percent.

The inclusion of bequests in the economy is also an important feature of the model. Wealth accumulation in a model without bequests would have to come strictly from the life-cycle savings of generations that are currently alive, this would lead to unrealistically high savings early in the life-cycle and equally unrealistic dissaving later on. Including a bequest motive of some type allows a large fraction of wealth accumulation to be generated by intergenerational transfers, and thus, leads to a more realistic pattern of life-cycle savings in the model. The analysis in this paper includes a simple “target bequest” motive that accounts for roughly 50 percent of total savings.

\(^{14}\)Auerbach (1996) estimates that existing corporate (non-corporate) capital carries a discount of approximately 8 (6) percent below market value due to accelerated depreciation allowances. Gravelle (1998) estimates that the average discount on capital has declined to 4 percent, since the length of asset lives for depreciation purposes, especially for structures, were increased in 1986 and 1993. I assume that capital in the rental sector carries a discount that is one percent smaller than in the non-housing sector, since the production of rental housing services is relatively structure intensive.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Consumers</strong></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Rate of time preference</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Intertemporal elasticity of substitution</td>
<td>0.26</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Intratemporal elasticity of substitution</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>Elasticity of substition for non-housing and housing</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>Elasticity of substitution for taxed and untaxed goods</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_5$</td>
<td>Elasticity of substitution for rental and owner housing</td>
<td>0.8</td>
</tr>
<tr>
<td>$\alpha_E$</td>
<td>Utility weight on leisure</td>
<td>0.46</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>Utility weight on composite consumption</td>
<td>0.54</td>
</tr>
<tr>
<td>$\alpha_G$</td>
<td>Utility weight on composite non-housing consumption</td>
<td>0.75</td>
</tr>
<tr>
<td>$\alpha_C_1$</td>
<td>Utility weight on the taxed non-housing good</td>
<td>0.56</td>
</tr>
<tr>
<td>$\alpha_C_2$</td>
<td>Utility weight on the untaxed non-housing good</td>
<td>0.44</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>Utility weight on composite housing consumption</td>
<td>0.25</td>
</tr>
<tr>
<td>$\alpha_O$</td>
<td>Utility weight on owner-occupied housing</td>
<td>0.76</td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>Utility weight on rental housing</td>
<td>0.24</td>
</tr>
<tr>
<td>$n$</td>
<td>Population growth rate</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td><strong>Technology</strong></td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>Technology growth rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Capital share in non-housing production</td>
<td>0.24</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Capital share in housing production</td>
<td>0.975</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Adjustment cost parameter</td>
<td>5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Non-housing adjustment cost parameter</td>
<td>0.0802</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>Housing adjustment cost parameter</td>
<td>0.0384</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Dividend payout ratio in the non-housing sector</td>
<td>0.68</td>
</tr>
<tr>
<td>$b$</td>
<td>Debt-to-capital ratio (in all three sectors)</td>
<td>0.35</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Economic depreciation in the non-housing sector</td>
<td>0.0601</td>
</tr>
<tr>
<td>$\delta^h$</td>
<td>Economic depreciation in the housing sector</td>
<td>0.0183</td>
</tr>
</tbody>
</table>
The income tax system in the base case equilibrium raises 2,118 billion in total tax revenue; 1,477 billion is raised by federal taxes and 640 billion is raised by the state and local government.\textsuperscript{15} The social security payroll tax is set so that revenue from the tax is equal to 538 billion, which is assumed to equal the amount social security benefits. The remaining federal taxes are raised by business income and personal taxes. Auerbach (1996) estimates that the effective corporate tax rate on non-residential investments is 31 percent and the effective non-corporate tax rate is 18 percent. Gravelle (1994) estimates that the effective tax rate on corporate and non-corporate rental housing is 40 percent and 20 percent respectively. Using these estimates of effective tax rates, the effective tax rate in the non-housing sector is set at 26 percent, and the effective tax rate in the rental housing sector is set at 22 percent.\textsuperscript{16} As reported by Auerbach (1996), estimates of the average marginal and average wage tax rates are 0.217 and 0.075 from the TAXSIM model of the National Bureau of Economic Research. The average marginal wage tax rate is equal to 0.217 and the average wage tax rate across all generations is 0.095. Following Auerbach (1996), the tax rate on individual interest income is set at 18.5 percent. the tax rate on dividends is set at 20 percent, capital gains in the non-housing sector are taxed at an effective annual accrual rate of 5 percent, and capital gains in the owner-occupied and rental housing sectors are untaxed.\textsuperscript{17} Federal government expenditures on goods and services are 79 percent of total federal revenues, and the remaining federal revenues

\begin{footnotesize}
\textsuperscript{15} Unless otherwise noted, the base case data on federal, state, and local government taxes and expenditures are taken from ERP 1999.

\textsuperscript{16} Note that 61.5 percent of the non-housing sector is incorporated, and 10 percent of the rental housing sector is incorporated.

\textsuperscript{17} The effective annual accrual tax rate on capital gains in the owner-occupied sector is assumed to equal zero, since the Taxpayer Relief Act of 1997 exempted gains up to $250,000 on the sale of a house.
\end{footnotesize}
are used to fund transfers. Government purchases of housing services are 3 percent of total government expenditures on goods and services.

The state and local government raises 209 billion in property taxes, 178 billion in retail sales taxes (excluding excises taxes), 34 billion in corporate income taxes, and 219 billion in personal taxes. Following Fullerton and Rogers (1993), the average property tax rate on non-residential capital is 0.0081, and the property tax rate on residential capital is 0.0171. The state tax rate on personal and business income is set at 0.04 in order to raise state business revenues consistent with the data in ERP. The state retail sales tax rate is set at 0.07; the retail sales tax base is assumed to include 56 percent of total non-housing consumption.\(^{18}\) It is assumed that state and local expenditures equal total state and local tax revenues.

The base case data on investment in each sector and consumption are taken from ERP 1999. Data on the value of capital in the non-housing, owner-occupied housing, and rental housing sectors is from the Survey of Current Business (1998). Table 2.3 characterizes the initial income tax equilibrium.

### 2.4 Simulation Results

In this section, simulation results are presented for a Flat Tax reform similar in nature to the proposal by Hall and Rabushka (1995). The primary focus is quantifying the two asset price effects that would accompany a switch from an income-based to a consumption-based tax system. The first of these, as discussed above, is the potential one-time windfall loss on the owners of existing capital, other than owner-occupied housing, that has been quantified in many of the studies discussed in section 1.4

\(^{18}\)See Zodrow (1999), who notes that on average roughly 40 percent of the state retail sales tax base is made up of business purchases. The taxation of business purchases under the retail sales tax is ignored in this model, and thus, the average state retail sales tax rate is overstated.
### Table 2.2 Federal and State Tax Rates and Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_d$</td>
<td>Dividend Tax Rate</td>
<td>0.20</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>Interest Income Tax Rate</td>
<td>0.185</td>
</tr>
<tr>
<td>$\tau_g$</td>
<td>Non-Housing Capital Gains Tax Rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>Non-Housing Business Tax Rate</td>
<td>0.268</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Social Security Tax Rate</td>
<td>0.102</td>
</tr>
<tr>
<td>$\tau_{wd}$</td>
<td>Tax Rate on TDA at Time of Withdrawal</td>
<td>0.135</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Parameter in Quadratic Wage Tax Function</td>
<td>-0.023</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Parameter in Quadratic Wage Tax Function</td>
<td>0.000004</td>
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<tr>
<td>$\tau_{rs}$</td>
<td>Rental Housing Tax Rate</td>
<td>0.25</td>
</tr>
<tr>
<td>$\tau_{gr}$</td>
<td>Rental Housing Capital Gains Tax Rate</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_{go}$</td>
<td>Owner Housing Capital Gains Tax Rate</td>
<td>0</td>
</tr>
</tbody>
</table>

#### Federal Taxes

#### State Taxes

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{st}$</td>
<td>Sales Tax Rate</td>
<td>0.07</td>
</tr>
<tr>
<td>$c$</td>
<td>Housing Property Tax Rate</td>
<td>0.0171</td>
</tr>
<tr>
<td>$c_{nh}$</td>
<td>Non-Housing Property Tax Rate</td>
<td>0.0081</td>
</tr>
<tr>
<td>$t_b^{st}$</td>
<td>Average Business Tax Rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$t_w^{st}$</td>
<td>Average Wage Tax Rate</td>
<td>0.04</td>
</tr>
</tbody>
</table>

### Table 2.3 Initial Income Tax Steady State Values for each Sector

<table>
<thead>
<tr>
<th></th>
<th>Non-Housing</th>
<th>Rental</th>
<th>Owner-Occupied</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td>7001.0</td>
<td>285.6</td>
<td>823.6</td>
<td>8110.0</td>
</tr>
<tr>
<td><strong>Capital</strong></td>
<td>8649.4</td>
<td>1912.8</td>
<td>6683.3</td>
<td>17245.6</td>
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<tr>
<td><strong>Wage Payments</strong></td>
<td>5320.8</td>
<td>5.5</td>
<td>22.2</td>
<td>5348.6</td>
</tr>
<tr>
<td><strong>Firm Value</strong></td>
<td>5275.6</td>
<td>1185.6</td>
<td>4344.1</td>
<td>10805.3</td>
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<tr>
<td><strong>Investment</strong></td>
<td>6936.8</td>
<td>73.5</td>
<td>256.7</td>
<td>1023.8</td>
</tr>
<tr>
<td><strong>Earnings</strong></td>
<td>1292.4</td>
<td>-</td>
<td>-</td>
<td>1292.4</td>
</tr>
<tr>
<td><strong>Net Service Flow</strong></td>
<td>-</td>
<td>76.3</td>
<td>275.1</td>
<td>351.4</td>
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</tbody>
</table>
and 2.1. However, many of these studies fail to distinguish between owner-occupied housing and other assets, and instead, simply focus on a single aggregate firm. The model employed for this study allows for a more detailed description of the effects of implementing a consumption based tax on the value of owner-occupied housing, rental housing, and non-housing capital. The second asset price effect results from adjustments in the size and allocation of capital across sectors in response to changes in the tax treatment of capital in different sectors. This asset price effect is most commonly discussed in connection with owner-occupied housing, since the switch from an income to a consumption tax would remove (or reduce depending on the specific reform) the tax advantage to owner-occupied housing under the current tax system. Simulations are reported for low ($\beta = 1$), moderate ($\beta = 5$), moderately-high ($\beta = 10$) and high ($\beta = 20$) levels of adjustment costs, since the costs of adjusting the capital stock play an important role in determining the timing of the reallocation of investment across sectors.

It is assumed that the payroll tax is maintained and that social security benefits are funded completely by the payroll tax. Note that simulation results for the most part are presented in terms of percentage changes from the initial income tax steady state values that would have occurred if the economy had continued along the initial steady state growth path.

2.4.1 The Flat Tax

In the base case the adjustment cost parameter ($\beta$) is equal to 5. This parameter value is chosen based on existing estimates by Cummins, Hasset and Hubbard (1994) and Summers (1981), and is allowed to vary widely over the range of estimates in the simulations presented in this study. Households are allowed personal exemptions and standard deductions in each year. The amount of the personal exemption and
standard deduction for each household equals $22,091 in the year of reform, and is assumed to grow at the steady state growth rate of the economy over time. As discussed above, owner-occupied housing is ignored for tax purposes under the Flat Tax.

Table 2.3 reports the effects of implementing a Flat Tax on the wage rate, the before-tax interest rate, gross domestic product (GDP), and output in the non-housing and housing sectors. The changes in the wage rate, GDP, and output in each sector are reported in terms of percentage changes from levels that would have occurred in the initial income tax steady state. The changes in the before-tax interest rate are reported in terms of percentage point changes from the initial income tax steady state values.

The general pattern of the wage rate and interest rate are similar for all levels of adjustment costs. The wage rate falls slightly in the year of reform and then rises at a decreasing rate during the transition to the long-run equilibrium. The wage rate falls in the year of reform because labor supply increases immediately as (1) the incentive to supply labor — the net-of-tax wage — increases and (2) the increase in the after-tax interest rate causes individuals to substitute future consumption and leisure for current consumption and leisure.\textsuperscript{19} The increase in the after-tax interest rate also reduces both the present value of future labor earnings and the present value of future consumption, and thus, has an ambiguous income effect. Following the initial 1.6 percent decrease, the wage rate rises over time, as increased saving leads to a higher capital-labor ratio, reaching a new equilibrium level that is 7.1 percent higher than in the initial income tax steady state.

\textsuperscript{19} Note that the net-of-tax wage actually decreases for generations that are near the age of retirement at the time of reform due to the “humped-back” wage profile and the progressive income tax rate structure in the initial equilibrium. Thus, generations nearing retirement may actually work less in the initial years after the reform. However, this effect is far out weighted by the increase in the net-of-tax wage for all younger and middle-aged generations.
The before-tax interest rate increases initially by 0.16 percentage points, because the aggregate demand for investment increases as the effective tax rates on non-housing and rental housing capital are reduced under the Flat Tax. The before-tax interest rate falls by 2 percentage points in the new equilibrium, with 50 percent of the decline occurring within 12 years after reform and 75 percent of the decline occurring within 21 years after reform.

GDP is 13.3 percent higher in the new steady state than in the initial income tax steady state. Most of the increase in GDP occurs within 20 years of the enactment of reform as labor supply and aggregate investment increase. Output in the non-housing sector increases initially by 5.1 percent and by 13.7 percent in the new equilibrium. This reflects the increase in labor supply, the increase in savings, and the reallocation capital from the housing to the non-housing sector. Output in the housing sector also increases. The value of output in the owner-occupied and rental housing sector is 1.0 percent higher one year after reform than in the initial income tax steady state and continues to increase gradually to a new steady state equilibrium value that is 7.4 percent higher than under the income tax. The initial increase in the value of output in the owner-occupied and rental housing sectors reflects a 3.2 percent increase in the rental price of housing services per unit in the first year after reform, relative to the initial income tax equilibrium. Following this increase, the rental price of housing services per unit falls gradually reaching a new steady state equilibrium value 1.5 percent less than in the initial income tax steady state. This implies that as the economy moves to the new steady state the output of housing services increases as labor supply and saving increases.

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20 As noted by Gravelle (1998), the range of estimated effects on GDP vary considerably even across models that are structurally similar. Thus, relying on the rather large increases in aggregate output reported in these types of studies for policy making is not advised.
Table 2.4  The Effects of Reform on the Wage Rate, Interest Rate, and Output

<table>
<thead>
<tr>
<th>Years After Reform</th>
<th>Wage Rate</th>
<th>Before-Tax Interest Rate</th>
<th>GDP</th>
<th>Non-Housing</th>
<th>Housing Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.6</td>
<td>0.16</td>
<td>4.9</td>
<td>5.1</td>
<td>0.12</td>
</tr>
<tr>
<td>1</td>
<td>-1.0</td>
<td>0.08</td>
<td>5.7</td>
<td>5.9</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
<td>-0.4</td>
<td>7.8</td>
<td>8.0</td>
<td>3.4</td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>-0.8</td>
<td>9.5</td>
<td>9.7</td>
<td>4.7</td>
</tr>
<tr>
<td>20</td>
<td>5.1</td>
<td>-1.5</td>
<td>11.5</td>
<td>11.8</td>
<td>6.2</td>
</tr>
<tr>
<td>50</td>
<td>6.8</td>
<td>-1.9</td>
<td>13.1</td>
<td>13.5</td>
<td>7.3</td>
</tr>
<tr>
<td>100</td>
<td>7.1</td>
<td>-2.0</td>
<td>13.3</td>
<td>13.7</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Replacing the federal income tax system with a Flat Tax that is revenue neutral yields a tax rate equal to 14.6 percent one year after reform. The tax rate gradually declines after the reform to 13.7 percent after 35 years and eventually reaches its new steady state equilibrium value 13.6 percent after approximately 60 years. Note that the payroll tax rate is 9.5 percent the year after reform and is 8.9 percent in the long-run equilibrium.

In each sector, the reform-induced effects on net investment, the capital stock, and the real value of equity capital of replacing the federal income tax system with a Flat Tax under the four levels of adjustment costs are depicted in Figures 2.1 - 2.17. As discussed above, the results explicitly account for important differences in the tax treatment of capital employed in the non-housing, rental housing, owner-occupied housing sectors.

The results reported in this section suggest that replacing the current federal income tax with a Flat Tax would induce significant increases in investment in the non-housing and rental housing sectors and significant decreases in investment in the owner-occupied housing sector. The increase in investment in the non-housing and rental housing sectors is a result of the reform-induced decrease in the user cost of
capital in each of these sectors. Moreover, the reduction of the relative tax advantage of capital employed in the owner-occupied housing sector encourages a reallocation of capital from the owner-occupied housing sector to the non-housing and rental housing sectors, and thus, also increases investment in these sectors in the short-run.

As discussed in Section 1.4, the user cost of owner-occupied housing would increase if a Flat Tax were implemented since the opportunity cost of the equity financed share of owner-occupied housing would increase since the after-tax return of investments in the non-housing and rental housing sectors would increase due to elimination of the tax on the normal rate of return. Furthermore, after-tax mortgage interest and property tax payments would also increase for households that itemize, since tax deductions for these payments would be eliminated. However, during the transition to the long run equilibrium, the decline in the before-tax interest rate (and thus the after-tax interest rate) over time would tend to lower the opportunity costs of equity financed investments in owner-occupied housing and mortgage interest payments. This implies that the user cost of owner-occupied housing capital increases initially, and then declines steadily as the increase in saving over time causes the after-tax interest rate to fall to a level near the original after-tax interest rate under the income tax system.

Figure 2.1 shows the effects of implementing a Flat Tax on net investment in each sector with moderate adjustment costs.\textsuperscript{21} Net investment in the non-housing sector is 136 percent higher in the year of reform than it would have been in the initial income tax steady state, and remains at least 70 (50) percent higher for 19 (35) years after reform. In the long-run equilibrium, net investment is 41.3 percent higher than it would have been under the income tax. In the year the Flat Tax is

\textsuperscript{21}The results are reported as percentage changes from the initial income tax steady state. Note that year 0 is the year of reform, year 1 is the first year after reform, year 2 is the second year after reform, and so on.
Figure 2.1 Effects of Flat Tax Reform on Net Investment, With Moderate Adjustment Costs ($\beta = 5$)

enacted, net investment in the rental sector increases by 50 percent relative to its level in the initial income tax equilibrium. Net investment in the rental sector continues to increase for 7 years after reform, reaching a level that is 57 percent higher 7 years after reform than it would have been in the initial income tax steady state. In the long-run equilibrium, net investment in rental housing is 27 percent higher than it would have been under the income tax. Net investment in owner-occupied housing decreases initially by 48 percent, and then increases rapidly for 18 years after the reform to a level that is 20 percent higher than the level that would have occurred under the initial income tax steady state. The rapid increase in net investment after the reform is a result of the decline in the interest rate after the enactment of the Flat Tax, since this decreases the user cost of capital in the owner-occupied sector.
In the long-run equilibrium, net investment in owner-occupied housing is 8.6 percent higher than it would have been under the income tax.

Figures 2.2 - 2.4 depict the effects on net investment in each sector under each of the alternative assumptions regarding the level of adjustment costs. The case of low adjustment costs ($\beta = 1$) is shown in figure 2.2, the case of moderately-high adjustment costs ($\beta = 10$) and high adjustment costs ($\beta = 20$) are shown in figures 2.3 and 2.4 respectively. In general, the lower (higher) the level of adjustment costs the more (less) rapid is the transition to the long-run equilibrium. For instance, in the low adjustment costs case, net investment in the non-housing and rental housing sectors in the year of reform is 266 and 62 percent higher, respectively, relative to the values that would have occurred in the initial income tax equilibrium. In the year of reform, net investment in the owner-occupied housing sector falls by 172 percent relative to the initial income tax steady state. By comparison, in the moderately-high adjustment costs case ($\beta = 10$), net investment in the non-housing sector increases initially by 95 percent and net investment in the rental sector increases initially by 40 percent relative to the values that would have occurred under the income tax. In the year of reform, net investment in the owner-occupied housing sector decreases by 25 percent relative to the level that would have occurred under the income tax. In the high adjustment costs case ($\beta = 20$), net investment in the non-housing and rental sectors increases initially by 62 and 28 percent, respectively, relative to the values that would have occurred under the income tax. With the high adjustment costs, net investment in the owner-occupied housing sector falls by 13 percent, in the year of reform, relative to the values in income tax steady state.

Figure 2.5 shows the reform-induced effects on the capital stock in each sector. The increase in net investment in the non-housing and rental housing sectors results in larger capital stocks relative to the income tax steady state in every year after
Figure 2.2  Effects of Flat Tax Reform on Net Investment, With Low Adjustment Costs (β = 1)

Figure 2.3  Effects of Flat Tax Reform on Net Investment, With Moderately-High Adjustment Costs (β = 10)
Figure 2.4 Effects of Flat Tax Reform on Net Investment. With High Adjustment Costs ($\beta = 20$)

Figure 2.5 Effects of Flat Tax Reform on Capital Stocks, With Moderate Adjustment Costs ($\beta = 5$)
the year of reform. In the non-housing (rental housing) sector 87 (83) percent of the increase in the size of the capital stock occurs within 30 years of the reform. In the owner-occupied sector the capital stock declines initially, relative to the stock of owner-occupied housing that would have occurred under the income tax, as capital is reallocated from the owner-occupied housing sector to the non-housing and rental housing sectors. However, the overall increase in saving, investment, and real GDP, along with a concomitant decline in the before-tax interest rate in the years following reform, leads to an eventual increase in the stock of owner-occupied housing capital relative to the level that would have occurred under the income tax. For instance, with moderate adjustment costs ($\beta = 5$), the stock of owner-occupied housing declines for 6 years following reform. In the sixth year after reform, the size of the capital stock employed in the owner-occupied housing sector is 2.6 percent smaller than it would have been under the income tax. The stock of capital in the owner-occupied housing sector reaches the level that would have occurred if the economy had continued to grow along the initial steady state path 16 years after reform. In the long run equilibrium, the increase in the size of the capital stock in each sector is equal to the increase in net investment in each sector.

In figures 2.6 - 2.8 the reform-induced effects on the capital stock in each sector are shown for the cases of low, moderately-high, and high level of adjustment costs. The case of low adjustment costs is shown in figure 2.6 and the case of high adjustment costs is shown in figure 2.8. Note that it takes 14 years in the low adjustment costs case, and 19 years in the high adjustment costs case, before the stock of owner-occupied housing returns to the level that would have occurred under the income tax.

One of the most important issues in predicting the reform induced welfare gains and losses across generations is the short-run impact of reform on existing asset
values; this includes the one-time windfall levy imposed on existing capital (other than owner-occupied housing) and the effects of reform of the value of capital in the owner-occupied housing sector. As noted above, there are a wide variety of factors associated with the implementation of a consumption-based tax that would act to offset the one-time windfall levy on existing assets. A partial list of the factors that would offset the effect of the one-time windfall tax on the value of non-housing and rental housing assets include: (1) the presence of adjustment costs which would allow the owners of capital to earn above normal returns on existing assets and new investments over the transition period to the new equilibrium; (2) a short-run increase in the after-tax rate of interest which would allow owners of capital to earn a higher after-tax rate of return on existing assets and new investments; and (3) the elimination or reduction of the expected tax on existing assets that were allowed accelerated depreciation allowances.
under the income tax. An additional factor, that is considered by some to be the principal controversy of implementing a consumption-based tax reform, is the effect of such a reform on the value of owner-occupied housing due to the elimination of the tax advantage of capital in the owner-occupied housing sector. In the presence of adjustment costs, the reallocation of capital across sectors that would accompany the elimination of the tax advantage to owner-occupied housing under a consumption-based tax reform would affect the value of non-housing assets as well as the value of housing assets.

Figure 2.9 shows the effects of replacing the income tax with the Flat Tax on the average real value of equity \((Q)\) in each sector. In the non-housing sector, with moderate adjustment costs \((\beta = 5)\), the average value of equity increases initially by 3.4 percent and then gradually declines over time. In the long-run equilibrium, the average value of equity in the non-housing sector decreases by 16.5 percent relative to the values that would have occurred under the income tax. The average real value of equity in the non-housing sector is higher than it was under the income tax in the year of reform and the first year after reform, and is lower in all remaining years. This implies that in this case, the offsetting factors described above, more than offset the one-time windfall levy on existing non-housing capital resulting in an increase in the average real value of equity capital immediately following reform.

In the rental housing sector, with moderate adjustment costs \((\beta = 5)\), the average value of equity \((QR)\) decreases initially by 12.3 percent, increases for the first four years after the year of reform, and then declines steadily over time. In the long-run equilibrium, the average value of equity in the rental sector is 17.8 percent smaller than it would have been under the income tax. At least two factors contribute to the difference in the changes in asset values across the non-housing and rental housing sectors. First, the model is calibrated so that in the initial income tax
Figure 2.7  Effects of Flat Tax on Capital Stocks, With Moderately-High Adjustment Costs ($\beta = 10$)

Figure 2.8  Effects of Flat Tax on Capital Stocks, With High Adjustment Costs ($\beta = 20$)
equilibrium capital in the non-housing sector carries a discount of 4 percent and capital in the rental sector carries a discount of 3 percent due to the existence of accelerated depreciation allowances. This implies that the reduction of the expected tax on the stream of earnings from rental assets, due to the existence of accelerated depreciation allowances, is smaller than in the non-housing sector. Second, the decline in the effective tax rate on capital in the non-housing sector would be larger than the decline of the effective tax rate on capital in the rental sector. This implies that there would be an incentive for capital to shift from the rental housing sector to the non-housing sector, and thus, offset the inflow of new investment into the rental sector from increased savings and the reallocation of capital out of the owner-occupied housing sector into the rental and non-housing sectors. Note that the non-housing sector would experience a surge of new investment from the increase in saving and the
reallocate capital from the rental housing and owner-occupied housing sectors. Thus, there would exist a greater potential for above normal returns on existing capital and new investment in the non-housing sector.

As shown in figure 2.5, the average real value of equity (QO) in the owner-occupied housing sector declines by 9.5 percent in the year of reform, and then increases reaching the value that occurred in the initial income tax steady state 7 years after the enactment of reform. The average real value of equity in the owner-occupied sector continues to increase until it is 3.4 percent higher than in the income tax steady state before declining gradually to its long-run equilibrium value that is equal to what it would have been under the income tax. The initial decrease in the average real value of equity in the owner-occupied housing sector is a result of the increased cost of owner housing capital which leads to a decrease in the demand for owner-occupied housing. The average value of equity in the owner-occupied housing sector begins to increase after its initial decrease as the interest rate declines over time. Again, this occurs because as the interest rate falls the user cost of owner-occupied housing falls, because the opportunity costs of equity financed investments in owner-occupied housing and mortgage interest payments decrease.

Figures 2.10 - 2.12 show the reform-induced effects on the average real value of equity capital in each sector for the low, moderately-high, and high adjustment costs cases, relative to the values that would have occurred under the income tax. In the low adjustment costs case (β = 1), shown in Figure 2.10, the average real value of equity in the non-housing sector decreases by 8.9 percent in the year of reform — this is similar to the findings of the other studies discussed above, however, the size of the decline in the average real value of equity for the aggregate capital stock is generally around 20 percent. In the moderately-high and high adjustment costs cases, shown in Figures 2.11 and 2.12, the average real value of equity in the non-
Figure 2.10  Effects of Flat Tax Reform on Average Real Values of Equity (Q), With Low Adjustment Costs ($\beta = 1$)

Figure 2.11  Effects of Flat Tax Reform on Average Real Values of Equity (Q), With Moderately-High Adjustment Costs ($\beta = 10$)
housing sector increases by 10.7 and 18.5 percent respectively, relative to the values that would have occurred under the initial income tax equilibrium. There is roughly a 20 percentage point difference in the estimates of the percentage changes in the average real value of non-housing equity in the low and moderately-high adjustment costs cases. This difference reflects the importance of the level of adjustment costs in determining the potential for above normal returns in the non-housing sector due to the delay of the inflow of new investment demand that results from increased saving and the reallocation of capital from the housing to the non-housing sector.

In the low adjustment costs case ($\beta = 1$), the initial decrease in the average real value of equity in the rental sector is 18.8 percent relative to the initial income tax steady state. With moderately-high adjustment costs ($\beta = 10$), the average real value of equity in the rental sector decreases by 5.5 percent. Most interestingly, variations
in the level of adjustment costs have a rather small effect on the initial decrease in the average value of equity in the owner-occupied housing sector, and only slightly more significant affects on the average real value of equity over the transition period. In the low adjustment costs case, the average value of equity in the owner-occupied sector is 8.7 percent lower than in the initial income tax equilibrium in the year of reform. By comparison, for the three higher levels of adjustment costs discussed in this paper, the average real value of equity in the owner-occupied sector in the year of reform is between 9.4 and 9.5 percent smaller than in the initial income tax equilibrium.

Figure 2.13 depicts the effects of implementing the Flat Tax on firm values in each sector. Changes in firm values are determined by the reform-induced effects on the average value of equity and net investment. In the first few years after the enactment of the Flat Tax, the reform-induced effects on the average real values of equity are the primary determinant of each firm's value. However, the reform-induced effects on investment eventually outweigh the effects on the average real values of equity in the long-run. This dependence is most easily seen in the results for the owner-occupied housing sector. In the year of enactment, the total equity value of owner-occupied housing falls by 9.5 percent relative to the value that would have occurred under the income tax. The initial decrease in the total equity value of owner-occupied housing is exactly equal to the decrease in the average value of equity in the owner-occupied sector.\textsuperscript{22} In the long-run, the average value of equity in the owner-occupied sector returns to the level that would have occurred under the income tax since only the relative tax treatment of owner-occupied housing is changed by the reform. Thus, in the long run, the change in the total equity value of owner-occupied housing will

\textsuperscript{22}In the year of reform, this must always be true since }$V = Q \cdot K$\textsuperscript{ and }$K$\textsuperscript{ is equal to the level that would have occurred under the income tax steady state.
depend solely on the reform-induced effects on investment in the owner-occupied sector. As shown above in Figures 2.1 - 2.4, net investment in the owner-occupied housing sector increases by 8.6 percent in the long-run relative to the level that would have occurred under the income tax steady state. The long-run increase in the total equity value of owner-occupied housing and the size of the capital stock in the owner-occupied sector is equal to 8.6 percent of the value that would have occurred under the initial income tax steady state.

The equity value of the non-housing firm increases initially by 3.4 percent and then rises gradually until in the long-run equilibrium the equity value of the firm is 18 percent larger than in the initial income tax equilibrium. The equity value of the rental firm falls by 12.3 percent in the year of reform, but returns to the value that would have occurred under the income tax steady state 17 years after the enactment
of reform. In the long-run equilibrium, the value of the rental firm is 4.3 percent higher than it would have been under the income tax.

Figure 2.14 - 2.16 show the reform-induced effects on real firm values in each sector for the low, moderately-high, and high adjustment costs cases, relative to the values that would have occurred under the income tax. In the low adjustment costs case (\(\beta = 1\)), shown in Figure 2.14, the equity value of the non-housing firm is 8.9 percent less than it was under the income tax; this case is consistent with the notion that the elderly would suffer a one-time capital levy under a consumption-based tax reform. For the moderately-high and high adjustment costs cases, the equity value of the non-housing firm is 10.7 and 18.5 percent higher than under the income tax. Note that with a moderate or higher level of adjustment costs the equity value of the non-housing firm actually increases. As shown in Figure 2.14, the equity value of the rental firm in the low adjustment costs case is 18.8 percent lower than under the income tax. In the high adjustment costs case (\(\beta = 20\)), the equity value of the rental firm falls by 3.3 percent relative to the levels that would have occurred under the income tax.

Figure 2.17 depicts the effects of replacing the income tax with a Flat Tax on the total equity value of owner-occupied housing for the two lowest adjustment costs cases. In the year of reform, the decrease in the total equity value of owner-occupied housing is larger in magnitude the higher the level of adjustment costs. However, in the 21 years following the year of reform, the decline in the total equity value of owner-occupied housing is smaller for the higher adjustment costs case. In particular, in the year of reform the total equity value of owner-occupied housing is roughly 1 percent smaller under the moderate adjustment cost case relative to the low adjustment costs case. The total value of equity is 2.1 smaller, 5 years after enactment of the reform, in the low adjustment costs case than it is in the moderate adjustment costs case.
Figure 2.14  Effects of Flat Tax on Real Firm Values, With Low Adjustment Costs ($\beta = 1$)

Figure 2.15  Effects of Flat Tax on Real Firm Values, With Moderately-High Adjustment Costs ($\beta = 10$)
Figure 2.16  Effects of Flat Tax on Real Firm Values, With High Adjustment Costs ($\beta = 20$)

Figure 2.17  Effects of Flat Tax Reform on Firm Value of Owner-Occupied Housing, Low and Moderate Adjustment Costs Cases ($\beta = 1$ and $\beta = 5$)
The total equity value of owner-occupied housing is 0.6 percent higher 30 years after reform under the low adjustment costs case than it is under the moderate adjustment costs case. These results suggest that the initial decline in the equity value of owner-occupied housing would be quite similar for all levels of adjustment costs, but that small differences in the equity value of owner-occupied housing would be likely to exist during the transition to the long-run equilibrium.

The intergenerational welfare effects of implementing a Flat Tax are shown in Figure 2.18 for all four levels of adjustment costs. The intergenerational welfare effects are presented in terms of the percentage changes in remaining lifetime resources across all generations, taking into account all reform-induced welfare gains and losses considered in the model.

The general pattern of welfare changes is similar under the varying levels of adjustment costs, however, the magnitude and sign of the net welfare change may vary considerably with the assumed level of adjustment costs for a particular generation. For example, the generation age 54 in the year of enactment suffers a net welfare loss of 1.8 percent of remaining lifetime resources in the lowest adjustment costs case ($\beta = 1$) and experiences net welfare gains ranging from 3.3 to 10.1 percent of remaining lifetime resources for each of the higher adjustment costs cases. The differences in welfare changes for each level of adjustment costs is reflective of the reform-induced effects on the total equity value of capital in the non-housing sector. As described above, in the lowest adjustment costs case ($\beta = 1$), the total value of equity in each sector decreases in the year of reform, and thus, the oldest generation alive at the time of reform suffers a welfare loss in their final year of life. For the three higher levels of adjustment costs, the one-time tax on existing capital in the non-housing sector is more than offset by the offsetting factors described in Section 1.3.2. In all of adjustment costs cases considered in this paper, the total equity value of owner-
occupied and rental housing assets decrease relative to the values that would have occurred under the income tax. Thus, it is possible that even though the value of the non-housing firm increases, the aggregate firm value — the sum of the value of the non-housing, rental housing, and owner-occupied housing firms — could still decrease. In this case, the oldest generations alive at the time of reform may experience a welfare loss in the year of reform. For example, with moderate adjustment costs ($\beta = 5$), there is a reform-induced increase in the total equity value of non-housing assets equal to 3.4 percent of the value that would have occurred in the income tax steady state. However, in this case, the aggregate value of all firms in the economy falls, relative to initial income tax steady tax, in the year of reform. This decline in the aggregate firm value is offset by an increase in the interest rate in the year of reform, relative to the interest rate that would have prevailed in the income tax
steady state, so that total financial wealth in the year of reform is larger than it would have been under the income tax. Thus, even though the aggregate value of equity in the economy falls, the oldest generation experiences a welfare gain after the enactment of reform, since the increase in after-tax interest earnings on existing financial wealth at the time of reform out weights the reform-induced decrease in the aggregate value of assets. In the moderately-high and high adjustment costs cases, there is a reform-induced increase in the aggregate value of equity in the economy, and thus, the welfare gains of the oldest generation increase relative to the lower and moderate adjustment costs cases.

For those who are retired at the time of reform, between the economic ages of 45 and 54, as age in the year of enactment decreases the net welfare gain increases (or the loss decreases) under the low and moderate levels of adjustment costs ($\beta = 1$ and $\beta = 5$). As discussed above, these results are due in part to the existence of inframarginal returns on existing capital and new investments in the non-housing and rental housing sectors during the transition, as well as the increase in the after-tax rate of return to capital that occurs in the years following reform. This occurs because the potential benefits from receiving a higher after-tax interest rate, and from earning inframarginal returns, increase with the length of the remaining life span at the time of reform.

The most striking feature in the moderate adjustment costs case ($\beta = 5$) is that all but a few of the young generations experience a net welfare gain. The generations born between three years before and after reform suffer small net welfare losses — the loss of each generation is less than 0.4 percent of lifetime resources. In the low adjustment costs case ($\beta = 1$), shown in Figure 2.18, the generation age 54 in the year of enactment suffers a net welfare loss equal to 1.8 percent of lifetime resources. This resembles the typical results of this type of study in which the elderly generations
suffer a net welfare loss due to the decrease in the value of their assets. However, the fact that every generation, other than the oldest generation, experiences reform-induced net welfare gains in the low adjustment costs case is quite different from the results of similar studies. For each of the higher adjustment cost cases ($\beta = 10$ and $\beta = 20$) the number of young generations that suffer a net welfare loss increases since the efficiency gains from reform do not take effect as rapidly as in the low and moderate adjustment costs cases. Also note that in the higher adjustment costs cases the elderly actually enjoy significant net welfare gains. For instance, the retired generations in the year of enactment enjoy net welfare gains that are between 6 and 10 percent of remaining lifetime resources under the high adjustment costs cases. Overall, the larger the level of adjustment costs the higher the net welfare gain for the generations between the ages 26 and 54 at the time of reform, and the smaller the net welfare gains (larger the net welfare losses) for the generations younger than age 26 at the time of reform.

The net welfare gains decline steadily for generations between the ages of 44 and 26 in the year of enactment for all levels of adjustment costs. This pattern of welfare gains is due in part to two major factors. First, replacing the federal income tax system with the Flat Tax actually raises the marginal tax rate on labor income for generations that will retire over the five year period after the reform is enacted, since the wage profile has a "humped-back" shape and the income tax rates are structured in a progressive manner in the initial income tax steady state. This implies that generations age 40 to 44 at the time of reform will face higher marginal tax rates after reform than they would have under the income tax, with the generations nearest the age of retirement suffering fewer years of higher marginal wage taxes than generations further away from retirement. This effect is diminished for middle aged and younger generations since they will benefit from lower marginal tax rates on wage income before reaching
the near retirement stage of life. Second, the initial decrease in the before-tax wage rate tends to have a larger impact on older and middle-aged generations since they have fewer years to benefit from the higher wage rates several years after reform, and during these years they are faced with the higher marginal tax rates on wage income near retirement.

As discussed above, older generations may pass a portion of the decline in asset values on to future generations by reducing the size of the bequest. This model employs a primitive target bequest, and thus, does not capture this type of transmission of welfare gains or losses across generations. However, one interesting result in the equivalent variations can be explained by examining the reform-induced effects on welfare changes for generations that receive their inheritance either before or after the time of reform. The generation age 25 in the year of enactment is the first generation to receive an inheritance after reform. As shown in Figure 2.20, the generation age 25 in the year of reform experiences a discontinuous change in net welfare that varies in magnitude and sign with the level of adjustment costs. In the low (high) adjustment costs case, the generation age 25 suffers a smaller (larger) welfare gain at the time of reform relative to the generation age 24 since a fraction of their inheritance is subject to the one-time windfall tax on existing capital. Since the size of the bequests is exogenous in the model, the equivalent variations by generation are affected based on whether the generation receives their inheritance before or after the reform.

The relatively large net welfare gains (small net welfare losses) in this study, as compared to other studies, are a direct result of the increased efficiency gains that result from explicitly including the distortions across the non-housing, rental housing, and owner-occupied housing sectors. Furthermore, with higher levels of adjustment costs the rate at which investment is reallocated across the three sectors tends to occur less rapidly. This leads to higher inframarginal returns and thus larger increases
Figure 2.19  Equivalent Variations of the Transition Generations

Percentage Change in Remaining Lifetime Resources

- EV (beta=1)
- EV (beta=5)
- EV (beta=10)
- EV (beta=20)

Age in the Year of Enactment

Figure 2.20  The Effect of Inheritances on Equivalent Variations by Generation During the Transition, With Low and High Adjustment Costs

The cohort age 25 at the time of reform receives their inheritance at the beginning of the period of reform.
(smaller decreases) in the total value of equity in the non-housing and rental housing sectors. This study suggests that these gains tend to more than offset the one-time tax on existing assets.

2.4.2 Sensitivity Analysis

Variations in the Intertemporal Elasticity of Substitution

The intertemporal elasticity of substitution ($\sigma_1$) governs the consumers' willingness to substitute present consumption for future consumption in response to changes in the relative price of consumption across periods, and thus, plays a major role in determining the responsiveness of savings to changes in the after-tax interest rate. In this paper the value of this elasticity is set so that it is consistent with empirical estimates in the literature. However, econometric estimates have failed to yield a consensus on the actual value of $\sigma_1$, with most estimates falling in the range from 0 to 1.$^{23}$ Given this, it is important to understand how varying the size of the intertemporal elasticity substitution would affect the results that were reported in the previous section.

Figure 2.21 shows the effects of implementing a Flat Tax on net investment given that the intertemporal elasticity of substitution is 0.1. In this case, the initial increase in net investment in the non-housing and rental housing sectors is dampened, since there is a smaller saving response compared to the benchmark simulation ($\sigma_1 = 0.26$). The increase in net investment in each of these sectors is also lower in the long run steady state. In the year of reform, net investment in the owner-occupied sector falls by 77 percent; this decrease in net investment is 29 percentage points larger than in the benchmark simulations. Moreover, the long run increase in net investment in the

Figure 2.21  Effects of Flat Tax Reform on Net Investment, With Low Intertemporal Elasticity of Substitution (\(\sigma = 0.1\))

The owner-occupied sector is 0.6 percent rather than 8.6 percent. Thus, assuming a lower value for the intertemporal elasticity of substitution has a dampening effect on the initial increase in investment in all three sectors. These results are consistent with the economic rationale underlying this elasticity, since a lower value of \(\sigma 1\) implies that saving and investment (consumption) will be less (more) responsive to changes in the after-tax interest rate relative to the benchmark case. For example, if \(\sigma 1 = 0.1\) composite consumption increases by 4.1 percent in the year of reform rather than 2.8 percent, however, the long run gains in consumption are smaller in this case — only 10 percent rather than 11.3 percent as in the benchmark case.

Figures 2.22 shows the effects of assuming a lower value of \(\sigma 1\) on the capital stock in each sector. The smaller increases in investment in the non-housing and rental housing sectors result in smaller capital stocks throughout the transition and in the
long run equilibrium relative to the benchmark simulations. Moreover, the lower the value of \( \sigma_1 \) the longer the adjustment process. To see this examine the adjustments in the size of the capital stock over time, relative to the size of the capital stock that would have occurred under the income tax, for the low and benchmark value of \( \sigma_1 \). In the owner-occupied housing sector the capital stock decreases by 7.2 percent (twelve years after reform) smaller rather than 2.6 percent (six years after the reform). The long run increase in the size of the owner-occupied capital stock is only 0.6 percent rather than 8.6. Note that the capital stock does not return to the level that would have occurred under the income tax until 117 years after the reform in this case.

Figure 2.23 depicts the effects of assuming a lower value of \( \sigma \) on the total value of equity in each sector. In the year of reform, the average real value of equity in the non-housing sector decreases by 4.5 percent rather than increasing by 3.4 percent.
as in the benchmark simulations. The average real value of equity, and thus the total value of equity, in the owner-occupied (rental) housing sector decreases by 14 (16.6) percent rather than 9.5 (12.3) percent in the year of reform. In this case, owners of existing non-housing and rental housing assets earn smaller above normal returns during the transition relative to the benchmark simulations. The long run increase in the total value of equity in the non-housing sector is only 7 percent rather than 18 percent as in the benchmark simulations. The long run increase in the total value of equity in the owner-occupied housing sector is 0.6 percent. In the rental housing sector, the smaller increases in net investment are more than offset by the decline in the average value of equity, and thus, the total equity value of the rental firm decreases by 4 percent in the long run equilibrium. The smaller increases in the total value of equity in the non-housing and rental housing sectors are a direct result
of the lower levels of investment (and slightly larger declines in the average value of equity) along the transition path and in the long run equilibrium.

By comparison, a relatively large value $\sigma$ of implies that saving is more responsive to changes in the after-tax interest rate, and thus implementing a Flat Tax would induce larger increases in saving and net investment, smaller short run increases in consumption, and larger long run increases in consumption. Figures 2.24 - 2.26 depict the effect of assuming a relatively high value of the intertemporal elasticity of substitution ($\sigma = 0.5$) on net investment, the capital stock, and real firm values in each sector. With $\sigma = 0.5$, the initial increase in consumption is only 1.9 percent, and the long run increase in consumption is 11.6 percent. This implies that the initial decrease (increase) in net investment is smaller (bigger) in the owner-occupied housing (non-housing or rental housing) sector relative to the benchmark simulations. As a result, the decrease in the owner-occupied capital stock is only 1 percent after 4 years (rather than 2.6 percent after six years), and in the long run, the capital stock increases by 10.8 percent (rather than 8.6 percent) relative to the benchmark simulations. The stock of capital in the non-housing and rental housing sectors increases more rapidly during the transition and are larger in the long run than in the benchmark simulations. As show in Figure 2.26, the total equity value of owner-occupied (rental) housing falls by 6.2 (9.1) percent in the year of reform and increases by 10.8 (6.4) percent in the long run. The total equity value of the non-housing firm increases by 8.8 percent in the year of reform and increases by 21 percent in the long run.

Thus, relatively smaller values of $\sigma$ reduce the responsiveness of saving and thus imply smaller (larger) increases (decreases) in net investment and firm values in the year of reform, and larger increases in consumption in the year of reform. The long run increases in net investment, firm values, and consumption are smaller the lower
Figure 2.24 Effects of Flat Tax Reform on Net Investment, With High Intertemporal Elasticity of Substitution ($\sigma = 0.5$)

Figure 2.25 Effects of Flat Tax Reform on Capital Stocks, With High Intertemporal Elasticity of Substitution ($\sigma = 0.5$)
Figure 2.26  Effects of Flat Tax Reform on Real Firm Values, With High Intertemporal Elasticity of Substitution ($\sigma = 0.5$)

Figure 2.27  Equivalent Variations by Generation, With Low and High Intertemporal Elasticities of Substitution
the value of $\sigma$. This has a significant impact on the net welfare gains and losses of current and future generations. Figure 2.27 shows the equivalent variations for the benchmark, low, and high values of the intertemporal elasticity of substitution. Note that with a low intertemporal elasticity of substitution the oldest generation suffers a net loss equal to 2 percent of remaining lifetime resources rather than a net gain of 3.2 percent. In addition, generations age 7 to 19 at the time of reform suffer net welfare losses between roughly 0 and 1.2 percent of remaining lifetime resources. The net welfare gains of the youngest generations are smaller the lower the value of the intertemporal elasticity of substitution, since net investment and thus the future capital stock is smaller in this case.

**Variation in the Intratemporal Elasticity of Substitution**

The intratemporal elasticity of substitution ($\sigma_2$) — defined as the degree that consumers substitute between leisure and consumption within a period — determines the responsiveness of labor supply to the net-of-tax wage rate. In particular, higher values of this elasticity imply that a larger labor supply response will occur for a given change in the net-of-tax wage. Thus, to the extent that enacting a Flat Tax would reduce the marginal tax rate on wage income, the intratemporal elasticity of substitution governs the consumer’s willingness to substitute work (consumption) for leisure in response to the higher after-tax wage rate. Following Auerbach and Kotlikoff (1987), the benchmark value of this elasticity is $\sigma_2 = 0.8$ which is consistent with the direct estimates of Ghez and Becker (1975) and is considered to be near the center of the range of plausible values for this parameter as shown by Auerbach, Kotlikoff and Skinner (1983). The alternative value for this parameter is $\sigma_2 = 0.4$.

Table 2.5 shows the reform-induced effects on the wage rate, labor supply, and GDP assuming that the intratemporal elasticity of substitution equals 0.4. The in-
Table 2.5 The Effects of Reform on the Wage Rate, Labor Supply and GDP: With a Low Intratemporal Elasticity of Substitution

<table>
<thead>
<tr>
<th>Years After Reform</th>
<th>Wage Rate</th>
<th>Labor Supply</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.1</td>
<td>4.9</td>
<td>3.3</td>
</tr>
<tr>
<td>1</td>
<td>-0.5</td>
<td>4.8</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>1.6</td>
<td>4.3</td>
<td>5.7</td>
</tr>
<tr>
<td>10</td>
<td>3.5</td>
<td>3.5</td>
<td>6.8</td>
</tr>
<tr>
<td>20</td>
<td>5.6</td>
<td>2.6</td>
<td>8.0</td>
</tr>
<tr>
<td>50</td>
<td>7.1</td>
<td>2.0</td>
<td>8.8</td>
</tr>
<tr>
<td>100</td>
<td>7.4</td>
<td>1.9</td>
<td>8.9</td>
</tr>
</tbody>
</table>

crease in the supply of labor and thus GDP are smaller during the transition and in the long run in this case, since a lower elasticity of substitution between consumption and leisure implies that consumers are less willing to substitute consumption for leisure in response to a change in the after-tax wage relative to the benchmark simulation. In the long run this leads to significantly smaller percentage increases in net investment and firm values relative to the benchmark simulations. For example, the percentage increase in the total value of equity (and thus net investment) in the owner-occupied housing sector is only 4.8 percent rather than 8.6 percent as in the benchmark simulations. The percentage change in the total value of equity in the non-housing (rental housing) sector is 13.6 (-0.3) percent rather than 18 (4.3) percent. Figure 2.28 shows that every generation younger than age 40 experiences smaller (larger) welfare gains (losses) with a lower intratemporal elasticity of substitution. This pattern occurs because smaller efficiency gains are achieved in the long run relative to the benchmark case since the overall labor supply response is response is smaller for a given change in the after-tax wage.
2.5 Conclusion and Future Research

This paper has examined the effects of implementing a Flat Tax on the value of assets in the non-housing, rental housing, and owner-occupied housing sectors. In particular, this paper focused on the two potential problems that often arise in discussions of the feasibility of consumption tax reform: (1) the potential negative effect of consumption taxes on the value of the existing housing stock, and (2) the tendency of such a reform to impose a one-time windfall loss on the owners of existing (non-owner-occupied housing) capital. The results suggest that the reform-induced one-time windfall loss on the owners of existing capital (other than owner-occupied housing) tends to be overstated in models that do not explicitly account for owner-occupied housing, and that the potential decline in the value of owner-occupied housing could be as large as 10 percent immediately after reform. The intergenerational redistribu-
tions associated with the implementation of a Flat Tax suggest that older generations actually gain after the reform since their assets earn above normal returns and the after-tax interest rate increases in the short-run. Younger generations also benefit more in this case because the efficiency gains from reform are larger than in the typical single production sector models. Higher adjustment costs increase the number of young and middle-aged generations that suffer losses, since the efficiency effects in this case tend to occur more slowly over time.

The distinguishing feature of the model used in this paper is the treatment of the housing market; the investment demands for owner-occupied and rental housing are endogenously determined, the tax advantage of owner-occupied housing relative to other assets under the current income tax is taken into account, and adjustment costs in the housing market are included explicitly. Thus, this model allows for a more detailed description of the effects of implementing a consumption-based tax reform on the housing and non-housing markets.

Single production sector models obviously cannot capture the efficiency gains from eliminating the distortions associated with the allocation of capital across sectors, which would tend to offset the reform-induced effects on the value of existing assets (other than owner-occupied housing). In particular, the current income tax seriously distorts the allocation of investment across owner-occupied and other business assets — an important effect since owner-occupied housing is the single largest component of the capital stock. In addition, the speed with which capital would move out of owner-occupied housing and into other production sectors would be affected by the magnitude of the costs of adjusting the capital stock. Since the reallocation involved could be sizable, the adjustment costs associated with moving capital out of owner-occupied housing and into other sectors could be quite important. Relatively large adjustment costs would imply that the owners of existing business assets would earn
inframarginal returns over a longer transition period, but that the efficiency gains of reform would be delayed.

One particularly interesting possibility for future research is the extension of the model to include 12 income classes similar to AAKSW. It is possible that including housing would reduce to some extent the welfare gains enjoyed by the highest income households. This seems likely since (1) they would experience a potentially significant welfare loss due to the decline in house values attributable to the reform-induced reduction in demand for expensive homes, but (2) they would benefit to a relatively small extent from the exclusion of owner-occupied housing from the capital levy attributable to the introduction of a consumption tax. Since the gains from a consumption tax reform are concentrated among the wealthy (see AAKSW), the resulting pattern of intragenerational redistributions could be less regressive than that predicted by an analysis that ignores owner-occupied housing.
Bibliography


Appendix A

Mathematical Description

A.1 Model Conventions

The following discussion provides the details of the model employed in examining the effects of implementing a Flat Tax. The model is constructed in discrete time and adopts the following conventions. All flow variables, including consumption, labor supply, saving, and investment occur at the end of the period. The individual’s utility level that is associated with current levels of consumption and leisure, and the issuance of debt and new share issues, which are related to saving and investment, occur at the end of the period. Note that the prices related to each of these flow variables are also determined at the end of the period.

All stock variables, including asset values, capital stocks, and the stock of debt are measured at the beginning of the period. That is, stock values at the beginning of the period reflect flow values at the end of the previous period.

Present values for consumption are measured at the end of period zero. Thus, period zero consumption is not discounted, and period one consumption is discounted for one full year.

Since present values are taken at the end of the period, asset values measured at the beginning of the period must be adjusted by interest for one period to be comparable with consumption, wages and other prices measured at the end of the period.

Since individual asset values are calculated at the beginning of the period, firm values must be calculated at the beginning of the period also; that is, present values for
production are measured at the beginning of the period. All firm cash flow variables, including investment, sales, labor purchases, etc., occur at the end of the period. Thus, in calculating the value of the firm, the first period term is discounted (measured at beginning of period with cash flows occurring at the end of the period).

Age \( a \) is defined at time \( t \), so it changes with time. Thus, an individual age 10 at \( t = 0 \) is age 30 at \( t = 20 \). Birthdays (and deaths) occur at the beginning of the period, as do inheritances and bequests. Thus, an individual is zero all of period zero and one all of period one.

Reform occurs at the beginning of period 1, thus affecting only \( C, S, I \) decisions made at the end of period 1, not in period zero. This implies that period zero \( C_0, S_0, I_0, K_0 \) and \( A_0 \) and period 1 \( K_1 \) are fixed in the initial income tax equilibrium. The first post-reform values are \( C_1, S_1 \) and \( I_1 \), and thus \( K_2 \). The enactment of reform at the beginning of period 1 is unexpected.

The \( q \) variable is measured at the beginning of the period (as are firm values and capital stocks). Thus, \( q_0 \) is fixed as the value of an increase in the capital stock in period zero under the income tax regime, and \( q_1 \) is the first new value of \( q \), measured at the beginning of period one (that is, when reform occurs). The first value of \( q \) relevant to new investment decisions, however, is \( q_2 \), since end-of-period-one investment \( I_1 \) is a function of beginning of period two \( q_2 \) and not \( q_1 \).

### A.2 Individual Behavior

Individual behavior is modeled using an overlapping generations framework that consists of 55 cohorts, denoted by ages that range from zero to 54, as the individual life span is known (with certainty) to be 55 years. Each generation is represented by a single individual, who has an economic life span of 55 years, works for the first 45 of those years, and is retired for the last 10 years. Individual tastes are identical so
that differences in behavior across generations are due solely to differences in lifetime budget constraints.

An individual age $a$ at the beginning of period $s$ has assets $A_s(a)$ which have been accumulated from the time of "economic birth" and are eventually used to fund (1) consumption over the individual’s lifetime, including that during the retirement period, and (2) the bequest. Assets include inheritances received. Inheritances are received at age 25 and bequests are given at the time of death, which implies that $A_s(55) = BQ_s$ where $INH_s = BQ_s = BQ_t(1 + n)^{s-t}(1 + g)^{s-t}$, where $n$ denotes the population growth rate and $BQ_t$ is the size of the bequest in period $t$, typically the initial equilibrium ($t = 0$). Note that this implies a relatively primitive "target model" of bequests, as bequests are assumed to be completely insensitive to changes in economic conditions, including changes in income.

In period $s$, an individual age $a$ chooses total consumption $\bar{C}_s(a)$ and leisure $E_s(a)$ for $s = t, ..., t + 54 - a$ to maximize rest-of-life utility $LU_s$.

$$LU_s(a) = \frac{1}{(1 - \frac{1}{\sigma_1})} \sum_{s=t}^{t+54-a} \frac{U_s(a)^{(1-\frac{1}{\sigma_1})}}{(1 + \rho)^{s-t}},$$  \hspace{1cm} (A.1)

where $\sigma_1$ is the intertemporal elasticity of substitution, $\rho$ is the pure rate of time preference, $U_s(a)$ is utility in each period $s$. Intraperiod utility is modelled as a CES function of total consumption $\bar{C}_s(a)$ and leisure $E_s(a)$,

$$U_s(a) = \left[ \alpha_\bar{C} \bar{C}_s(a)^{\frac{\sigma_2 - 1}{\sigma_2}} + \alpha_E E_s(a)^{\frac{\sigma_2 - 1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_2 - 1}},$$  \hspace{1cm} (A.2)

where $\sigma_2$ is the intertemporal elasticity of substitution between total consumption and leisure in any period, $\alpha_\bar{C}$ and $\alpha_E$ measure the relative intensities of household preferences for leisure and consumption (with $\alpha_\bar{C} + \alpha_E = 1$), and leisure is defined as $E_s(a) = H_T - L_s(a)$, where $H_T$ is the total number of hours available in a period for either labor supply $L_s(a)$ or leisure.
The composite consumption good is modelled as a CES function of non-housing goods and housing services,

\[ \bar{C}_s(a) = \left[ \alpha_G^{\frac{1}{\sigma_3}} C_s^{\frac{\sigma_3 - 1}{\sigma_3}} + \alpha_H^{\frac{1}{\sigma_3}} H_s^{\frac{\sigma_3 - 1}{\sigma_3}} \right]^{\frac{\sigma_3}{\sigma_3 - 1}}, \tag{A.3} \]

where \( C_s \) is the composite good including all non-housing related goods in period \( s \), \( H_s \) is the composite good of housing services in period \( s \), \( \alpha_G \) and \( \alpha_H \) measure the relative intensities of household preferences for non-housing goods and housing goods, and \( \sigma_3 \) is the elasticity of substitution between housing and non-housing goods.\(^{24}\)

The composite consumption good is modelled as a CES function of the quantities of the taxed and non-taxed goods in excess of the minimum required purchases,

\[ C_s(a) = \left[ \alpha_1^{\frac{1}{\sigma_4}} (C_{1s} - b_{1s})^{\frac{\sigma_4 - 1}{\sigma_4}} + \alpha_2^{\frac{1}{\sigma_4}} (C_{2s} - b_{2s})^{\frac{\sigma_4 - 1}{\sigma_4}} \right]^{\frac{\sigma_4}{\sigma_4 - 1}}, \tag{A.4} \]

where \( C_{is} \) is the consumption of good \( i \) in period \( s \), \( b_{is} \) is the minimum required purchase of good \( i \) in each period \( s \), and \( \sigma_4 \) is the elasticity of substitution between "above-minimum" quantities of the taxed and non-taxed goods.

Composite consumption of housing services, \( H_s \), are given by a CES function of the quantities of rental housing services and owner-occupied housing services in excess of the minimum required purchases,

\[ H_s(a) = \left[ \alpha_O^{\frac{1}{\sigma_5}} (OH_s - b_{Os})^{\frac{\sigma_5 - 1}{\sigma_5}} + \alpha_R^{\frac{1}{\sigma_5}} (RH_s - b_{Rs})^{\frac{\sigma_5 - 1}{\sigma_5}} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}, \tag{A.5} \]

where \( OH_s(a) \) is the quantity of owner housing services consumed by an individual age \( a \) in period \( s \), \( RH_s(a) \) is the quantity of rental housing service consumed by an individual age \( a \) in period \( s \), \( b_{Os} \) and \( b_{Rs} \) represent the minimum purchase requirements of owner-occupied and rental housing, \( \sigma_5 \) is the elasticity of substitution between "above-minimum" quantities owner-occupied housing and rental housing

\(^{24}\)The age subscript has been dropped in order to simplify the notation.
services, and $\alpha_O$ and $\alpha_R$ measure the relative intensities of household preferences for owner-occupied and rental housing (with $\alpha_O + \alpha_R = 1$).

The lifetime utility function $LU_s(a)$ is maximized subject to the lifetime budget constraint (with present values taken at the end of period $s$)

\[
0 = TW_s(a) - \sum_{s=t}^{t+54-a} \frac{p_s(1 + \tau_{cs})(C_{1s} - b_{1s}) + p_s(C_{2s} - b_{2s})}{\Pi_{u=t+1}^s [1 + i_u(1 - \tau_{iu})]}
- \sum_{s=t}^{t+54-a} \frac{p_{Os}(OH_s - b_{Os}) + p_{Rs}(RH_s - b_{Rs})}{\Pi_{u=t+1}^s [1 + i_u(1 - \tau_{iu})]}
\]

where $TW_s(a)$ is total wealth available for consumption expenditures at the end of period $s$ for an individual age $a$ in period $s$. $p_s$ is the real price of the consumption good — which is chosen to be the numeraire so that $p_s = 1$, $\tau_c$ is the state retail sales tax rate, $p_{Os}$ is the unit price of owner-occupied housing services, and $p_{Rs}$ is the unit price of rental housing services. Individuals discount at the after-tax discount rate $r_s$

\[r_s = i_s(1 - \tau_{is}),\]

where $\tau_{is}$ is the tax rate on interest income under the income tax in period $s$.

Total wealth available for discretionary consumption expenditures (those in excess of minimum required expenditures) for an individual age $a$ at the end of period $s$ is given by

\[
TW_s(a) = A_s(a)(1 + r_s) + TDA_s(1 + i_s) - \frac{BQ_{t+55-a}}{\Pi_{u=t+1}^{t+54-a}(1 + r_u)}
+ \sum_{s=t}^{t+54-a} \frac{[w_s(a)(H_T - E_s) - SD_s](1 - \tau_{ws} - \tau_{ss}) + SSB_s}{\Pi_{u=t+1}^s (1 + r_u)}
+ \sum_{s=t}^{t+54-a} \frac{WD_s(1 - \tau_{ws}) + TR_s + LSR_s}{\Pi_{u=t+1}^s (1 + r_u)}
- \sum_{s=t}^{t+54-a} \frac{p_s(1 + \tau_{cs})b_{1s} + p_s b_{2s} + p_{Os} b_{Os} + p_{Rs} b_{Rs}}{\Pi_{u=t+1}^s (1 + r_u)},
\]
where $A_s(a)$ is the value of taxable assets (including the value of the inheritance) in period $s$ for an individual age $a$, $\text{TDA}_s(a)$ is the value of tax-deferred assets at time $s$ for an individual age $a$, $BQ_{t+55-a}$ is the exogenous bequest, $w_s(a)$ is the wage rate earned by an individual of age $a$ in period $s$, $L_s$ is labor supplied in period $s$, $SD_s$ is the mandatory tax-deferred savings requirement in period $s$, which is equal to $SD$ during working years and is zero during the retirement years, $WD_s$ is the withdrawal from the tax-deferred saving account in each of the retirement years, $t_{ws}$ is the average tax rate on wage income in period $s$, $t_{ss}$ is the social security tax on wage income in period $s$. $SSB_s$ represents social security benefits received in period $s$, $TR_s$ is transfers received in period $s$ (which are modeled as uniformly distributed across all 55 generations and growing at the rate of technological growth in the economy $(g)$), and $p_i b_i$ is the cost of meeting the minimum purchase requirement for the consumption good $i$ in period $s$. Note that, following Auerbach and Kotlikoff (1987), the wage rate is defined as $w_s(a) = w_s e(a)$, where $w_s$ is the economy-wide equilibrium wage and $e(a)$ is the individual's "human capital profile" that varies in a "hump-backed" fashion to reflect changes in productivity over the life cycle.

Dropping the notation for age and letting $[U^*, \bar{C}^*, \bar{C}]$ equal the expressions in the brackets in the definitions of $U_s$, $\bar{C}_s$, and $C_s$ the first order conditions with respect to $C_{1s}$ and $C_{2s}$ imply

$$
\frac{U_s^{\frac{1}{s}} [U^*]^{\frac{s-1}{2s-1}} \alpha C_s^{\frac{1}{2s}} \bar{C}_s^{\frac{1}{2s}} [\bar{C}^*]^{\frac{s-1}{2s}} \alpha \bar{C}_s^{\frac{1}{2s}} C^{\frac{1}{2s}} [C^*]^{\frac{s-1}{2s}} \alpha C_s^{\frac{1}{2s}} (C_{1s} - b_{1s})^{\frac{s}{2s}}}{(1 + \rho)^{s-1}} = \lambda \left\{ \frac{p_s (1 + \tau_{cs})}{\prod_{u=t+1}^s (1 + r_u)} \right\}
$$

$$
\frac{U_s^{\frac{1}{s}} [U^*]^{\frac{s-1}{2s-1}} \alpha \bar{C}_s^{\frac{1}{2s}} C_s^{\frac{1}{2s}} [\bar{C}^*]^{\frac{s-1}{2s}} \alpha C_s^{\frac{1}{2s}} C^{\frac{1}{2s}} [C^*]^{\frac{s-1}{2s}} \alpha \bar{C}_s^{\frac{1}{2s}} (C_{2s} - b_{2s})^{\frac{s}{2s}}}{(1 + \rho)^{s-1}} = \lambda \left\{ \frac{p_s}{\prod_{u=t+1}^s (1 + r_u)} \right\}.
$$

Taking the ratio of these two expressions yields

$$
\frac{\frac{1}{\alpha C_s^{\frac{1}{2s}}} (C_{1s} - b_{1s})^{\frac{s}{2s}}}{\frac{1}{\alpha C_s^{\frac{1}{2s}}} (C_{2s} - b_{2s})^{\frac{s}{2s}}} = (1 + \tau_{cs}),
$$
this expression can be written as

$$(C_{2s} - b_{2s}) = (C_{1s} - b_{1s}) \frac{a_{C2}}{a_{C1}} (1 + \tau_{cs})^{\sigma_4}.$$ 

Substituting into the definition of $C_s$ yields

$$C_s = \left[ \alpha_{C1}^{\frac{1}{\sigma_4}} (C_{1s} - b_{1s}) \right]^{\frac{\sigma_4 - 1}{\sigma_4}} + \alpha_{C2}^{\frac{1}{\sigma_4}} (C_{2s} - b_{2s}) \right]^{\frac{\sigma_4 - 1}{\sigma_4}} \right]^{\frac{\sigma_4}{\sigma_4 - 1}}$$

$$= \left[ \alpha_{C1}^{\frac{1}{\sigma_4}} (C_{1s} - b_{1s}) \right]^{\frac{\sigma_4 - 1}{\sigma_4}} + \alpha_{C2}^{\frac{1}{\sigma_4}} \left\{ (C_{1s} - b_{1s}) \frac{a_{C2}}{a_{C1}} (1 + \tau_{cs})^{\sigma_4} \right\}^{\frac{\sigma_4 - 1}{\sigma_4 - 1}} \right]^{\frac{\sigma_4}{\sigma_4 - 1}}$$

$$= \left( C_{1s} - b_{1s} \right) \left[ \alpha_{C1}^{\frac{1}{\sigma_4}} + \alpha_{C2} \frac{a_{C2}}{a_{C1}} \left( 1 + \tau_{cs} \right)^{\sigma_4 - 1} \right]^{\frac{\sigma_4}{\sigma_4 - 1}}$$

solving for $(C_{1s} - b_{1s})$

$$(C_{1s} - b_{1s}) = C_s \left[ \alpha_{C1}^{\frac{1}{\sigma_4}} + \alpha_{C2} \frac{a_{C1}}{a_{C2}} \left( 1 + \tau_{cs} \right)^{\sigma_4 - 1} \right]^{\frac{\sigma_4}{\sigma_4 - 1}},$$

$$= C_s \left\{ \alpha_{C1}^{\frac{1}{\sigma_4}} \left[ a_{C1} + \alpha_{C2} (1 + \tau_{cs})^{\sigma_4 - 1} \right] \right\}^{\frac{\sigma_4}{\sigma_4 - 1}},$$

$$= C_s \alpha_{C1} \left[ a_{C1} + \alpha_{C2} (1 + \tau_{cs})^{\sigma_4 - 1} \right]^{\frac{\sigma_4}{\sigma_4 - 1}},$$

$$= C_s \alpha_{C1} \left\{ (1 + \tau_{cs})^{\sigma_4 - 1} \left[ a_{C1} (1 + \tau_{cs})^{1 - \sigma_4} + \alpha_{C2} \right] \right\}^{\frac{\sigma_4}{\sigma_4 - 1}},$$

$$= C_s \alpha_{C1} (1 + \tau_{cs})^{1 - \sigma_4} \left[ a_{C1} (1 + \tau_{cs})^{1 - \sigma_4} + \alpha_{C2} \right]^{\frac{\sigma_4}{1 - \sigma_4}}.$$

Solving next for $(C_{2s} - b_{2s})$, recall that

$$(C_{2s} - b_{2s}) = (C_{1s} - b_{1s}) \frac{a_{C2}}{a_{C1}} (1 + \tau_{cs})^{\sigma_4},$$

$$= \left\{ C_s \alpha_{C1} (1 + \tau_{cs})^{\sigma_4} \left[ a_{C1} (1 + \tau_{cs})^{1 - \sigma_4} + \alpha_{C2} \right]^{\frac{\sigma_4}{1 - \sigma_4}} \right\} \frac{a_{C2}}{a_{C1}} (1 + \tau_{cs})^{\sigma_4},$$

$$= C_s \alpha_{C2} \left[ a_{C1} (1 + \tau_{cs})^{1 - \sigma_4} + \alpha_{C2} \right]^{\frac{\sigma_4}{1 - \sigma_4}},$$

$$= C_s \alpha_{C2} \left[ a_{C1} (1 + \tau_{cs})^{1 - \sigma_4} + \alpha_{C2} \right]^{\frac{\sigma_4}{1 - \sigma_4}}.$$

Thus, given total non-housing related consumption $C_s$, the allocation of consumption between taxed and untaxed goods is

$$(C_{1s} - b_{1s}) = C_s \alpha_{C1} (1 + \tau_{cs})^{1 - \sigma_4} \left[ a_{C1} (1 + \tau_{cs})^{1 - \sigma_4} + \alpha_{C2} \right]^{\frac{\sigma_4}{1 - \sigma_4}}, \quad (A.8)$$
\[(C_{2s} - b_{2s}) = C_s \alpha C_2 (\alpha C_1 (1 + \tau_{cs})^{1 - \sigma^4} + \alpha C_2)^{\frac{\sigma^4}{1 - \sigma^4}}. \]  

(A.9)

These two terms can be substituted into (A.6), the expression for total discretionary wealth, to yield

\[TW_s(a) = \sum_{s=t}^{t+54-a} \frac{F_s C_s + p_{Os} (O_s - b_{Os}) + p_{Rs} (R_s - b_{Rs})}{\prod_{u=t+1}^{s} (1 + r_u)},\]

where \(F_{Cs}\) is given by

\[F_{Cs} = p_s [\alpha C_1 (1 + \tau_{cs})^{1 - \sigma^4} + \alpha C_2]^{\frac{1}{1 - \sigma^4}},\]

which is a function of parameters and the consumption tax rate in period \(s\). Note that when there are no untaxed goods, \(F_s\) reduces to \(p_s (1 + \tau_{cs})\).

The representative agent also must choose the optimal quantity of owner-occupied and rental housing services in order to maximize lifetime utility. Dropping the notation for age and letting \([U^*], [C^*],\) and \([H^*]\) equal the expressions in the brackets in the definitions of \(U_s, \bar{C}_s,\) and \(\bar{H}_s\) the first order conditions with respect to \(OH_s\) and \(RH_s\) imply

\[\frac{U_s^{\frac{1}{\sigma_s^2}} [U^*]^{\frac{1}{\sigma^2}} \alpha C_2^{\frac{1}{3}} \bar{C}_s^{\frac{1}{3}} [C^*]^{\frac{1}{\sigma_s^2 - 1}} \alpha H^{\frac{1}{3}} \bar{H}^{\frac{1}{3}} [H^*]^{\frac{1}{\sigma_s^2 - 1}} \alpha \bar{O}_s^{\frac{1}{3}} (O_s - b_{Os})^{\frac{1}{3}}}{(1 + \rho)^{s-t}} = \lambda \frac{p_{Os}}{\prod_{u=t+1}^{s} (1 + r_u)}\]

and

\[\frac{U_s^{\frac{1}{\sigma_s^2}} [U^*]^{\frac{1}{\sigma^2}} \alpha C_2^{\frac{1}{3}} \bar{C}_s^{\frac{1}{3}} [C^*]^{\frac{1}{\sigma_s^2 - 1}} \alpha H^{\frac{1}{3}} \bar{H}^{\frac{1}{3}} [H^*]^{\frac{1}{\sigma_s^2 - 1}} \alpha \bar{R}_s^{\frac{1}{3}} (R_s - b_{Rs})^{\frac{1}{3}}}{(1 + \rho)^{s-t}} = \lambda \frac{p_{Rs}}{\prod_{u=t+1}^{s} (1 + r_u)}\].

Taking the ratio of these two expressions yields

\[\frac{\alpha \bar{O}_s^{\frac{1}{3}} (O_s - b_{Os})^{\frac{1}{3}}}{\alpha \bar{R}_s^{\frac{1}{3}} (R_s - b_{Rs})^{\frac{1}{3}}} = \frac{p_{Os}}{p_{Rs}},\]

this expression can be written as

\[(R_s - b_{Rs}) = (O_s - b_{Os}) \frac{\alpha \bar{R}_s (p_{Os})^{\sigma_s^2}}{\alpha \bar{O}_s (p_{Rs})^{\sigma_s^2}} .\]
Substituting into the definition of \( H_s \) yields

\[
H_s = \left[ \frac{1}{\alpha_O} (O_s - b_{O_s})^{\frac{\sigma_5}{\sigma_5}} + \frac{1}{\alpha_R} (R_s - b_{R_s})^{\frac{\sigma_5}{\sigma_5}} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

\[
= \left[ \frac{1}{\alpha_O} (O_s - b_{O_s})^{\frac{\sigma_5}{\sigma_5}} + \frac{1}{\alpha_R} (O_s - b_{O_s})^{\frac{\sigma_5}{\sigma_5}} \left( \frac{\alpha_R}{\alpha_O} \right)^{\frac{\sigma_5}{\sigma_5}} \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5 - 1} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

\[
= (O_s - b_{O_s}) \left[ \frac{1}{\alpha_O} + \frac{1}{\alpha_R} \left( \frac{\alpha_R}{\alpha_O} \right)^{\frac{\sigma_5}{\sigma_5}} \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5 - 1} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

solving for \((O_s - b_{O_s})\)

\[
(O_s - b_{O_s}) = H_s \left[ \frac{1}{\alpha_O} + \frac{1}{\alpha_R} \frac{P_{O_s}}{P_{R_s}}^{\frac{\sigma_5}{\sigma_5}} \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5 - 1} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

\[
= H_s \left[ \frac{1}{\alpha_O} \left[ \frac{P_{O_s}}{P_{R_s}} \right]^{\sigma_5 - 1} + \frac{1}{\alpha_R} \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5 - 1} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

\[
= H_s \left[ \frac{P_{O_s}}{P_{R_s}}^{\sigma_5 - 1} + \frac{1}{\alpha_O} \left[ \frac{P_{O_s}}{P_{R_s}} \right]^{\sigma_5 - 1} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

Solving next for \((R_s - b_{R_s})\), recall that

\[
(R_s - b_{R_s}) = (O_s - b_{O_s}) \frac{\alpha_R}{\alpha_O} \frac{P_{O_s}}{P_{R_s}}^{\sigma_5}
\]

\[
= \left\{ H_s \frac{P_{O_s}}{P_{R_s}}^{\sigma_5 - 1} \left[ \frac{P_{O_s}}{P_{R_s}} \right]^{\sigma_5 - 1} + \frac{1}{\alpha_R} \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5} \right\} \frac{\alpha_R}{\alpha_O} \frac{P_{O_s}}{P_{R_s}}^{\sigma_5}
\]

\[
= H_s \alpha_R \left[ \alpha_O \frac{P_{O_s}}{P_{R_s}}^{\sigma_5 - 1} + \frac{1}{\alpha_R} \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5} \right]^{\frac{\sigma_5}{\sigma_5 - 1}}
\]

Thus, given total housing consumption \( H_s \), the allocation of consumption between owner-occupied and rental housing is given by

\[
(O_s - b_{O_s}) = H_s \alpha_O \frac{P_{O_s}}{P_{R_s}}^{\sigma_5 - 1} \left[ \alpha_O \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5} + \alpha_R \right]^{\frac{\sigma_5}{\sigma_5 - 1}} \quad \text{(A.10)}
\]

\[
(R_s - b_{R_s}) = H_s \alpha_R \left[ \alpha_O \left( \frac{P_{O_s}}{P_{R_s}} \right)^{\sigma_5} + \alpha_R \right]^{\frac{\sigma_5}{\sigma_5 - 1}}. \quad \text{(A.11)}
\]
Substituting these terms into the expression for total discretionary wealth yields

$$TW_s(a) = \sum_{s=t}^{t+54-a} \frac{F_{Cs}C_s + F_{Hs}H_s}{\prod_{u=t+1}^s (1 + r_u)}, \tag{A.12}$$

where $F_{Hs}$, the price index for the composite housing good, is defined as

$$F_{Hs} = p_{Rs} [\alpha_{O} \left( \frac{P_{Oa}}{p_{Rs}} \right)^{1-\sigma_5} + \alpha_{R}]^{\frac{1}{1-\sigma_5}}.$$

The consumer chooses the optimal amounts of the non-housing and housing composite goods by maximizing utility with respect to the $C_s$ and $H_s$. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{(1 - \frac{1}{\sigma})} \sum_{s=t}^{t+54-a} \frac{U_s \left( 1 - \frac{1}{\sigma} \right)}{(1 + \rho)^{s-t}} + \lambda \left\{ \sum_{s=t}^{t+54-a} \frac{F_{Cs}C_s + F_{Hs}H_s}{\prod_{u=t+1}^s (1 + r_u)} \right\},$$

where, repeating from above,

$$U_s = [\alpha_{C}^\frac{1}{\sigma_2} C_s^\frac{\sigma_2-1}{\sigma_2} + \alpha_{E}^\frac{1}{\sigma_2} E_s^\frac{\sigma_2-1}{\sigma_2}]^\frac{\sigma_2}{\sigma_2-1} = [U^*_s]^\frac{\sigma_2}{\sigma_2-1},$$

$$\tilde{C}_s = A_1 [\alpha_{C}^\frac{1}{\sigma_3} C_s^\frac{\sigma_3-1}{\sigma_3} + \alpha_{H}^\frac{1}{\sigma_3} H_s^\frac{\sigma_3-1}{\sigma_3}]^\frac{\sigma_3}{\sigma_3-1} = [	ilde{C}_s^*]^\frac{\sigma_3}{\sigma_3-1},$$

$$H_s = [\alpha_{O}^\frac{1}{\sigma_5} (O_H - b_{Os})^\frac{\sigma_5-1}{\sigma_5} + \alpha_{R}^\frac{1}{\sigma_5} (R_H - b_{Rs})^\frac{\sigma_5-1}{\sigma_5}]^\frac{\sigma_5}{\sigma_5-1} = [H^*_s]^\frac{\sigma_5}{\sigma_5-1}.$$

The first order condition with respect to $C_s$ is

$$\frac{[U^*_s]^\frac{\sigma_2}{\sigma_2-1} [U^*_s]^\frac{\sigma_2}{\sigma_2-1} \alpha_{C}^\frac{1}{\sigma_2} \tilde{C}_s^\frac{\sigma_2}{\sigma_2-1} [\tilde{C}_s^*]^\frac{\sigma_3}{\sigma_3-1} \alpha_{C}^\frac{1}{\sigma_3} C_s^\frac{\sigma_3}{\sigma_3-1}}{(1 + \rho)^{s-t}} = \lambda \left\{ \frac{F_{Cs}}{\prod_{u=t+1}^s (1 + r_u)} \right\}$$

while, similarly, the first order condition with respect to $H_s$ is

$$\frac{[U^*_s]^\frac{\sigma_2}{\sigma_2-1} [U^*_s]^\frac{\sigma_2}{\sigma_2-1} \alpha_{C}^\frac{1}{\sigma_3} \tilde{C}_s^\frac{\sigma_3}{\sigma_3-1} [\tilde{C}_s^*]^\frac{\sigma_3}{\sigma_3-1} \alpha_{H}^\frac{1}{\sigma_3} H_s^\frac{\sigma_3}{\sigma_3-1}}{(1 + \rho)^{s-t}} = \lambda \left\{ \frac{F_{Hs}}{\prod_{u=t+1}^s (1 + r_u)} \right\}.$$
Taking the ratio of these two conditions yields
\[
\frac{\alpha_C^G C_s^{\sigma_3}}{\alpha_H H_s^{\sigma_3}} = \frac{F_{C_s}}{F_{H_s}}.
\]
Solving for \(H_s\)
\[
H_s = \left(\frac{F_{C_s}}{F_{H_s}}\right)^{\sigma_3} \frac{\alpha_H}{\alpha_G} C_s,
\]
and substituting into the definition of \(\bar{C}_s\) yields
\[
\bar{C}_s = A_1 \left[ \alpha_C^G C_s^{\sigma_3 - 1} + \alpha_H H_s^{\sigma_3 - 1} \right]^{\frac{\sigma_3}{\sigma_3 - 1}}
\]
\[
= A_1 \left[ \alpha_C^G C_s^{\sigma_3 - 1} + \alpha_H \left( \frac{F_{C_s}}{F_{H_s}} \right)^{\sigma_3 - 1} \frac{\alpha_H}{\alpha_G} C_s \right]^{\frac{\sigma_3}{\sigma_3 - 1}}
\]
\[
= C_s A_1 \left[ \alpha_C^G + \alpha_H \alpha_G^{\sigma_3 - 1} \left( \frac{F_{C_s}}{F_{H_s}} \right)^{\sigma_3 - 1} \right]^{\frac{\sigma_3}{\sigma_3 - 1}}
\]
solving for \(C_s\)
\[
C_s = \frac{\bar{C}_s}{A_1} \left[ \alpha_C^G + \alpha_H \alpha_G^{\sigma_3 - 1} \left( \frac{F_{C_s}}{F_{H_s}} \right)^{\sigma_3 - 1} \right]^{\frac{\sigma_3}{\sigma_3 - 1}}
\]
\[
= \frac{\bar{C}_s}{A_1} \left[ \alpha_C^G \left( \frac{F_{C_s}}{F_{H_s}} \right)^{\sigma_3 - 1} \alpha_G \left( \frac{F_{C_s}}{F_{H_s}} \right)^{1-\sigma_3} + \alpha_H \right]^{\frac{\sigma_3}{\sigma_3 - 1}}
\]
Solving next for \(H_s\), recall that
\[
H_s = \left(\frac{F_{C_s}}{F_{H_s}}\right)^{\sigma_3} \frac{\alpha_H}{\alpha_G} C_s,
\]
\[
= \left(\frac{F_{C_s}}{F_{H_s}}\right)^{\sigma_3} \frac{\alpha_H}{\alpha_G} \bar{C}_s \left[ \alpha_G \left( \frac{F_{C_s}}{F_{H_s}} \right)^{1-\sigma_3} + \alpha_H \right]^{\frac{\sigma_3}{\sigma_3 - 1}}
\]
\[
= \frac{\bar{C}_s}{A_1} \alpha_H \left[ \alpha_G \left( \frac{F_{C_s}}{F_{H_s}} \right)^{1-\sigma_3} + \alpha_H \right]^{\frac{\sigma_3}{\sigma_3 - 1}}.
\]
Thus, given total composite consumption \(\bar{C}_s\), the allocation of consumption between non-housing and housing goods is given by
\[
C_s = \frac{\bar{C}_s}{A_1} \alpha_G \left( \frac{F_{C_s}}{F_{H_s}} \right)^{1-\sigma_3} \left[ \alpha_G \left( \frac{F_{C_s}}{F_{H_s}} \right)^{1-\sigma_3} + \alpha_H \right]^{\frac{\sigma_3}{\sigma_3 - 1}},
\]  
(A.13)
\[ H_s = \frac{\tilde{C}_s}{A_1} \alpha_H (\frac{F_{C_s}}{F_{H_s}})^{1-\sigma_3} + \alpha_H \frac{\sigma^3}{1-\sigma_3}. \]  

(A.14)

Substituting these terms into the expression for total discretionary wealth yields

\[ TW_s(a) = \sum_{s=t}^{t+54-a} \frac{F_{C_s} C_s + F_{H_s} H_s}{\prod_{u=t+1}^s [1 + r_u]}, \]

\[ TW_s(a) = \sum_{s=t}^{t+54-a} \frac{\tilde{C}_s F_{C_s} \alpha_G (\frac{F_{C_s}}{F_{H_s}})^{-\sigma_3} + F_{H_s} \alpha_H [\alpha_G (\frac{F_{C_s}}{F_{H_s}})^{1-\sigma_3} + \alpha_H]^{\frac{\sigma^3}{1-\sigma_3}}}{\prod_{u=t+1}^s [1 + r_u]}, \]

\[ TW_s(a) = \sum_{s=t}^{t+54-a} \frac{\tilde{C}_s F_{H_s} [\alpha_G (\frac{F_{C_s}}{F_{H_s}})^{1-\sigma_3} + \alpha_H]^{\frac{\sigma^3}{1-\sigma_3}}}{\prod_{u=t+1}^s [1 + r_u]}, \]

where

\[ F_s = \frac{F_{H_s} [\alpha_G (\frac{F_{C_s}}{F_{H_s}})^{1-\sigma_3} + \alpha_H]^{\frac{1}{\sigma_3}}} {A_1}, \]

and, repeating

\[ F_{C_s} = p_s [\alpha_{C1} (1 + \tau_{cs})^{1-\sigma_4} + \alpha_{C2}]^{\frac{1}{1-\sigma_4}}, \]

\[ F_{H_s} = p_{Rs} [\alpha_O (\frac{p_{O_s}}{p_{Rs}})^{1-\sigma_5} + \alpha_R]^{\frac{1}{1-\sigma_5}}. \]

The Lagrangian for the consumer optimization problem of choosing \( \tilde{C}_s \) and \( E_s \) is

\[ \mathcal{L} = \frac{1}{(1 - \frac{1}{\sigma_I})} \sum_{s=t}^{t+54-a} \frac{U_s (1 - \frac{1}{\sigma_I})}{(1 + \rho)^{s-t}} + \lambda \{ TW_s - \sum_{s=t}^{t+54-a} \frac{F_s \tilde{C}_s}{\prod_{u=t+1}^s [1 + r_u]} \}. \]

The first order condition with respect to \( \tilde{C}_s \) is

\[ \frac{[U^*_s]^\frac{1}{\sigma_1} \sigma_1^2 \frac{\sigma_2^2}{\sigma_1} [U^*_s]^\frac{1}{\sigma_2} \sigma_2^2 \frac{\sigma^2}{\sigma_2} \alpha_{C1} \tilde{C}_s^{\frac{1}{\sigma_1} \sigma_1^2} }{(1 + \rho)^{s-t}} = \lambda \left\{ \frac{F_s}{\prod_{u=t+1}^s [1 + r_u]} \right\}, \]

\[ \frac{[U^*_s]^\frac{1}{\sigma_2} \sigma_2^2 \frac{\sigma_1^2}{\sigma_2} \alpha_{C2} \tilde{C}_s^{\frac{1}{\sigma_2} \sigma_2^2} }{(1 + \rho)^{s-t}} = \lambda \left\{ \frac{F_s}{\prod_{u=t+1}^s [1 + r_u]} \right\}. \]
while, similarly, the first order condition with respect to \( E_s \) is

\[
\frac{[U_s^*]^{(\frac{1}{\sigma_s^2} - \frac{1}{\sigma_s^{2-1}})}}{(1 + \rho)^{s-t}} \alpha_E^{\frac{1}{\sigma_s^2}} E_s^{\frac{1}{\sigma_s^{2-1}}} \alpha_E^{\frac{1}{\sigma_s^{2-1}}} E_s^{\frac{1}{\sigma_s^{2-1}}} = \lambda \left\{ \frac{w_s(1 - \tau_{ws} - \tau_{ss})}{\prod_{u=t+1}^s [1 + r_u]} \right\}.
\]

Taking the ratio of these two conditions yields

\[
\frac{\alpha_C^{\frac{1}{\sigma_s^2}} C_s^{\frac{1}{\sigma_s^{2-1}}}}{\alpha_E^{\frac{1}{\sigma_s^2}} E_s^{\frac{1}{\sigma_s^{2-1}}}} = \frac{F_s}{w_s(1 - \tau_{ws} - \tau_{ss})}.
\]

Solving for \( E_s \)

\[
E_s = \left( \frac{F_s}{w_s(1 - \tau_{ws} - \tau_{ss})} \right)^{\sigma^2} \left( \frac{\alpha_E}{\alpha_C} \right) C_s.
\]

Substituting into \([U_s^*]\) yields

\[
[U_s^*] = \left[ \alpha_E^{\frac{1}{\sigma_s^2}} C_s^{\frac{1}{\sigma_s^{2-1}}} + \alpha_E^{\frac{1}{\sigma_s^2}} E_s^{\frac{1}{\sigma_s^{2-1}}} \right] = \alpha_C^{\frac{1}{\sigma_s^2}} C_s^{\frac{1}{\sigma_s^{2-1}}} + \alpha_E^{\frac{1}{\sigma_s^2}} \left\{ \frac{F_s}{w_s(1 - \tau_{ws} - \tau_{ss})} \right\}^{\sigma^2} \left( \frac{\alpha_E}{\alpha_C} \right) C_s^{\frac{1}{\sigma_s^{2-1}}} = \alpha_C^{\frac{1}{\sigma_s^2}} + \alpha_E^{\left( \frac{1}{\sigma_s^2} - 1 \right)} C_s^{\frac{1}{\sigma_s^{2-1}}} = \alpha_C^{\frac{1}{\sigma_s^2}} + \alpha_E^{\left( \frac{1}{\sigma_s^2} - 1 \right)} \left( \frac{F_s}{w_s(1 - \tau_{ws} - \tau_{ss})} \right)^{\sigma^2 - 1} \left( -1 \right) C_s^{\frac{1}{\sigma_s^{2-1}}} = \alpha_C^{\frac{1}{\sigma_s^2}} + \alpha_E^{\left( \frac{1}{\sigma_s^2} - 1 \right)} \frac{w_s(1 - \tau_{ws} - \tau_{ss})}{F_s} \left( 1 - \sigma^2 \right) \alpha_C^{\frac{1}{\sigma_s^2}} C_s^{\frac{1}{\sigma_s^{2-1}}}
\]

or

\[
[U_s^*] = FU_s \alpha_C^{\left( \frac{1}{\sigma_s^2} - 1 \right)} C_s^{\frac{1}{\sigma_s^{2-1}}}
\]

where

\[
FU_s = \alpha_C + \alpha_E \left( \frac{w_s(1 - \tau_{ws} - \tau_{ss})}{F_s} \right)^{(1 - \sigma^2)}.
\]

Substituting back into the first order condition for \( C_s \) yields

\[
\frac{[U_s^*]^{(\frac{1}{\sigma_s^2} - \frac{1}{\sigma_s^{2-1}})} A_C^{\frac{1}{\sigma_s^2}} C_s^{\left( \frac{1}{\sigma_s^2} \right)}}{(1 + \rho)^{s-t}} = \lambda \left\{ \frac{F_s}{\prod_{u=t+1}^s [1 + r_u]} \right\}
\]

\[
[FU_s \alpha_C^{\left( \frac{1}{\sigma_s^2} - 1 \right)} C_s^{\frac{1}{\sigma_s^{2-1}}}] A_C^{\frac{1}{\sigma_s^2}} C_s^{\left( \frac{1}{\sigma_s^2} \right)} = \lambda \left\{ \frac{F_s}{\prod_{u=t+1}^s [1 + r_u]} \right\}
\]
Thus, the first order conditions for total consumption in any two periods \( \bar{C}_s \) and \( \bar{C}_{s-1} \) imply that

\[
\frac{FU_s \alpha C_s^{(\frac{1}{\eta_s} - 1)} \bar{C}_s^{\frac{1}{\eta_s}}}{(1 + \rho)^{s-t}} \alpha \bar{C}_s^{\frac{1}{\eta_s}} \bar{C}_s^{\frac{1}{\eta_s}} = \lambda \left\{ \frac{F_s}{\prod_{u=t+1}^{s} [1 + r_u]} \right\}
\]

\[
\frac{FU_{s-1} \alpha C_{s-1}^{(\frac{1}{\eta_{s-1}} - 1)} \bar{C}_{s-1}^{\frac{1}{\eta_{s-1}}}}{(1 + \rho)^{s-t-1}} \alpha \bar{C}_{s-1}^{\frac{1}{\eta_{s-1}}} \bar{C}_{s-1}^{\frac{1}{\eta_{s-1}}} = \lambda \left\{ \frac{F_{s-1}}{\prod_{u=t}^{s-1} [1 + r_u]} \right\}
\]

Taking the ratio of these two equations and multiplying by \((1 + \rho)\) yields

\[
\frac{\bar{C}_s^{(\frac{1}{\eta_s})}}{\bar{C}_{s-1}^{(\frac{1}{\eta_{s-1}})}} \frac{FU_s^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}{FU_{s-1}^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1} = \frac{(1 + \rho)}{(1 + r_s)} \frac{F_s}{F_{s-1}}
\]

\[
\frac{\bar{C}_s^{(\frac{1}{\eta_s})}}{\bar{C}_{s-1}^{(\frac{1}{\eta_{s-1}})}} = \frac{(1 + r_s)}{(1 + \rho)} \frac{F_s}{F_{s-1}} \frac{FU_s^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}{FU_{s-1}^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}
\]

\[
\bar{C}_s = \frac{(1 + r_s)}{(1 + \rho)} \frac{F_s}{F_{s-1}} \frac{FU_s^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}{FU_{s-1}^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}
\]

which implies the optimal time path of \( \bar{C}_s(a) \)

\[
\bar{C}_s(a) = \left\{ \frac{(1 + r_s)}{(1 + \rho)} \frac{F_s}{F_{s-1}} \frac{FU_s^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}{FU_{s-1}^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1} \right\} \bar{C}_{s-1}(a) \tag{A.15}
\]

\[
= \left\{ \frac{(1 + r_s)}{(1 + \rho)} \frac{F_s}{F_{s-1}} \right\} \sigma_1 \{ RFU_s \} \bar{C}_{s-1}(a)
\]

where

\[
RFU_s = \left[ \frac{FU_s^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1}{FU_{s-1}^{[(\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}) \frac{\eta_{s-1}}{\eta_s}]} - 1} \right].
\]

Substituting (A.15) into (A.12) repeatedly and continuing this process until \( s = t \) yields

\[
TW_t(a) = \sum_{s=t}^{t+54-a} \frac{F_s \left\{ \frac{(1 + r_s)}{(1 + \rho)} \frac{F_{s-1}}{F_s} \right\} \sigma_1 \bar{C}_{s-1}(a)}{\prod_{u=t+1}^{s} [1 + r_u]}
\]

\[
= \sum_{s=t}^{t+54-a} \frac{(1 + r_s)^{\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}} (F_{s-1})^{\sigma_1 (F_s)^{(1 - \sigma_1)} R F U_s} \bar{C}_{s-1}(a)}{[1 + r_s]^{(1 - \sigma_1) \prod_{u=t+1}^{s} [1 + r_u]}}
\]

\[
= \sum_{s=t}^{t+54-a} \frac{(1 + r_s)^{\frac{1}{\eta_s} - \frac{1}{\eta_{s-1}}} (F_{s-1})^{\sigma_1 (F_s)^{(1 - \sigma_1)} R F U_s} \left\{ \frac{F_{s-1}^{\frac{1}{\eta_{s-1}}}}{(1 + \rho)} \bar{C}_{s-1}(a) \right\}}{[1 + r_s]^{(1 - \sigma_1) \prod_{u=t+1}^{s} [1 + r_u]}}
\]

\[
= \sum_{s=t}^{t+54-a} \frac{(1 + r_s)^{-2 \sigma_1 (F_s) \sigma_1 (F_s)^{(1 - \sigma_1)}} R F U_s \bar{C}_{s-2}(a)}{\prod_{u=s-1}^{s} [1 + r_u]^{(1 - \sigma_1) \prod_{u=t+1}^{s} [1 + r_u]}} \left( \prod_{u=s-1}^{s} R F U_u \right) \bar{C}_{s-2}(a)...
\]
This can be written as

\[ TW_t(a) = \left\{ \sum_{s=t}^{t+54-a} \frac{(1 + \rho)^{-(s-t)\sigma}(F_{s-2})^{\sigma}(F_s)^{(1-\sigma)}}{\prod_{u=t+1}^s (1 + r_u)^{(1-\sigma)}} \right\} (\prod_{u=t+1}^s RFU_u) \tilde{C}_t(a). \]

Defining

\[ \Psi_t(a) = \left\{ \sum_{s=t}^{t+54-a} \frac{(1 + \rho)^{-(s-t)\sigma}(F_{s-2})^{\sigma}(F_s)^{1-\sigma}}{\prod_{u=t+1}^s (1 + i_u(1 - \tau_{iu}))^{(1-\sigma)}} \right\} (\prod_{u=t+1}^s RFU_u) \]

implies

\[ \tilde{C}_t(a) = \frac{TW_t(a)}{\Psi_t(a)}. \] (A.16)

The consumption path for the remainder of the life for an individual age \( a \) is specified by A.16, and the consumption levels of taxed non-housing goods, untaxed non-housing goods, owner-occupied housing, and rental housing are given by

\[ C_{1s} - b_{1s} = \tilde{C}_s \alpha C_1 [\alpha C_1 + \alpha C_2 (1 + \tau_{cs})^{\sigma_3 - 1} \left( \frac{P_s}{P_s} \right)^{\sigma_3 - 1}]^{\frac{s-3}{1-\sigma_3}}, \]

\[ C_{2s} - b_{2s} = \tilde{C}_s \alpha C_2 [\alpha C_1 (1 + \tau_{cs})^{1-\sigma_3} \left( \frac{P_s}{P_s} \right)^{1-\sigma_3} + \alpha C_2]^{\frac{s-3}{1-\sigma_3}}. \]

\[ (O_s - b_{Os}) = H_s \alpha O [\alpha O + \alpha R \left( \frac{PO_s}{P_R} \right)^{\sigma_5 - 1}]^{\frac{s_5}{1-\sigma_5}}, \]

\[ (R_s - b_{Rs}) = H_s \alpha R [\alpha O \left( \frac{PO_s}{P_R} \right)^{1-\sigma_5} + \alpha R]^{\frac{s_5}{1-\sigma_5}}. \]

A.2.1 Tax Deferred Assets

As is well known, the current income tax includes a variety of features similar to consumption-based taxes. In particular, a variety of tax deferred saving plans that allow current deductions for saving and tax free accumulation, coupled with full taxation of all withdrawals, are available under current law; these include saving in
pension funds and in vehicles such as 401 (k) and 403 (b) plans. Keogh plans, and, to a lesser extent, IRAs. Accordingly, the availability of tax deferred assets under the income tax is explicitly taken into account in the model, using the following formulation — which is fairly simple but nevertheless rich enough to capture the most important transition effects of the reform. In each period of an individual’s working life, a fixed amount $SD$ is assumed to be deposited in a tax deferred account. Thus, for an individual who is age $a$ in period $t$ (when reform is enacted) and is in the labor force (and so is between the ages of $a = 0$ and $a = 44$), the value of tax deferred assets at time $t$ is

$$TD_{A_t}(a) = \sum_{v=t-a}^{t-1} SD_v \prod_{u=v+1}^{t-1} (1 + i_u).$$

where $SD_v$ represents the amount of wage earnings that are deposited in the tax deferred asset account in period $v$ and thus earn interest at the tax-free rate of return $i_u$ until they are withdrawn from the account. (Recall that wages and thus saving in the tax deferred account does not occur until the end of the period, while the value of assets is measured at the beginning of the period.)

Upon reaching the retirement age of 45, individuals begin making withdrawals from their tax deferred accounts, with assets remaining in the account continuing to accumulate at the before-tax rate of return. Specifically, individuals are assumed to make ten equal withdrawals at the end of the periods in which they are of age 45 through 54. Each of these ten equal payments is denoted $WD$. To calculate $WD$, suppose that an individual retires in period $t^*$ (which could be before or after the period in which reform is enacted $t$). Following the first year of retirement (at the beginning of period $t^* + 1$ in which the individual has an economic age of 46 years), the value of the tax deferred asset account is

$$TD_{A_{t^*+1}}(a) = TD_{A_{t^*}}(a)(1 + i_{t^*}) - WD.$$
Similarly, after the second year of retirement, \( t^* + 2 \), the value of the tax deferred asset account equals

\[
TDA_{t^*+2}(a) = TDA_{t^*+1}(a)(1 + i_{t^*+1}) - WD
\]
\[
= [TDA_{t^*}(a)(1 + i_{t^*}) - WD](1 + i_{t^*+1}) - WD
\]
\[
= TDA_{t^*}(a)(1 + i_{t^*})(1 + i_{t^*+1}) - WD[1 + (1 + i_{t^*+1})],
\]

Continuing in this fashion until period \( t^* + 10 \) when the tax deferred assets by assumption are completely depleted (the final withdrawal occurs at the end of period \( t^* + 9 \)) implies

\[
TDA_{t^*+10}(a) = 0 = TDA_{t^*+9}(a)(1 + i_{t^*+9}) - WD
\]
\[
= TDA_{t^*+9}(a)(1 + i_{t^*+9}) - WD
\]
\[
= TDA_{t^*} \prod_{u=t^*}^{t^*+9} (1 + i_u) - WD\{1 + \sum_{v=t^*+1}^{t^*+9} \prod_{u=v}^{t^*+9} (1 + i_u)\}.
\]

Thus, the amount of the withdrawal in each period of retirement is

\[
WD = \frac{TDA_{t^*} \prod_{u=t^*}^{t^*+9} (1 + i_u)}{1 + \sum_{v=t^*+1}^{t^*+9} \prod_{u=v}^{t^*+9} (1 + i_u)}
\]

where the value of assets in the year of retirement \( t^* \) is

\[
TDA_{t^*}(a) = \sum_{v=t^*-45}^{t^*-1} SD_v \prod_{u=v+1}^{t^*-1} (1 + i_u).
\]

Generalizing to the case of an individual of any age \( a \) (ranging from negative values for unborn generations to an individual of age 54) at time \( t \) when reform is enacted yields

\[
WD = \frac{TDA_{t^*+45-a} \prod_{u=t^*-45-a}^{t^*-44-a} (1 + i_u)}{1 + \sum_{v=t^*-46-a}^{t^*-45-a} \prod_{u=v}^{t^*-44-a} (1 + i_u)}
\]

where

\[
TDA_{t^*+45-a}(a) = \sum_{v=t^*-a}^{t^*-44-a} SD_v \prod_{u=v+1}^{t^*-44-a} (1 + i_u).
\]
Note that $SD$ is assumed to be grow at the rate of productivity growth, so that although with population growth the amount of savings deposited in tax deferred accounts increases at the growth rate of the economy.

### A.2.2 The Tax System

The income tax is modeled as a progressive tax on labor income after the tax deferred saving deposit, $SD_s$, is deducted, coupled with flat rate taxes on capital income. The total tax burden on taxable wage income is defined as $[\psi + (\frac{\chi}{2})(w_sL_s - SD_s)](w_sL_s - SD_s)$, where $\chi > 0$ and $(w_sL_s - SD_s)$ is the individual's wage tax base in period $s$. This implies an average tax rate on labor income of

$$\tau_{wags} = \psi + (\frac{\chi}{2})(w_sL_s - SD_s)$$

and a marginal tax rate of

$$\tau_{ws} = \psi + \chi(w_sL_s - SD_s).$$

When $\chi = 0$, the income tax system is proportional.

Capital income is assumed to be taxed at flat rates $\tau_{ds}$ on dividends, $\tau_{id}$ on interest and $\tau_{gs}$ on capital gains. The tax rate on capital gains is an effective annual accrual rate, taking into account the benefits of tax deferral until gains are realized and tax exemption of gains transferred at death.

### A.2.3 Social Security

The social security benefit, $SSB_s$, is received by those who reach an economic age of 45 in year $s$ until death at age 54. The benefit payment depends on average wage earnings over the 45 working years and an assumed rate of replacement, $RSS$. 
Average wage earnings over the first 45 working years of the individuals life equal

\[ AV_\varepsilon_s = \sum_{a=1}^{45} w_{s-45+a}(h_{s-45+a}L_{e_{s-45+a}})/45 \]

where \( w_{s-45+a} \) represents the wage in year \( s + 45 - a \) and \( L_{e_{s-45+a}} \) represents effective labor in year \( s - 45 + a \) of individuals that reached age 45 in year \( s \). The benefit received is thus related to average earnings by

\[ SSB_s = RSS(AV_\varepsilon_s). \]

Assuming that social security is self financing (in each period) implies that annual social security taxes must equal annual benefit payments:

\[ \frac{\tau_{bs} \sum_{a=1}^{45} w_{s-45+a} (h_{s-45+a}L_{e_{s-45+a}})}{(1 + n)^{(a-1)}} = \frac{SSB_{s-u}}{(1 + n)^{(45+u)}}. \]

### A.3 Firm Behavior

#### A.3.1 Non-Housing Firm

The analysis assumes that firm managers act to maximize the value of the firm in a perfectly competitive environment in the absence of uncertainty. The approach utilized is based on Tobin's "\( q \)" theory of investment, as extended to include adjustment costs by Hayashi (1982). It is similar to the firm modeling approaches used by Goulder and Summers (1989) and Keuschnigg (1990).

The non-housing production sector is characterized by a Cobb-Douglas production function

\[ F(K_s, L_e) = (K_s)^{\alpha_1}(L_e)^{1-\alpha_1}, \quad 0 < \alpha_1 < 1, \quad (A.17) \]

where \( K_s \) denotes inputs of capital used for production in period \( s \), \( L_e \) denotes effective labor used for production in period \( s \), and \( \alpha_1 \) is the capital share parameter in the
Cobb-Douglas production function. The labor supply of a representative individual of age \( b \), \( L_s(b) \), is exogenous.

The size of the population age \( b \) in year \( s \), \( P_s(b) \), is assumed to grow at a constant rate \( n \), this implies that the entire population grows at a the same rate. The equation representing the size of the population age \( b \) is

\[
P_s(b) = P_t(b)(1 + n)^{s-t}.
\]

This implies that in any period \( s \), the total effective labor force is

\[
L_e_s = \sum_{b=0}^{44} P_s(b) L_s(b)
\]

Initial conditions for \( L_e_s \) and \( P_s \) are established at the beginning of period zero \( (s = 0) \), with reform occurring at the beginning of period one \( (t = s = 1) \).

Gross investment in period \( s \), \( I_s \), equals

\[
I_s = K_{s+1} - (1 - \delta) K_s
\]

where \( \delta \) is the rate of depreciation of the capital stock. Note that the investment good and the non-housing consumption good are assumed to be identical (i.e., produced using a single production function).

Following Goulder and Summers (1989), adjustment costs per unit of investment are assumed to be

\[
\Phi_s \left( \frac{I_s}{K_s} \right) = \frac{p_s \left( \frac{\beta}{2} \right) \left( \frac{I_s}{K_s} - \mu \right)^2}{\left( \frac{I_s}{K_s} \right)}
\]

where \( \beta \) and \( \mu \) are the adjustment cost parameters; higher values of \( \beta \) and lower values of \( \mu \) imply higher adjustment costs. The value of \( \mu \) is set equal to the steady state ratio of gross investment to capital; that is, \( \mu = \delta + n \). The price of the production good, \( p_s \), is assumed to be the numeraire. Note that for this adjustment cost function, the total derivative with respect to the ratio of investment to capital, \( \frac{I_s}{K_s} \), is
\[
\Phi'(\frac{I_s}{K_s}) = \frac{\partial \Phi_s}{\partial (\frac{I_s}{K_s})} = p_s(\frac{\beta}{2})[1 - (\frac{\mu}{K_s})^2], \tag{A.21}
\]
and the partial derivatives with respect to investment and the capital stock are

\[
\frac{\partial \Phi_s}{\partial I_s} = p_s(\frac{\beta}{2K_s})[1 - (\frac{\mu}{K_s})^2] = \frac{\Phi'}{K_s},
\]

and

\[
\frac{\partial \Phi_s}{\partial K_s} = -p_s(\frac{\beta}{2K_s})(\frac{I_s}{K_s^2})[1 - (\frac{\mu}{K_s})^2] = -\frac{I_s}{K_s^2}\Phi'. \tag{A.22}
\]

Finally, for future reference, note that

\[
\Phi_s + (\frac{I_s}{K_s})\Phi' \quad = \quad p_s(\frac{\beta}{2})(\frac{I_s}{K_s} - \mu)^2 + p_s(\frac{\beta}{2})(\frac{I_s}{K_s})[1 - (\frac{\mu}{K_s})^2]
\]

\[
= \quad p_s[(\frac{\beta}{2})(\frac{I_s}{K_s}) - \mu \beta + (\frac{\beta}{2})(\frac{\mu^2}{K_s}) + (\frac{\beta}{2})(\frac{I_s}{K_s}) - (\frac{\beta}{2})(\frac{\mu^2}{K_s})] - \mu \beta
\]

\[
= \quad p_s \beta(\frac{I_s}{K_s}) - \mu \beta
\]

\[
= \quad p_s \beta[(\frac{I_s}{K_s}) - \mu].
\]

Investment can be financed with either debt or equity. Firms are assumed to maintain a constant debt-capital ratio, \( b \), so that

\[
B_s = b[K_s], \tag{A.24}
\]

where \( B_s \) is the stock of outstanding debt at time \( s \). New bond issues in period \( s \), \( BN_s \), are the difference in bonds outstanding in two consecutive periods, or \( BN_s = B_{s+1} - B_s \). Combining equations (A.19) and (A.24) yields

\[
BN_s = b(K_{s+1} - K_s) = b(I_s - \delta K_s) \tag{A.25}
\]

Note that this formulation implies that existing loans are repaid at the rate of depreciation of the existing capital stock; that is, loans outstanding in period \( s + 1 \) are
\[ B_{s+1} = bK_{s+1} = bK_s - b\delta K_s + bI_s, \] so that the existing stock of debt outstanding equals \( bK_s \), \( b\delta K_s \) is the amount of existing debt that is retired in period \( s \), and \( bI_s \) is the amount of new debt that is accumulated in period \( s \).

Total equity earnings in period \( s \), \( EARN_s \), are defined as the value of output less labor costs and real interest payments on the total level of indebtedness \( B_s \), or

\[ EARN_s = p_s F(K_s, L_e) - w_s L_e - i_s B_s. \tag{A.26} \]

Dividends paid are assumed to equal a constant fraction \( (\zeta) \) of the firm’s after-tax earnings net of economic depreciation, or

\[ DIV_s = \zeta[EARN_s - TE_s - p_s\delta K_s] \tag{A.27} \]

where \( TE_s \) denotes total corporate taxes paid in period \( s \). Assuming that adjustment costs are fully deductible and that the corporate business tax rate is \( \tau_{bs} \), total corporate taxes are defined as

\[ TE_s = \tau_{bs}[p_s F(K_s, L_e) - w_s L_e - f_1 I_s(1 + \Phi_s) - f_2 i_s B_s + f_3 bI_s - f_4 b\delta K_s - f_5 \delta^T K_s^T]. \tag{A.28} \]

Under an income tax \( f_1 = f_3 = f_4 = 0 \) and \( f_2 = f_5 = 1 \). Under a “R-based” cash flow business tax \( f_1 = 1 \) and \( f_2 = f_3 = f_4 = f_5 = 0 \), while under a “R+F based” cash flow business tax \( f_5 = 0 \) and \( f_1 = f_2 = f_3 = f_4 = 1 \). Assuming no cash accumulation on the part of the firm, cash inflows in period \( s \) must equal total disbursements, or

\[ EARN_s + BN_s + VN_s = DIV_s + I_s(1 + \Phi_s) + TE_s. \tag{A.29} \]

Following Goulder and Summers, the model assumes individual level arbitrage. The after-tax nominal return on bonds is \( \tau_s = (1 - \tau_{is})i_s \) so that

\[ \tau_s = (1 - \tau_{is})i_s = \frac{(1 - \tau_{ds})DIV_s + (1 - \tau_{gs})(V_{s+1} - V_s - VN_s)}{V_s}; \tag{A.30} \]
that is, the effective tax rate on equity income at the individual level is a weighted average of the tax rate on dividends $\tau_d$, the effective annual accrual tax rate on capital gains $\tau_g$ (taking into account the benefits of deferral and exemption of gains transferred at death and the cost of the taxation of nominal gains), $V_s$ is the value of the firm, $VN_s$ is new share issues, $(V_{s+1} - V_s - VN_s)$ is the capital gain on outstanding shares, and $DIV_s$ is dividends paid. The treatment of equity finance follows the "traditional" view of the effects of dividend taxation. As noted above, dividends are a fixed fraction of earnings after taxes and depreciation. Investments are financed from the remaining retained earnings, or with new share issues if retained earnings are insufficient to finance the desired level of investment. (If desired investment is less than retained earnings, the firm repurchases shares without paying a dividend tax.)

Rearranging and simplifying equation (A.30) yields

$$\begin{align*}
(1 - \tau_{gs})V_{s+1} &= (1 - \tau_{gs})(V_s + VN_s) - (1 - \tau_{ds})DIV_s + [(1 - \tau_{is})i_s]V_s, \\
V_{s+1} &= V_s(1 + \theta_s) + VN_s - \frac{1 - \tau_{ds}}{1 - \tau_{gs}}DIV_s, \\
\text{Equation (A.31)}
\end{align*}$$

where $\theta_s = \frac{(1-\tau_{is})i_s}{(1-\tau_{gs})}$. Equation (A.31) can be used to obtain the expression for the value of the firm by repeatedly substituting for $V_{s+i-1}$ as follows. Note that $V_{s+2}$ can be written as

$$\begin{align*}
V_{s+2} &= V_{s+1}(1 + \theta_{s+1}) + VN_{s+1} - \frac{(1 - \tau_{ds+1})}{(1 - \tau_{gs+1})}DIV_{s+1},
\end{align*}$$

so that substituting for $V_{s+1}$ yields

$$\begin{align*}
V_{s+2} &= \{V_s(1 + \theta_s) + VN_s - \frac{(1 - \tau_{ds})}{(1 - \tau_{gs})}DIV_s\}(1 + \theta_{s+1}) \\
&\quad + VN_{s+1} - \frac{(1 - \tau_{ds+1})}{(1 - \tau_{gs+1})}DIV_{s+1},
\end{align*}$$

$$\begin{align*}
V_{s+2} &= V_s(1 + \theta_s)(1 + \theta_{s+1}) + VN_s(1 + \theta_{s+1}) + VN_{s+1}
\end{align*}$$
\[- \frac{(1 - \tau_{ds})}{(1 - \tau_{gs})} DIV_s (1 + \theta_{s+1}) - \frac{(1 - \tau_{ds+1})}{(1 - \tau_{gs+1})} DIV_{s+1}. \]

Similarly, \( V_{s+3} \) can be written as

\[ V_{s+3} = V_{s+2} (1 + \theta_{s+2}) + VN_{s+2} - \frac{(1 - \tau_{ds+2})}{(1 - \tau_{gs+2})} DIV_{s+2}, \]

so that substituting for \( V_{s+2} \) yields

\[ V_{s+3} = \{ V_s (1 + \theta_s) (1 + \theta_{s+1}) + VN_s (1 + \theta_{s+1}) + VN_{s+1} \]
\[- \frac{(1 - \tau_{ds})}{(1 - \tau_{gs})} DIV_s (1 + \theta_{s+1}) - \frac{(1 - \tau_{ds+1})}{(1 - \tau_{gs+1})} DIV_{s+1} \} (1 + \theta_{s+2}) \]
\[ + VN_{s+2} - \frac{(1 - \tau_{ds+2})}{(1 - \tau_{gs+2})} DIV_{s+2}, \]

\[ V_{s+3} = V_s (1 + \theta_s) (1 + \theta_{s+1}) (1 + \theta_{s+2}) + VN_s (1 + \theta_{s+1}) (1 + \theta_{s+2}) \]
\[ + VN_{s+1} (1 + \theta_{s+2}) + VN_{s+2} - \frac{(1 - \tau_{ds})}{(1 - \tau_{gs})} DIV_s (1 + \theta_{s+1}) (1 + \theta_{s+2}) \]
\[- \frac{(1 - \tau_{ds+1})}{(1 - \tau_{gs+1})} DIV_{s+1} (1 + \theta_{s+2}) - \frac{(1 - \tau_{ds+2})}{(1 - \tau_{gs+2})} DIV_{s+2}. \]

Continuing this process indefinitely yields

\[ V_{T+1} = V_s \prod_{u=s}^T (1 + \theta_u) + \sum_{u=s}^T VN_u \prod_{j=s+1}^T (1 + \theta_j) - \sum_{u=s}^T \frac{(1 - \tau_{du})}{(1 - \tau_{gu})} DIV_u \prod_{j=s+1}^T (1 + \theta_j), \]

which can be solved for \( V_s \)

\[ V_s = \frac{V_{T+1}}{\prod_{v=s}^T (1 + \theta_v)} + \frac{\sum_{u=s}^T \frac{(1 - \tau_{du})}{(1 - \tau_{gu})} DIV_u - VN_u \prod_{j=s+1}^T (1 + \theta_j)}{\prod_{v=s}^T (1 + \theta_v)}. \]

Taking the limit as \( T \to \infty \) and imposing the transversality condition

\[ \lim_{T \to \infty} V_{T+1} \prod_{u=t}^T \frac{1}{(1 + \theta_u)} = 0, \]

which rules out the possibility that the firm may become infinitely large, yields

\[ V_s = \sum_{u=s}^{\infty} \frac{\frac{(1 - \tau_{du})}{(1 - \tau_{gu})} DIV_u - VN_u}{\prod_{v=s}^T (1 + \theta_v)}, \tag{A.32} \]
for \( s \geq t \). That is, \( V_s \) equals the present value of all future net distributions to shareholders. Note that under the assumption of individual arbitrage, the firm's discount rate \( \theta_s = \frac{(1 - \tau_{tu})i_s}{(1 - \tau_{tu})} \) is increased to reflect the fact that dividend distributions avoid the capital gains tax that arises when the firm retains earnings. Note also that this is a perfect foresight condition in that the firm must predict all future values of the interest rate (in \( \theta_s \)) and the tax rate variables. The firm's manager chooses \( L_s \) and \( I_s \) to maximize its market value, as defined in (A.32), subject to the constraints (A.17-A.29).

To simplify the expression for the market value of the firm substitute from (A.24), (A.26) and (A.28) into expression (A.27) and gathering similar terms yields

\[
DIV_s = \zeta[(1 - \tau_{ts})(p_s F(K_s, L_e, w_s L_s) - (1 - \tau_{ts} f_2)i_s B_s - \tau_{ts} f_1 I_s (1 + \Phi_s) - f_3 \tau_{ts} b I_s + f_4 \tau_{ts} \delta b K_s + f_5 \tau_{ts} \delta^T K_s^T - p_s \delta K_s)].
\]

Using equation (A.27) to define dividends paid out as a share of after-tax earnings net of depreciation as follows:

\[
\zeta = \frac{DIV_s}{EARN_s - TE_s - p_s \delta K_s}.
\]

Subtracting one from both sides and dividing by \( \zeta \) yields

\[
\frac{\zeta - 1}{\zeta} = \frac{DIV_s - EARN_s - TE_s - p_s \delta K_s}{EARN_s - TE_s - p_s \delta K_s},
\]

\[
DIV_s \left( \frac{\zeta - 1}{\zeta} \right) = DIV_s - EARN_s + TE_s + p_s \delta K_s. \tag{A.34}
\]

Rearranging the firm’s cash flow equation, given in equation (A.29), to express new share issues in terms of other variables yields

\[
VN_s = DIV_s - EARN_s + TE_s + \delta K_s - \delta K_s + I_s(1 + \Phi_s) - BN_s.
\]

Using equations (A.34) and (A.25) to substitute for terms in the equation above yields

\[
VN_s = DIV_s \left( \frac{\zeta - 1}{\zeta} \right) - \delta K_s + I_s(1 + \Phi_s) - b(I_s - \delta K_s). \tag{A.35}
\]
Substituting equation (A.35) into equation (A.32) yields

\[ V_s = \sum_{u=s}^{\infty} \frac{DIV_u [\Omega_u] - I_u (1 + \Phi_u) + \delta K_u + b (I_u - \delta K_u)}{\prod_{v=s}^{u} (1 + \theta_v)}, \]

where

\[ \frac{\Omega_u}{\zeta} = \frac{(1 - \tau_{du})}{(1 - \tau_{gu})} - \frac{\zeta - 1}{\zeta}. \]

so that

\[ \Omega_u = \frac{[\zeta (1 - \tau_{du}) + (1 - \zeta) (1 - \tau_{gu})]}{(1 - \tau_{gu})}. \]

Substituting for the value of dividends, as defined in equation (A.33), and recalling that \( B_s = b K_s \), implies that

\[ V_s = \sum_{u=s}^{\infty} \left[ \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} \right] \left[ (1 - \tau_{bs}) \Omega_u (p_s F(K_s, L_s) - w_s L_s) \right. \]

\[ - (1 - \tau_{bs} f_2) \Omega_u i_s b K_s + f_1 \tau_{bs} \Omega_u I_s (1 + \Phi_s) - f_3 \tau_{bs} \Omega_u b I_s \]

\[ + f_4 \tau_{bs} \Omega_u \delta b K_s + f_5 \tau_{bs} \Omega_u \delta^{r} K_s^{r} - p_s \Omega_u \delta K_s \]

\[ - I_u (1 + \Phi_u) + \delta K_u + b (I_u - \delta K_u) \}. \]

The term \( f_5 \tau_{bs} \Omega_u \delta^{r} K_s^{r} \) reflects the amount of tax savings from depreciation allowances, which include both those attributable to past investments (previous to time \( t \)) and those attributable to future investments made after time \( t \). In order to maximize the value of the firm, the firm’s manager chooses the optimal path of future investments. Accordingly, it is useful to distinguish between the present value of depreciation allowances on old capital and the present value of depreciation allowances on future investments.

To separate out these two effects from the term it is necessary to trace out the time path of the tax basis, \( K^r \). This path differs from the true capital accumulation path due to the modestly accelerated depreciation allowances under current law. For
\( s \geq t \), the time path of the tax basis, \( K^\tau \), follows the equation

\[
K_s^\tau = I_{s-1} + (1 - \delta^\tau)K_{s-1}^\tau.
\]

Substituting for \( K_{s-1}^\tau = I_{s-2} + (1 - \delta^\tau)K_{s-2}^\tau \) into the previous equation yields

\[
K_s^\tau = I_{s-1} + (1 - \delta^\tau)I_{s-2} + (1 - \delta^\tau)^2K_{s-2}^\tau.
\]

Similarly, substituting for \( K_{s-2}^\tau = I_{s-3} + (1 - \delta^\tau)K_{s-2} \) yields

\[
K_s^\tau = I_{s-1} + (1 - \delta^\tau)I_{s-2} + (1 - \delta^\tau)^2I_{s-3} + (1 - \delta^\tau)^3K_{s-3}^\tau.
\]

In general (i.e., for \( s = t - 3 \) above, and noting that the summation over \( v \) below begins at \( v = t = s - (s - t) \) and ends at \( v = s - 1 \)),

\[
K_s^\tau = (1 - \delta^\tau)^{s-t}K_t^\tau + \sum_{j=t}^{s-1}(1 - \delta^\tau)^{s-1-j}I_j;
\]

that is, the tax basis at time \( s \) reflects the basis existing at the beginning of the period in the year of enactment (\( t \)) plus all new investment since time \( t \) through time \( s - 1 \) (but not \( s \), since the capital stock is measured at the beginning of the period). Thus, the term in equation (A.36) that reflects the present value of tax savings from all past and future depreciation allowances,

\[
\sum_{u=s}^{\infty} \left( \prod_{u=s}^{u} \frac{1}{(1 + \theta_v)} \right) \tau_{bu} \Omega_u \delta^\tau K_u^\tau
\]

(A.37)

can be rewritten as the sum of two terms as follows

\[
\{ \sum_{u=s}^{\infty} \left( \prod_{u=s}^{u} \frac{1}{(1 + \theta_v)} \right) \tau_{bu} \Omega_u \delta^\tau \} K_u^\tau
\]

(A.38)

\[
= \left\{ \sum_{u=s}^{\infty} \tau_{bu} \Omega_u \delta^\tau (1 - \delta^\tau)^{u-t} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} \right\} K_t^\tau
\]

\[
+ \sum_{u=s}^{\infty} \tau_{bu} \Omega_u \delta^\tau \sum_{j=t}^{u-1} \left( (1 - \delta^\tau)^{u-1-j} I_j \right) \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} \}.
\]
The second term in equation (A.38) represents the tax savings from accelerated depreciation allowances on future investments, and can be expanded (noting that there are no terms in the sum when \( s = t \)) to yield

\[
\sum_{u=s}^{\infty} \left\{ \tau_{bu} \Omega_u \delta^r \sum_{j=t}^{u-1} [(1 - \delta^r)^{u-1-j} I_j] \prod_{v=s}^{u} \frac{1}{1 + \theta_v} \right\}
\]

\[
= \tau_{bs+1} \Omega_{s+1} \delta^r \prod_{v=s}^{s+1} \frac{1}{1 + \theta_v} [I_s]
\]

\[
+ \tau_{bs+2} \Omega_{s+2} \delta^r \prod_{v=s}^{s+2} \frac{1}{1 + \theta_v} [I_{s+1} + I_s(1 - \delta^r)]
\]

\[
+ \tau_{bs+3} \Omega_{s+3} \delta^r \prod_{v=s}^{s+3} \frac{1}{1 + \theta_v} [I_{s+2} + I_{s+1}(1 - \delta^r) + I_s(1 - \delta^r)^2]
\]

\[
+ \tau_{bs+4} \Omega_{s+4} \delta^r \prod_{v=s}^{s+4} \frac{1}{1 + \theta_v} [I_{s+3} + I_{s+2}(1 - \delta^r) + I_{s+1}(1 - \delta^r)^2 + I_s(1 - \delta^r)^3]...
\]

Collecting terms with \( I_t \) yields

\[
I_s \{ \tau_{bs+1} \Omega_{s+1} \delta^r \prod_{v=s}^{s+1} \frac{1}{1 + \theta_v} + (1 - \delta^r) \tau_{bs+2} \Omega_{s+2} \delta^r \prod_{v=s}^{s+2} \frac{1}{1 + \theta_v} \\
+ (1 - \delta^r)^2 \tau_{bs+3} \Omega_{s+3} \delta^r \prod_{v=s}^{s+3} \frac{1}{1 + \theta_v} + (1 - \delta^r)^3 \tau_{bs+4} \Omega_{s+4} \delta^r \prod_{v=s}^{s+4} \frac{1}{1 + \theta_v} \}...
\]

\[
= I_u \left[ \sum_{u=s+1}^{\infty} \tau_{bu} \Omega_u \delta^r (1 - \delta^r)^{u-s-1} \prod_{v=s}^{u} \frac{1}{1 + \theta_v} \right] .
\]

Similarly, collecting terms with \( I_{t+1} \) yields

\[
I_{t+1} \{ \tau_{bs+2} \Omega_{s+2} \delta^r \prod_{v=s}^{s+2} \frac{1}{1 + \theta_v} + (1 - \delta^r) \tau_{bs+3} \Omega_{s+3} \delta^r \prod_{v=s}^{s+3} \frac{1}{1 + \theta_v} \\
+ (1 - \delta^r)^2 \tau_{bs+4} \Omega_{s+4} \delta^r \prod_{v=s}^{s+4} \frac{1}{1 + \theta_v} \}...
\]

\[
= I_{s+1} \left[ \sum_{u=s+1}^{\infty} \tau_{bu} \Omega_u \delta^r (1 - \delta^r)^{u-s-1} \prod_{v=s}^{u} \frac{1}{1 + \theta_v} \right] .
\]
Summing over all such \( I_s \) terms yields

\[
= \sum_{j=s}^{\infty} \left\{ \sum_{u=j}^{\infty} \tau_{bu} \Omega_u \delta^u (1 - \delta)^{u-j-1} \prod_{v=u}^{u} \frac{1}{(1 + \theta_v)} \right\}
\]

which can be rewritten as

\[
= \sum_{j=s}^{\infty} \left\{ \sum_{u=j}^{\infty} \tau_{bu} \Omega_u \delta^u (1 - \delta^{u-j-1}) \prod_{v=j}^{u} \frac{1}{(1 + \theta_v)} \prod_{v=s}^{j} \frac{1}{(1 + \theta_v)} \right\}.
\]

Repeating from equation (A.38) above and substituting the new expression for the second term yields

\[
\sum_{u=s}^{\infty} \left[ \prod_{v=u}^{j} \frac{1}{(1 + \theta_v)} \right] \tau_{bu} \Omega_u \delta^u K^\tau_u
\]

\[
\sum_{u=s}^{\infty} \left[ \prod_{v=u}^{j} \frac{1}{(1 + \theta_v)} \right] \tau_{bu} \Omega_u \delta^u (1 - \delta^{u-t}) K^\tau_t
\]

\[
+ \sum_{j=s}^{\infty} \left\{ \sum_{u=j}^{\infty} \tau_{bu} \Omega_u \delta^u (1 - \delta^{u-j-1}) \prod_{v=j}^{u} \frac{1}{(1 + \theta_v)} \prod_{v=s}^{j} \frac{1}{(1 + \theta_v)} \right\},
\]

which can be written as

\[
\sum_{u=s}^{\infty} \left[ \prod_{v=u}^{j} \frac{1}{(1 + \theta_v)} \right] \tau_{bu} \Omega_u \delta^u K^\tau_u = X_t + \sum_{j=s}^{\infty} [I_j Z_{j+1} \prod_{v=s}^{j} \frac{1}{(1 + \theta_v)}]
\]

(A.39)

\[
= X_t + \sum_{j=s}^{\infty} [I_j Z_{j+1} \prod_{v=s}^{j} \frac{1}{(1 + \theta_v)}],
\]

where

\[
Z_j = \sum_{u=j}^{\infty} \tau_{bu} \Omega_u \delta^u (1 - \delta^{u-j}) \prod_{v=j}^{u} \frac{1}{(1 + \theta_v)}
\]

and \( X_t = Z_t K^\tau_t \). The first term — \( X_t \) — is the value of future depreciation deductions on "old" capital existing at the time of reform and the second term is the value of depreciation deductions on all investment made after the enactment of reform.
Substituting from equation (A.39) into (A.36) yields

\[ V_s = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} \Gamma_u + f_5 X_t, \quad (A.40) \]

where

\[
\Gamma_u = (1 - \tau_{bu}) \Omega_u [p_u F(K_u, L_e u) - w_u L_u] \\
- K_u [\Omega_u (1 - \tau_{bu} f_2) i_u b - \delta (1 - b - \Omega_u (1 - f_4 \pi_{bs} b))] \\
- I_u [1 - b - \Omega_u \pi_{bs} (f_1 - f_3 b) - f_5 Z + \Phi_u (1 - \tau_{bu} \Omega_u)].
\]

The firm maximizes (A.40) subject to the constraints

\[ K_{s+1} = I_s + (1 - \delta) K_s \]

and

\[ \lim_{T \to \infty} K_T \geq 0. \]

Given this, the Lagrangian can be defined as

\[
\mathcal{L} = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} \Gamma_u + f_5 X_t + q_{u+1}^* [I_u + (1 - \delta) K_u - K_{u+1}]
\]

\[
= \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} \{ \Gamma_u + f_5 X_t + q_{u+1}^* [I_u + (1 - \delta) K_u - K_{u+1}] \},
\]

where

\[ q_{u+1}^* = [\prod_{v=s}^{u} \frac{1}{(1 + \theta_v)}] q_{u+1}. \]

The necessary conditions for a maximum are derived as follows. First, the necessary condition with respect to the optimal choice of labor in each period,

\[
\frac{\partial \mathcal{L}}{\partial L_e s} = (1 - \tau_{bs}) \Omega_s [p_s \frac{\partial F}{\partial L_e s} - w_s] \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v)} = 0,
\]
implies

$$w_s = p_s F_L. \quad (A.41)$$

This is simply the well known result that in equilibrium the wage rate must be equal to the value of the marginal product of labor, $p_s F_L$.

Second, the necessary condition with respect to the optimal choice of investment in each period is given by

$$\frac{\partial \mathcal{L}}{\partial I_s} = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \left(1 + \theta_v\right) \left[-1 + b + f_5 Z_{s+1} + \Omega_u \tau_{bs} (f_1 - f_3 b) \right.$$

$$\left.- \Phi_s (1 - \tau_{bs} \Omega_s) - I_s (1 - \tau_{bs} \Omega_s) \frac{\partial \Phi_s}{\partial I_s} + q_{s+1}\right] = 0$$

Recalling that $\frac{\partial \Phi_s}{\partial I_s} = \frac{\Phi'}{K_s}$, allows us to write this expression as

$$q_{s+1} = 1 - b - f_5 Z_{s+1} - \Omega_u \tau_{bs} (f_1 - f_3 b) + (1 - \tau_{bs} \Omega_s) [\Phi_s + \frac{I_s}{K_s} \Phi'] \quad (A.42)$$

This describes the variable commonly known as Tobin's $q$—the ratio of the market value of capital to its replacement cost. It demonstrates that the shadow price of additional capital goods ($q_{s+1}$) must equal the after-tax marginal cost of capital goods (the right hand side). Since the investment good is the numeraire, the first term in the equation indicates that the shadow price is simply one in the absence of debt and taxes. The second term reflects the financing of a fraction $b$ of the cost of the investment with debt. The third term reflects the reduction in the shadow price of new capital goods due to tax deductions for depreciation. The last term reflects the costs of installing new capital goods with immediate expensing of such adjustment costs. Recalling $[\Phi_s + \frac{I_s}{K_s} \Phi'] = p_s \beta [\frac{I_s}{K_s}] - \mu$, this equation can be solved to give the optimal investment rate for the firm as

$$\frac{I_s}{K_s} = \frac{(q_{s+1} - 1 + b + f_5 Z_{s+1} + \Omega_u \tau_{bs} (f_1 - f_3 b))}{p_s \beta (1 - \tau_{bs} \Omega_s)} + \mu \quad (A.43)$$
It is desirable to express investment demand as a function the value of the firm, \( V_s \), rather than of \( q_{s+1} \). To do this, note that, as shown by Hayashi (1982) or Keuschnigg (1990), the assumptions of linear homogeneity of the production function and homogeneity of degree zero of the adjustment cost function in investment and capital, imply the following relationship between marginal \( q \) and average \( q \) (denoted as \( Q \)):

\[
q_s = \frac{[V_s - X_s]}{K_s}, \quad Q_s = \frac{V_s}{K_s}.
\]  

(A.44)

Thus, the investment demand function can be written as

\[
\frac{I_s}{K_s} = \frac{\frac{[V_{s+1} - X_{s+1}]}{K_{s+1}} - 1 + b + f_5 Z_{s+1} + \Omega_u \tau_{bs} (f_1 - f_3 b)}{p_s \beta (1 - \tau_{bs} \Omega_s)} + \mu
\]  

(A.45)

Third, noting that the \( q_{s+1} \) term in \( \mathcal{L} \) above has two terms with \( K_s \), one in the current period and one in the next period, the necessary condition with respect to the optimal capital stock in each period is given by

\[
\frac{\partial \mathcal{L}}{\partial K_s} = \sum_{u=0}^{\infty} \sum_{v=0}^{\infty} \frac{1}{(1 + \theta_u)} \{ \Omega_s (1 - \tau_{bs}) p_s \frac{\partial F}{\partial K_s} - \left[ \Omega_s (1 - \tau_{bs} f_2) i_s b - \delta (1 - b - \Omega_s (1 - f_4 \tau_{bs} b)) \right] \}
\]

\[
- I_s (1 - \tau_{bs} \Omega_s) \frac{\partial \Phi_s}{\partial K_s} \}
\]

+ \prod_{u=t}^{s} \frac{q_{s+1}}{(1 + \theta_u)(1 - \delta)} - \prod_{u=t}^{s-1} \frac{q_s}{(1 + \theta_u)} = 0.
\]

which implies

\[
\Omega_s (1 - \tau_{bs} f_2) i_s b = \Omega_s (1 - \tau_{bs}) p_s \frac{\partial F}{\partial K_s} + \delta (1 - b - \Omega_s (1 - f_4 \tau_{bs} b)) - I_s (1 - \tau_{bs} \Omega_s) \frac{\partial \Phi_s}{\partial K_s}
\]

\[
+ q_{s+1} (1 - \delta) - q_s (1 + \theta_s).
\]

Recall that \( \theta_s = \frac{(1 - \tau_{bs} i_s)}{(1 - \tau_{gs})} \), and thus,

\[
i_s \left[ \Omega_s (1 - \tau_{bs} f_2) b + q_s \frac{(1 - \tau_{is})}{(1 - \tau_{gs})} \right]
\]

\[
= \Omega_s (1 - \tau_{bs}) p_s \frac{\partial F}{\partial K_s} + \delta (1 - b - \Omega_s (1 - f_4 \tau_{bs} b))
\]

\[
- I_s (1 - \tau_{bs} \Omega_s) \frac{\partial \Phi_s}{\partial K_s} + q_{s+1} (1 - \delta) - q_s.
\]
Substituting equations (A.21) and (A.22) into the equation above yields

$$i_s = \frac{\Omega_s(1 - \tau_{bs})p_s \frac{\partial F}{\partial K_s} + \delta(1 - b - \Omega_s(1 - f_4 \tau_{bs} b)) + q_{s+1}(1 - \delta) - q_s}{\Omega_s(1 - \tau_{bs} f_2) b + q_s \frac{(1-\tau_{bs})}{(1-\tau_{gs})}}$$

Equation (A.46) is the Euler equation. It can be written as the following difference equation in $q_t$:

$$q_{s+1} (1 - \delta) = q_s + [\Omega_s(1 - \tau_{bs} f_2) b + q_s \frac{(1-\tau_{ta})}{(1-\tau_{gs})}] i_s - \Omega_s(1 - \tau_{bs}) p_s \frac{\partial F}{\partial K_s} - \delta(1 - b - \Omega_s(1 - f_4 \tau_{bs} b)) - (1 - \tau_{bs} \Omega_s) p_s \frac{\beta}{2}[(\frac{I_s}{K_s})^2 - \mu^2]$$

or

$$q_{s+1} (1 - \delta) = q_s [1 + \frac{(1-\tau_{ta})}{(1-\tau_{gs})}] - \{ - \Omega_s(1 - \tau_{bs} f_2) i_s b + \Omega_s(1 - \tau_{bs}) p_s \frac{\partial F}{\partial K_s} + \delta(1 - b - \Omega_s(1 - f_4 \tau_{bs} b)) + (1 - \tau_{bs} \Omega_s) p_s \frac{\beta}{2}[(\frac{I_s}{K_s})^2 - \mu^2]$$

or

$$q_{s+1} (1 - \delta) = q_s (1 + \theta_s) - \{ - \Omega_s(1 - \tau_{bs} f_2) i_s b + \Omega_s(1 - \tau_{bs}) p_s \frac{\partial F}{\partial K_s} + \delta(1 - b - \Omega_s(1 - f_4 \tau_{bs} b)) + (1 - \tau_{bs} \Omega_s) p_s \frac{\beta}{2}[(\frac{I_s}{K_s})^2 - \mu^2]$$

which can be solved to yield

$$q_s = \sum_{u=s}^{\infty} \left\{ \prod_{v=s}^{u-1} \frac{1}{(1+\theta_v)} \right\} (1 - \delta)^{u-1} \left\{ \Omega_u(1 - \tau_{bu}) p_u \frac{\partial F}{\partial K_u} - \Omega_u(1 - \tau_{bu} f_2) i_u b + \delta(1 - b - \Omega_u(1 - f_4 \tau_{bu} b)) + (1 - \tau_{bu} \Omega_u) p_u \frac{\beta}{2}[(\frac{I_u}{K_u})^2 - \mu^2] \right\}.$$
This equation implies that the shadow price of new capital — \( q_t \) — equals the present value of future income, reflecting the productivity of the asset, depreciation allowances, savings in future installation costs and future interest payments. Since \( q_s = \frac{V_s - X_s}{K_s} \), the value of "marginal" \( q \) can be used to solve for average \( Q \) as follows

\[
Q_s = \frac{V_s}{K_s} = q_s + \frac{X_s}{K_s};
\]

that is, average \( Q \) equals marginal \( q \) plus an adjustment for future depreciation deductions on existing assets. It is also interesting to note that equation (A.46) can be solved for the user cost of capital developed by Jorgenson (1963) — the minimum return an investment must yield in order to provide the investor with the same rate of return that would be received from lending at the after-tax interest rate, or

\[
F_K = \frac{\Omega_u(1 - \tau_{bu}f_2)i_b(1 - \delta(1 - b - \Omega_a(1 - f_4\tau_{bo}b))) + q_s(1 + \theta_s)}{\Omega_s(1 - \tau_{bo})p_s} + \frac{-q_{s+1}(1 - \delta) - (1 - \tau_{bu}\Omega_u)p_u\left(\frac{\mu}{K_u}\right)^2 - \mu^2}{\Omega_s(1 - \tau_{bo})p_s}
\]

Finally, the optimal solution must satisfy

\[
\lim_{T \to \infty} K_{T+1} \geq 0 \quad \lim_{T \to \infty} q_{T+1}^* \geq 0 \quad \text{and} \quad \lim_{T \to \infty} K_{T+1}q_{T+1}^* = 0.
\]

(A.49)

### A.3.2 The Housing Sector

The analysis of the housing sector is based on Tobin's \( q \) theory of investment extended to include adjustment cost as formalized by Hayashi (1982). The approach used to model the housing sector is similar to the work of Goulder (1989) and Goulder and Summers (1989). The model extends upon the work of Goulder and Summers by explicitly calculating assets values in the rental housing and owner-occupied housing sectors. These calculations account for the existing differences in the tax treatment of the returns to rental and owner-occupied housing under current law. It is assumed
that owner-occupiers and landlords act to maximize the value of their housing capital in perfectly competitive markets with no uncertainty. Note that the superscript, \( l = O, R \), is used to denote owner-occupied or rental housing.

The production of housing services is characterized by a Cobb-Douglas production function

\[
F(K^l_s, L^l_s) = (K^l_s)^{\alpha_2}(L^l_s)^{1-\alpha_2}, \quad 0 < \alpha_2 < 1, \tag{A.50}
\]

where \( K^l_s \) denotes inputs of capital used in the production of housing services in period \( s \), \( L^l_s \) denotes effective labor used in the production of housing services in period \( s \), and \( \alpha_2 \) is the capital share parameter in the Cobb-Douglas production function.

Gross investment in period \( s \), \( I^l_s \), equals

\[
I^l_s = K^l_{s+1} - (1 - \delta^l)K^l_s. \tag{A.51}
\]

where \( \delta^l \) is the economic rate of depreciation of capital employed in the housing sector. Similar to the firm sector above, adjustment cost per unit of investment are represented as

\[
\Phi^l_s \left( \frac{I^l_s}{K^l_s} \right) = \frac{p^l_s (\beta/2)(I^l_s/K^l_s - \mu)^2}{(I^l_s/K^l_s)}, \tag{A.52}
\]

where \( p^l_s \) is the price of housing services. Corresponding to the discussion of adjustment costs for the firm, \( \beta \) and \( \mu \) are the adjustment cost parameters; higher values of \( \beta \) and lower values of \( \mu \) imply higher adjustment costs. The value of \( \mu \) is set equal to the steady state ratio of gross investment to capital, which is \( \delta + n \).\(^{25}\)

The framework for determining the value of housing services to homeowners or landlords is similar to that employed for the non-housing firm, however, in the housing

\(^{25}\)The derivatives of the adjustment cost function are identical to the results obtained above in the firm behavior section.
sector dividends — the service flows from housing — are paid out in full and new share issues are not available to finance investment in the housing sector.\textsuperscript{26} This implies, that investment in the housing sector is financed by either debt or equity. Homeowners and landlords are assumed to maintain a constant debt to capital ratio, denoted by \( b' \), this implies that

\[
B_s^l = b'(K_s^l),
\]  

(A.53)

where \( B_s^l \) is the stock of outstanding debt at time \( s \). New bond issues in period \( s \), \( BN_s^l \), equals the difference between bonds outstanding in the two consecutive periods \( s \) and \( s + 1 \), or

\[
BN_s^l = B_{s+1}^l - B_s^l.
\]

Combining the equation for new bond issues and equation (A.53) yields

\[
BN_s^l = b'(K_{s+1}^l - K_s^l) = b'(I_s^l - \delta^l K_s^l).
\]  

(A.54)

Note that this formulation implies that existing loans are repaid at the rate of deprecation of the existing capital stock in the housing sector; that is, loans outstanding in period \( s + 1 \) are \( B_{s+1}^l = b'K_{s+1}^l = b'K_s^l - b'\delta^l K_s^l + b'I_s^l \), so that the amount of old debt is \( b'K_s^l \), \( b'\delta^l K_s^l \) is the amount of this existing debt that is retired, and \( b'I_s^l \) is the amount of new debt that is accumulated.

Earnings in the owner-occupied and rental housing sectors in period \( s \), \( EARN_s^l \), are defined as the value of housing services less labor cost, real interest payments on total indebtedness, and property tax payments. The earnings equation in each sector is represented as

\[
EARN_s^l = p_s^l F(K_s^l, L_s^l) - w_s L_s^l - i_s B_s^l - cK_s^l,
\]  

(A.55)

\textsuperscript{26}Only 4.3 percent of the net capital stock in the rental housing sector is owned by corporations. Thus, it is assumed that new share issues are not used to finance rental housing capital.
where $c$ is the property tax rate imposed on the value of housing capital. Following
Goulder (1989), the labor input in the owner-occupied housing sector represents labor
used in the production or maintenance of owner-occupied housing services. Earnings
in the owner-occupied housing sector are not taxed, but owner-occupiers receive in-
come tax deductions for mortgage interest and property tax payments. The value of
these deductions are calculated according to

$$ TE_s^O = \vartheta \tau_{ts} [-i_s B_s^O - cK_s^O] $$  \hspace{1cm} (A.56)

where $\tau_{ts}$ is the tax rate on individual income and $\vartheta$ is the fraction of homeowners
that itemize. Assuming that adjustment costs are fully deductible, the total tax
liability of landlords in period $s$ is given by

$$ TE_s^R = \tau_s^R [p_s^K F(K_s^R, L_s^R) - w_s L_s^R - f_2 i_s B_s^R - f_2 m K_s^R$$

$$ - f_1 I_s^R (1 + \Phi_s^R) + f_3 b^R I_s^R - f_4 \delta^R b^R K_s^R - cK_s^R - f_5 \delta^R K_s^R] $$

where the tax rate on rental housing income, $\tau_s^R$, is a weighted average of the corporate
tax rate and the non-corporate tax rate on landlord profits, $m$ is the percentage
of maintenance expenditures required to maintain the rental capital stock in each
period, $\delta^R$ is the accelerated rate of depreciation, and $K_s^R$ is the remaining basis of
the rental housing capital stock for tax purposes. In the initial income tax steady
state $f_1 = f_3 = f_4 = 0$ and $f_2 = f_5 = 1$. Under the Flat Tax, the “R-based” cash
flow business tax is modelled by setting $f_1 = 1$ and $f_2 = f_3 = f_4 = f_5 = 0$. Note
that the rental housing firm is treated as a strictly non-corporate firm, and thus is
not allowed to issue stock, since only a small fraction of rental housing is corporate
owned. However, the tax rate on rental housing does reflect the fact that a small
fraction of the rental housing market is corporate owned. The tax rate on rental
housing income, $\tau_s^R$, is a weighted average of the marginal corporate tax rate ($\tau_c^R$)
and the marginal tax rate on landlord profits ($\tau^R_s$). The tax rate on the net income of the owners of rental housing in period $s$ is represented as follows

$$\tau^R_s = \left( \frac{a_l}{a_l + a_c} \right) \tau^R_{ls} + \left( \frac{a_c}{a_l + a_c} \right) \tau^R_{cs},$$

where $a_l$ and $a_c$ are the shares of housing services produce in the landlord-owned and corporate-owned sectors respectively. Also, note that landlords, but not owner-occupiers, are allowed a tax deduction for maintenance and repair expenditures.

Following Goulder and Summers (1989), it is assumed that neither owner-occupied or rental housing owners accumulate cash; this implies that cash inflows in period $s$ must be equal to total disbursements in the owner-occupied and rental housing markets, or

$$EARN^l_s + BN^l_s = S^l_s + I^l_s(1 + \Phi^l_s) + TE^l_s$$

(A.58)

for $l = O, R$. The net service flow to owner-occupants and landlords are given by $S^O_s$ and $S^R_s$ respectively. Solving equation (A.58) for the net service flow of owner-occupied or rental housing, $S^l_s$, yields

$$S^l_s = EARN^l_s + BN^l_s - I^l_s(1 + \Phi^l_s) - TE^l_s.$$  

(A.59)

where $BN^l_s$ is new bonds issued in period $s$. Note that a fraction of all marginal investments in the owner-occupied and rental housing sectors must be financed by reductions in the net service flow to owner-occupants and landlords, since all housing owners are assumed to maintain a constant debt to capital ratio and are not allowed to issue shares of stock.

Assuming individual level arbitrage implies that the after-tax nominal return to bonds, is equal to the net return of either owning and occupying or renting out a house. This condition is represented as

$$(1 - \tau_{ls})\pi_s = \frac{S^l_s + (1 - \tau^l_{gs})(V^l_{s+1} - V^l_s)}{V^l_s},$$

(A.60)
where $V_s^l$ is the value of the owner-occupied or rental firm, $(V_{s+1}^l - V_s^l)$ represents the capital gain on owner-occupied or rental housing, and $\tau_{gs}$ is the effective annual accrual tax rate on capital gains in the owner-occupied or rental housing sector.\(^{27}\)

Solving the difference equation in (A.60) subject to the following transversality condition

$$
\lim_{T \to \infty} V_{T+1}^l \prod_{u=t}^{T} \frac{1}{(1 + \theta_u^l)} = 0, \tag{A.61}
$$

where $\theta_u^l = \frac{(1 - \gamma_u) i_s}{(1 - \tau_{gs})}$, yields

$$
V_s^l = \sum_{u=s}^{T} \{ \prod_{v=s}^{u} \left[ \frac{1}{(1 + \theta_v^l)} \right] \left[ \frac{1}{(1 - \tau_{gs})} \right] S_u^l \}. \tag{A.62}
$$

That is, the value of owner-occupied or rental housing equals the present value of all future net service flows that owner-occupants or landlords receive. The transversality condition implies the value of the owner-occupied or rental housing firm is not allowed to become infinitely large in the future.

**Rental Market**

Landlords choose investment $(I_s^R)$ in order to maximize the market value of their rental housing capital (A.62) subject to constraints (A.50) through (A.58).

To find the net service flow in the rental housing sector substitute equations (A.54), (A.55), and (A.57) into equation (A.59). The net service flow in the rental housing sector is given by

$$
S_s^R = p_s^R F(K_s^R, L_e^R) - w_s L_e^R - i_s B_s^R - c(p_s^R K_s^R) + b(I_s^R - \delta_s^R K_s^R) - I_s^R (1 + \Phi_s^R) - \tau [p_s^R F(K_s^R, L_e^R) - w_s L_e^R - f_1 I_s^R (1 + \Phi_s^R)] - f_2 i_s B_s^R + f_3 b I_s^R - f_4 \delta b K_s^R - c(p_s^R K_s^R) - f_5 p_s^R (\delta R^R K_s^R)],
$$

\(^{27}\)This takes into account the benefits of deferral of capital gains.
rearranging terms yields

\[ S_s^R = (1 - \tau^R)[p_s^R F(K_s^R, Le_s^R) - w_s Le_s^R - c(p_s^R K_s^R)] + b(I_s^R - \delta^R K_s^R) - I_s^R(1 + \Phi_s^R) + f_1 \tau^R I_s^R(1 + \Phi_s^R) - (1 - \tau^R f_2)i_s B_s^R - f_3 \tau^R b I_s^R + f_4 \tau^R \delta b K_s^R + f_5 \tau^R p_s^R (\delta^{Rr} K_s^{Rr})]. \] (A.63)

Substituting equation (A.63) into (A.62), and defining \( \Omega_u^R = [1/(1 - \tau_{gu}^R)] \) yields

\[ V_s^R = \sum_{u=s}^T \left\{ \prod_{v=s}^u \left[ \frac{1}{(1 + \theta_s^R)} \right] \Omega_u^R(1 - \tau^R)[p_s^R F(K_s^R, Le_s^R) - w_s Le_s^R - c(p_s^R K_s^R)] + b\Omega_u^R(I_s^R - \delta^R K_s^R) - I_s^R \Omega_u^R(1 + \Phi_s^R) + f_1 \tau^R \Omega_u^R I_s^R(1 + \Phi_s^R) - (1 - \tau^R f_2)\Omega_u^R i_s B_s^R - f_3 \tau^R \Omega_u^R b I_s^R + f_4 \tau^R \Omega_u^R \delta b K_s^R + f_5 \tau^R p_s^R \Omega_u^R (\delta^{Rr} K_s^{Rr})] \right\}. \] (A.64)

The term \( f_5 \tau_s^R p_s^R \Omega_s^R \delta^{Rr} K_s^{Rr} \) reflects the value of tax savings from depreciation allowances on capital employed in the rental housing sector, including both those attributable to past investments (previous to time \( t \)) and those attributable to future investments made after time \( t \). In maximizing the value of rental housing, the landlords choose the optimal path of future investments. Accordingly, it is useful to distinguish between the present value of depreciation allowances on old rental housing capital and the present value of depreciation allowances on future investments in rental housing.

To separate out these two effects from the term \( f_5 \tau_s^R p_s^R \Omega_s^R \delta^{Rr} K_s^{Rr} \) it is necessary to trace out the time path of the tax basis of \( K^{Rr} \). This path differs from the true capital accumulation path due to the modestly accelerated depreciation allowances provided to rental housing under current law. For \( s \geq t \), the time path of tax basis \( K^{Rr} \) follows the equation

\[ K_s^{Rr} = I_{s-1}^R + (1 - \delta^{Rr}) K_{s-1}^{Rr}. \]
Substituting \( K_{s-1}^{Rr} = I_{s-2}^R + (1 - \delta^{Rr})K_{s-2}^{Rr} \) into the previous equation yields

\[
K_{s}^{Rr} = I_{s-1}^R + (1 - \delta^{Rr})I_{s-2}^R + (1 - \delta^{Rr})^2 K_{s-2}^{Rr}.
\]

Similarly, substituting \( K_{s-2}^{Rr} = I_{s-3}^R + (1 - \delta^{Rr})K_{s-3}^{Rr} \) yields

\[
K_{s}^{Rr} = I_{s-1}^R + (1 - \delta^{Rr})I_{s-2}^R + (1 - \delta^{Rr})^2 I_{s-3}^R + (1 - \delta^{Rr})^3 K_{s-3}^{Rr}.
\]

In general (i.e., for \( s = t - 3 \) above, and noting that the summation over \( v \) below begins at \( v = t = s - (s - t) \) and ends at \( v = s - 1 \),

\[
K_{s}^{Rr} = (1 - \delta^{Rr})^{s-t} K_{t}^{Rr} + \sum_{j=t}^{s-1} (1 - \delta^{Rr})^{s-1-j} I_{j}^R,
\]

that is, the tax basis at time \( s \) reflects the basis existing at the beginning of the period in the year of enactment (\( t \)) plus all new investment since time \( t \) through time \( s - 1 \) (but not \( s \), since the capital stock is measured at the beginning of the period).

Thus, the term \( (f_s R_r) P_s R_s \Omega_s R_s \delta^{Rr} K_{s}^{Rr} \) that reflects the present value of depreciation allowances on past and future investments,

\[
\sum_{u=s}^{\infty} \left[ \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^{R})} \right] \tau_u^R P_u R_u \Omega_u R_u \delta^{Rr} K_{u}^{Rr},
\]

can be rewritten as the sum of two terms as follows

\[
\{ \sum_{u=s}^{\infty} \left[ \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^{R})} \right] \tau_u^R P_u R_u \Omega_u R_u \delta^{Rr} \} K_{u}^{Rr}
\]

\[
= \{ \sum_{u=s}^{\infty} \left[ \tau_u^R P_u R_u \Omega_u R_u \delta^{Rr} (1 - \delta^r) \right] \left( \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^{R})} \right) \} K_{t}^{Rr}
\]

\[
+ \sum_{u=s}^{\infty} \left\{ \sum_{j=t}^{u-1} \left[ \tau_u^R P_u R_u \Omega_u R_u \delta^{Rr} (1 - \delta^{Rr}) \right] \left( \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^{R})} \right) \right\}.
\]

The second term in the equation above, the tax savings from depreciation allowances on future investments, can be expanded (noting that there are no terms in the sum when \( s = t \)) to yield
\[
\sum_{u=s}^{\infty} \left\{ \tau_u^R \rho_u^R \Omega_u^R \delta^{Rr} \sum_{j=t}^{u-1} [(1 - \delta^{Rr})^{u-1-j}] I_j^R \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^R)} \right\} \\
= \tau_{s+1}^R \rho_{s+1}^R \Omega_{s+1}^R \delta^{Rr} \prod_{u=s}^{s+1} \frac{1}{(1 + \theta_u^R)} [I_s^R] \\
+ \tau_{s+2}^R \rho_{s+2}^R \Omega_{s+2}^R \delta^{Rr} \prod_{u=s}^{s+2} \frac{1}{(1 + \theta_u^R)} [I_{s+1}^R + I_s^R (1 - \delta^{Rr})] \\
+ \tau_{s+3}^R \rho_{s+3}^R \Omega_{s+3}^R \delta^{Rr} \prod_{u=s}^{s+3} \frac{1}{(1 + \theta_u^R)} [I_{s+2}^R + I_{s+1}^R (1 - \delta^{Rr}) + I_s^R (1 - \delta^{Rr})^2]...
\]

Collecting terms with \( I_t^R \) yields

\[
I_t^R \left\{ \tau_{s+1}^R \rho_{s+1}^R \Omega_{s+1}^R \delta^{Rr} \prod_{u=s}^{s+1} \frac{1}{(1 + \theta_u^R)} \\
+ (1 - \delta^{Rr}) \tau_{s+2}^R \rho_{s+2}^R \Omega_{s+2}^R \delta^{Rr} \prod_{u=s}^{s+2} \frac{1}{(1 + \theta_u^R)} \\
+ (1 - \delta^{Rr})^2 \tau_{s+3}^R \rho_{s+3}^R \Omega_{s+3}^R \delta^{Rr} \prod_{u=s}^{s+3} \frac{1}{(1 + \theta_u^R)} \\
+ (1 - \delta^{Rr})^3 \tau_{s+4}^R \rho_{s+4}^R \Omega_{s+4}^R \delta^{Rr} \prod_{u=s}^{s+4} \frac{1}{(1 + \theta_u^R)} \right\} \\
= I_u^R \left\{ \sum_{u=s+1}^{\infty} \tau_u^R \rho_u^R \Omega_u^R \delta^{Rr} (1 - \delta^{Rr})^{u-s-1} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^R)} \right\}.
\]

Similarly, collecting terms with \( I_{t+1}^R \) yields

\[
I_{t+1}^R \left\{ \tau_{s+2}^R \rho_{s+2}^R \Omega_{s+2}^R \delta^{Rr} \prod_{u=s}^{s+2} \frac{1}{(1 + \theta_u^R)} \\
+ (1 - \delta^{Rr}) \tau_{s+3}^R \rho_{s+3}^R \Omega_{s+3}^R \delta^{Rr} \prod_{u=s}^{s+3} \frac{1}{(1 + \theta_u^R)} \\
+ (1 - \delta^{Rr})^2 \tau_{s+4}^R \rho_{s+4}^R \Omega_{s+4}^R \delta^{Rr} \prod_{u=s}^{s+4} \frac{1}{(1 + \theta_u^R)} \right\} \\
= I_{s+1}^R \left\{ \sum_{u=s+1}^{\infty} \tau_u^R \rho_u^R \Omega_u^R \delta^{Rr} (1 - \delta^{Rr})^{u-s-1} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^R)} \right\}.
\]
Summing over all such \( I_s^R \) terms yields
\[
= \sum_{j=s}^{\infty} \left\{ I_j^R \left[ \sum_{u=j+1}^{\infty} \tau_u^R P_u^R \Omega_u^R \delta^{R_R} (1 - \delta^{R_R})^{u-j-1} \prod_{v=j+1}^{u} \frac{1}{(1 + \theta_v^R)} \right] \right\},
\]

which can be written as
\[
= \sum_{j=s}^{\infty} \left\{ I_j^R \left[ \sum_{u=j+1}^{\infty} \tau_u^R P_u^R \Omega_u^R \delta^{R_R} (1 - \delta^{R_R})^{u-j-1} \prod_{v=j+1}^{u} \frac{1}{(1 + \theta_v^R)} \prod_{v=s}^{j} \frac{1}{(1 + \theta_v^R)} \right] \right\}.
\]

Repeating from equation (A.65) above and substituting the new expression for the second term yields
\[
\sum_{u=s}^{j} \left[ \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^R)} \tau_u^R P_u^R \Omega_u^R \delta^{R_R} K_u^{R_R} \right]
\]

\[
= X_t^R + \sum_{j=s}^{\infty} \left[ I_j^R Z_{j+1}^R \prod_{v=s}^{j} \frac{1}{(1 + \theta_v^R)} \right]
\]

\[
= X_t^R + \sum_{j=s}^{\infty} \left[ I_j^R Z_{j+1}^R \prod_{v=s}^{j} \frac{1}{(1 + \theta_v^R)} \right]
\]

where
\[
Z_j^R = \sum_{u=j}^{\infty} \left[ \tau_u^R P_u^R \Omega_u^R \delta^{R_R} (1 - \delta^{R_R})^{u-j-1} \prod_{v=j+1}^{u} \frac{1}{(1 + \theta_v^R)} \right]
\]

and \( X_t^R = Z_t^R K_t^{R_R} \). The first term — \( X_t^R \) — is the value of future depreciation deductions on “old” housing capital in the rental sector existing at the time of reform and the second term is the value of depreciation deductions on investments made after the enactment of reform.

Substituting equation (A.66) into (A.64) yields
\[
V_s = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta_v^R)} \tau_u^R P_u^R + f_s X_t^R,
\]

\[
(A.67)
\]

where
\[ \Gamma^R_u = \Omega^R_u (1 - \tau^R_u) [p^R_u F(K^R_u, L^R_u) - w_u L^R_u] - K^R_u [\Omega^R_u (1 - \tau^R_u) c p^R_u + b^R_u \Omega^R_u \delta^R - \Omega^R_u (1 - \tau^R_u) f_2 i_u b^R - f_4 \tau^R_u \Omega^R_u \delta b^R] \\ - I^R_u [\Omega^R_u - b^R \Omega^R_u - \tau^R \Omega^R_u (f_1 - f_3 b^R) + \Omega^R_u (1 - \tau^R_u) \Phi^R_s + f_5 Z^R_{u+1}]. \]

Thus, landlords maximize (A.67) subject to the constraints

\[ K^R_{s+1} = I^R_s + (1 - \delta^R) K^R_s \]

and

\[ \lim_{\tau \to -\infty} K^R_I \geq 0. \]

The Lagrangian for the rental housing market is

\[ L^R = \sum_{u=s}^{\infty} \prod_{u=s}^{u} \frac{1}{(1 + \theta^R)} \Gamma^R_u + f_5 \chi^R_i + q^R_{u+1} [I^R_u + (1 - \delta^R) K^R_u - K^R_{u+1}] \]

\[ = \sum_{u=s}^{\infty} \prod_{u=s}^{u} \frac{1}{(1 + \theta^R)} \{ \Gamma^R_u + f_5 \chi^R_i + q^R_{u+1} [I^R_u + (1 - \delta^R) K^R_u - K^R_{u+1}] \}, \]

where

\[ q^R_{u+1} = \prod_{u=s}^{u} \frac{1}{(1 + \theta^R)} q^R_{u+1}. \]

The necessary conditions for a maximum are derived as follows.

First, the necessary condition with respect to the optimal choice of investment in each period is

\[ \frac{\partial L}{\partial I^R_s} = \sum_{u=s}^{\infty} \prod_{u=s}^{u} \frac{1}{(1 + \theta^R)} \{ b^R \Omega^R_u - \Omega^R_u + \tau^R \Omega^R_u (f_1 - f_3 b^R) \\ + f_5 Z^R_{s+1} - (1 - \tau^R_s) \Omega^R_s \Phi^R_s - (1 - \tau^R_s) \Omega^R_s \frac{\partial \Phi^R_s}{\partial I^R_s} + q^R_{s+1} \}. \]
Recall that $\frac{\partial \phi'}{\partial I^R} = \frac{\phi'}{K^R}$, this implies

$$q_{s+1}^R = \Omega_u^R - b^R \Omega_u^R - \tau^R \Omega_u^R (f_1 - f_3 b^R) - f_5 Z_{s+1}^R + (1 - \tau_s^R) \Omega_s^R \Phi_s + \Phi_s' \frac{\Phi_s'}{K_s^R}. \quad (A.68)$$

Again, this describes the variable commonly known as Tobin’s $q$. It demonstrates that the shadow price of additional rental housing capital ($q_{s+1}^R$) must equal the after-tax marginal cost of rental housing capital (the right hand side). The first term indicates that the shadow price is simply one in the absence of debt and taxes, since the investment good is the numeraire. The second term reflects the financing of a fraction $b$ of the cost of the investment with debt. The third term reflects the reduction in the shadow price of new capital goods due to tax deductions for depreciation. The last term reflects the costs of installing new capital goods with immediate expensing of such adjustment costs. Recalling that $[\Phi_s + \frac{L_s}{K_s} \Phi_s'] = p_s \beta ([L_s^R] - \mu)$, this equation can be solved to give the optimal investment rate for the firm as

$$\frac{I_s^R}{K_s^R} = \frac{q_{s+1}^R - \Omega_u^R + b^R \Omega_u^R + \tau^R \Omega_u^R (f_1 - f_3 b^R) + f_5 Z_{s+1}^R}{(1 - \tau_s^R) \Omega_s^R p_s \beta} + \mu. \quad (A.69)$$

In order to express investment demand as a function of the value of the firm, $V_s^R$, rather than $q_{s+1}^R$, the relationship between marginal $q$ and average $q$ (denoted as $Q$), as shown by Hayashi (1982), is defined as

$$q_s^R = \frac{[V_s^R - X_s^R]}{K_s^R}, \quad Q_s^R = \frac{V_s^R}{K_s^R}.$$

Thus, the investment demand function can be written as

$$\frac{I_s^R}{K_s^R} = \frac{[V_{s+1}^R - X_{s+1}^R]}{K_{s+1}^R} - \Omega_u^R + b^R \Omega_u^R + \tau^R \Omega_u^R (f_1 - f_3 b^R) + f_5 Z_{s+1}^R}{(1 - \tau_s^R) \Omega_s^R p_s \beta} + \mu.$$

Noting that the $q_{s+1}^R$ term in the Lagrangian above has two terms with $K_s^R$, one in the current period and one in the next period, the necessary condition with respect to the optimal capital stock in each period,
\[
\frac{\partial L}{\partial K_s} = \sum_{u=s}^{\infty} \left[ \prod_{v=s}^{u-1} \frac{1}{(1 + \theta_v^R)} \right] \{ \Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} - [c p_u^R + b^R \Omega_u^R \delta^R - \Omega_u^R (1 - \tau_u^R) i_u b^R] \\
- I_s^R \Omega_s^R (1 - \tau_s^R) \frac{\partial \Phi_s^R}{\partial K_s^R} \} + \prod_{u=t}^{s} \frac{q_{s+1}^R}{(1 + \theta_u^R)} (1 - \delta^R) - \prod_{u=t}^{s-1} \frac{q_u^R}{(1 + \theta_u^R)} = 0,
\]

which implies

\[
\Omega_u^R (1 - \tau_u^R) i_u b^R = -\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} + c p_u^R + b^R \Omega_u^R \delta^R \\
+ I_s^R \Omega_s^R (1 - \tau_s^R) \frac{\partial \Phi_s^R}{\partial K_s^R} - q_{s+1}^R (1 - \delta^R) + q_s^R (1 + \theta_s^R)).
\]

Recall that \(\theta_s^R = \frac{\tau_s^R}{(1 - \tau_s^R)}\), this implies

\[
i_s[\Omega_s^R (1 - \tau_s^R) b^R + q_s^R \frac{1 - \tau_s^R}{(1 - \tau_s^R)}] = -\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} + c p_u^R + b^R \Omega_u^R \delta^R \\
+ I_s^R \Omega_s^R (1 - \tau_s^R) \frac{\partial \Phi_s^R}{\partial K_s^R} - q_{s+1}^R (1 - \delta^R) + q_s^R.
\]

Noting that

\[
\frac{\partial \Phi_s^R}{\partial K_s^R} = -\left( \frac{I_s^R}{(K_s^R)^2} \right) \Phi_s^R
\]

and

\[
\Phi_s^R \left( \frac{I_s^R}{K_s^R} \right) = \frac{\partial \Phi_s^R}{\partial \left( \frac{I_s^R}{K_s^R} \right)} = p_s^R \left( \frac{\beta}{2} \right) [1 - \left( \frac{\mu}{I_s^R K_s^R} \right)^2],
\]

this can be written as

\[
i_s = \frac{-\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} + c p_s^R + b^R \Omega_s^R \delta^R - q_{s+1}^R (1 - \delta^R) + q_s^R}{\Omega_s^R (1 - \tau_s^R) b^R + q_s^R \frac{(1 - \tau_s^R)}{(1 - \tau_{gs}^R)}} \tag{A.70}
\]

\[
+ \frac{\Omega_s^R (1 - \tau_s^R) p_s^R \left( \frac{\beta}{2} \right) \left[ \left( \frac{I_s^R}{K_s^R} \right)^2 - \mu^2 \right]}{\Omega_s^R (1 - \tau_s^R) b^R + q_s^R \frac{(1 - \tau_s^R)}{(1 - \tau_{gs}^R)}} \tag{A.71}
\]
Equation (A.70) is the Euler equation for the rental housing sector. It can be written as the following difference equation in \( q_t \),

\[
q_{s+1}^R (1 - \delta^R) = q_s^R - \Omega_s^R (1 - \tau_s^R) b_s^R + q_s^R \left( \frac{1 - \tau_s^R}{1 - \tau_{gs}^R} \right) i_s
\]

\[-\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} + c p_u^R + b^R \Omega_u^R \delta^R
\]

\[+\Omega_s^R (1 - \tau_s^R) p_s^R \left( \frac{\beta}{2} \right) \left( \frac{I_s^R}{K_s^R} \right)^2 - \mu^2 \],

or

\[
q_{s+1}^R (1 - \delta^R) = q_s^R \left[ 1 - \left( \frac{1 - \tau_i^R}{1 - \tau_{gs}^R} \right) i_s \right] - \Omega_s^R (1 - \tau_s^R) b_s^R i_s
\]

\[-\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} + c p_u^R + b^R \Omega_u^R \delta^R
\]

\[+\Omega_s^R (1 - \tau_s^R) p_s^R \left( \frac{\beta}{2} \right) \left( \frac{I_s^R}{K_s^R} \right)^2 - \mu^2 \],

or

\[
q_{s+1}^R (1 - \delta^R) = q_s^R \left[ 1 - \theta_s^R \right]
\]

\[-\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} + c p_u^R + b^R \Omega_u^R \delta^R
\]

\[+\Omega_s^R (1 - \tau_s^R) p_s^R \left( \frac{\beta}{2} \right) \left( \frac{I_s^R}{K_s^R} \right)^2 - \mu^2 \],

which can be solved to yield

\[
q_s^R = \sum_{u=s}^{\infty} \left\{ \prod_{v=s}^{u-1} \frac{1}{1 + \theta_v^R} \right\} (1 - \delta^R)^{u-t} \left[ -\Omega_s^R (1 - \tau_s^R) p_s^R \frac{\partial F}{\partial K_s^R} - \Omega_s^R (1 - \tau_s^R) b_s^R i_s
\]

\[+c p_u^R + b^R \Omega_u^R \delta^R + \Omega_s^R (1 - \tau_s^R) p_s^R \left( \frac{\beta}{2} \right) \left( \frac{I_s^R}{K_s^R} \right)^2 - \mu^2 \} \].

This equation implies that the shadow price of new rental housing capital — \( q_t^R \) — equals the present value of future income, reflecting the productivity of the asset,
depreciation allowances, savings in future installation costs, property tax cost, and future interest payments. Since \( q^R_s = \frac{V^R_s - X^R_s}{K^R_s} \), this value of “marginal” \( q \) can be used to solve for average \( Q \) as follows

\[
Q^R_s = \frac{V^R_s}{K^R_s} = q^R_s + \frac{X^R_s}{K^R_s};
\]

that is, average \( Q \) equals marginal \( q \) plus an adjustment for future depreciation deductions on existing assets. It is also interesting to note that equation (A.46) can be solved for the user cost of capital developed by Jorgenson (1963) — the minimum return the investment must yield in order to provide the investor with the same rate of return that would be received from lending at the after-tax interest rate, or

\[
F_K = \frac{\Omega_u(1 - \tau_{bu})i_u b - \delta(1 - b - \Omega_u) + q_s(1 + \theta_s) - q_{s+1}(1 - \delta)}{\Omega_s(1 - \tau_{bs})p_s} - \frac{(1 - \tau_{bu} \Omega_u)p_u(\frac{\delta}{K_s})[(\frac{b}{K_s})^2 - \mu^2]}{\Omega_s(1 - \tau_{bs})p_s}.
\]  

(A.72)

Finally, the optimal solution must satisfy

\[
\lim_{T \to \infty} K^R_{T+1} \geq 0; \lim_{T \to \infty} q^R_{T+1} \geq 0 \text{ and } \lim_{T \to \infty} K^R_{T+1} q^R_{T+1} = 0.
\]

(A.73)

**Owner-Occupied Housing**

To find the expression for the net service flow in the owner-occupied sector substitute equations (A.53) and (A.56) into (A.59) to yield

\[
S^o_s = p^o_s F(K^o_s; L^o_s) - w_s L^o_s - (1 - \tau_s)[b^o(K^o_s) + c(p^o_s K^o_s)] + b(I^o_s - \delta^o K^o_s) - L^o_s(1 + \Phi^o_s).
\]

(A.74)

Substituting equation (A.74) into (A.62) and defining \( \Omega^o_u = [(1 - \tau^o_u)/(1 - \tau^o_{gu})] \) yields

\[
V_s = \sum_{u=s}^{\infty} \prod_{v=s}^{u-1} \frac{1}{(1 + \theta^o_v)\Gamma^o_u},
\]

(A.75)
where
\[
\Gamma^o_u = \Omega_u^{o}[p^o_s F(K^o_s, L^o_s) - w_s L^o_s] - K^o_s[\Omega_u^o (1 - \tau_s) i_s b^o + \Omega_u^o (1 - \tau_s) c b^o p^o_s - \Omega_u^o b^o] \\
+ I^o_s[\Omega_u^o b^o - \Omega_u^o (1 + \Phi^o_s)]
\]

Owner-occupiers maximize (A.75) subject to the constraints
\[
K^o_{s+1} = I^o_s + (1 - \delta^o) K^o_s
\]
and
\[
\lim_{r \to \infty} K^o_r \geq 0.
\]

The Lagrangian for the owner-occupied housing market is
\[
L^o = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta^o_v)} \Gamma^o_u + q^o_{u+1}[I^o_u + (1 - \delta^o) K^o_u - K^o_{u+1}]
\]
\[
= \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta^o_v)} \{\Gamma^o_u + q^o_{u+1}[I^o_u + (1 - \delta^o) K^o_u - K^o_{u+1}]\},
\]
where
\[
q^o_{u+1} = \left[ \prod_{v=s}^{u} \frac{1}{(1 + \theta^o_v)} \right] q^o_{u+1}.
\]

The necessary conditions for a maximum are derived as follows.

First, the necessary condition with respect to the optimal choice of investment in each period is
\[
\frac{\partial L^o}{\partial I^o_s} = \sum_{u=s}^{\infty} \prod_{v=s}^{u} \frac{1}{(1 + \theta^o_v)} \left\{ \Omega_u^o b^o - \Omega_u^o (1 + \Phi^o_s) - I^o_s \Omega_u^o \frac{\partial \Phi^o_s}{\partial I^o_s} + q^o_{u+1} \right\} = 0.
\]
where \( \frac{\partial \Phi^o_s}{\partial I^o_s} = \frac{\Phi^o_s}{K^o_s} \), this implies
\[
q^o_{s+1} = \Omega_u^o - b^o \Omega_u^o + \Omega_s^o [\Phi^o_s + \left( \frac{I^o_s}{K^o_s} \right) \Phi^o_s].
\]

Again, this describes the variable commonly known as Tobin's \( q \). It demonstrates that the shadow price of additional owner-occupied housing capital \( (q^o_{s+1}) \) must equal
the after-tax marginal cost of owner-occupied housing capital (the right hand side). Since the investment good is the numeraire, the shadow price is simply one in the absence of debt and taxes. The second term reflects the financing of a fraction $b$ of the cost of the investment with debt. The third term reflects the reduction in the shadow price of new capital goods due to tax deductions for depreciation. The last term reflects the costs of installing new capital goods with immediate expensing of such adjustment costs. Recalling that \[ \Phi_s + \frac{1}{K_s} \Phi_s' = p_s^o \beta [\frac{r_s^o}{K_s^o} - \mu], \] this equation can be solved to give the optimal investment rate for the firm according to

\[ \frac{I_s^o}{K_s^o} = \left[ q_{s+1}^o - \frac{\Omega_u^o + b^o \Omega_u^o}{\Omega_s^o p_s^o \beta} \right] + \mu. \]  

(A.77)

In order to express investment demand as a function of the value of the firm, $V_s^o$, rather than $q_{s+1}^o$, the relationship between marginal $q$ and average $q$ (denoted as $Q$) as shown by Hayashi (1982) or Keuschnigg (1990) and described previously in the description of firm behavior is used. This implies

\[ q_s^o = \frac{[V_s^o - X_s^o]}{K_s^o}, \quad Q_s^o = \frac{V_s^o}{K_s^o}. \]

Thus, the investment demand function can be written as

\[ \frac{I_s^o}{K_s^o} = \left[ \frac{[V_s^o - X_{s+1}^o]}{K_{s+1}^o} - \frac{\Omega_u^o + b^o \Omega_u^o}{\Omega_s^o p_s^o \beta} \right] + \mu. \]

Noting that the $q_{s+1}^o$ term in the Lagrangian above has two terms with $K_s^o$, one in the current period and one in the next period, the necessary condition with respect to the optimal capital stock in each period,

\[ \frac{\partial \mathcal{L}}{\partial K_s^o} = \sum_{u=s}^{\infty} \prod_{v=s}^{u-1} \frac{1}{(1 + \theta_v^o)} \left\{ \Omega_s^o p_s^o \frac{\partial F}{\partial K_s^o} - [\Omega_u^o (1 - \tau_u) \delta u^o b^o + \Omega_u^o (1 - \tau_u) c b^o p_s^o - \Omega_u^o b^o \delta^o] \right\} \]

\[ -I_s^o \Omega_s^o \frac{\partial \Phi_s^o}{\partial K_s^o} + \prod_{u=t}^{s} \frac{q_{s+1}^o}{(1 + \theta_u^o)} (1 - \delta^o) - \prod_{u=t}^{s-1} \frac{q_s^o}{(1 + \theta_u^o)} = 0, \]
which implies

\[
\Omega^o_s(1 - \tau^o_s)\imath u b^o = \Omega^o_s p^o_s \frac{\partial F}{\partial K^o_s} - \Omega^o_u(1 - \tau_{iu}) cb^o p^o_u + \Omega^o_s b^o \delta^o + I^o_s \Omega^o_s \frac{\partial \Phi^o_s}{\partial K^o_s} - q^o_{s+1}(1 - \delta^o) + q^o_s(1 + \theta^o_s).
\]

Recall that \( \theta^o_s = \frac{r^o_s}{(1 - \tau^o_s)} \), and thus,

\[
i_s[\Omega^o_s(1 - \tau^o_s)b^o + q^o_s(1 - \tau_{is})] = \Omega^o_s p^o_s \frac{\partial F}{\partial K^o_s} - \Omega^o_u(1 - \tau_{iu}) cb^o p^o_u + \Omega^o_s b^o \delta^o + I^o_s \Omega^o_s \frac{\partial \Phi^o_s}{\partial K^o_s} - q^o_{s+1}(1 - \delta^o) + q^o_s.
\]

Since

\[
\frac{\partial \Phi^o_s}{\partial K^o_s} = -\frac{I^o_s}{((K^o_s)^2)} \Phi^o_s
\]

and

\[
\Phi^o_s(\frac{I^o_s}{K^o_s}) = \frac{\partial \Phi^o_s}{\partial (\frac{I^o_s}{K^o_s})} = p^o_s(\frac{\beta}{2})[1 - (\frac{\mu}{I^o_s})^2],
\]

this can be written as

\[
i_s = \frac{\Omega^o_s p^o_s \frac{\partial F}{\partial K^o_s} - \Omega^o_u(1 - \tau_{iu}) cb^o p^o_u + \Omega^o_s b^o \delta^o - q^o_{s+1}(1 - \delta^o) + q^o_s - \Omega^o_s p^o_s \left[\frac{\beta}{2}\right][\left(\frac{I^o_s}{K^o_s}\right)^2 - \mu]}{\Omega^o_s(1 - \tau^o_s)b^o + q^o_s \left(1 - \tau_{is}\right)/\left(1 - \tau^o_s\right)}.
\]  

(A.78)

Equation (A.78) is the Euler equation for owner-occupied housing. It can be written as the following difference equation in \( q_t \),

\[
q^o_{s+1}(1 - \delta^o) = q^o_s - [\Omega^o_s(1 - \tau^o_s)b^o + q^o_s \left(1 - \tau_{is}\right)/\left(1 - \tau^o_s\right)]i_s
\]

\[
-\Omega^o_s p^o_s \frac{\partial F}{\partial K^o_s} + \Omega^o_u(1 - \tau_{iu}) cb^o p^o_u + \Omega^o_s b^o \delta^o + \Omega^o_s p^o_s \left[\frac{\beta}{2}\right][\left(\frac{I^o_s}{K^o_s}\right)^2 - \mu^2],
\]
or

\[ q_{s+1}(1 - \delta^o) = q_s^o[1 - \frac{(1 - \tau_{is})}{(1 - \tau_{is})}] - \Omega_s^o(1 - \tau_s^o)b^o i_s \]

\[-\Omega_s^o p_s^o \frac{\partial F}{\partial K_s^o} + \Omega_s^o (1 - \tau_{iu}) c b^o p_u^o + \Omega_s^o b^o \delta^o \]

\[+\Omega_s^o p_s^o \frac{\beta}{2} \left[ \left( \frac{I_s^o}{K_s^o} \right)^2 - \mu^2 \right], \]

or

\[ q_{s+1}(1 - \delta^o) = q_s^o[1 - \theta_s^o] - \Omega_s^o(1 - \tau_s^o)b^o i_s \]

\[-\Omega_s^o p_s^o \frac{\partial F}{\partial K_s^o} + \Omega_s^o (1 - \tau_{iu}) c b^o p_u^o + \Omega_s^o b^o \delta^o \]

\[+\Omega_s^o p_s^o \frac{\beta}{2} \left[ \left( \frac{I_s^o}{K_s^o} \right)^2 - \mu^2 \right], \]

which can be solved to yield

\[ q_s^o = \sum_{u=s}^{\infty} \left\{ \prod_{u=s}^{\infty} \frac{1}{(1 + \theta_u^o)}(1 - \delta^o)^{u-t} \left[ \Omega_u^o p_u^o \frac{\partial F}{\partial K_u^o} - \Omega_u^o (1 - \tau_u^o)b^o i_s \right. \right. \]

\[+\Omega_u^o (1 - \tau_{iu}) c b^o p_u^o + \Omega_u^o b^o \delta^o + \Omega_u^o p_u^o \frac{\beta}{2} \left[ \left( \frac{I_u^o}{K_u^o} \right)^2 - \mu^2 \right] \} \] \]

This equation implies that the shadow price of new owner-occupied housing capital — \( q_t^o \) — equals the present value of future income, reflecting the productivity of the asset, depreciation allowances, savings in future installation costs, property tax cost, and future interest payments. Since \( q_s^o = \frac{V_s^o - X_s^o}{K_s^o} \), this value of “marginal” \( q \) can be used to solve for average \( Q \) as

\[ Q_s^o = \frac{V_s^o}{K_s^o} = q_s^o + \frac{X_s^o}{K_s^o}, \]

that is, average \( Q \) equals marginal \( q \) plus an adjustment for future depreciation deductions on existing assets. It is also interesting to note that equation (A.78) can be solved for the user cost of capital developed by Jorgenson (1963) — the minimum
return the investment must yield in order to provide the investor with the same rate of return that would be received from lending at the after-tax interest rate, or

\[
F_K = \frac{\Omega_u (1 - \tau_{bu})u - \delta (1 - b - \Omega_u) + q_s (1 + \theta_s) - q_{s+1} (1 - \delta)}{\Omega_s (1 - \tau_{bs})p_s} \Omega_s (1 - \tau_{bs})p_s
\]

Finally, the optimal solution must satisfy

\[
\lim_{T \to \infty} K_{T+1}^o \geq 0; \lim_{T \to \infty} q_{T+1}^o \geq 0 \text{ and } \lim_{T \to \infty} K_{T+1}^o q_{T+1}^o = 0. \tag{A.80}
\]

### A.4 Measuring Flat Tax Reform-Induced Welfare Changes

The changes in welfare that occur when the income tax is replaced with the Flat Tax are calculated as described below; this method follows Auerbach and Kotlikoff (1987) and Seidman (1983). The approach is based on calculating the change in “rest-of-life resources” that would be needed under the reformed tax system to return an individual to the lifetime utility level that would have been obtained in the absence of reform (that is, if the income tax had been maintained indefinitely). Since the lifetime utility function is homothetic, this change in wealth is proportional to the percentage change in annual consumption, each year, under the retail sales tax that would yield the same rest-of-life” lifetime utility level as obtained under the income tax. To convert this change to a relative concept, it is expressed as a percent of total lifetime resources under continued maintenance of the income tax regime. For a representative individual of cohort “a”, this welfare change is determined as follows.

If the income tax regime were maintained indefinitely, rest-of-life” utility $LU_Y(a)$ for an individual age $a$ at the time of reform, would have been

\[
LU_Y(a) = \frac{1}{(1 - \frac{1}{\delta})} \sum_{s=t}^{54+t} \frac{\bar{C}_{sY}(a) (1 - \frac{1}{\delta})}{(1 + \rho)^{s-t}},
\]
where $\bar{C}_sY$ is consumption in period $s$ under the income tax system. The expressions for consumption obtained above indicate that a proportionate increase in total discretionary wealth gives rise to an equiproportionate increase in consumption in each period — that is, the utility function chosen is homothetic. Recall that total discretionary wealth is defined as

$$TW_s = A_s(1 + r_s) + TDA_s(1 + i_s) - \frac{BQ_{t+54-a}}{\prod_{u=t+1}^{t+54-a}(1 + r_u)}$$

$$+ \sum_{s=t}^{t+54-a} \frac{w_s(H_T - E_s) - SD_s(1 - \tau_{wages} - \tau_{ss})}{\prod_{u=t+1}^{s}(1 + r_u)}$$

$$+ \sum_{s=t}^{t+54-a} \frac{SSB_s + WD_s(1 - \tau_{w}) + TR_s + LSR_s}{\prod_{u=t+1}^{s}(1 + r_u)}$$

$$- \sum_{s=t}^{t+54-a} \frac{p_s(1 + \tau_{cs})b_{1s} + p_s^2b_{2s} + p_{O_s}b_{O_s} + p_{R_s}b_{R_s}}{\prod_{u=t+1}^{s}(1 + r_u)}.$$

The desired proportionate increase in wealth and consumption denoted as $DW(a)$, is that value which will satisfy

$$LU_Y(a) = \frac{1}{(1 - \frac{1}{a})} \sum_{s=t}^{54+t-a} \bar{C}_s(a)^{(1 - \frac{1}{a})},$$

where

$$\bar{C}_sY = \bar{C}_s(a)[1 + DW(a)],$$

where $\bar{C}_s(a)$ is the value under the retail sales tax reform as calculated above. Thus,

$$LU_Y(a) = [1 + DW(a)]^{1-1/a} LU(a)$$

where $LU(a)$ is the rest-of-life” utility level obtained under the retail sales tax reform. Thus,

$$DW(a) = \left[ \frac{LU_Y(a)}{LU(a)} \right]^{1-1/a} - 1,$$

and the required change in rest-of-life resources needed to attain the same utility level as that which would have been obtained under the income tax is simply
$DW(a)TW_t(a)$. This is easily converted to the analogous percentage change in lifetime resources, measured in present values calculated in the year of birth ($t^*$) of an individual age $a$ in the time of reform, which is denoted as $DLR(a)$, as

$$DLR(a) = \left[ \frac{DW(a)TW_t(a)}{TLR_{t^*}(a) \prod_{u=t^*+1}^{t^*+54} (1 + r_u)} \right]$$

where $TLR_{t^*}(a)$ is total lifetime resources (including those used to make minimum required consumption purchases, and the required bequest - government bonds need not be added since they do not add to net wealth) of an individual age $a$ at the time of reform, calculated in the year of birth $t^*$, or

$$TLR_{t^*}(a) = [INH_t(a)](1+r_{t^*}) + \sum_{s=t^*}^{t^*+54} \left[ w_s(a)L_s(1 - \tau_{ws} - \tau_{ss}) + SSB_s + TR_s + LSR_s \right] \prod_{u=t^*+1}^{t^*+54} (1 + r_u)$$

Thus, the percentage changes in lifetime resources induced by the implementation of the sales tax reform are $DLR(a)$, and these are used as the measure of reform-induced welfare changes in the analysis.