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Exchange-Rate-Based Stabilization Syndrome: Credible Disinflation, Capital Inflows, and the Domestic Banking System

by

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Houston, Texas

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy

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Houston, Texas
April, 1998
Abstract

Exchange-Rate-Based Stabilization Syndrome: Credible Disinflation, Capital Inflows, and the Domestic Banking System

by

Yuri V. Sobolev
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Interactions between the banking sector and an open capital account are investigated as rationalizations for the empirical regularities that characterize disinflation programs anchored by fixing the exchange rate. Financial intermediation and bank money creation are formalized within a dynamic general equilibrium model with multiple monetary aggregates and a financial system characterized by imperfections such as incomplete markets and an externality in the bank lending process. Financial markets are incomplete in the sense that bank loans are the sole source of external finance for nonfinancial firms, and bank deposits are the only form of household savings. The bank lending externality arises because individual banks do not internalize the effect of their lending decisions on the quality of information about potential borrowers received by other banks, and therefore extend more credit than they otherwise would.

Simulation of the model economy's equilibrium dynamics shows that an initial increase in the supply of loanable funds resulting from remonetization of the economy in the wake of disinflation can translate into a further rapid expansion of bank credit
financed by short-term capital inflows. A credit-driven boom results, accompanied by a currency overvaluation and current account deficits. Together, these generate systemic financial fragilities and make the economy vulnerable to a small shock that can trigger banking and balance-of-payment crises. The model is thus capable of replicating the empirical regularities observed in exchange-rate-based stabilization programs without relying on imperfect credibility or nominal rigidities.

Accounting for the role of the banking sector can help to explain why even well-designed exchange-rate-based stabilization programs may set in motion a dynamic process that can lead to a financial crisis and the program’s collapse. The policy implication for developing countries is that the authorities should pay attention not only to the design of monetary and exchange rate policies but also to the framework of monetary and financial institutions. The results of the study suggest that diversifying the source of investment finance away from banks and reducing externalities associated with bank lending may be essential preconditions for implementing successful stabilization programs.
Acknowledgements

I am very grateful to my advisor, Peter Hartley, and to Alain Ize, George Kanatas, and Jeromin Zettelmeyer for helpful comments and suggestions. I would also like to thank Yusuke Horiguchi for his suggestion to investigate the macroeconomic implications of capital inflows as the topic of my doctoral research.
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Chapter 1

Exchange-Rate-Based Stabilization Syndrome: Empirical Evidence and Existing Theories

1.1 Introduction

Stabilizing high inflation by fixing the exchange rate has gained new prominence as a result of its widespread use in the previously centrally planned economies of Eastern Europe and the former Soviet Union. Exchange-rate-based stabilization (ERBS) programs also have a long history in many countries in Latin America.

There are a number of reasons why an exchange-rate-based stabilization may be preferable to a money-based stabilization. Unlike the money supply, a nominal exchange rate anchor is readily observed by all participants in the economy and may be more effective in reducing high inflation in the face of unstable money demand, large movements in velocity, and a high degree of dollarization. A fixed exchange rate also may induce greater financial discipline on the part of the authorities since it places their foreign reserve holdings at risk. Indeed, evidence presented by Sahay and Végh (1996) suggests that exchange rate anchors have generally been superior to money anchors in reducing inflation in Eastern Europe. On the other hand, if the underlying fiscal and political conditions are not right, a fixed exchange rate strategy can quickly lead to major distortions and defeat the basic objective of the program.

Yet even well-engineered ERBS programs that succeed in bringing inflation down for a sustained period—a year and more—may set in motion a dynamic process that can lead to a financial crisis and the program’s collapse. These disruptive dynamics can be summarized as an “exchange-rate-based stabilization syndrome”: an initial
expansion in economic activity, financed largely by capital inflows and accompanied by sharp real appreciation and widening external imbalances, is followed by a balance-of-payments crisis and forced devaluation. The most recent illustration of this phenomenon is the collapse of the Mexican peso in December 1994.

Three major explanations emerged from a large literature that sought to rationalize the empirical regularities observed in ERBS programs: (1) the lack of credibility (or temporariness) theory; (2) the sticky inflation (or nominal rigidities) theory; and (3) the wealth effects (or equilibrium) theory. None of these theories posited a role for the banking sector in disinflation programs or explained the behavior of important financial sector variables such as broad money and banking credit. Indeed, the existing theories have largely rationalized the ERBS syndrome by relying on imperfect credibility and nominal rigidities in an otherwise classical framework where markets are complete and market failures are absent. In a classical world, money has no real effects and the financial sector becomes irrelevant.  

Yet the financial system in developing countries is characterized by two critical imperfections: incomplete financial markets and an externality in the bank lending process. Financial markets are incomplete in the sense that bank loans are the dominant source of external finance for nonfinancial firms, and bank deposits are the most important form of household savings. The bank lending externality arises because individual banks do not internalize the effect of their lending decisions on the quality of information about potential borrowers received by other banks, and therefore extend more credit than they otherwise would. Accounting for the role of the banking

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1Several studies—notably Goldfajn and Valdés (1997), Edwards and Végh (1997), Kaminsky and Reinhart (1996), McKinnon and Pill (1996), and Velasco (1987)—have focused on financial intermediation in a broader context of exchange rate crises. However, none of them analyzed the ERBS syndrome as such. Also, none of them examined the externality in the lending process or substitution between inside and outside money that we examine in this paper.
sector can therefore help to explain why even well-designed ERBS programs may end up in financial and balance-of-payments crises.

This thesis develops an alternative explanation for the ERBS syndrome. It emphasizes the role of the domestic banking system as the major conduit for capital inflows in economies undertaking exchange-rate-based stabilization and market-oriented structural reforms. In particular, the thesis shows that, in financial systems where banks dominate and there is an externality in the lending process, an initial increase in the supply of loanable funds resulting from remonetization\(^2\) of the economy in the wake of disinflation can translate into a rapid expansion of bank credit financed by short-term capital inflows. A credit-driven boom results, accompanied by a currency overvaluation and current account deficits even within an environment of perfect credibility and price flexibility. Together, these generate systemic financial fragilities and make the economy vulnerable to a small shock that can trigger banking and balance-of-payment crises.

The thesis is organized as follows. Chapter 1 presents a brief overview of the stylized facts of ERBS programs and theories that have been advanced to explain them. Chapter 2 offers an alternative explanation with financial intermediation playing a central role, and shows how financial intermediation can be incorporated within a dynamic general equilibrium model with multiple monetary aggregates. Chapter 3 studies the sensitivity of the model's predictions to alternative parameter values and structural specifications of the financial sector, and draws policy lessons. Chapter 4 describes the solution method employed in the thesis. Finally, Appendix A presents a computer program for computing dynamic equilibrium and impulse response functions.

\(^2\)The term "remonetization" refers to a rise in real holdings of domestic money.
1.2 Stylized Facts of ERBS Programs

Numerous inflation stabilization programs have been undertaken in high inflation countries during the past 30 years. For the empirical analysis, this study concentrates on the ten major stabilization plans which represent clearly defined and lasting episodes. Table 1.1 on page 11 describes the main features of the plans under study. As is evident from Table 1.1 and Figures 1.1 and 1.2 on pages 14 and 15, the stabilization plans differed markedly in four main respects: (1) the degree of exchange rate "flexibility"—ranging from crawling bands to a currency board; (2) the reliance on supplementary price and wage controls (incomes policies); (3) the magnitude of the accompanying fiscal consolidation (expenditure reduction and revenue collection measures aimed at reducing the fiscal deficit); and (4) the nature of structural reforms undertaken before or during the plan. In spite of the high degree of variations in these key program characteristics, the pattern of macroeconomic effects across the stabilization plans is strikingly similar. Figures 1.3 through 1.13 on pages 16 - 26 illustrate the dynamics associated with the ten major stabilization plans, and Tables 1.2 and 1.3 on pages 12 and 13 summarize the main macroeconomic effects of the plans.

---


4 The average in the bottom panel of Figure 1.2 was calculated as a mean of the observations available in year $t + k$, where $t$ is the first year of the program and integer $k \in [-3, 5]$, and in year $T + j$, where $T$ is the last year of the program and integer $j \in [1, 3]$. All index numbers were normalized to unity in year $t = 0$. (The program duration for Israel was arbitrarily set to six years.)

5 The representative dynamics depicted in Figure 1.13 were derived by calculating the averages of the ten major ERBS programs listed in Table 1.1. The average for each series was calculated as a mean of the observations available in year $t + k$, where $t$ is the first year of the program and integer
As is evident from Tables 1.2 and 1.3 and Figure 1.13, the key empirical regularities that have characterized the exchange-rate-based stabilization programs in high inflation countries include:

1. Remonetization of the economy occurs, accompanied by a strong increase in private sector credit even when measured relative to real economic activity.

2. The rate of inflation converges slowly to the new lower rate of devaluation, and is accompanied by a rise in the relative price of nontraded goods—that is, an appreciation of the real exchange rate.\(^6\)

3. The trade balance and the current account of the balance of payments deteriorate, with the current account deficits being financed by large capital inflows.

4. There is an initial expansion in economic activity (output and investment) relative to trend, which is accompanied by a private consumption boom and an increase in real wages.\(^7\)

5. There is a boom-bust "cycle" in the sense that the stabilization program, more often than not, culminates in a financial crisis, capital flight, and forced devaluation of the currency followed by a severe recession.

---

\(^k\in[-3,5]\), and in year \(T+j\), where \(T\) is the last year of the program and integer \(j\in[1,3]\). All index numbers were normalized to unity in year \(t=0\). (The program duration for Israel was arbitrarily set to six years.)

\(^6\)The appreciation of the real exchange rate is evident in the figures as the gap between the rate of inflation and the rate of devaluation.

\(^7\)Reinhart and Végh (1995b) use the fixed effects (least squares dummy variable) model to estimate GDP growth and its composition during ERBS plans for the period 1964–1994. They report that, during an early phase of a stabilization plan, growth rates of real GDP, private consumption, and fixed investment are significantly above their historic means: real GDP growth is 2.4 percent higher, while real private consumption and fixed investment growth rates are 5.1 and 5.9 percent higher, respectively.
Figures 1.14 to 1.16 on pages 27 - 29 show the behavior of interest rates in six stabilization plans for which data were available. Figure 1.17 on page 30 plots the average of the six plans for each series. As is evident from the figure, on average, nominal interest rates fall with inflation while ex-post real interest rates rise. Both nominal and real interest rate spreads decline.

To facilitate comparison with the simulation results reported in Chapters 2 and 3, Figures 1.18 and 1.19 on pages 31 and 32 present the representative dynamics of ERBS programs and the average behavior of interest rates expressed as percentage deviations of the variables from their values at the beginning of a stabilization program.

Figures 1.21 through 1.23 on pages 35 - 37 depict the macroeconomic effects of three ERBS programs in selected transition economies—the Czech Republic, Estonia, and Lithuania. (See Tables 1.4 and 1.5 on page 33 and Figure 1.20 on page 34 for a short description of the main program characteristics and their macroeconomic effects.) These economies are undergoing deep structural transformations in their transition from central planning to a market-based economic system. The macroeconomic effects of these structural changes are likely to dominate the effects of exchange-rate-based stabilization. For this reason, these stabilization episodes are reported separately and were not used in the derivation of the representative dynamics.

---


1.3 Existing Theories

1.3.1 Lack of Credibility

According to this theory, pioneered by Calvo (1986b), economic agents expect the disinflation program to be discontinued in the future. The anticipated future higher inflation in the context of a nominal interest rate fixed by interest parity lowers the effective price of consumption now versus consumption later. Via intertemporal substitution, this induces an initial consumption and output boom, and produces real appreciation.\(^{10}\)

Reinhart and Végh (1995a) assess the empirical relevance of the lack of credibility hypothesis in a perfect foresight framework. They conclude that, given low intertemporal elasticities of substitution in consumption, sharp declines in nominal interest rates—larger than have been registered in several programs—are needed to produce consumption booms of the order of magnitude that has been observed.\(^{11}\)

1.3.2 Sticky Inflation

According to this theory, first proposed by Dornbusch (1982) and Rodriguez (1982), fixing the exchange rate reduces the nominal interest rate as a result of the interest parity arbitrage condition. Persistent inflation arising from adaptive expectations and backward-looking indexation and contracts then reduces the real interest rate.\(^{12}\) This in turn causes an aggregate demand boom. Higher demand, plus the inertia in inflation rates combined with the currency peg, results in overvaluation of the

---

\(^{10}\)Calvo and Végh (1994a) and Calvo and Végh (1993) extend the basic model to incorporate currency substitution and sticky prices.

\(^{11}\)Mendoza and Uribe (1996) improve on the quantitative predictions of the temporariness hypothesis by relaxing the perfect foresight assumption. They model the lack of credibility as the probability of abandoning the currency peg with the date of collapse being a random variable with finite support.

\(^{12}\)See Fischer (1986) for a theoretical analysis of the effects of indexation on macroeconomic stability and the problem of disinflation.
domestic currency. Overvaluation in turn causes output to decline and, eventually, leads to a speculative attack on the exchange rate.

Calvo and Végh (1994b) question the explanatory power of this hypothesis from an analytical perspective. They argue that the initial fall in the real interest rate will cause an initial expansion only if the intratemporal elasticity of substitution between traded and nontraded goods is lower than the intertemporal elasticity of substitution in consumption, which they claim to be empirically implausible. From an empirical standpoint, Khamis (1996) points out that ex-post real interest rates rose sharply upon the initiation of heterodox\textsuperscript{13} stabilization plans in the mid-1980s, which undermines the main building block of the sticky inflation hypothesis.\textsuperscript{14} Moreover, backward-looking contracts may themselves only persist when the disinflation policy lacks credibility.\textsuperscript{15}

1.3.3 Wealth Effects

According to this theory, the dynamics of a disinflation program anchored by fixing the nominal exchange rate are driven by wealth effects generated by one of the following mechanisms: (1) intergenerational redistribution of wealth (as in Helpman and Razin (1987)); (2) fiscal consolidation (as in Rebelo (1994)); and (3) supply-side response (as in Roldós (1995)).

(1) Helpman and Razin (1987) examine a model where individuals have finite horizons, and hence Ricardian equivalence does not hold. In that setting, freezing the nominal exchange rate reduces the inflation tax and provides the generation currently

\textsuperscript{13}The so-called heterodox ERBS programs include incomes policies to control prices and wages as opposed to orthodox ERBS programs that rely exclusively on the nominal exchange rate.

\textsuperscript{14}Nevertheless, Dornbusch and Werner (1994) use the notion of sticky inflation to analyze the 1987 Mexican heterodox stabilization. On a similar note, see Dornbusch and Edwards (1994).

\textsuperscript{15}One could contrast persistent backward-looking expectations with the rapid fall in inflation at the end of the German hyperinflation following credible institutional reform.
alive with a capital gain that is not offset by future tax liabilities. As a result, higher consumption by the current generation, which future generations pay for in the form of lower consumption, brings about a real exchange rate appreciation and worsening of the current account.

(2) Rebelo (1994) considers a situation where the initial fiscal position is unsustainable in the long run. A fiscal adjustment undertaken in conjunction with adopting a fixed exchange rate strategy then creates a positive wealth effect by lowering the present value of future tax liabilities perceived by private agents. These liabilities represent the real resources the government eventually would have to extract from the private sector to bring the fiscal situation under control. As a result of the wealth effect, economic expansion takes place in spite of increased tax rates, and is accompanied by real appreciation and deterioration of the trade balance.¹⁶ Reinhart and Végh (1995b) make a tentative conclusion that the fiscal effects explanation may be discarded based on its poor predictive power and the fact that fiscal policy varied greatly across stabilization plans.¹⁷

(3) Roldós (1995) assumes a differential supply response in the traded goods sector vis-à-vis the nontraded goods sector, with capital being used and accumulated only in the traded goods sector. A permanent reduction in the devaluation rate leads to a higher real rate of return on domestic assets, and thus to a higher desired consumption and capital stock in the long run. Upon implementation of the policy, the increase in capital accumulation and consumption produces a real exchange rate appreciation and current account deficit. Subsequently, new capital installed in the traded goods sector attracts labor away from the nontraded goods sector, and thus leads to further real appreciation. At the same time, the trade balance continuously improves as tradables

¹⁶See Drazen and Helpman (1988) and Drazen and Helpman (1987) for a closely related analysis of the effects of anticipated changes in fiscal policy on ERBS dynamics.
¹⁷See Figures 1.1 and 1.2.
output increases in response to the growing capital stock. Thus, in transition to a new steady state, the economy experiences improving trade balances along with persistent real exchange rate appreciation.\textsuperscript{18} This hypothesis therefore views real appreciation and current account deficits as an equilibrium response of the economy to a credible disinflation. Viewing them as equilibrium responses, however, makes an occurrence of financial or balance-of-payments crises highly unlikely.

1.4 Concluding Remarks about Evidence and Theories

Inflation stabilization programs anchored by fixing the exchange rate have been characterized by a series of empirical regularities that can be summarized as an “exchange-rate-based stabilization syndrome”: an initial expansion in economic activity, financed largely by capital inflows and accompanied by sharp real appreciation and widening external imbalances, is followed by a balance-of-payments crisis and forced devaluation. The existing theories suffer from various shortcomings in explaining the behavior of key economic variables in observed episodes of exchange-rate-based stabilizations. A point of departure for our analysis, however, is that the theories that allow for a possibility of a financial crisis—namely, the lack of credibility and sticky inflation theories—have used a framework where the banking sector plays no role, even though some studies (for instance, Rebelo and Végh (1995)) cite a rapid increase in the ratio of M1 to GDP as a stylized fact of ERBS programs. These theories thus cannot explain the role of important financial variables such as broad money and banking credit. By incorporating an explicit role for these variables into our model economy, we hope to better explain the stylized facts of ERBS programs. This is the central task of Chapter 2.

\textsuperscript{18}See also Roldós (1997) and Uribe (1995).
Table 1.1 Ten Major Exchange-Rate-Based Stabilizations. Program Characteristics

<table>
<thead>
<tr>
<th>Program</th>
<th>Exchange Rate Arrangement</th>
<th>Incomes Policies</th>
<th>Fiscal Consolidation</th>
<th>Structural Reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil 1964:3–1968:8</td>
<td>Peg</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Argentina 1967:3–1970:5</td>
<td>Peg</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes. Incentives for capital inflows</td>
</tr>
<tr>
<td>Uruguay 1968:5–1971:12</td>
<td>Peg</td>
<td>Yes, initially</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Chile 1978:2–1982:6</td>
<td>Crawl, then peg</td>
<td>No</td>
<td>Yes</td>
<td>Yes. Trade and capital account liberalization</td>
</tr>
<tr>
<td>Uruguay 1978:10–1982:10</td>
<td>Crawl</td>
<td>No</td>
<td>Yes</td>
<td>Yes. Trade liberalization</td>
</tr>
<tr>
<td>Argentina 1978:12–1981:2</td>
<td>Crawl</td>
<td>No</td>
<td>Yes, moderate</td>
<td>Yes. Trade and capital account liberalization</td>
</tr>
<tr>
<td>Israel 1985:7–present</td>
<td>Peg, then crawl, then band, then crawling band</td>
<td>Yes</td>
<td>Yes, major</td>
<td>Yes. Financial sector deregulation</td>
</tr>
<tr>
<td>Mexico 1987:12–1994:12</td>
<td>Peg, then crawl, then band</td>
<td>Yes</td>
<td>Yes, major</td>
<td>Yes. Privatization, trade and capital account liberalization</td>
</tr>
<tr>
<td>Uruguay 1991:1–present</td>
<td>Crawl, then band</td>
<td>No</td>
<td>Yes, initially</td>
<td>—</td>
</tr>
<tr>
<td>Argentina 1991:4–present</td>
<td>Currency board</td>
<td>No</td>
<td>Yes, major</td>
<td>Yes. Privatization, trade liberalization</td>
</tr>
</tbody>
</table>

Notes: The program dates comprise the months during which the program was in effect; "Peg" means fixed exchange rate with infrequent adjustments; "Crawl" means preannounced rate of devaluation; "Band" means exchange rate floating within a fixed band; "Crawling band" means preannounced rate of devaluation of the upper and lower bounds; "Currency board" means unequivocal commitment of the central bank to supply or redeem, without limit, its monetary liabilities at predetermined exchange rate.
### Table 1.2 Ten Major Exchange-Rate-Based Stabilizations.
**Macroeconomic Effects, Financial and External Sectors**

<table>
<thead>
<tr>
<th>Program</th>
<th>Remonetization</th>
<th>Lending Boom</th>
<th>Real Exchange Rate</th>
<th>Trade Balance</th>
<th>Capital Inflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil 1964:3–1968:8</td>
<td>Yes, in late phase</td>
<td>Yes, in late phase</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge in late phase</td>
</tr>
<tr>
<td>Argentina 1967:3–1970:5</td>
<td>Yes</td>
<td>Yes</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
<tr>
<td>Uruguay 1968:5–1971:12</td>
<td>Not clear</td>
<td>Not clear</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
<tr>
<td>Chile 1978:2–1982:6</td>
<td>Yes</td>
<td>Yes</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
<tr>
<td>Uruguay 1978:10–1982:10</td>
<td>Yes, in late phase</td>
<td>Yes, in late phase</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
<tr>
<td>Argentina 1978:12–1981:2</td>
<td>Not clear</td>
<td>Yes, in late phase</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge in early phase</td>
</tr>
<tr>
<td>Israel 1985:7–present</td>
<td>Not clear</td>
<td>No</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge in early phase</td>
</tr>
<tr>
<td>Mexico 1987:12–1994:12</td>
<td>Yes</td>
<td>Yes</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
<tr>
<td>Uruguay 1991:1–present</td>
<td>No</td>
<td>No</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
<tr>
<td>Argentina 1991:4–present</td>
<td>Yes</td>
<td>Yes</td>
<td>Appreciates</td>
<td>Deteriorates</td>
<td>Surge</td>
</tr>
</tbody>
</table>

**Notes:** The program dates comprise the months during which the program was in effect; Qualitative effects reported are based on author’s calculations using data from IMF, International Financial Statistics; “Remonetization” refers to rising ratios of broad money to base money, broad money to GDP, and ratio of deposits to currency.
<table>
<thead>
<tr>
<th>Program</th>
<th>Consumption Boom</th>
<th>Investment Boom</th>
<th>Real GDP</th>
<th>Real Wages</th>
<th>Ended in Crisis?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil 1964:3–1968:8</td>
<td>Yes</td>
<td>Yes, in late phase</td>
<td>Expands</td>
<td>—</td>
<td>No</td>
</tr>
<tr>
<td>Argentina 1967:3–1970:5</td>
<td>Yes</td>
<td>Yes</td>
<td>Expands</td>
<td>—</td>
<td>Yes</td>
</tr>
<tr>
<td>Uruguay 1968:5–1971:12</td>
<td>Yes</td>
<td>Yes</td>
<td>Expands</td>
<td>—</td>
<td>Yes</td>
</tr>
<tr>
<td>Chile 1978:2–1982:6</td>
<td>Yes</td>
<td>Yes</td>
<td>Expands</td>
<td>Rise</td>
<td>Yes</td>
</tr>
<tr>
<td>Uruguay 1978:10–1982:10</td>
<td>No</td>
<td>No</td>
<td>Expands in early phase</td>
<td>—</td>
<td>Yes</td>
</tr>
<tr>
<td>Argentina 1978:12–1981:2</td>
<td>Yes, in early phase</td>
<td>Yes, in early phase</td>
<td>Expands</td>
<td>—</td>
<td>Yes</td>
</tr>
<tr>
<td>Israel 1985:7–present</td>
<td>Yes</td>
<td>Yes, in early phase</td>
<td>Expands</td>
<td>Rise</td>
<td>No</td>
</tr>
<tr>
<td>Mexico 1987:12–1994:12</td>
<td>Yes</td>
<td>Yes</td>
<td>Expands</td>
<td>Rise</td>
<td>Yes</td>
</tr>
<tr>
<td>Uruguay 1991:1–present</td>
<td>Yes</td>
<td>Yes</td>
<td>Expands</td>
<td>—</td>
<td>No</td>
</tr>
<tr>
<td>Argentina 1991:4–present</td>
<td>Yes</td>
<td>Yes</td>
<td>Expands</td>
<td>Rise</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: The program dates comprise the months during which the program was in effect; Qualitative effects reported are based on author's calculations using data from IMF, International Financial Statistics.
Figure 1.1 Ratio of Fiscal Deficit to GDP. (Note: Decline in index represents improvement of fiscal position; Index for Uruguayan October 1978 plan scaled by factor 0.1)
Figure 1.2 Ratio of Fiscal Deficit to GDP. (Note: Decline in index represents improvement of fiscal position; Average excludes Uruguayan October 1978 plan)
Figure 1.3  Brazil, March 1964 Stabilization Plan
Figure 1.4  Argentina, March 1967 Stabilization Plan
Figure 1.5  Uruguay, May 1968 Stabilization Plan
Figure 1.6 Chile, February 1978 Stabilization Plan
Figure 1.7 Uruguay, October 1978 Stabilization Plan
Figure 1.8  Argentina, December 1978 Stabilization Plan
Figure 1.9  Israel, July 1985 Stabilization Plan. (Note: Capital Inflows index is scaled by factor 0.1)
Figure 1.10  Mexico, December 1987 Stabilization Plan
Figure 1.11  Uruguay, January 1991 Stabilization Plan.
(Note: Trade Balance index is scaled by factor 0.1)
Figure 1.12 Argentina, April 1991 Stabilization Plan. (Note: Capital Inflows index is scaled by factor 0.05; The vertical line indicates the March 1995 banking crisis in the aftermath of the Mexican peso devaluation)
Figure 1.13  Representative Dynamics of Exchange-Rate-Based Stabilization. (Note: The shaded area represents the average duration of a stabilization program)
Figure 1.14 Nominal Interest Rates
Figure 1.15  Ex-Post Real Interest Rates
Figure 1.16  Interest Rate Spreads
Figure 1.17  Average Interest Rates and Interest Rate Spreads. (Note: The shaded area represents the average duration of a stabilization program)
Figure 1.18  Representative Dynamics of Exchange-Rate-Based Stabilization. (*Percentage Deviation from the Value at the Beginning of a Program*) (Note: The shaded area represents the average duration of a stabilization program)
Figure 1.19  Average Interest Rates and Interest Rate Spreads. 
(Percentage Deviation from the Value at the Beginning of a Program) (Note: 
The shaded area represents the average duration of a stabilization program)
### Table 1.4 Selected Transition Economies. Program Characteristics of Exchange-Rate-Based Stabilizations

<table>
<thead>
<tr>
<th>Program</th>
<th>Exchange Rate Arrangement</th>
<th>Incomes Policies</th>
<th>Fiscal Position</th>
<th>Structural Reforms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Czech Republic 1991:1–1997:5</td>
<td>Peg</td>
<td>Yes</td>
<td>Sound</td>
<td>Yes, rapid progress</td>
</tr>
<tr>
<td>Estonia 1992:6–present</td>
<td>Currency Board</td>
<td>Yes</td>
<td>Sound</td>
<td>Yes, rapid progress</td>
</tr>
<tr>
<td>Lithuania 1994:4–present</td>
<td>Currency Board</td>
<td>Yes</td>
<td>Improves</td>
<td>Yes, moderate progress</td>
</tr>
</tbody>
</table>

Notes: The program dates comprise the months during which the program was in effect; “Peg” means fixed exchange rate with infrequent adjustments; “Currency board” means unequivocal commitment of the central bank to supply or redeem, without limit, its monetary liabilities at predetermined exchange rate.

### Table 1.5 Selected Transition Economies. Macroeconomic Effects of Exchange-Rate-Based Stabilizations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Remonetization</td>
<td>Not clear</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Lending Boom</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>Appreciates</td>
<td>Appreciates</td>
<td>Appreciates</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>Deteriorates</td>
<td>Deteriorates</td>
<td>Deteriorates</td>
</tr>
<tr>
<td>Capital Inflows</td>
<td>Surge</td>
<td>Surge</td>
<td>Surge</td>
</tr>
<tr>
<td>Consumption</td>
<td>Increases</td>
<td>Bottoms out</td>
<td>—</td>
</tr>
<tr>
<td>Investment</td>
<td>Increases</td>
<td>Booms</td>
<td>—</td>
</tr>
<tr>
<td>Real GDP</td>
<td>Bottoms out</td>
<td>Bottoms out</td>
<td>Expands</td>
</tr>
<tr>
<td>Real Wages</td>
<td>Rise</td>
<td>Fall, then rise</td>
<td>Rise</td>
</tr>
<tr>
<td>Ended in Crisis?</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: The program dates comprise the months during which the program was in effect; Qualitative effects reported are based on author’s calculations using data from IMF, International Financial Statistics; “Remonetization” refers to rising ratios of broad money to base money, broad money to GDP, and ratio of deposits to currency.
Figure 1.20  Selected Transition Economies. Fiscal Balance (Percentage of GDP) and Ex-Post Real Interest Rates
Figure 1.21  Czech Republic, January 1991 Stabilization Plan.
(Note: Data prior to 1993 are for Czechoslovakia)
Figure 1.22  Estonia, June 1992 Stabilization Plan. (Note: Capital Inflows index is scaled by factor 0.1)
Figure 1.23 Lithuania, April 1994 Stabilization Plan
Chapter 2

An Alternative Mechanism

2.1 Money Matters

As the discussion in Chapter 1 suggests, the existing theories have largely rationalized the ERBS syndrome by relying on imperfect credibility and nominal rigidities in an otherwise classical framework where markets are complete and market failures are absent. In a classical world, the competitive outcome is Pareto efficient: the optimization problem is equivalent to a single representative agent optimizing subject to a real resource constraint, and, since money has no real effects, the financial sector becomes irrelevant. Indeed, as shown in Calvo (1986b), a permanent reduction in the rate of devaluation would be entirely neutral in a model of the representative household in an economy with flexible prices and fully credible policies: the economy would remain at the initial steady state, and inflation would adjust immediately to the lower equilibrium level.

Yet the financial system in developing countries is hardly free from significant market imperfections; and when it is not, money is no longer just a “veil”. As the model developed in Section 2.3 shows, in financial systems characterized by imperfections such as incomplete markets and an externality in the lending process, an initial increase in the supply of loanable funds resulting from remonetization of the economy in the wake of disinflation can translate into a rapid expansion of bank credit financed by short-term capital inflows. A credit-driven boom results, accompanied by a currency overvaluation and current account deficits, even within an environment of
perfect credibility and price flexibility. Together, these generate systemic financial fragilities and make the economy vulnerable to a small shock that can trigger banking and balance-of-payment crises. These results stand in stark contrast to the predictions of a classical model. We show in Chapter 3 that the presence of an externality in the lending process and incomplete financial markets are both critical for the results obtained in this study.

The rest of this section discusses the market imperfections that characterize the financial system in developing countries. It then develops an alternative explanation for the ERBS syndrome in which banks play a major role as financial intermediaries. The section concludes by discussing the systemic financial fragilities produced by a bank lending boom, and the possible channels through which banking and balance-of-payment crises can occur.

2.1.1 Incomplete Markets and the Special Role of Banks

The weak state of the legal framework and the inadequate accounting standards in developing countries do not allow investors to evaluate corporate cash flow and thus the creditworthiness of most potential borrowers. Hence, the potential pool of issuers is not large enough to create a liquid market for nonbank liabilities such as commercial paper. As a result, there are no issuers of perfect substitutes for bank deposits such as mutual funds.

At the same time, liquidity becomes a primary proof of solvency because the deficient legal and accounting systems make other forms of evaluating creditworthiness difficult. Consequently, investors prefer to hold short-term liquid assets, and

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19In the context of this study, perfect credibility is defined as the belief on the part of private agents that the government will carry out its commitment to maintaining the exchange rate peg. This implies, inter alia, that the government follows a set of consistent policies and the fiscal position at the outset of stabilization is sound.
borrowers—who are forced by lenders to remain liquid—are restricted to short-term funds.

Banks constitute the payments system and are the only nongovernmental issuers of liquid short-term liabilities that are also accepted as a means of payment. Banks therefore are uniquely positioned to intermediate between borrowers and lenders and thus to provide liquidity transformation services to portfolio investors. As a result, bank loans are the dominant source of external finance for nonfinancial firms, and bank deposits are the most important form of household savings. This explains why equity and corporate bond markets are insignificant in developing countries and commercial banks play the dominant role in the financial system.

2.1.2 Externality in Bank Lending

Banks rely on market signals and private information about their borrowers to evaluate creditworthiness and make lending decisions. In developing countries, the weak state of accounting and information disclosure frameworks limit the ability of banks to collect and process information. Furthermore, market signals received by individual banks can be misleading during periods of buoyant economic activity. Credit-market booms therefore hinder the ability of banks to appraise loans and credit risks.

In a competitive market for bank services, each bank does not take account of the effect its lending decisions have on the quality of information received by other banks. Specifically, liquidity is the primary proof of solvency in financial systems with inadequate legal and accounting infrastructures. If borrowers can prove to banks they

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20 For example, in APEC developing countries, portfolio flows—of which bond issues account for a dominant share—have accounted for a high and rising portion of total private capital inflows. Commercial banks have been the major issuers of international bonds in these countries: from 1990 to 1994, banks accounted for 33 percent of total issues. (See Ishii and Dunaway (1995).)

are liquid, and the source of liquidity is not observable, they can effectively prove they are solvent. However, when more credit is available, it is easier for borrowers to remain liquid.

This is the information externality in bank lending stressed by Gavin and Hausmann (1996): banks do not internalize the adverse impact of their lending on other banks' information, and therefore extend more credit than they otherwise would. As a result, the aggregate amount of loans granted by the banking system is excessive.

2.2 Propagation and Amplification

Suppose a country embarks on a well-designed stabilization program anchored by fixing the nominal exchange rate.\textsuperscript{22} The ensuing disinflation increases the demand for domestic monetary assets and improves the liquidity position of the banking sector. A remonetization of the economy occurs, increasing the supply of loanable funds. Consequently, bank lending rises—but, because of the lending externality, the aggregate amount of loans granted by banks is excessive. The resulting increase in inside money and bank lending are associated with increasing investment, output, and consumption demand for both traded and nontraded goods. In response to the increase in the demand for nontraded goods, labor used to produce nontradables has to rise. The increase in the total labor supply would not be sufficient to meet the increased demand for nontraded goods even if it were all allocated to the production of nontradables. As a result, the relative price of nontraded goods has to rise to shift labor from the traded to nontraded goods sector—that is, the real exchange rate appreciates.

The appreciation of the real exchange rate and the movement of labor out of the traded goods sector reduce the production of tradables. Together with the increase

\textsuperscript{22}By "well-designed" we mean that monetary and fiscal policies have also been adjusted to support the fixed exchange rate.
in consumption, this results in a deterioration of the trade balance, with the resulting current account deficit being financed by capital inflows intermediated through the banking system.

The private expenditure boom also increases the demand for bank deposits as a medium of exchange, and further stimulates inside money creation. This process of rising expenditure, financed by capital inflows intermediated through the banking system, can continue until the real currency appreciation and monetary expansion reach levels threatening domestic financial stability.

At this stage, however, a financial crisis need not emerge. Real currency appreciation and multiple expansion of the banking system’s balance sheet do not generate a crisis by themselves. Rather, they render the banking system fragile and make the economy as a whole vulnerable to a small shock that can trigger banking and balance-of-payments crises through a number of mechanisms described below.

2.2.1 Financial Fragilities

1. Suppose the central bank can act as a lender of last resort to avert a banking crisis. If there is a negative liquidity shock to the banking system (an adverse shock to the demand for deposits, for instance), discount-window loans to troubled banks will expand the monetary base relative to holdings of foreign exchange. Given the past inflation record, central bank lending to the banking system is likely to rekindle inflationary expectations and cause a loss of confidence in the exchange rate peg. This will in turn lead to capital outflows and depletion.

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23 *Inside money* is money such as bank deposits that is based on private sector debt created through lending to private sector borrowers. *Outside money* includes currency and private bank reserves. It represents claims on central bank assets such as government debt and foreign exchange reserves.

24 It is worth mentioning here that the expenditure boom may also produce a boom in the real estate market, and a rise in the prices of assets that are used as a collateral for bank loans, thus further facilitating borrowing. This particular aspect of market behavior is not considered in this study.
of international reserves. As soon as the lower stock of international reserves becomes widely known, devaluation will be considered unavoidable, provoking a speculative attack against the domestic currency.

2. Now suppose that the ability of the central bank to act as a lender of last resort is limited by a credible commitment to a currency board.\textsuperscript{25} Under such circumstances, the inability of a single bank to honor its liabilities (say, because of default by a large debtor) can trigger a bank run. The run on banks will immediately translate into a run against the domestic currency as investors will fear that the real costs of the impending banking system collapse will force the government to abandon the currency board and extend domestic credit to prevent massive bank insolvency.\textsuperscript{26}

3. Real exchange rate appreciation, which can cumulate into overvaluation,\textsuperscript{27} reduces the profitability and debt-servicing capacity of bank borrowers, and thus raises the probability of bank default. This degrades the quality of bank assets, and makes the occurrence of a crisis through the first two mechanisms more likely.

4. Finally, irrespectively of whether the central bank acts as a lender of last resort or not, an external shock—such as a rise in interest rates in industrial countries, a change in the perception of the country’s fundamentals, or an adverse shift in investor confidence caused by contagion effects in emerging markets—can

\textsuperscript{25}That is, the commitment to maintain a constant ratio between the domestic monetary base and foreign reserves prevents the central bank from altering the monetary base.

\textsuperscript{26}This scenario effectively requires that the central bank lacks credibility with the international community. Otherwise, as in the case of Argentina in March 1995, an emergency financing loan might be extended to allow the exchange rate to be maintained.

\textsuperscript{27}Since some real appreciation is equilibrating, there is inevitably a high degree of uncertainty in the financial markets as to whether the currency is overvalued. One can consider the real exchange rate overvalued if the current account is deemed unsustainable in the long run.
trigger both currency and banking crises if the real exchange rate is overvalued. An overvalued currency requires the current account deficit to be offset by continued capital inflows, which makes the economy highly vulnerable to a sudden cessation or reversal of the inflows. When a shock causes capital inflows to cease or reverse, a drop in the demand for the domestic currency produces a sharp contraction of bank deposits, downward pressure on the exchange rate, and hence pressure to devalue. A fear of devaluation and an incipient speculative attack provokes a run against liabilities denominated in the domestic currency, and hence a run on banks.²⁸

Since bank and currency runs go hand in hand in all the cases, the liquid domestic liabilities of commercial banks—the amount of monetary assets that economic agents fleeing the domestic currency will try to convert into foreign exchange—represent a contingent claim on the foreign exchange reserves of the central bank. However, the preceding rapid expansion of banks' balance sheets will produce a stock of broad money greatly in excess of the stock of foreign exchange reserves. The government then has two options when a speculative attack gets under way: either to devalue or to endure a deep and prolonged recession caused by a banking system collapse.²⁹ Since it is rather unlikely that the government will choose the latter option, the exchange rate peg has to be abandoned and a balance-of-payments crisis follows.

In all the cases, the fragile state of the banking system circumscribes the ability of the central bank to defend the domestic currency with higher interest rates. Higher interest rates will adversely affect bank borrowers' cash flow and debt-servicing ca-

²⁸The run on banks will occur if: (1) there is a lack of confidence in the soundness of the banking sector adversely affected by currency overvaluation and a rise in nonperforming loans; or (2) deposits denominated in a foreign currency are not allowed.

²⁹Assuming the third option of an emergency financing loan from the international community is not available.
pacity, and thus can precipitate increases in loan losses and raise the probability of a
systemic failure of the banking system.

Modeling a crisis is beyond the scope of this study which shows only the transition
of the economy from an initial (pre-stabilization) steady state to a new dynamic
equilibrium characterized by high financial fragilities.

2.3 The Baseline Model

A small open economy is populated by a large number of infinitely-lived households.
Apart from households, the economy also contains nonfinancial firms and commercial
banks. Perfect competition and goods mobility are assumed to prevail. There are two
goods-producing sectors. The first sector produces a tradable good that can be con-
sumed or invested. The second sector produces a nontradable good that can be used
for consumption only. The banking sector produces demand deposits and makes
loans to finance working capital and purchases of investment goods. Households
supply labor that is perfectly mobile across sectors.

The nominal exchange rate is defined as the domestic currency price of foreign
currency. The foreign currency price of the traded good is given exogenously by the
foreign price of the good. For convenience, the foreign price of the traded good is
assumed constant, and is normalized to unity. The real exchange rate is defined as
the relative price of nontraded goods in terms of traded goods.

\[30\] We can think of the nontradable good as services, including housing and utilities.

\[31\] This follows from the assumptions that the open economy is small and the external inflation rate
is zero.
2.3.1 Nonfinancial Firms

Purchases of goods and labor services by firms are subject to money-in-advance constraints. As discussed in Section 2.1.1, financial markets are incomplete and firms are restricted to borrowing from domestic banks. To pay for inputs—labor and capital—in advance of production, firms borrow from banks at the beginning of the period. The nominal interest rate paid by firms on bank loans is $r_t^f$.

The Traded Goods Sector

The technology for producing tradable goods is given by

$$Y_t^T = T_t (K_t)^{\alpha} \left( N_t^T \right)^{1-\alpha}$$

(2.1)

where $T_t > 0$ is a level parameter, $\alpha \in (0, 1)$, and $K_t$ and $N_t^T$ denote time $t$ inputs of capital and labor into production of tradables, respectively.

The representative firm maximizes its time $t$ profits, $\Pi_t^T$, defined as sales revenue minus bank loan repayments:

$$E_t Y_t^T - \left( 1 + r_t^f \right) \left( E_t K_t + w_t N_t^T \right)$$

(2.2)

by choice of $K_t$ and $N_t^T$ subject to (2.1), and taking the nominal wage rate, $w_t$, as given. Here, $E_t$ denotes the nominal exchange rate. Under the assumptions of small open economy, zero external inflation, and perfect goods mobility, $E_t$ is also the domestic currency price of the traded good. The total supply of tradables—both imported and domestically produced—is thus infinitely elastic.

For simplicity, the stock of capital is assumed to depreciate completely at the end of the period. The capital stock is therefore equal to the investment flow.\footnote{Roldós (1995) studies the effects of capital accumulation on ERBS dynamics in a different model. While including such dynamics in our model would alter quantitatively the adjustment path to a shock, accounting for these effects is not central to our main concern.}
The Nontraded Goods Sector

The production technology for nontradelables reflects the fact that the nontraded goods sector requires significant investments in infrastructure with long gestation lags. For simplicity, the stock of infrastructure capital is taken as fixed. Therefore, the supply of nontradelables is inelastic—at least in the short to medium run—relative to tradables. The production function is given by

$$Y_t^N = H \left( N_t^N \right)^\xi$$

(2.3)

where $H > 0$ is a level parameter, $\xi \in (0, 1)$, and $N_t^N$ is time $t$ labor employed in the nontraded goods sector.

The representative firm maximizes its time $t$ profits, $\Pi_t^N$, defined as sales revenue minus bank loan repayments:

$$P_t^N Y_t^N - (1 + r_t^l) w_t N_t^N$$

(2.4)

by choice of $N_t^N$ subject to (2.3), and taking $w_t$ as given. Here, $P_t^N$ denotes the domestic currency price of the nontraded good. Since labor is assumed to be perfectly mobile, the wage rate $w_t$ is perfectly flexible and equalized across the sectors.

At the end of the period, firms sell their output to households and use the revenue to discharge their liabilities to banks. The representative household owns both firms and receives the profits at the end of the period. Under the assumption of perfect competition, the equilibrium profits of the traded goods sector are zero. Since the production technology for nontradelables exhibits decreasing returns to scale, the equilibrium profits in the nontraded goods sector are equal to the return to the infrastructure capital:

$$\Pi_t^N = (1 - \xi) P_t^N Y_t^N$$

(2.5)
2.3.2 Commercial Banks

Banks intermediate between households (lenders) and firms (borrowers) taking the number of banks in the banking sector, \( n_t \), and the prevailing interest rates—lending rate, \( r^l_t \), and deposit rate, \( r^d_t \)—as given. Assume all banks are identical so that in equilibrium \( n_t \tilde{Z}_t = Z_t \), where \( \tilde{Z}_t \) is an individual bank variable and \( Z_t \) is the banking sector total.

Each period, banks decide how much of the domestically available funds to put aside as reserves, and how much to lend to firms. If the sum of loans, \( \tilde{L}_t \), and reserves, \( \tilde{R}_t \), chosen by banks—banks' assets—exceeds the banks' domestic liabilities, \( \tilde{D}_t \), banks borrow abroad to close the financing gap by issuing one-period bonds, \( \tilde{B}_{t+1} \), denominated in foreign currency.

If banks survive through the period, they earn interest on their loans and pay foreign lenders the principal plus interest at the time invariant rate \( r^* \).\(^{33}\) Foreign lenders are willing to lend as long as the banks pay off their last period's debt. If banks do not survive through the period because of a banking or currency crisis, their loans will not mature, they will be unable to service their debt, and foreign lenders will not be willing to lend to them in the next period. Therefore, the constraint on bank borrowing abroad is a nonzero probability that foreign funds will not be forthcoming in the next period. The availability of funds depends in turn on the probability of banks' surviving through the period as defined below.

As discussed in Section 2.2.1, bank and currency runs go hand in hand: one triggers the other. A bank run can be triggered directly by a single bank failure, or

---

\(^{33}\)The cost of borrowing abroad is inconsequential if banks can roll over debt indefinitely. Hence the simplifying assumption of the constant interest rate \( r^* \). What matters is that foreign lending may abruptly go to zero at any interest rate. This will be assumed to occur beyond the time horizon for our model—that is, we model the development of a crisis situation but not what actually happens when the crisis hits and in its aftermath.
it can be triggered indirectly by a run against the currency and hence on deposits denominated in the domestic currency.

Profit maximizing banks account for the effect of their own lending decisions on the probability of their own failure stemming from their domestic operations by choosing the ratio of reserves to deposits, $\frac{a_t}{D_t}$. However, in a competitive bank market, each bank takes the ratio of base money to broad money, $\frac{M_0}{M_2}$, which is also a function of the equilibrium level of $\frac{a_t}{D_t}$, as given. This captures the externality in bank lending discussed in Section 2.1.2: banks do not internalize the effect of their lending decisions on the total stock of loans outstanding in the economy and hence on the monetary aggregates, which is observationally equivalent to the situation where banks do not internalize the adverse impact of their lending on other banks’ information.

Furthermore, the bank survival probability depends on the sustainability of the current account position as measured by the ratio $\frac{TB_t}{F_t}$, where $TB_t$ is time $t$ net exports and $F_t \equiv B_{t+1} - B_t - r^*B_t$ is net capital inflow in period $t$ less interest payment on the stock of foreign liabilities outstanding at the beginning of the period. The ratio $\frac{TB_t}{F_t}$ is used as a proxy for the sustainability of the current account: the lower the ratio, the lower the survival probability. The sustainability of the current account thus depends on the cost of servicing the banking sector’s debt, $r^*B_t$, which is directly affected by banks’ borrowing decisions, and on net exports, $TB_t$. Yet, in a perfectly competitive market, banks take net exports as well as other banks’ borrowing and lending decisions as given: they recognize that the probability of collapse increases with deterioration in the current account position (which incorporates real

---

34 Note that the model is solved in terms of percentage deviations of the decision variables from their pre-stabilization steady state values (see Section 2.4.1). This does not allow us to use the direct measure of the current account, $TB_t - r^*B_t$, because both current and capital accounts of the balance of payments are zero in a steady state.

35 The solution technique employed in the study precludes sign switching in situations where the trade balance, for instance, goes from surplus to deficit.
appreciation)—but each bank, being small in the market, rationally ignores the effect of its lending on economy-wide variables.

Explicitly, the survival probability of an individual bank is given by

\[
q_t = \frac{a \left( 1 - e^{-c \left( \frac{R_t}{D_t} + \varphi \frac{M_t}{M_{t-1}} + \psi \frac{T_B_t}{F_t + \left( 1 - \frac{a}{b} \right) F_t} \right)} \right)}{a + be^{-c \left( \frac{R_t}{D_t} + \varphi \frac{M_t}{M_{t-1}} + \psi \frac{T_B_t}{F_t + \left( 1 - \frac{a}{b} \right) F_t} \right)}}
\]

(2.6)

where \( \varphi \) and \( \psi \) are level parameters. The function \( q_t(\cdot) \) is twice continuously differentiable, and lies in the interval \([0,1]\) for a nonnegative argument. The structural parameters \( a, b, \) and \( c \) represent a wide range of paths for the probability of survival. At this stage of the analysis, we have only examined the special case \( a = b = c = 1 \).

\( \tilde{F}_t \) is determined from the bank’s balance sheet:\(^{36}\)

\[
\tilde{F}_t = \frac{\tilde{L}_t + \tilde{R}_t - \tilde{D}_t}{E_t}
\]

(2.7)

Banks are assumed to operate costlessly. A bank’s losses in case a run occurs are limited to its reserves \( \tilde{R}_t \). The representative bank’s problem is to maximize its time \( t \) expected profits

\[
q_t \left( \tilde{R}_t, \tilde{L}_t, \tilde{S}_t, n_t \right) \cdot \left( \tau^t_{L_t} \tilde{L}_t - \tau^d_{S_t} \tilde{S}_t \right) - \left( 1 - q_t \right) \tilde{R}_t
\]

(2.8)

subject to (2.6) and (2.7) by choice of \( \tilde{R}_t, \tilde{L}_t, \) and \( \tilde{S}_t. \(^{37}\) Here, \( \tilde{S}_t \) denotes households’ interest-bearing deposits with the bank.

Under the assumption of perfect competition, expected profits are zero in equilibrium:

\[
q_t \left( \tilde{R}_t, \tilde{L}_t, \tilde{S}_t, n_t \right) \cdot \left( \tau^t_{L_t} \tilde{L}_t - \tau^d_{S_t} \tilde{S}_t \right) - \left( 1 - q_t \right) \tilde{R}_t = 0
\]

(2.9)

\(^{36}\)Note that this eliminates \( \tilde{B}_{t+1} \) from the list of decision variables, and makes the banking sector's problem static.

\(^{37}\)By choosing \( \tilde{S}_t \), banks effectively choose \( \tilde{D}_t \) but the equilibrium condition \( n_t \tilde{D}_t = n_t \tilde{S}_t + w_t N_t^M \) (see (2.19) on page 53) is redundant by Walras law.
The representative household owns the bank and receives bank profits at the end of the period. Since entry into the banking sector occurs until expected profits are zero, in periods when banks survive, profits will be strictly positive:

\[ \Pi_t^B = n_t \left( r_t^L \bar{L}_t - r_t^d \bar{S}_t \right) \]  \hspace{1cm} (2.10)

2.3.3 The Central Bank

The only function of the central bank in this model is to engage in nonsterilized intervention.\textsuperscript{38} That is, the central bank ensures costless conversion of foreign currency into domestic currency (and vice versa) at any moment at the specified exchange rate, \( E_t \), which evolves according to

\[ E_{t+1} = (1 + \varepsilon_{t+1}) E_t \]  \hspace{1cm} (2.11)

where \( \varepsilon_{t+1} \) is the rate of devaluation preannounced by the central bank.

The monetary base, \( M_0_t \), defined as the sum of currency held by households, \( M_t \), and bank reserves, \( R_t \), is therefore determined by the central bank’s stock of foreign exchange reserves, \( X_t \), at the beginning of the period:\textsuperscript{39}

\[ M_0_t \equiv M_t + R_t = E_t X_t \]  \hspace{1cm} (2.12)

\textsuperscript{38}This assumption is motivated by the fact that sterilization—the exchange of domestic (public) bonds for foreign exchange—may not be a viable policy option, especially in the longer term, because it entails high interest rates and large quasi-fiscal costs stemming from the difference between the interest paid on domestic bonds and the return earned on foreign reserves invested in international financial markets. In addition to sterilization conducted through open market operations, the central bank can conduct sterilization by increasing reserve requirements. High reserve requirements, however, lead to financial disintermediation and lax lending standards (see Rojas-Suárez and Weisbrod (1995)). In transition economies, sterilization may not be feasible because of the lack of financial instruments.

\textsuperscript{39}This is equivalent to a currency board arrangement.
Broad money, $M2_t$, is defined as the sum of currency and total bank deposits:\footnote{The only distinction between checking and interest-bearing (saving) accounts in this model is that the latter earn interest. Therefore, $M2$ and $M1$ in the model are the same. $M2$ was chosen over $M1$ to facilitate calibration (see Section 2.4.2).}

$$M2_t = M_t + D_t$$ (2.13)

The time $t$ flow of foreign exchange reserves at the central bank is determined by the balance of payments:\footnote{For simplicity, it is assumed that the central bank earns no interest on its foreign exchange reserves, pays no interest on bank reserves, and incurs no operating costs.}

$$\frac{X_{t+1} - X_t}{\text{Official Reserve Transactions}} = \frac{B_{t+1} - B_t}{\text{Capital Account Balance}} + \frac{TB_t - r^*B_t}{\text{Current Account Balance}}$$ (2.14)

$$\text{Balance of Payments}$$

The fixed exchange rate regime commits the central bank to exchange domestic money for foreign exchange. Therefore, the central bank—the “government” in this model—effectively owns all foreign liabilities, $B_t$, and has to raise tax to redeem them with interest. Specifically, there is a central bank (government) budget constraint:

$$\tau_t + E_t B_{t+1} + \pi_{t+1} \left( M_{t+1} + S_{t+1} \right) \frac{P_t}{P_{t+1}} = E_t (1 + r^*) B_t$$ (2.15)

$$\text{Inflation Tax}$$

Part of the tax revenue comes from the inflation tax on households’ nominal asset holdings, $M_{t+1} + S_{t+1}$. Households also effectively pay the balance in the form of a lump sum tax $\tau_t$.\footnote{The legal incidence of the tax could be on firms or commercial banks but, ultimately, only households—the owners of firms and banks—pay the tax.} Here, $P_t$ denotes time $t$ price level defined as

$$P_t = (E_t)^\gamma \left( P_t^N \right)^{1-\gamma}$$ (2.16)

where $\gamma$ is the share of tradables consumption expenditure in total consumption expenditure, and $\pi_{t+1}$ is the rate of inflation given by $\frac{P_{t+1}}{P_t} - 1$. 

2.3.4 Households

The representative household seeks to maximize its discounted lifetime utility defined over sequences of consumption of traded goods, $C_t^T$, and nontraded goods, $C_t^N$, using a time separable utility function:

$$
\sum_{t=0}^{\infty} \frac{\beta^t}{1 - \frac{1}{\sigma}} \left[ \left( (C_t^T)^{\gamma} (C_t^N)^{1-\gamma} \right)^{1-\frac{1}{\sigma}} - 1 \right]
$$

(2.17)

where $\beta \in (0, 1)$ is the time discount factor, $\sigma > 0$ is the elasticity of intertemporal substitution in consumption, and $\gamma \in (0, 1)$ is a share parameter.

The household enters the period with nominal assets in the form of currency holdings, $M_t$, and interest-bearing demand deposits, $S_t$, which earn interest at the rate $r_t^d$.

The household divides $N$ units of time supplied inelastically each period into time spent working in the marketplace, $N_t^M$, and time spent acquiring consumption goods, $N_t^S$:

$$
N = N_t^M + N_t^S
$$

(2.18)

Labor income is deposited by firms directly into households' checking accounts at the beginning of the period. Hence, total nominal checkable (demand) deposits of the representative household are the sum of interest-bearing deposits, $S_t$, and labor income, $w_t N_t^M$:

$$
D_t = S_t + w_t N_t^M
$$

(2.19)

Households use currency, demand deposits, and time to purchase consumption goods. The amount of time spent shopping, $N_t^S$, is an increasing function of total consumption expenditure, $E_t C_t^T + P_t^N C_t^N$, and a decreasing function of both nominal currency holdings, $M_t$, and demand deposits, $D_t$. The transactions technology is
given by\(^{43}\)

\[
N_t^S = J \left( \frac{E_t C_t^T + P_t^N C_t^N}{M_t^S D_t^{1-\theta}} \right)
\]  

(2.20)

where \(J > 0\) is a level parameter, and \(\theta \in (0, 1)\) is a share parameter.

The representative household’s end of period flow budget constraint is

\[
E_t C_t^T + P_t^N C_t^N + M_{t+1} + S_{t+1} = M_t + \left(1 + r_t^d\right) S_t + w_t N_t^M + \Pi_t^N + \Pi_t^B - \tau_t
\]  

(2.21)

The household’s problem is to maximize (2.17) subject to (2.18), (2.19), (2.20), and (2.21) by choice of sequences of \(C_t^T, C_t^N, N_t^M, M_{t+1}\), and \(S_{t+1}\) for \(t > 0\).

2.3.5 Market Clearing Conditions

The equilibrium conditions for the goods, labor, and loan markets are given by the equality of the supply and demand for

Traded goods:

\[
Y_t^T - TB_t = C_t^T + K_t
\]  

(2.22)

Nontraded goods:

\[
Y_t^N = C_t^N
\]  

(2.23)

Labor:

\[
N_t^M = N_t^T + N_t^N
\]  

(2.24)

Loans:

\[
L_t = E_t K_t + w_t N_t^M
\]  

(2.25)

Equilibrium in the market for bank reserves is determined by equations 2.12 and 2.14 (supply) and the first order condition 4.4 (demand).

\(^{43}\)This specification of the transactions technology is used by Chari et al. (1995).
2.4 Quantitative Properties of the Model

2.4.1 Approximate Solution Procedure

The model does not have a closed form solution for the decision variables. To investigate the behavior of the model, this study employs the method suggested by King et al. (1988), which obtains an approximate solution by linearizing the Euler equations around the steady state. The solution results are then expressed as percentage deviations of the decision variables from their steady state values. Using the approximate optimal paths for the decision variables, other variables of interest can then be expressed as percentage deviations from their steady state values.

As in Rebelo and Végh (1995), the numerical approximation procedure involves linearizing the model around a steady state to which the economy never returns. This feature of the solution method compromises how much emphasis can be put on quantitative predictions of the model. The linearization gives the direction of movement (first derivative) away from an initial steady state. However, the further the economy moves away from that steady state, the less accurate the linear approximation becomes. The ultimate extent of the movement depends on the behavior of the nonlinear system (that is, on higher order derivatives).

In the initial steady state, all real variables are constant and all nominal variables are growing at the rate of inflation equal to the rate of devaluation. The steady state for the economy follows from zero profit condition 2.9, the central bank's balance sheet 2.12, the balance of payments 2.14, the set of resource constraints 2.18, 2.19, 2.20, 2.21, market clearing conditions 2.22, 2.23, 2.24, 2.25, and a set of conditions below that follow from the Euler equations.

The optimal allocation of labor between the sectors is determined by

\[(1 - \alpha) ETK^\alpha (N^T)^{-\alpha} = \xi HP^N (N^N)^{\xi-1}\]
The optimal consumption mix is given by

$$\gamma P^N C^N = (1 - \gamma) EC^T$$

The ratio of currency to deposits follows from

$$\frac{M}{D} = \frac{\theta}{1 - \theta} \left( 1 - \frac{\beta r^d}{1 + \pi - \beta} \right)$$

2.4.2 Parameter Values

To simulate the linearized model, it is necessary to assign values to the model structural parameters. The model was calibrated to the Argentine economy, which experienced high nominal volatility during the decade prior to the 1991 Convertibility Plan. Argentina nevertheless was chosen as the reference country because the Convertibility Plan did not rely on price and wage controls and obligated the central bank to issue the domestic currency against foreign exchange reserves at a fixed exchange rate. These two features of the Convertibility plan fit well with the model’s assumptions of perfect price flexibility and the use of a currency board as a policy rule for base money creation.

A natural reference period for the calibration exercise would be the period between the 1982 Latin American debt crisis and the introduction of the Convertibility Plan in the second quarter of 1991. However, 1983, 1989, and 1990 were discarded as they were years of extreme volatility in nominal variables.

The model was calibrated as follows. The values for the capital share in the tradables sector, \( \alpha \), labor share in the nontradables sector, \( \xi \), time discount factor, \( \beta \),

---

44Thus, we must have that \( \frac{1 + r^d}{1 + \pi} < \frac{1}{\beta} = 1 + r^* \) for \( \frac{M}{D} \) to be positive in the steady state. Otherwise, households would accumulate bank deposits as a form of “real” saving as well as for transaction purposes. The household sector’s optimality conditions imply that, on the margin, a change in \( \pi \) causes a greater relative change in the cost of holding deposits than in the cost of holding currency. Therefore, \textit{ceteris paribus}, \( \frac{M}{D} \) is increasing in \( \pi \).
the consumption share parameter, $\gamma$, the elasticity of intertemporal substitution in consumption, $\sigma$, and the foreign real rate of interest, $r^*$, were taken from Rebelo and Végh (1995).

Inflation, lending, and deposit rates—$\pi$, $r^l$, and $r^d$, respectively—were set to the sample averages of the corresponding variables calculated using data from the IMF, *International Financial Statistics* (IFS). The transactions technology share parameter, $\theta$, the survival probability level parameters, $\varphi$ and $\psi$, and the model period$^{45}$ were set using the equations reported in Section 2.4.1 so that the steady state properties of the model replicate the average values of some key ratios for the Argentine economy during the mid-1980s that were calculated using data from the IFS. The household’s non-leisure time endowment, $N$, was normalized to unity with 95% of the time, $N^M$, spent working in the marketplace. The number of banks in the banking sector, $n$, was set at approximately half the number of the largest bank in the mid-1990s reported in IMF (1995).

Finally, the values for the level parameters in the tradables and nontradables production functions, $T$ and $H$, and the transactions technology level parameter, $J$, were set given the parameter values discussed above.$^{46}$

The model structural parameters and the quantitative properties of the pre-stabilization steady state are reported in Table 2.1 on page 62. The variables with time dimension are expressed at annual rates. Prior to solving and simulating the model, they were converted to units corresponding to the model period.

---

$^{45}$The model period, expressed as a fraction of a year, is approximately the inverse of the number of transactions made by households in this model economy during a year.

$^{46}$It should be noted that the model solution is invariant to the values of $T$, $H$, and $J$ since they are incorporated into the values of $Y^T$, $Y^N$, and $N^S$, respectively.
2.4.3 Simulation Results

This section presents the dynamic response of the model economy to a credible disinflation anchored by fixing the exchange rate: first in isolation, and then coupled with a productivity increase in the traded goods sector. The model was simulated over 40 model periods, which is approximately 5 years. In period 0, the economy is at its pre-stabilization steady state. The stabilization is not anticipated but is perfectly perceived once it is implemented in period 1.\textsuperscript{47} The results are expressed as percentage deviations of the variables of interest from their pre-stabilization steady state values. All nominal variables were converted into real values using the price level $P_t$ as the deflator.

The solid lines in Figures 2.1 through 2.4 (see pages 63 - 66) depict the response of the economy to a permanent 100% reduction in the rate of devaluation.\textsuperscript{48} Fixing the exchange rate leads to a drop in the rate of inflation and a rise in real currency holdings and demand deposits (Figure 2.1). The ensuing increase in the demand for domestic money improves the liquidity position of banks. The ratio of deposits to currency increases and bank loans rise in real terms (Figure 2.1). Consequently, the money multiplier, defined as the ratio of $M2$ to $M0$, and the ratio of $M2$ to GDP rise (Figure 2.1). The resulting increase in inside money and bank lending are associated with increasing investment, output, and consumption demand for both traded and nontraded goods (Figure 2.2). The increase in the total labor supply (not shown) is not sufficient to meet the increased demand for nontraded goods—even if it were all allocated to the production of nontradables—and the relative price of nontraded

---

\textsuperscript{47} Initial drop or jump in period 1 in some figures reflects the impact effect of the shock. The linearized model then behaves monotonically from period 1 on.

\textsuperscript{48} Note that the quantitative response of the economy to disinflation in the simulation exercise depends on the steady state values of the model structural parameters and key ratios which were calibrated to Argentina. This may compromise a direct quantitative comparison of the model's predictions with the average behavior shown in Figures 1.18 and 1.19.
goods rises to shift labor from the traded to nontraded goods sector. As a result, the real wage rate rises, the rate of inflation in the nontraded goods sector (and hence overall inflation\textsuperscript{49}) converges slowly to the rate of devaluation, and the real exchange rate appreciates (Figure 2.3). The consumption boom, together with the decline in the output of traded goods, causes a sharp deterioration of the trade balance accompanied by massive capital inflows to finance the resulting current account deficit (Figure 2.3).

Now suppose that, along with fixing the exchange rate, the authorities implement structural reforms—such as privatization, liberalization, and deregulation—that increase productivity in the traded goods sector. The dashed lines in Figures 2.1 through 2.3 show the response of the economy to a permanent disinflation combined with a permanent 5% productivity gain in the traded goods sector. The real economy adjusts immediately to the new level of productivity: in the impact period of the shock, the capital stock rises and labor moves into the traded goods sector. The output of tradables increases and the trade balance improves. In period 2, the dynamics associated with capital inflows and bank money creation set in. Qualitatively, from period 2 on, the model exhibits the same transition path as in the case of disinflation alone described above. Quantitatively, the economy's response to disinflation is amplified by the productivity gain.

The model thus replicates the stylized facts of ERBS programs described in Section 1.2. Remonetization of the economy, the strong growth in bank credit, and the slow convergence of the inflation rate to the devaluation rate are evident from Figures 2.1 and 2.3. The private consumption boom is shown in Figure 2.2. Figure 2.3 depicts the real exchange rate appreciation and the deterioration of the trade balance.

\textsuperscript{49}Recall that $\pi_t$ is determined from (2.16) as

\[ (1 + \varepsilon_t)^\gamma (1 + \pi_t^N)^{1-\gamma} - 1 \]

where $\pi_t^N$ is the rate of inflation in the nontraded goods sector.
The model reproduces the initial expansion in economic activity: real GDP and investment (Figure 2.2), and the real wage rate (Figure 2.3) rise in response to fixing the exchange rate.

Figure 2.3 shows the high degree of financial fragility and vulnerability of the economy to a reversal of the capital inflows. The trade balance and the current account are in deficit and must be offset by continued capital inflows: the stock of net foreign liabilities grows without bound. The current account is definitely unsustainable in the long run, and the program is bound to collapse when the “right” shock comes along.

The behavior of interest rates shown in Figure 2.4 differs from the actual behavior observed in ERBS programs in that the nominal interest rates decline only gradually and do not fall in line with inflation immediately upon fixing the exchange rate. As a result, the real interest rates increase dramatically in the impact period of the shock and then converge to new higher equilibrium levels from above. A possible explanation for such behavior of interest rates in the model is that, upon implementation of a stabilization program, the real interest rates adjust instantaneously from low (negative, in case of the deposit rate) levels in a financially repressed environment to high levels that reflect the true cost of the funds. As the volume of financial intermediation and financial saving grows over time, the real interest rates decline—albeit to higher new equilibrium levels that reflect the increased probability of collapse.

50 Recall that the simulation results are expressed as percentage deviations of the variables from their steady state values. Since the capital account balance, $B_{t+1} - B_t$, is zero in the initial steady state, we cannot show the evolution of net capital inflows. Instead, we show the evolution of the stock of net foreign liabilities, $B_t$.

51 Financial fragility is also revealed by the financial sector ratios in Figure 2.1.
A shortcoming of the model is that it does not address explicitly the subsequent downturn in economic activity documented in the later stage of ERBS programs. Kiguel and Liviatan (1992) point to the inability to finance the growing current account deficit as the immediate reason for the end of the boom. The beginning of the recessionary phase would thus coincide with the reversal of the capital inflows and the onset of banking and balance-of-payment crises along the lines described in Section 2.2.1. The flight from domestic currency and the associated contraction of bank deposits would bring about a credit crunch and rising real interest rates, and cause a contraction in the real economy.

\[52\text{There is a certain irrationality on the part of households. Rational households would realize that there is only probability } q_t \text{ of the economy surviving until the next period. They would maximize expected utility, and the determinants of } q_t \text{ (in particular, the time of collapse and the values of the state variables at the time of collapse) would enter the household's maximization problem. If utility in the collapse state also depends on the values of the state variables at the time of collapse, then we would also need to model explicitly what happens in the collapse state.}

\[\text{However, to do the full analysis of a financial meltdown, we would need to employ a different solution method. By linearizing the model around a steady state, we are focusing on how the system behaves as it moves away from the initial steady state. Since the onset of a crisis represents a regime switch, linearization would no longer suffice as a solution technique.}\]
Table 2.1  Baseline Model Parameter Values and Quantitative Properties of Pre-Stabilization Steady State

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital Share, Traded Goods Sector</td>
<td>0.52</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time Discount Factor</td>
<td>0.96</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Share Parameter, Momentary Utility</td>
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</tr>
<tr>
<td>$\xi$</td>
<td>Labor Share, Nontraded Goods Sector</td>
<td>0.63</td>
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<tr>
<td>$\sigma$</td>
<td>Elasticity of Intertemporal Substitution</td>
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</tr>
<tr>
<td>$r^*$</td>
<td>International Real Interest Rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Share Parameter, Transactions Technology</td>
<td>0.68</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Level Parameter, Survival Probability</td>
<td>4.34</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Level Parameter, Survival Probability</td>
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</tr>
<tr>
<td>$T$</td>
<td>Level Parameter, Traded Goods Sector</td>
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<td>$H$</td>
<td>Level Parameter, Nontraded Goods Sector</td>
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<tr>
<td>$J$</td>
<td>Level Parameter, Transactions Technology</td>
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Quantitative Properties of Pre-Stabilization Steady State:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>Inflation Rate</td>
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<tr>
<td>$r^l$</td>
<td>Lending Rate</td>
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<tr>
<td>$r^d$</td>
<td>Deposit Rate</td>
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<td>$N$</td>
<td>Household's Non-Leisure Time Endowment</td>
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<td>$N^M$</td>
<td>Time Spent Working in Marketplace</td>
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<td>$n$</td>
<td>Number of Banks in Banking Sector</td>
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<td>$TB_{GDP}$</td>
<td>Ratio of Trade Balance to GDP</td>
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<tr>
<td>$TB_{Y^T}$</td>
<td>Ratio of Trade Balance to Output of Tradables</td>
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<tr>
<td>$TB_{CT}$</td>
<td>Ratio of Trade Balance to Consumption of Tradables</td>
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<tr>
<td>$S_D$</td>
<td>Ratio of Interest-Bearing Deposits to Total Deposits</td>
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<td>$R_D$</td>
<td>Ratio of Reserves to Deposits</td>
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<td>Ratio of Currency to Deposits</td>
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<tr>
<td>$M_{M0}$</td>
<td>Ratio of Currency to Monetary Base</td>
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<tr>
<td>$M_{M2}$</td>
<td>Ratio of Monetary Base to Broad Money</td>
<td>0.44</td>
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</tbody>
</table>

Model period (fraction of year): 0.13

Note: Variables with time dimension expressed at annual rates
Figure 2.1  Response to a Permanent 100% Reduction in the Rate of Devaluation Coupled with a Permanent 5% Productivity Increase in the Traded Goods Sector. *(Percentage Deviation from the Pre-Stabilization Steady State)*
Figure 2.2  Response to a Permanent 100% Reduction in the Rate of Devaluation Coupled with a Permanent 5% Productivity Increase in the Traded Goods Sector. (Percentage Deviation from the Pre-Stabilization Steady State)
Figure 2.3  Response to a Permanent 100% Reduction in the Rate of Devaluation Coupled with a Permanent 5% Productivity Increase in the Traded Goods Sector. *(Percentage Deviation from the Pre-Stabilization Steady State)*
Figure 2.4  Response to a Permanent 100% Reduction in the Rate of Devaluation. *(Percentage Deviation from the Pre-Stabilization Steady State)*
Chapter 3

Sensitivity Analysis and Policy Implications

3.1 Sensitivity Analysis

3.1.1 Alternative Parameterizations of the Baseline Model

The baseline model has three free parameters (not determined by calibration to "fit" average data for Argentina in the 1980s): the share parameter in the utility function, $\gamma$, the share of household’s non-leisure time endowment spent working in the marketplace, $N^M$, and a level parameter in the probability of survival function, $\psi$. The value of $\psi$ is set by informal search so that the transversality condition 4.23 (see page 90) is satisfied. There is a narrow range of parameter values of $\psi$ of an order of magnitude of $10^{-3}$ for which the transversality condition is satisfied. However, the model is insensitive to variations in $\psi$ of such an order of magnitude. Therefore, we consider variations in the values of $\gamma$ and $N^M$ only.

The benchmark value of $\gamma$ equal to 0.5 implies that the share of traded goods in the total private consumption expenditure is 50 percent. A limiting value of $\gamma$ equal to 1 would imply that households consume, and the economy produces, only tradable goods. A limiting value of $\gamma$ equal to 0 would imply that the economy consumes only nontraded goods and produces traded goods only for capital investment purposes.

Since we do not have a point estimate of $\gamma$, we consider two alternative values of $\gamma$—0.4 and 0.6—below and above the benchmark value. Figures 3.1 through 3.3 on pages 74 - 76 report the results.

A higher value of $\gamma$ implies a higher share of traded goods in GDP, and hence a higher labor input in the tradables sector, $N^T$, in the steady state. Consequently, a
greater share of the increase in the demand for traded goods is met with tradables produced domestically: the decline in the labor input in the traded goods sector and thus in the output of tradables is lower relative to the benchmark case. As a result, the trade deficit and hence the level of net foreign liabilities are also lower. A greater labor input into the production of tradables implies a lower marginal product of labor in the traded goods sector which in turn leads to a higher capital stock and hence to a higher level of real GDP. A smaller reallocation of labor from the traded to nontraded goods sector requires a lower real wage rate. Therefore, the rate of inflation in the nontradables sector is lower relative to the benchmark case. The combined effect of a lower nontradables inflation and a higher share of traded goods—whose price is fixed—in GDP is faster disinflation and, consequently, less appreciated real exchange rate. Finally, compared with the benchmark case, a higher level of real domestic activity leads to a stronger demand for bank loans and deposits and thus to more financial intermediation. For a lower value of \( \gamma \), all the effects analyzed above are reversed.

Figures 3.4 through 3.6 on pages 77 - 79 show the sensitivity of the baseline model’s predictions to different values of \( N^M \): 0.91, 0.99, and the benchmark value of 0.95. A higher value of \( N^M \) implies less room for increase in the total labor supply.\(^{53}\) Consequently, both the supply-side response of the economy to disinflation and the wealth effect induced by disinflation via the reduction in the shopping time are weaker. As a result, the demand for both traded and nontraded goods and the supply of labor in both sectors are lower relative to the benchmark case. A lower labor supply leads to a lower capital stock and in turn to a lower output of tradables and a lower level of real GDP. The trade deficit and the level of net foreign liabilities are

\(^{53}\)Recall that \( N^M \) is the difference between the household’s non-leisure time endowment, \( N \), and the time spent shopping, \( N^S \).
therefore higher. The real wage rate rises relative to the benchmark case to induce the required reallocation of labor from the traded goods sector and into the production of nontradables. However, because of the lower demand for nontraded goods, the rate of inflation in the nontradables sector is lower, which in turn leads to less appreciation of the real exchange rate. Finally, compared with the benchmark case, a lower level of real domestic activity leads to a weaker demand for bank loans and deposits, and thus to less financial intermediation. For a lower value of $N^M$, all the effects analyzed above are reversed.

The above analysis suggests that variations in the model's free parameters alter the economy's response to disinflation quantitatively in the predictable way, but not qualitatively. The baseline model's predictions are thus robust to alternative values of the free parameters.

3.1.2 Alternative Structural Specifications of the Financial Sector

The Lending Externality Partially Internalized

Recall that the bank survival probability in the baseline model was defined as

$$
\theta_t = \frac{a \left( 1 - e^{-c \left( \frac{R_t}{D_t} + \frac{M_0}{M_2} + \psi \frac{T_B}{F_t + (1 - \frac{1}{\alpha_t}) F_t} \right) } \right)}{a + be^{-c \left( \frac{R_t}{D_t} + \frac{M_0}{M_2} + \psi \frac{T_B}{F_t + (1 - \frac{1}{\alpha_t}) F_t} \right)}}
$$

To capture the bank lending externality discussed in Section 2.1.2, we assumed that banks were choosing the ratio of reserves to deposits, $\frac{R_t}{D_t}$, taking the ratio of monetary base to broad money, $\frac{M_0}{M_2}$, which was also a function of the equilibrium level of $\frac{R_t}{D_t}$, as given. That is, in the baseline model, banks do not internalize the effect of their lending decisions on the total stock of loans outstanding in the economy and

\[54\] See page 50.
hence on the monetary aggregates, which is observationally equivalent to the situation where banks do not internalize the adverse impact of their lending on other banks' information.

In this section, we relax this assumption and consider a situation where banks internalize the effect of their lending decisions on the domestic monetary aggregates. We internalize the lending externality further by assuming that an individual bank, which still takes the number of banks in the banking sector as given, can force its behavior upon all other banks in the system. The survival probability of an individual bank now takes the following form:

\[
\varrho_t = \frac{a \left( 1 - e^{-\left( \frac{n_t \theta_t}{n_t D_t} + \frac{n_t + n_t \theta_t + \psi M_t + \psi T B_t}{M_t + n_t D_t} \right)} \right)}{a + b e^{-\left( \frac{n_t \theta_t}{n_t D_t} + \frac{n_t + n_t \theta_t + \psi M_t + \psi T B_t}{M_t + n_t D_t} \right)}}
\]

However, to the extent that banks take net exports, \(TB_t\), in the functional specification of \(\varrho_t\) as given, there is still an element of externality in bank lending.

Figures 3.7 through 3.9 on pages 80 - 82 report the results. Accounting for the effect of bank lending on broad money dampens the quantitative response of the economy to disinflation but does not alter the qualitative response as long as there is an initial spur to bank money creation caused by the remaining externality. This outcome is a direct result of the functional form chosen for the probability of survival, \(\varrho_t\): the probability of collapse, \(1 - \varrho_t\), can approach 1 asymptotically but never becomes 1 unless the economy is hit by an adverse shock. Therefore, internalizing the effect of bank lending on the monetary aggregates can only slow down the process of domestic monetary expansion and thus postpone the day when the current account deficit reaches a level perceived unsustainable by foreign lenders. Hence, in order to evaluate the relative importance of the process of financial intermediation in ERBS programs, the next section considers a version of the model without banks.
A Stylized Model without Banks

To evaluate the importance of the banking sector and bank money creation in ERBS programs, we remove the banking sector from the baseline model. In the absence of uncertainty and market imperfections, domestic money and foreign exchange become a "veil". The model can thus be reduced to a stylized real business cycle model of intertemporal consumption and investment. To economize on notation, we assume that households directly operate the economy's technology. The representative household's end of period flow budget constraint is\(^{55}\)

\[
E_t C_t^T + P_t^N C_t^N + E_t K_{t+1} + (1 + r^*) E_t B_t = E_t B_{t+1} + E_t Y_t^T + P_t^N Y_t^N \tag{3.1}
\]

The household's problem is to maximize (2.17) subject to (2.1), (2.3), (3.1), and the time endowment constraint, \(N_t^T + N_t^N = N\), by choice of sequences of \(C_t^T, C_t^N, N_t^T, N_t^N, K_{t+1}\), and \(B_{t+1}\) for \(t > 0\). In addition, market clearing conditions 2.23 and 2.22 continue to hold. The only function of the central bank in this model is to devalue the exchange rate—that is, the price of traded goods—according to (2.11).

The dotted lines in Figures 3.7 through 3.9 on pages 80 - 82 depict the response of the economy to a permanent 100\% reduction in the rate of devaluation. Fixing the exchange rate leads to an immediate disinflation: the rate of inflation in the nontraded goods sector drops by 100\% in the impact period of the shock, and overall inflation rate is zero from period 1 on. The real economy remains at the initial steady state.\(^{56}\) These results differ dramatically from the predictions of the baseline model and speak for the importance of the banking sector and bank money creation in ERBS programs.

\(^{55}\)We continue to assume complete depreciation. In the absence of the banking sector, the stock of capital, \(K_t\), becomes a state variable.

\(^{56}\)Calvo (1986b) obtains the same disinflation neutrality result in a cash-in-advance model.
3.2 Summary and Conclusions

Disinflation programs in chronic inflation countries anchored by fixing the exchange rate have been characterized by a series of empirical regularities which can be summarized as an "exchange-rate-based stabilization syndrome": an initial expansion in economic activity, financed largely by capital inflows and accompanied by sharp real appreciation and widening external imbalances, is followed by a balance-of-payments crisis and forced devaluation. A number of theories have emerged from a large literature that sought to explain the stylized facts associated with ERBS programs, yet the role of the banking sector and its interactions with an open capital account have not received much attention.

This thesis developed an alternative explanation for the ERBS syndrome. It emphasized the role of the domestic banking system as the financial intermediary and major conduit for capital inflows in economies undertaking exchange-rate-based stabilization and market-oriented structural reforms. The thesis investigated the linkages between capital inflows and bank money creation within a dynamic general equilibrium model with multiple monetary aggregates and a financial system characterized by imperfections such as incomplete markets and an externality in the bank lending process. Financial markets are incomplete in the sense that bank loans are the sole source of external finance for nonfinancial firms, and bank deposits are the only form of household savings. The bank lending externality arises because individual banks do not internalize the effect of their lending decisions on the quality of information about potential borrowers received by other banks, and therefore extend more credit than they otherwise would.

Simulation of the model economy's equilibrium dynamics shows that an initial increase in the supply of loanable funds resulting from remonetization of the economy in the wake of disinflation can translate into a further rapid expansion of bank credit
financed by short-term capital inflows. A credit-driven boom results, accompanied by a currency overvaluation and current account deficits. Together, these generate systemic financial fragilities and make the economy vulnerable to a small shock that can trigger banking and balance-of-payment crises. The model is thus capable of replicating the empirical regularities observed in ERBS programs, and rationalizes the syndrome without relying on imperfect credibility or nominal rigidities.

The main conclusion of the thesis is that the banking sector may play an important role in ERBS programs. Accounting for the role of the banking sector can help to explain why even well-designed disinflation programs may set in motion a dynamic process that can lead to a financial crisis and the program's collapse. Consequently, the policy implication for developing countries is that the authorities should pay attention not only to the design of monetary and exchange rate policies but also to the framework of monetary and financial institutions. In particular, the results of the study suggest that diversifying the source of investment finance away from banks and reducing externalities associated with bank lending may be essential preconditions for implementing successful stabilization programs. In turn, these institutional changes would require appropriate structural policy measures such as: (1) reforming the legal, accounting, and disclosure frameworks of the financial system; and (2) facilitating a free flow of timely, accurate, and comprehensive information on the aggregate state of the economy.
Figure 3.1  Sensitivity of Baseline Model’s Predictions
to Alternative Values of Share Parameter $\gamma$
Figure 3.2  Sensitivity of Baseline Model's Predictions to Alternative Values of Share Parameter $\gamma$
Figure 3.3  Sensitivity of Baseline Model’s Predictions
   to Alternative Values of Share Parameter $\gamma$
Figure 3.4  Sensitivity of Baseline Model's Predictions to Alternative Values of Total Labor Supply Parameter $N^M$
Figure 3.5  Sensitivity of Baseline Model's Predictions to Alternative Values of Total Labor Supply Parameter $N^M$
Figure 3.6  Sensitivity of Baseline Model’s Predictions to Alternative Values of Total Labor Supply Parameter $N^M$
Figure 3.7  Sensitivity of Baseline Model’s Predictions to Alternative Structural Specifications of Financial Sector
Figure 3.8  Sensitivity of Baseline Model’s Predictions to Alternative Structural Specifications of Financial Sector
Figure 3.9  Sensitivity of Baseline Model's Predictions to Alternative Structural Specifications of Financial Sector
Chapter 4

Approximate Solution Method: A Detailed Explanation

This chapter describes in detail, using the baseline model as an example, the approximate solution method employed in this thesis to study the quantitative properties of the models constructed in Chapters 2 and 3.

4.1 The Baseline Model: First Order Conditions

Nonfinancial firms

\[ K_t: \quad \alpha T_t K_t^{\alpha - 1} \left( N_t^T \right)^{1-\alpha} = (1 + r_t^i) \]  \hspace{1cm} (4.1)

\[ N_t^T: \quad (1 - \alpha) E_t T_t K_t^\alpha \left( N_t^T \right)^{-\alpha} = (1 + r_t^i) w_t \]  \hspace{1cm} (4.2)

\[ N_t^N: \quad H_t \xi P_t^N \left( N_t^N \right)^{\xi-1} = (1 + r_t^i) w_t \]  \hspace{1cm} (4.3)

Commercial Banks

\[ R_t: \quad \left( \frac{1}{D_t} - \frac{\psi E_t T_B_t}{n_t (L_t + R_t - D_t)^2} \right) \left( r_t^L L_t - r_t^S S_t + R_t \right) = \frac{a + b}{c(a + b \varrho_t)} \]  \hspace{1cm} (4.4)

\[ L_t: \quad \frac{\psi E_t T_B_t}{n_t (L_t + R_t - D_t)^2} \left( r_t^L L_t - r_t^S S_t + R_t \right) = r_t^L \frac{\varrho_t (a + b)}{c(a + b \varrho_t) (1 - \varrho_t)} \]  \hspace{1cm} (4.5)

where

\[ \varrho_t = \frac{a \left( 1 - e^{-c \left( \frac{R_t + \phi M_t^B + \psi E_t T_B_t}{D_t + \phi M_t^B + \psi E_t T_B_t + R_t - D_t} \right)} \right)}{a + b e^{-c \left( \frac{R_t + \phi M_t^B + \psi E_t T_B_t}{D_t + \phi M_t^B + \psi E_t T_B_t + R_t - D_t} \right)}} \]
Households

The Lagrangian:

\[
\sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1-\sigma} \left( \left( (C^T_t)^\gamma (C^N_t)^{1-\gamma} \right)^{1-\frac{1}{\sigma}} - 1 \right) + \eta_t \left( N - N^M_t - J \left( \frac{E_t C^T_t + P^N_t C^N_t}{M^\theta_t (S_t + w_t N^M_t)^{1-\theta}} \right) \right) \right. \\
\left. + \lambda_t \left( M_t + \left( 1 + r^d_t \right) S_t + w_t N^M_t + \Pi^N_t + \Pi^B_t - \tau_t \right. \right. \\
\left. - E_t C^T_t - P^N_t C^N_t - (M_{t+1} + S_{t+1}) \frac{(E_{t+1})^\gamma (P^N_{t+1})^{1-\gamma}}{(E_{t+1})^\gamma (P^N_{t+1})^{1-\gamma}} (1 + \pi_{t+1}) \right] 
\]

\[C^T_t:
\gamma (C^T_t)^{1+\gamma-\frac{1}{\sigma}} (C^N_t)^{1-\gamma-\frac{1-\gamma}{\sigma}} = \lambda_t \frac{E_t}{(E_t)^\gamma (P^N_t)^{1-\gamma}} + \eta_t \frac{J E_t}{M^\theta_t (S_t + w_t N^M_t)^{1-\theta}} \quad (4.6)
\]

\[C^N_t:
(1 - \gamma) (C^T_t)^{\gamma-\frac{1}{\sigma}} (C^N_t)^{\gamma-\frac{1-\gamma}{\sigma}} = \lambda_t \frac{P^N_t}{(E_t)^\gamma (P^N_t)^{1-\gamma}} + \eta_t \frac{J P^N_t}{M^\theta_t (S_t + w_t N^M_t)^{1-\theta}} \quad (4.7)
\]

\[N^M_t:
\frac{\lambda_t w_t}{\eta_t (E_t)^\gamma (P^N_t)^{1-\gamma}} = 1 - \frac{(1 - \theta) J w_t (E_t C^T_t + P^N_t C^N_t)}{M^\theta_t (S_t + w_t N^M_t)^{2-\theta}} \quad (4.8)
\]

\[\eta_t:
N^M_t + J \left( \frac{E_t C^T_t + P^N_t C^N_t}{M^\theta_t (S_t + w_t N^M_t)^{1-\theta}} \right) = N \quad (4.9)
\]

\[M_{t+1}:
\frac{\beta J (E_{t+1} C^T_{t+1} + P^N_{t+1} C^N_{t+1})}{\eta_{t+1} M^\theta_{t+1} (S_{t+1} + w_{t+1} N^M_{t+1})^{1-\theta}} = \lambda_t \frac{(1 + \pi_{t+1}) - \beta \lambda_{t+1}}{(E_{t+1})^\gamma (P^N_{t+1})^{1-\gamma}} \quad (4.10)
\]

\[S_{t+1}:
\frac{\beta J (1 - \theta) (E_{t+1} C^T_{t+1} + P^N_{t+1} C^N_{t+1})}{\eta_{t+1} M^\theta_{t+1} (S_{t+1} + w_{t+1} N^M_{t+1})^{2-\theta}} = \lambda_t \frac{(1 + \pi_{t+1}) - \beta \lambda_{t+1} (1 + r^d_{t+1})}{(E_{t+1})^\gamma (P^N_{t+1})^{1-\gamma}} \quad (4.11)
\]

\[\lambda_t:
E_t C^T_t + P^N_t C^N_t + (M_{t+1} + S_{t+1}) \frac{(E_t)^\gamma (P^N_t)^{1-\gamma}}{(E_{t+1})^\gamma (P^N_{t+1})^{1-\gamma}} (1 + \pi_{t+1}) \quad (4.12)
\]

\[= M_t + \left( 1 + r^d_t \right) S_t + w_t N^M_t + \Pi^N_t + \Pi^B_t - \tau_t
\]
The set of optimality conditions 4.1 through 4.12 together with zero profit condition 2.9 and the equilibrium conditions listed in Section 2.3.5 has no closed form solution for the decision variables. The method proposed by King et al. (1988) involves approximating the optimality conditions in the neighborhood of the steady state by a set of linear equations in the unknowns and then solving the resulting linear dynamic system.

The rest of this chapter proceeds as follows. Section 4.2 presents a solution algorithm for a general linear quadratic stochastic optimization problem. Section 4.3 constructs a system of nonlinear difference equations which are to be linearized around the steady state. Section 4.4 describes a linear approximation procedure and presents the linearized system in matrix form.

4.2 Solution Algorithm

The exposition in this section borrows heavily from Burnside (1995).

4.2.1 Dynamic Equilibrium

Consider the general problem of choosing a contingency plan for \( \{u_t\}_{t=0}^{\infty} \) to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ x'_t A_{xx} x_t + 2 x'_t A_{xu} u_t + 2 x'_t A_{xz} z_t + u'_t A_{uu} u_t + 2 u'_t A_{uz} z_t + z'_t A_{zz} z_t \right]
\]

subject to

\[
B_n x_{t+1} = B_x x_t + B_u u_t + B_z z_t
\]

where \( x_t \) is \( n_s \times 1 \) vector of state variables, \( u_t \) is \( n_c \times 1 \) vector of control variables, \( z_t \) is \( n_{ex} \times 1 \) vector of exogenous variables, and \( B_n \) is \( n_{cs} \times 1 \) matrix where \( n_{cs} \leq n_s \).
The Lagrangian for the problem is

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ x_t' A_{xx} x_t + 2x_t' A_{zu} u_t + 2x_t' A_{xz} z_t + u_t' A_{uu} u_t + 2u_t' A_{uz} z_t + z_t' A_{zz} z_t \right] + 2\lambda_t' (B_x x_t + B_u u_t + B_z z_t - B_n x_{t+1}) \]

where \( \lambda_t \) is \( n_{cs} \times 1 \) vector of costate variables.

The Euler equations are

\[ u_t: \quad \begin{equation} A_{uu} u_t + A_{zu} u_t + A_{uz} z_t + B_u' \lambda_t = 0 \end{equation} \]

\[ x_{t+1}: \quad \begin{equation} -B_n' \lambda_t + \beta E_t (A_{xx} x_{t+1} + A_{zu} u_{t+1} + A_{xz} z_{t+1} + B_z' \lambda_{t+1}) = 0 \end{equation} \]

\[ \lambda_t: \quad \begin{equation} B_x x_t + B_u u_t + B_z z_t - B_n x_{t+1} = 0 \end{equation} \]

Rewrite the Euler equation for \( u_t \) in matrix form as

\[ A_{uu} u_t = (-A_{zu} - B_u') \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} + (-A_{uz}) z_t \]

Rewrite the Euler equations for \( x_{t+1} \) and \( \lambda_t \) in matrix form as

\[ \begin{pmatrix} \beta A_{xx} & \beta B_z' \\ -B_n' & 0 \end{pmatrix} E_t \begin{pmatrix} x_{t+1} \\ \lambda_{t+1} \end{pmatrix} + \begin{pmatrix} 0 & -B_n' \\ B_z & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} = \begin{pmatrix} -\beta A_{zu} \\ 0 \end{pmatrix} E_t u_{t+1} + \begin{pmatrix} 0 \\ -B_u \end{pmatrix} u_t + \begin{pmatrix} -\beta A_{xz} \\ 0 \end{pmatrix} E_t z_{t+1} + \begin{pmatrix} 0 \\ -B_z \end{pmatrix} z_t \]

Rewrite the matrix equations above using the notation of King et al. (1988):

\[ M_{ee} u_t = M_{es} \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} + M_{ee} z_t \]
\[ M_{ss0} E_t \begin{pmatrix} x_{t+1} \\ \lambda_{t+1} \end{pmatrix} + M_{ss1} \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} = M_{sc0} E_t u_{t+1} + M_{sc1} u_t + M_{se0} E_t z_{t+1} + M_{se1} z_t \]

where the matrices \( M_{cc}, M_{cs}, \) and \( M_{ce} \) relate controls to controls, states, and exogenous variables, respectively, and \( M_{ss}, M_{sc}, \) and \( M_{se} \) relate states to states, controls, and exogenous variables, respectively.

Solve these matrix equations for \( u_t, x_{t+1}, \) and \( \lambda_{t+1} \):

\[ u_t = M_{cc}^{-1} M_{cs} \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} + M_{cc}^{-1} M_{ce} z_t \quad (4.13) \]

\[ E_t \begin{pmatrix} x_{t+1} \\ \lambda_{t+1} \end{pmatrix} = W \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} + RE_t z_{t+1} + Q z_t \quad (4.14) \]

where

\[
W = - (M_{ss0} - M_{sc0} M_{cc}^{-1} M_{cs})^{-1} (M_{ss1} - M_{sc1} M_{cc}^{-1} M_{cs})
\]

\[
R = (M_{ss0} - M_{sc0} M_{cc}^{-1} M_{cs})^{-1} (M_{se0} + M_{sc0} M_{cc}^{-1} M_{ce})
\]

\[
Q = (M_{ss0} - M_{sc0} M_{cc}^{-1} M_{cs})^{-1} (M_{se1} + M_{sc1} M_{cc}^{-1} M_{ce})
\]

Form the diagonalization, \( W = PA P^{-1} \), where \( \Lambda \) is a diagonal matrix with the eigenvalues of \( W \) on its diagonal, and \( P \) is a matrix whose columns are linearly independent eigenvectors of \( W \).\(^{57}\) Multiplying (4.14) through by \( P^{-1} \) we obtain

\[ E_t \begin{pmatrix} \tilde{x}_{t+1} \\ \tilde{\lambda}_{t+1} \end{pmatrix} = \Lambda \begin{pmatrix} \tilde{x}_t \\ \tilde{\lambda}_t \end{pmatrix} + P^{-1} R E_t z_{t+1} + P^{-1} Q z_t \quad (4.15) \]

where\(^{58}\)

\[
\begin{pmatrix} \tilde{x}_t \\ \tilde{\lambda}_t \end{pmatrix} = \begin{pmatrix} P^{11} & P^{12} \\ P^{21} & P^{22} \end{pmatrix} \begin{pmatrix} x_t \\ \lambda_t \end{pmatrix} \]

\(^{57}\)This requires that \( n_s + n_{cs} \) linearly independent eigenvectors exist for \( W \).

\(^{58}\)See the partitioning of \( P \) below.
and

$$\Lambda = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}$$

with all elements of $\Lambda_1$ less than 1 in absolute value, and all elements of $\Lambda_2$ greater than 1 in absolute value.\(^{59}\)

Partition the matrices $W$, $R$, $Q$, $P$, and $P^{-1}$ as follows:

$$W = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}, \quad R = \begin{pmatrix} R_x \\ R_\lambda \end{pmatrix}, \quad Q = \begin{pmatrix} Q_x \\ Q_\lambda \end{pmatrix},$$

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}, \quad P^{-1} = \begin{pmatrix} P^{11} & P^{12} \\ P^{21} & P^{22} \end{pmatrix}$$

Since $W = P \Lambda P^{-1}$ we have

$$\begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} = \begin{pmatrix} P_{11} \Lambda_1 P^{11} + P_{12} \Lambda_2 P^{21} & P_{11} \Lambda_1 P^{12} + P_{12} \Lambda_2 P^{22} \\ P_{21} \Lambda_1 P^{11} + P_{22} \Lambda_2 P^{21} & P_{21} \Lambda_1 P^{12} + P_{22} \Lambda_2 P^{22} \end{pmatrix} \quad (4.17)$$

Solving (4.15) backward for $\tilde{x}_{t+1}$ and forward for $\tilde{\lambda}_{t+1}$ we get

$$E_t \tilde{x}_{t+1} = \Lambda_1 \tilde{x}_t + \left( P^{11} R_x + P^{12} R_\lambda \right) E_t z_{t+1} + \left( P^{11} Q_x + P^{12} Q_\lambda \right) z_t \quad (4.18)$$

$$E_t \tilde{\lambda}_{t+1} = \Lambda_2 \tilde{\lambda}_t + \left( P^{21} R_x + P^{22} R_\lambda \right) E_t z_{t+1} + \left( P^{21} Q_x + P^{22} Q_\lambda \right) z_t \quad (4.19)$$

From (4.14) we have

$$E_t x_{t+1} = W_{11} x_t + W_{12} \lambda_t + R_x E_t z_{t+1} + Q_x z_t$$

\(^{59}\)Normally, we need to have $n_{cs}$ eigenvalues of $W$ greater than 1 in modulus, and $n_s$ eigenvalues of $W$ less than 1 in modulus.
Using (4.16) and (4.17) and the partitioned inverse formula, rewrite the above equation as

\[ E_t x_{t+1} = \left( P_{11} \Lambda_1 P_{11}^{-1} \right) x_t + \left( P_{11} \Lambda_1 P_{12} + P_{12} \Lambda_2 P_{22} \right) \left( P_{22} \right)^{-1} \tilde{\lambda}_t + R_x E_t z_{t+1} + Q_z z_t \]

This expresses the solution for \( x_{t+1} \) as a function of the past state, and, given the solution for \( \tilde{\lambda}_t \) derived below, as a function of current and expected future values of the exogenous variables, \( z_t \).

Choosing an AR(1) representation for \( z_t \), so that \( z_{t+1} = \Pi z_t + \epsilon_{t+1} \), we obtain the solution for \( \tilde{\lambda}_t \) from (4.19):

\[
\tilde{\lambda}_t = \Lambda_2^{-1} E_t \tilde{\lambda}_{t+1} - \Lambda_2^{-1} \left( P^{21} R_x + P^{22} R_\lambda \right) E_t z_{t+1} - \Lambda_2^{-1} \left( P^{21} Q_x + P^{22} Q_\lambda \right) z_t
\]

\[
= - \sum_{j=0}^{\infty} \Lambda_2^{-(j+1)} \left( \Phi_0 E_t z_{t+1+j} + \Phi_1 E_t z_{t+j} \right)
\]

\[
= - \left[ \sum_{j=0}^{\infty} \Lambda_2^{-(j+1)} \left( \Phi_0 \Pi + \Phi_1 \right) \Pi^j \right] z_t
\]

\[
= \Psi z_t
\]

where

\[
\Phi_0 = \left( P^{21} R_x + P^{22} R_\lambda \right) \quad \quad \Phi_1 = \left( P^{21} Q_x + P^{22} Q_\lambda \right)
\]

Defining \( \Lambda_2 \), as the \( i \)th diagonal element of \( \Lambda_2 \), and \( \Phi_j \), as the \( i \)th row of \( \Phi_j \), \( j = 0, 1 \), we obtain the \( i \)th row of \( \Psi \):

\[
\Psi_i = - \left[ \sum_{j=0}^{\infty} \Lambda_2^{-(j+1)} \left( \Phi_0 \Pi + \Phi_1 \right) \Pi^j \right]
\]

\[\text{The inverse of a matrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \text{ is} \]

\[
\begin{pmatrix}
E^{-1} & -E^{-1}BD^{-1} \\
-D^{-1}CE^{-1} & D^{-1} + D^{-1}CE^{-1}BD^{-1}
\end{pmatrix}
\]

where \( E = A - BD^{-1}C \).

\[\text{This requires that the eigenvalues of} \ \Pi \text{ are less than 1 in modulus.}\)
\[ \begin{align*}
&= -\Lambda_{2,1}^{-1}(\Phi_0, \Pi + \Phi_{1,1}) \sum_{j=0}^{\infty} \Lambda_{2,1}^{-j} \Pi^j \\
&= -\Lambda_{2,1}^{-1}(\Phi_0, \Pi + \Phi_{1,1}) \left( I_{n_{ax}} - \Lambda_{2,1}^{-1} \Pi \right)^{-1}
\end{align*} \]

The solution for \( x_{t+1} \) then becomes

\[ x_{t+1} = \Upsilon_{xx} x_t + T_{xx} z_t \tag{4.20} \]

where

\[ \Upsilon_{xx} = P_{11} \Lambda_1 P_{11}^{-1} \quad \Upsilon_{xz} = \left( P_{11} \Lambda_1 P_{12} + P_{12} \Lambda_2 P_{22} \right) \left( P_{22} \right)^{-1} \Psi + R_x \Pi + Q_x \]

Using (4.16) and the solution for \( \bar{\lambda}_t \), we obtain the solution for \( \lambda_t \):

\[ \lambda_t = \Upsilon_{\lambda x} x_t + \Upsilon_{\lambda z} z_t \tag{4.21} \]

where

\[ \Upsilon_{\lambda x} = -\left( P_{22} \right)^{-1} P_{21} \quad \Upsilon_{\lambda z} = \left( P_{22} \right)^{-1} \Psi \]

Finally, the solution for \( u_t \) is obtained from (4.13):

\[ u_t = \Upsilon_{ux} x_t + \Upsilon_{uz} z_t \tag{4.22} \]

where

\[ \Upsilon_{ux} = M_{cc}^{-1} M_{cs} \begin{pmatrix} I \cr -\left( P_{22} \right)^{-1} P_{21} \end{pmatrix} \quad \Upsilon_{uz} = M_{cc}^{-1} M_{cs} \begin{pmatrix} 0 \cr \left( P_{22} \right)^{-1} \Psi \end{pmatrix} + M_{cc}^{-1} M_{cs} \]

The solution we obtained above for a general linear quadratic optimization problem (equations 4.20, 4.21, and 4.22) is unique if the following transversality condition is satisfied:

\[ \lim_{t \to -\infty} E_0 \beta^t \left[ x_t' A_{xx} x_t + 2x_t' A_{xz} u_t + 2x_t' A_{zz} z_t + 2x_t' B_x' \lambda_t \right] = 0 \tag{4.23} \]

This condition is satisfied if the eigenvalues of \( \Lambda_2 \) are less than \( \beta^{-\frac{1}{2}} \) in absolute value.\(^{62}\)

\(^{62}\)See Burnside (1995), Chapter 5 for a detailed discussion of uniqueness and boundedness.
4.2.2 Impulse Response Functions

An impulse response function describes the response of endogenous variables in a model to innovations in the exogenous variables in the model. In order to describe this response, we derive the moving average representation of all variables of interest—controls, states, and additional variables expressed as linear combinations of the controls, states, and exogenous variables—in terms of current and past innovations in the vector of exogenous variables, $z_t$.

We can rewrite the solution for a general linear quadratic optimization problem—equations 4.20, 4.21, and 4.22—as

$$s_{t+1} = Ms_t + \tilde{e}_{t+1}$$

where

$$s_t = \begin{pmatrix} x_t \\ z_t \end{pmatrix} \quad \tilde{e}_t = \begin{pmatrix} 0 \\ \epsilon_t \end{pmatrix} \quad M = \begin{pmatrix} \Upsilon_{xx} & \Upsilon_{xz} \\ 0 & \Pi \end{pmatrix}$$

and the vector $\epsilon_t$ is the vector of innovations to the exogenous variables. Also, we have

$$\lambda_t = \begin{pmatrix} \Upsilon_{\lambda x} & \Upsilon_{\lambda z} \end{pmatrix} s_t$$

and

$$u_t = \begin{pmatrix} \Upsilon_{ux} & \Upsilon_{uz} \end{pmatrix} s_t$$

Let $f_t$ be the vector of additional variables of interest which can be expressed as a linear combination of the controls, states, and exogenous variables:

$$f_t = F_c u_t + F_x x_t + F_e z_t$$

$$= \left[ F_c \begin{pmatrix} \Upsilon_{ux} & \Upsilon_{uz} \end{pmatrix} + \begin{pmatrix} F_x & F_e \end{pmatrix} \right] s_t$$

where the matrices $F_c$, $F_x$, and $F_e$ relate the additional variables to the controls, states, and exogenous variables, respectively. Hence, we can represent the costates,
controls, and additional variables in the following form:

$$\begin{pmatrix}
\lambda_t \\
u_t \\
f_t
\end{pmatrix} = Hs_t$$

where

$$H = \begin{pmatrix}
\gamma_{\lambda x} & \gamma_{\lambda z} \\
\gamma_{uz} & \gamma_{uz} \\
F_c \left( \begin{pmatrix}
\gamma_{uz} \\
\gamma_{uz}
\end{pmatrix} + \begin{pmatrix}
F_x & F_{\varepsilon}
\end{pmatrix}
\right)
\end{pmatrix}$$

The moving average representation for all the variables, including the states, is

$$s_t = Ms_{t-1} + \bar{\varepsilon}_t$$

### 4.3 A System of Nonlinear Difference Equations

The system of nonlinear difference equations to be linearized around the steady state consists of

The firm sector optimality conditions 4.1, 4.2, and 4.3:

$$\alpha T_t K_t^{\alpha-1} (N_t^T)^{1-\alpha} = (1 + r_t^f) \tag{4.24}$$

$$(1 - \alpha) E_t T_t K_t^{\alpha} (N_t^T)^{-\alpha} = (1 + r_t^f) w_t \tag{4.25}$$

$$H \xi P_t^N (N_t^N)^{\xi-1} = (1 + r_t^f) w_t \tag{4.26}$$

The nontraded goods sector equilibrium condition 2.23:

$$H (N_t^N)^{\xi} = C_t^N \tag{4.27}$$

The labor market equilibrium condition 2.24:

$$N_t^T + N_t^N = N_t^M \tag{4.28}$$
The household sector optimality conditions 4.6, 4.7, 4.8, and 4.9:\(^{63}\)

\[
\gamma \left( C_t \right)^{1+\gamma - \frac{1}{\sigma}} \left( C_t \right)^{1-\gamma - \frac{1}{\sigma}} = \lambda_t \frac{E_t}{(E_t)^\gamma (P_t^N)^{1-\gamma}} + \eta_t \frac{J E_t}{M_t^{\theta} (S_t + w_t N_t^M)^{1-\theta}} \tag{4.29}
\]

\[
\gamma P_t^N C_t^N = (1-\gamma) E_t C_t^T \tag{4.30}
\]

\[
\frac{\lambda_t}{\eta_t} \frac{w_t}{(E_t)^\gamma (P_t^N)^{1-\gamma}} = 1 - \frac{(1-\theta) J w_t (E_t C_t^T + P_t^N C_t^N)}{M_t^{\theta} (S_t + w_t N_t^M)^{2-\theta}} \tag{4.31}
\]

\[
J \left( \frac{E_t C_t^T + P_t^N C_t^N}{M_t^{\theta} (S_t + w_t N_t^M)^{1-\theta}} \right) = N - N_t^M \tag{4.32}
\]

The banking sector optimality conditions 4.4 and 4.5:\(^{64}\)

\[
\frac{1}{\psi} \frac{E_t \left( T_t K_t \alpha N_t^{1-\alpha} - C_t^T - K_t \right)}{n_t \left( L_t + R_t - (S_t + w_t N_t^M) \right)^2} \left( r_t L_t - r_t^d S_t + R_t \right) = \frac{\theta_t (a + b)}{c (a + b \theta_t) (1 - \theta_t)} \tag{4.33}
\]

The banking sector zero profit condition 2.9:

\[
\theta_t \left( r_t L_t - r_t^d S_t + R_t \right) = R_t \tag{4.34}
\]

where

\[
\theta_t = \frac{-c \left( \frac{R_t}{S_t + w_t N_t^M} + \varphi \frac{M_t + R_t}{M_t + S_t + w_t N_t^M} + \psi \frac{E_t \left( T_t K_t \alpha N_t^{1-\alpha} - C_t^T - K_t \right)}{L_t + R_t - (S_t + w_t N_t^M)} \right)}{a + b \theta_t}
\]

Loan market equilibrium condition 2.25:

\[
L_t = E_t K_t + w_t N_t^M \tag{4.35}
\]

---

\(^{63}\)Equation 4.30 is obtained by dividing (4.7) by (4.6).

\(^{64}\)Equation 4.33 is obtained by dividing (4.4) by (4.5).
The household sector optimality conditions 4.10, 4.11, and 4.12: \(^{65}\)

\[
\frac{\beta J \theta \left( E_{t+1} C_{t+1}^T + P_{t+1}^N C_{t+1}^N \right)}{M_{t+1}^{1+\theta} \left( S_{t+1} + w_{t+1} N_{t+1}^M \right)^{1-\theta}} = \frac{\lambda_t (1 + \pi_{t+1}) - \beta \lambda_{t+1}}{(E_{t+1})^\gamma (P_{t+1}^N)^{1-\gamma}} \tag{4.37}
\]

\[
\frac{(1 - \theta) M_{t+1}}{\theta (S_{t+1} + w_{t+1} N_{t+1}^M)} = 1 - \frac{\beta \lambda_{t+1} r_{t+1}^d}{\lambda_t (1 + \pi_{t+1}) - \beta \lambda_{t+1}} \tag{4.38}
\]

\[
E_t C_t^T + P_t^N C_t^N + (M_{t+1} + S_{t+1}) \frac{(E_t)^\gamma (P_t^N)^{1-\gamma}}{(E_{t+1})^\gamma (P_{t+1}^N)^{1-\gamma}} \tag{4.39}
\]

\[
= M_t + (1 - \xi) P_t^N H \left( N_t^N \right)^{1-\xi} + \left( 1 + r_t^N \right) L_t + R_t
\]

The balance-of-payments condition 2.14 combined with the central bank balance sheet 2.12 and the traded goods sector equilibrium condition 2.22:

\[
\frac{M_{t+1} + R_{t+1}}{E_{t+1}} - \frac{M_t + R_t}{E_t} - \frac{L_t + R_t - (S_t + w_t N_t^M)}{E_t} \tag{4.40}
\]

\[
= T_t (K_t)^\alpha (N_t^T)^{1-\alpha} - C_t^T - K_t
\]

Additional variables which include:

Real currency holdings:

\[
\frac{M_t}{P_t} = \frac{M_t}{(E_t)^\gamma (P_t^N)^{1-\gamma}} \tag{4.41}
\]

Real bank deposits:

\[
\frac{D_t}{P_t} = \frac{S_t + w_t N_t^M}{(E_t)^\gamma (P_t^N)^{1-\gamma}} \tag{4.42}
\]

Ratio of deposits to currency:

\[
\frac{D_t}{M_t} = \frac{S_t + w_t N_t^M}{M_t} \tag{4.43}
\]

Ratio of loans to GDP:

\[
\frac{L_t}{GDP_t} = \frac{L_t}{E_t T_t (K_t)^\alpha (N_t^T)^{1-\alpha} + P_t^N C_t^N} \tag{4.44}
\]

\(^{65}\)Equation 4.38 is obtained by dividing (4.11) by (4.10). Equation 4.39 is obtained by combining (4.12) with (2.5), (2.3), (2.10), (2.15), and (2.7).
Ratio of broad money to monetary base (money multiplier):

\[ \frac{M2_t}{M0_t} = \frac{M_t + S_t + w_t N_t^M}{M_t + R_t} \]  (4.45)

Ratio of broad money to GDP:

\[ \frac{M2_t}{GDP_t} = \frac{M_t + S_t + w_t N_t^M}{E_t T_t (K_t)^\alpha (N_t^T)^{1-\alpha} + P_t^N C_t^N} \]  (4.46)

Real GDP:

\[ \frac{GDP_t}{P_t} = \frac{E_t T_t (K_t)^\alpha (N_t^T)^{1-\alpha} + P_t^N C_t^N}{(E_t)^\gamma (P_t^N)^{1-\gamma}} \]  (4.47)

Real wage rate:

\[ \frac{w_t}{P_t} = \frac{w_t}{(E_t)^\gamma (P_t^N)^{1-\gamma}} \]  (4.48)

Output of tradables:

\[ Y_t^T = T_t (K_t)^\alpha (N_t^T)^{1-\alpha} \]  (4.49)

Inflation rate in the nontradables sector:

\[ \pi_t^N = \frac{P_t^N}{P_{t-1}^N} - 1 \]  (4.50)

Trade balance in foreign currency terms:

\[ TB_t = T_t K_t^\alpha N_t^{T1-\alpha} - C_t^T - K_t \]  (4.51)

Real exchange rate defined as the relative price of nontradables in terms of tradables:

\[ \epsilon_t = \frac{P_t^N}{E_t} \]  (4.52)

Net capital inflow in foreign currency terms:

\[ F_t = \frac{L_t + R_t - (S_t + w_t N_t^M)}{E_t} \]  (4.53)

The equation not included into the system

Net foreign liabilities, stock:

\[ NFL_{t+1} = (1 + r^*) NFL_t + F_t \]  (4.54)
The next step in obtaining the system of linear difference equations is to approximate equations 4.24 through 4.54 linearly around the steady state.

4.4 Linear Approximation

To linearize equations 4.24 through 4.54 around the steady state, we totally differentiate the equations at the steady state values. Following King et al. (1988), we define the percentage deviation of variable \( z_t \) from its steady state value \( z \) as \( \hat{z}_t = \frac{dz_t}{z} \) and then rewrite each equation in the system in terms of percentage deviations of the variables from their steady state values. Note that, in the steady state, \( \pi = \varepsilon > 0 \)—that is, there are no steady state levels for \( E_t \) and \( P_t^N \). Therefore, we express the linearized system in terms of first differences of the percentage deviations, \( \hat{z}_t - \hat{z}_{t-1} \).\(^{66}\)

In what follows, we utilize the result

\[
\frac{E_t - E_{t-1}}{E_t} = \frac{\varepsilon}{1 + \varepsilon} \hat{E}_t
\]

where \( \varepsilon \) is the steady state rate of devaluation and \( \hat{E}_t \) is the percentage deviation of the devaluation rate from its steady state level.\(^{67}\) The linearized system expressed in terms of first differences of percentage deviations of the variables from their stationary levels is presented below.

---

\(^{66}\)Note, however, that the simulation results in Section 2.4.3 are presented in terms of percentage deviations, \( \hat{z}_t \). This is done by calculating the cumulative sum of \( \hat{z}_t - \hat{z}_{t-1} \) for each simulated series.

\(^{67}\)Hence, fixing the exchange rate amounts to setting \( \varepsilon_t = -100 \) in the simulation exercise.
Controls, equations 4.24 - 4.36:

\[
M_{cc} \cdot \begin{pmatrix}
  \tilde{K}_t - \tilde{K}_{t-1} \\
  \tilde{N}^T_t - \tilde{N}^T_{t-1} \\
  \tilde{N}^N_t - \tilde{N}^N_{t-1} \\
  \tilde{C}^T_t - \tilde{C}^T_{t-1} \\
  \tilde{C}^N_t - \tilde{C}^N_{t-1} \\
  \tilde{N}^M_t - \tilde{N}^M_{t-1} \\
  \tilde{P}^N_t - \tilde{P}^N_{t-1} \\
  \tilde{\omega}_t - \tilde{\omega}_{t-1} \\
  \tilde{R}_t - \tilde{R}_{t-1} \\
  \tilde{L}_t - \tilde{L}_{t-1} \\
  \tilde{r}^l_t - \tilde{r}^l_{t-1} \\
  \tilde{r}^d_t - \tilde{r}^d_{t-1} \\
  \tilde{\eta}_t - \tilde{\eta}_{t-1}
\end{pmatrix} = M_{cs} \cdot \begin{pmatrix}
  \tilde{M}_t - \tilde{M}_{t-1} \\
  \tilde{S}_t - \tilde{S}_{t-1} \\
  \tilde{\lambda}_t - \tilde{\lambda}_{t-1} \\
  \tilde{\eta}_t - \tilde{\eta}_{t-1}
\end{pmatrix} + M_{ce} \cdot \begin{pmatrix}
  \tilde{T}_t - \tilde{T}_{t-1} \\
  \tilde{\varepsilon}_t
\end{pmatrix}
\]
States/costates, equations 4.37 - 4.40:

\[
(M_{ss0} + M_{ss1}) \cdot \begin{pmatrix}
\hat{M}_{t+1} - \hat{M}_t \\
\hat{S}_{t+1} - \hat{S}_t \\
\hat{\lambda}_{t+1} - \hat{\lambda}_t \\
\hat{\eta}_{t+1} - \hat{\eta}_t \\
\hat{K}_{t+1} - \hat{K}_t \\
\hat{N}^T_{t+1} - \hat{N}^T_t \\
\hat{N}^N_{t+1} - \hat{N}^N_t \\
\hat{C}^T_{t+1} - \hat{C}^T_t \\
\hat{C}^N_{t+1} - \hat{C}^N_t \\
\hat{N}^M_{t+1} - \hat{N}^M_t \\
\hat{P}^N_{t+1} - \hat{P}^N_t \\
\hat{w}_{t+1} - \hat{w}_t \\
\hat{R}_{t+1} - \hat{R}_t \\
\hat{L}_{t+1} - \hat{L}_t \\
\hat{r}^l_{t+1} - \hat{r}^l_t \\
\hat{r}^d_{t+1} - \hat{r}^d_t \\
\hat{n}_{t+1} - \hat{n}_t
\end{pmatrix}
\]

\[
= (M_{sc0} + M_{sc1}) \cdot \begin{pmatrix}
\hat{T}_{t+1} - \hat{T}_t \\
\hat{\varepsilon}_{t+1}
\end{pmatrix}
\]
Additional variables, equations 4.41 - 4.53:

\[
\begin{pmatrix}
\frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \\
\frac{D_t}{P_t} - \frac{D_{t-1}}{P_{t-1}} \\
\frac{D_t}{M_t} - \frac{D_{t-1}}{M_{t-1}} \\
\frac{L_t}{GDP_t} - \frac{L_{t-1}}{GDP_{t-1}} \\
\frac{M_0t}{N_0t} - \frac{M_{0t-1}}{N_{0t-1}} \\
\frac{GDP_t}{P_t} - \frac{GDP_{t-1}}{P_t} \\
\frac{w_t}{P_t} - \frac{w_{t-1}}{P_{t-1}} \\
\bar{Y}_t^T - \bar{Y}_{t-1}^T \\
\bar{P}^N_t - \bar{P}^N_{t-1} \\
\bar{R}_t - \bar{R}_{t-1} \\
\bar{L}_t - \bar{L}_{t-1} \\
\bar{T} - \bar{T}_{t-1} \\
\bar{F}_t - \bar{F}_{t-1}
\end{pmatrix}
= f_c \cdot \begin{pmatrix}
\bar{K}_t - \bar{K}_{t-1} \\
\bar{N}_t^T - \bar{N}_{t-1}^T \\
\bar{N}_t^N - \bar{N}_{t-1}^N \\
\bar{C}_t^T - \bar{C}_{t-1}^T \\
\bar{C}_t^N - \bar{C}_{t-1}^N \\
\bar{P}_t^N - \bar{P}_{t-1}^N \\
\bar{w}_t - \bar{w}_{t-1} \\
\bar{R}_t - \bar{R}_{t-1} \\
\bar{L}_t - \bar{L}_{t-1} \\
\bar{F}_t - \bar{F}_{t-1}
\end{pmatrix}
+ f_x \cdot \begin{pmatrix}
\bar{M}_t - \bar{M}_{t-1} \\
\bar{S}_t - \bar{S}_{t-1} \\
\bar{\lambda}_t - \bar{\lambda}_{t-1} \\
\bar{\eta}_t - \bar{\eta}_{t-1}
\end{pmatrix}
+ f_e \cdot \begin{pmatrix}
\bar{T}_t - \bar{T}_{t-1}
\end{pmatrix}
\]
where

\[ M_{cc}(1, 1) = \alpha - 1 \]
\[ M_{cc}(1, 2) = 1 - \alpha \]
\[ M_{cc}(1, 11) = -\frac{\nu}{1 + \nu} \]
\[ M_{cc}(2, 1) = -\alpha \]
\[ M_{cc}(2, 2) = \alpha \]
\[ M_{cc}(2, 8) = 1 \]
\[ M_{cc}(2, 11) = \frac{r_l}{1 + r_l} \]
\[ M_{cc}(3, 3) = \xi - 1 \]
\[ M_{cc}(3, 7) = 1 \]
\[ M_{cc}(3, 8) = -1 \]
\[ M_{cc}(3, 11) = -\frac{r_l}{1 + r_l} \]
\[ M_{cc}(4, 3) = \xi \]
\[ M_{cc}(4, 5) = -1 \]
\[ M_{cc}(5, 2) = \frac{N^T}{N^M} \]
\[ M_{cc}(5, 3) = \frac{N}{N^M} \]
\[ M_{cc}(5, 6) = -1 \]
\[ M_{cc}(6, 4) = -1 + \gamma - \frac{1}{\sigma} \]
\[ M_{cc}(6, 5) = 1 - \gamma - \frac{1}{\sigma} \]
\[ M_{cc}(6, 6) = (1 - \theta) \frac{r}{1 + r} \frac{w}{D^M} \]
\[ M_{cc}(6, 7) = (1 - \gamma) \frac{r}{1 + r} \]
\[ M_{cc}(6, 8) = (1 - \theta) \frac{r}{1 + r} \frac{w}{D^M} \]
\[ M_{cc}(7, 4) = -1 \]
\[ M_{cc}(7, 5) = 1 \]
\[ M_{cc}(7, 7) = 1 \]
\[ M_{cc}(8, 4) = \Theta \gamma \]
\[ M_{cc}(8, 5) = \Theta (1 - \gamma) \]
\[ M_{cc}(8, 6) = -\Theta (2 - \theta) \frac{w}{D^M} \]
\[ M_{cc}(8, 7) = (1 - \gamma) (\Theta - 1) \]
\[ M_{cc}(8, 8) = \Theta - \Theta (2 - \theta) \frac{w}{D^M} + 1 \]
\[ M_{cc}(9, 4) = \gamma \]
\[ M_{cc}(9, 5) = 1 - \gamma \]
\[ M_{cc}(9, 6) = \frac{N^M}{N^E} - (1 - \theta) \frac{w}{D^M} \]
\[ M_{cc}(9, 7) = 1 - \gamma \]
\[ M_{cc}(9, 8) = - (1 - \theta) \frac{w}{D^M} \]
\[ M_{cc}(10, 1) = -\nu \zeta \]
\[ M_{cc}(10, 2) = - (1 - \alpha) \frac{E \gamma^T}{E \gamma B} \zeta \]
\[ M_{cc}(10, 4) = \frac{E \gamma^T}{E \gamma B} \zeta \]
\[ M_{cc}(10, 6) = \Omega \]
\[ M_{cc}(10, 8) = \Omega \]
\[ M_{cc}(10, 9) = \frac{1}{1 - \gamma} \Psi + 2 \zeta \frac{R}{E_F} \]
\[ M_{cc}(10, 10) = -\frac{\nu}{E_F} \left( \frac{1}{1 - \gamma} \Sigma - 2 \zeta \right) \]
\[ M_{cc}(10, 11) = 1 \]
\[ M_{cc}(10, 13) = \zeta \]
\[ M_{cc}(11, 1) = -\nu \zeta \]
\[ M_{cc}(11, 2) = - (1 - \alpha) \frac{E \gamma^T}{E \gamma B} \zeta \]
\[ M_{cc}(11, 4) = \frac{E \gamma^T}{E \gamma B} \zeta \]
\[ M_{cc}(11, 6) = \nu \Phi - 2 \frac{w}{E_F} \]
\[ M_{cc}(11, 8) = \nu \Phi - 2 \frac{w}{E_F} \]
\[ M_{cc}(11, 9) = 2 \frac{R}{E_F} + \nu \Psi - \nu \]
\[ M_{cc}(11, 10) = 2 \frac{R}{E_F} - \nu \Sigma - \frac{L}{E_F} - \nu \]

\[ M_{cc}(11, 1) = -\nu \zeta \]
\[ M_{cc}(11, 2) = - (1 - \alpha) \frac{E \gamma^T}{E \gamma B} \zeta \]
\[ M_{cc}(11, 4) = \frac{E \gamma^T}{E \gamma B} \zeta \]
\[ M_{cc}(11, 6) = \nu \Phi - 2 \frac{w}{E_F} \]
\[ M_{cc}(11, 8) = \nu \Phi - 2 \frac{w}{E_F} \]
\[ M_{cc}(11, 9) = 2 \frac{R}{E_F} + \nu \Psi - \nu \]
\[ M_{cc}(11, 10) = 2 \frac{R}{E_F} - \nu \Sigma - \frac{L}{E_F} - \nu \]
\[ M_{cc} (11, 11) = 1 - \nu \]
\[ M_{cc} (11, 12) = \kappa \]
\[ M_{cc} (11, 13) = 1 \]
\[ M_{cc} (12, 1) = -\Sigma \nu \]
\[ M_{cc} (12, 2) = -\Sigma (1 - \alpha) \frac{EY_T}{ETB} \]
\[ M_{cc} (12, 4) = \Sigma \frac{E\gamma T}{ETB} \]
\[ M_{cc} (12, 6) = \Phi \]
\[ M_{cc} (12, 8) = \Phi \]
\[ M_{cc} (12, 9) = \Psi + \omega - 1 \]
\[ M_{cc} (12, 10) = \nu - \Sigma \frac{L}{EF} \]
\[ M_{cc} (12, 11) = \nu \]
\[ M_{cc} (12, 12) = -\kappa \]
\[ M_{cc} (13, 1) = \frac{EK}{L} \]
\[ M_{cc} (13, 6) = \frac{wM}{L} \]
\[ M_{cc} (13, 8) = \frac{wM}{L} \]
\[ M_{cc} (13, 10) = -1 \]
\[ M_{cs} (6, 1) = -\theta \frac{\Gamma}{1+\Gamma} \]
\[ M_{cs} (6, 2) = -(1 - \theta) \frac{\Gamma}{1+\Gamma} \frac{S}{D} \]
\[ M_{cs} (6, 3) = \frac{1}{1+\Gamma} \]
\[ M_{cs} (6, 4) = \frac{\Gamma}{1+\Gamma} \]
\[ M_{cs} (8, 1) = \Theta \theta \]
\[ M_{cs} (8, 2) = \Theta (2 - \theta) \frac{S}{D} \]
\[ M_{cs} (8, 3) = \frac{1}{1+\Gamma} \]
\[ M_{cs} (8, 4) = 1 \]
\[ M_{cs} (9, 1) = \theta \]
\[ M_{cs} (9, 2) = (1 - \theta) \frac{S}{D} \]
\[ M_{cs} (10, 1) = -\frac{1}{1+\epsilon} \Xi \]
\[ M_{cs} (10, 2) = 2\zeta \frac{S}{EF} + \zeta \frac{S}{D} - \frac{1}{1+\epsilon} \gamma \]
\[ M_{cs} (11, 1) = -\nu \Xi \]
\[ M_{cs} (11, 2) = 2\frac{S}{EF} - \nu \gamma - \kappa \]
\[ M_{cs} (12, 1) = -\Xi \]
\[ M_{cs} (12, 2) = \kappa - \gamma \]
\[ M_{ce} (1, 1) = -1 \]
\[ M_{ce} (2, 1) = 1 \]
\[ M_{ce} (2, 2) = \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (6, 2) = \frac{1-\gamma+\Gamma}{1+\Gamma} \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (7, 2) = \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (8, 2) = \gamma (1 - \Theta) \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (9, 2) = -\gamma \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (10, 1) = \frac{EY_T}{ETB} \zeta \]
\[ M_{ce} (10, 2) = -\left(1 - \Theta \right) \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (11, 1) = \frac{EY_T}{ETB} \theta \]
\[ M_{ce} (11, 2) = -(\nu \gamma - 1) \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (12, 1) = \frac{\epsilon}{1+\epsilon} \]
\[ M_{ce} (13, 2) = -\frac{EK}{L} \frac{\epsilon}{1+\epsilon} \]
\[ M_{ss0} (1, 1) = 1 + \theta \]
\[ M_{ss0} (1, 2) = (1 - \theta) \frac{S}{D} \]
\[ M_{ss0} (1, 3) = -\frac{\beta}{1+\gamma+\beta} \]
\[ M_{ss0} (1, 4) = -1 \]
\[ M_{ss0} (2, 1) = 1 \]
\[ M_{ss0} (2, 2) = -\frac{S}{D} \]
\[ M_{s0}(2, 3) = \Lambda \]
\[ M_{s0}(3, 1) = \Delta \frac{M}{L} \]
\[ M_{s0}(3, 2) = \Delta \frac{S}{L} \]
\[ M_{s0}(4, 1) = -\frac{M}{E_F} \]
\[ M_{s0}(1, 3) = \frac{1+\pi}{1+\pi-\beta} \]
\[ M_{s0}(2, 3) = -\Lambda \]
\[ M_{s0}(3, 1) = -\Delta \frac{M}{L} \]
\[ M_{s0}(4, 1) = \frac{M}{E_F} \]
\[ M_{s0}(4, 2) = -\frac{S}{E_F} \]
\[ M_{s0}(1, 4) = \gamma \]
\[ M_{s0}(1, 5) = 1 - \gamma \]
\[ M_{s0}(1, 6) = -(1 - \theta) \frac{wN^M}{D} \]
\[ M_{s0}(1, 7) = (1 - \gamma) \frac{1+\pi-2\beta}{1+\pi-\beta} \]
\[ M_{s0}(1, 8) = -(1 - \theta) \frac{wN^M}{D} \]
\[ M_{s0}(2, 6) = \frac{wN^M}{D} \]
\[ M_{s0}(2, 7) = (1 - \gamma) \Lambda \]
\[ M_{s0}(2, 8) = \frac{wN^M}{D} \]
\[ M_{s0}(2, 12) = -\frac{\beta r d}{1+\beta(1+\beta)} \]
\[ M_{s0}(3, 7) = \Delta (1 - \gamma) \left( \frac{M}{L} + \frac{S}{L} \right) \]
\[ M_{s0}(4, 9) = \frac{R}{E_F} \]
\[ M_{s1}(1, 7) = (1 - \gamma) \frac{1+\pi}{1+\pi-\beta} \]
\[ M_{s1}(2, 7) = -(1 - \gamma) \Lambda \]
\[ M_{s1}(3, 4) = -\frac{\xi(1-\gamma)}{\gamma + \xi(1-\gamma)} \]
\[ M_{s1}(3, 5) = -\frac{\xi(1-\gamma)}{\gamma + \xi(1-\gamma)} \]
\[ M_{s1}(3, 7) = -\frac{\xi(1-\gamma)}{\gamma + \xi(1-\gamma)} - \frac{\Delta(1-\gamma)(M+S)}{L} \]
\[ M_{s1}(3, 9) = \Delta \frac{R}{L} \]
\[ M_{s1}(3, 10) = \Delta \left( 1 + r^d \right) \]
\[ M_{s1}(3, 11) = \Delta r^d \]
\[ M_{s1}(4, 1) = \nu \]
\[ M_{s1}(4, 2) = (1 - \alpha) \frac{EY^T}{ETB} \]
\[ M_{s1}(4, 4) = -\frac{EY^T}{ETB} \]
\[ M_{s1}(4, 6) = \frac{wN^M}{E_F} \]
\[ M_{s1}(4, 8) = \frac{wN^M}{E_F} \]
\[ M_{s1}(4, 9) = -2 \frac{R}{E_F} \]
\[ M_{s1}(4, 10) = -\frac{L}{E_F} \]
\[ M_{s0}(1, 2) = \gamma \frac{1+\pi-2\beta}{1+\pi-\beta} \frac{\epsilon}{1+\epsilon} \]
\[ M_{s0}(2, 2) = \gamma A \frac{\epsilon}{1+\epsilon} \]
\[ M_{s0}(3, 2) = \Delta \gamma \left( \frac{M}{L} + \frac{S}{L} \right) \frac{\epsilon}{1+\epsilon} \]
\[ M_{s0}(4, 2) = -(\frac{M}{E_F} + \frac{R}{E_F}) \frac{\epsilon}{1+\epsilon} \]
\[ M_{s1}(1, 2) = \gamma \frac{1+\pi}{1+\pi-\beta} \frac{\epsilon}{1+\epsilon} \]
\[ M_{s1}(2, 2) = -\gamma A \frac{\epsilon}{1+\epsilon} \]
\[ M_{s1}(3, 2) = -\frac{\epsilon}{\gamma + \xi(1-\gamma) + \Delta(1-\gamma)(M+S)} \frac{L}{1+\epsilon} \]
\[ M_{s1}(4, 1) = \frac{EY^T}{ETB} \]
\[ M_{s1}(4, 2) = \left( 1 + \frac{M}{E_F} + \frac{R}{E_F} \right) \frac{\epsilon}{1+\epsilon} \]
\[ f_c(1, 7) = \gamma - 1 \]
\[ f_c(2, 6) = \frac{wN^M}{D} \]
\[ f_c(2, 7) = \gamma - 1 \]
\[ f_c(2, 8) = \frac{wN^M}{D} \]
\[ f_c(3, 6) = \frac{wN^M}{D} \]
\[ f_c(3, 8) = \frac{wN^M}{D} \]
\[ f_c(4, 1) = -\alpha \frac{EY^T}{GDP} \]
\[ f_c(4, 2) = -(1 - \alpha) \frac{EY^T}{GDP} \]
\[ f_c(4, 5) = -\frac{\beta N Y^N}{GDP} \]
\[ f_c(4, 7) = -\frac{\beta N Y^N}{GDP} \]
\[ f_c(4, 10) = 1 \]
\[ f_c(5, 6) = \frac{\dot{w} N M}{M_2} \]
\[ f_c(5, 8) = \frac{\dot{w} N M}{M_2} \]
\[ f_c(5, 9) = -\frac{R}{M_0} \]
\[ f_c(6, 1) = -\alpha \frac{\dot{E} Y^T}{GDP} \]
\[ f_c(6, 2) = -(1 - \alpha) \frac{\dot{E} Y^T}{GDP} \]
\[ f_c(6, 5) = -\frac{\beta N Y^N}{GDP} \]
\[ f_c(6, 6) = \frac{\dot{w} N M}{M_2} \]
\[ f_c(6, 7) = -\frac{\beta N Y^N}{GDP} \]
\[ f_c(6, 8) = \frac{\dot{w} N M}{M_2} \]
\[ f_c(7, 1) = \alpha \frac{\dot{E} Y^T}{GDP} \]
\[ f_c(7, 2) = (1 - \alpha) \frac{\dot{E} Y^T}{GDP} \]
\[ f_c(7, 5) = \frac{\beta N Y^N}{GDP} \]
\[ f_c(7, 7) = \frac{\beta N Y^N}{GDP} - 1 + \gamma \]
\[ f_c(8, 7) = \gamma - 1 \]
\[ f_c(8, 8) = 1 \]
\[ f_c(9, 1) = \alpha \]
\[ f_c(9, 2) = 1 - \alpha \]
\[ f_c(10, 7) = \frac{1 + \pi p^N}{\pi P^N} \]
\[ f_c(11, 1) = \nu \]
\[ f_c(11, 2) = (1 - \alpha) \frac{\dot{E} Y^T}{ETB} \]
\[ f_c(11, 4) = -\frac{\dot{E} C^T}{ETB} \]
\[ f_c(12, 7) = 1 \]
\[ f_c(13, 6) = -\frac{\dot{w} N M}{EF} \]
\[ f_c(13, 8) = -\frac{\dot{w} N M}{EF} \]
\[ f_c(13, 9) = \frac{R}{EF} \]
\[ f_c(13, 10) = \frac{F}{EF} \]
\[ f_x(1, 1) = 1 \]
\[ f_x(2, 2) = \frac{S}{D} \]
\[ f_x(3, 1) = -1 \]
\[ f_x(3, 2) = \frac{S}{D} \]
\[ f_x(5, 1) = \frac{M}{M_2} - \frac{M}{M_0} \]
\[ f_x(5, 2) = \frac{S}{M_2} \]
\[ f_x(6, 1) = \frac{M}{M_2} \]
\[ f_x(6, 2) = \frac{S}{M_2} \]
\[ f_x(13, 2) = -\frac{S}{EF} \]
\[ f_e(1, 2) = -\gamma \frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(2, 2) = -\gamma \frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(4, 1) = -\frac{\dot{E} Y^T}{GDP} \]
\[ f_e(4, 2) = -\frac{\dot{E} Y^T}{GDP} \frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(6, 1) = -\frac{\dot{E} Y^T}{GDP} \]
\[ f_e(6, 2) = -\frac{\dot{E} Y^T}{GDP} \frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(7, 1) = \frac{\dot{E} Y^T}{GDP} \]
\[ f_e(7, 2) = \left( \frac{\dot{E} Y^T}{GDP} - \gamma \right) \frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(8, 2) = -\gamma \frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(9, 1) = 1 \]
\[ f_e(11, 1) = \frac{\dot{E} Y^T}{ETB} \]
\[ f_e(11, 2) = -\frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(12, 2) = -\frac{\varepsilon}{1 + \varepsilon} \]
\[ f_e(13, 2) = -\frac{\varepsilon}{1 + \varepsilon} \]
Net foreign liabilities, equation 4.54:

\[ N\tilde{F}L_{t+1} = (1 + r^*) \tilde{N}FL_t - r^* \tilde{F}_t \]

where

\[ \pi^{PN} = \left( \frac{1 + \pi}{1 + \varepsilon} \right) \frac{1}{\gamma} - 1 \]

\[ N^T = N^M \frac{1 - \alpha}{\frac{1}{\gamma} \text{ECT}_Y + 1 - \alpha} \]

\[ N^N = N^M - N^T \]

\[ \theta = \frac{R}{r^d - \frac{S}{L} + \frac{R}{L}} \]

\[ \varphi = -\frac{\frac{R}{D} + \psi \frac{TB}{F} + \frac{1}{c} \ln \frac{a(1 - \varphi)^{\frac{1}{a + b \varphi}}}{\frac{M_0}{M_2}}}{\frac{M_0}{M_2}} \]

\[ \theta = \frac{(1 + \pi - \beta) \frac{M}{D}}{1 + \pi - \beta (1 + r^d) + (1 + \pi - \beta) \frac{M}{D}} \]

\[ \Gamma = \frac{1}{N^M \frac{(\text{ECT}_{P^NCN})}{wN^M} - (1 - \theta) \frac{(\text{ECT}_{P^NCN})}{D}} \]

\[ \Theta = \frac{1}{\frac{(1 - \theta) N^M}{wN^M} - 1} \]

\[ \Lambda = \frac{\beta r^d (1 + \pi)}{(1 + \pi - \beta (1 + r^d))(1 + \pi - \beta)} \]

\[ \Delta = \frac{1}{1 + r^d + \frac{R}{L} - \frac{S}{L}} \]

\[ \Sigma = c \frac{(1 - \varphi)(a + b \varphi)}{(a + b \varphi) \varphi} \frac{TB}{F} \]

\[ \Phi = \frac{c (1 - \varphi)(a + b \varphi)}{(a + b \varphi)} \left( \psi \frac{TB}{F} \frac{wN^M}{EF} - \frac{R}{D} \frac{wN^M}{D} - \frac{\varphi M_0}{M_2} \frac{wN^M}{M_2} \right) \]

\[ \Psi = \frac{c (1 - \varphi)(a + b \varphi)}{(a + b \varphi)} \left( \frac{R}{D} + \varphi \frac{R}{M_2} - \psi \frac{TB}{F} \frac{R}{EF} \right) \]

\[ \Xi = \frac{c (1 - \varphi)(a + b \varphi)}{(a + b \varphi)} \varphi \frac{M}{M_2} \left( 1 - \frac{M_0}{M_2} \right) \]
\[ \gamma = \frac{c(1 - \varrho)(a + b\varrho)}{(a + b)\varrho} \left( \psi \frac{TB}{F} \frac{S}{EF} - \frac{R}{D} \frac{S}{D} - \varphi \frac{M_0}{M_2} \frac{S}{M_2} \right) \]

\[ \zeta = \frac{1}{1 - \varrho} \Sigma + \frac{1}{1 + \psi \frac{TB}{F} \frac{D}{EF}} \]

\[ \zeta = \frac{1}{1 - \psi \frac{TB}{nF} \frac{D}{EF}} \]

\[ \Omega = \frac{1}{1 - \varrho} \Phi - 2\kappa \frac{w N^M}{EF} - \kappa \frac{w N^M}{D} \]

\[ \psi = \frac{1 + b \varrho^2}{(1 + b \varrho)(1 - \varrho)} \]

\[ \vartheta = \psi \Sigma + 1 \]

\[ \iota = \frac{r^l}{r^l - r^d s_L + \frac{R}{L}} \]

\[ \kappa = \frac{r^d s_L}{r^l - r^d s_L + \frac{R}{L}} \]

\[ \omega = \frac{\frac{R}{L}}{r^l - r^d s_L + \frac{R}{L}} \]

\[ \nu = \alpha \frac{E Y^T}{ETB} - \frac{E K}{ETB} \]
Bibliography


ments*. IMF Staff Country Report No. 95/110: International Monetary Fund, Washington, D.C.


and Business Cycles: Technical Appendix.” *mimeo*, University of Rochester.


tional Capital Flows: The “Overborrowing Syndrome.” In *Financial Deregula-


Appendix A

Computer Program

This appendix presents a computer program written in MATLAB code which utilizes the solution algorithm described in Section 4.2 to compute the dynamic equilibrium and impulse response functions. This program is generic and can be used to solve numerically any model that has the general linear quadratic form described in Section 4.2.

%DYNAMIC SYSTEM IN NORMAL FORM

\[ w = -\text{inv}(\text{mss0}-\text{msc0}\text{*inv}(\text{mcc})\text{*mcs})\text{*}(\text{mss1}-\text{msc1}\text{*inv}(\text{mcc})\text{*mcs}); \]
\[ r = \text{inv}(\text{mss0}-\text{msc0}\text{*inv}(\text{mcc})\text{*mcs})\text{*}(\text{mse0}+\text{msc0}\text{*inv}(\text{mcc})\text{*mce}); \]
\[ q = \text{inv}(\text{mss0}-\text{msc0}\text{*inv}(\text{mcc})\text{*mcs})\text{*}(\text{mse1}+\text{msc1}\text{*inv}(\text{mcc})\text{*mce}); \]

%compute eigenvalues of w and verify they are real

[\text{p0,}\text{lamb}] = \text{eig}(w);
\text{lamb0} = \text{diag}(\text{lamb});

\text{if sum(\text{abs(}\text{imag(}\text{lamb0})) > 1e-10}
\text{disp('Error: some eigenvalues of the state-costate transition matrix are complex numbers')}\]

end

%sort eigenvalues in ascending order

\text{pr} = \text{real(}\text{p0});
\text{lambr} = \text{real(}\text{lamb0});
\text{alamb} = \text{abs(}\text{lambr});

[\text{alambsorted,}\text{lambz}] = \text{sort(}\text{alamb});
lambs = lambr(lambz,1);
lambda = diag(lambs);
p = pr(:,lambz);

%partition matrices lambda, p, r, and q
lamb1 = lambda(1:ns,1:ns);
lamb2 = lambda(ns+1:ns+ncs,ns+1:ns+ncs);
p11 = p(1:ns,1:ns);
p12 = p(1:ns,ns+1:ns+ncs);
p21 = p(ns+1:ns+ncs,1:ns);
p22 = p(ns+1:ns+ncs,ns+1:ns+ncs);
ps = inv(p);
ps11 = ps(1:ns,1:ns);
ps12 = ps(1:ns,ns+1:ns+ncs);
ps21 = ps(ns+1:ns+ncs,1:ns);
ps22 = ps(ns+1:ns+ncs,ns+1:ns+ncs);
rxe = r(1:ns,1:nex);
rlc = r(ns+1:ns+ncs,1:nex);
qxe = q(1:ns,1:nex);
q1c = q(ns+1:ns+ncs,1:nex);

%check transversality condition
for k = 1:ncs
    if abs(lamb2(k,k))>beta^(-0.5)
        disp('Transversality condition not satisfied')
    end
    k = k+1;
end
%generate matrices m and h
phi0 = ps21*rxe+ps22*rle;
phi1 = ps21*qxe+ps22*qle;
psi = zeros(ncs, nex);
for i = 1:ncs
    psi(i,:) = -(phi0(i,:)*pai+phi1(i,:))
                *inv(eye(nex)-pai/lamb2(i,i))/lamb2(i,i);
i = i+1;
end
xx = p11*lamb1*inv(p11);
x = (p11*lamb1*ps12+p12*lamb2*ps22)*inv(ps22)*psi+rxe*pai+qxe;
solx = [xx xe];
x = -inv(ps22)*ps21;
lex = inv(ps22)*psi;
soll = [lx lex];
cxl = inv(mcc)*mcs;
ce = inv(mcc)*mce;
solc = [[cxl*[eye(ns);lx]] [cxl*[zeros(ns, nex);lex]+ce]];
solf = [fx fe]+fc*solc;
m = [solx;[zeros(nex, ns) pai]];
h = [soll;solc;solf];