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Intertemporal Arbitrage, Speculative Balances, and the Liquidity Effect

by

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A THESIS SUBMITTED
IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

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Abstract

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This thesis explores money manager intertemporal arbitrage as an explanation of the liquidity effect. We develop a theoretical model of optimal portfolio adjustment by professional money managers, and show that they engage in intertemporal asset price arbitrage; they reduce their holdings of financial assets when they expect asset prices to fall, and increase their holdings when they expect prices to rise. Since a reduction in financial assets can only be accomplished through an increase in money holdings, a connection exists between intertemporal price arbitrage and speculative balances. We show that in equilibrium, money manager behavior causes market liquidity shocks to be accompanied by a form of asset price overshooting in which asset prices first rise above their long-run value and then slowly return as speculative balances are lent out to borrowers and absorbed into transactions balances. Such asset price overshooting is precisely the liquidity effect, stated in terms of asset prices rather than interest rates. This shadows the result established by Hartley (1990), who showed that the combination of sector-specific liquidity shocks and trading rigidities across sectors will cause general price overshooting in those sectors closest to the money supply injection. The second part of this thesis attempts to identify an empirical relationship between speculative balances and asset prices as a means of verifying the hypothesis that money manager intertemporal price arbitrage generates the liquidity effect. It is not possible to estimate this relationship on an aggregate level because no means exist to identify speculative balances relative to the total money supply. However, it is possible to test if individual money managers engage in intertemporal price arbitrage. We do so by estimating individual institutional investor demand for speculative balances. The data source we use is the Flow of Funds, which gives money holdings for various groups of institutional investors. We find an elasticity of 0.1 for speculative balances with respect to the stock market price-earnings ratio.
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Chapter 1: Introduction

Speculative balances are money holdings which function as an asset rather than as a transactions medium. They necessarily exist whenever the quantity of money in the economy exceeds what is needed for transaction purposes, and they have long been thought to play a role in determining the prices of stocks and bonds. Keynes, in 1935, wrote that "the rate of interest and the price of bonds have to be fixed at the level at which the desire on the part of certain individuals to hold cash (because at that level they feel 'bearish' of the future of bonds) is exactly equal to the amount of cash available for the speculative-motive".¹

This thesis ties the existence and magnitude of speculative balances to the intertemporal arbitrage activities of professional money managers, and shows how such intertemporal arbitrage activities can generate the liquidity effect. We start in Chapter 2 by reviewing existing empirical research on the liquidity effect, defined as a temporary fall in short-term interest rates caused by an increase in the money supply. Existing research is generally geared towards observing if a liquidity effect exists and gauging its magnitude, rather than testing theories of how the liquidity effect is generated.

The obvious way to test if a liquidity effect exists is to investigate if changes in the supply of money affect the interest rate. However, if transactions balances depend only on nominal income (in particular, if they do not depend on the interest rate), and nominal income is fixed in the short-run, then the liquidity effect can also be tested for by
investigating either i) if aggregate money demand depends negatively on the interest rate. or ii) if speculative balances depend negatively on the interest rate. The latter two approaches derive from Keynes, who theorized that the liquidity effect operates through the speculative balance component of money demand. This assertion is based on assumptions stated earlier. That is, prices and national income are the only determinants of transactions balances and they are fixed in the short run. If these assumptions hold then the only way the demand for money can increase in the short-run, to match an increase in supply, is through a fall in the interest rate which increases the demand for speculative balances.

Regardless of which approach is used to test for the existence of a liquidity effect, the results are only valid if the supply of money is exogenous with respect to asset prices. If the money supply is not exogenous, then it is not clear whether a negative relationship between interests rates and money is due to a liquidity effect, or simply to the negative response of money supply to the interest rate. The latter is called the reverse causality problem, and sets the stage for our own empirical work. Since we use disaggregated data on speculative balance holdings, the reverse causality problem does not appear — the interest rate is exogenous for any individual group of institutional investors (this assumes the units of observation are small enough that their portfolio decisions do not affect the equilibrium interest rate) so that causality always runs from the interest rate to speculative holdings of money. Our empirical work thus makes a unique contribution to the question of whether or not a liquidity effect exists.

Chapter 3 reviews the portfolio rigidity literature on which our theoretical model is loosely based. Ever since Friedman (1968) introduced the helicopter drop to model
increases in the money supply, a standing challenge has existed for economists to try and identify those institutional features responsible for the liquidity effect. A helicopter drop of money means that all agents in the economy instantaneously see their money holdings increase in proportion to the overall increase in the money supply. This method of modeling a money supply increase allows the general price level to change but not the interest rate.

The portfolio rigidity literature advances beyond the helicopter drop method of modeling money supply changes by focusing on the following two institutional features:

1. Only a subset of agents see their money holdings directly affected by a change in the money supply.

2. At least some agents do not immediately adjust their borrowing and saving in response to a change in the interest rate.

Three approaches to implementing these features have appeared in the literature. The first approach is demonstrated by the models of Grossman and Weiss (1983) and Rotemberg (1984), where only banks are directly affected by a change in the money supply and only a subset of agents can adjust their lending and borrowing in any given period in response to a change in the interest rate. The second approach is demonstrated by the model of Hartley (1990), where only a subset of consumer–traders is directly affected by a change in the money supply and only a subset of consumer–traders can adjust their money holdings in any given period. The third approach is demonstrated by Fuerst (1992), where banks are again the ones directly affected by a change in the money supply.
supply, and households can only adjust their lending and borrowing with a one period lag following an initial change in the interest rate (firms can adjust immediately). Chapter 3 gives a detailed summary of these four models, showing the manner in which the institutional constraints give rise to a liquidity effect.

Chapter 4 presents our model showing how intertemporal arbitrage by professional money managers can generate a liquidity effect, and demonstrating inter alia how speculative balances and the interest rate are interrelated. We start by considering the portfolio adjustment problem of professional money managers with an infinite horizon. Their goal is to maximize total returns to their portfolio, subject to the expected future path of asset prices, and the expected future path of interest rates. It is shown that professional money managers will increase their holdings of speculative balances when the interest rate falls. In general equilibrium, this causes market liquidity shocks to be accompanied by a form of asset price overshooting in which asset prices first rise above their long-run value and then slowly return as speculative balances are lent out to borrowers and absorbed into transactions balances.²

Chapter 5 uses Flow of Funds data on institutional investor money holdings to estimate the demand for speculative balances as a function of various asset prices. Our goal is to find evidence of intertemporal price arbitrage on the part of money managers. Intertemporal price arbitrage occurs when money managers delay their purchases of financial assets because they expect a price decline. Their demand for financial assets thus shifts from the current period to future periods. The magnitude of this shift can be measured by money managers' holdings of speculative balances. By definition, speculative balances are money holdings not used to purchase financial assets in the
current period, but which are being held over to purchase assets in future periods. If the
demand for speculative balances increases with the price of financial assets, empirical
support will be provided for the model developed in chapter 4.

---

1 The General Theory of Employment, Interest and Money, p. 171. Speculators are 'bearish' on the future of bonds if they expect the price of bonds to fall.

2 Asset price overshooting is a preliminary rise in asset prices which occurs when subsequent depreciation of the asset will be needed to maintain equilibrium in some markets.
Chapter 2: Previous Empirical Work on The Liquidity Effect

Review

Soon after Keynes published his General Theory, researchers became interested in identifying speculative balances and quantifying their effect on interest rates. The earliest investigator was Brown (1939), who identified speculative balances as

\[ S_t = M_t - \alpha Y_t \]  

(1)

with \( M_t \) and \( Y_t \) being the real money supply and real income, respectively, and \( \alpha \) being the lowest observed ratio of total money balances to income over the sample period. He found a negative relationship between speculative balances and the interest rate for Great Britain. Similar results were obtained for the United States by Tobin (1947).

After these two early attempts, researchers ceased trying to explicitly identify speculative balances. Attention turned towards estimating the standard money demand equation posited by Keynesian theory:

\[ M_t = \beta_s + \beta Y_t + \beta r_t + \varepsilon_t \]  

(2)

where \( M_t \) is total money holdings, whether held for transactions purposes, speculative purposes, or precautionary purposes. \( Y_t \) is a transactions variable such as national income. \( r_t \) is the interest rate, and all variables are in real terms. The effect of the interest rate on money demand, as given by \( \beta_2 \), is generally regarded as being due to adjustments in
speculative balances. Equations of this type have been estimated by many researchers. One prominent example is Meltzer (1963). Using the interest rate on twenty-year bonds, he found its estimated coefficient to be negative, statistically significant, and stable over ten-year intervals. Another example is Laidler (1966), who included short- and long-term interest rates in the same equation, finding both to be statistically significant with the interest rate elasticity of money demand being -.7 for the long-term rate and -.15 for the short-term rate.

Some notable variations on the standard money demand equation were, first, the attempt to take account of the fact that money itself often earns a return. The appropriate interest rate to include in money demand equations would then be the difference between the market interest rates and the return earned on money. Among examples of the latter approach are Lee (1967), who used the interest rate differential between demand deposits and various other assets. He found a statistically significant negative relationship between the interest rate differential and money demand. Klein (1974) used separate variables for the rate of return of money and the rate of return on other assets. He found that they took coefficients of opposite signs but similar orders of magnitude. Barro and Santomero (1972) did similar work which supported Klein's results.

Another variation on the standard money demand equation is the stock adjustment approach. This was initiated by Chow (1967), and involves estimating an equation of the form

$$M_t = \beta_0 + \beta_1 Y_t + \beta_2 r_t + \beta_3 M_{t-1} + \varepsilon_t$$  (3)
which is meant to reflect the fact that it takes time for agents to attain their desired stock of money.

One of the primary purposes of the empirical money demand literature was to measure how changes in interest rates affected the demand for money. This could then be used to gauge the magnitude of the liquidity effect. To make a connection between the liquidity effect and the interest sensitivity of money demand, we must assume that prices and national income respond with a lag to increases in the money supply. Under such conditions, the only way the quantity of money demanded can increase to match an increase in the quantity supplied in the short-run is if the interest rate adjusts. The coefficient on the interest rate in the money demand equation thus defines the magnitude of the liquidity effect.

An alternative approach to investigating the liquidity effect is to directly relate changes in the money supply to changes in the interest rate without making any assumptions about how this relationship comes about. This is called the reduced form approach. The simplest reduced form equation is

$$r_t = \beta(L) M_t + \varepsilon_t$$  \hspace{1cm} (4)

where $\beta(L)$ is a polynomial in the lag operator.\textsuperscript{3} The slope coefficients are generally interpreted as providing evidence on the liquidity effect. Cagan and Gandolfi (1969) using monthly data, employed this type of equation to relate the level of interest rates to the growth rate of the money stock, defined to include time deposits. Their estimates imply that it requires one and a half years for interest rates to return to their initial level following a change in the money supply.
Barro (1977) initiated a new line of research in the money demand literature when he tested the idea that only unanticipated money growth has real effects (defined as a change in the unemployment rate). The idea was subsequently applied to interest rates and most recently appeared in studies by Leeper and Gordon (1992) and Christiano and Eichenbaum (1992). Leeper and Gordon use quarterly data over the period 1954-1990 to regress the federal funds rate on a distributed lag of money shocks. They assume money is generated by a univariate autoregressive process, the residual from which is identified as the monetary shock. They find a significant liquidity effect when either of M0 or M1 is used as the measure of money. This result continues to hold when the data are divided into subperiods, with the exception that an inverse liquidity effect appears in the period 1973-1979 for M0 — an increase in M0 causing an increase in the federal funds rate. The authors then repeat the regressions controlling for consumer prices, industrial production, and the lagged federal funds rate. Under these conditions, the liquidity effect disappears, with an inverse liquidity effect appearing in its place regardless of the period and regardless of whether M0 or M1 is used as the measure of money.

Christiano and Eichenbaum (1992) use quarterly data over the period 1966-1991 to regress the federal funds rate on money innovations and find a significant liquidity effect when non-borrowed reserves is used as the measure of money but an inverse liquidity effect when either M1 or M0 is used. They measure money innovations as the residual from a regression of money on lagged values of real GNP, the GNP deflator, money, and the federal funds rate, with all variables in logs.
Problems Establishing Causality

A problem which is pervasive in both the money demand approach and the reduced form approach to the liquidity effect is that aggregate money holdings are used during estimation and it is not clear that the aggregate supply of money is independent of the interest rate. To see how this causes problems, consider again the simple money demand equation

\[ M_t = \beta_0 + \beta_1 Y_t + \beta_2 r_t + \epsilon \]  \hspace{1cm} (5)

We wish to interpret the coefficient \( \beta_2 \) as capturing the response of money demand to changes in the interest rate. If the supply of money is exogenous, this interpretation is correct because the relationship between \( M_t \) and \( r_t \) will pick up pure demand side effects. However, if the money supply responds to the interest rate, then it is not clear if the relationship between \( M_t \) and \( r_t \) is due to demand-side effects or supply-side effects. If it is the latter, using the estimate of \( \beta_2 \) to draw inference on the liquidity effect is erroneous.

The same problem appears in reduced form estimation. Consider the reduced form equation

\[ r_t = \beta(L) M_t + \epsilon_t \]  \hspace{1cm} (6)

In order to interpret \( \beta(L) \) as capturing the response of the interest rate to changes in the money supply (as is required to establish the existence of a liquidity effect), the supply of money cannot itself respond to changes in the interest rate. If the latter does occur, using the estimate of \( \beta(L) \) to draw inferences on the liquidity effect is erroneous.
The above considerations are more than academic — interest rates can affect the money supply through several avenues. One avenue is bank lending. When interest rates increase, it becomes profitable to lend to marginal customers (customers with high default rates). An expansion in bank lending occurs, which leads to a general expansion of any money supply components that include bank liabilities as a component. Depending on the marginal propensities of agents to hold currency, demand deposits, and different types of savings deposits, all measures of money — M1, M2, and M3 — will increase. Borrowed reserves and M0 will also increase if the higher interest rates make it profitable for banks to engage in arbitrage between the discount rate and the bank lending rate.

Another manner in which interest rates can affect the money supply is through Central Bank policy. If the Central Bank follows a policy of output stabilization, it will tend to increase the money supply when interest rates fall and decrease the money supply when interest rates rise. If the Central Bank follows a policy of money demand accommodation, it will tend to decrease the money supply when interest rates fall and increase the money supply when interest rates rise. The Central Bank controls the money supply by manipulating the amount of reserves in the banking system. This affects the volume of M0 directly, and affects other measures of money due to impact of M0 on bank lending. It is worth noting that causality from the interest rate to the money supply arising from Central Bank behavior will be even more pronounced when the interest rate is used as the proximate target.

The foregoing sets the stage for our own empirical work. Since we use disaggregated data on speculative balance holdings, the reverse causality problem does not appear. The interest rate is exogenous for any individual group of institutional
investors (this assumes the units of observation are small enough that their portfolio decisions do not affect the equilibrium interest rate) so that causality always runs from the interest rate to speculative holdings of money. To see this more clearly let the \( i \)-th professional money manager purchase the quantity \( F'(P_t) \) of financial assets in period \( t \) as a function of the asset price \( P_t \). Let the market supply of new financial assets be given by

\[
F'_t = F'(P_t) + \delta_t \quad \text{(7)}
\]

The market equilibrium price then satisfies

\[
\int F'(P_t) = F'(P_t) + \delta_t \quad \text{(8)}
\]

The assumption that no individual investor can affect the price of financial assets is equivalent to the assumption that a change in any individual \( F'_t \) will not affect the value of the integral on the left-hand side of (8).

\[ 1_{M_{1,M_t}} - \rho_0 \sum_{i=1}^{M_t} \rho_{M_i,M_t} \]
Chapter 3: Theoretical Models

Grossman and Weiss' Model

In Grossman and Weiss's model (1983), portfolio rigidity takes the form of agents not being able to visit the bank in every period. Consumers are divided into two groups and their visits to the bank are staggered — consumers in the first group can withdraw money only in odd periods and consumers in the second group can withdraw money only in even periods. Output is given exogenously and is sold to the household for cash. The only means of obtaining cash is through bank withdrawals. Revenue from the sale of output is divided equally among the households and accrues directly as interest-earning bank deposits.

The lifetime budget constraint, in terms of monetary withdrawals, of a household which visits the bank in even periods is

$$\sum_{i=0}^{c} \alpha_{2i} \frac{M_{2i}}{P_{2i}} = \sum_{i=0}^{c} \alpha_{i} Y_{i}$$

(9)

where $\alpha_{i} = \sum_{t=0}^{i} 1 + r$, with $r$, being the objective rate of interest offered by banks in period $i$. The budget constraint for a household which visits the bank in odd periods is

$$\sum_{i=0}^{c} \alpha_{2i+1} \frac{M_{2i+1}}{P_{2i+1}} = \sum_{i=0}^{c} \alpha_{i} Y_{i} + M_{i}$$

(10)
where \( M_0 \) is the initial money holdings of a household which visits the bank in odd periods (this will be spent in period 0), and \( Y_t \) is nominal income in period \( t \). The budget constraint reflects the fact that the household withdraws money only every second period.

The cash-in-advance constraint for a household which visits the bank at time \( t \) is

\[
M_t = P_t C_t + P_{t+1} C_{t+1}
\]

(11)

Assuming logarithmic utility, the household objective is

\[
\max \sum_{t=0}^{\infty} \beta^t \ln C_t
\]

subject to (11) and (10) or (9), where \( \beta \) is the subjective rate of time discounting.

Taking the path of prices and interest rates as given, the optimal path of consumption is found in two steps. First, \( C_t \) and \( C_{t+1} \) are found by maximizing

\[
\ln C_t + \beta \ln C_{t+1}
\]

subject to (11). Then the optimal sequence of monetary withdrawals is found by substituting \( C_t(M_t) \) into (12) and carrying out the optimization with respect to \( M_t \).

The general equilibrium solution consists of sequences of prices and interest rates which induce agents to exactly consume the exogenously given output of each period and which cause the amount of money which the households at the bank in period \( t \) wishes to hold to be exactly equal to the amount of money the bank has available to lend.

Now consider the steady state of the economy. For convenience we will deal with a representative household of each type, and we will set output each period equal to unity. In the steady state, prices are constant at \( \bar{P}_t \).
Each household consumes $\bar{C}_h$ in the periods in which it goes to the bank, and $\bar{C}_x$ in the other periods. The household which has just gone to the bank has money holdings

$$M_h = \bar{P} \bar{C}_h + \bar{P} \bar{C}_x = \bar{P}$$

(14)

and the household which will go to the bank next period has money holdings

$$M_x = \bar{P} \bar{C}_x$$

(15)

Now consider the effects of an open market operation. The Central Bank injects money into the economy by distributing an equal amount into the bank account of each household. This adds to the quantity of money available to be lent out. General equilibrium requires the that the household which happens to be at the bank at the time of the injection withdraws the entire increase in the money supply, $\Delta M$, along with its usual withdrawal $\bar{M}_h$. Refer to the household which was at the bank at the time of the injection as household $a$ and refer to the other household as household $b$. Let the injection occur at time $t=0$. In the period of the injection we have

$$M'_a = \bar{M}_h + \Delta M > \bar{M}_h$$

(16)

With its larger money holdings, household $a$ will find it optimal to spend more on first period consumption than it was spending in the steady state, so that $P_aC'_a > \bar{P} \bar{C}_h$. The household which will go to the bank next period has not had its money holdings affected and will continue to spend $\bar{M}_x = \bar{P} \bar{C}_x = P_oC'_o$. Thus, we have the relationship between current expenditure and steady state expenditure

$$P_oC'_o + P_oC'_b > \bar{P} \bar{C}_h + \bar{P} \bar{C}_x$$

(17)

Which implies $P_o > \bar{P}$. 
Given that the price level has increased but the household not at the bank still has the steady state level of money holdings, consumption of the household not at the bank must be below its steady state level and consumption of the household at the bank must be at its steady state level.

Grossman and Weiss show that the economy will eventually return to a steady state following the monetary injection. The new steady state will be given by $P = (1 + \Delta M/M) \bar{P}$, $C_i = \bar{C}_i$, $C_j = \bar{C}_j$, with consumption of both households returning to its initial steady state level. This means that household a's increase in consumption is only temporary. In order to induce the household to increase current consumption relative to future consumption, the interest rate must fall. Thus, a monetary injection causes a fall in the interest rate due to the fact that only a subset of households are at the bank at the time the monetary injection occurs, and a fall in the interest rate is necessary to induce these households to absorb the entire injection.

*Rotemberg's Model*

In Grossman and Weiss's model, output is exogenous, so monetary shocks can only affect prices. Rotemberg (1984) employs the same basic set-up, but makes output endogenous through the addition of a capital stock, thus opening an avenue for monetary shocks to have real effects.

As before, household income accrues directly as interest-earning savings deposits. The bank invests these savings by purchasing capital. The timing in the model is as follows. At the beginning of period $t$ the bank issues as much money as desired by the
households at the bank. There is no restriction on the amount of money which the bank can have outstanding within the period, but between periods the quantity of money issued by the bank cannot exceed the quantity of high-powered money $M_t$.

After households at the bank have withdrawn their money, the firms borrow the stock of capital held by the bank and combine it with the inelastically supplied labor of the households to produce output. The production technology exhibits constant returns to scale and the capital stock depreciates completely each period.

After production has been completed, the households use their money to make purchases. The firm uses part or all of its monetary revenue from the households, along with, if necessary, part of the output which was not sold to households, to repay with interest the capital stock which it borrowed from the bank. The remainder, if any, of the firm's monetary revenue and unsold output is split equally among the households and is deposited directly into their savings accounts. The bank thus ends up holding all of the output which has not been consumed. In equilibrium, this will be equal to the amount of capital which the bank desires to bring into the next period.\(^5\)

As before, the solution strategy starts by taking $K_t$, $r_t$, and $P_t$ as given and solving the two period problem given by (13) and (11), using the fact that $Y_t = 0.5 f_t(K_t) \bar{L}$. Then the optimal sequence of money withdrawals is solved for using (12) and (9) or (10). The first order conditions are

$$C_{t+1} = \frac{\beta P_t C_t}{P_{t-1}} = \frac{\beta M_t}{1 + \beta P_{t-1}} = 0$$

(18)
\[-\frac{1}{M_{t-k_1}/P_{t-k_1}} + \frac{\beta^2(1+r_{t-k_2})(1+r_{t-k_1})}{M_{t-k_2}/P_{t-k_2}} = 0 \]  

(19)

The general equilibrium solution consists of a sequence of interest rates, capital stocks, and prices which satisfy (18), (19), and

\[1 + r_t = f'(K_t)\]

\[K_{t+1} = f(K_t) - P_t(c_{t-1} + c_t')\]

\[P_{t+1}C_{t+1} = M_t\]

(20)

The first condition in the immediately preceding trio says that the rate of interest offered by banks must equal the rate of return on the capital stock. The second condition says that the desired future capital stock, \(K_{t+1}\), must be equal to the de facto quantity of capital which the bank finds itself holding at the end of the period. The third condition says that the quantity of money which an agent who withdraws at time \(t\) desires to bring into the next period must be equal to the stock of high-powered money.\(^7\)

Substituting the general equilibrium conditions into (19) and using (18) results in a difference equation governing the evolution of capital.

\[f(K_{t+1}) - K_{t+1} = \beta^2 \frac{1 + \beta(M_{t+2}/M_{t+1})}{1 + \beta(M_t/M_{t+1})} f'(K_{t+1})f''(K_{t+2})[f(K) - K_t + 1] \]

(21)

Now consider the effects of a Central Bank money injection, starting from the steady state. As before, the households at the bank will withdraw the whole of the injection, the price level will increase, and households not at the bank will decrease their consumption below its steady state level.

In contrast to Grossman and Weiss, the households at the bank do not have to increase their consumption to compensate for reduced consumption for those not at the
bank. Excess output can now be used to increase the capital stock. Since the households at the bank effectively receive a wealth transfer when the monetary injection occurs, they might desire to engage in consumption smoothing by increasing the capital stock.

Rotemberg calibrates (21) using \( f(K_r) = K_r^{0.75} \), \( \beta = 0.99 \), and a steady state capital stock of 0.30394. He finds that an unexpected 2 percent increase in the money stock results in a two-tenths of a percent increase in the capital stock the following period, from which point the capital stock gradually declines back to its steady state level. Since the rate of interest is always equal to the marginal product of capital, the interest rate falls in the period following the money injection and then slowly rises to its initial level. Notice that the interest rate does not change in the period of the monetary injection, due to the capital stock being fixed.

**Fuerst's Model**

In Rotemberg's model, as in that of Grossman and Weiss, the group of agents which happens to be at the bank at the time of the monetary injection, and which therefore adjusts more quickly to the injection, is randomly chosen. A refinement of this approach would be to have agents' ability to adjust to money supply shocks be related to certain characteristics of the agents. In Fuerst (1992), agents are characterized as either firms or households, and firms are assumed to be able to adjust more quickly to monetary shocks. This has interesting implications for labor supply, output, and the general price level.
In Fuerst (1992), portfolio rigidity takes the form of households being able to adjust their lending and borrowing less frequently than firms. Since all households are identical and only firms have access to productive capital, only households lend and only firms borrow. Thus, the portfolio adjustment constraint could be recast as a constraint that lenders have slower adjustment speeds than borrowers.

The fact that monetary injections affect households uniformly means that the wealth effects found in Grossman and Weiss, and Rotemberg are absent. Fuerst points out that the elimination of wealth effects lessens the persistence of monetary shocks.

The economy consists of a representative household and a representative firm. The household’s labor supply is endogenized to allow output to respond to monetary injections. The objective of the household is

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, L_t)$$

where $c_t$ is consumption and $L_t$ is the quantity of labor supplied. The endogeneity of household labor supply will allow output to respond to monetary injections.

The firm owns all capital in the economy, which it combines with labor to produce output according to the production function

$$y_t = \theta_t K_t^{1-a} H_t^a$$

The capital stock depreciates completely each period and all inputs must be paid for in cash.

The timing of monetary transactions is as follows. The household enters the period with the quantity of cash $M_t$ and deposits an amount $N_t$ in the financial intermediary. After the household makes its deposit, the productivity shock and money
supply shock are revealed. The firm then goes to the intermediary and borrows working capital, \( B_i \), with which to pay wages and purchase investment goods.

The cash-in-advance constraints are

\[
M_i - N_i \geq P_i C_i
\]  
(24)

\[
B_i \geq P_i k_{i-1} + W_i H_i
\]  
(25)

The firm sells some of its output to the household and purchases the rest of it as investment. The firm then repays its working capital loan, with interest, and pays out any remaining funds to the household. The households cash holdings thus evolve according to

\[
M_{i+1} = [M_i + N_i R_i + W_i L_i - P_i C_i] + [X_i (1 + R_i)]
\
+ [P_i Y_i - W_i H_i - P_i k_{i-1} - B_i R_i]
\]  
(26)

The general equilibrium conditions are

\[
L = H
\]  
(27)

\[
B = N + X
\]  
(28)

\[
C = k + y
\]  
(29)

The associated value function is

\[
J(M, K) = \max_{\eta} \max_{c, l, b, h, k} [U(c, l) + \beta J(M', K')]
\]  
(30)

Let \( \lambda_1 \) and \( \lambda_2 \) be the multipliers associated with (24) and (25). The first order conditions of interest are

\[
E \lambda_1 = E \beta J_{M} (M', K') R
\]  
(31)

\[
\lambda_2 = \beta J_{K} (M', K') R
\]  
(32)
The variable \( \lambda_1 \) is the value of money in the goods market and \( \lambda_2 \) is the value of money in the bonds market. Since the household must choose the quantity of money brought into the goods market prior to the shock, it cannot ensure that money will be equally valuable in the goods market and the bond market after the shock has occurred. The best it can do is to equate \( E\lambda_i = \lambda_2 \), based on the expected distribution of the money shock (which is injected into the bond market). If the money supply shock turns out to be larger than expected, money will be less valuable in the bond market than in the goods market ex post, while if the money shock turns out to be smaller than expected, money will be less valuable in the goods market.

Money supply shocks affect the interest rate according to the relation

\[
[1 + R] = \frac{[\lambda_2 - \lambda_1]}{M_t} + \frac{U'(c_t) / P_t}{\beta EU'(c_{t-1})P_{t-1}}
\]  

(33)

Fuerst refers to \( (\lambda_2 - \lambda_1)/M_t \) as the liquidity effect. It represents how valuable money is in the bonds market relative to the goods market. A monetary injection lowers the value of money in the bond market relative to the goods market, and lowers the interest rate. This stimulates output because working capital is cheaper, inducing firms to expand production. Firms will find it optimal to offer a higher nominal wage\(^4\) to attract more workers, and to purchase more investment goods with the currently available cheap working capital. Fuerst shows that a monetary injection increases investment, output, and the investment-output ratio, and decreases the nominal interest rate. He also shows that the general price level can either increase or decrease.
**Hartley's Model**

The models of Fuerst, Rotemberg, and Grossman and Weiss ignore one important aspect of money supply shocks. That is, as increases in the money supply filter through the economy, some sectors are affected sooner than others. The issue is addressed by Hartley (1990).

In Hartley's model, portfolio rigidity takes the form of each agent being able to trade only with a subset of other agents. The structure of transactions in the model can be visualized by imagining that agents are arranged around a circle. Agents may purchase goods only from the agent on their right and may sell goods only to the agent on their left. All purchases must be paid for in cash. Each agent is endowed with one unit of output each period. There is no lending or borrowing in the economy, so the only way money can be transferred among agents is through transactions.

If the Central Bank makes a monetary transfer to only one of the agents, the other agents can not immediately adjust their own money holdings, but must wait for the cash to be passed around the trading circle. The trading structure may thus be regarded as a form of portfolio rigidity.

Formally, the optimization problem faced by agent $j$ is to choose a sequence $c'_t$ and $M'_{t+1}$ to

$$\max \sum_{t=0}^\infty \beta^t U(c'_t)$$

subject to the constraints

$$M'_{t+1} = M'_t + p_{t+1} c'_{t+1} - p_t c'_t$$

(34)
\[ p'_i c'_i \leq M'_i \]  

(36)

The general equilibrium condition is \( c'_i = 1 \) for all \( j \) and \( t \). Assume the economy is initially in an equilibrium where agents have identical levels of cash balances and each agent spends her entire cash balances each period. The price of output in each market will be \( P'_i = M'_i \) (recall that output is equal to unity). The economy will be in an equilibrium where, each period, agents pass their cash balances to the agent on their right and receive an equal quantity of cash balances from the agent on their left.

Now consider a Central Bank transfer of cash to agent 1 in period 0. Assume initially that agent 1 continues to spend all her cash each period. The price of output in market 1 will rise in proportion to the increase in agent 1's cash balances, and agent 2, who sells output in market 1, will receive all of agent 1's cash. At the same time, agent 1, who sells output in market m, will receive all of agent m's cash, which agent 1 will then spend in period 1. Since agent 1 has less money in period 1 (agent m was not a beneficiary in the Central Bank transfer), the price of output in market 1 will fall back to its initial level.

However, since agent 1 will have the insight to see that the price of output in market 1 is abnormally high in period 0, and that it will fall in period 1, she will actually save part of the initial monetary injection to spend in period 1, when the price jump which occurred in period 0 has dissipated. That is, agent 1 will hoard some of the initial monetary injection. If we think about market 1 as an asset market, such money hoardings function as speculative balances.
Assuming market 1 as an asset market, Hartley's model makes clear predictions about the demand for speculative balances. Let $\mu$ be the initial money balances of each agent and let $\tau$ be the size of the monetary injection. Hartley shows that, conditional on the ratio $\tau/\mu$ and the number of markets, $m$, the quantity of cash hoarded by the recipients of the transfer varies in the same direction as the price of the output which they purchase. He shows that in an economy with $m$ markets and with $\tau/\mu$ satisfying

$$\frac{1 + \beta + \ldots + \beta^s - (s + 1)\beta^{s+1}}{\beta^{s+1}} \geq \frac{\tau}{\mu} \geq \frac{1 + \beta + \ldots + \beta^s - s\beta^s}{\beta^s}$$

(37)

the price of output purchased by the first market follows

$$P_0 = \frac{(s + 1)\mu + \tau}{1 + \beta + \ldots + \beta^s},$$

(38)

$$P_t = \beta P_{t-1},$$

(for $t=0,\ldots,s-1$) and the cash hoarded by first market agents is

$$\tau + (\mu - P_0) + (\mu - \beta P_0) + \ldots + (\mu - \beta^s P_0)$$

(39)

(for $t \leq s-2$) which is decreasing in $t$, given that $\beta P_0 > \mu$ for $\tau/\mu$ satisfying (37). It can be seen that prices increase in the period of the transfer and then decline by a factor of $\beta$ in each succeeding period over the interval during which speculative balances are held.

Notice that we have omitted the case $\tau/\mu < 1 + \beta$. This range of values will result in no hoarding. The recipient of the transfer spends the entire amount, along with her other money holdings, in the period of the transfer. Prices will increase proportionately in the period of the transfer and fall in the subsequent period. However, the price effects are sufficiently small that they do not offset the agent's preference for current
consumption over next period consumption. Thus, for sufficiently small injections, no speculative balances are held.

\[\text{Note that portfolio rigidity does not imply that agents are "credit-constrained". Once an agent is at the bank he may withdraw or deposit an unlimited amount of money subject to his long-run budget constraint.}\]

\[\text{The manner in which the economy operates can best be understood if we first consider an analogous economy with no firms and no money. Again, let the bank hold the capital stock on behalf of the households, but now let the bank also undertake production. After production has occurred, let the households withdraw whatever amount of output they want to consume, leaving the rest for next period's capital stock. Now, go one step further and assume that instead of producing output themselves, banks give the capital stock to firms, who produce output. Let the banks also give poker chips to the households. Let the firms accept the poker chips in exchange for output and then pass the chips along to the banks where they are destroyed. Let the firm give to the banks the output which was not purchased with poker chips, which the bank holds for next period's capital stock.}\]

\[\text{The latter fact is merely a technicality, note that the timing of income does not affect the first order condition for an optimum.}\]

\[\text{The equation assumes the cash-in-advance constraint always binds. It also incorporates the fact that a household which visited the bank at period t-1 does not bring any money into period t-1.}\]

\[\text{The real wage cannot increase, but workers are willing to increase their labor supply in order to rebuild their real cash balances in the face of higher future prices.}\]
Chapter 4: The Portfolio Investment Problem of Professional Money Managers

Preliminary Concepts

Professional money managers are entities such as banks, mutual funds, pension funds, and insurance funds, which engage in large-scale purchases of stocks, bonds and other financial assets. They are specialized in the buying and selling of such assets. The group of all professional money managers taken together will be referred to as the financial sector.

Individual money managers obtain investment funds, in the form of money, through transfers from small investors who wish to take advantage of the money manager's superior investment acumen. The problem the money manager faces is to maximize the total return on these funds. Total return is the sum of interest payments, dividend payments, and capital gains. The money manager can hold financial assets and/or money in her portfolio. Generally, she will prefer to hold the entire portfolio as financial assets, since these earn a rate of return and money does not. However, if she thinks financial assets are about to undergo a capital loss large enough to wipe out the income realized through interest and dividend receipts, she will prefer to hold money.

An important difference exists between the money holdings of an individual money manager and the money holdings of the financial sector: although an individual money manager can always reduce her money holdings by purchasing assets, the
financial sector as a whole cannot. This is because with most purchases of financial assets, the selling party is another money manager. The reduction in money holdings on the part of one money manager (the buyer of the financial asset) is thus matched by an increase in money holdings on the part of another money manager (the seller of the financial asset), with the result that the money holdings of the financial sector as a whole are unchanged.

The money holdings of the financial sector can only be reduced when the outstanding stock of financial assets increases through new issues. A money manager purchase of newly issued assets transfers money to agents outside of the financial sector, namely, the firms or other entities which issued the assets. The money holdings of the financial sector as a whole are thus decreased. \textit{Liquidity} is the money holdings of an agent in the financial sector. \textit{Market liquidity} is the aggregate money holdings of the financial sector, also called \textit{financial sector liquidity}.

A sudden one-period increase in market money holdings will be referred to as a \textit{market liquidity shock}. Market liquidity shocks may arise from the private sector or from the Central Bank.

\textit{Model}

We will consider a representative professional money manager who is risk neutral. For simplicity we will assume all financial assets are perpetuity bonds which pay a coupon of $c$ dollars per period. The price of bonds is denoted by $P_t$. 
The money manager enters period \( t \) with money holdings \( M_t \) and bonds of face value \( B_t \). At the beginning of the period, she receives cash payments \( cB_t \) on bonds held in her portfolio and also receives cash inflows \( N_t \) from customers. The money manager then chooses the quantity of bonds \( B_{t+1} \) which she desires to bring into the next period.

The money manager's money holdings evolves according to:

\[
M_{t+1} = M_t + cB_t + N_t - P_t(B_{t+1} - B_t)
\]

(1)

The value of bonds purchased, \( P_t(B_{t+1} - B_t) \), must satisfy the cash-in-advance constraint:

\[
P_t(B_{t+1} - B_t) \leq M_t + cB_t + N_t
\]

(2)

The objective of the money manager is to choose a stream of bond holdings and money holdings to maximize the present discounted value of the stream of total returns to her portfolio:

\[
\max \sum_{t=0}^{\infty} \beta^t \left[ cB_t + (P_t - P_{t-1})B_t \right]
\]

(3)

subject to (1) and (2) with \( M_0, B_0 \) and \( N_t \) exogenously given. The term \( \beta \) is the subjective discount factor of the fund manager.

**Market Equilibrium**

Market equilibrium requires that the quantity of bonds purchased by the representative money manager equal the aggregate supply of such bonds. The value of new bonds supplied to the market is a function of \( P_t \), denoted by

\[
P_tB_t^* = F(P_t)
\]

(4)

where \( F(P_t) \) strictly increases in \( P_t \) and satisfies the steady state condition \( F(P_t) = 0 \).

Setting (4) equal to \( P_t(B_{t+1} - B_t) \) we get the market equilibrium condition
\[ P_t(B_{t+1} - B_t) = F(P_t) \]  

(5)

In market equilibrium we must also make the substitution

\[ N_t = h(P_t) - cB_t, \]

(6)

which defines aggregate cash inflows from customers to the financial sector. The first term, \( h(P_t) \), is the desired gross supply of new investment funds on the part of customers. It is assumed to be a function of the bond price \( P_t \), with the function \( h \) assumed strictly decreasing in \( P_t \) and to satisfy the steady-state condition \( h(P_t^*) = 0 \). To get from the gross supply of new investment funds to the net supply, we must adjust for the fact that part of the desired increase in financial investment will be met by accrued returns on funds previously invested. This is \( cB_t \), the interest earned on the fund’s bond holdings, which must be subtracted from \( h(P_t) \) to get actual cash inflows from customers.

**Definition 1** The *capital gain* in period \( t \) on a bond is \( P_t - P_{t-1} \).

**Definition 2** Initial *liquidity* in period \( t \) is \( M_t \).

**Comment** Initial liquidity is the quantity of money brought into the period.

**Definition 3** Speculative *balances* of the professional money manager in time \( t \) are

\[
S_t = M_t + cB_t + N_t - P_t(B_{t+1} - B_t) \\
= M_{t+1}
\]

**Comment** Speculative balances are money balances left over at the end of the period. They are equal to next period’s initial liquidity.

**Definition 4** The *liquidity withdrawal function* is
\[ L(P_t) = F(P_t) - h(P_t) \]

Comment This pertains to market equilibrium. \( F(P_t) \) is asset-price induced borrowing by firms, which is a flow of money out of the financial sector, and \( h(P_t) \) is the asset-price induced flow of gross investment funds to the financial sector from households. \( L(P_t) \) gives the overall effect on financial sector liquidity of these two flows.

**Assumption 1** \( L(P_t) \) is strictly increasing in \( P_t \).

**Assumption 2** \( L(P_t) > 0 \) for \( P_t > P^* \)

**Preliminary Solution Concepts**

To simplify notation, let the sequence \( \{P_t\}^\infty_{t=1} \) be denoted by \( \bar{P}_t \).

**Definition 5** The value function \( V(B_t, M_t, \bar{P}_{t-1}) \) is the maximized value of (3) subject to the constraints (1) and (2).

Since (3) is additively separable over time, \( V(B_t, M_t, \bar{P}_{t-1}) \) satisfies the Bellman equation:

\[ V(B_t, M_t, \bar{P}_{t-1}) = \max_{b_{t-1}, M_{t-1}} (c + P_t - P_{t-1})B_t + \beta V(B_{t-1}, M_{t-1}, \bar{P}_t) \quad (7) \]

where the maximization is subject to

\[ P_t(B_{t+1} - B_t) \leq M_t + cB_t + N_t \quad (8) \]

\[ M_{t+1} = M_t + cB_t + N_t - P_t(B_{t+1} - B_t) \quad (9) \]
with $M_0$ and $B_0$ given. Note that the only bond prices relevant for choosing current holdings are this period's price and next period's price. Substituting (9) into (7), the choice variable $M_{t+1}$ can be eliminated from the objective function. The market equilibrium condition is

$$P_t(B_{t+1} - B_r) = F(P_t)$$

and the definition of $N_t$ is

$$N_t = h(P_t) - cB_t$$

The first order condition for the optimal choice of $B_{t+1}$ is

$$-\lambda_t P_t + \alpha V_B(B_{t+1}, M_{t+1}, \bar{P}_t) - P_t\alpha V_M(B_{t+1}, M_{t+1}, \bar{P}_t) = 0$$

From the envelope theorem, the value function derivatives in the preceding equation must satisfy

$$V_B(B_t, M_t, \bar{P}_{t-1}) = (c + P_t - P_{t-1}) + \lambda_t [c + P_t] + \alpha V_M(B_{t-1}, M_{t-1}, \bar{P}_t)[c + P_t]$$

$$V_M(B_t, M_t, \bar{P}_{t-1}) = \lambda_t + \alpha V_M(B_{t-1}, M_{t-1}, \bar{P}_t)$$

Equations (8), (9), (12), (13) and (14) give the first order conditions to the money manager's optimization problem. They constitute five equations in the five unknowns $M_{t+1}, B_{t+1}, \lambda_t, V_B,$ and $V_M$, with $N_t$ and $\bar{P}_t$ assumed to be exogenous (recall that $\bar{P}_t$ is our notation for $\{P_t\}_0^\infty$).

The market equilibrium is found by substituting (10) and (11) into the set of equations defining the first order conditions, giving five equations in the new set of variables: $M_{t+1}, P_t, \lambda_t, V_B, \text{and } V_M$, where $P_t$ now replaces $B_{t+1}$. The latter substitution
transforms $P_i$ from an exogenous state variable to an endogenous variable. Once the market equilibrium solution is found, $B_{t+1}$ can be obtained using (9).

**Interpretation of First Order Conditions**

The term $\lambda_i$ is to be interpreted as the value to the money manager of receiving in the current period an extra dollar which would otherwise be received next period.

Equation (12) determines the optimal choice of $B_{t+1}$, the stock of bonds brought into next period. Increasing $B_{t+1}$ by one unit benefits the money manager by an amount $\beta \nu_{t+1}$. However, the additional bond must be purchased with cash, which reduces money holdings by $P_i$, imposing a cost of $P_i [\lambda_i + \beta \nu_{t+1}]$. Optimality requires the costs and benefits of purchasing an additional unit of bonds to cancel each other out, giving us equation (12).

Equation (13) says that the value of bringing an extra unit of bonds into the current period is, first of all, the return $c$ plus the capital gain $P_i - P_{t-1}$. In addition, with $B_{t+1}$ fixed, an extra unit of $B_i$ allows the professional money manager to purchases one less bond in the current period and still attain $B_{t+1}$. This results in an implicit cash saving of $P_i$, which, along with the cash return $c$ constitutes the implicit increase in cash holdings realized by the money manager as a result of bringing an extra unit of bonds into the current period. The value of this increase in cash holdings is $[\lambda_i + \beta \nu_{t+1}] [c + P_i]$. 
Condition (14) says that the value of bringing an extra dollar into time $t$ is the value of having an extra dollar in time $t$ rather than in time $t+1$, given by $\lambda_t$, plus the value of having an extra dollar in time $t+1$, given by $\beta V_{M_t+1}$.

**Steady State**

The steady-state is characterized by a constant constant market liquidity of zero ($M^\ast = 0$), a constant stock of bonds $B^\ast$, and a constant bond price. For convenience, we will set $P^\ast = 1$, making the steady-state rate of return on bonds equal to their absolute return $c$.

**Assumption 3** $L(P^\ast) = 0$

**Comment** This follows from the assumptions $F(P^\ast) = 0$ and $h(P^\ast) = 0$.

**Value Function Solution for a Special Case**

We are going to examine the effect of a money shock $M_t$ at time $t = 0$, assuming we are starting from a steady-state. Imposing market equilibrium, the dynamic solution to the representative professional money manager’s optimization problem consists of functions $P_t(M_t, \bar{P}_{t-1})$, $M_t(M_t, P_{t-1})$, $V_{M_t}(M_t, \bar{P}_{t-1})$, $V_{M_t}(M_t, \bar{P}_{t-1})$, and $\lambda_t(M_t, \bar{P}_{t-1})$ which satisfy the constraints (8) through (11) and the first order conditions (12) through (14).
Our primary object of concern is the function $P_t(M_t, \bar{P}_{t-1})$. The remainder of this chapter centers on guessing a solution for the foregoing price function and exploring its properties. Our guess for the equilibrium price function, taking initial money holdings $M_0$ as given, is presented below in Definition 6. We state several facts concerning the behavior of the price function given by Definition 6. These are supported by lemmas and proofs given in Appendix II. We also posit guesses for $M_t(M_t, \bar{P}_{t-1})$, $V_{R_t}(M_t, \bar{P}_{t-1})$, $V_{M_t}(M_t, \bar{P}_{t-1})$, and $\lambda_t(M_t, \bar{P}_{t-1})$. The proof that our guesses for the preceding four functions, along with our guess for $P_t(M_t, \bar{P}_{t-1})$ as given by Definition 6, satisfy the first order conditions for the professional money manager's optimization problem are given in Appendix II, as is the proof that the implied solution for the value function satisfies Bellman's equation.

If the representative money manager enters period $t$ with money holdings $M_t$, then money holdings at the start of period $t+n$, $n \geq 1$ (starting balances in period $t+n+1$), will be

$$M_{t-n} = M_t - \sum_{i=0}^{n-1} L(P_{t-i})$$

(15)

and speculative balances in period $t+n$ will be

$$S_{t-n} = M_t - \sum_{i=0}^{n} L(P_{t-i})$$

(16)

These statements are proven in Appendix II in Lemma 1 and Lemma 2. Note that $S_{t-n}$ differs from $M_{t-n}$ by the quantity of liquidity withdrawn in period $t+n$:

$$S_{t-n} = M_{t-n} - L(P_{t-n})$$

(17)
Equilibrium Bond Price Sequence

We will now posit a guess for the equilibrium price function, taking initial money holdings \( M_0 \) as given, and assuming the economy is in a steady-state before \( M_0 \) occurs.

Definition 6

Given \( M_0 \), the equilibrium price sequence is

\[
P_t = P^* + (n-t)c + d \quad \text{for } t = 0, \ldots, n \tag{18}
\]

\[
P_t = P^* \quad \text{for } t > n \tag{19}
\]

where \( d \) and \( n \) satisfy

\[
\sum_{t=0}^{n} L[P^* + (n-t)c + d] = M_0 \tag{20}
\]

with \( n \in \{0,1,2,3,\ldots\} \) and

\[
d \in \begin{cases} 
(0,c] \quad \text{for } M_0 \geq 0 \\
[-P^*,0] \quad \text{for } M_0 < 0
\end{cases} \tag{21}
\]

Note that \( n \) and \( d \) in Definition 6 are a function of \( M_0 \). Also note that \( d \) must be greater than zero. Equation (18) says that in periods 1 through \( n \) bonds undergo a capital loss of \( c \), defined as the difference between last period's price and this period's price. This can be demonstrated by simply taking the difference \( P_{t-1} - P_t \). Equation (20) says that the liquidity shock \( M_0 \) will have been eliminated from the market after \( n \) periods.

Definition 6 indicates that following a positive liquidity shock, asset prices are initially above their steady-state level, and undergo capital losses so as to keep the return
on bonds equal to the return on money (zero). This state of affairs persists until all liquidity has been withdrawn from the market.

The case of a negative liquidity shock is not symmetrical to a positive shock because professional money managers cannot hold negative money balances. Unlike the case of positive money balances, which can be gradually dissipated over time, (incipient) negative money balances must be dissipated immediately. This happens through adjustment in the price of assets, which falls sufficiently for liquidity equal to the negative shock to flow into the financial sector. In terms of (20), we will have \( n = 0 \) and \( d \) taking on a negative value.

**Example of Equilibrium Bond Price Sequence**

To illustrate Definition 6, assume that for initial money holdings \( M_0 \), the definition is satisfied by \( n=3 \) and \( d=0.5c \). The bond price will evolve as follows:

\[
\begin{align*}
P_0 &= P^* + 3.5c \\
P_1 &= P^* + 2.5c \\
P_2 &= P^* + 1.5c \\
P_3 &= P^* + 0.5c \\
\vdots
\end{align*}
\]

Market liquidity at the end of \( t+3 \) will be

\[
M_0 - L(P^* + 3.5c) - L(P^* + 2.5c) - L(P^* + 1.5c) - L(P^* + 0.5c) = 0
\]

As illustrated, the equilibrium price sequence is such that the bond price falls by an amount \( c \) in periods 1 through 3, and each period's price induces an amount of liquidity
$L(P_t)$ to be withdrawn such that all liquidity is withdrawn at exactly the same time that
$P_t$ ceases to be above its steady-state value.

**Facts About $P_t(M_t, B_t)$ Proven in Appendix II**

Money manager liquidity will be characterized by the following three properties:
i) following a positive liquidity shock, initial liquidity is positive for the first $n$
   periods following the shock.
ii) following a positive liquidity shock, liquidity declines monotonically. and
iii) following a positive or negative liquidity shock, initial liquidity is zero for
    all periods after the $n^{th}$.

These properties are proved in Appendix II in Lemma 3, Lemma 4, and Lemma 5.

In market equilibrium, the price of bonds falls by an amount $c$ whenever money
managers are holding speculative balances ($S_t > 0$), and never falls by more than $c$. This
is proven in Lemma 6 in Appendix II.

Lemma 7 and Lemma 9 in Appendix II give conditions on $L(P_t)$ which ensure
there exist $\hat{n}$ and $\hat{d}$ satisfying Definition 6. The Lemmas prove that the first part of the
equilibrium price sequence $\{P_t\}^\#_0$ exists for a positive or negative liquidity shock
occurring at time $t = 0$. The second part of the price sequence, $\{P_t\}^{\tau+1}_{\tau+1}$, is given by
$P_t = P^*$ and needs no further exposition. Lemma 10 and Lemma 11 in Appendix II
show that the price sequence established in Lemma 7 and Lemma 9, with $P_t = P^*$ for
t > $\hat{n}$, is feasible, that is, liquidity withdrawals in any period do not exceed the quantity
of liquidity held by the professional money manager in that period.
So far, we have restricted our attention to the fact that $P_t$ exists as a function of $M_0$. Lemma 11 in Appendix II shows that $P_t$ actually depends only on $M_t$, making it compatible with the value function approach which will later be applied to the money manager’s optimization problem.

Lemma 11 allows us to consider a sequence of optimization problems corresponding to initial liquidity of $M_0, M_1, M_2, \ldots$ etc., where $M_0$ is given and the rest of the sequence is generated by the initial liquidity transition function, which is obtained by combining (9) with the market equilibrium conditions (10) and (11) to get

$$M_{t+1} = M_t - L(P_t)$$

Thus we can express the price function as $P(M_t)$, making it compatible with the value function approach, where controls depend only on current state variables.

For future reference, note that the sequences $\hat{n}_0, \hat{n}_1, \hat{n}_2, \ldots$ and $\hat{d}_0, \hat{d}_1, \hat{d}_2, \ldots$ will satisfy

$$\hat{n}_t = \begin{cases} \hat{n}_{t-1} - 1 & \text{if } \hat{n}_{t-1} > 0 \\ 0 & \text{if } \hat{n}_{t-1} = 0 \end{cases} \quad (23)$$

$$\hat{d}_t = \hat{d}_{t-1} \quad (24)$$

The equilibrium price sequence can thus be expressed as

$$P_t = \begin{cases} P^* + \hat{d}_t + (\hat{n}_t - 1)c & \text{if } \hat{n}_{t-1} > 0 \\ P^* & \text{if } \hat{n}_{t-1} = 0 \end{cases} \quad (25)$$

$$\hat{n}_t = \begin{cases} \hat{n}_{t-1} - 1 & \text{if } \hat{n}_{t-1} > 0 \\ 0 & \text{if } \hat{n}_{t-1} = 0 \end{cases} \quad (26)$$

$$\hat{d}_t = \hat{d}_{t-1} \quad (27)$$
Lemma 13 in Appendix II establishes that the price function defined in Definition 6. and as expressed by (25) through (27), is the equilibrium price function which is part of the solution to the money manager's optimization problem. First we must posit formulations for $M_{t-1}(M_t, \bar{P}_{t-1}), V_B(M_t, \bar{P}_{t-1}), V_M(M_t, \bar{P}_{t-1}), \text{and } \lambda(M_t, \bar{P}_{t-1})$. This is done in Appendix I.

**An Example**

We will solve for the price function associated with a quadratic quarterly liquidity withdrawal function, with money holdings restricted to the interval [-5, 30]. Define $L(P_t)$ as

$$L(P_t) = \begin{cases} (P_t - 1)^2 \left(\frac{1}{.024}\right)^2 & \text{if } P_t \geq 1 \\ -(1 - P_t)^2 \left(\frac{1}{.024}\right)^2 & \text{if } P_t < 1 \end{cases}$$

(28)

Figure 1 shows the liquidity withdrawal function, giving the amount of liquidity withdrawn from the market in the current quarter as a function of the bond price for that quarter. We must find the equilibrium price function, which will be of the form

$$P(M_t) = P^* + n_c + d_t$$

(29)

with the pair $d_t$ and $n_t$ satisfying

$$\sum_{i=0}^{n_t} L[P^* + (n_i - i) + d_t] = M_t$$

(30)

where $d_t \in [-5, c]$ and $n_t \in \{0, 1, 2, 3, \ldots\}$. For $M_t \geq 0$ equation (29) takes the form

$$\sum_{i=0}^{n_t} (d_t + ic)^2 \left(\frac{1}{.024}\right)^2 = M_t$$

(31)
which can be rewritten as

\[
\left(\frac{1}{0.024}\right)^2 \left[ (n_t + 1) d_t^2 + c d_t (n_t^2 + n_t + 1) + \frac{c^2 (n_t + 1)(2n_t + 1)}{6} \right] = M_t, \tag{32}
\]

Let \( c = 0.024 \), which is the quarterly counterpart to an annual rate of 10 percent. For any given \( M_t \), \( n_t \), and \( d_t \) can be found by setting \( d_t = 0 \) in (31) and solving for \( \bar{n}_t \), which will not necessarily be an integer. Setting \( n_t \) to the integer portion of \( \bar{n}_t \) and substituting into (31) then allows us to solve for \( d_t \). For practical purposes, \( n_t \) can be found directly as:

\[
n_t = \begin{cases} 
0. & M_t \in [-5.1) \\
1. & M_t \in [1.5) \\
2. & M_t \in [5.14) \\
3. & M_t \in [14.30) 
\end{cases} \tag{33}
\]

The price function for the quadratic liquidity withdrawal function is shown in Figure 2. Initial liquidity is the unobserved (as of time \( t \)) quantity \( M_t \). Speculative balances correspond to the observed quantity \( M_t - L(P_t) \). The latter is shown in Figure 3.

**Yield Curves in Non-Stochastic Environment**

Bonds of any maturity can be priced at time \( t \) once we know the perpetuity price at time \( t \). Denoting the latter by \( P_t \), the price in time \( t \) of a bond maturing \( k \) periods in the future and paying coupon \( c \) is given by

\[
P_t^k = \begin{cases} 
1 + kc & \text{if } k \leq n_t \\
1 + n_t c + d_t = P_t & \text{if } k > n_t
\end{cases} \tag{34}
\]
Figure 1: Liquidity Withdrawal Function
Figure 2: Bond Price Versus Initial Liquidity
Figure 3: Speculative Balance Demand Curve
Equation (34) reflects the fact that bonds of any maturity must fall in price by \( c \) in each period of excess liquidity. If \( k \leq n \), so that the bond matures before all liquidity is dissipated, then the price of the bond is simply its face value of unity plus \( c \) times the number of periods \( k \) remaining before the bond matures. If \( k > n \), so that the bond matures after all liquidity has dissipated, the price of the bond will be \( P_t^k = 1 + d_t + n, c \), which is the same as the price of the perpetuity.

The implicit interest rate on a bond of maturity \( k \) is given as the solution to

\[
P_t^k = \sum_{i=1}^{k} \frac{c}{(1 + r_k)^i} + \frac{1}{(1 + r_k)^k}
\]

(35)

To interpret (35), consider a borrower who is trying to sell newly issued bonds maturing in \( k \) periods. The rate of interest \( r_k \) will give the newly-issued bonds the same yield as existing \( k \)-period bonds which pay \( c \) percent interest and sell at \( P_t^k \) percent of face value. Equation (35) implies that the interest rate on perpetuities is

\[
r_k = \frac{c}{P_t}
\]

(36)

Yield curves for various levels of initial money shock \( M_0 \) are shown in Figure 4. From top to bottom, the yield curves correspond to initial liquidity of -5, -3, -1, 2, 10, 28. For positive \( M_0 \), the yield curve is decreasing, and it decreases more quickly the smaller is initial liquidity. For negative initial liquidity, the yield curve is increasing, and it increases more quickly the more negative is initial liquidity. Note that for \( M_0 = 0 \), the yield curve would be a horizontal line.
Figure 4: Yield Curves for $M_t = -5, -3, -1, 2, 10, 28$
Characteristics of Solution

The model presented in this chapter and the foregoing example demonstrates that intertemporal arbitrage on the part of money managers will cause market liquidity shocks to be accompanied by asset price overshooting. Asset prices must undergo a preliminary increase of \( d, + n, c \) so that they can depreciate each period by an amount \( c \) for as long as excess liquidity persists in the financial market. These results are based on the assumption that the amount of liquidity withdrawn from the financial sector is increasing in the bond price.

An interesting feature of the asset price path is that it induces voluntary hoarding or money among money managers. Hoarding is necessary because it takes time for the supply of financial assets to meet the increase in demand brought on by an increase in market liquidity. Money managers willingly hoard because the change in price between any two periods is sufficient to make hoarding as attractive as holding bonds.

Does Money Manager Liquidity Reflect Market Liquidity?

In order for the representative agent approach to be valid for the professional money manager's problem, it must be the case that the liquidity of individual money managers reflects market liquidity. Only then is it valid to treat the liquidity of an individual money manager as equal to total market liquidity.
It will be instructive to begin our treatment of this problem by looking at how it has been handled previously. Briefly, previous investigations into money demand assumed that money managers have continuous liquidity preference. To illustrate, consider a positive liquidity shock such as a surprise increase in the money supply. The Central Bank increases the money supply by purchasing government securities in an open market operation. The particular money manager who sold government securities to the Central Bank in the open market sale will have too much liquidity relative to interest earning assets. The money manager will try to get rid of the liquidity by using it to purchase financial assets. However, these financial assets must be purchased from some other money manager. That money manager will then be stuck with excess liquidity and must in turn get rid of it by purchasing financial assets from someone else, and so on¹.

As the increase in the money supply gets passed around from money manager to money manager, asset prices get bid up higher and higher. This puts the market on the road to attaining a new equilibrium, because as asset prices increase, money managers will desire to hold additional liquidity. Thus, those money managers with excess liquidity will start to find willing sellers of financial assets among those money managers who wish to increase their liquidity, and the upward pressure on asset prices will start to subside.

In the final equilibrium, asset prices will rise no further and all money managers will have attained their desired portfolio composition. If money managers have similar liquidity preference, the increase in the money supply will at this point be distributed in a balanced manner among the portfolios of all money managers. (Otherwise some money
managers would not yet be satisfied with their liquidity position and more trading would be required for the market to attain equilibrium).

The traditional approach assumes that liquidity preference varies continuously with asset prices. While we have made no assumptions about liquidity preference, it is still possible to argue that market liquidity will tend to end up being evenly distributed among money managers. Consider the case where, instead of money managers having some preferred level of liquidity which they are trying to attain, they instead simply try to get rid of all their liquidity as fast as possible during any period in which the expected return on assets is positive (they get rid of their liquidity by using it to purchase assets) and hold on to their liquidity during any period in which the expected return on assets is zero or negative.

For now we will assume that the price for which the expected return on assets is zero depends on market liquidity. Following a liquidity shock, the interest rate does not immediately jump to its new equilibrium level because it takes time for money managers to become aware of the liquidity of the market. As money managers buy and sell assets from each other during the normal process of portfolio diversification, they gradually become aware of the new state of market liquidity. At some point money managers reach full awareness of market liquidity, asset prices converge on their equilibrium value, and money managers hold onto whatever liquidity they happen to have in their portfolios. How exactly will liquidity be distributed among agents at this point? On average, the quantity of liquidity each agent ends up holding will be proportional to that agent's trading activity. For example, if a particular money manager accounts for two percent of the total value of assets traded in any particular period, we expect that money
manager on average to be in possession of two percent of total liquidity. If each agent's trading activity is a constant proportion of total market trading activity, then the liquidity of individual money managers will vary in proportion to the liquidity of the market. This is a second approach which may be used to justify our use of the representative agent model.

Implications for the Liquidity Effect

The model presented in this chapter implies that if money managers control all financial investment in the economy, and if the rate of liquidity withdrawal from the financial sector is an increasing function of the price of bonds, then a money shock will cause asset price overshooting, with money managers holding speculative balances while the overshooting occurs.

Asset price overshooting is simply the liquidity effect, stated in terms of asset prices rather than interest rates. One implication of the liquidity effect in our model is that the volume of speculative balances held by money managers will be positively correlated with the price of bonds (negatively correlated with the interest rate). This implication is tested in the next chapter.

---

1 Note that these are newly issued bonds.

2 Strictly speaking, asset price overshooting only occurs when the market liquidity shock is larger than \( t_1 p^{t_1} \) This is shown by the asset price function given in Definition 5.

3 Although it may appear in this example that what we are calling liquidity are actually transactions balances, in that they are used to make purchases, such is not the case. Liquidity is defined relative to the economy as a whole. Although any individual money manager may be able to get rid of their liquidity by purchasing assets, the market as a whole cannot get rid of liquidity. For the market as a whole, liquidity is distinguished from transactions balances by the fact that the former are a determinant of asset prices while the latter are not.
Appendix I

Derivation of $M_{t+1}(M_t, \bar{P}_{t-1}), V_n(M_t, \bar{P}_{t-1}), V_M(M_t, \bar{P}_{t-1}),$ and $\lambda(M_t, \bar{P}_{t-1})$

In this appendix we will dispense with the hats over $n, d,$ in order to simplify the notation.

Consider the initial liquidity transition function

$$M_{t+1} = M_t + cB_t + N_t - P_t(B_{t+1} - B_t)$$  \hspace{1cm} (1)

Substituting in $N_t = h(P_t) - cB_t$ and the market equilibrium condition

$$P_t(B_{t+1} - B_t) = F(P_t, Z_t)$$  \hspace{1cm} (2)

we get

$$M_{t+1} = M_t + cB_t + h(P_t) - cB_t - F(P_t)$$

$$= M_t - L(P_t)$$  \hspace{1cm} (3)

The foregoing will be our guess for $M_{t+1}(M_t, \bar{P}_{t-1})$.

Now we will guess a formulation for $V_n(M_t, \bar{P}_{t-1})$. Consider the value of bringing an extra dollar’s worth of bonds into the first period. The contribution to the value function in the first period is

$$c + P_t - P_{t-1}$$  \hspace{1cm} (4)

and the agent has an extra quantity of money $c$ which can be re-invested in bonds. Notice that when the budget constraint is not binding, there is no difference between purchasing bonds in period $t$ and purchasing them in period $t+1$. To see that it makes no difference
when bonds are purchased, observe that spending $SM$ in period 1 results, at the end of period 2, in bond holdings of

$$B = \frac{M}{P_1} \left(1 + \frac{c}{P_2}\right)$$  \hspace{1cm} (5)$$

where we have used the fact that the return on first-period bond holdings may be reinvested in the second period. Spending $SM$ in period 2 results in bond holdings of

$$B = \frac{M}{P_2}$$  \hspace{1cm} (6)$$
equating (5) and (6) we get

$$\frac{M}{P_1} \left(1 + \frac{c}{P_2}\right) = \frac{M}{P_2}$$  \hspace{1cm} (7)$$

Multiplying through by $P_1P_2$ and rearranging we get

$$MP_1 - MP_2 = cM$$  \hspace{1cm} (8)$$

which will be true when the budget constraint is not binding, so that the equality $P_1 - P_2 = c$ holds.

Since there is no difference between purchasing bonds in period $t$ and purchasing them in any period between $t$ and $t + n$, inclusive, we lose nothing by assuming the investor sells her dollar of bonds in time $t$ at price $P_t$, adds the money to the $c$ dollars of income, and purchases bonds in period $t + n$. Thus, bringing an additional $\$1$ worth of bonds into period $t$ results in additional bonds in period $t + n$, of

$$\Delta B_{t+n} = \frac{P_t + c}{P_{t+n}}$$  \hspace{1cm} (9)$$
These bonds will start earning a return in period \( t + n_t + 1 \). Discounted to period \( t \), the total return from time \( t + n_t + 1 \) onward per dollar (face value) of bonds is

\[
\text{Return} = \beta^{n_t+1}c + \beta^{n_t+2}(1+c)c + \beta^{n_t+3}(1+c)^2c + \cdots \\
= \frac{\beta^{n_t+1}c}{1 - \beta(1+c)}
\]  

(10)

Combining (4), (9) and (10) we get

\[
V_R(M_t, \tilde{P}_{t-1}) = c + P_t - P_{t-1} + \left[ \frac{P_t + c}{P_{t-1}} \right] \left[ \frac{\beta^{n_t+1}c}{1 - \beta(1+c)} \right]
\]  

(11)

Now consider the value of bringing an extra dollar of money into the current period. The dollar can purchase \( 1/P_t \) worth of bonds, which will start earning interest next period. As before, we will lose nothing by assuming the dollar is not spent on bonds until period \( t + n_t \). This gives us

\[
V_M(M_t, \tilde{P}_{t-1}) = \left[ \frac{1}{P_{t-n_t}} \right] \left[ \frac{\beta^{n_t+1}c}{1 - \beta(1+c)} \right]
\]  

(12)

We will now find a formula for the multiplier by looking at the difference in value between a dollar in time \( t \) and a dollar in time \( t+1 \). Defining \( \lambda_t = V_M(M_t, \tilde{P}_{t-1}) - \beta V_M(M_{t-1}, \tilde{P}_{t-1}) \). First note that for \( n_t > 0 \) we have. from updating (12),

\[
V_M(M_{t+1}, P_t) = \left[ \frac{1}{P_{t+1-n_t}} \right] \left[ \frac{\beta^{n_t+1}c}{1 - \beta(1+c)} \right]
\]  

(13)

Using the fact that \( n_{t+1} = n_t - 1 \) we get

\[
V_M(M_{t+1}, P_t) = \left[ \frac{1}{P_{t-n_t}} \right] \left[ \frac{\beta^n c}{1 - \beta(1+c)} \right]
\]  

(14)

Multiplying (14) by \( \beta \) we get the present value of a dollar in time \( t + 1 \).
\[ \beta V_M(M_{t-1}, P_t) = \left[ \frac{1}{P_{t-n}} \right] \left[ \frac{\beta^{n-1}c}{1 - \beta(1 + c)} \right] \] (15)

which is identical to \( V_M(M_t, P_{t-1}) \). Thus, for \( n_t > 0 \), the multiplier will be zero. Now consider the case of \( n_t = 0 \). We have

\[ V_M(M_t, P_{t-1}) = \frac{1}{P^*} \frac{\beta c}{1 - \beta(1 + c)} \] (16)

\[ \beta V_M(M_{t-1}, P_{t-1}) = \frac{1}{P^*} \frac{\beta^2 c}{1 - \beta(1 + c)} \] (17)

Using (16) and (17) and defining \( \lambda_t = V_{M,t} - \beta V_{M,t-1} \) we get

\[ \lambda_t = \left[ \frac{\beta - \beta^2}{P^*} \right] \left[ \frac{c}{1 - \beta(1 + c)} \right] \] (18)
**Appendix II**

**Lemma 1**

Assume the representative money manager enters period $t$ with money holdings $M_t$ and initial bond holdings $B_t$. Let the path of bond prices from time $t$ onwards be $\{P_t, P_{t+1}, P_{t+2}, \ldots\}$. Then the representative money manager will enter period $t+n$, $n \geq 1$, with money holdings

$$M_{t+n} = M_t - \sum_{i=0}^{n-1} L(P_{t+i})$$  \hspace{1cm} (1)

**Proof**  From the money balance transition equation we have

$$M_{t+1} = M_t + cB_t + N_t - P_t(B_{t+1} - B_t)$$  \hspace{1cm} (2)

Updating one period we get

$$M_{t+2} = M_{t+1} + cB_{t+1} + N_{t+1} - P_{t+1}(B_{t+2} - B_{t+1})$$  \hspace{1cm} (3)

Using (2) to substitute out $M_{t+1}$ in (3) we get

$$M_{t+2} = M_t + cB_t + cB_{t+1} + N_t + N_{t+1} - P_t(B_{t+1} - B_t) - P_{t+1}(B_{t+2} - B_{t+1})$$  \hspace{1cm} (4)

Continuing in this fashion we get

$$M_{t+n} = M_t + cB_t + cB_{t+1} + \ldots + cB_{t+n-1} + N_t + N_{t+1} + \ldots + N_{t+n-1} - P_t(B_{t+1} - B_t) - P_{t+1}(B_{t+2} - B_{t+1}) - \ldots - P_{t+n-1}(B_{t+n} - B_{t+n-1})$$  \hspace{1cm} (5)

which can be rewritten as

$$M_{t+n} = M_t + \sum_{i=0}^{n-1} \left[ cB_{t+i} + N_{t+i} - P_{t+i}(B_{t+i+1} - B_{t+i}) \right]$$  \hspace{1cm} (6)

Using the definition of $N_t$ we get
\[ M_{t+n} = M_t + \sum_{i=0}^{n-1} \left\{ h(P_{t+i}) - P_{t+i} (B_{t+i+1} - B_{t+i}) \right\} \]  

(7)

and imposing the bond market equilibrium condition we have

\[ M_{t+n} = M_t + \sum_{i=0}^{n-1} \left\{ h(P_{t+i}) - F(P_{t+i}) \right\} \]  

(8)

Using Definition 4 we get

\[ M_{t+n} = M_t - \sum_{i=0}^{n-1} L(P_{t+i}) \]  

(9)

Q.E.D.

**Lemma 2**

Assume the representative money manager enters period \( t \) with money holdings \( M_t \) and initial bond holdings \( B_t \). Let the future path of bond prices from time \( t \) onwards be \( \{P_t, P_{t+1}, P_{t+2}, \ldots\} \). Then speculative balances in period \( t+n \) will be

\[ S_{t-n} = M_t - \sum_{i=0}^{n} L(P_{t+i}) \]  

(10)

**Proof** From Definition 3 and Lemma 1 we have

\[ S_{t-n} = M_{t+n-1} \]

\[ = M_t - \sum_{i=0}^{n} L(P_{t+i}) \]  

(11)

Q.E.D.
Lemma 3

Let \( \{P_i\}_0^\infty \) satisfy Definition 6. If \( M_0 > 0 \) then \( M_i > 0 \) for \( i \leq n \).

Proof. From Lemma 1 we have

\[
M_i = M_0 - \sum_{k=0}^{i-1} L(P_k)
\]

Using Definition 6 to substitute out \( M_0 \) and \( P_k \) we get

\[
M_i = \sum_{k=0}^{n} L\left[P^* + (n-k)c + d\right] - \sum_{k=0}^{i-1} L\left[P^* + (n-k)c + d\right]
\]

which gives us

\[
M_i = \sum_{k=i}^{n} L\left[P^* + (n-k)c + d\right]
\]

Now note that

\[
k \leq n \Rightarrow P_k = \left[P^* + (n-k)c + d\right] > P^*
\]

\[
P_k > P^* \Rightarrow L(P_k) > 0
\]

Since the right-hand side of (14) is a sum over positive terms, the left-hand side

must be positive, establishing the desired result.

Q.E.D.

Lemma 4

If \( M_0 \geq 0 \) then \( M_i > M_{i+1} \) for \( i \leq n \)

Proof. As in the proof for Lemma 3 we have
\[ M_t = \sum_{k=t}^{n} L(P_k) \]

\[ M_{t+1} = \sum_{k=t+1}^{n} L(P_k) \]

\[ M_t - M_{t+1} = L(P_t) \]

Now note that

\[ t \leq n \Rightarrow P_t = \left[ P^* + (n-t)c + d \right] > P^* \]

where we have used the fact that \( d > 0 \). As in the proof for Lemma 3 we have

\[ P_t > P^* \Rightarrow L(P_t) > 0 \]

Q.E.D.

Lemma 5

\( M_t = 0 \) for \( t > n \)

Proof From Lemma 1 we have

\[ M_t = M_0 - \sum_{k=0}^{t-1} L(P_k) \]

\[ = M_0 - \sum_{k=0}^{n} L(P_k) - \sum_{k=n+1}^{t-1} L(P_k) \]

Using Definition 6 and Assumption 3 we get

\[ M_t = M_0 - M_0 - \sum_{k=n+1}^{t-1} 0 \]

\[ = 0 \]

Q.E.D.
Lemma 6

For $M_0 > 0$, the equilibrium price sequence satisfies

\[ P_t - c = P_{t+1} \quad \text{for } S_t > 0 \]
\[ P_t - c \leq P_{t+1} \quad \text{for } S_t = 0 \]

(22)

**Proof** From Definition 3, Lemma 3, and Lemma 5 we have

\[ S_t > 0 \Rightarrow t + 1 \leq n \]
\[ S_t = 0 \Rightarrow t + 1 > n \]

(23)

The proof will use the following facts

\[ t + 1 \leq n \Rightarrow P_{t+1} = \left[ P^* + (n - t - 1)c + d \right] \]

(24)

\[ t + 1 > n \Rightarrow P_{t+1} = P^* \]

(25)

**CASE I:** $[S_t > 0, t + 1 \leq n]$

\[ P_{t+1} - P_t = \left[ P^* + (n - t - 1)c + d \right] - \left[ P^* + (n - t)c + d \right] = -c \]

(26)

**CASE II:** $[S_t = 0, t + 1 = n + 1]$

\[ P_{t+1} - P_t = P_{n+1} - P_n \]
\[ = P^* - P^* + d \]
\[ = -d \]
\[ \geq -c \]

(27)

**CASE III:** $[S_t = 0, t + 1 > n + 1]$

\[ P_{t+1} - P_t = P^* - P^* \]
\[ = 0 \]
\[ \geq -c \]

(28)

*Q.E.D.*
Lemma 7

Let $M_0 > 0$. If $L(P^*_{1})$ is continuous, strictly increasing in $P^*_{1}$, positive for $P^*_{1} > P^*_{0}$, and satisfies the steady state condition $L(P^*_{0}) = 0$, then there exist $\hat{n} \in \{0, 1, 2, 3, \ldots\}$ and $\hat{d} \in (0, c]$ satisfying

$$
\sum_{t=0}^{\hat{n}} L[P^*_{1} + (\hat{n} - t)c + \hat{d}] = M_0
$$

(29)

Proof Define

$$
W(n, d) = \sum_{t=0}^{n} L[P^*_{1} + (n - t)c + d]
$$

(30)

Let $\hat{n}$ satisfy

$$
W(\hat{n}, c) = \sum_{t=0}^{\hat{n}} L[P^*_{1} + (\hat{n} - t)c + c] \geq M_0
$$

(31)

$$
W(\hat{n} - 1, c) = \sum_{t=0}^{\hat{n} - 1} L[P^*_{1} + (\hat{n} - 1 - t)c + c] < M_0
$$

(32)

where such an $\hat{n}$ is guaranteed to exist by the fact that

$$
W(\hat{n}, c) - W(\hat{n} - 1, c) > 0
$$

(33)

as established in Lemma 8 below. Allowing equality to hold in (31) but not in (32) ensures $\hat{d} > 0$, as will become clear shortly. Now note the following

$$
W(\hat{n}, 0) = \sum_{t=0}^{\hat{n}} L[P^*_{1} + (\hat{n} - t)c]
$$

$$
= \sum_{t=0}^{\hat{n}} L[P^*_{1} + (\hat{n} - 1 - t)c + c]
$$

(34)

$$
= L(P^*_{1}) + \sum_{t=0}^{\hat{n} - 1} \rho L[P^*_{1} + (\hat{n} - 1 - t)c + c]
$$

$$
= 0 + W(\hat{n} - 1, c) < M_0
$$
We thus have $W(\hat{n}, c) \geq M_0$ and $W(\hat{n}, 0) < M_0$. Since $W(n, d)$ is continuous in $d$ the intermediate value theorem tells us there must exist some $\hat{d} \in (0, c]$ for which $W(\hat{n}, \hat{d}) = M_0$.

Q.E.D.

Lemma 8

$W(\hat{n}, c) - W(\hat{n} - 1, c) > 0$

Proof From the previous lemma we have

\[
W(\hat{n}, c) - W(\hat{n} - 1, c) = \sum_{r=0}^{\hat{n}} L[P^\ast + (\hat{n} - t)c + c] - \sum_{r=0}^{\hat{n} - 1} L[P^\ast + (\hat{n} - t)c + c]
\]

\[
= \sum_{r=0}^{\hat{n}} L[P^\ast + (\hat{n} - t)c + c] - \sum_{r=1}^{\hat{n}} L[P^\ast + (\hat{n} - t)c + c]
\]

\[
= L[P^\ast + (\hat{n} - 0)c + c] + L[P^\ast + (\hat{n} + 1)c] > 0
\]

where we have used the fact that $L(P_r)$ is strictly increasing in $P$, and positive for $P_r > P^\ast$.

Q.E.D.

Lemma 9

Let $M_0 \leq 0$. If $L(P_r)$ satisfies $L(P_r) \rightarrow -\infty$ as $P_r \rightarrow 0$. and satisfies the steady state condition $L(P^\ast) = 0$, then there exist $\hat{d} \in [-P^\ast, 0]$ and $\hat{n} \in \{0, 1, 2, 3, \ldots\}$ for which

\[
\sum_{r=0}^{\hat{n}} L[P^\ast + (\hat{n} - t)c + \hat{d}] = M_0
\]

Proof Let $\hat{n} = 0$ and define $\hat{d}$ such that
\[ L(P^* + \hat{d}) = M_0 \quad (37) \]

Such a \( \hat{d} \) is guaranteed to exist by the assumption that \( L(P_i) \to -\infty \) as \( P_i \to 0 \).

The pair \((0, \hat{d})\) then satisfies the conditions of the lemma.

\textit{Q.E.D.}

\textbf{Lemma 10}

\( \{P_i\}_0^{\hat{n}} \) as in Definition 6 above satisfies

\[ L(P_i) = L\left[ P^* + (\hat{n} - t)c + \hat{d} \right] \leq M_i \quad (38) \]

\textit{Proof} For \( M_0 \leq 0 \) the result is self-evident, with equality holding in (38). We will proceed with the case \( M_0 > 0 \). From Lemma 1 we have

\[ M_i = M_0 - \sum_{k=0}^{t-1} L\left[ P^* + (\hat{n} - t)c + \hat{d} \right] \quad (39) \]

Using Definition 6 to substitute out \( M_0 \), we get

\[ M_i = \sum_{k=0}^{\hat{n}} L\left[ P^* + (\hat{n} - t)c + \hat{d} \right] - \sum_{k=0}^{t-1} L\left[ P^* + (\hat{n} - t)c + \hat{d} \right] \]

\[ = \sum_{k=0}^{\hat{n}} L\left[ P^* + (\hat{n} - t)c + \hat{d} \right] \quad (40) \]

Now recall that \( L\left[ P^* + (\hat{n} - t)c + \hat{d} \right] > 0 \) for \( t \leq \hat{n} \). Since the right-hand side of (40) is a sum over positive numbers, the left-hand side must be at least as great as any one of them, establishing the desired result.

\textit{Q.E.D.}
Lemma 11

\( \{ P_r \}_{r=0}^{n} \) as in Definition 6 satisfies

\[
L(P_r) \leq M_r
\]

\( r = 0, 1, \ldots, n \)

(41)

Proof Substituting in for \( P_r \), we get

\[
L(P^r) = 0 \leq M_r = 0
\]

where the fact that \( M_r = 0 \) for \( t > n \) was established in Lemma 5.

Q.E.D.

Lemma 12

Let \( \hat{M}_0 \) be given and let \( \{ \hat{P}_r \}_{r=0}^{n} \) satisfy Definition 6 with \( \hat{n} \) and \( \hat{d} \) denoting the values taken on by the indicated variables. Now define \( \hat{M}_0 \) such that \( \hat{M}_0 = \hat{M}_k \), with \( \hat{M}_k \) generated by the initial liquidity transition function and shown in Lemma 1 to be representable as

\[
\hat{M}_k = \hat{M}_0 - \sum_{r=0}^{k-1} L(P_r)
\]

(43)

Let \( \{ \tilde{P}_r \}_{r=0}^{n} \) satisfy Definition 6 with \( \tilde{M}_0 \) given, and with \( \tilde{n} \) and \( \tilde{d} \) denoting the values taken on by the indicated variables. Then \( \tilde{P}_r = \hat{P}_{r-k} \).

Proof Part I of the proof handles the case \( 0 \leq k \leq \hat{n} \), and Part II of the proof handles the case \( k > \hat{n} \).

Part I \( [0 \leq k \leq \hat{n}] \) First note that we have \( \hat{M}_0 = \hat{M}_k > 0 \) (for \( k \leq \hat{n} \)) so that \( \tilde{d} \) is restricted to the interval \( (0, c] \). This part of the proof will proceed in two halves. First, it
will be show that the values $\bar{n} = \hat{n} - k$ and $\bar{d} = \hat{d}$ satisfy the liquidity withdrawal equation in Definition 6. Then it will be shown that substituting these values for $\bar{n}$ and $\bar{d}$ into the price function in Definition 6 gives $\bar{P}_t = \hat{P}_{t+k}$.

The numbers $\bar{n}$ and $\bar{d}$ are defined (from Definition 6) by

$$\bar{M}_0 = \sum_{i=0}^{\bar{n}} L \left[ P^* + (\bar{n} - i)c + \bar{d} \right]$$ \hspace{1cm} (44)

Substituting our hypothesized values for $\bar{n}$ and $\bar{d}$ gives us

$$\bar{M}_0 = \sum_{i=0}^{\bar{n}-k} L \left[ P^* + (\bar{n} - i - k)c + \bar{d} \right]$$ \hspace{1cm} (45)

which we must show to be true. Evaluating we get

$$\bar{M}_0 = \sum_{i=k}^{\bar{n}} L \left[ P^* + (\bar{n} - i)c + \bar{d} \right]$$

$$= \hat{M}_k$$ \hspace{1cm} (46)

Which is true by construction. Now for the second half of Part I. Two possible cases may occur.

**CASE I** \[ t \leq \bar{n} \]. When $t \leq \bar{n}$, the first price equation in Definition 6 applies. Substituting we get

$$\bar{P}_t = P^* + (\bar{n} - t)c + \bar{d}$$

$$= P^* + (\bar{n} - k - t)c + \hat{d}$$

$$= P^* + (\bar{n} - (t + k))c + \hat{d}$$

$$= \hat{P}_{t+k}$$ \hspace{1cm} (47)

**CASE II** \[ t > \bar{n} \]. When $t > \bar{n}$, equation the second price equation in Definition 6 applies. Substituting we get
\[ \bar{p}_t = p^t \]
\[ = P_{t+k} \quad \text{if} \quad t + k > \hat{n} \]  

(48)

Now note that

\[ t > \bar{n} \Rightarrow t > \hat{n} + k \]
\[ \Rightarrow t + k > \hat{n} \]

(49)

**Part I** \([k > \hat{n}]\) First note that we have \( \tilde{M}_0 = \tilde{M}_k = 0 \) so that \( \tilde{d} \) is restricted to the interval \([- P^t, 0]\). The numbers \( \bar{n} \) and \( \tilde{d} \) are defined (from Definition 6) by

\[ 0 = \sum_{t=0}^{\bar{n}} L \left[ P^t + (\bar{n} - t) c + \tilde{d} \right] \]

(50)

which is satisfied by \( \bar{n} = \tilde{d} = 0 \). Two possible cases may occur.

**CASE I** \([t \leq \bar{n}]\). Since we have already established that \( \bar{n} = 0 \), this case only occurs when \( t = 0 \). Starting from the first price equation in Definition 6 and substituting we get

\[ \bar{p}_t = P^t + (\bar{n} - t) c + \tilde{d} \]
\[ = P^t + (0 - 0) c + 0 \]
\[ = P^t \]
\[ = \hat{p}_{t+k} \quad \text{if} \quad t + k > \hat{n} \]

(51)

Now recall that the current part of the proof (Part II) deals with the case \( k > \hat{n} \).

Combining this with the fact that \( t \) is non-negative gives us \( t + k > \hat{n} \) as required.

**CASE II** \([t > \bar{n}]\). When \( t > \bar{n} \), the second price equation in Definition 6 applies.

Substituting we get

\[ \bar{p}_t = P^t \]
\[ = \hat{p}_{t-k} \quad \text{if} \quad t + k > \hat{n} \]

(52)

which has already been shown to be true.

Q.E.D.
Lemma 13

The first order conditions

\[ V_M(M_t, \bar{P}_{t-1}) = \lambda_t + \beta V_M(M_{t-1}, \bar{P}_t) \]  
\[ V_B(M_t, \bar{P}_{t-1}) = (c + P_t + P_{t-1}) + \lambda_t [c + P_t] \cdots 
+ [c + P_t] \beta V_M(M_{t-1}, \bar{P}_t) \]
\[ - \lambda_t P_t + \beta V_B(M_{t-1}, \bar{P}_t) - P_t \beta V_M(M_{t-1}, \bar{P}_t) = 0 \]

along with the constraints

\[ M_{t+1} = M_t + cB_t + N_t - P_t (B_{t-1} - B_t) \]
\[ P_t (B_{t-1} - B_t) \leq M_t + cB_t + N_t \]
\[ P_t (B_{t-1} - B_t) = F(P_t) \]
\[ N_t = h(P_t) - cB_t \]

are satisfied by

\[ M_{t+1} = M_t - L(P_t) \]
\[ V_B(M_t, \bar{P}_{t-1}) = c + P_t - P_{t-1} + \left[ \frac{P_t + c}{P_{t+1}} \right] \beta^{n_t-1} \frac{c}{1 - \beta(1+c)} \]
\[ V_M(M_t, \bar{P}_{t-1}) = \left[ \frac{1}{P_{t+1}} \right] \beta^{n_t-1} \frac{c}{1 - \beta(1+c)} \]

\[ \lambda_t = \begin{cases} 0 & n_t > 0 \\ \left[ \frac{\beta - \beta^2}{P^*} \right] \left[ \frac{c}{1 - \beta(1+c)} \right] & n_t = 0 \end{cases} \]
\[ P_t = \begin{cases} P^* + d_t + n_t c & n_{t-1} > 0 \\ P_s & n_{t-1} = 0 \end{cases} \]
\[ n_r = \begin{cases} n_{r-1} - 1 & \text{if } n_{r-1} > 0 \\ 0 & \text{if } n_{r-1} = 0 \end{cases} \]  \hspace{1cm} (65)

\[ d_r = d_{r-1} \]  \hspace{1cm} (66)

With \( M_0 \) given, and with \( d_0 \) and \( n_0 \) satisfying Definition 6 for \( M_0 \).

\textbf{Proof}  \hspace{0.5cm} \text{Part I of the proof shows that the constraints are satisfied. Part II of the proof shows that the other optimality conditions hold when } n_r > 0, \text{ while Part III shows this for } n_r = 0. \text{ Part I} \hspace{0.5cm} \text{The four constraints can be reduced to two by substituting (58) and (59) into (56) and (57), giving us}

\[ M_{r+1}(M_r, \bar{P}_{r-1}) = M_r - L(P_r) \]  \hspace{1cm} (67)

\[ L(P_r) \leq M_r \]  \hspace{1cm} (68)

Equation (67) is true by construction, being identical to our guess for \( M_{r+1}(M_r, \bar{P}_{r-1}) \). To see that equation (68) holds, note that given the state variable \( M_r \), equation (68) depends only on \( P_r \). Our method of constructing \( P_r \) ensured inter alia that equation (68) holds, as proven in Lemma 10.

\textbf{Part II}  \hspace{0.5cm} [\( n_r > 0 \)]  \hspace{0.5cm} \text{We will start by finding expressions for } V_n(M_{r+1}, \bar{P}_r) \text{ and } V_M(M_{r+1}, \bar{P}_r). \text{ Updating (61) we have}

\[ V_n(M_{r+1}, \bar{P}_r) = c + P_{r+1} - P_r + \left[ \frac{P_{r+1} + c}{P_{r} + \frac{\beta^n c}{1 - \beta(1 + c)}} \right] \]  \hspace{1cm} (69)

where we have used the fact that \( n_{r+1} = n_r - 1 \). Employing the latter fact and updating (62) we have
\[ V_M(M_{t-1}, \bar{p}_t) = \frac{1}{P_{t-n}} \left[ \frac{\beta^n c}{1 - \beta(1 + c)} \right] \]  \hspace{1cm} (70)

**i) proof that F.O.C. (53) holds:** Substituting (62), (63), and (70) into (53) we get

\[ \frac{1}{P_{t-n}} \left[ \frac{\beta^{n-1} c}{1 - \beta(1 + c)} \right] = \beta \frac{1}{P_{t-n}} \left[ \frac{\beta^n c}{1 - \beta(1 + c)} \right] \]  \hspace{1cm} (71)

which is identically true.

**ii) proof that F.O.C. (54) holds:** Substituting (61), (63), and (70) into (54) we get

\[
c + P_t - P_{t-1} + \left[ \frac{P_t + c}{P_{t-n}} \right] \left[ \frac{\beta^{n-1} c}{1 - \beta(1 + c)} \right] = c + P_t - P_{t-1} \quad \cdots
\]

\[ + [c + P_t] \beta \frac{1}{P_{t-n}} \left[ \frac{\beta^n c}{1 - \beta(1 + c)} \right] \]  \hspace{1cm} (72)

which is identically true.

**iii) proof that F.O.C. (55) holds:** Substituting (63), (69), and (70) into (55) we get

\[
\beta \left( c + P_{t-1} - P_t + \left[ \frac{P_{t+1} + c}{P_{t-n}} \right] \frac{\beta^n c}{1 - \beta(1 + c)} \right) - P_t \beta \left[ \frac{1}{P_{t-n}} \right] \frac{\beta^n c}{1 - \beta(1 + c)} = 0 \]  \hspace{1cm} (73)

which is also identically true, taking account of the fact that \( P_{t-1} = P_t - c \) for \( n_t > 0 \).

**Part III \( [n_t = 0] \).** We will start by finding expressions for \( V_B(M_{t-1}, \bar{p}_t) \) and \( V_M(M_{t-1}, \bar{p}_t) \). Updating (61), and using the fact that \( P_{t-1} = P_t = P^* = 1 \) when \( n_t = 0 \), we have

\[ V_B(M_{t-1}, \bar{p}_t) = c + (1 + c) \frac{\beta c}{1 - \beta(1 + c)} \]  \hspace{1cm} (74)

Updating (62) we have
\[ V_M(M_{i+1}, \bar{p}_i) = \frac{\beta c}{1 - \beta(1 + c)} \]  

(75)

**i) proof that F.O.C. (53) holds:** Substituting (62) and (75) into (53) we get

\[
\frac{\beta c}{1 - \beta(1 + c)} = (\beta - \beta^2) \frac{c}{1 - \beta(1 + c)} + \beta \left[ \frac{\beta c}{1 - \beta(1 + c)} \right]
\]

(76)

which is identically true.

**ii) proof that F.O.C. (54) holds:** Substituting (61) and (75) into (54) we get

\[
c + (1 + c) \frac{\beta c}{1 - \beta(1 + c)} = c + \left[ (\beta - \beta^2) \frac{c}{1 - \beta(1 + c)} \right] [c + 1] \cdots
\]

\[+ [c + 1] \beta \left[ \frac{\beta c}{1 - \beta(1 + c)} \right] \]

(77)

which is identically true.

**iii) proof that F.O.C. (55) holds:** Finally, substituting (74) and (75) into (55) and reducing we get

\[
(\beta^2 - \beta) \frac{c}{1 - \beta(1 + c)} + \beta \left[ c + (1 + c) \frac{\beta c}{1 - \beta(1 + c)} \right] - \beta \left[ \frac{c}{1 - \beta(1 + c)} \right] = 0
\]

\[
\frac{\beta^2 c}{1 - \beta(1 + c)} - \frac{\beta c}{1 - \beta(1 + c)} + \beta c \left[ \frac{\beta(1 + c)}{1 - \beta(1 + c)} \right] - \frac{\beta^2 c}{1 - \beta(1 + c)} = 0
\]

(78)

which is identically true.

\textit{Q.E.D.}
Chapter 5: Speculative Balances and Asset Prices:

Evidence From Institutional Money Holdings

This chapter uses Flow of Funds data on institutional investor money holdings to estimate the demand for speculative balances as a function of asset prices. It was shown in the previous chapter that if money managers engage in intertemporal price arbitrage, nominal speculative balances will be negatively related to asset prices. The equation to be estimated is:

\[ S_t = \beta_0 + \beta_1 r_t + \beta_2 \pi_t + \gamma Z_t + \varepsilon_t \]  

(1)

where \( S_t \) is real speculative balances, \( r_t \) is the interest rate, \( \pi_t \) is the inflation rate and \( Z_t \) is a vector of control variables. A statistically significant coefficient on \( r_t \) will be interpreted as evidence that money managers engage in intertemporal price arbitrage.

The remainder of this chapter proceeds as follows. First, we define what institutional investors are and discuss why their money holdings can be regarded as speculative balances. This is important because in general, money holdings have a transactions component which cannot be distinguished from the speculative balance component. We argue that the nature of activities undertaken by money managers eliminates the need for transactions balances. Following this, we present the data which will be used for estimation and discuss relevant econometric issues. We finish by presenting the estimation procedure and results.
Institutional Investors and Speculative Balances

Institutional investors are entities such as banks, mutual funds, pension funds, and insurance funds, which engage in large-scale financial investment. For our purposes, the key feature of institutional investors is that they are specialized in buying and selling financial assets. Their transactions need for money is thus negligible and their entire holdings of money can be regarded as speculative balances. This makes them an ideal data source for estimating the demand for speculative balances.

To see how specialization in financial asset trade eliminates the need for transactions balances, we will consider how the transactions demand for money arises in an agent not specialized in financial asset trade. We will then argue that the elements of the environment which cause these agents to hold transactions balances do not pertain to institutional investors.

For convenience, agents not specialized in financial asset trade will be referred to as households. Households hold transactions balances for two reasons. First, asset market transactions costs make it optimal for them to liquidate (sell) interest-bearing assets only periodically. Otherwise, households would hold interest-bearing assets instead of money, and would liquidate their assets in the exact amount needed to meet each expense as it came due. Second, households' expenses tend to be randomly spread out over time. If each household could arrange its expenses so that the latter all occurred immediately after household members received their paychecks, then no transactions balances would be needed. The latter argument also applies if households do not earn any income, but instead finance all of their expenses from savings. If expenses could be
arranged so that they were all bunched together, they could be paid immediately after households had undertaken their periodic liquidation of interest-bearing assets. Households would then have no reason to hold money in the time intervals between their periodic sessions of asset-liquidation-and-payment-of-expenses.

The aspects of the environment which cause households to hold transactions balances are thus asset market transactions costs and random timing of expenses. Now consider institutional investors. The primary activity of institutional investors is the buying and selling of financial assets. This activity is undertaken with the goal of maximizing return. Funds being held by the institutional investors are subject to withdrawal by households at any time.

Institutional investors thus have two types of expenses for which they require the use of money: purchases of financial assets, and payment to outside parties who are withdrawing their funds. The first thing to note about this environment is that only the latter expenses are randomly spread out over time. The former type of expenses, purchases of financial assets, do not occur randomly. In order to maximize the return on funds, institutional investors will want to purchase financial assets immediately upon receipt of such funds. Such immediate purchases are made possible by the presence of a secondary market for financial assets. The only reason a professional investor would wish to delay such a purchase is if the asset price were considered too high, in which case the money balances being held in lieu of making the purchase are properly classified as speculative balances rather than as transactions balances. Thus, no transactions demand for money arises from the activity of purchasing financial assets.
This still leaves us with the issue of fund withdrawals, which, unlike asset purchases, tend to be random. However, recall that the household's need for transactions balances arose not only from the fact that their expenses were random, but also from the fact that it was costly for them to engage in asset market trade. This made it unfeasible for them to liquidate assets for each individual expense as it became due. In contrast, institutional investors are specialized in asset market trade. The marginal cost of additional trades is extremely low. Thus, even with random expenses, institutional investors have no need for transactions balances.

Preliminary Evidence

Figure 1 shows deflated money holdings for a particular group of institutional investors, government retirement funds. The pattern of money holdings in Figure 1 supports our contention that transactions holdings are negligible — the fluctuations do not correspond to what we would expect from variations in transactions needs, either seasonal or trend. The pattern of variation cannot be explained by size effects either, as shown by Figure 2, where the money holdings of government retirement funds are divided by the latter's real asset holdings. Given their variation, it is plausible that Figures 1 and 2 reflect speculative holdings of money.

In Figure 3, the six-month treasury bill rate is superimposed on the money-asset ratio of government retirement funds. A strong negative correlation is evident, indicating the existence of a particular type of liquidity effect, in which excess liquidity in the financial sector (which includes institutional investors) is associated with a decline in the interest rate. The relationship between money holdings and the interest rate can be
Figure 1: Government Retirement Fund Real Money Holdings
(in billions of 1987 dollars)
Figure 2: Government Retirement Fund Money-Asset Ratio
Figure 3: Money-Asset Ratio vs. T-Bill Rate
regarded as a demand function for speculative balances. These demand schedules are what we will be estimating.

**Data**

**Speculative Balances**

The *Flow of Funds* contains data on the following institutional investors:

1. commercial banks
2. savings and loan associations
3. mutual savings banks
4. credit unions
5. life insurance companies
6. private pension funds
7. state and local government retirement funds
8. other insurance companies
9. finance companies
10. real estate investment trusts
11. mutual funds (open-end investment companies)
12. money market mutual funds
13. security brokers and dealers
End of quarter money holdings, defined as currency plus checkable deposits, is available from the *Flow of Funds* for each of the categories listed. It should be noted that "the flow of funds accounts are calculated for the single last day of each quarter rather than for weekly, monthly, or quarterly averages," and "the one-day numbers...are frequently very different from the averages because of specific events of even the day of the week that end a quarter." (*Introduction to the Flow of Funds* p.40). Apart from the errors-in-variables problem, this fact shouldn't affect our estimates, as long as the biases imparted to the data are independent of the systematic influences we are investigating. The presence or errors-in-variables will tend to bias the estimated coefficients towards zero and inflate the standard errors.

Money holdings for depository institutions were defined as total reserves minus legally required reserves. This is the appropriate measure for our purposes because it gives the amount of money that depository institutions are holding as idle cash — the amount they would prefer to invest in financial assets. Data on required reserves was not readily available for depository institutions other than commercial banks, so these institutions — Savings and loan associations, Mutual savings banks, and Credit Unions — were eliminated from the analysis. Data on commercial bank reserves was obtained from *Citibase*.

Finance companies, real estate investment trusts, and money market mutual funds were also eliminated from the analysis. The first two were eliminated because all their assets are in the form of mortgages, consumer credit, and other loans. These were not tradable credit market instruments until recently, and institutions do not actively trade have no reason to hold speculative balances. Money market mutual funds were
eliminated from analysis because their aggregate reported money holdings are negative for a large number of periods. This phenomenon reflects draw downs on bank overdrafts.

Figure 4 shows quarterly real money holdings, where the GDP deflator was used for deflation, from 1959 to mid-1991 for the institutions included in our analysis other then government retirement funds, for which data was presented earlier. Some types of institutional investors show a marked seasonal pattern in their money holdings. These are: life insurance companies, mutual funds, and commercial banks. The seasonality may indicate the existence of transactions balances. Filtering out the seasonal component should leave us with a measure of money holdings containing only speculative balances. To further characterize the data, Table 1 presents some descriptive statistics for seasonally-adjusted real money holdings. The seasonal adjustment was carried out with the Census XII procedure in SAS.

Prior to estimation, the money holdings for all groups of institutional investors were deflated with the GDP deflator and then seasonally-adjusted with the Census XII procedure in SAS. The money holdings of security brokers and dealers was left in unadjusted form because the Census XII procedure generated negative values. We took this as evidence that the data lack a seasonal component, so no adjustment for it is necessary. The method used to filter out the growth component of money holdings is discussed later. The need for such filtering is also discussed.

**Interest Rate**

We use three different measures of the interest rate: the six-month treasury bill rate, the Federal funds rate, and the stock market price-earnings ratio. The latter is
Figure 4: Institutional Investor Money Holdings (billions of $)
Table 1: Descriptive Statistics
real seasonally adjusted money holdings
billions of dollars, 1959-1991

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<th></th>
<th>( \mu )</th>
<th>( \sigma )</th>
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<td>Life Insurance Company</td>
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<tr>
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</table>

obtained by dividing the Standard and Poors' stock price index by the Standard and Poors' earning index. All series are taken from the Citibank data set. Of the three interest rates, the price-earnings ratio can be expected to perform best at picking up speculative movements. This is true for three reasons:
1. the price-earnings ratio filters out movements in the real interest rate

2. the price-earnings ratio is not affected by inflationary expectations

3. stocks carry with them a well-perceived risk of capital loss

To these issues we will now turn.

**Controlling for Real Interest Rate Movements**

The optimal measure of asset prices would be one which picks up liquidity effects but ignores movements in the rate of return on physical capital. This frees us from having to control for the latter in our regressions. The price-earnings ratio, as a measure of stock prices, fulfills this condition, provided that earnings follow a random walk. Theoretical arguments can be used under these conditions to show that the price-earnings ratio picks up only liquidity effects. Assume stock prices can be represented as the product of their fundamental value and a liquidity component:

\[ p_t = p_t^f \rho_t \]  \hspace{1cm} (2)

The fundamental value of a stock is the present discounted value of expected future earnings:

\[ p_t^f = \sum_{\tau=1}^{\infty} \frac{E_{t} (e_{t+\tau})}{(1 + r)^\tau} \]  \hspace{1cm} (3)

Assuming earnings follow a random walk

\[ e_t = \mu + e_{t-1} + \varepsilon_t \quad \varepsilon_t \approx iidN(0, \sigma^2) \]  \hspace{1cm} (4)

the fundamental stock value will be given by

\[ p_t^f = \left( \frac{1 + g}{r - g} \right) e_t \]  \hspace{1cm} (5)
\[ 1 + g = \exp[\mu + \sigma^2/2] \]  
(6)

The price of the stock will then be given by

\[ p_t = \left( \frac{1+g}{r-g} \right) e_t p'_t \]  
(7)

and the price-earnings ratio will be

\[ \frac{p_t}{e_t} = k p'_t \]  
(8)

Where \( p'_t \) is the liquidity component. If \( p'_t \) is a stationary stochastic process, the price-earnings ratio will be too. In addition, movements in the price-earnings ratio reflect departures of price from its fundamental value. Such movements are pure liquidity effects.

**Controlling for Inflationary Expectations**

The optimal interest rate measure would also ignore inflationary expectations. This is certainly true for the price-earnings ratio. Expected future inflation does not affect current earnings. It also does not affect the current stock price because investors know that any future inflation which does occur will be matched by increased earnings. There is thus no need to compensate for expected future inflation by paying less for stocks. The same is not true of bonds. While expected future inflation will affect the price of bonds, the effect will be weaker for shorter maturities. Since the two bond interest rates we use are of short maturity, they should not be much affected by inflationary expectations.
The Role of Asset Maturity

In order for the relationship between money balances and an asset price to reflect speculative activity, the asset must have a long enough maturity that investors perceive a risk of capital loss. To illustrate the problem we run into when this is not the case, assume we regress institutional money holdings on the overnight deposit rate. If we interpret the result as a demand schedule for speculative balances, then we must interpret the predicted values from the regression as reflecting the amount of money investors hold to speculate on overnight deposits. But why would investors speculate at all on an overnight asset? The probability of taking a capital loss on overnight assets is effectively zero, so investors should be willing to put an unlimited amount of cash in overnight deposits rather than hold money, which earns a zero return.

Investors will hold money only if the effective rate of return on overnight assets is zero. This does not mean that the observed rate of return must be zero, but the observed rate of return must be offset by transactions costs. The latter fact may cause a spurious inverse relationship to exist between short-term interest rates and speculative balances. This is because the lower the interest rate, the more quickly investors will find that transactions costs wipe out the gains to be made from buying short-term assets rather than holding money, and the larger the amount of money they will hold. This results in a well-defined inverse relationship between money and the interest rate, but it is not a relationship between speculative balances and the interest rate. Rather, it is a relationship between residual money holdings and the interest rate/transaction costs.
where residual money holdings are money holdings which it is not profitable to invest because transactions costs exceed the potential return.

Table 2 gives a representative sample of yields on overnight and longer term assets. Even in liquid markets rates of return can differ among overnight assets because different transactions costs are involved. Table 2 indicates that in order for investors to hold speculative balances, the transactions cost of putting an additional dollar into a savings account ($2.500 min.) for one day must exceed .007 cents." Likewise, the transactions cost of putting an additional dollar into overnight repos must exceed .015 cents.

<table>
<thead>
<tr>
<th>Table 2: Annual Percentage Yields 3/9/98</th>
</tr>
</thead>
<tbody>
<tr>
<td>checking</td>
</tr>
<tr>
<td>regular savings</td>
</tr>
<tr>
<td>savings. $2.500 min.</td>
</tr>
<tr>
<td>savings. $100.000 min.</td>
</tr>
<tr>
<td>overnight repo*</td>
</tr>
<tr>
<td>Fed Funds rate</td>
</tr>
<tr>
<td>3-mo. T-bill</td>
</tr>
<tr>
<td>6-mo. T-bill</td>
</tr>
</tbody>
</table>

*6-mo T-Bill

Sources: rates on bank deposits are as quoted by Apple Bank. All other rates are from Bloomberg.

We have included yields on longer term assets in the table to illustrate the pitfalls of using the difference between the rate of return on assets and the rate of return on
money as the asset returns variable in money demand equations. The first problem which becomes evident is that money has many different rates of return and it is unclear which one to use. The money rates of return given in Table 2 vary from 1.01% to 5.45%. The second and larger problem is that Table 2 does not give the true rate of return on money. The true rate of return is the nominal return minus transactions costs. In liquid markets, the true rate of return should be zero.

From Table 2 we can see that blindly subtracting the rate of return on money from the interest rate could result in a scenario where the rate of return on 6-month T-bills is actually negative, even without accounting for inflation. (This example uses the overnight repo rate as the rate of return on money.) Such a perverse result arises from the failure to take account of transactions costs; in this case failing to take account of the fact that transactions costs on repos (relative to their maturity) are much higher than on 6-month T-Bills.

In order for the relationship between speculative balances and an asset price to reflect speculative activity, the asset must have a long enough maturity that investors perceive a risk of capital loss. However, even these regressions will only show the relationship between residual money holdings and the interest rate. Can these be interpreted as demand schedules for speculative balances? The answer is yes. Regardless of how the quantity of speculative balances is determined, the fact remains that this quantity of money is available to be invested in financial assets. (This assumes transactions costs on longer term assets are low enough that any quantity of money can be profitably invested in them under normal conditions; investors will thus never be holding money simply because the transactions costs on all assets is too high.)
The problem of having transactions costs reflected in asset prices does not disappear altogether when longer term assets are used because all assets involve some transactions costs. However, as asset maturities increase, transactions costs become a less important price determinant and speculative activity becomes more important. In this regard, the Federal Funds rate can be expected to perform extremely poorly at picking up speculative movements. The six-month treasury bill rate will perform somewhat better and the stock price-earnings ratio will perform very well.

**Control Variables**

The equilibrium bond price path depends in part on investors expectations about the speed of liquidity withdrawal. If liquidity withdrawal is expected to be especially strong, the bond price will not appreciate by as much (less inducement is needed to get firms to borrow) and vice versa. Since such expectations clearly affect the relationship between the interest rate and the quantity of speculative balances being held by money managers, it is important to control for them. The control variable we use is the capacity utilization rate taken from Citibase.

**Econometric Issues**

**Controlling for Size Effects**

Everything we have said so far about the demand for speculative balances assumes a non-growing economy. In actual economies, variables are growing over time. Failure to control for this growth can mask the relationship between speculative balances
and asset prices. The manner in which this may occur can best be illustrated by carrying out a hypothetical experiment in which the economy suddenly doubles overnight. Everything in the old economy will be present in the new economy, along with an exact double. Every worker will have an exact double, every factory will have an exact double, every investment project will have an exact double, and so forth. With all physical objects having doubled, we can expect economic activities to be carried on at twice their previous magnitude. The demand for loanable funds will be twice as large. Open market operations will be twice as large. The amount of excess liquidity withdrawn from the financial sector each period will be twice as large. And so forth.

Clearly speculative balances will also be twice as large in the new economy as in the old. But what about asset prices? Notice that the doubling process occurs in a manner such that for any interest rate all quantities are double what they were. For example, for every investment project that was profitable in the old economy at a given interest rate, there now exists a double, which is also profitable at the same interest rate. In effect, asset prices stay fixed while everything around them doubles. Clearly the demand for speculative balances will be double what it was at every interest rate. Thus the doubling process distorts the original relationship between speculative balances and the interest rate.

The foregoing example was meant to convey the general problem posed by economic growth. Economic growth causes the observed relationship between speculative balances and asset prices to be a distorted version of the true relationship, and may wipe out the latter all together. In order to prevent this, growth effects must be
removed from the data. Various methods exist for doing so. These methods will be presented in terms of general statistical theory.

Let a time series be represented by

$$X_t = e^{\eta t} \varepsilon_t$$

(9)

where $\eta$ is the growth rate, $e^{\eta t}$ is the growth component of the series, and $\varepsilon_t$ is the cyclical component of the series. The series of interest is $\varepsilon_t$, which we wish to extract from $X_t$. Since most statistical theory deals with series having additive components, we will consider with the logarithmic form of the foregoing:

$$\ln X_t = \eta + \ln \varepsilon_t$$

(10)

in statistical terminology. $\ln X_t$ is a non-stationary series. It is composed of a pure non-stationary part, $\eta$, and a stationary part, $\ln \varepsilon_t$.

The series $\ln X_t$ is usually classified as trending or integrated (or both), depending on the properties of its non-stationary part. If $\eta$ is a deterministic function of time, $\ln X_t$ is said to be trending (or trend-stationary) and $\eta$ is said to be the trend. Presuming the form of the trend is known, we can recover $\varepsilon_t$ by regressing $\ln X_t$ on the relevant time terms and taking the antilog of the residual.

An alternative to trend stationarity is $\ln X_t$ being integrated of arbitrary order. When a series is integrated, it is not clear how to separate the growth and cyclical components. Any method must rely on assumptions rather than on observed properties of the series. Two frequently used methods are the Beveridge Nelson (1981) decomposition, which assumes $\eta$ and $\ln \varepsilon_t$ are perfectly correlated, and the unobserved components method which assumes $\eta$ and $\ln \varepsilon_t$ are perfectly uncorrelated.
A problem in applying the foregoing methods of extracting $\varepsilon_i$ is that we must first ascertain whether $\ln X_i$ is trend stationary, integrated, or both.\footnote{The statistical theory on testing for the type of non-stationarity is currently very limited and deals with only a handful of cases (although arguably the most important ones).} It is convenient to ignore the issue and apply a filter which eliminates the trend and integrated components simultaneously. While this is only a second-best solution, the state of statistical theory at this time is such that a first-best solution does not exist. We use the Hodrick-Prescott filter. This involves minimizing

$$\sum_{t=1}^{T} (y_t - s_t)^2 + 1600 \sum_{t=1}^{T} [(s_{t+1} - s_t) - (s_t - s_{t-1})]^2$$

(11)

with respect to $s_t$ for $t=1$ to $T$, where $y_t$ is the original series. The sequence of $s_t$'s which minimizes the preceding equation is to be interpreted as the non-stationary component of $y_t$. The Hodrick-Prescott filter removes trend components and nonstationary components which are integrated of order four or less (Baxter and King, 1994).

**Consistent Regression**

Hodrick-Prescott filtering ensures that the money variables in our regressions are stationary. In order for existing statistical theory to be applicable, the interest rate terms must also be stationary. Visual inspection shows that none of our interest rates are trending, but they may be integrated. Integration of the interest rate terms must be tested for, and if not rejected, some method of filtering out the nonstationary component must be applied. For consistency, this method may as well be the Hodrick-Prescott filter.
Integration Test Procedure

The conventional procedure for determining whether or not a series is integrated is to test for a unit root in the autoregressive representation. Assume a series can be represented by a first order autoregressive process

\[ y_t = \rho y_{t-1} + \varepsilon_t, \quad \varepsilon_t \approx iid(0, \sigma^2) \quad (12) \]

If \(|\rho| = 1\) then \(y_t\) is said to have a unit root. In most economic settings, \(\rho = -1\) does not occur so we will define a unit root as \(\rho = 1\). The test for a unit root proceeds by obtaining OLS estimates of \(\rho\) and \(\sigma^2\), and forming the statistic \(\dd = \hat{\sigma}^{-1}(\hat{\rho} - 1)\). Under the null hypothesis of \(\rho = 1\), the statistic \(\dd\) has a Dickey-Fuller distribution. The critical values for this distribution are available in various sources. We use Banerjee et. al.

It is convenient to subtract \(y_{t-1}\) from both sides of (12) and rewrite it as

\[ \Delta y_t = (\rho - 1)y_{t-1} + \varepsilon_t \quad (13) \]

The statistic \(\dd\) can then be easily obtained as the t-statistic for the hypothesis that the regression coefficient in (13) equals zero. The representation (13) generalizes nicely to the case where \(y_t\) is generated by a higher order autoregressive process. Assume

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t \quad (14) \]

Although multiple unit roots may exist, it is conventional to limit testing to the case of a single unit root. The null hypothesis is \(\phi + \phi_2 + \ldots + \phi_p = 1\). The counterpart of (13) for this case is

\[ \Delta y_t = \theta_1 y_{t-1} + \theta_2 \Delta y_{t-1} + \ldots + \theta_{p-1} \Delta y_{t-p-1} + \varepsilon_t \quad (15) \]
Again, the t-statistic for the hypothesis that $\theta_i = 0$ can be used for $\tilde{d}$. The latter has the same distribution as it did for the case of a first order autoregressive process.

The formulation (15) is called the augmented Dickey-Fuller test. If the representation (13) is used when a higher order autoregressive process is appropriate, it will show up as serial correlation in the error term which violates the required stochastic assumptions. It is thus wise to always use the augmented form of the Dickey-Fuller test. The appropriate value for $p$ can be found by minimizing Akaike's information criterion.

$$AIC(p) = \ln \left( \frac{e'e}{T} \right) + \left[ \frac{2p}{T} \right]$$  

(16)

Integration Test Results

We tested each of the price variables for a unit root by estimating (15) with $p=\delta$. Table 3 gives the estimated augmented Dickey-Fuller statistics for the four equations.

<table>
<thead>
<tr>
<th>Fed Funds</th>
<th>T-Bill</th>
<th>P/E Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.539**</td>
<td>-3.7252**</td>
<td>-2.488*</td>
</tr>
</tbody>
</table>

**unit root rejected at .01 level of significance
*unit root rejected at .025 level of significance

In all cases, a unit root can be rejected. This means that for statistical purposes we can treat the interest rate terms as being stationary. Their estimated coefficients will be consistent and asymptotically normally distributed, provided the error term obeys the
assumptions of one of the classical central limit theorems. Note that the other explanatory variables may be non-stationary. The consistency and distribution of their estimated coefficients will then be unknown, but the properties of the coefficient estimates on the stationary variables will not be affected.

**Regression Stability**

The relationship between speculative balances and asset prices may not be stable across different Federal Reserve Board operating regimes. For one thing, the distribution of liquidity shocks may differ, and we have seen that this will change the quantity of money which portfolio investors wish to hold at every interest rate. The Federal Reserve Bank has been subject to various operational restrictions (some self-imposed) throughout its history. The following brief summary is based on material in Meulendyke (1989).

**1942, April:** At request of the Treasury. Fed starts formally pegging the rate on 90-day Treasury Bills. with less formal pegging on longer term government debt.

**1951:** Fed is given permission to resume an independent monetary policy.

**1951—1953:** Fed gradually withdraws interest rate pegging.

**1953—1960:** Open market operations become an active tool of monetary policy. Fed purposely does not use interest rates as a policy guide. Bank credit is formal policy guide, but day-to-day operations are based on free reserves.
mid-60s: Interbank market expands to the point where the Federal Fund rate is a useful indicator of money market conditions.

late-60s: Fed starts paying more attention to money growth, business conditions, total reserves, and the monetary base.

1970: Fed formally adopts monetary targets. Federal Funds rate is primary guide in day-to-day operations.

1979, Oct.: Non-borrowed reserves becomes operational guide.

1983—present: Mix of Federal Funds rate targeting and borrowed reserve targeting.

The present study is based on the sample period 1982-1991. The Federal Reserve operating procedures have not undergone any major shifts during this period.

Estimation

For any single institutional group, the demand for speculative balances is

\[ \ln S_t = \beta_0 + \beta_1 \ln r_t + \beta_2 \pi_t + \beta_3 CAPU_t + \varepsilon_t \]  \hspace{2cm} (17)
where \( S_t \) is speculative balances, \( r_t \) is the interest rate, \( \pi_t \) is the inflation rate between \( t-1 \) and \( t \) and \( \text{CAPU} \) is the capacity utilization rate. The specification employing \( \ln r_t \) rather than \( r_t \) was chosen because a priori it seems sensible to assume that the elasticity of \( S_t \) with respect to \( r_t \) is constant (the elasticity is given by \( \beta_1 \)). We arranged all seven institutional groups into panel data form and estimated

\[
\ln S_{i,t} = \beta_0 + \beta_1 \ln r_t + \beta_2 \pi_t + \beta_3 \text{CAPU}_t + \varepsilon_{i,t} \tag{18}
\]

where \( i = 1, 2, \ldots, 7 \) indexes the institutional group. During estimation we allowed for groupwise heteroscedasticity, contemporaneous cross group correlation, and group specific first-order autocorrelation. The stochastic assumptions are

\[
E[\varepsilon_t^2] = \sigma_u \tag{19}
\]

\[
\text{Cov}[\varepsilon_{i,t}, \varepsilon_{j,t}] = \sigma_u \tag{20}
\]

\[
\varepsilon_t = \rho \varepsilon_{t-1} + u_t \tag{21}
\]

We tested for parameter constancy across groups by including group specific intercepts and slope terms in the panel data model and then checking t-statistics. In order to preserve the degrees of freedom, the groups were tested one-by-one rather than simultaneously. Based on these tests, the final panel data models allowed group specific coefficients for two institutional groups.

The LIMDEP estimation procedure we used tested for three forms of heteroscedasticity:

1. homoscedastic
2. groupwise heteroskcedastic
3. groupwise heteroskedastic and cross group correlated
The procedure also tested for three forms of autocorrelation:

1. nonautocorrelated
2. common autocorrelation coefficient
3. group specific autocorrelation coefficients

In all, nine models are estimated, with the three contemporaneous covariance specifications crossed with the three autocorrelation specifications. Diagnostic statistics are generated for testing type (2) heteroscedasticity against type (3), and for testing type (1) against type (2). No specific test is generated for the autocorrelation types, but we tested the significance of the estimated correlations themselves by referring

\[(T - 1)r / (1 - r) \approx \chi^2[1]\]

to the value 3.84, which is the 95 percent critical value from the chi-squared distribution with one degree of freedom. Based on the generated test statistics, we choose the fully unrestricted model, with groupwise heteroskedastic, cross group correlated errors and group specific autocorrelation coefficients.

Estimation results are presented in Table 9. As expected, the demand for speculative balances is related more strongly to the price-earning ratio than to the other two measures of the interest rate. One percent increase in the price-earnings ratio causes a one-tenth of a percent increase in speculative money holdings. The regression using the T-Bill rate also reveals a statistically significant demand for speculative balances. The positive coefficient on the capacity utilization rate indicates that an increase in capacity utilization causes institutional investors to hold more speculative balances at every
interest rate. The t-statistics on the intercepts and the interest rate coefficients were indistinguishable from zero, and hence were not reported.
<table>
<thead>
<tr>
<th></th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Equation 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>0.01 (1.84)</td>
<td>0.01 (1.44)</td>
<td>0.01 (1.41)</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(P/E Ratio) Pooled</td>
<td>0.09* (3.33)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(P/E Ratio) Commercial</td>
<td>-0.89 (-1.85)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(P/E Ratio) Mutual</td>
<td>0.77 (1.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Funds</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Fed Funds Rate) Pooled</td>
<td></td>
<td>-0.03 (-1.51)</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Fed Funds Rate)</td>
<td></td>
<td>-0.57 (-1.21)</td>
<td></td>
</tr>
<tr>
<td>Commercial Banks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Fed Funds Rate)</td>
<td></td>
<td>1.10 (1.43)</td>
<td></td>
</tr>
<tr>
<td>Mutual Funds</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(6-Mo T-Bill Rate)</td>
<td></td>
<td></td>
<td>-0.04* (-1.95)</td>
</tr>
<tr>
<td>Rate) Pooled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(6-Mo T-Bill Rate)</td>
<td></td>
<td></td>
<td>-0.63 (-1.33)</td>
</tr>
<tr>
<td>Commercial Banks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(6-Mo T-Bill Rate)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mutual Funds</td>
<td></td>
<td></td>
<td>1.05 (1.38)</td>
</tr>
</tbody>
</table>

1 Significant at two-sided 5% level (t975=1.9).
1 Pooled coefficients are elasticities of money demand with respect to the interest rate variables.
2 Estimated coefficient is the difference between money demand elasticity for indicated group and pooled elasticity.
**Specification Tests**

Specification tests were performed to see if the money-asset ratio is a more appropriate right-hand side variable than money alone. The reasoning behind this hypothesis is that portfolio managers do not choose speculative balances outright, rather, they choose a proportion of their portfolio to hold as speculative balances. Letting \( A \) be portfolio size, we test the specification

\[
\ln \frac{S_{i,t}}{A_{i,t}} = \beta_0 + \beta_1 \ln r_i + \beta_2 \pi_i + \beta_3 \text{CAPU}_i + \varepsilon_{i,t}
\]  

(22)

against (17). Two measures of portfolio size were used. One was value of total assets, and the other was value of actively traded assets. The rationale for the latter definition was given in the previous chapter where it was noted that group \( i \)'s speculative balances might be expected to be proportional to group \( i \)'s volume of financial market trade, which can be measured by group \( i \)'s holdings of actively traded assets. We define actively-traded assets as those for which a well-developed secondary market exists. Following is our breakdown of assets into actively and non-actively traded:

**Actively Traded**

1. Money market fund shares
2. Corporate equities
3. Treasury securities
4. Federal agency securities
Non-Actively Traded

1. Time and savings accounts
2. Fed funds and security RPs
3. State and local government securities
4. Corporate and foreign bonds
5. Open-market paper
6. Life insurance reserves
7. Pension fund reserves
8. Mortgages
9. Consumer credit
10. Bank loan n.e.c.
11. Other loans
12. Security credit
13. Trade credit
14. Taxes payable
15. Miscellaneous

An issue which needed to be dealt with is that the Flow of Funds data nets out intrasectoral transactions. For example, if a money market fund buys shares in another money market fund, the transaction is canceled out in the consolidated money market fund data so that total assets and total liabilities for that sector are not affected by the transaction. This fact becomes important when we are trying to measure agents' volume
of financial market trade. The latter will be a function of intrasectoral trades just as much as it will be a function of trade between sectors. The omission from an agent's stock of actively-traded assets of those assets issued within the sector could bias our results.

How significant is the omission? To answer this question, we will assume an asset is traded intra-sectorally if it shows up in both the assets and the liabilities of a sector. The only asset for which this is true is the federal funds holdings of commercial banks: purchases and sales of federal funds occur primarily within the commercial banking sector and are thus netted out in the data. The stock of federal funds assets reported for commercial banks thus understates the total. Since we do not regard federal funds as an actively traded asset, this fact will not affect our results.

The test of (17) against (22) can be carried out by performing the regression

$$\ln S_{i,j} = \gamma \ln A_{i,j} + \beta_0 + \beta_1 \ln r_i + \beta_2 \pi_i + \beta_3 \text{CAPU}_i + \varepsilon_{i,j}$$

and using the t-statistic on $\gamma$ to test the significance of $\ln A_{i,j}$.

Before estimating (23) we must transform $S_{i,j}$ and $A_{i,j}$ to be stationary. Notice that while $S_{i,j} / A_{i,j}$ may be stationary, $S_{i,j}$ and $A_{i,j}$ need not be individually stationary.

Let $S_{i,j}^*$ and $A_{i,j}^*$ be the results of stationarity-inducing transformations on $S_{i,j}$ and $A_{i,j}$. A priori, it seems desirable to have the stationarity-inducing transformation preserve the relationship of interest between speculative balances and asset holdings. We will thus impose the restriction

$$\frac{S_{i,j}^*}{A_{i,j}^*} = \frac{S_{i,j}}{A_{i,j}}$$

(24)
This was accomplished by performing stationarity transformations (deflation, deseasonalization, and detrending) on \( S_{t,\tau} \) and then calculating \( A_{t,\tau}^* \) as

\[
A_{t,\tau}^* = \frac{S_{t,\tau}^*}{S_{t,\tau}} A_{t,\tau}
\]  
(25)

\( S_{t,\tau}^* \) and \( A_{t,\tau}^* \) were then used to estimate (23). This procedure was undertaken with \( A_{t,\tau} \) being defined as total assets, and then again with \( A_{t,\tau} \) being defined as actively-traded assets. In both cases the estimated coefficient on \( A_{t,\tau} \) was insignificant. Thus the specification (17) seems to be the appropriate one.

**Extensions**

**Definition of Money**

During empirical work we defined money to be the sum of currency and checkable deposits — M1. However, when we think about why institutional investors hold money, this definition is clearly incomplete. Institutional investors hold money because they want to avoid taking a capital loss on alternative assets. In this context, the components of M1 are no different from any other assets which can be liquidated on demand with no loss in capital value. (By *liquidated* we mean converted to currency or checkable deposits.) The distinction here is between assets on which it is possible to take a capital loss — non-monetary assets — and assets on which it is not possible to take a capital loss — monetary assets.
Given the forgoing considerations, the first item we should add to our definition of money is savings deposits. Even though savings deposits require a certain number of days notification prior to making a withdrawal, these requirements are not enforced. Savings deposits may de facto be liquidated on demand with no loss in capital value. They thus function as a monetary asset with respect to the definition given in the previous paragraph.

The next items we should include in our definition of money are *money market mutual fund* balances and *money market deposit account* balances. These are essentially checkable deposits with restricted checking privileges. For example, deposit holders may be limited to two checks per month with a minimum check size of $5,000. These restrictions do not impede the ability of such accounts to function as part of speculative money holdings; they maintain their capital value and they can be liquidated on demand.

It may be desirable to broaden our definition of money even further and include assets with very short maturities. Presumably, investors would be willing to hold any asset for speculative purposes as long as the asset could be liquidated in time to purchase non-monetary assets at the bottom of the market. Although this is true for assets which can be liquidated on demand, it is also true for assets with very short maturities. For non-banks, two categories of assets have very short maturities: repurchase agreements and overnight Eurodollar deposits. "Repurchase agreements are short-term collateralized loans in which the borrower sells securities to the lender with a provision to repurchase them on a specified future date at a specified price. Most repurchase agreements are for very brief periods of time, overnight to a few days." (Kaufman, p.63) Overnight
Eurodollar deposits are dollar-denominated overnight deposits in European branches of American banks.

Commercial banks have access to an additional short-maturity asset, federal funds. However, it may be reasonable to exempt this from our definition of money even if we allow other very short term assets. Why? Commercial banks are able to speculate on the federal funds market, and including federal funds themselves as part of the money holdings used to speculate on the capital value of federal funds would be inconsistent.

Speculation on the federal funds market is possible because of the peculiar manner in which required reserve accounting is carried out. To meet their required reserve mandate, banks must hold an average level of reserves over each two-week period equal to a specified proportion of average deposits over the same two-week period. If a bank thinks the federal funds rate will rise over a particular two-week reserve accounting period, it can hold sufficient excess reserves at the beginning of the period to meet its entire required reserve mandate. This frees the bank of the need to hold reserves during the latter part of the period. If the bank was correct in its expectations, the federal funds rate will increase before the end of the two-week reserve accounting period. Since the bank has already satisfied its reserve mandate, it can lend out all of its money holdings at a higher rate.

The redefinition of money to include savings deposits, money market deposit account balances, and repurchase agreements affects only a subset of institutional investors. The following list gives the types of monetary assets held by each type of institutional investor:
I. Life Insurance Companies
   A. Money Market Mutual Fund Shares

II. Other Insurance Companies
   A. Repurchase Agreements

III. Private Pension Funds
   A. Money Market Mutual Fund Shares
   B. Small Time Deposits
   C. Large Time Deposits

IV. Government Retirement Funds
   A. Large Time Deposits


Conclusion

The empirical relationship between speculative balances and the interest rate was estimated using quarterly Flow of Funds data on the money holdings of various institutional investors over the years 1982-1991. We estimated a panel data model using three separate measures of the interest rate: the stock market price-earnings ratio, the six-month treasury bill rate, and the Federal Funds rate. The demand for speculative balances was found to be related most strongly to the price-earning ratio. A one percent increase in the price earnings ratio causes a one-tenth of a percent increase in speculative money holdings. The regression using the T-Bill rate also revealed a statistically significant demand for speculative balances. We interpret these findings as evidence that money
managers engage in intertemporal price arbitrage, thus providing a for the model of asset price overshooting presented in the previous chapter.

1 The choice of six-month treasury bills over other maturities is justified in later sections.

2 Depository institutions are institutions which accept deposits from the public.

3 Total and required reserves are available in a form adjusted for changes in reserve requirements. In this form, each dollar of reserves supports the same quantity of deposits regardless of the time period under consideration. We do not use the data in this form because whether or not the reserve ratio varies is irrelevant for our purposes. Adjusting for it would only serve to mask true money holdings. A recent discussion of some of the issues surrounding adjusted versus unadjusted reserves appears in Haslag and Scott (1995).

4 Because speculative balances are potential loanable funds, we expect their volume to decrease as the rate of return on physical capital increases and lending becomes more profitable, so that failure to control for movements in the rate of return on physical capital will give rise to a spurious liquidity effect, with the interest rate moving in the opposite direction of speculative balances.

5 The fundamental value formula used here assumes investors use a constant discount rate.

6 As derived by Kleidon (1986).

7 As a first approximation, we ignore default risk.

8 An example of transactions costs: purchases and sales of federal funds must occur in minimum lots of ten million (Kohn, p. 229).

9 This is the daily equivalent of the rate quoted in the table.

10 Both these methods are discussed in Harvey (1993).

11 Methods for trend-stationary and integrated series can be applied sequentially to the same series.

12 This assumption will be inadequate insofar as institutions which trade both sides of the market do not have dual open positions at the particular point in time that the Flow of Funds records balances outstanding.
Chapter 6: Conclusion

This thesis investigated the role of professional money manager intertemporal arbitrage in explaining the liquidity effect. We started by developing a model of the professional money manager optimization problem, showing how their portfolio choices in equilibrium give rise not only to a liquidity effect, but also to the existence of speculative balances. Our model also revealed that the liquidity effect and speculative balances are related in magnitude. The model can be summarized by illustrating how money managers respond to liquidity shocks.

Example: Money Manager Response to Liquidity Shocks

First, consider a positive market liquidity shock. Money managers suddenly find themselves with an undesired increase in money holdings — undesired because the return on money is zero. Money managers will try to get rid of the money by purchasing bonds. The price of bonds is bid up, increasing outside borrowing, and resulting in money being withdrawn from the financial sector.

Suppose asset prices are bid up high enough that all money is withdrawn from the financial sector in the initial period. In the following period, there will no longer be excess liquidity pushing up bond prices, so the latter will fall and money managers will take a capital loss on their bond holdings. If the capital loss is large enough to outweigh the income earned on bonds, money managers will take a net loss. Money managers will of course not allow this scenario to happen, that is, they will not be willing to buy bonds
if the price is so high that it causes all excess liquidity will be withdrawn from the market in the first period. The highest price money managers will be willing to pay for bonds is that which induces just enough liquidity to be withdrawn from the market to in turn generate a net return of zero on bonds (net bond return is zero when price falls by exactly enough to offset interest).

Continuing with this argument, bond prices must fall in each period that money managers are holding excess liquidity, returning to their new steady state level at the same time that all excess liquidity is withdrawn from the market. Working backwards, we see that the particular sequence of capital losses required to reach a new steady state dictates how high the initial bond price must be immediately following the liquidity shock. Actually, the initial bond price and the sequence of capital losses are determined simultaneously, such that the sum of liquidity withdrawals induced by each bond price on the path to the new steady state adds up to the initial liquidity shock.

The initial increase in the money supply thus causes a form of asset price overshooting, with asset prices first rising above their long-run value and then slowly returning to it as excess liquidity is withdrawn from the market. Asset price overshooting can be regarded as interest rate undershooting — which is the liquidity effect if the initiating liquidity shock is an increase in the money supply.

If the financial market faces a negative liquidity shock, professional money managers will face a liquidity crunch; they will not have enough money to meet the desired withdrawals of households and firms. In their attempts to obtain money, professional money managers will sell bonds, causing the price of the latter to fall. But as bond prices fall, the quantity of money which firms and households wish to withdraw
declines and then becomes negative, at which point money flows into the financial sector. The bond price will fall until induced money inflows are sufficient to offset the initial liquidity shortfall.

Summary of Empirical Procedure and Results

The second part of this thesis tested a key implication of the model: that professional money managers engage in intertemporal arbitrage. This is analogous to the existence of a well-defined demand for speculative balances, which is what we directly test for. The empirical relationship between speculative balances and the interest rate was estimated using quarterly Flow of Funds data on the money holdings of various institutional investors over the years 1982-1991. We estimated a panel data model using three separate measures of the interest rate: the stock market price-earnings ratio, the six-month treasury bill rate, and the Federal Funds rate. The demand for speculative balances was found to be related most strongly to the price-earnings ratio. A one percent increase in the price earnings ratio causes a one-tenth of a percent increase in speculative money holdings. The regression using the T-Bill rate also revealed a statistically significant demand for speculative balances. We interpret these findings as evidence that money managers engage in intertemporal price arbitrage, thus providing evidence for our equilibrium model of asset price overshooting.

Our finding of a statistically significant interest elasticity for speculative balances provides evidence in favor of the existence of a liquidity effect which is free of the reverse causality problem.
References


