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Ascent Performance Feasibility for
Next-Generation Spacecraft

by

Salvatore Mancuso

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IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

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Abstract

Ascent Performance Feasibility for

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This thesis deals with the optimization of the ascent trajectories for single-stage suborbital (SSSO), single-stage-to-orbit (SSTO), and two-stage-to-orbit (TSTO) rocket-powered spacecraft. The maximum payload weight problem has been solved using the sequential gradient-restoration algorithm. For the TSTO case, some modifications to the original version of the algorithm have been necessary in order to deal with discontinuities due to staging and the fact that the functional being minimized depends on interface conditions.

The optimization problem is studied for different values of the initial thrust-to-weight ratio in the range 1.3 to 1.6, engine specific impulse in the range 400 to 500 sec, and spacecraft structural factor in the range 0.08 to 0.12. For the TSTO configuration, two subproblems are studied: uniform structural factor between stages and nonuniform structural factor between stages.

Due to the regular behavior of the results obtained, engineering approximations have been developed which connect the maximum payload weight to the engine specific impulse and spacecraft structural factor; in turn, this leads to useful design considerations.
Also, performance sensitivity to the scale of the aerodynamic drag is studied, and it is shown that its effect on payload weight is relatively small, even for drag changes approaching ± 50%.

The main conclusions are that: the design of a SSSO configuration appears to be feasible; the design of a SSTO configuration might be comfortably feasible, marginally feasible, or unfeasible, depending on the parameter values assumed; the design of a TSTO configuration is not only feasible, but its payload appears to be considerably larger than that of a SSTO configuration.

Improvements in engine specific impulse and spacecraft structural factor are desirable and crucial for SSTO feasibility; indeed, it appears that aerodynamic improvements do not yield significant improvements in payload weight.

Key Words. Flight mechanics, rocket-powered spacecraft, single-stage-to-orbit spacecraft, two-stage-to-orbit spacecraft, optimal trajectories, ascent trajectories.
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1. Introduction

After more than thirty years of development of multi-stage-to-orbit (MSTO) spacecraft, the natural continuation for a modern space program is the development of two-stage-to-orbit (TSTO) and then single-stage-to-orbit (SSTO) spacecraft (Refs. 1-3). The first step toward the latter goal is the development of single-stage suborbital (SSSO) rocket-powered spacecraft which must take-off vertically, reach given suborbital altitude and speed, and then land horizontally.

Within the above frame, this thesis deals with the optimization of the ascent trajectories for three different configurations: a SSSO configuration, an example of which is the Lockheed-Martin X-33 spacecraft; a SSTO configuration, an example of which is the Lockheed-Martin Venture Star spacecraft; and a TSTO configuration. Here, the optimization criterion is the maximum payload weight; realistic constraints are imposed on tangential acceleration, dynamic pressure, and heating rate.

The optimization is done employing the sequential gradient-restoration algorithm for optimal control problems (SGRA, Refs. 4-6), developed and perfected by the Aero-Astronautics Group of Rice University. SGRA has the major property of being a robust algorithm, and it has been employed with success to solve a wide variety of aerospace problems (Refs. 7-21), including interplanetary trajectories (Ref. 7), flight in windshear (Refs. 8-11), aerospace plane trajectories (Refs. 12-14), and aeroassisted orbital transfer (Refs. 15-21). For the TSTO case, some modifications to the original version of the algorithm have been necessary in order to deal with discontinuities due to staging and the fact that the functional being minimized depends on interface conditions.
For the SSO configuration, the maximum payload weight problem is studied for initial thrust-to-weight ratio $\sigma$ in the range 1.4 to 1.6, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and spacecraft structural factor $\varepsilon$ in the range 0.08 to 0.12. For the SSTO configuration, the maximum payload weight problem is studied for initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and spacecraft structural factor $\varepsilon$ in the range 0.08 to 0.12. For the TSTO configuration, the maximum payload weight problem is studied first for initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and uniform structural factor $\hat{\varepsilon} = \bar{\varepsilon}$ in the range 0.08 to 0.12, with $\hat{\varepsilon}$ denoting the structural factor of Stage 1 and $\bar{\varepsilon}$ denoting the structural factor of Stage 2; then, for a particular value of the initial thrust-to-weight ratio ($\sigma = 1.4$), the maximum payload weight problem is studied for nonuniform structural factor $\tilde{\varepsilon} = k\hat{\varepsilon}$, with $k \geq 1$, by varying $k$ and hence $\tilde{\varepsilon}$, while keeping $\hat{\varepsilon}$ fixed.

From the results obtained, engineering approximations are developed which connect the maximum payload weight to the engine specific impulse and spacecraft structural factor; in turn, this leads to useful design considerations. In particular, with reference to the $(I_{sp}, \varepsilon)$-domain, zero-payload lines are constructed separating feasibility from unfeasibility; thus, for given specific impulse, one can predict the maximum value of the structural factor that guarantees feasibility; conversely, for given structural factor, one can predict the minimum value of the specific impulse that guarantees feasibility.

With the above study completed, the investigation is then extended to assess the sensitivity of the results to the scale of the aerodynamic drag. The maximum payload
weight problem is solved again, for particular ($\sigma$, $l_{sp}$, $\varepsilon$)-combinations, assuming drag changes of $\pm 50\%$ with respect to that of the baseline configuration, while leaving the lift unchanged.

The organization of the thesis is as follows. In Sections 2-3, we present a summary of the computational algorithm employed, the sequential gradient-restoration algorithm, first for the continuous case (Section 2), and then for the discontinuous case (Section 3). In Section 4, we present the system description. In Section 5, we formulate the optimization problem and give results for single-stage suborbital configurations. In Section 6, we formulate the optimization problem and give results for single-stage-to-orbit configurations. In Section 7, we formulate the optimization problem and give results for two-stage-to-orbit configurations. Section 8 contains some design considerations. Section 9 assesses the effect of drag changes. Finally, Section 10 contains the conclusions.
2. Computational Algorithm: Continuous Case

The sequential gradient-restoration algorithm (SGRA) is an iterative technique, developed by Miele et al during the years 1970 to 1990, which includes a sequence of two-phase cycles, each composed of a gradient phase and a restoration phase. This technique is designed to achieve a decrease in the functional being minimized at the end of each cycle, while the constraints are satisfied to a predetermined accuracy. For SSSO and SSTO problems, the original SGRA algorithm can be used without change.

2.1. Problem Formulation. For the continuous case, it is convenient to normalize the interval of integration to unity; the actual final time \( \tau \) is treated as a component of a vector parameter \( \pi \) to be optimized. Therefore, the optimal control problem can be formulated as follows:

\[
\min I = \int_0^1 f(x,u,\pi,t)dt + [g(x,\pi)]_1,
\]  \( \quad (1) \)

s. t. \( \dot{x} = \phi(x,u,\pi,t), \)  \( \quad (2a) \)

\[
x_0 = \text{given},
\]  \( \quad (2b) \)

\[
[\psi(x, \pi)]_1 = 0.
\]  \( \quad (2c) \)

Here, \( I \) is the functional being minimized; \( t \) denotes the normalized time, \( 0 \leq t \leq 1 \); the dot superscript denotes derivative with respect to the normalized time; \( f \) and \( g \) are scalar functions, \( \phi \) is a \( n \)-vector function, and \( \psi \) is a \( q \)-vector function. The dependent variables are the \( n \)-vector state \( x(t) \), the \( m \)-vector control \( u(t) \), and the \( p \)-vector parameter \( \pi \). Any trajectory satisfying the constraints \( (2) \) is called a feasible trajectory.
2.2. First-Order Conditions. From calculus of variations, it is known that the previous problem is one of the Bolza type. It can be recast as

$$\min \ J = \int_0^1 [f + \lambda^T (x - \phi)] dt + [g + \mu^T \psi]_1,$$

s. t. \( \dot{x} = \phi \),

\( x_0 = \text{given} \),

\([\psi]_1 = 0 \).

Here, \( J \) is the augmented functional, the \( n \)-vector \( \lambda(t) \) is a variable Lagrange multiplier, and the \( q \)-vector \( \mu \) is a constant Lagrange multiplier.

After integration by parts, the first variation of the augmented functional can be written as

$$\delta J = \int_0^1 (f_x - \phi_x \lambda - \dot{\lambda})^T \Delta x dt + \int_0^1 (f_u - \phi_u \lambda)^T \Delta u dt$$

$$+ \left[ \int_0^1 (f_{\pi} - \phi_{\pi} \lambda) dt + (g_{\pi} + \psi_{\pi} \mu) \right]^T \Delta \pi + \left[ (g_x + \psi_x \mu + \lambda)^T \Delta x \right]_1.$$  \hspace{1cm} (5)

The optimal trajectory must satisfy not only the constraint equations (4) but also the first-order optimality conditions

$$\dot{\lambda} = f_{\tau} - \phi_{\tau} \lambda,$$ \hspace{1cm} (6a)

$$f_u - \phi_u \lambda = 0,$$ \hspace{1cm} (6b)

$$\int_0^1 (f_{\pi} - \phi_{\pi} \lambda) dt + (g_{\pi} + \psi_{\pi} \mu)_1 = 0,$$ \hspace{1cm} (6c)

$$(g_{\pi} + \psi_{\pi} \mu + \lambda)_1 = 0.$$ \hspace{1cm} (6d)

Satisfaction of (6) ensures the vanishing of the first variation (5) for any system of variations. Any trajectory satisfying (4), (6) is called an extremal trajectory.
2.3. Approximate Solutions. In general, the system (4), (6) is nonlinear, and sometimes strongly nonlinear. Therefore, approximate methods must be used to seek a solution iteratively.

Let the norm squared of a vector be defined as

\[ N(v) = v^T v. \]  

(7)

Then, under the assumption that the initial condition (4b) is satisfied at every stage of SGRA, the functionals

\[ P = \int_0^t N(\dot{x} - \phi) dt + N(\psi)_1, \]  

(8)

\[ Q = \int_0^t N(\dot{\lambda} - f_x + \phi_x \lambda) dt + \int_0^t N(f_u - \phi_u \lambda) dt \]

\[ + N\left[ \int_0^t (f_x - \phi_x \lambda) dt + (g_x + \psi_x \mu)_1 \right] + N(g_x + \psi_x \mu + \lambda)_1, \]  

(9)

represent the constraint error and the optimality condition error, respectively. For an exact optimal solution,

\[ P = 0, \]  

(10a)

\[ Q = 0. \]  

(10b)

For an approximation to the optimal solution,

\[ P \leq \zeta_1, \]  

(11a)

\[ Q \leq \zeta_2, \]  

(11b)

where \( \zeta_1 \) and \( \zeta_2 \) are small preselected numbers.

2.4. Sequential Gradient-Restoration Algorithm. As explained, SGRA involves cycles including a gradient phase and a restoration phase. The gradient phase is started
whenever Ineq. (11a) is satisfied and involves a single iteration. In this gradient iteration, the objective is to reduce the augmented functional $J$, while the constraints are satisfied to first order. The restoration phase is started whenever Ineq. (11a) is violated and involves one or several iterations. In each restorative iteration, the objective is to reduce the functional $P$, while the constraints are satisfied to first order and the norm squared of the variations of the control and the parameter is minimized. The restoration phase is terminated whenever Ineq. (11a) is satisfied. SGRA terminates when Ineqs. (11) are both satisfied.

2.5. System of Variations. For any iteration of the gradient phase or the restoration phase, the following terminology is adopted: $x(t)$, $u(t)$, $\pi$ denote the nominal functions; $x(t)$, $u(t)$, $\pi$ denote the varied functions; $\Delta x(t)$, $\Delta u(t)$, $\Delta \pi$ denote the perturbations of the nominal functions; $A(t)$, $B(t)$, $C$ denote the perturbations per unit stepsize $\alpha$. Therefore,

$$x(t) = x(t) + \Delta x(t) = x(t) + \alpha A(t), \quad (12a)$$

$$u(t) = u(t) + \Delta u(t) = u(t) + \alpha B(t), \quad (12b)$$

$$\pi = \pi + \Delta \pi = \pi + \alpha C. \quad (12c)$$

Concerning the functionals (3), (8), the terminology is as follows: $J$, $P$ denote the nominal functionals; $\mathcal{J}$, $\mathcal{P}$ denote the varied functionals; $\Delta J$, $\Delta P$ denote the total variations of these functionals. Therefore,

$$\mathcal{J} = J + \Delta J, \quad \mathcal{P} = P + \Delta P. \quad (13)$$

The perturbations per unit stepsize $A(t)$, $B(t)$, $C$ must be chosen so that, at each iteration,
either $\Delta J < 0$ or $\Delta P < 0$. \hfill (14)

To enforce (14), SGRA is constructed so that, at each iteration,

\begin{equation}
\text{either } \delta J < 0 \text{ or } \delta P < 0, \hfill (15)
\end{equation}

where $\delta J$ is given by (5) and $\delta P$ is given by

\begin{equation}
\delta P = 2\alpha \int_0^1 (\dot{x} - \phi)^T (\dot{A} - \phi_x^T A - \phi_u^T B - \phi_x^T C) dt
+ 2\alpha \left[ \psi_x^T A + \psi_x^T C \right]. \hfill (16)
\end{equation}

\textbf{2.6. Gradient Phase.} As explained, the gradient phase consists of a single iteration designed to decrease the augmented functional (3).

Suppose that the nominal functions $x(t)$, $u(t)$, $\pi$ satisfy the feasibility conditions (4). To first order, the perturbations per unit stepsize $A(t)$, $B(t)$, $C$ must satisfy the linearized constraint equations

\begin{equation}
\dot{A} - \phi_x^T A - \phi_u^T B - \phi_x^T C = 0, \hfill (17a)
\end{equation}

\begin{equation}
(A)_0 = 0, \hfill (17b)
\end{equation}

\begin{equation}
(\psi_x^T A + \psi_x^T C)_1 = 0. \hfill (17c)
\end{equation}

By inspection of (5), we see that $\delta J$ can be made negative through the following choice of variations per unit stepsize:

\begin{equation}
B = -(f_u - \phi_u \lambda), \hfill (18a)
\end{equation}

\begin{equation}
\begin{aligned}
C &= -\left[ \int_0^1 (f_x - \phi_x \lambda) dt + (g_x + \psi_x \mu)_1 \right], \hfill (18b)
\end{aligned}
\end{equation}

with $A(t)$ consistent with (17) and multipliers $\lambda(t)$, $\mu$ consistent with
\[ \dot{\lambda} = f_x - \phi_x \lambda, \]  
\[ (g_x + \psi_x \mu + \lambda)_1 = 0. \]  
(18c)  
(18d)

With this choice, the first variations of the functionals under consideration become

\[ \delta J = -\alpha Q, \quad \delta P = 0, \]  
(19)

with optimality condition error given by

\[ Q = \int_0^1 B^T B dt + C^T C. \]  
(20)

Since \( Q > 0 \), the first of Eqs. (19) shows that \( \delta J < 0 \). Hence, for \( \alpha \) sufficiently small, the decrease in the augmented functional \( J \) is guaranteed.

The system (17)-(18) is linear and nonhomogeneous in the unknowns \( A(t) \), \( B(t) \), \( C \) and \( \lambda(t) \), \( \mu \) and is independent of the stepsize \( \alpha \). The latter is to be determined a posteriori in such a way that the descent requirement \( \Delta J < 0 \) is enforced.

The linear two-point boundary-value problem (17)-(18) can be solved via the method of particular solutions (MPS). The number of integration required is \( n + p + 1 \) if a forward integration scheme is used and \( q + 1 \) if a backward-forward integration scheme is used. For details, see Ref. 4.

Once \( A(t) \), \( B(t) \), \( C \) and \( \lambda(t) \), \( \mu \), are known, one can form the one-parameter family of solutions (12) for which the augmented functional (3) and the constraint error (8) are functions of the form

\[ J = J(\alpha), \quad P = P(\alpha). \]  
(21)

Then, some one-dimensional search scheme must be employed, and \( \alpha \) must be selected in such a way that the following inequalities are satisfied:
\[ J(\alpha) < J(0), \]  
\[ P(\alpha) < P_* . \]  

Concerning (22a), its satisfaction is guaranteed by the descent property of the gradient phase. Concerning (22b), \( P_* \) is a preselected number, not necessarily small, which limits the constraint violation at the end of the gradient phase.

In summary, the gradient stepsiz e must be chosen so that Ineqs. (22) are satisfied. Should any violation occur, then a smaller value of \( \alpha \) must be employed and can be obtained with a bisection process, starting from a suitable chosen reference stepsiz e \( \alpha_0 \). In turn, \( \alpha_0 \) is obtained via a scanning process followed by cubic interpolation.

2.7. Restoration Phase. At the end of a gradient phase, the varied constraint error \( P \) can be computed with (8). If Ineq. (11a) is satisfied, then a new gradient phase is executed; otherwise, a restoration phase must be executed. In general, the restoration phase consists of several restorative iterations, each decreasing the constraint error until Ineq. (11a) is satisfied. When this occurs, the restoration phase ends and a new gradient phase is started.

Let \( x(t), u(t), \pi \) denote nominal functions violating Ineq. (11a). By inspection of (16), we see that \( \delta P \) can be made negative via variations per unit stepsize satisfying the linearized constraint equations

\[ \dot{A} - \phi_\pi^T A - \phi_\pi^T B - \phi_\pi^T C + (x - \phi) = 0, \]  

\[ (A)_0 = 0, \]
\[(\psi_x^\top A + \psi_x^\top C + \psi)_1 = 0.\] (23c)

Any solution \(A(t), B(t), C\) of Eqs. (23) is such that the first variation of the constraint error becomes
\[
\delta P = -2\alpha P. \tag{24}
\]

Since \(P > 0\), Eq. (24) shows that \(\delta P < 0\). Hence, for \(\alpha\) sufficiently small, the decrease in the constraint error is guaranteed.

Note that Eqs. (23) admit an infinite number of solutions. We render the solution unique by determining the perturbations per unit stepsize \(A(t), B(t), C\) so that the following functional is minimized:
\[
K = \int_0^1 B^\top B dt + C^\top C, \tag{25}
\]
subject to the linearized constraints (23). The solution of the ensuing linear-quadratic optimal control problem is characterized by the following variations per unit stepsize:
\[
B = \phi_u \lambda, \tag{26a}
\]
\[
C = \int_0^1 \phi_x \lambda dt - (\psi_x \mu)_1. \tag{26b}
\]

with \(A(t)\) consistent with (23) and multipliers \(\lambda(t), \mu\) consistent with
\[
\dot{\lambda} + \phi_x \lambda = 0, \tag{26c}
\]
\[
(\psi_x \mu + \lambda)_1 = 0. \tag{26d}
\]

The system (23), (26) is linear and nonhomogenous in \(A(t), B(t), C\) and \(\lambda(t), \mu\) and is independent of the stepsize \(\alpha\). The latter is to be determined a posteriori in such a way that the descent requirement \(\Delta P < 0\) is enforced.

The linear two-point boundary-value problem (23), (26) can be solved via the
method of particular solutions (MPS). For a restorative iteration, the number of integrations required is the same as for a gradient iteration. For details, see Ref. 4.

Once $A(t), B(t), C$ and $\lambda(t), \mu$ are known, one can form the one-parameter family of solutions (12) for which the constraint error (8) is a function of the form

$$ P = P(\alpha) . $$

(27)

Then, some one-dimensional search scheme must be employed, and $\alpha$ must be selected in such a way that the following inequality is satisfied:

$$ P(\alpha) < P(0) . $$

(28)

Should a violation occur, then a smaller value of $\alpha$ must be employed and can be obtained with a bisection process starting from $\alpha = 1$.

2.8. Gradient-Restoration Cycle. Let $I_1$ denote the value of the functional (1) at the beginning of the gradient phase; let $I_2$ denote the value of (1) at the end of the gradient phase; let $I_3$ denote the value of (1) at the end of the restoration phase. Note that $I_1$ and $I_2$ are not comparable, since the constraints are not satisfied to the same accuracy. On the other hand, $I_1$ and $I_3$ are comparable, since the constraints are satisfied to the same accuracy. For SGRA to be stable on a digital computer, one must require satisfaction of the inequality

$$ I_3 < I_1 . $$

(29)

If Ineq. (29) is satisfied, the gradient-restorative cycle is complete, and the next cycle is started. If Ineq. (29) is violated, one must return to the previous gradient phase and reduce the gradient stepsize until, after restoration, Ineq. (29) is satisfied. This property is guaranteed in SGRA.
2.9. Summary of the Algorithm. For the continuous case, SGRA can be summarized via Steps 1 to 4 below.

Step 1. Assume nominal functions \( x(t), u(t), \pi \) consistent with the initial conditions (4b). Compute the constraint error via (8). If \( P \) satisfies Ineq. (11a), go to Step 2. If \( P \) violates Ineq. (11a) go to Step 3.

Step 2. Gradient Phase. This phase consists of a single iteration described by Steps 2a to 2e below.

Step 2a. Compute \( (f_x, f_u, f_\pi), (\phi_x, \phi_u, \phi_\pi), (g_x, g_\pi)_1, (\psi_x, \psi_u, \psi_\pi)_1 \).

Step 2b. Solve the linear two-point boundary-value problem (17)-(18) with the method of particular solutions. Obtain \( A(t), B(t), C \) and \( \lambda(t), \mu \).

Step 2c. Compute the optimality condition error via (20). If \( Q \) satisfies Ineq. (11b), stop; a solution has been found and SGRA terminates. If \( Q \) violates Ineq. (11b), go to Step 2d.

Step 2d. Compute the gradient stepsize by a one-dimensional search on the augmented functional \( J(\alpha) \). First, determine the reference stepsize \( \alpha_0 \) via a scanning process followed by a cubic interpolation process. Then, perform a bisection process on \( \alpha \), starting from \( \alpha_0 \), until Ineqs. (22) are both satisfied.

Step 2e. Once the gradient stepsize is known, compute the varied functions \( x(t), u(t), \pi \) via (12). Return to Step 1 by setting the new nominal functions \( x(t), u(t), \pi \) equal to the varied functions just computed.

Step 3. Restoration Phase. This phase consists of several restorative iterations, each described by Steps 3a to 3d below.
Step 3a. Compute \( \dot{x} = \phi \cdot (\phi_\lambda, \phi_\mu, \phi_\pi), (\psi, \psi_\lambda, \psi_\mu) \).

Step 3b. Solve the linear two-point boundary-value problem (23), (26) with the method of particular solutions. Obtain \( A(t), B(t), C \) and \( \lambda(t), \mu \).

Step 3c. Compute the restoration stepsize by a one-dimensional search on the constraint error \( P(\alpha) \). Perform a bisection process on \( \alpha \), starting from \( \alpha = 1 \), until Ineq. (28) is satisfied.

Step 3d. Once the restoration stepsize is known, compute the varied functions \( x(t), u(t), \pi \) via (12). Compute the constraint error via (8). If \( P \) violates Ineq. (11a), return to Step 3a by setting the new nominal functions \( x(t), u(t), \pi \) equal to the varied functions just computed. If \( P \) satisfies Ineq. (11a), stop; the restoration phase ends. Go to Step 2 if the restoration phase just ended is part of an incomplete gradient-restoration cycle (this can only occur at the beginning of SGRA); otherwise, go to Step 4.

Step 4. Complete Gradient-Restoration Cycle. For each complete gradient-restoration cycle, verify whether Ineq. (29) is satisfied, that is, whether the value of the functional \( I \) at the end of the cycle is smaller than that at the beginning of the cycle. If Ineq. (29) is satisfied, go to Step 2. If Ineq. (29) is violated, return to Step 2d of the previous gradient iteration and bisect the gradient stepsize as many times as needed until, after restoration, Ineq. (29) is satisfied. Then, go to Step 2.
3. Computational Algorithm: Discontinuous Case

For TSTO problems, algorithmic modifications to SGRA are necessary to account for state variable discontinuities due to staging and the fact that the functional being minimized depends on interface conditions.

3.1. Problem Formulation. For the discontinuous case, it is convenient to normalize the interval of integration of each subarc to unity; the actual time lengths of the first subarc \( \bar{\tau} \) and the second subarc \( \bar{\tau} \) are treated as components of a vector parameter \( \pi \) to be optimized; the final time is given by \( \tau = \bar{\tau} + \bar{\tau} \). Therefore, the optimal control problem can be formulated as follows:

\[
\min L = \int_0^2 f(x, u, \pi, t) dt + [h(x_-, x_+, \pi)]_1 + [g(x, \pi)]_2 ,
\]

subject to:

\[
x = \phi(x, u, \pi, t), \tag{31a}
\]

\[
x_0 = \text{given}, \tag{31b}
\]

\[
[\gamma(x_-, x_+, \pi)]_1 = 0, \tag{31c}
\]

\[
[\psi(x, \pi)]_2 = 0 \tag{31d}
\]

Here, \( L \) is the functional being minimized; \( \tau \) denotes the normalized time, \( 0 \leq \tau \leq 1 \) for the first subarc and \( 1 \leq \tau \leq 2 \) for the second subarc; the dot superscript denotes derivative with respect to the normalized time; \( f, h, g \) are scalar functions, \( \phi \) and \( \gamma \) are \( n \)-vector functions, and \( \psi \) is a \( q \)-vector function; the subscript \(-\) denotes a quantity immediately before the discontinuity, and the subscript \(+\) denotes a quantity immediately after the discontinuity. The dependent variables are the \( n \)-vector state \( x(t) \), the \( m \)-vector control \( u(t) \), and the
\( p \)-vector parameter \( \pi \). Any trajectory satisfying the constraints (31) is called a feasible trajectory.

3.2. First-Order Conditions. From calculus of variations, it is known that the previous problem is one of the Bolza type. It can be recast as

\[
\min J = \int_0^1 \left[ f + x^T (x - \phi) \right] dt + [h + v^T \gamma]_1 + [g + \mu^T \psi]_2,
\]

s. t. \( \dot{x} = \phi \), \hspace{1cm} (33a)
\( x_0 = \text{given}, \) \hspace{1cm} (33b)
\( [\gamma]_1 = 0, \) \hspace{1cm} (33c)
\( [\psi]_2 = 0. \) \hspace{1cm} (33d)

Here, \( J \) is the augmented functional, the \( n \)-vector \( \lambda(t) \) is a variable Lagrange multiplier, the \( n \)-vector \( v \) and the \( q \)-vector \( \mu \) are constant Lagrange multipliers.

After integration by parts, the first variation of the augmented functional can be written as

\[
\delta J = \int_0^1 (f_x - \phi_x \lambda - \dot{\lambda})^T \Delta x dt + \int_0^1 (f_u - \phi_u \lambda)^T \Delta u dt
\]
\[
+ \left[ \int_0^1 (f_x - \phi_x \lambda) dt + (h_x + \gamma \nu)_1 + (g_x + \psi_x \mu)_2 \right]^T \Delta \pi + \left[ (h_{x-} + \gamma_{x-} \nu + \lambda_-) \Delta x_+ \right]_1
\]
\[
+ \left[ (h_{x+} + \gamma_{x+} \nu - \lambda_+) \Delta x_- \right]_1 + [(g_x + \psi_x \mu + \lambda) \Delta x]_2.
\]

(34)

The optimal trajectory must satisfy not only the constraint equations (33) but also the first-order optimality conditions

\[
\dot{\lambda} = f_x - \phi_x \lambda,
\]

(35a)
\[ f_u - \phi_u \lambda = 0 , \quad (35b) \]
\[ \int_0^2 (f_x - \phi_x \lambda) dt + (h_x + \gamma_x \nu)_1 + (g_x + \psi_x \mu)_2 = 0 , \quad (35c) \]
\[ (h_{x-} + \gamma_{x-} \nu + \lambda_1)_1 = 0 , \quad (35d) \]
\[ (h_{x+} + \gamma_{x+} \nu - \lambda_1)_1 = 0 , \quad (35e) \]
\[ (g_x + \psi_x \mu + \lambda)_2 = 0 . \quad (35f) \]

Satisfaction of (35) ensures the vanishing of the first variation (34) for any system of variations. Any trajectory satisfying (33), (35) is called an extremal trajectory.

### 3.3. Approximate Solutions

In general, the system (33), (35) is nonlinear, and sometimes strongly nonlinear. Therefore, approximate methods must be used to seek a solution iteratively.

Let the norm squared of a vector be defined as
\[ N(\nu) = \nu^T \nu . \quad (36) \]

Then, under the assumption that the initial condition (33b) is satisfied at every stage of SGRA, the functionals
\[ P = \int_0^2 N(\dot{x} - \dot{\phi}) dt + N(\gamma)_1 + N(\psi)_2 , \quad (37) \]
\[ Q = \int_0^2 N(\dot{\lambda} - f_x + \phi_x \lambda) dt + \int_0^2 N(f_u - \phi_u \lambda) dt \]
\[ + N\left[ \int_0^2 (f_x - \phi_x \lambda) dt + (h_x + \gamma_x \nu)_1 + (g_x + \psi_x \mu)_2 \right] \]
\[ + N(h_{x-} + \gamma_{x-} \nu + \lambda_1)_1 + N(h_{x+} + \gamma_{x+} \nu - \lambda_1)_1 + N(g_x + \psi_x \mu + \lambda)_2 . \quad (38) \]

represent the constraint error and the optimality condition error, respectively. For an exact optimal solution,
\[ P = 0, \] \hspace{1cm} (39a)
\[ Q = 0. \] \hspace{1cm} (39b)

For an approximation to the optimal solution,
\[ P \leq \zeta_1, \] \hspace{1cm} (40a)
\[ Q \leq \zeta_2, \] \hspace{1cm} (40b)

where \( \zeta_1 \) and \( \zeta_2 \) are small preselected numbers.

3.4. Sequential Gradient-Restoration Algorithm. As explained, SGRA involves cycles including a gradient phase and a restoration phase. The gradient phase is started whenever Ineq. (40a) is satisfied and involves a single iteration. In this gradient iteration, the objective is to reduce the augmented functional \( J \), while the constraints are satisfied to first order. The restoration phase is started whenever Ineq. (40a) is violated and involves one or several iterations. In each restorative iteration, the objective is to reduce the functional \( P \), while the constraints are satisfied to first order and the norm squared of the variations of the control and the parameter is minimized. The restoration phase is terminated whenever Ineq. (40a) is satisfied. SGRA terminates when Ineqs. (40) are both satisfied.

3.5. System of Variations. For any iteration of the gradient phase or the restoration phase, the following terminology is adopted: \( x(t), u(t), \pi \) denote the nominal functions; \( \Delta x(t), \Delta u(t), \Delta \pi \) denote the perturbations of the nominal functions; \( A(t), B(t), C \) denote the perturbations per unit stepsize \( \alpha \). Therefore,
\[ x(t) = x(t) + \Delta x(t) = x(t) + \alpha A(t), \quad (41a) \]

\[ u(t) = u(t) + \Delta u(t) = u(t) + \alpha B(t), \quad (41b) \]

\[ \pi = \pi + \Delta \pi = \pi + \alpha C. \quad (41c) \]

Concerning the functionals (32), (37), the terminology is as follows: \( J, P \) denote the nominal functionals; \( \tilde{J}, \tilde{P} \) denote the varied functionals; \( \Delta J, \Delta P \) denote the total variations of these functionals. Therefore,

\[ \tilde{J} = J + \Delta J, \quad \tilde{P} = P + \Delta P. \quad (42) \]

The perturbations per unit stepsize \( A(t), B(t), C \) must be chosen so that, at each iteration,

either \( \Delta J < 0 \) or \( \Delta P < 0. \quad (43) \]

To enforce (43), SGRA is constructed so that, at each iteration,

either \( \delta J < 0 \) or \( \delta P < 0, \quad (44) \]

where \( \delta J \) is given by (34) and \( \delta P \) is given by

\[
\delta P = 2\alpha \int_0^T (\dot{x} - \phi) (\dot{A} - \phi^T A - \phi^T B - \phi^T C) dt
+ 2\alpha \left[ \gamma^T (\gamma_{x-} A(-) + \gamma_{x+} A(+) + \gamma_{x}^T C) \right]_i + 2\alpha \left[ \psi^T (\psi_x^T A + \psi_x^T C) \right]_i. \quad (45)
\]

### 3.6. Gradient Phase.

As explained, the gradient phase consists of a single iteration designed to decrease the augmented functional (32).

Suppose that the nominal functions \( x(t), u(t), \pi \) satisfy the feasibility conditions (33). To first order, the perturbations per unit stepsize \( A(t), B(t), C \) must satisfy the linearized constraint equations

\[ \dot{A} - \phi_x^T A - \phi_u^T B - \phi_{\pi}^T C = 0, \quad (46a) \]
\[(A)_0 = 0, \quad (46b)\]
\[(\gamma^T A_+ + \gamma^T A_+ + \gamma^T C)_1 = 0, \quad (46c)\]
\[(\psi^T A + \psi^T C)_2 = 0. \quad (46d)\]

By inspection of (34), we see that \(\delta J\) can be made negative through the following choice of variations per unit stepsize:

\[B = -(f_u - \phi_u \lambda), \quad (47a)\]
\[C = -\left[\int_0^t (f_x - \phi_x \lambda)dt + (h_x + \gamma_x \nu)_1 + (g_x + \psi_x \mu)_2\right], \quad (47b)\]

with \(A(t)\) consistent with (46) and multipliers \(\lambda(t), \nu, \mu, \mu\) consistent with

\[\dot{\lambda} = f_x - \phi_x \lambda, \quad (47c)\]
\[(h_x + \gamma_x \nu + \lambda)_1 = 0, \quad (47d)\]
\[(h_x + \gamma_x \nu - \lambda)_1 = 0, \quad (47e)\]
\[(g_x + \psi_x \mu + \lambda)_2 = 0. \quad (47f)\]

With this choice, the first variations of the functionals under considerations become

\[\delta J = -\alpha Q, \quad \delta P = 0, \quad (48)\]

with optimality condition error given by

\[Q = \int_0^t \dot{B}^T B dt + C^T C. \quad (49)\]

Since \(Q > 0\), the first of Eqs. (48) shows that \(\delta J < 0\). Hence, for \(\alpha\) sufficiently small, the decrease in the augmented functional \(J\) is guaranteed.

The system (46)-(47) is linear and nonhomogeneous in the unknowns \(A(t), B(t), C\) and \(\lambda(t), \mu, \nu\) and is independent of the stepsize \(\alpha\). The latter is to be determined a posteriori in such a way that the descent requirement \(\Delta J < 0\) is enforced.
The linear two-point boundary-value problem (46)-(47) can be solved via the method of particular solutions (MPS). The number of integration required is \( n + p + 1 \) if a forward integration scheme is used and \( q + 1 \) if a backward-forward integration scheme is used. However, in this case integration must be stopped at \( t = 1 \) and then restarted after having used the linearized interface condition (46c), (47d), (47e).

Once \( A(t), B(t), C \) and \( \lambda(t), v, \mu \) are known, one can form the one-parameter family of solutions (41) for which the augmented functional (32) and the constraint error (37) are functions of the form

\[
I = I(\alpha), \quad P = P(\alpha).
\]

Then, some one-dimensional search scheme must be employed, and \( \alpha \) must be selected in such a way that the following inequalities are satisfied:

\[
I(\alpha) < I(0), \tag{51a}
\]

\[
P(\alpha) < P_*. \tag{51b}
\]

Concerning (51a), its satisfaction is guaranteed by the descent property of the gradient phase. Concerning (51b), \( P_* \) is a preselected number, not necessarily small, which limits the constraint violation at the end of the gradient phase.

In summary, the gradient stepsize must be chosen so that Ineqs. (51) are satisfied. Should any violation occur, then a smaller value of \( \alpha \) must be employed and can be obtained with a bisection process, starting from a suitable chosen reference stepsize \( \alpha_0 \). In turn, \( \alpha_0 \) is obtained via a scanning process followed by cubic interpolation.
3.7. Restoration Phase. At the end of a gradient phase, the varied constraint error \( P \) can be computed with (37). If Ineq. (40a) is satisfied, then a new gradient phase is executed; otherwise, a restoration phase must be executed. In general, the restoration phase consists of several restorative iterations, each decreasing the constraint error until Ineq. (40a) is satisfied. When this occurs, the restoration phase ends and a new gradient phase is started.

Let \( x(t), u(t), \pi \) denote nominal functions violating Ineq. (40a). By inspection of (45), we see that \( \delta P \) can be made negative via variations per unit stepsize satisfying the linearized constraint equations

\[
\dot{\phi}_x^T A - \phi_x^T B - \phi_{\pi}^T C + (\dot{x} - \phi) = 0, \tag{52a}
\]

\[
(A)_0 = 0, \tag{52b}
\]

\[
(\gamma_{\pi}^T A + \gamma_{\pi}^T B + C + \gamma)_1 = 0, \tag{52c}
\]

\[
(\psi_{\pi}^T A + \psi_{\pi}^T C + \psi)_2 = 0. \tag{52d}
\]

Any solution \( A(t), B(t), C \) of Eqs. (52) is such that the first variation of the constraint error becomes

\[
\delta P = -2\alpha P. \tag{53}
\]

Since \( P > 0 \), Eq. (53) shows that \( \delta P < 0 \). Hence, for \( \alpha \) sufficiently small, the decrease in the constraint error \( P \) is guaranteed.

Note that Eqs. (52) admit an infinite number of solutions. We render the solution unique by determining the perturbations per unit stepsize \( A(t), B(t), C \) so that the following functional is minimized:
\[ K = \int_0^2 B^T B dt + C^T C , \]  

(54)

subject to the linearized constraints (52). The solution of the ensuing linear-quadratic optimal control problem is characterized by the following variations per unit stepsize:

\[ B = \phi_u \lambda , \]  

(55a)

\[ C = \int_0^2 \phi_x \lambda dt - (\gamma_x \nu)_1 - (\psi_x \mu)_2 , \]  

(55b)

with \( A(t) \) consistent with (52) and multipliers \( \lambda(t) \), \( \nu, \mu \) consistent with

\[ \dot{\lambda} + \phi_x \lambda = 0 , \]  

(55c)

\[ (\gamma_x \nu + \lambda)_1 = 0 , \]  

(55d)

\[ (\gamma_x \nu - \lambda)_1 = 0 , \]  

(55e)

\[ (\psi_x \mu + \lambda)_2 = 0 , \]  

(55f)

The system (52), (55) is linear and nonhomogenous in \( A(t) \), \( B(t) \), \( C \) and \( \lambda(t) \), \( \nu, \mu \) and is independent of the stepsize \( \alpha \). The latter is to be determined a posteriori in such a way that the descent requirement \( \Delta P < 0 \) is enforced.

The linear two-point boundary-value problem (52), (55) can be solved via the method of particular solutions (MPS). For a restorative iteration, the number of integration required is the same as for a gradient iteration. In this case, integration must be stopped at \( t = 1 \) and then restarted after having used the linearized interface condition (52c), (55d), (55e).

Once \( A(t) \), \( B(t) \), \( C \) and \( \lambda(t) \), \( \nu, \mu \) are known, one can form the one-parameter family of solutions (41) for which the constraint error (37) is a function of the form

\[ P = P(\alpha) . \]  

(56)
Then, some one-dimensional search scheme must be employed, and $\alpha$ must be selected in such a way that the following inequality is satisfied:

$$P(\alpha) < P(0).$$  \hspace{1cm} (57)

Should a violation occur, then a smaller value of $\alpha$ must be employed and can be obtained with a bisection process starting from $\alpha = 1$.

### 3.8. Gradient-Restoration Cycle

Let $I_1$ denote the value of the functional (30) at the beginning of the gradient phase; let $I_2$ denote the value of (30) at the end of the gradient phase; let $I_3$ denote the value of (30) at the end of the restoration phase. Note that $I_1$ and $I_2$ are not comparable, since the constraints are not satisfied to the same accuracy. On the other hand, $I_1$ and $I_3$ are comparable, since the constraints are satisfied to the same accuracy. For SGRA to be stable on a digital computer, one must require satisfaction of the inequality

$$I_3 < I_1.$$  \hspace{1cm} (58)

If Ineq. (58) is satisfied, the gradient-restoration cycle is complete, and the next cycle is started. If Ineq. (58) is violated, one must return to the previous gradient phase and reduce the gradient stepsize until, after restoration, Ineq. (58) is satisfied. This property is guaranteed in SGRA.

### 3.9. Summary of the Algorithm

For the discontinuous case, SGRA can be summarized via Steps 1 to 4 below.

Step 1. Assume nominal functions $x(t)$, $u(t)$, $\pi$ consistent with the initial conditions (33b). Compute the constraint error via (37). If $P$ satisfies Ineq. (40a), go to
Step 2. If $P$ violates Ineq. (40a) go to Step 3.

Step 2. Gradient Phase. This phase consists of a single iteration described by Steps 2a to 2e below.

Step 2a. Compute $(f_x, f_u, f_x), (\phi_x, \phi_u, \phi_x), (h_x, h_x, h_x)_1$, $(\gamma, \gamma_x, \gamma_x, \gamma_x)_1, (g_x, g_x)_2$, $(\psi, \psi_x, \psi_x)_2$.

Step 2b. Solve the linear two-point boundary-value problem (46)-(47) with the method of particular solutions. Obtain $A(t), B(t), C$ and $\lambda(t), \nu, \mu$.

Step 2c. Compute the optimality condition error via (49). If $Q$ satisfies Ineq. (40b), stop; a solution has been found and SGRA terminates. If $Q$ violates Ineq. (40b), go to Step 2d.

Step 2d. Compute the gradient stepsize by a one-dimensional search on the augmented functional $J(\alpha)$. First, determine the reference stepsize $\alpha_0$ via a scanning process followed by a cubic interpolation process. Then, perform a bisection process on $\alpha$, starting from $\alpha_0$, until Ineqs. (51) are both satisfied.

Step 2e. Once the gradient stepsize is known, compute the varied functions $x(t), u(t), \pi$ via (41). Return to Step 1 by setting the new nominal functions $x(t), u(t), \pi$ equal to the varied functions just computed.

Step 3. Restoration Phase. This phase consists of several restorative iterations, each described by Steps 3a to 3d below.

Step 3a. Compute $\dot{x} - \phi, (\phi_x, \phi_u, \phi_x), (\gamma, \gamma_x, \gamma_x, \gamma_x)_1, (\psi, \psi_x, \psi_x)_2$.

Step 3b. Solve the linear two-point boundary-value problem (52), (55) with the method of particular solutions. Obtain $A(t), B(t), C$ and $\lambda(t), \nu, \mu$. 
Step 3c. Compute the restoration stepsize by a one-dimensional search on the constraint error $P(\alpha)$. Perform a bisection process on $\alpha$, starting from $\alpha = 1$, until Ineq. (57) is satisfied.

Step 3d. Once the restoration stepsize is known, compute the varied functions $x(t), u(t), \pi$ via (41). Compute the constraint error via (37). If $P$ violates Ineq. (40a), return to Step 3a by setting the new nominal functions $x(t), u(t), \pi$ equal to the varied functions just computed. If $P$ satisfies Ineq. (40a), stop: the restoration phase ends. Go to Step 2 if the restoration phase just ended is part of an incomplete gradient-restoration cycle (this can only occur at the beginning of SGRA); otherwise, go to Step 4.

Step 4. Complete Gradient-Restoration Cycle. For each complete gradient-restoration cycle, verify whether Ineq. (58) is satisfied, that is, whether the value of the functional $I$ at the end of the cycle is smaller than that at the beginning of the cycle. If Ineq. (58) is satisfied, go to Step 2. If Ineq. (58) is violated, return to Step 2d of the previous gradient iteration and bisect the gradient stepsize as many times as needed until, after restoration, Ineq. (58) is satisfied. Then, go to Step 2.
4. System Description

In this thesis, three configurations are studied: a single-stage suborbital (SSSO), a single-stage-to-orbit (SSTO), and a two-stage-to-orbit (TSTO) rocket-powered spacecraft. The following assumptions are employed: (A1) the flight takes place in a vertical plane over a spherical Earth; (A2) the Earth rotation is neglected; (A3) the gravitational field is central and obeys the inverse square law; (A4) the thrust is directed along the spacecraft reference line; hence, the thrust angle of attack is the same as the aerodynamic angle of attack; (A5) the spacecraft is controlled via the angle of attack and power setting.

4.1. Differential System. With the above assumptions, the motion of the spacecraft is described by the following differential system for the altitude $h$, velocity $V$, flight path angle $\gamma$, and reference weight $W$ (Ref. 22):

\[ \dot{h} = V \sin \gamma , \]  
\[ \dot{V} = (T / W) g_e \cos \alpha - (D / W) g_e - g \sin \gamma , \]  
\[ \dot{\gamma} = (T g_e / WV) \sin \alpha - (L g_e / WV) + (V / r - g / V) \cos \gamma , \]  
\[ \dot{W} = -T / I_{sp} , \]

in which the dot denotes derivative with respect to the time $t$. Here, $0 \leq t \leq \tau$, where $\tau$ is the final time. The quantities appearing on the right-hand side of (59) are the thrust $T$, drag $D$, lift $L$, radial distance $r$, local acceleration of gravity $g$, sea level acceleration of gravity $g_e$, angle of attack $\alpha$, and engine specific impulse $I_{sp}$. 
4.2. Functional Relations. In the system (59), the radius and local acceleration of gravity depend on the altitude via the relations

\begin{align*}
    r &= r_e + h, \quad (60a) \\
    g &= \frac{\mu}{r^2} = \frac{\mu}{(r_e + h)^2}, \quad (60b)
\end{align*}

where \( r_e \) is the Earth radius and \( \mu = ge^2 \) the Earth gravitational constant. Also, the specific impulse is related to the exit velocity of the gases \( V_{ex} \) by the simple expression

\begin{equation}
    I_{sp} = \frac{V_{ex}}{g_e}. \quad (60c)
\end{equation}

The reference weight is the so-called sea-level weight

\begin{equation}
    W = mg_e. \quad (60d)
\end{equation}

which is the product of the instantaneous mass \( m \) and the sea-level acceleration of gravity \( g_e \). The reference weight differs from the local weight

\begin{equation}
    W' = mg, \quad (60e)
\end{equation}

which is the product of the instantaneous mass \( m \) and the local acceleration of gravity \( g \).

Generally speaking, the aerodynamic forces can be represented by the functional relations

\begin{align*}
    D &= D(h, V, \alpha), \quad (61a) \\
    L &= L(h, V, \alpha), \quad (61b)
\end{align*}

with \( \alpha \) the angle of attack. The powerplant performance is described by the functional relations

\begin{align*}
    T &= T(h, V, \beta), \quad (62a) \\
    I_{sp} &= I_{sp}(h, V, \beta), \quad (62b)
\end{align*}

with \( \beta \) the power setting.
4.3. Inequality Constraints. Inspection of the system (59) in light of (60)-(62) shows that the time history of the state $h(t)$, $V(t)$, $\gamma(t)$, $W(t)$ can be computed by forward integration for given initial conditions, given controls $\alpha(t)$ and $\beta(t)$, and given final time $\tau$. In turn, the controls are subject to the two-sided inequality constraints

$$\alpha_L \leq \alpha \leq \alpha_U, \quad (63a)$$

$$\beta_L \leq \beta \leq \beta_U. \quad (63b)$$

which must be satisfied everywhere along the interval of integration. These inequality constraints can be accounted for via the trigonometric transformations

$$\alpha = (1/2)(\alpha_L + \alpha_U) + (1/2)(\alpha_U - \alpha_L) \sin x, \quad (64a)$$

$$\beta = (1/2)(\beta_L + \beta_U) + (1/2)(\beta_U - \beta_L) \sin y, \quad (64b)$$

where $x$ and $y$ are regarded as auxiliary control variables, providing the values of $\alpha$, $\beta$ are not fixed at any point of the interval of integration.

If the values of $\alpha$ and $\beta$ are fixed at the initial point and/or at the final point (this is the case with the particular problem under consideration), then $\alpha$, $\beta$ and for that matter $x$, $y$ must be treated as auxiliary state variables. This is achieved via the supplementary transformations

$$\dot{x} = u, \quad (65a)$$

$$\dot{y} = w, \quad (65b)$$

or

$$\dot{x} = u, \quad (66a)$$

$$\dot{y} = w^2, \quad (66b)$$

with $x$, $y$ auxiliary state variables and $u$, $w$ auxiliary control variables. The representation
(65) allows $\alpha$ and $\beta$ to increase or decrease as needed along the trajectory. The representation (66) allows $\alpha$ to increase or decrease as needed, but forces $\beta$ to decrease monotonically along the trajectory; see the boundary conditions (78f) and (79f) for SSSO spacecraft, (90f) and (91f) for SSTO spacecraft, (103f) and (104f) for TSTO spacecraft. We employ (66) preferentially to (65) so as to force the power setting to decrease monotonically along the trajectory.

4.4. Aerodynamic Forces. The aerodynamic forces (61) are given by

$$D = (1/2)C_D \rho(h)SV^2,$$

$$L = (1/2)C_L \rho(h)SV^2,$$  \hspace{1cm} (67a) \hspace{1cm} (67b)

where $C_D$ is the drag coefficient, $C_L$ the lift coefficient, $S$ a reference surface area, and $\rho$ the air density (Ref. 23). Disregarding the dependence on the Reynolds number, the aerodynamic coefficients can be represented via polynomial functions of the angle of attack, specifically,

$$C_D = A_0(M) + A_1(M)\alpha + A_2(M)\alpha^2,$$$$

$$C_L = B_0(M) + B_1(M)\alpha + B_2(M)\alpha^2,$$  \hspace{1cm} (68a) \hspace{1cm} (68b)

with coefficients $A_i(M)$ and $B_i(M)$ depending only on the Mach number $M = V/a(h)$, where $a$ is the speed of sound. These coefficients are computed via least-square fit of available aerodynamic data at various Mach numbers and angles of attack (Refs. 12-14). Here, the assumptions $A_1 = 0$, $B_2 = 0$ are employed; the remaining coefficients $A_0$, $A_2$ and $B_0$, $B_1$ are shown in Figs. 1-4 versus the Mach number.

As explained, the angle of attack is subject to Ineq. (63a). For the present
configuration, the bounds in (63a) are given by

$$\alpha_L = -3.0 \text{ deg,} \quad \alpha_U = +3.0 \text{ deg,} \quad (69a)$$

hence

$$-3.0 \leq \alpha \leq +3.0 \text{ deg,} \quad (69b)$$

with the implication that (64a) and (66a) become

$$\alpha = K \sin x, \quad K = 3 \text{ deg} = 0.05236 \text{ rad,} \quad (70a)$$

$$x = u, \quad (70b)$$

with \(x\) an auxiliary state variable and \(u\) an auxiliary control variable.

4.5. Powerplant Model. For the rocket powerplant under consideration, the following simplified form of Eqs. (62) is assumed:

$$T = \beta T_*, \quad (71a)$$

$$I_{sp} = I_{sp*}, \quad (71b)$$

where \(\beta\) is the power setting, \(T_*\) a reference thrust (thrust for \(\beta = 1\), and \(I_{sp*}\) a reference specific impulse. The fact that \(T_*\) and \(I_{sp*}\) are assumed to be constant means that the weak dependence of \(T\) and \(I_{sp}\) on altitude and Mach number, relevant to a precision study, is disregarded within the present feasibility study.

As explained, the power setting is subject to Ineq. (63b). For the present configuration, the bounds in (63b) are assumed to be

$$\beta_L = 0, \quad \beta_U = 1, \quad (72a)$$

hence
0 \leq \beta \leq 1, \quad (72b)

implying that \( T_0 \) is the same as the maximum thrust and further implying that Eqs. (64b) and (66b) become

\[
\begin{align*}
\beta &= (1/2) (1 + \sin y) , \quad (73a) \\
\dot{y} &= w^2 , \quad (73b)
\end{align*}
\]

with \( y \) an auxiliary state variable and \( w \) an auxiliary control variable.

### 4.6. Path Constraints

Let the tangential acceleration \( a_T \), dynamic pressure \( q \), and heating rate \( Q \) per unit time and unit surface area be described by the relations

\[
\begin{align*}
a_T &= (T / W) g_e \cos \alpha - (D / W) g_e - g \sin \gamma , \quad (74a) \\
q &= (1/2) \rho(h)V^2 , \quad (74b) \\
Q &= C \sqrt{\left[ \rho(h) / \rho(h_0) \right] (V / V_0)^{3.07}} . \quad (74c)
\end{align*}
\]

In Eq. (74c), \( h_0 = 100 \text{ kft} \) is a reference altitude and \( V_0 = 10 \text{ kft/sec} \) a reference velocity; under the assumption that the nose radius is \( r_n = 1.0 \text{ ft} \), the constant in (74c) has the value \( C = 102 \text{ BTU/ft}^2\text{sec} \). In addition to the control inequality constraints (69b) and (72b), converted into state equality constraints via the transformations (70) and (73), some path constraints can be imposed on the quantities (74), for instance,

\[
\begin{align*}
a_T &\leq 3 \ g_e , \quad (75a) \\
q &\leq 700 \ lb/\text{ft}^2 , \quad (75b) \\
Q &\leq 150 \text{ BTU/ft}^2\text{sec} . \quad (75c)
\end{align*}
\]
4.7. Atmospheric Model. The atmospheric model used is the 1976 US Standard Atmosphere (Ref. 23). In this model, the values of the density are tabulated at discrete altitudes. For intermediate altitudes, the density is computed by assuming an exponential fit for the function $\rho(h)$. This is equivalent to assuming that the atmosphere behaves isothermally between any two contiguous altitudes tabulated in Ref. 23.

4.8. Supplementary Data. The following data have been used in the numerical experiments:

radius of Earth $r_e = 20925$ kft, \hspace{1cm} (76a)

Earth gravitational constant $\mu = 0.14076 \text{ E+08 kft}^2/\text{sec}^2$, \hspace{1cm} (76b)

sea-level acceleration of gravity $g_e = 32.174 \text{ ft/sec}^2$. \hspace{1cm} (76c)
5. SSSO Configuration

For the SSSO spacecraft, the maximum payload weight problem is studied for initial thrust-to-weight ratio $\sigma$ in the range 1.4 to 1.6, engine specific impulse $I_{\text{sp}}$ in the range 400 to 500 sec, and structural factor $\varepsilon$ in the range 0.08 to 0.12. The value assumed for the initial wing loading is

$$W_i/S = 175 \text{ lb/ft}^2,$$  \hspace{1cm} (77)

close to that of the Lockheed-Martin X-33 spacecraft.

5.1. Boundary Conditions. The initial conditions ($t = 0$, subscript $i$) are

$$h_i = 0 \text{ kft} = 0 \text{ km},$$  \hspace{1cm} (78a)

$$V_i = 0 \text{ kft/sec} = 0 \text{ km/sec},$$  \hspace{1cm} (78b)

$$\gamma_i = \pi/2,$$  \hspace{1cm} (78c)

$$W_i/W_r = 1,$$  \hspace{1cm} (78d)

$$\alpha_i = 0 \Rightarrow x_i = 0,$$  \hspace{1cm} (78e)

$$\beta_i = 1 \Rightarrow \gamma_i = \pi/2.$$  \hspace{1cm} (78f)

The final conditions ($t = \tau$, free, subscript $f$) are

$$h_f = 250 \text{ kft} = 76.2 \text{ km},$$  \hspace{1cm} (79a)

$$V_f = 14.13 \text{ kft/sec} = 4.307 \text{ km/sec},$$  \hspace{1cm} (79b)

$$\gamma_f = 0,$$  \hspace{1cm} (79c)

$$W_f/W_r = \text{free},$$  \hspace{1cm} (79d)

$$\alpha_f = \text{free} \Rightarrow x_f = \text{free},$$  \hspace{1cm} (79e)
\[ \beta_r = \text{free} \Rightarrow y_r = \text{free.} \]  

In Eqs. (78)-(79), the reference weight \( W_r \) is the same as the initial weight \( W_i \). The value \( V_r = 14.13 \text{ kft/sec} \) corresponds to \( M_r = 15 \) at \( h_r = 250 \text{ kft} \).

### 5.2. Weight Distribution

Once the final weight \( W_f \) is known for a given initial weight \( W_i \), one can compute the propellant weight \( W_p \), structural weight \( W_s \), and payload weight \( W_* \) using the following relations:

\[
W_i = W_f + W_p , \tag{80a}
\]

\[
W_f = W_s + W_* , \tag{80b}
\]

\[
\varepsilon W_p = (1 - \varepsilon) W_s , \tag{80c}
\]

where

\[
\varepsilon = W_i / (W_s + W_p) \tag{81}
\]

denotes the structural factor. These relations admit the solutions

\[
W_p = W_i - W_f , \tag{82a}
\]

\[
W_s = \varepsilon (W_i - W_f) / (1 - \varepsilon) , \tag{82b}
\]

\[
W_* = (W_f - \varepsilon W_i) / (1 - \varepsilon) , \tag{82c}
\]

which in normalized form become

\[
W_p/W_i = 1 - W_f/W_i , \tag{83a}
\]

\[
W_s/W_i = \varepsilon (1 - W_f/W_i) / (1 - \varepsilon) , \tag{83b}
\]

\[
W_*/W_i = (W_f/W_i - \varepsilon) / (1 - \varepsilon) . \tag{83c}
\]
5.3. Optimization Problem. For the SSSO configuration, the maximum payload problem can be formulated as follows:

$$\max J = W_* = (W_f - \varepsilon W_i) / (1 - \varepsilon),$$  \hfill (84a)

s. t. \ (59), (70), (73), (75), (78), (79). \hfill (84b)

The unknowns are the functions

$$h(t), V(t), \gamma(t), W(t), x(t), y(t), u(t), w(t), \tau,$$  \hfill (85)

which include the original state variables $h, V, \gamma, W$, the auxiliary state variables $x, y$, the auxiliary control variables $u, w$, and the parameter $\tau$. With the solution known, the original control variables $\alpha, \beta$ can be recovered via (70a) and (73a).

(i) Concerning the path constraints (75), the nature of the problem is such that (75c) is satisfied always with strict inequality. Therefore, only (75a) and (75b) are accounted for via penalty functionals added to the functional being minimized.

(ii) In treating problem (84), it is convenient to normalize the dimensional time $t$ via the transformation

$$\theta = t / \tau,$$  \hfill (86a)

where $\theta$ is the dimensionless time and $\tau$ the final time. Clearly,

$$0 \leq \theta \leq 1, \quad \text{if } 0 \leq t \leq \tau.$$  \hfill (86b)

(iii) At the initial point, a special singularity is present in Eq. (59c), due to the fact that $V_i = 0$. For this reason, we split the trajectory into two phases. Phase 1 is flown with constraints

$$\gamma = \pi/2,$$  \hfill (87a)

$$\beta = 1.$$  \hfill (87b)
In this phase, the angle of attack is small so that

\[ \sin \alpha \approx \alpha. \quad (87c) \]

In this way, the system (59) becomes a 3rd order system, the singularity is avoided, and \( \alpha \) is uniquely determined in Phase 1. This phase is terminated when the velocity reaches the value 0.3 kft/sec. At this point, Phase 2 is started and the complete system (59) is treated as a 4th order system. Typically, the time length of Phase 1 is in the order of \( 1/10 \) of the time length of Phase 2.

(iv) The optimization problem under consideration is of the Bolza type and can be solved via the sequential gradient-restoration algorithm for optimal control problems (SGRA, Refs. 4-6). Since SGRA is a gradient-type algorithm, it can handle singular optimal control problems, while this is not the case with second-order algorithms.

5.4. Results. For the SSSO configuration, a parametric study with respect to the parameters \( \sigma \) (initial thrust-to-weight ratio), \( I_{sp} \) (engine specific impulse), and \( \varepsilon \) (spacecraft structural factor) has been carried out. The following parameter values have been considered:

\[ \sigma = 1.4, 1.5, 1.6, \quad (88a) \]

\[ I_{sp} = 400, 420, 440, 460, 480, 500 \text{ sec}, \quad (88b) \]

\[ \varepsilon = 0.08, 0.10, 0.12 . \quad (88c) \]

For each \((\sigma, I_{sp}, \varepsilon)\)-combination, the maximum payload weight problem (84) has been solved with SGRA. The results for the final weight \( W_f \), propellant weight \( W_p \), structural weight \( W_s \), and payload weight \( W_\ast \) associated with the various \((\sigma, I_{sp}, \varepsilon)\)-combinations
are given in Tables 1-3. In Figs. 5-7 the normalized payload weight $W_i/W_i$ is plotted versus the specific impulse $I_{sp}$ for nine $(\sigma, \varepsilon)$-combinations. From Tables 1-3 and Figs. 5-7, the following comments can be made.

(i) The normalized payload weight increases as the engine specific impulse increase, but decreases as the spacecraft structural factor increases.

(ii) The design of the SSSO configuration is feasible for all the parameter combinations considered.

(iii) For given values of $\sigma$ and $I_{sp}$, feasibility (payload weight positive) can be achieved providing the structural factor is below a threshold value. For $\sigma = 1.5$ and $I_{sp} = 420$ sec, the threshold value is $\varepsilon_{max} = 0.255$ (not shown in Fig. 6). Should this threshold value be exceeded, it would become impossible for the spacecraft to reach the desired final Mach number $M_f = 15$.

For a particular SSSO combination ($\sigma = 1.5, I_{sp} = 420$ sec, $\varepsilon = 0.10$), perhaps close to that of the Lockheed-Martin X-33 spacecraft, Figs. 8-17 contain the time histories of the altitude $h$, velocity $V$, flight path angle $\gamma$, normalized weight $W/W_i$, angle of attack $\alpha$, power setting $\beta$, thrust-to-weight ratio $T/W$, tangential acceleration $a_T$, dynamic pressure $q$, and heating rate $Q$. 
6. SSTO Configuration

For the SSTO spacecraft, the maximum payload weight problem is studied for initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5, engine specific impulse $I_{sp}$ in the range 400 to 500 sec and structural factor $\varepsilon$ in the range 0.08 to 0.12. The value assumed for the initial wing loading is

$$W_i/S = 387 \text{ lb/ft}^2,$$

perhaps close to that of the Lockheed-Martin Venture Star spacecraft.

6.1. Boundary Conditions. The initial conditions ($t = 0$, subscript $i$) are

$$h_i = 0 \text{ kft} = 0 \text{ km},$$

$$V_i = 0 \text{ kft/sec} = 0 \text{ km/sec},$$

$$\gamma_i = \pi/2,$$

$$W_i/W_i = 1,$$

$$\alpha_i = 0 \Rightarrow x_i = 0,$$

$$\beta_i = 1 \Rightarrow y_i = \pi/2.$$

The final conditions ($t = \tau = \text{free}$, subscript $f$) are

$$h_f = 1519 \text{ kft} = 463 \text{ km},$$

$$V_f = 25.04 \text{ kft/sec} = 7.632 \text{ km/sec},$$

$$\gamma_f = 0,$$

$$W_f/W_i = \text{free},$$

$$\alpha_f = \text{free} \Rightarrow x_f = \text{free},$$

$$\beta_f = \text{free} \Rightarrow y_f = \text{free},$$

$$\alpha_f = \text{free} \Rightarrow x_f = \text{free},$$

$$\beta_f = \text{free} \Rightarrow y_f = \text{free}.$$
\( \beta_t = 0 \quad \Rightarrow \quad \gamma_r = 3\pi/2. \) \hspace{1cm} (91f)

In Eqs. (90)-(91), the reference weight \( W_r \) is the same as the initial weight \( W_i \). The velocity (91b) is the circular velocity at the altitude (91a).

6.2. Weight Distribution. Once the final weight \( W_f \) is known for a given initial weight \( W_i \), one can compute the propellant weight \( W_p \), structural weight \( W_s \), and payload weight \( W_\star \) using the following relations:

\[
W_i = W_f + W_p, \tag{92a}
\]

\[
W_f = W_s + W_\star, \tag{92b}
\]

\[
\varepsilon W_p = (1 - \varepsilon) W_s, \tag{92c}
\]

where

\[
\varepsilon = W_s / (W_s + W_p) \tag{93}
\]

denotes the structural factor. These relations admit the solutions

\[
W_p = W_i - W_f, \tag{94a}
\]

\[
W_s = \varepsilon (W_i - W_f) / (1 - \varepsilon), \tag{94b}
\]

\[
W_\star = (W_f - \varepsilon W_i) / (1 - \varepsilon), \tag{94c}
\]

which in normalized form become

\[
W_p/W_i = 1 - W_f/W_i, \tag{95a}
\]

\[
W_s/W_i = \varepsilon (1 - W_f/W_i) / (1 - \varepsilon), \tag{95b}
\]

\[
W_\star/W_i = (W_f/W_i - \varepsilon) / (1 - \varepsilon). \tag{95c}
\]
6.3. Optimization Problem. For the SSTO configuration, the maximum payload problem can be formulated as follows:

\[ \max J = W_\ast = \left( W_f - \varepsilon W_i \right) / (1 - \varepsilon), \quad (96a) \]

s. t. \((59), (70), (73), (75), (90), (91)\). \quad (96b)

The unknowns are the functions

\[ h(t), V(t), \gamma(t), W(t), x(t), y(t), u(t), w(t), \tau, \quad (97) \]

which include the original state variables \(h, V, \gamma, W\), the auxiliary state variables \(x, y\), the auxiliary control variables \(u, w\), and the parameter \(\tau\). With the solution known, the original control variables \(\alpha, \beta\) can be recovered via \((70a)\) and \((73a)\).

(i) Concerning the path constraints \((75)\), the nature of the problem is such that \((75b)\) and \((75c)\) are satisfied always with strict inequality. Therefore, only \((75a)\) is accounted for via a penalty functional added to the functional being minimized.

(ii) In treating problem \((96)\), it is convenient to normalize the dimensional time \(t\) via the transformation

\[ \theta = t / \tau, \quad (98a) \]

where \(\theta\) is the dimensionless time and \(\tau\) the final time. Clearly,

\[ 0 \leq \theta \leq 1, \quad \text{if} \quad 0 \leq t \leq \tau. \quad (98b) \]

(iii) At the initial point, a special singularity is present in Eq. \((59c)\), due to the fact that \(V_i = 0\). For this reason, we split the trajectory into two phases. Phase 1 is flown with constraints \((87a), (87b)\). In this phase, the angle of attack is small so that approximation \((87c)\) holds. In this way, the system \((59)\) becomes a 3rd order system, the singularity is avoided, and \(\alpha\) is uniquely determined in Phase 1. This phase is terminated when the
velocity reaches the value 0.3 kft/sec. At this point, Phase 2 is started and the complete system (59) is treated as a 4th order system. Typically, the time length of Phase 1 is in the order of 1/50 of the time length of Phase 2.

(iv) The optimization problem under consideration is of the Bolza type and can be solved via the sequential gradient-restoration algorithm for optimal control problems (SGRA, Refs. 4-6). Since SGRA is a gradient-type algorithm, it can handle singular optimal control problems, while this is not the case with second-order algorithms.

6.4. Results. For the SSTO configuration, a parametric study with respect to the parameters $\sigma$ (initial thrust-to-weight ratio), $I_{sp}$ (engine specific impulse), and $\varepsilon$ (spacecraft structural factor) has been carried out. The following parameter values have been considered:

$$\sigma = 1.3, 1.4, 1.5,$$  \hspace{1cm} (99a)

$$I_{sp} = 400, 420, 440, 460, 480, 500 \text{ sec},$$ \hspace{1cm} (99b)

$$\varepsilon = 0.08, 0.10, 0.12.$$ \hspace{1cm} (99c)

For each ($\sigma$, $I_{sp}$, $\varepsilon$)-combination, the maximum payload weight problem (96) has been solved with SGRA.

The results for the final weight $W_f$, propellant weight $W_p$, structural weight $W_s$, and payload weight $W_\ast$ associated with the various ($\sigma$, $I_{sp}$, $\varepsilon$)-combinations are given in Tables 4-6. In Figs. 18-20, the normalized payload weight $W_\ast/W_i$ is plotted versus the specific impulse $I_{sp}$ for nine ($\sigma$, $\varepsilon$)-combinations. From Tables 4-6 and Figs. 18-20, the following comments can be made.
(i) The normalized payload weight increases as the engine specific impulse increase, but decreases as the spacecraft structural factor increases.

(ii) The design of the SSTO configuration might be comfortably feasible, marginally feasible, or unfeasible, depending on the parameter values assumed. Only for $\varepsilon = 0.08$, the SSTO spacecraft is feasible (payload weight positive) in the whole range of initial thrust-to-weight ratios and specific impulses considered.

(iii) For given values of $\sigma$ and $\varepsilon$, feasibility can be achieved providing the engine specific impulse is above a threshold value. For $\sigma = 1.4$, the threshold value is $(I_{sp})_{\text{min}} = 425$ sec if $\varepsilon = 0.10$, $(I_{sp})_{\text{min}} = 464$ sec if $\varepsilon = 0.12$, and $(I_{sp})_{\text{min}} = 505$ sec if $\varepsilon = 0.14$. Note that the latter value of $(I_{sp})_{\text{min}}$ is well in excess of the values associated with the Space Shuttle engines, which are at most 450 sec.

(iv) For given values of $\sigma$ and $I_{sp}$, feasibility can be achieved providing the structural factor is below a threshold value. For $\sigma = 1.4$, this threshold value is $\varepsilon_{\text{max}} = 0.087$ if $I_{sp} = 400$ sec, $\varepsilon_{\text{max}} = 0.107$ if $I_{sp} = 440$ sec, and $\varepsilon_{\text{max}} = 0.128$ if $I_{sp} = 480$ sec.

For a particular SSTO combination ($\sigma = 1.4$, $I_{sp} = 440$ sec, $\varepsilon = 0.10$), perhaps close to that of the Lockheed-Martin Venture Star spacecraft, Figs. 21-30 contain the time histories of the altitude $h$, velocity $V$, flight path angle $\gamma$, normalized weight $W/W_0$, angle of attack $\alpha$, power setting $\beta$, thrust-to-weight ratio $T/W$, tangential acceleration $a_T$, dynamic pressure $q$, and heating rate $Q$. 
7. TSTO Configuration

For the TSTO spacecraft, the maximum payload weight problem is studied first for initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and uniform structural factor $\tilde{e} = \bar{e}$ in the range 0.08 to 0.12 with $\tilde{e}$ denoting the structural factor of Stage 1 and $\bar{e}$ denoting the structural factor of Stage 2. Then, for a particular value of the initial thrust-to-weight ratio ($\sigma = 1.4$), the maximum payload weight problem is studied for nonuniform structural factor by varying $\tilde{e} = k\bar{e}$, with $k \geq 1$, while keeping $\tilde{e}$ fixed. The value assumed for the initial wing loading is

$$\frac{W_i}{S} = 387 \text{ lb/ft}^2,$$

perhaps close to that of the Lockheed-Martin Venture Star spacecraft.

Let a caret superscript denote Stage 1; let a tilde superscript denote Stage 2. The differential system (59) applies to both Stage 1 and Stage 2; however, owing to weight discontinuity, integration must be stopped at the end of Stage 1 and restarted at the beginning of Stage 2. The unknowns are the functions

$$\hat{h}(t), \hat{V}(t), \hat{\gamma}(t), \hat{\bar{W}}(t), \hat{\alpha}(t), \hat{\beta}(t), \hat{\tau}.$$ (101a)

$$\bar{h}(t), \bar{V}(t), \bar{\gamma}(t), \bar{W}(t), \bar{\alpha}(t), \bar{\beta}(t), \bar{\tau}.$$ (101b)

which include the state variables $\hat{h}, \hat{V}, \hat{\gamma}, \hat{\bar{W}}$ and $\bar{h}, \bar{V}, \bar{\gamma}, \bar{W}$, the control variables $\hat{\alpha}, \hat{\beta}$ and $\bar{\alpha}, \bar{\beta}$, and the parameters $\hat{\tau}$ and $\bar{\tau}$. The final time from takeoff to orbit is given by

$$\tau = \hat{\tau} + \bar{\tau}.$$ (102)

The inequality constraints (69b) and (72b) apply to both Stage 1 and Stage 2. These are converted into equality constraints via the trigonometric transformations (70a), (73a)
employed in conjunction with the differential constraints (70b), (73b). As a result, the replacements

\[ \bar{\alpha}(t), \bar{\beta}(t) \to \hat{x}(t), \hat{y}(t), \hat{u}(t), \hat{w}(t), \]  
\[ \bar{\alpha}(t), \bar{\beta}(t) \to \bar{x}(t), \bar{y}(t), \bar{u}(t), \bar{w}(t), \] 

must be introduced in (101) with \( \hat{x}, \hat{y} \) and \( \bar{x}, \bar{y} \) denoting auxiliary state variables and with \( \hat{u}, \hat{w} \) and \( \bar{u}, \bar{w} \) denoting auxiliary control variables. The path constraints (75) apply to both Stage 1 and Stage 2. At the interface between stages, there is a weight discontinuity due to staging; in turn, this induces a thrust discontinuity due to the requirement that the tangential acceleration be kept unchanged.

### 7.1. Boundary Conditions.

Equations (90) must be understood as initial conditions for Stage 1; hence,

\[ \hat{h}_i = 0 \text{kft} = 0 \text{ km}, \]  
\[ \hat{V}_i = 0 \text{kft/sec} = 0 \text{ km/sec}, \]  
\[ \hat{y}_i = \pi/2, \]  
\[ \hat{W}_i/W_r = 1, \]  
\[ \hat{\alpha}_i = 0 \Rightarrow \hat{x}_i = 0, \]  
\[ \hat{\beta}_i = 1 \Rightarrow \hat{y}_i = \pi/2. \]  

Equations (91) must be understood as final conditions for Stage 2; hence,

\[ \bar{h}_f = 1519 \text{ kft} = 463 \text{ km}, \]  
\[ \bar{V}_f = 25.04 \text{ kft/sec} = 7.632 \text{ km/sec}, \]
\( \ddot{y}_f = 0, \quad (105c) \)

\( \ddot{w}_f / w_f = \text{free}, \quad (105d) \)

\( \ddot{a}_f = \text{free} \implies \ddot{x}_f = \text{free}, \quad (105e) \)

\( \ddot{\beta}_f = 0 \implies \ddot{\gamma}_f = 3\pi/2. \quad (105f) \)

In Eqs. (104)-(105), the reference weight \( W_f \) is the same as the initial weight \( W_i \). The velocity (105b) is the circular velocity at the altitude (105a).

### 7.2. Interface Conditions

At the interface between stages, the state variables \( h, V, \gamma, x \) behave continuously, while the state variables \( \dot{W}, y \) behave discontinuously. The discontinuity in \( \dot{W} \) is due to structural weight ejection due to staging. The discontinuity in \( y \), hence discontinuity in power setting \( \beta \) or thrust \( T \), is due to the requirement that the tangential acceleration \( a_T \) be kept unchanged across the interface between stages. To sum up, the interface conditions (subscript \( i \) for Stage 2, subscript \( f \) for Stage 1) are as follows:

\[ h_i = h_f, \quad (106a) \]

\[ V_i = V_f, \quad (106b) \]

\[ \dot{y}_i = \dot{y}_f, \quad (106c) \]

\[ \ddot{W}_i = \ddot{W}_f - \ddot{W}_i \implies \ddot{w}_i / w_i = (\ddot{w}_f / w_f - \ddot{e}) / (1 - \ddot{e}), \quad (106d) \]

\[ \ddot{a}_i = \ddot{a}_f \implies \ddot{x}_i = \ddot{x}_f, \quad (106e) \]

\[ \ddot{a}_T i = \ddot{a}_T f \implies (\ddot{a}_T i \cos \ddot{a}_i - \ddot{D}_i) / \ddot{W}_i = (\ddot{a}_T f \cos \ddot{a}_f - \ddot{D}_f) / \ddot{W}_f. \quad (106f) \]

Concerning (106d), see Eq. (112); concerning (106f), see Eq. (74a).
7.3. **Weight Distribution.** Relations (80)-(83) and (92)-(95), written for the SSSO and SSTO configurations, are valid for the TSTO configuration with the following understanding: each relation must be rewritten with a caret superscript for Stage 1 and a tilde superscript for Stage 2. Concerning the structural factor, we assume that

\[ \tilde{\varepsilon} = k \hat{\varepsilon}, \quad (107a) \]

with

\[ k \geq 1. \quad (107b) \]

In particular, for Stage 1, we have [see (82) or (94)]

\[ \hat{W}_p = \hat{W}_i - \hat{W}_f, \quad (108a) \]

\[ \hat{W}_s = \hat{\varepsilon} (\hat{W}_i - \hat{W}_f) / (1 - \hat{\varepsilon}), \quad (108b) \]

\[ \hat{W}_s = (\hat{W}_f - \hat{\varepsilon} \hat{W}_i) / (1 - \hat{\varepsilon}), \quad (108c) \]

with

\[ \hat{W}_i = \hat{W}_p + \hat{W}_s + \hat{W}_s. \quad (109) \]

For Stage 2, we have [see (82) or (94)]

\[ \tilde{W}_p = \tilde{W}_i - \tilde{W}_f, \quad (110a) \]

\[ \tilde{W}_s = \tilde{\varepsilon} (\tilde{W}_i - \tilde{W}_f) / (1 - \tilde{\varepsilon}), \quad (110b) \]

\[ \tilde{W}_s = (\tilde{W}_f - \tilde{\varepsilon} \tilde{W}_i) / (1 - \tilde{\varepsilon}), \quad (110c) \]

with

\[ \tilde{W}_i = \tilde{W}_p + \tilde{W}_s + \tilde{W}_s. \quad (111) \]

The transition condition at the interface between stages is

\[ \tilde{W}_i = \tilde{W}_s = \tilde{W}_f - \tilde{W}_s = (\tilde{W}_f - \tilde{\varepsilon} \tilde{W}_i) / (1 - \tilde{\varepsilon}). \quad (112) \]
For the TSTO configuration as a whole, the following definitions hold:

\[ W_i = \tilde{W}_i, \quad W_f = \tilde{W}_f, \quad (113a) \]
\[ W_p = \hat{W}_p + \tilde{W}_p, \quad W_s = \hat{W}_s + \tilde{W}_s, \quad W_* = \tilde{W}_*. \quad (113b) \]

with the implication that

\[ W_i - W_p + W_s + W_* , \quad (114) \]

as can be seen by adding (109) and (111), and then accounting for (108c), (112), (113).

7.4. Optimization Problem. For the TSTO configuration, the maximum payload weight problem can be formulated as follows:

\[ \max J = W_* = \tilde{W}_* = (\tilde{W}_f - \tilde{e} \tilde{W}_i) / (1 - \tilde{e}), \quad (115a) \]

s. t. (59), (70), (73), (75), (104), (105), (106). \quad (115b)

The unknowns are the functions

\[ \hat{h}(t), \hat{V}(t), \hat{\gamma}(t), \hat{W}(t), \hat{x}(t), \hat{y}(t), \hat{u}(t), \hat{w}(t), \hat{\tau}, \quad (116a) \]
\[ \tilde{h}(t), \tilde{V}(t), \tilde{\gamma}(t), \tilde{W}(t), \tilde{x}(t), \tilde{y}(t), \tilde{u}(t), \tilde{w}(t), \tilde{\tau}. \quad (116b) \]

with the caret superscript denoting Stage 1 and tilde superscript denoting Stage 2. The unknowns (116) include the original state variables \( \hat{h}, \hat{V}, \hat{\gamma}, \hat{W} \) and \( \tilde{h}, \tilde{V}, \tilde{\gamma}, \tilde{W} \), the auxiliary state variables \( \hat{x}, \hat{y} \) and \( \tilde{x}, \tilde{y} \), the auxiliary control variables \( \hat{u}, \hat{w} \) and \( \tilde{u}, \tilde{w} \), and the parameters \( \hat{\tau} \) and \( \tilde{\tau} \). With the solution known, the original control variables \( \hat{\alpha}, \hat{\beta} \) and \( \tilde{\alpha}, \tilde{\beta} \), can be recovered via (70a) and (73a). Also, the total time from takeoff to orbit is

\[ \tau = \hat{\tau} + \tilde{\tau}. \quad (117) \]
(i) Concerning the path constraints (75), the nature of the problem is such that (75b) and (75c) are satisfied always with strict inequality. Therefore, only (75a) is accounted for via a penalty functional added to the functional being minimized.

(ii) In treating problem (115), it is convenient to normalize the dimensional time $t$ via the transformations

$$\theta = t / \hat{\tau}, \quad 0 \leq t \leq \hat{\tau},$$

$$\theta = 1 + (t - \hat{\tau}) / \bar{\tau}, \quad \hat{\tau} \leq t \leq \hat{\tau} + \bar{\tau} = \tau,$$

where $\theta$ is the dimensionless time, $\hat{\tau}$ the time length of Stage 1, $\bar{\tau}$ the time length of Stage 2, and $\tau$ the final time. Clearly,

$$0 \leq \theta \leq 1, \quad \text{if } 0 \leq t \leq \hat{\tau},$$

$$1 \leq \theta \leq 2, \quad \text{if } \hat{\tau} \leq t \leq \hat{\tau} + \bar{\tau} = \tau.$$

Therefore, the dimensionless time has the value $\theta = 0$ at the beginning of Stage 1, $\theta = 1$ at the interface between stages, and $\theta = 2$ at the end of Stage 2.

(iii) At the initial point, a special singularity is present in Eq. (59c), due to the fact that $\hat{V}_i = 0$. For this reason, we split the trajectory into two phases. Phase 1 is flown with constraints (87a), (87b). In this phase, the angle of attack is small so that approximation (87c) holds. In this way, the system (59) becomes a 3rd order system, the singularity is avoided, and $\alpha$ is uniquely determined in Phase 1. This phase is terminated when the velocity reaches the value 0.3 kft/sec. At this point, Phase 2 is started and the complete system (59) is treated as a 4th order system. Typically, the time length of Phase 1 is in the order of $1/50$ of the time length of Phase 2.

(iv) The optimization problem under consideration is of the Bolza type and can be
solved via the sequential gradient-restoration algorithm for optimal control problems (SGRA, Refs. 4-6). Since SGRA is a gradient-type algorithm, it can handle singular optimal control problems, while this is not the case with second-order algorithms. For the present configuration (TSTO), an additional complication exists, namely, the switch time \( \hat{\tau} \) from Stage 1 to Stage 2 must be determined in addition to the final time \( \tau \). A further complication is the presence of the interface conditions (106) in addition to the initial conditions (104) and final conditions (105). With reference to (106), note that (106a), (106b), (106c), (106e) are continuity conditions, while (106d) and (106f) are discontinuity conditions: (106d) yields a discontinuity in weight consistent with the fact that structural weight is being ejected due to staging; (106f) yields a discontinuity in thrust consistent with the requirement that the tangential acceleration be continuous across the interface between stages.

7.5. Results: Uniform Structural Factor. In this study, the following parameter values have been considered:

\[
\begin{align*}
\sigma &= 1.3, 1.4, 1.5, \\
I_{sp} &= 400, 420, 440, 460, 480, 500 \text{ sec}, \\
\hat{\varepsilon} = \bar{\varepsilon} &= 0.08, 0.10, 0.12.
\end{align*}
\]

For each \((\sigma, I_{sp}, \hat{\varepsilon} = \bar{\varepsilon})\)-combination, the maximum payload weight problem (115) has been solved with SGRA. The results for the final weight \( W_f \), propellant weight \( W_p \), structural weight \( W_s \), and payload weight \( W_\ast \) associated with the various \((\sigma, I_{sp}, \hat{\varepsilon} = \bar{\varepsilon})\)-combinations are given in Tables 7-9. In Figs. 31-33, the normalized payload weight
$W_p/W_i$ is plotted versus the specific impulse $I_{sp}$ for nine $(\sigma, \hat{\varepsilon} = \tilde{\varepsilon})$-combinations. In Figs. 34-36, which refer to $\sigma = 1.4$ and three values of the structural factor, a comparison is made between the normalized payload weights of TSTO and SSTO configurations.

From Tables 7-9 and Figs. 31-36, the following comments can be made.

(i) The normalized payload weight increases as the engine specific impulse increase, but decreases as the spacecraft structural factor increases.

(ii) The design of the TSTO configuration is feasible for all of the parameter combinations considered.

(iii) For initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5 and uniform structural factor $\hat{\varepsilon} = \tilde{\varepsilon}$ in the range 0.14 to 0.18 (not shown in Figs. 31-33), feasibility of the TSTO spacecraft can be achieved even with present-day engines having specific impulse around 450 sec.

(iv) The most striking impression that one gets from Figs. 34-36 is the clear superiority of the TSTO configuration vis-a-vis the SSTO configuration; indeed, the normalized payload gap between the two configurations increases as the structural factor increases.

(v) To sum up, for the same structural factor, the normalized payload of a TSTO configuration is considerably larger than that of a SSTO configuration. Conversely, for the same normalized payload, one can design a TSTO configuration having a much larger structural factor than the SSTO configuration, with the implication of increased safety and reliability.

For a particular TSTO configuration ($\sigma = 1.4$, $I_{sp} = 440$ sec, $\hat{\varepsilon} = \tilde{\varepsilon} = 0.10$),
Figs. 37-46 show the time histories of the altitude \( h \), velocity \( V \), flight path angle \( \gamma \), normalized weight \( W/W_i \), angle of attack \( \alpha \), power setting \( \beta \), thrust-to-weight ratio \( T/W \), tangential acceleration \( \alpha_T \), dynamic pressure \( q \), and heating rate \( Q \).

### 7.6. Results: Nonuniform Structural Factor.

In this study, the following parameter values have been considered:

\[
\begin{align*}
\sigma &= 1.4, & (121a) \\
I_{sp} &= 400, 420, 440, 460, 480, 500 \text{ sec}, & (121b) \\
\hat{e} &= 0.08, 0.10, 0.12, & (121c) \\
\bar{e} &= k \hat{e}, \quad k = 1.0, 1.5, 2.0, 2.5, 3.0. & (121d)
\end{align*}
\]

For each \((\sigma, I_{sp}, \hat{e}, \bar{e} = k \hat{e})\)-combination, the maximum payload weight problem (115) has been solved with SGRA. The results for the final weight \( W_f \), propellant weight \( W_p \), structural weight \( W_s \), and payload weight \( W_s \) associated with various \((\sigma, I_{sp}, \hat{e}, \bar{e} = k \hat{e})\)-combinations are given in Tables 7-21. In Figs. 31-33 and 47-50, the normalized payload weight \( W_s/W_i \) is plotted versus the specific impulse for the values (121c) of \( \hat{e} \) and (121d) of \( k \).

From Tables 7-21 and Figs. 31-33 and 47-50, the following comments can be made.

(i) The normalized payload weight increases as the engine specific impulse increases, but decreases as the Stage 1 structural factor increases and as the parameter \( k \) increases, hence as the Stage 2 structural factor increases.

(ii) Even if the Stage 2 structural factor is more than twice the Stage 1 structural
factor, the TSTO configuration is feasible.

(iii) For $I_{sp} = 440$ sec, the maximum value of the parameter $k = \tilde{e} / \hat{e}$ for which feasibility (payload weight positive) can be guaranteed is $k = 3.5$ for $\hat{e} = 0.08$, $k = 2.8$ for $\hat{e} = 0.10$, and $k = 2.3$ for $\hat{e} = 0.12$; this corresponds to a Stage 2 structural factor $\tilde{e} \equiv 0.28$, nearly independent of the Stage 1 structural factor $\hat{e}$. 
8. Design Considerations

From Tables 1-21 and Figs. 5-7, 18-20, 31-33, 47-50, we see that, for the SSSO configuration, the normalized payload weight behaves almost linearly with respect to the engine specific impulse and almost quadratically with respect to the spacecraft structural factor; for the SSTO configuration, the normalized payload weight behaves almost linearly with respect to the engine specific impulse and the spacecraft structural factor; for the TSTO configuration, the same behavior as that of the SSTO configuration holds as long as the ratio $k$ of the Stage 2 structural factor to the Stage 1 structural factor is kept constant. These findings lead to useful design considerations.

8.1. SSSO Configuration. For this configuration, the assumed representation of the results is the following:

$$
\frac{W_s}{W_i} = A + B\epsilon + C\epsilon^2 + (D + E\epsilon) \frac{I_{sp}}{I_{spr}} .
$$

(122)

where $A$, $B$, $C$, $D$, $E$ are dimensionless constants and $I_{spr} = 1000$ sec is a reference specific impulse. The representation (122) implies that the zero-payload condition ($W_s = 0$) separating feasibility ($W_s > 0$) from unfeasibility ($W_s < 0$) is given by

$$
\frac{I_{sp}}{I_{spr}} = \frac{(F + G\epsilon + H\epsilon^2)/(1 + K\epsilon)}{1 + K\epsilon},
$$

(123)

with

$$
F = -A/D, \quad G = -B/D, \quad H = -C/D, \quad K = E/D .
$$

(124)

The constants in Eq. (122) can be obtained via a least-square fit of the computed normalized payload weights $W_s/W_i$, treated as available data in the least-square process.
For $\sigma = 1.5$, the following values have been obtained for the constants in Eq. (122):

$$
A = -0.0600, \quad B = -1.1253, \quad C = -1.4438, \quad D = 0.7304, \quad E = 1.2359, \quad (125)
$$

implying that the constants in Eq. (123) are given by

$$
F = 0.0821, \quad G = 1.5407, \quad H = 1.9767, \quad K = 1.6921. \quad (126)
$$

The normalized payload weights predicted via (122) and (125) are in excellent agreement with the available data, as shown in Fig. 51. Consistently, with reference to the $(I_{sp}, \varepsilon)$-plane, the zero-payload condition predicted via (123) and (126) is shown in Fig. 52. From this graph, one obtains the largest structural factor which guarantees feasibility for given specific impulse, or the smallest specific impulse which guarantees feasibility for given structural factor. In Fig. 52, the limiting line $W_* = 0$ is nonconservative, because it disregards the need of propellant for space maneuvers, reentry and landing maneuvers, and reserve margin for emergency; hence, an actual design must lie wholly inside the feasibility region of Fig. 52.

### 8.2. SSTO Configuration.

For this configuration, the assumed linear representation of the results is the following:

$$
W/W_i \equiv A + B\varepsilon + C I_{sp}/I_{spr}, \quad (127)
$$

where $A$, $B$, $C$ are dimensionless constants and $I_{spr} = 1000$ sec is a reference specific impulse. The representation (127) implies that the zero-payload condition ($W_* = 0$) separating feasibility ($W_* > 0$) from unfeasibility ($W_* < 0$) is given by

$$
\varepsilon \equiv D + E I_{sp}/I_{spr}, \quad (128)
$$
with

\[ D = -\frac{A}{B}, \quad E = -\frac{C}{B}. \]  \hspace{1cm} (129)

The constants in Eq. (127) can be obtained via a least-square fit of the computed normalized payload weights \( \frac{W_i}{W} \), treated as available data in the least-square process.

For \( \sigma = 1.4 \), the following values have been obtained for the constants in Eq. (127):

\[ A = -0.1282, \quad B = -1.0958, \quad C = 0.5590, \]  \hspace{1cm} (130)

implying that the constants in Eq. (128) are given by

\[ D = -0.1170, \quad E = 0.5101. \]  \hspace{1cm} (131)

The normalized payload weights predicted via (127) and (130) are in excellent agreement with the available data, as shown in Fig. 53. Consistently, with reference to the \((I_{sp}, \varepsilon)\)-plane, the zero-payload condition predicted via (128) and (131) is shown in Fig. 54. From this graph, one obtains the largest structural factor which guarantees feasibility for given specific impulse, or the smallest specific impulse which guarantees feasibility for given structural factor. In Fig. 54, the limiting line \( W_s = 0 \) is nonconservative, because it disregards the need of propellant for space maneuvers, reentry and landing maneuvers, and reserve margin for emergency; hence, an actual design must lie wholly inside the feasibility region of Fig. 54.

8.3. TSTO Configuration: Uniform Structural Factor. The representation (127)-(128) can be extended to a TSTO configuration with uniform structural factor if one sets

\[ \varepsilon = \hat{\varepsilon} = \bar{\varepsilon}. \]  \hspace{1cm} (132)
For $\sigma = 1.4$, the constants (130) must be replaced by

$$A = -0.1059, \quad B = -0.4500, \quad C = 0.4852,$$

(133)

with the implication that the constants (131) must be replaced by

$$D = -0.2353, \quad E = 1.0782.$$

(134)

The normalized payload weights predicted via (127) and (133) are in excellent agreement with the available data, as shown in Fig. 55. Consistently, with reference to the $(I_{sp}, e)$-plane, the zero-payload condition predicted via (128) and (134) is shown in Fig. 56. From this graph, one obtains the largest structural factor which guarantees feasibility for given specific impulse, or the smallest specific impulse which guarantees feasibility for given structural factor. In Fig. 56, the limiting line $W_* = 0$ is nonconservative, because it disregards the need of propellant for space maneuvers, reentry and landing maneuvers, and reserve margin for emergency; hence, an actual design must lie wholly inside the feasible region of Fig. 56.

8.4. TSTO Configuration: Nonuniform Structural Factor. The representation (127)-(128) can be further extended to a TSTO configuration with nonuniform structural factor if one sets

$$\hat{e} = e, \quad \bar{e} = k \hat{e},$$

(135)

where $k$ is a parameter. In this case, the constants in Eq. (127) must be replaced by functions of the parameter $k$, so that the representation becomes

$$W_* / W_i = A(k) + B(k)e + C(k)I_{sp} / I_{spr},$$

(136)
with the implication that the zero-payload condition \((W_*=0)\) separating feasibility \((W_*>0)\) from unfeasibility \((W_*<0)\) is given by

\[\varepsilon \equiv D(k) + E(k) \frac{I_{sp}}{I_{spr}}, \quad (137)\]

with

\[D(k) = -\frac{A(k)}{B(k)}, \quad E(k) = -\frac{C(k)}{B(k)} \quad (138)\]

Five values of the parameter \(k\) have been considered, namely,

\[k = 1.0, 1.5, 2.0, 2.5, 3.0. \quad (139)\]

For \(\sigma = 1.4\), the constants (133)-(134) must be replaced by

\[A(1.0) = -0.1059, \quad B(1.0) = -0.4500, \quad C(1.0) = 0.4852, \quad (140a)\]

\[A(1.5) = -0.1122, \quad B(1.5) = -0.5917, \quad C(1.5) = 0.4952, \quad (140b)\]

\[A(2.0) = -0.1193, \quad B(2.0) = -0.7792, \quad C(2.0) = 0.5171, \quad (140c)\]

\[A(2.5) = -0.1266, \quad B(2.5) = -1.0000, \quad C(2.5) = 0.5429, \quad (140d)\]

\[A(3.0) = -0.1325, \quad B(3.0) = -1.2542, \quad C(3.0) = 0.5686, \quad (140e)\]

and

\[D(1.0) = -0.2353, \quad E(1.0) = 1.0782, \quad (141a)\]

\[D(1.5) = -0.1896, \quad E(1.5) = 0.8369, \quad (141b)\]

\[D(2.0) = -0.1531, \quad E(2.0) = 0.6636, \quad (141c)\]

\[D(2.5) = -0.1266, \quad E(2.5) = 0.5429, \quad (141d)\]

\[D(3.0) = -0.1056, \quad E(3.0) = 0.4534. \quad (141e)\]

In turn, \(D\) and \(E\) behave almost quadratically with respect to the parameter \(k\). Hence, the assumed representation is the following:
\[ D(k) = D_0 + D_1 k + D_2 k^2, \]  
\[ E(k) = E_0 + E_1 k + E_2 k^2. \]

(142a) \hspace{1cm} (142b)

A least-square fit of the available data (141) with the quadratic functions (142) produces the following constants:

\[ D_0 = -0.3501, \quad D_1 = +0.1320, \quad D_2 = -0.0169, \]  
\[ E_0 = +1.6886, \quad E_1 = -0.7156, \quad E_2 = +0.1017. \]

(143a) \hspace{1cm} (143b)

For each of the values (139) of the parameter \( k \), the normalized payload weights predicted via (136) and (140) are in excellent agreement with the available data, as shown in Figs. 55, 57, 59, 61, 63. Consistently, with reference to the \((I_{sp}, \varepsilon)\)-plane, for each of the values (139) of the parameter \( k \), the zero-payload condition predicted via (137) and (141) is shown in Figs. 56, 58, 60, 62, 64. From these graphs, one obtains the largest structural factor which guarantees feasibility for given specific impulse, or the smallest specific impulse which guarantees feasibility for given structural factor. In Figs. 56, 58, 60, 62, 64, the limiting line \( W_* = 0 \) is nonconservative, because it disregards the need of propellant for space maneuvers, reentry and landing maneuvers, and reserve margin for emergency; hence, an actual design must lie wholly inside the feasibility region of Figs. 56, 58, 60, 62, 64.

As a final observation, with reference to the \((I_{sp}, \varepsilon)\)-plane of Figs. 56, 58, 60, 62, 64, we note that the zero-payload lines \( W_* = 0 \) shift downward as the parameter \( k \) increases, thereby shrinking the size of the feasibility region as \( k \) increases.
9. Effect of Drag Changes

After completing the previous optimization studies on SSSO, SSTO, and TSTO configurations (Sections 5-7), an additional topic was considered, namely, the effect of drag reduction or increase on payload weight. Suppose that the drag and lift of the spacecraft are embedded into a one-parameter family of the form

\[ D' = \eta D, \quad L' = L, \]  \hspace{1cm} (144)

where \( \eta \) is the drag factor. Clearly, \( \eta = 1 \) yields the drag and lift of the baseline configuration; \( \eta = 0.5 \) reduces the drag by 50\%, while keeping the lift unchanged; \( \eta = 1.5 \) increases the drag by 50\%, while keeping the lift unchanged.

The maximum payload weight problem was solved again for \( \eta = 0.5, \eta = 1.0, \) and \( \eta = 1.5 \). The following parameter values have been considered:

(SSSO) \( \sigma = 1.5, \) \hspace{1cm} (145a)
\[ I_{sp} = 420 \text{ sec}, \]  \hspace{1cm} (145b)
\[ \varepsilon = 0.10, \]  \hspace{1cm} (145c)

(SSTO) \( \sigma = 1.4, \) \hspace{1cm} (146a)
\[ I_{sp} = 440 \text{ sec}, \]  \hspace{1cm} (146b)
\[ \varepsilon = 0.10, \]  \hspace{1cm} (146c)

(TSTO) \( \sigma = 1.4, \) \hspace{1cm} (147a)
\[ I_{sp} = 440 \text{ sec}, \]  \hspace{1cm} (147b)
\[ \varepsilon = \ddot{\varepsilon} = \dddot{\varepsilon} = 0.10, \]  \hspace{1cm} (147c)
with (147c) indicating that a uniform structural factor is being considered for the TSTO configuration.

The results for the final weight \( W_f \), propellant weight \( W_p \), structural weight \( W_s \), and payload weight \( W_\ell \) are shown in Tables 22-24. In Figs. 65-67, the normalized payload weight \( W_\ell/W_1 \) is plotted versus the drag factor \( \eta = D'/D \) for the parameter choices (145)-(147). The results show that changing the drag by \( \pm 50\% \) produces relatively small changes in final weight and payload weight.

One must conclude that the final weight and payload weight are not too sensitive to the aerodynamic model of the spacecraft, or equivalently that the aerodynamic forces do not have a large influence on propellant consumption. Indeed, should an energy balance be made, one would find that the largest part of the energy produced by the rocket powerplant is spent in accelerating the spacecraft to the final velocity; only a minor part is spent in overcoming aerodynamic and gravitational effects.

For TSTO configurations, these results justify having neglected in the analysis drag changes due to staging, and hence having assumed that the drag function of Stage 2 is the same as the drag function of Stage 1.
10. Summary and Conclusions

In this thesis, the maximum payload weight problem has been solved for a single-stage suborbital (SSSO), a single-stage-to-orbit (SSTO), and a two-stage-to-orbit (TSTO) rocket-powered spacecraft.

For the SSSO configuration, the maximum payload weight problem has been studied for initial thrust-to-weight ratio $\sigma$ in the range 1.4 to 1.6, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and structural factor $\varepsilon$ in the range 0.08 to 0.12. For the SSTO configuration, the maximum payload weight problem has been studied for initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and structural factor $\varepsilon$ in the range 0.08 to 0.12. For the TSTO configuration with uniform structural factor, the maximum payload weight problem has been studied for initial thrust-to-weight ratio $\sigma$ in the range 1.3 to 1.5, engine specific impulse $I_{sp}$ in the range 400 to 500 sec, and structural factor $\varepsilon = \hat{\varepsilon} = \tilde{\varepsilon}$ in the range 0.08 to 0.12, with $\hat{\varepsilon}$ denoting the structural factor of Stage 1 and $\tilde{\varepsilon}$ denoting the structural factor of Stage 2. For the TSTO configuration with nonuniform structural factor ($\hat{\varepsilon} = \varepsilon$ and $\tilde{\varepsilon} = k\varepsilon$, $k \geq 1$) and for a particular combination ($\sigma = 1.4$ and $I_{sp} = 440$ sec), the maximum payload weight problem has been studied again by varying $\tilde{\varepsilon}$, while keeping $\hat{\varepsilon}$ fixed.

With the above study completed, the investigation has been then extended to assess the sensitivity of the results to the scale of the aerodynamic drag. The maximum payload weight problem has been solved again, for particular parameter combinations, assuming drag changes of $\pm 50\%$ with respect to that of the baseline configuration.
while leaving the lift unchanged.

General results and particular optimal trajectories are shown in Tables 1-24 and in Figs. 1-67. From these tables and figures, the following main conclusions emerge.

(i) Feasibility of SSSO configurations can be achieved with all the parameter combinations considered.

(ii) Feasibility of SSTO configurations can be achieved with only part of parameter combinations considered.

(iii) Feasibility of TSTO configurations can be achieved with all the parameter combinations considered.

(iv) For the case of uniform structural factor, comparison of the TSTO results with SSTO results shows that the maximum payload of the former is several times that of the latter for all the parameter combinations considered. Only for the largest values of the specific impulse, a good margin of feasibility can be guaranteed for the SSTO configuration.

(v) For the case of nonuniform structural factor, let us compare the TSTO configuration with a SSTO configuration having the same payload, same initial thrust-to-weight ratio, and same specific impulse. Assume that the Stage 1 structural factor of the TSTO configuration is the same as that of the SSTO configuration. For the same payload, the Stage 2 structural factor is considerably larger than the Stage 1 structural factor, implying increased safety and reliability of the TSTO configuration vis-a-vis the SSTO configuration.

(vi) For the case of nonuniform structural factor, the fact that $\tilde{c}$ can be larger than $\hat{c}$ suggests that an attractive TSTO design might be a first-stage structure made of only
tanks and a second-stage structure made of engines, tanks, electronics, and so on.

(vii) For SSSO configurations, the maximum payload weight behaves almost linearly with respect to the engine specific impulse and almost quadratically with respect to the spacecraft structural factor. For SSTO configurations, the maximum payload weight behaves almost linearly with respect to the engine specific impulse and the spacecraft structural factor; this property holds also for TSTO configurations as long as the ratio of the Stage 2 structural factor to Stage 1 structural factor is held constant. With reference to the specific impulse/structural factor domain, these properties lead to the construction of zero-payload lines separating the feasibility region (positive payload) from the unfeasibility region (negative payload). From these lines, one obtains the largest structural factor which guarantees feasibility for given specific impulse, or the smallest specific impulse which guarantees feasibility for given structural factor.

(viii) For SSTO spacecraft to became reality, improvements in engine specific impulse and spacecraft structural factor are desirable, indeed crucial. Paradoxically, aerodynamic improvements are less important in that they yield relatively small improvements in payload weight.

(ix) To sum up, while the design of a SSSO spacecraft appears to be feasible, the design of a SSTO spacecraft, although attractive from a practical point of view (complete reusability of the spacecraft), might be unfeasible depending on the parameter values considered. Indeed, prudence suggests that a TSTO spacecraft be given concurrent consideration, especially if it is not possible to achieve major improvements in engine specific impulse.
References


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List of Tables

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Table 3. SSSO spacecraft: Normalized weight distribution, 
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Table 4. SSTO spacecraft: Normalized weight distribution, 
\[ \varepsilon = 0.08. \]

Table 5. SSTO spacecraft: Normalized weight distribution, 
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Table 12. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \  \hat{e} = 0.08, \ k = 2.5, \ \text{nonuniform structural factor}. \]

Table 13. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \  \hat{e} = 0.08, \ k = 3.0, \ \text{nonuniform structural factor}. \]

Table 14. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \  \hat{e} = 0.10, \ k = 1.5, \ \text{nonuniform structural factor}. \]

Table 15. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \  \hat{e} = 0.10, \ k = 2.0, \ \text{nonuniform structural factor}. \]

Table 16. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \  \hat{e} = 0.10, \ k = 2.5, \ \text{nonuniform structural factor}. \]

Table 17. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \  \hat{e} = 0.10, \ k = 3.0, \ \text{nonuniform structural factor}. \]

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\[ \sigma = 1.4, \  \hat{e} = 0.12, \ k = 2.0, \ \text{nonuniform structural factor}. \]

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\[ \sigma = 1.4, \  \hat{e} = 0.12, \ k = 2.5, \ \text{nonuniform structural factor}. \]

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\[ \sigma = 1.4, k = 1.0, \text{uniform structural factor, zero-payload line}. \]

Fig. 57. TSTO spacecraft: Normalized payload weight vs specific impulse,
\[ \sigma = 1.4, k = 1.5, \text{nonuniform structural factor, least square fit}. \]

Fig. 58. TSTO spacecraft: Structural factor vs specific impulse,
\[ \sigma = 1.4, k = 1.5, \text{nonuniform structural factor, zero-payload line}. \]

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\[ \sigma = 1.4, k = 2.0, \text{nonuniform structural factor, least square fit}. \]

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$\sigma = 1.4$, $I_{sp} = 440$ sec, $\varepsilon = 0.10$.

TSTO spacecraft: Normalized payload weight vs drag factor $\eta = D'/D$.

$\sigma = 1.4$, $I_{sp} = 440$ sec, $\dot{\varepsilon} = \ddot{\varepsilon} = 0.10$, $k = 1.0$, uniform structural factor.
Table 1. SSSO spacecraft: Normalized weight distribution,

\[ \varepsilon = 0.08. \]

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Table 2. SSSO spacecraft: Normalized weight distribution,

\( \varepsilon = 0.10 \).

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Table 3. SSSO spacecraft: Normalized weight distribution,

\( \varepsilon = 0.12. \)

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Table 4. SSTO spacecraft: Normalized weight distribution, $\varepsilon = 0.08$.

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Table 5. SSTO spacecraft: Normalized weight distribution,

$\varepsilon = 0.10$.

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Table 6. SSTO spacecraft: Normalized weight distribution.

\( \varepsilon = 0.12. \)

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Table 7. TSTO spacecraft: Normalized weight distribution,

\( \hat{\varepsilon} = \bar{\varepsilon} = 0.08, k = 1.0, \) uniform structural factor.

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Table 8. TSTO spacecraft: Normalized weight distribution.

$\tilde{\epsilon} = \bar{\epsilon} = 0.10$, $k = 1.0$, uniform structural factor.

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Table 9. TSTO spacecraft: Normalized weight distribution,

\[ \hat{\varepsilon} = \bar{\varepsilon} = 0.12, \ k = 1.0, \ \text{uniform structural factor}. \]

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<td>0.054</td>
</tr>
<tr>
<td>1.5</td>
<td>440</td>
<td>0.089</td>
<td>0.831</td>
<td>0.114</td>
<td>0.055</td>
</tr>
<tr>
<td>1.3</td>
<td>460</td>
<td>0.089</td>
<td>0.825</td>
<td>0.113</td>
<td>0.062</td>
</tr>
<tr>
<td>1.4</td>
<td>460</td>
<td>0.094</td>
<td>0.825</td>
<td>0.112</td>
<td>0.063</td>
</tr>
<tr>
<td>1.5</td>
<td>460</td>
<td>0.098</td>
<td>0.823</td>
<td>0.112</td>
<td>0.065</td>
</tr>
<tr>
<td>1.3</td>
<td>480</td>
<td>0.098</td>
<td>0.818</td>
<td>0.111</td>
<td>0.071</td>
</tr>
<tr>
<td>1.4</td>
<td>480</td>
<td>0.103</td>
<td>0.816</td>
<td>0.111</td>
<td>0.073</td>
</tr>
<tr>
<td>1.5</td>
<td>480</td>
<td>0.106</td>
<td>0.816</td>
<td>0.111</td>
<td>0.073</td>
</tr>
<tr>
<td>1.3</td>
<td>500</td>
<td>0.106</td>
<td>0.810</td>
<td>0.111</td>
<td>0.079</td>
</tr>
<tr>
<td>1.4</td>
<td>500</td>
<td>0.112</td>
<td>0.808</td>
<td>0.110</td>
<td>0.082</td>
</tr>
<tr>
<td>1.5</td>
<td>500</td>
<td>0.117</td>
<td>0.806</td>
<td>0.110</td>
<td>0.084</td>
</tr>
</tbody>
</table>
Table 10. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \; \hat{e} = 0.08, \; k = 1.5, \; \text{nonuniform structural factor}. \]

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( \frac{W_d}{W_i} )</th>
<th>( \frac{W_p}{W_i} )</th>
<th>( \frac{W_s}{W_i} )</th>
<th>( \frac{W_f}{W_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.073</td>
<td>0.874</td>
<td>0.088</td>
<td>0.038</td>
</tr>
<tr>
<td>420</td>
<td>0.083</td>
<td>0.864</td>
<td>0.088</td>
<td>0.048</td>
</tr>
<tr>
<td>440</td>
<td>0.093</td>
<td>0.854</td>
<td>0.087</td>
<td>0.059</td>
</tr>
<tr>
<td>460</td>
<td>0.100</td>
<td>0.847</td>
<td>0.085</td>
<td>0.068</td>
</tr>
<tr>
<td>480</td>
<td>0.110</td>
<td>0.837</td>
<td>0.085</td>
<td>0.078</td>
</tr>
<tr>
<td>500</td>
<td>0.120</td>
<td>0.827</td>
<td>0.084</td>
<td>0.089</td>
</tr>
</tbody>
</table>

Table 11. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \; \hat{e} = 0.08, \; k = 2.0, \; \text{nonuniform structural factor}. \]

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( \frac{W_d}{W_i} )</th>
<th>( \frac{W_p}{W_i} )</th>
<th>( \frac{W_s}{W_i} )</th>
<th>( \frac{W_f}{W_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.073</td>
<td>0.873</td>
<td>0.102</td>
<td>0.025</td>
</tr>
<tr>
<td>420</td>
<td>0.082</td>
<td>0.864</td>
<td>0.101</td>
<td>0.035</td>
</tr>
<tr>
<td>440</td>
<td>0.092</td>
<td>0.855</td>
<td>0.099</td>
<td>0.046</td>
</tr>
<tr>
<td>460</td>
<td>0.100</td>
<td>0.846</td>
<td>0.098</td>
<td>0.056</td>
</tr>
<tr>
<td>480</td>
<td>0.110</td>
<td>0.837</td>
<td>0.096</td>
<td>0.067</td>
</tr>
<tr>
<td>500</td>
<td>0.119</td>
<td>0.828</td>
<td>0.095</td>
<td>0.077</td>
</tr>
</tbody>
</table>
Table 12. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \ \hat{\epsilon} = 0.08, \ k = 2.5, \ \text{nonuniform structural factor.} \]

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_c/W_i )</th>
<th>( W_d/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.072</td>
<td>0.873</td>
<td>0.116</td>
<td>0.011</td>
</tr>
<tr>
<td>420</td>
<td>0.081</td>
<td>0.865</td>
<td>0.114</td>
<td>0.021</td>
</tr>
<tr>
<td>440</td>
<td>0.091</td>
<td>0.855</td>
<td>0.113</td>
<td>0.032</td>
</tr>
<tr>
<td>460</td>
<td>0.100</td>
<td>0.846</td>
<td>0.111</td>
<td>0.043</td>
</tr>
<tr>
<td>480</td>
<td>0.109</td>
<td>0.837</td>
<td>0.109</td>
<td>0.054</td>
</tr>
<tr>
<td>500</td>
<td>0.118</td>
<td>0.828</td>
<td>0.107</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Table 13. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \ \hat{\epsilon} = 0.08, \ k = 3.0, \ \text{nonuniform structural factor.} \]

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_c/W_i )</th>
<th>( W_d/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.071</td>
<td>0.874</td>
<td>0.131</td>
<td>-0.005</td>
</tr>
<tr>
<td>420</td>
<td>0.080</td>
<td>0.865</td>
<td>0.129</td>
<td>0.006</td>
</tr>
<tr>
<td>440</td>
<td>0.089</td>
<td>0.856</td>
<td>0.127</td>
<td>0.017</td>
</tr>
<tr>
<td>460</td>
<td>0.098</td>
<td>0.847</td>
<td>0.125</td>
<td>0.028</td>
</tr>
<tr>
<td>480</td>
<td>0.108</td>
<td>0.838</td>
<td>0.122</td>
<td>0.040</td>
</tr>
<tr>
<td>500</td>
<td>0.117</td>
<td>0.829</td>
<td>0.120</td>
<td>0.051</td>
</tr>
</tbody>
</table>
Table 14. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \, \hat{\varepsilon} = 0.10, \, k = 1.5 \], nonuniform structural factor.

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_s/W_i )</th>
<th>( W_o/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.070</td>
<td>0.861</td>
<td>0.112</td>
<td>0.027</td>
</tr>
<tr>
<td>420</td>
<td>0.078</td>
<td>0.854</td>
<td>0.110</td>
<td>0.036</td>
</tr>
<tr>
<td>440</td>
<td>0.087</td>
<td>0.845</td>
<td>0.109</td>
<td>0.046</td>
</tr>
<tr>
<td>460</td>
<td>0.097</td>
<td>0.836</td>
<td>0.107</td>
<td>0.057</td>
</tr>
<tr>
<td>480</td>
<td>0.105</td>
<td>0.828</td>
<td>0.106</td>
<td>0.066</td>
</tr>
<tr>
<td>500</td>
<td>0.114</td>
<td>0.819</td>
<td>0.105</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Table 15. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \, \hat{\varepsilon} = 0.10, \, k = 2.0 \], nonuniform structural factor.

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_s/W_i )</th>
<th>( W_o/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.069</td>
<td>0.861</td>
<td>0.129</td>
<td>0.010</td>
</tr>
<tr>
<td>420</td>
<td>0.077</td>
<td>0.854</td>
<td>0.126</td>
<td>0.020</td>
</tr>
<tr>
<td>440</td>
<td>0.086</td>
<td>0.845</td>
<td>0.125</td>
<td>0.030</td>
</tr>
<tr>
<td>460</td>
<td>0.095</td>
<td>0.837</td>
<td>0.122</td>
<td>0.041</td>
</tr>
<tr>
<td>480</td>
<td>0.104</td>
<td>0.828</td>
<td>0.121</td>
<td>0.051</td>
</tr>
<tr>
<td>500</td>
<td>0.113</td>
<td>0.819</td>
<td>0.120</td>
<td>0.061</td>
</tr>
</tbody>
</table>
Table 16. TSTO spacecraft: Normalized weight distribution,

$\sigma = 1.4$, $\hat{\varepsilon} = 0.10$, $k = 2.5$, nonuniform structural factor.

<table>
<thead>
<tr>
<th>$I_{sp}$ [sec]</th>
<th>$W_f/W_i$</th>
<th>$W_p/W_i$</th>
<th>$W_s/W_i$</th>
<th>$W_r/W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.067</td>
<td>0.863</td>
<td>0.146</td>
<td>-0.009</td>
</tr>
<tr>
<td>420</td>
<td>0.075</td>
<td>0.857</td>
<td>0.142</td>
<td>0.001</td>
</tr>
<tr>
<td>440</td>
<td>0.084</td>
<td>0.846</td>
<td>0.142</td>
<td>0.012</td>
</tr>
<tr>
<td>460</td>
<td>0.094</td>
<td>0.837</td>
<td>0.140</td>
<td>0.023</td>
</tr>
<tr>
<td>480</td>
<td>0.103</td>
<td>0.828</td>
<td>0.138</td>
<td>0.034</td>
</tr>
<tr>
<td>500</td>
<td>0.111</td>
<td>0.820</td>
<td>0.135</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Table 17. TSTO spacecraft: Normalized weight distribution,

$\sigma = 1.4$, $\hat{\varepsilon} = 0.10$, $k = 3.0$, nonuniform structural factor.

<table>
<thead>
<tr>
<th>$I_{sp}$ [sec]</th>
<th>$W_f/W_i$</th>
<th>$W_p/W_i$</th>
<th>$W_s/W_i$</th>
<th>$W_r/W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.065</td>
<td>0.864</td>
<td>0.166</td>
<td>-0.030</td>
</tr>
<tr>
<td>420</td>
<td>0.073</td>
<td>0.856</td>
<td>0.163</td>
<td>-0.019</td>
</tr>
<tr>
<td>440</td>
<td>0.082</td>
<td>0.847</td>
<td>0.161</td>
<td>-0.008</td>
</tr>
<tr>
<td>460</td>
<td>0.092</td>
<td>0.838</td>
<td>0.158</td>
<td>0.004</td>
</tr>
<tr>
<td>480</td>
<td>0.101</td>
<td>0.829</td>
<td>0.156</td>
<td>0.015</td>
</tr>
<tr>
<td>500</td>
<td>0.109</td>
<td>0.821</td>
<td>0.152</td>
<td>0.027</td>
</tr>
</tbody>
</table>
Table 18. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \; \dot{e} = 0.12, \; k = 1.5, \] nonuniform structural factor.

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( W_d/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_f/W_i )</th>
<th>( W_d/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.066</td>
<td>0.849</td>
<td>0.135</td>
<td>0.016</td>
</tr>
<tr>
<td>420</td>
<td>0.073</td>
<td>0.842</td>
<td>0.133</td>
<td>0.025</td>
</tr>
<tr>
<td>440</td>
<td>0.082</td>
<td>0.834</td>
<td>0.132</td>
<td>0.034</td>
</tr>
<tr>
<td>460</td>
<td>0.091</td>
<td>0.825</td>
<td>0.131</td>
<td>0.044</td>
</tr>
<tr>
<td>480</td>
<td>0.100</td>
<td>0.817</td>
<td>0.128</td>
<td>0.055</td>
</tr>
<tr>
<td>500</td>
<td>0.108</td>
<td>0.809</td>
<td>0.127</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Table 19. TSTO spacecraft: Normalized weight distribution,

\[ \sigma = 1.4, \; \dot{e} = 0.12, \; k = 2.0, \] nonuniform structural factor.

<table>
<thead>
<tr>
<th>( I_{sp} ) [sec]</th>
<th>( W_d/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_f/W_i )</th>
<th>( W_d/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.063</td>
<td>0.850</td>
<td>0.156</td>
<td>-0.006</td>
</tr>
<tr>
<td>420</td>
<td>0.071</td>
<td>0.843</td>
<td>0.152</td>
<td>0.005</td>
</tr>
<tr>
<td>440</td>
<td>0.080</td>
<td>0.834</td>
<td>0.152</td>
<td>0.014</td>
</tr>
<tr>
<td>460</td>
<td>0.089</td>
<td>0.826</td>
<td>0.149</td>
<td>0.025</td>
</tr>
<tr>
<td>480</td>
<td>0.098</td>
<td>0.817</td>
<td>0.148</td>
<td>0.035</td>
</tr>
<tr>
<td>500</td>
<td>0.106</td>
<td>0.810</td>
<td>0.144</td>
<td>0.046</td>
</tr>
</tbody>
</table>
Table 20. TSTO spacecraft: Normalized weight distribution, 

$\sigma = 1.4$, $\hat{\varepsilon} = 0.12$, $k = 2.5$, nonuniform structural factor.

<table>
<thead>
<tr>
<th>$I_p$ [sec]</th>
<th>$W_d/W_i$</th>
<th>$W_p/W_i$</th>
<th>$W_f/W_i$</th>
<th>$W_f'/W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.060</td>
<td>0.852</td>
<td>0.177</td>
<td>-0.029</td>
</tr>
<tr>
<td>420</td>
<td>0.068</td>
<td>0.845</td>
<td>0.174</td>
<td>-0.019</td>
</tr>
<tr>
<td>440</td>
<td>0.077</td>
<td>0.836</td>
<td>0.172</td>
<td>-0.008</td>
</tr>
<tr>
<td>460</td>
<td>0.087</td>
<td>0.827</td>
<td>0.170</td>
<td>0.003</td>
</tr>
<tr>
<td>480</td>
<td>0.095</td>
<td>0.819</td>
<td>0.167</td>
<td>0.014</td>
</tr>
<tr>
<td>500</td>
<td>0.104</td>
<td>0.811</td>
<td>0.164</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 21. TSTO spacecraft: Normalized weight distribution, 

$\sigma = 1.4$, $\hat{\varepsilon} = 0.12$, $k = 3.0$, nonuniform structural factor.

<table>
<thead>
<tr>
<th>$I_p$ [sec]</th>
<th>$W_d/W_i$</th>
<th>$W_p/W_i$</th>
<th>$W_f/W_i$</th>
<th>$W_f'/W_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.060</td>
<td>0.852</td>
<td>0.203</td>
<td>-0.055</td>
</tr>
<tr>
<td>420</td>
<td>0.067</td>
<td>0.845</td>
<td>0.200</td>
<td>-0.045</td>
</tr>
<tr>
<td>440</td>
<td>0.075</td>
<td>0.838</td>
<td>0.196</td>
<td>-0.034</td>
</tr>
<tr>
<td>460</td>
<td>0.083</td>
<td>0.829</td>
<td>0.193</td>
<td>-0.022</td>
</tr>
<tr>
<td>480</td>
<td>0.092</td>
<td>0.821</td>
<td>0.189</td>
<td>-0.010</td>
</tr>
<tr>
<td>500</td>
<td>0.102</td>
<td>0.812</td>
<td>0.186</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Table 22. SSSO spacecraft: Effect of drag change on normalized weights, \( \varepsilon = 0.10. \)

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( I_\text{sp} ) [sec]</th>
<th>( \eta )</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_s/W_i )</th>
<th>( W_s/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>420</td>
<td>0.5</td>
<td>0.264</td>
<td>0.736</td>
<td>0.082</td>
<td>0.182</td>
</tr>
<tr>
<td>1.5</td>
<td>420</td>
<td>1.0</td>
<td>0.255</td>
<td>0.745</td>
<td>0.083</td>
<td>0.172</td>
</tr>
<tr>
<td>1.5</td>
<td>420</td>
<td>1.5</td>
<td>0.247</td>
<td>0.753</td>
<td>0.084</td>
<td>0.163</td>
</tr>
</tbody>
</table>

Table 23. SSTO spacecraft: Effect of drag change on normalized weights, \( \varepsilon = 0.10. \)

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( I_\text{sp} ) [sec]</th>
<th>( \eta )</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_s/W_i )</th>
<th>( W_s/W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>440</td>
<td>0.5</td>
<td>0.108</td>
<td>0.892</td>
<td>0.099</td>
<td>0.009</td>
</tr>
<tr>
<td>1.4</td>
<td>440</td>
<td>1.0</td>
<td>0.107</td>
<td>0.893</td>
<td>0.099</td>
<td>0.008</td>
</tr>
<tr>
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<td>0.105</td>
<td>0.895</td>
<td>0.099</td>
<td>0.006</td>
</tr>
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</table>

Table 24. TSTO spacecraft: Effect of drag change on normalized weights, \( \hat{\varepsilon} = \bar{\varepsilon} = 0.10, k = 1.0, \) uniform structural factor.

<table>
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<tr>
<th>( \sigma )</th>
<th>( I_\text{sp} ) [sec]</th>
<th>( \eta )</th>
<th>( W_i/W_i )</th>
<th>( W_p/W_i )</th>
<th>( W_s/W_i )</th>
<th>( W_s/W_i )</th>
</tr>
</thead>
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<td>0.5</td>
<td>0.091</td>
<td>0.843</td>
<td>0.094</td>
<td>0.064</td>
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<td>1.0</td>
<td>0.089</td>
<td>0.844</td>
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<td>0.062</td>
</tr>
<tr>
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<td>1.5</td>
<td>0.088</td>
<td>0.844</td>
<td>0.094</td>
<td>0.061</td>
</tr>
</tbody>
</table>
Fig. 1. Coefficient $A_0$ vs Mach number.

Fig. 2. Coefficient $A_2$ vs Mach number.
Fig. 3. Coefficient $B_0$ vs Mach number.

Fig. 4. Coefficient $B_1$ vs Mach number.
Fig. 5. SSSO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$.

Fig. 6. SSSO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.5$.

Fig. 7. SSSO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.6$. 
Fig. 8. SSSO spacecraft: Altitude vs time, 
\[ \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10. \]

Fig. 9. SSSO spacecraft: Velocity vs time, 
\[ \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10. \]
Fig. 10. SSSO spacecraft: Flight path angle vs time, \( \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10. \)

Fig. 11. SSSO spacecraft: Normalized weight vs time, \( \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10. \)
Fig. 12. SSSO spacecraft: Angle of attack vs time,
\( \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10 \).

Fig. 13. SSSO spacecraft: Power setting vs time,
\( \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10 \).
Fig. 14. SSSO spacecraft: Thrust-to-weight ratio vs time, 
$\sigma = 1.5, I_{sp} = 420$ sec, $\varepsilon = 0.10$.

Fig. 15. SSSO spacecraft: Tangential acceleration vs time, 
$\sigma = 1.5, I_{sp} = 420$ sec, $\varepsilon = 0.10$. 
Fig. 16. SSSO spacecraft: Dynamic pressure vs time, 
\[ \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10. \]

Fig. 17. SSSO spacecraft: Heating rate vs time, 
\[ \sigma = 1.5, I_{sp} = 420 \text{ sec}, \varepsilon = 0.10. \]
Fig. 18. SSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.3$.

Fig. 19. SSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$.

Fig. 20. SSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.5$. 
Fig. 21. SSTO spacecraft: Altitude vs time, 
\( \sigma = 1.4, I_p = 440 \text{ sec, } \varepsilon = 0.10 \).

Fig. 22. SSTO spacecraft: Velocity vs time, 
\( \sigma = 1.4, I_p = 440 \text{ sec, } \varepsilon = 0.10 \).
Fig. 23. SSTO spacecraft: Flight path angle vs time, 
\[ \sigma = 1.4, \quad I_{sp} = 440 \text{ sec}, \quad \varepsilon = 0.10. \]

Fig. 24. SSTO spacecraft: Normalized weight vs time, 
\[ \sigma = 1.4, \quad I_{sp} = 440 \text{ sec}, \quad \varepsilon = 0.10. \]
Fig. 25. SSTO spacecraft: Angle of attack vs time, 
$\sigma = 1.4, I_{sp} = 440$ sec, $\varepsilon = 0.10$.

Fig. 26. SSTO spacecraft: Power setting vs time, 
$\sigma = 1.4, I_{sp} = 440$ sec, $\varepsilon = 0.10$. 
Fig. 27. SSTO spacecraft: Thrust-to-weight ratio vs time,
\(\sigma = 1.4, I_{sp} = 440 \text{ sec}, \varepsilon = 0.10.\)

Fig. 28. SSTO spacecraft: Tangential acceleration vs time,
\(\sigma = 1.4, I_{sp} = 440 \text{ sec}, \varepsilon = 0.10.\)
Fig. 29. SSTO spacecraft: Dynamic pressure vs time, 
\( \sigma = 1.4, I_{sp} = 440 \text{ sec}, \varepsilon = 0.10. \)

Fig. 30. SSTO spacecraft: Heating rate vs time, 
\( \sigma = 1.4, I_{sp} = 440 \text{ sec}, \varepsilon = 0.10. \)
Fig. 31. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.3, k = 1.0 \), uniform structural factor.

Fig. 32. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.4, k = 1.0 \), uniform structural factor.

Fig. 33. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.5, k = 1.0 \), uniform structural factor.
Fig. 34. SSTO/TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $\varepsilon = 0.08$, $\hat{\varepsilon} = \bar{\varepsilon} = 0.08$, $k = 1.0$, uniform structural factor.

Fig. 35. SSTO/TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $\varepsilon = 0.10$, $\hat{\varepsilon} = \bar{\varepsilon} = 0.10$, $k = 1.0$, uniform structural factor.

Fig. 36. SSTO/TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $\varepsilon = 0.12$, $\hat{\varepsilon} = \bar{\varepsilon} = 0.12$, $k = 1.0$, uniform structural factor.
Fig. 37. TSTO spacecraft: Altitude vs time,
\[ \sigma = 1.4, I_{\text{sp}} = 440 \text{ sec}, \hat{\varepsilon} = \bar{\varepsilon} = 0.10, k = 1.0, \text{ uniform structural factor}. \]

Fig. 38. TSTO spacecraft: Velocity vs time,
\[ \sigma = 1.4, I_{\text{sp}} = 440 \text{ sec}, \hat{\varepsilon} = \bar{\varepsilon} = 0.10, k = 1.0, \text{ uniform structural factor}. \]
Fig. 39. TSTO spacecraft: Flight path angle vs time,
\[ \sigma = 1.4, I_\text{sp} = 440 \text{ sec}, \dot{\varepsilon} = \ddot{\varepsilon} = 0.10, k = 1.0, \text{ uniform structural factor.} \]

Fig. 40. TSTO spacecraft: Normalized weight vs time,
\[ \sigma = 1.4, I_\text{sp} = 440 \text{ sec}, \dot{\varepsilon} = \ddot{\varepsilon} = 0.10, k = 1.0, \text{ uniform structural factor.} \]
Fig. 41. TSTO spacecraft: Angle of attack vs time,
\[ \sigma = 1.4, I_{sp} = 440 \text{ sec}, \dot{\varepsilon} = \ddot{\varepsilon} = 0.10, k = 1.0, \text{ uniform structural factor.} \]

Fig. 42. TSTO spacecraft: Power setting vs time,
\[ \sigma = 1.4, I_{sp} = 440 \text{ sec}, \dot{\varepsilon} = \ddot{\varepsilon} = 0.10, k = 1.0, \text{ uniform structural factor.} \]
Fig. 43. TSTO spacecraft: Thrust-to-weight ratio vs time,
\[ \sigma = 1.4, \ I_{sp} = 440 \text{ sec}, \ \bar{\dot{e}} = \bar{\dot{\epsilon}} = 0.10, \ k = 1.0, \ \text{uniform structural factor}. \]

Fig. 44. TSTO spacecraft: Tangential acceleration vs time,
\[ \sigma = 1.4, \ I_{sp} = 440 \text{ sec}, \ \bar{\dot{e}} = \bar{\dot{\epsilon}} = 0.10, \ k = 1.0, \ \text{uniform structural factor}. \]
Fig. 45. TSTO spacecraft: Dynamic pressure vs time,
\[ \sigma = 1.4, \ I_{sp} = 440 \ \text{sec}, \ \hat{\epsilon} = \bar{\epsilon} = 0.10, \ k = 1.0, \ \text{uniform structural factor.} \]

Fig. 46. TSTO spacecraft: Heating rate vs time,
\[ \sigma = 1.4, \ I_{sp} = 440 \ \text{sec}, \ \hat{\epsilon} = \bar{\epsilon} = 0.10, \ k = 1.0, \ \text{uniform structural factor.} \]
Fig. 47. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.4, k = 1.5 \), nonuniform structural factor.

Fig. 48. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.4, k = 2.0 \), nonuniform structural factor.
Fig. 49. TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $k = 2.5$, nonuniform structural factor.

Fig. 50. TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $k = 3.0$, nonuniform structural factor.
Fig. 51. SSSO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.5$, least square fit.

Fig. 52. SSSO spacecraft: Structural factor vs specific impulse, $\sigma = 1.5$, zero-payload line.
Fig. 53. SSTO spacecraft: Structural factor vs specific impulse, \( \sigma = 1.4 \), least square fit.

Fig. 54. SSTO spacecraft: Structural factor vs specific impulse, \( \sigma = 1.4 \), zero-payload line.
Fig. 55. TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $k = 1.0$, uniform structural factor, least square fit.

Fig. 56. TSTO spacecraft: Structural factor vs specific impulse, $\sigma = 1.4$, $k = 1.0$, uniform structural factor, zero-payload line.
Fig. 57. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.4, k = 1.5 \), nonuniform structural factor, least square fit.

Fig. 58. TSTO spacecraft: Structural factor vs specific impulse, \( \sigma = 1.4, k = 1.5 \), nonuniform structural factor, zero-payload line.
Fig. 59. TSTO spacecraft: Normalized payload weight vs specific impulse, \( \sigma = 1.4, k = 2.0 \), nonuniform structural factor, least square fit.

Fig. 60. TSTO spacecraft: Structural factor vs specific impulse, \( \sigma = 1.4, k = 2.0 \), nonuniform structural factor, zero-payload line.
Fig. 61. TSTO spacecraft: Normalized payload weight vs specific impulse, $\sigma = 1.4$, $k = 2.5$, nonuniform structural factor, least square fit.

Fig. 62. TSTO spacecraft: Structural factor vs specific impulse, $\sigma = 1.4$, $k = 2.5$, nonuniform structural factor, zero-payload line.
Fig. 63. TSTO spacecraft: Normalized payload weight vs specific impulse,  
\( \sigma = 1.4, k = 3.0 \), nonuniform structural factor, least square fit.

Fig. 64. TSTO spacecraft: Structural factor vs specific impulse,  
\( \sigma = 1.4, k = 3.0 \), nonuniform structural factor, zeropayload line.
Fig. 65. SSSO spacecraft: Normalized payload weight vs drag factor $\eta = D'/D$, $\sigma = 1.5$, $I_{sp} = 420$ sec, $\varepsilon = 0.10$.

Fig. 66. SSTO spacecraft: Normalized payload weight vs drag factor $\eta = D'/D$, $\sigma = 1.4$, $I_{sp} = 440$ sec, $\varepsilon = 0.10$.

Fig. 67. TSTO spacecraft: Normalized payload weight vs drag factor $\eta = D'/D$, $\sigma = 1.4$, $I_{sp} = 440$ sec, $\hat{\varepsilon} = \tilde{\varepsilon} = 0.10$, $k = 1.0$, uniform structural factor.