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Experimental and Analytical Results for Longitudinal Electromagnetic Levitation

by

Rod William Shampine

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

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ABSTRACT
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Electromagnetic levitation offers the possibility of working with metals in a containerless fashion. In order to realize this on a commercial basis, a solid theoretical understanding of the phenomena is needed, coupled with experimental work validating the theoretical models. In the case of the conventional conical electromagnetic levitator, models have been proposed for the forces, heating, and torque experienced by a levitated specimen. Experimental work has been primarily focused on measuring the forces.

A new type of levitator is proposed in the first part of this work. The proposed levitator is suitable for use on earth as well as in micro gravity and it overcomes almost all of the drawbacks that are inherent in currently used levitation melting devices. This levitator can support samples that are an order of magnitude more massive than those that can be supported by existing devices. Further, it can levitate liquid metal samples of arbitrary shapes and provide control over the position, movement, and the rate of heat generation in them. The new levitator has the potential to become a "containerless" manufacturing process.

An analysis of the currents induced in specimens supported in a longitudinal electromagnetic levitator is presented. Expressions for the forces, heating, and torque are developed. The predictions are compared
with experimental measurements and are found to be in excellent agreement. Inductance, optimum specimen to coil size ratio, and effect of the number of poles are discussed in connection with the design of this class of levitator.

Future directions for research with longitudinal levitators are discussed. These include finding the shape of the levitated specimen after it melts, levitation casting of significant quantities of metal, and demonstrating continuous processing of materials where only the molten portion is supported.

The final part of this work fills in a significant gap in the understanding of conical electromagnetic levitators by presenting experimental results for the heating in spherical specimens, and compares these with theoretical predictions. It is found that the heating may be predicted with good accuracy, even using a simplified model.
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The work on the longitudinal levitator is supported in part by the Texas Advanced Technology Program under grant number 003604-041 and by the National Science Foundation under grant number CTS-9312379.
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NOMENCLATURE

A  Vector potential
a  Specimen radius (m)
a_0  Loop radius (m)
B  Magnetic flux density (T)
C  Capacitance (F)
E  Electric field intensity (v/m)
c_p  Heat capacity (J/kg K)
f  Frequency (Hz)
F  Force (N)
g  Acceleration of gravity (9.8 m/s^2)
H_l  Skin depth function for power absorbed
h_c  Critical height of liquid metal (m)
I_{l+1/2}  Modified Bessel function
I_0  Peak coil current (A)
I_{n.l,m}  Source function for the nth current source (A)
l  Sample length (m)
J  Current density (A/m)
J_n  Bessel function
j  Imaginary number $\sqrt{-1}$
L  Inductance (H)
P  Power (W)
P_0  Characteristic power (W)
P_s  Power dissipated in the sphere (W)
\( P_l^m \)  Associated Legendre polynomial of the first kind
\( q \)  Ratio of radius to skin depth
\( R_s \)  Sphere radius (m)
\( T \)  Temperature (K)
\( \dot{v} \)  Volume flow rate (m\(^3\)/s)
\( x_{l+1/2,k} \)  kth real root of \( J_{l+1/2}(x_{l+1/2,k}) = 0 \)
\( Y_l^m \)  Spherical harmonic
\( \delta_{l,m} \)  Kronecker delta function

**Greek Symbols**

\( \vartheta \)  Variation
\( \delta \)  Skin depth (m)
\( \varepsilon \)  Permitivity (F/m)
\( \tau \)  Surface tension (N/m)
\( \rho \)  Density (kg/m\(^3\))
\( \rho_v \)  Free charge density (C/m\(^3\))
\( \mu_0 \)  Permeability (H/m)
\( \sigma \)  Conductivity (S/m)

**Superscripts**

\*  Per unit length

**Subscripts**

\( n \)  Due to the nth current source
\( s \)  Surface
I. INTRODUCTION TO ELECTROMAGNETIC LEVITATION

The conical electromagnetic levitation device was invented in 1952 (Okress, et alia) and has seen little change since then. A typical electromagnetic levitation device consists of a few turns of hollow copper tubing wound over the length of a few centimeters (the levitation coil) and topped by a few reverse wound turns (the capping coil), as illustrated in Figure 1. A large, high frequency current (~ 400 to 600 A, 400 kHz) is allowed to flow through the coil and set up an alternating magnetic field. By virtue of its geometry, the coil creates a magnetic field with a region of minimum field strength (known as the 'potential well') in the gap between the levitation and the capping coils. When an electrically conducting sample is placed in this gap, eddy currents are induced in the sample, similar to the currents that are induced in the secondary of a transformer by the primary. As these currents flow they cause heating due to the resistivity of the sample. They also interact with the external field and exert a Lorentz force on the sample that tends to move it to areas of low field gradient, which are arranged to be above the sample so that a force is developed in a direction opposite to that of gravity. The sample therefore levitates and melts within a hundred seconds or so (Sneyd and Moffatt (1982)). There are several advantages of suspending a sample of liquid metal in this fashion (Bayazitoglu (1996)). First, it does away with a physical crucible and provides a solution to the problem of contamination due to containers. Second, it is a valuable diagnostic tool. For example, study of the heat transfer from the sample reveals information about the
Figure 1  A typical existing levitator.
thermal diffusivity of the material and its optical properties such as surface emissivity (Bayazitoglu, Suryanarayana, and Sathuvalli (1990), Murphy and Bayazitoglu (1992), Bayazitoglu (1995). In addition, by analyzing the dynamics of a suspended liquid metal droplet, we obtain the surface tension and viscosity of the liquid metal (Soda, Mclean, and Miller (1997), Bayazitoglu and Suryanarayana (1992), Suryanarayana and Bayazitoglu (1991), Bayazitoglu, Sathuvalli, and Mitchell, 1995).

Since Okress, et alia demonstrated levitation melting in 1952, different aspects of levitation melting have been analyzed. Most of the literature on levitation in a conical levitator has been concerned with one or more of the following aspects: calculations of levitation forces on a conducting body in an alternating magnetic field and of the power absorbed by it (Brisley and Thornton (1963), El-Kaddah and Szekely (1983), Rony (1964), Smith (1965), Bocian and Young (1971), Lohofer (1989), Bayazitoglu and Cerny (1994), Sathuvalli and Bayazitoglu (1994), Bayazitoglu and Sathuvalli (1996)), stability and flow phenomena in the levitating sample (Volko (1962), Bocian and Young (1971), Holmes (1978), Sneyd and Moffatt (1982), Mestel (1982), Garnier and Moreau (1983)), and experimental studies (Okress, et alia, Harris and Jenkins (1959), Colgate, Furth, and Halliday (1960), Bunshah and Juntz (1964), Fromm and Jahn (1965), Shampine, Sathuvalli, and Bayazitoglu, (1996)).

One of the earliest analyses of levitation studies concerns the force on a conducting sphere in a uniform unidirectional alternating magnetic field (Rony). Fromm and Jahn calculate the forces on conducting specimens in the field produced by circular current carrying loops and compares the
results with experimental data. The magnetic fields of axisymmetric levitation coils are studied by Bayazitoglu and Sathuvalli (1993) and Sathuvalli and Bayazitoglu (1993). Brisley and Thornton obtain a solution of the electromagnetic field problem for a conducting sphere on the axis of a set of coaxial current carrying loops. Smith derives an expression for the levitation force on a sphere in terms of the effective inductance of the levitation coil. Bocian and Young considers the stability of a levitating specimen by using a circuit equivalent model of the excitation coil and the levitating sample. An analysis of a sphere in an arbitrary magnetic field shows that the induced current density by it can be expressed in terms of certain "source functions" (Lohofer (1989)). Bayazitoglu and Sathuvalli (1994) presents a method to calculate these "source functions" for an arbitrary coil geometry and shows the role of the nonhomogeneity of the field.

Levitation melting in microgravity has been pursued with the hope of eventually developing the levitator into a manufacturing tool and also for using it to measure the thermophysical properties of reactive or high-purity materials. The typical electromagnetic levitation melting device suffers from several drawbacks inherent in its design:

i) **Sample shapes:** The coils and the sample posses a degree of axisymmetry about the coil axis which restricts the kinds of samples that can be levitated. For example, samples with large aspect ratios such as cylinders cannot be levitated in currently available levitators. To the best
of our knowledge, only small samples (that can be circumscribed by a sphere) have been levitated in this device.

ii) **Visual access**: When these levitators are used as diagnostic instruments to measure the thermophysical properties they suffer from the problem of sample visibility and control. The coil design is such that it obstructs the view of the sample and makes data collection difficult. In experiments that measure surface tension or viscosity, visual access is very important since the shape oscillations of the sample must be recorded photographically (Krishnan, Hansen, Hauge, and Margrave (1988), Egry, Lohofer, Neuhaus, and Sauerland (1992)). In experiments that involve heat transfer, optical pyrometric techniques are used to measure the temperature.

iii) **Position control of the sample**: Existing levitators are essentially static devices, i.e., they allow very little manipulation of the sample. The region of equilibrium of the sample is limited to the volume of the "potential well" that is created in the gap between the levitation and capping coils.

iv) **Sample instability**: The levitated sample is very unstable. In existing levitators the coil is a helix that is wound on the surface of a cone. Even when the coil is tightly wound there is a finite transverse component of the magnetic field at points on the axis. This is responsible for a small but non-zero transverse component of the Lorentz force when the sample is
along the axis. In addition, these non-zero components of the field create a
torque that causes rapid rotation of the sample. Other unavoidable
asymmetries in the coil (for example, the conductor that connects the
levitation and capping coils) can be responsible for unstable motion of the
sample (Egry, Lohofer, Neuhaus, and Sauerland, and Cummings and
Blackburn (1991)).

v) Control of sample heating: The drawbacks identified so far are
inevitable in the design of the levitator. In addition, when these levitators
are used to simulate microgravity experiments on earth, they have other
disadvantages. The heating and the Lorentz force cannot be controlled
independently in a levitator that operates on earth. At the frequency, $f$, of
the levitating field, for a given coil current, the parameter that controls the
heat generation and the Lorentz force is the ratio ($q$) of the sample size ($a$)
to its skin depth ($\delta$). In the range of operation ($\sim 10^5$ Hz) the rate of heat
generation varies linearly with the parameter $q$. On the other hand, the
Lorentz force reaches an asymptotic value at high frequencies (i.e. large
values of $q$). At low frequencies, however, both the heat generation and
the Lorentz force vary as the fourth power of $q$. This was predicted by
Rony in 1969, but there has been little experimental work to verify the
heating predictions. This lack is addressed in chapter IV. The differing
heating and force functions imply that the field may not exert enough force
on the sample if the frequency is too low. If the frequency is too high, the
control on the heat generation is effectively lost. At a given frequency, the
Lorentz force and rate of heat generation depend on the square of the coil
current. In microgravity, the situation is saved by the fact that lower frequencies and lower currents are required for levitation. To a certain extent, this removes the upper limit on the sample size and provides a greater degree of control on the heating since it can be controlled by auxiliary sources of heat. Therefore, the small forces and the accompanying low rates of joule heating encountered in space cannot be simulated on earth.

vi) The "magnetic hole": When the frequency of magnetic field is high, the induced currents flow essentially on the surface of the sample. The Lorentz force manifests itself as a surface pressure distribution. In conical levitation devices, the magnetic pressure on the sample surface vanishes at the top and bottom points. The molten metal is prevented from leaking out of the "magnetic hole" at the lowest point by surface tension (Sneyd and Moffatt). Thus, there is an upper limit on the mass that can be levitated and melted without "leakage" in this kind of a levitator (Bocian and Young). There is a critical sample height $h_c$ given by $h_c = 2\pi/\rho g \delta$, where $\tau$ and $\rho$ are the surface tension and density of the sample respectively and $g$ is the acceleration due to gravity. Above the critical height, the surface tension at the bottom of the sample cannot support the hydrostatic pressure and the metal begins to drip. The magnetic hole can be closed by using multiple frequencies, but this also multiplies the cost of the process (Sagardia and Segsworth (1977)). These constraints may have led Garnier (1987) to state that "...levitation melting will never be used on an industrial scale." In microgravity this situation is, of course, unimportant.
vii) **Magnetic stirring**: Finally, the induction in the sample is responsible for "electromagnetic stirring" of the molten fluid. In general, the Lorentz force on the sample has a non-zero curl and is responsible for fluid motion in the sample. If $\delta \ll a$, the driving force is confined to the skin depth close to the surface of the sample. If $\delta$ is of the order of the sample size or greater, the sample is transparent to the magnetic field and the driving force is more uniformly distributed across its volume. If the frequency of the magnetic field is increased from zero to infinity, it has been found that the "stirring efficiency" (measured in terms of the kinetic energy of the stirred fluid) exhibits a peak at an optimum frequency and is very small at high frequencies (Moffatt (1991)). In microgravity, the frequencies of the levitating magnetic field are less than those required on earth. This results in a more uniform distribution of the stirring force (and movement of the fluid). However, the magnitude of the stirring force is correspondingly smaller due to the smaller currents in the coils.

These disadvantages imply a need for a device that can be used to levitate large samples and control them precisely during the process of levitation. Due to the high cost of performing a study in microgravity, it is necessary to carefully consider the phenomena to be studied in orbit. Before going into space, the proposed experimental study must be thoroughly investigated on earth. Only those aspects of the problem that cannot be addressed or simulated on earth must be investigated in microgravity. Clearly, conical levitators do not permit such a preliminary study.
We now introduce a new class of electromagnetic levitators that alleviates most of the problems suffered by the currently used levitators and that can be used for containerless manufacturing processes both on earth and in microgravity. In the Department of Mechanical Engineering and Materials Science at Rice University, we have levitated and melted copper, aluminum, and brass samples up to 622 grams. The levitator is a radical departure from the existing designs and has the following features:

1) It is capable of supporting spherical and non-spherical sample shapes, such as cylinders with a large aspect ratio, rectangular blocks, etc.

2) The new design has very good visual access to the sample.

3) The position of the sample in the levitator can be controlled precisely.

4) The sample is very stable and does not rotate or vibrate. This feature makes the design ideal for thermophysical property determination.

Furthermore, under normal gravity conditions on earth:

1) Despite the large loads that can be levitated, the heating rates are not very large. The temperature of the sample can be controlled independently by using an auxiliary heat source.

2) It can already support samples that are an order of magnitude more massive than the samples that can supported using existing levitators and can be scaled up, thus overcoming the problem of the "magnetic hole."
3) Multiple specimens can be simultaneously levitated and controlled. This leads to the possibility of creating and studying alloys in a containerless manner, a yet unrealized goal of the manufacturing industry.

I. LONGITUDINAL ELECTROMAGNETIC LEVITATOR

As shown by Figure 2, an eight pole levitation coil (Bayazitoglu and Shampine (1996, 1997)) is formed by bending copper tubing so that a set of parallel conductors are formed, with neighboring conductors passing current in opposite directions. These conductors form a cylinder about an axis. The longitudinal conductors can also be curved along their length, and the surface they define is not limited to a cylinder. Either bends may be used to connect the ends in series, or the ends may be connected together in parallel, depending on the desired impedance of the coil. Further, the coil may be arranged with multiple sections to permit connection to multiple power sources, possibly differing in phase and/or frequency. Although it is not necessary to position both tube ends on the same end of the coil, it facilitates connection to the generator.

Experimental coils have been made with 1/4 and 3/16 inch (6.3 mm and 4.7 mm) diameter copper tubing, through which cooling water is passed. The cooling water is provided to prevent the coils from overheating due to the high current they carry, and has no effect on specimens in the levitator.
Figure 2  An 8 pole longitudinal electromagnetic levitator.
DISCUSSION

In the cross-section of the levitator in Figure 2, shown in Figure 3, the current in adjacent straight sections flows in opposite directions, forming four pairs of opposed magnetic poles. For purposes of illustration, the absolute magnitude of the magnetic flux density at various points in a cross-sectional plane through a levitator with six poles is shown as elevation in Figure 4. The levitator creates a range of magnetic flux density values ranging from zero around the axis of the levitator and outside of it to a maximum value on the surface of the conductors. The points of zero value at the center of each peak are when conductors are located, as there is no magnetic flux inside the conductors. The magnitude and direction of the force on a conducting specimen within the present levitation coil is related to the slope of this graph. A useful analogy is to imagine placing a marble on this surface. The marble will experience forces analogous to those experienced by the specimen. These levitation coils have two types of stable regions; on axis, and between conductors. The region, or well, centered on the axis is stable, having a restoring force in all directions. The points situated midway between each pair of adjacent conductors are metastable; that is, they have restoring forces pushing away from the conductors, but no restoring force directed radially either into or out of the coil.

The well extends along the axis of the levitator, forming a levitation "tunnel" approximately the same length as the levitator. This tunnel may be closed on the ends by positioning end turns across it, but this is not
Figure 3  A cross-sectional view of Figure 5 showing the direction of current flow and the magnetic field lines.
Figure 4  A schematic representation of the magnitude of magnetic flux density in the cross-sectional plane of a six pole longitudinal levitator.
required. Specimens may be freely moved along the length of this tunnel by tilting the levitator or by applying a small external force along the axis of the levitator. By making the levitator axis wavy rather than straight, a series of stable positions or potential wells within the levitator can be created. Slight adjustments to the angle of the levitator or the application of external forces (such as those supplied by a second coil operating at a different frequency) can move one or more specimens along or amongst these well(s) under close control. By using multiple phases, it is also possible to cause the specimen to rotate under control. Both rotary and translational motion within the levitator experience no frictional effects, permitting extremely high velocities.

The present levitator has multiple lines of zero force between the conductors and parallel to the axis of the levitator, similar to the point of zero force at the center of conical levitators. These zero force lines may be eliminated by using multiple frequencies or multiple phases, allowing additional height in the specimen. Even without eliminating the zero force lines, the present levitator is able to vastly exceed the lifting capacity of conventional levitators by increasing the volume of the specimen without increasing its height. The volume increase is made possible by increasing the length and width of the specimen. Wider specimens require sufficient conductors under the specimen to maintain it within the levitator.

Because of the large area of low magnetic flux density within this levitator, it produces less heating in levitated specimens than other types of levitators, allowing control of the temperature of the specimen using the heat produced by the levitator or by the application of external heating. As
a specimen is moved away from the axis of the levitator, the heat produced in it increases.

The specimen temperature is dependent on: 1) the rate of induction heating in the specimen, 2) the rate of heat loss to the surrounding atmosphere, if any, and 3) the energy provided by any external heat source. Because relatively little heat is transferred to the specimen by the present levitator, it is less likely that the specimen will be molten in the absence of an external heat source. Even when the specimen is molten in the absence of an external heat source, as in the case of low melting point metals, the rate of induction heating is low enough that the specimen can be maintained below its melting point by misting it with an air/water spray mist or similar cooling medium. The low heat input produced by this levitator more closely approximates the conditions experienced in microgravity, and also permits the frequency to be lowered to increase electromagnetic stirring while maintaining levitation.

A molten specimen in the longitudinal levitator is repulsed by the field gradient surrounding each conductor. In the absence of gravity, the molten specimen is shaped only by the magnetic flux density and its own surface tension and therefore assumes a slightly star-shaped or lobed cross-section similar to what would be seen if a liquid with high surface tension were poured into the center of Figure 4. In the presence of gravity, the cross-section of the specimen is somewhat flattened, as the top poles of the levitator play a less important role in containing the specimen. The cross sectional shape of the specimen can be modified by changing the configuration of the poles. Hence, it is possible to produce near net shape
specimens of arbitrary shape requiring less machining than the generally pear shaped specimens produced by prior levitators.

Coil inductance increases with levitator length and diameter, and decreases with the number of poles. Higher inductance reduces the current that can be passed through the coil which, in turn, reduces the levitation force. More poles produce a more even distribution of forces around the specimen and improve side-to-side stability at the expense of added complexity. The top poles in an earthbound levitator do not contribute to levitation force, but instead act to increase the magnetic field gradient in comparison to a coil without them.

This levitator can be used in either batch or continuous mode. In batch mode, individual specimens or groups of specimens are separately processed in the levitator and the levitator is emptied between batches. If power to the levitator is off when a batch is placed in the device, the specimen or specimens comprising the batch will rest on the lower coils and must be large enough to be mechanically supported by the coils. If the levitator is already turned on when the specimen is placed inside it, the specimen will levitate immediately. When the levitator is operated continuously, new material is fed into the levitation zone and processed material is removed from the levitation zone continuously. Unlike prior levitators, the present levitators are particularly well suited to processing in the continuous mode because they provide unobstructed access to the levitation zone at each end of the levitation "tunnel." Material can be fed into one end of the levitator, moved through the levitation zone along its axis, and be removed at the opposite end of the levitation zone.
Alternatively, because the amount of heat imparted to the specimen by the present levitator is so small, the specimen can be suspended its solid state and an external heat source can be used to cause localized heating of the specimen. The molten region can be moved along the specimen by moving the heat source, or the specimen itself can be moved relative to the heat source. If desired, the coil can be configured so that changes to the shape of the specimen are produced in the molten region and reflected in the cast solid when the specimen is cooled.

**LEVITATION RESULTS**

The lifting capacity of multipole levitators differs for solid and liquid specimens. For solid specimens, there is a maximum diameter of specimen that can be lifted, limited both by physically interfering with the coil and by the downward force exerted by the upper turns. We have had good results with specimens up to 60% of the coil diameter. For liquid metals, the relationship between metallostatic head and surface tension dictate the maximum force per unit length that can be applied. It is possible to expand the molten specimen both along the axis of the levitator, and also horizontally. We have levitated molten specimens that crossed up to three minimas in the force field (Figure 5).

We have experimentally determined the maximum mass and force per unit length that three representative coils, consisting of four, six, and eight poles, can deliver. Their cross sections are shown in Figure 6. These coil configurations were arrived at empirically, and represent the most
Figure 5  Levitated shapes of molten metal crossing one and three force field minima.
Figure 6  Levitation coils; a) 4 pole, b) 6 pole, and c) 8 pole
successful examples of a larger group. Their lifting limits are shown in Table 1. The six pole coil, which was constructed last, represents the best tradeoff between number of poles, coil diameter, and length, in terms of lifting capacity.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Material,</th>
<th>Diameter (cm)</th>
<th>Mass (g)</th>
<th>Force (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Pole</td>
<td>Cu, solid</td>
<td>1.27</td>
<td>153.05</td>
<td>12.8</td>
</tr>
<tr>
<td>6 Pole</td>
<td>Cu, solid</td>
<td>2.54</td>
<td>622.2</td>
<td>43.9</td>
</tr>
<tr>
<td></td>
<td>Al, solid and liquid</td>
<td>2.54</td>
<td>180.4</td>
<td>11.6</td>
</tr>
<tr>
<td>8 Pole</td>
<td>Al, solid</td>
<td>2.54</td>
<td>206.8</td>
<td>13.29</td>
</tr>
<tr>
<td></td>
<td>Al, solid and liquid</td>
<td>1.27</td>
<td>50</td>
<td>3.2</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

The longitudinal levitator described here represents a significant improvement over previous levitators. Its lifting capacity is already an order of magnitude greater (620 grams) and can be scaled up. With this comes reduced sample heating, radically improved stability, positioning, and visual access. Further, it is capable of supporting and forming objects of arbitrary shape, especially those with large aspect ratios. Its simplest form is a set of parallel conductors on the surface of a horizontal cylinder. Experimental results for lifting capacity of multiple coils are presented, along with heating and force measurements from multiple specimen sizes as they are moved perpendicularly to the axis of a coil.
II. THEORETICAL ANALYSIS OF THE LONGITUDINAL LEVITATOR

Although conical levitators have been studied extensively, there is very little discussion of the longitudinal geometry in the literature. Vutsens (1971) shows some experimental results for two and four pole levitators using liquid sodium. Okress, et alia gives some experimental results for a two pole levitator with the currents directed in the same direction in both conductors, and Sneyd and Moffatt (1982) analyze this case. Finally, Piggott and Nix (1966) present an analysis and experimental results for a two pole levitator with the currents flowing in opposite directions.

In order to find the electromagnetic effects on a specimen, we must first know the distribution of currents induced in it by the magnetic field. The relationship between electric field intensity (\( \mathbf{E} \)), magnetic flux density (\( \mathbf{B} \)), and current density (\( \mathbf{J} \)) is described by Maxwell’s equations.

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} = \mu \left( \mathbf{J} + \mathbf{e} \frac{\partial }{\partial t} \mathbf{E} \right) \\
\n\nabla \cdot \mathbf{E} = \rho_v / \varepsilon \\
\n\nabla \cdot \mathbf{B} = 0 \\
\n\mathbf{J} = \sigma \mathbf{E}
\]

In solving this set of equations, it is convenient to introduce a vector potential \( \mathbf{A} \) defined such that
\[ \nabla \times \mathbf{A} = \mathbf{B} \]
\[ \nabla \cdot \mathbf{A} = 0 \]

The vector potential allows Maxwell's equations to be reduced to a single governing equation.

\[ \nabla^2 \mathbf{A} + \sigma \frac{\partial \mathbf{A}}{\partial t} - \mu \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0 \]

If we make the assumption that the time dependence is of the form \( e^{i\omega t} \) (sinusoidal), as is the case for most generators capable of producing the current required for levitation, the time dependence of the above equation can be factored out.

\[ \nabla^2 \mathbf{A} - j\omega \sigma \mathbf{A} + \omega^2 \mu \varepsilon \mathbf{A} = 0 \]

For non-conducting media with structures much smaller than the electromagnetic wavelength, this reduces to

\[ \nabla^2 \mathbf{A} = 0. \]

Two types of boundary conditions are important here. First is the interface between a conductor and a loss-less dielectric. For this case, both the electric field and the magnetic flux density are continuous across the surface, and the normal component of the electric field is zero. The distance over which the electromagnetic field and, equivalently, the eddy
currents, decays to 1/e is referred to as the skin depth, and can be found from

$$\delta = 1/\sqrt{\pi f \mu_0 \sigma},$$

where $f$ is the frequency, $\mu_0$ is the permeability, and $\sigma$ is the conductivity.

The second type of boundary is one between two dielectric media on which a surface current flows. Again, the electric field is continuous, and the normal component of the magnetic flux density is also continuous. The tangential component of the magnetic flux density has a discontinuity equal to

$$\hat{a}_n \times (\Delta B_{\tan}) = \mu J_s.$$

A z-directed line current at $b_m, \alpha_m$ may be represented by a Fourier series of surface currents on the cylinder $r=b_m$.

$$J_m(\theta) = \sum_{-\infty}^{\infty} J_{n,m} e^{jn\theta}$$

$$= (1/2\pi b_m) \lim_{\delta \to 0} \int_{\alpha_m - \delta}^{\alpha_m + \delta} (\hat{i}_m/2\delta) e^{-jn\theta} d\theta,$$

$$= (\hat{i}_m/2\pi b_m) e^{-jn\alpha_m}$$

where $\hat{i}_m$ is the peak current in the $m$th filament, positive for currents in the z direction. Figure 7 shows the coordinate system.
Figure 7  Coordinate System
In order to find the current density in the cylinder, Maxwell's equation needs to be solved in three regions, and the boundary conditions need to be applied to find the constants. In the first region, within the cylinder, the conductivity is non-zero, requiring the full Maxwell's equation to be solved.

Drawing on Piggott and Nix, a solution of the following form is appropriate:

\[ A = \sum_{n} C_n J_n(k_l r)e^{jn\theta}. \]

Where the constant \( k_l \) is defined as \( k_l = \sqrt{-j\omega\mu_0\sigma} \). The \( n = 0 \) term represents a net current flowing along the axis of the cylinder. As there is no return path for this current, and as it has not been applied externally, \( C_0 \) must be zero. For the region between the cylinder and the current filament \( m, a < r < b_m \), a different form is used to solve the simpler form of Maxwell's equation that applies.

\[ A = \sum_{n} \left(D_n r^n + E_n r^{-n}\right)e^{jn\theta} \]

The region outside the current filament uses the same solution, modified by the condition that it must go to zero at infinity; i.e., there are no positive powers of \( r \).

\[ A = \sum_{n} F_n r^{-n}e^{jn\theta} + \sum_{n} G_n r^{n}e^{jn\theta} \]
Applying the boundary conditions at \( r = a \) yields two equations:

\[
D_n a^n + E_n a^{-n} + C_n J_n(k_1a) = 0
\]
\[
D_n a^n - E_n a^{-n} + (k_1 a/n) C_n J'_n(k_1a) = 0
\]

Where \( J'_n(k_1r) \) is the derivative of \( J_n(k_1r) \) with respect to \( k_1r \). The surface currents \( J_{n,m} \) affect the boundary conditions at \( r = b_m \), which are complicated by the division into positive and negative \( n \).

\[
D_n b^n + E_n b^{-n} = F_n b^{-n}, n > 0
\]
\[
D_n b^n + E_n b^{-n} = G_n b^n, n < 0
\]
\[
D_n b^n - E_n b^{-n} + F_n b^{-n} = (b\mu/n) J_{n,m}, n > 0
\]
\[
D_n b^n - E_n b^{-n} - G_n b^n = (b\mu/n) J_{n,m}, n < 0
\]

Solving for \( C_n \), we find

\[
C_n = (a/b_m)^{n-1} \mu J_{n,m}/k_1 J_{n-1}(k_1a), n > 0
\]
\[
C_n = (-1)^n (a/b_m)^{n-1} \mu J_{n,m}/k_1 J_{n-1}(k_1a), n < 0
\]

Adding in the contributions of all of the \( p \) current filaments, we finally get

\[
C_n = \mu \sum_{m=1}^{p} (a/b_m)^{n-1} \mu J_{n,m}/k_1 J_{n-1}(k_1a), n > 0
\]
\[
C_n = \mu (-1)^n \sum_{m=1}^{p} (a/b_m)^{n-1} \mu J_{n,m}/k_1 J_{n-1}(k_1a), n < 0
\]
and can evaluate

\[ B_r = \sum_{n=0}^{\infty} \left( \frac{j_n}{r} \right) C_n J_n(k_1 r) e^{i n \theta} \]

\[ B_\theta = -k_1 \sum_{n=0}^{\infty} C_n J'_n(k_1 r) e^{i n \theta} \]

\[ E_z = -\left( k_1^2 / \mu_0 \sigma \right) \sum_{n=0}^{\infty} C_n J'_n(k_1 r) e^{i n \theta} \]

**FORCE**

The net Lorentz force on an object is the integral of the cross product of the current density and the magnetic flux density. In evaluating this it is convenient to use complex coordinates, where a point in the Cartesian plane is represented by \((x+jy)\). Force per unit length, \(F^*\), is found by integrating

\[ F^* = \text{Re} \int_0^{2\pi} \int_0^a \mathbf{J} \times \mathbf{B} \ r \ dr \ d\theta. \]

For \(z\) directed currents and a magnetic flux with only \(r\) and \(\theta\) components, this reduces to

\[ F^* = \text{Re} \int_0^{2\pi} \int_0^a j J_z (B_r (\cos \theta + \sin \theta) + B_\theta (\cos \theta - \sin \theta)) \ r \ dr \ d\theta. \]

Introducing the expressions for current density and magnetic flux density found earlier and gathering like terms, we find
\[
F' = \text{Re} \left[ \int_0^{2\pi} \sum_{n=-\infty}^{\infty} C_n e^{in\theta} \left( \sum_{q=-\infty}^{\infty} \bar{C}_q e^{-iq\theta} \left( \begin{pmatrix} -jq/(r \cos \theta + \sin \theta) \\ -k_l/(r \cos \theta - \sin \theta) \end{pmatrix} \right) \right) d\theta \right] J_n(k_l r) |^r dr
\]

Integration yields

\[
F' = \left( \pi \omega a / 2 \right) \sum_{n=-\infty}^{\infty} C_n C_{n+1} J_n(k_l a) J_{n+1}(k_l a).
\]

Define \( c_m \) to be the position of the \( m \)th filament in the complex coordinate system and \( w \) to be the center of the cylinder. Rearrangement of the series yields a readily evaluated expression for the force per unit length on the cylinder.

\[
F' = \frac{\mu_0}{4 \pi a} \sum_{n=1}^{\infty} \left\{ \sum_{m=1}^{p} \left( \frac{a}{c_m - z} \right)^n \hat{I}_m \right\} \left\{ \sum_{m=1}^{p} \left( \frac{a}{\bar{c}_m - \bar{z}} \right)^{n+1} \hat{I}_m \right\} \text{Re} \left[ \frac{J_{n+1}(k_l a)}{J_{n-1}(k_l a)} \right]
\]

(1)

**POWER**

The instantaneous power dissipated in the cylinder by the eddy currents can be found by integrating the dot product of the electric field and the current density over the cylinder, equivalent to voltage times current in an ordinary electric circuit. Taking the root mean square (RMS) of the instantaneous power yields the equivalent continuous power. In the case of sinusoidal waves, this is equivalent to dividing by 2. RMS power per unit length is found by integrating the following.
\[ P^* = \frac{(\sigma/2)}{2^{\pi a}} \int_0^\frac{2\pi}{a} \int_0^\infty |E_z|^2 r \, dr \, d\theta \]

Integration yields

\[ P^* = \pi \sigma \omega^2 \sum_{n=-\infty}^{\infty} |C_n|^2 \frac{a}{2k_i} \left[ J_n(k_i a)J_{n-1}(k_i a) - jJ_n(k_i a)J_{n+1}(k_i a) \right]. \]

Rearranging the series leads to

\[ P^* = \frac{\omega \mu}{4\pi} \sum_{n=1}^{\infty} \left\{ \sum_{m=1}^{n} \left( \frac{a}{c_m - w} \right)^n \right\}^2 \frac{1}{n} \text{Im} \frac{J_{n+1}(k_i a)}{J_{n-1}(k_i a)}. \]

**TORQUE**

The longitudinal levitator bears a great resemblance to a single phase motor with \( p/2 \) poles. Such a motor runs at a speed up to \( p\omega/4 \). In the case of an eight pole levitator operated at 300 kHz, the synchronous speed is 36 million revolutions per second. So, a crucial question is whether the levitated specimen will start to rotate. Following Fitzgerald, Kingsley, and Kusko (1961), a single phase motor can be modeled as two magnetic fields rotating in opposite directions at synchronous speed. The net torque is the difference between the torque delivered by the two fields which, in turn is equivalent to the power delivered to the rotor divided by the synchronous speed.
\[ T = T_f - T_b = \left( P_{gf} - P_{gb} \right)/\omega_s \]

The rotor is slower than the forward field and faster than the backward field. The slip, \( S \), is defined as \( \omega_s = (1 - S)\omega_f \). A slip of one means the rotor is not turning. The electrical model of a levitation coil in motor terms is shown in Figure 8. The \( \phi \) components are the resistance and inductance of the coil. \( x_2 \) represents the air gap, the \( r_2/2S \) resistor is where the forward power is dissipated, and the other \( r_2 \) resistor is where the backward power is dissipated. For the levitation coil used in the experimental portion of this work, the measured values are as follows.

\[ r_\phi = 0.0168\Omega \]
\[ I_m = 424A_{rms} \]
\[ V = 467V_{rms} \]
\[ P_{gf} + P_{gb} \leq 394W_{rms} \]

Assuming a sinusoidal distribution of current around the cylinder which is confined to a layer equal to the skin depth, the rotor resistance can be approximated by

\[ r_2 = 2\rho p^2 l/\pi^2 d\delta = 0.00413\Omega. \]

Using this, \( x_2 \) can be found to be 1.475 Ohms, and \( x_\phi \) to be 4.07 Ohms. The equivalent forward and backward field resistances, referred to the input current are given by
Figure 8  Equivalent Circuit for Levitation Coil Modeled as a Motor
\[ x_{22} = x_2 + x_\phi \]
\[ Q_2 = x_{22}/r_2 \]
\[ R_f = \frac{x_\phi^2}{x_{22}} \left( \frac{1}{SQ_2 + (1/SQ_2)} \right) \]
\[ R_b = \frac{x_\phi^2}{x_{22}} \left( \frac{1}{(2-S)Q_2 + (1/(2-S)Q_2)} \right) \]

Knowing these resistances, the net torque can be found from

\[ T = I^2 (R_f - R_b)/2\omega_z. \]

Using these formulae, it can be shown that the torque is zero for slip equal to one or zero. In other words, the rotor will not start by itself. Inserting the experimental values shows that a rotor turning at 170 RPM, faster than could be accidentally achieved, experiences only 2.2\times10^{-12} Newton-meters of torque. Experimentally, all specimens have stopped rotating within a few minutes after being started spinning at considerable speed.

**IMPEDEANCE**

In order to find the impedance of a multipole coil we will model it to as a transmission line. Using the relationship \( LC = \mu e \), we can perform the simple calculation of capacitance and then use it to find inductance. For two parallel wires of radius \( a \), with their centers \( D \) apart, the capacitance between them is
\[ C^* = \pi \varepsilon / \cosh^{-1}(D/2a) \]

measured in Farads per meter. Numbering the \( p \) conductors such that the even numbered conductors have the opposite sense from the odd conductors, we sum the parallel capacitors between each conductor and its neighbors. This can be simplified by knowing that there is no capacitance between conductors at the same potential, which is the case for the set of odd conductors, and separately for the set of even conductors. Using the notation that \( D_{i,j} \) represents the distance between the \( i \) and \( j \) conductors, the capacitance and inductance per unit length for levitators with their conductors connected in parallel is

\[
C^*_{\text{parallel}} = \pi \varepsilon \left[ \sum_{i=1}^{p/2} \sum_{j=1}^{p/2} 1/cosh^{-1}(D_{2i,2j-1}/2a) \right]
\]

\[
L^*_{\text{parallel}} = \frac{\mu}{\pi} \left[ \sum_{i=1}^{p/2} \sum_{j=1}^{p/2} 1/cosh^{-1}(D_{2i,2j-1}/2a) \right].
\]

If the conductors are connected in series, as is normally done, the inductance per unit length will be

\[
L^*_{\text{series}} = \frac{n^2 \mu}{4\pi} \left[ \sum_{i=1}^{p/2} \sum_{j=1}^{p/2} 1/cosh^{-1}(D_{2i,2j-1}/2a) \right].
\]

For conductors spaced evenly on a circle, as the number of conductors increases, the inductance decreases. This decrease is much more rapid for
a parallel connection than a series connection. The resistance per unit length is that of a tube one skin depth thick:

\begin{align*}
R_{\text{parallel}}^* &= 4/\pi\sigma p (2a\delta - \delta^2) \\
R_{\text{series}}^* &= \rho/\pi\sigma (2a\delta - \delta^2)
\end{align*}

As the number of poles increases, the force on a given specimen increases. Starting with the eight conductor coil used in the experimental measurements and using a 6 mm diameter specimen, Figure 9 shows the variation in maximum force with the number of poles for evenly spaced coils of constant radius. The curves for series and parallel connection take in to account the parameters of the radio frequency generator and the change in inductance of the coil with the number of poles. For a reasonable number of poles, say four to ten, the parallel connection results in an increase in the current per conductor. However, for large numbers of conductors, the increase in the terminal current is offset by the number on conductors it is divided amongst, and the generator approaches its short circuit current. Also, coils with a large numbers of poles are much more difficult to fabricate, and they restrict the visual access to the specimen.

Having used the force and heating predictions detailed above to arrive at a coil geometry, current, and frequency for a particular job, the impedance per unit length and the parameters of the generator can be used to determine how long the coil can be.
Figure 9  Variation of Force per Unit Length with respect to number of poles
III. EXPERIMENTAL RESULTS WITH THE LONGITUDINAL LEVITATOR

POWER MEASUREMENTS

In order to produce measurements of power and force in multipole levitators that can be compared with theoretical predictions, a coil was constructed in which the positions of the conductors were known very accurately. It consisted of 8 conductors of 0.479 cm diameter with their centerlines evenly spaced on a 2.985 cm radius circle. The straight portion of the conductors was 17.221 cm long. To prevent the conductors from moving due to the magnetic forces between them, the two ends were secured inside Nylon rings. Connected through a step down transformer to the radio frequency generator, this coil carried 600 amps peak at 284 kHz. Water cooled specimens were traversed through the coil using a laboratory jack with a ruler attached to measure the position. The relationship between the measured position and the coil was determined by analyzing the data. Water flow was measured with a variable area flow meter. This flow meter was calibrated at the two operating points to eliminate error due to non-linearity in the meter. Inlet and outlet temperature were measured with an electronic thermocouple meter reading in tenths of a degree Celsius. One meter lengths of rubber hose were used to isolate the thermocouple reader from the RF energy.
Knowing the temperature difference $\Delta T$, the flow rate $\dot{V}$, the length of the specimen $l$, and the heat capacity of water $c_p$, the power dissipated in it per unit length can be calculated.

$$P = \frac{\Delta T \dot{V} c_p \rho}{l}$$

The ratio of radius to skin depth, $q$, can be found from

$$q = a \sqrt{\frac{\pi f \mu_0 \sigma}{}}$$

where $f$ is the frequency, $a$ is the specimen diameter, $\mu_0$ is the permeability, and $\sigma$ is the conductivity. Tests were carried out with copper tubes of 4.125 cm, 0.960 cm, and 0.643 cm. The equivalent $q$ values are 164, 38, and 25. The wall thickness of the tubes was about 1.6 mm, 6.5 times thicker than the skin depth. This means that at the 284 kHz operating frequency, the tubes behave exactly the same as a solid cylinder of the same diameter, with the exception of being lighter.

The two smallest specimens were traversed completely though the coil, entering between the bottom two conductors, passing through the center of the coil, and exiting between the top pair of conductors. The center of the coil can be easily located from the data, and this position is declared to be zero height. Dividing the height of the specimen by the radius of the conductor centerlines gives us a normalized position. As can be seen in Figure 10, the heating peaks just inside the coil radius, where the specimen is between two conductors. The heating shows a minimum at the axis and
Figure 10 Specimen Heating Versus Position
obvious symmetry. For the smaller two specimens, the heating approaches zero over much of the interior of the coil, while the large specimen is close enough to the coil even at the center to still experience significant heating.

The solid lines represent the heating predicted by theory. The predictions match extremely well for the smaller specimens, and have some divergence at the extremes for the larger specimen. This may be due to the difficulty of establishing the maximum distance that can be moved off axis. As the large specimens approach the coil, there are considerable forces between the two, making these position measurements somewhat less reliable.

**FORCE MEASUREMENTS**

Force measurements were taken using the same conditions used for the power measurements. The specimens were suspended in the coil so that they could not move vertically relative to the laboratory jack, but could freely swing horizontally. This was done so that the forces applied by the coil would automatically put the specimen on a plane passing between opposite pairs of conductors. An electronic balance was chosen for these experiments because its pan does not move in response to a change in load. The scale was zeroed with the specimen in place, and this zero was checked between each force measurement. Force per unit length was calculated by converting the "grams" reading on the scale to Newtons, and dividing by the specimen length.
Table 2. Specimen Geometry and Ratio of Radius to Skin Depth ($q$)

<table>
<thead>
<tr>
<th>Number</th>
<th>Diameter (cm)</th>
<th>Length (cm)</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.125</td>
<td>7.757</td>
<td>163.9</td>
</tr>
<tr>
<td>2</td>
<td>3.169</td>
<td>7.568</td>
<td>125.9</td>
</tr>
<tr>
<td>3</td>
<td>2.560</td>
<td>7.896</td>
<td>101.7</td>
</tr>
<tr>
<td>4</td>
<td>1.947</td>
<td>7.583</td>
<td>77.3</td>
</tr>
<tr>
<td>5</td>
<td>0.960</td>
<td>7.68</td>
<td>38.2</td>
</tr>
<tr>
<td>6</td>
<td>0.636</td>
<td>7.654</td>
<td>25.5</td>
</tr>
<tr>
<td>7</td>
<td>0.479</td>
<td>7.720</td>
<td>19.0</td>
</tr>
<tr>
<td>8</td>
<td>1.947</td>
<td>1.363</td>
<td>77.3</td>
</tr>
<tr>
<td>9</td>
<td>1.947</td>
<td>2.560</td>
<td>77.3</td>
</tr>
<tr>
<td>10</td>
<td>1.947</td>
<td>3.937</td>
<td>77.3</td>
</tr>
<tr>
<td>11</td>
<td>1.947</td>
<td>5.155</td>
<td>77.3</td>
</tr>
<tr>
<td>12</td>
<td>1.947</td>
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<td>77.3</td>
</tr>
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<td>8.928</td>
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</tr>
<tr>
<td>14</td>
<td>1.947</td>
<td>12.757</td>
<td>77.3</td>
</tr>
</tbody>
</table>

Twelve different specimens were used, varying both diameter and length independently, and are shown in Table 2. All the specimens had approximately 1.6 mm thick walls. The resultant graphs of force versus position are clearly a family of similar curves, and can be collapsed into one curve by dividing by $4.6394 \times 10^{-4} q^2 + 9.146 \times 10^{-11} q^{5.6}$. The result of this is shown in Figure 11. While this equation was determined numerically, by rearranging Eq. 1 as shown, it is clear that the effect of specimen diameter is separable from position and coil geometry. Only the
Figure 11 Normalized Force per Unit Length

\[ F^* \left/ \left( 4.6394 \times 10^{-4} q^2 + 9.146 \times 10^{-11} q^6 \right) \right. \]
first four or five terms are important, but this function involves multiple ratios of Bessel functions, and does not lend itself to design work.

\[ F^* = \frac{\mu_0}{4\pi} \sum_{n=1}^{\infty} a^{2n} \text{Re} \frac{J_{n+1}(k_i a)}{J_{n-1}(k_i a)} \left\{ \sum_{m=1}^{n} \frac{1}{c_m - z} \right\}^n \hat{j}_m \left\{ \sum_{m=1}^{n} \frac{1}{c_m - z} \right\}^{n+1} \hat{i}_m \]

Only the three smallest specimens could be moved completely out of the coil, but they demonstrated that while these coils have a stable force minima on the axis, they also have unstable minima between each pair of neighboring conductors. It is possible to suspend a small specimen outside of the levitator, but the horizontal stability is very poor, whereas inside the levitator it is excellent. Again, the theoretical predictions are shown as lines, and the agreement is excellent.

Figure 12 shows that as the specimen diameter increases, the maximum force applied to it also increases. However, the volumetric force has a maximum, implying that there is an optimum ratio of specimen diameter to coil diameter of 0.35 for maximum levitation capacity. This is just larger than the largest specimen that can pass between the conductors. Theoretical predictions, shown as lines, agree well with the measured data, despite the difficulty of locating the point of maximum force for specimens that can touch the coil. The lower theoretical limit for specimen size is on the order of \( q=1 \), or the radius equal to the skin depth.

**ERROR ANALYSIS**
Figure 12  Maximum Force per Unit Length and Force per Unit Volume Versus Specimen Size
The uncertainty analysis is based on the method outlined by Bragg (1974). Uncertainty in the derived quantities is found from the experimental uncertainties, shown in Table 3.

Table 3. Measurement Uncertainty

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Worst case value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>-</td>
<td>±2.54x10⁻⁵</td>
</tr>
<tr>
<td>Vertical position (m)</td>
<td>-</td>
<td>±2.54x10⁻⁴</td>
</tr>
<tr>
<td>Force (N)</td>
<td>9.8x10⁻⁴</td>
<td>9.8x10⁻⁴</td>
</tr>
<tr>
<td>Temperature difference (°C)</td>
<td>±0.1</td>
<td>±0.1</td>
</tr>
<tr>
<td>Flow rate (m³/s)</td>
<td>2.46x10⁻⁶</td>
<td>±5%</td>
</tr>
<tr>
<td>Frequency (kHz)</td>
<td>284</td>
<td>±0.1</td>
</tr>
<tr>
<td>Current (Amps peak)</td>
<td>600</td>
<td>±5%</td>
</tr>
</tbody>
</table>

The length measurements are limited by the resolution of the measurement implement used, while force and temperature are limited to plus or minus one count. Flow rate may be more accurate than shown, due to calibration being performed at both operating points, but variable area flow meters are typically ±5% instruments, leading to our pessimistic choice. Frequency uncertainty is limited by the frequency drift in the generator; as the parts of the oscillating circuit heat up, their values change, changing the operating frequency. Current measurement at these power levels is difficult, and is made radically more complicated by the high frequency. We feel that the ±5% accuracy is an appropriate choice, as a comparison between measurements on a radio frequency shunt and our current transformer agreed within 2%. 
The derived quantities are the ratio of specimen radius to skin depth, force per unit length $F^*$, and power per unit length $P^*$. The uncertainty in $q$ is found from

$$\partial q = \left( \left( \sqrt{\pi \mu_0 \sigma \delta a} \right)^2 + \left( a \sqrt{\pi \mu_0 \sigma / f \delta f} \right)^2 \right)^{1/2}.$$ 

This is ±0.2049, which is one percent error in the worst case. Force and power per unit length are found from

$$\partial F^* = \left( \left( \delta F / l \right)^2 + \left( F \delta l / l^2 \right)^2 \right)^{1/2},$$

$$\partial P^* = \left( \left( \delta V \delta (\Delta T) / l c_p \right)^2 + \left( T \delta V \delta l / l^2 c_p \right)^2 + \left( T \delta \Delta T l / l^2 c_p \right)^2 \right)^{1/2}.$$ 

The uncertainty in $F^*$ is ±0.0129 N/m which is the same as one count on the scale divided by the longest specimen's length. Power per unit length is known to ±7.9x10⁻¹⁰ W/m, limited mainly by the smallest measured temperature difference being 0.1°C.

**CONCLUSIONS**

Equations for the forces and power produced by a longitudinal levitator have been given and shown to agree well with experimental results. The optimum ratio of specimen size to coil size is approximately 0.35 for an eight pole coil. Expressions are given to find the impedance of a longitudinal levitation coil, which can be used to find the length of coil that
a given generator can drive. It is shown that forces increase with increasing numbers of poles, but this is offset by increased difficulty of fabrication for large numbers of poles. Torque on levitated specimens is shown to be zero for non-rotating specimens, and experimental results confirm that no accidental rotation is sufficient to cause the specimen to exhibit continuous rotation near synchronous speed. This information permits a longitudinal levitator coil to be designed for a particular specimen and generator.

V. FUTURE DIRECTIONS

Some areas pertinent to the longitudinal electromagnetic levitator have yet to be dealt with. In the theoretical arena, the shape assumed by a molten specimen needs to be predicted. This could be dealt with using a numerical minimization routine, but the possibility exist for an analytical solution based on the method of image currents. This technique allows difficult boundary problems to be solved by assuming additional currents on the inactive side of the boundary such that the boundary conditions are satisfied. The forces can be found simply by finding the forces between the line currents. The Lorentz forces on the surface are less obvious, but the gravitational and surface tension forces can be evaluated.

The experimental side still needs results for the molten shape for future predictions to be compared to. Demonstration of levitation casting of significant quantities of metal has yet to be done. This will involve either an additional heat source or a carefully tailored alloy so that the solid - liquid - solid transition can be accomplished. Continuous processing, where
a long bar has part of its length melted within a levitator as it travels through the levitator is another promising project. Surface tension and viscosity measurements can be done using either longitudinal waves or shape deformations (coupled with the proper analytical solutions).

A final challenge is levitating more than a kilogram of metal, both solid and liquid.
IV. ELECTROMAGNETIC HEATING OF SPHERES

A commonly encountered problem in induction heating and levitation work is the determination the heat generated in the workpiece or, conversely, the determination of the coil and generator parameters necessary for a specified heat input. In this chapter we describe a series of experiments to verify theoretical predictions of the heating in conducting spheres.

Induction heating and electromagnetic levitation use coils of water cooled copper tubing that carry large high frequency currents (more than 100 amps, on the order of 500 kHz) surrounding a conducting specimen. The magnetic field generated by this coil induces eddy currents in the specimen, which then dissipate energy in ohmic heating. These currents also interact with the applied field and produce Lorentz forces. However, in this paper, we are only concerned with the heat generated.

EXPERIMENTAL APPARATUS

The experimental apparatus was designed to vary the ratio of radius to skin depth, $q_n$, over as wide a range as possible above and below one and measure the heat generated in a metal sphere. The experimental setup, shown schematically in Figure 13, consisted of a well insulated sphere with a thermocouple in it surrounded by a water cooled coil. The coil, with a parallel resonant capacitor was driven by a power amplifier which, in turn, was fed by a stable sine-wave oscillator. Coil current, coil frequency.
Figure 13 Experimental Apparatus for Measuring Electromagnetic Heating
change in temperature of the sphere, and time were measured during the experiment.

A coil with only a few turns, as is commonly used in induction heating, would have been impractical, as the heating rates we could achieve would have been too low to measure with the available current. More turns imply higher effective currents and thus higher heating, however this lowers the maximum frequency of operation. A 100 turn coil was chosen, with a 12.74 ±0.01 cm inside diameter, a 16.00 cm outside diameter, and a height of 2.02 cm with a nearly square cross section, yielding an equivalent single turn radius of 7.18 cm. The coil was water cooled to eliminate an error term due to the coil heating significantly over room temperature as seen in Shampine et alia (1996). To increase the current in the coil, parallel resonant capacitors were used. These capacitors carry significant alternating currents, and must be carefully chosen to survive this application. Ideally, this combination draws no current when driven at its resonant frequency. In reality, it draws current to make up the losses in the coil, capacitor, and sphere. The resonant frequency $f_0$ is

$$f_0 = \frac{1}{2\pi \sqrt{LC}},$$

and can be located by searching for the minimum current drawn by the system. At the resonant frequency, most of the current drawn will be at twice the applied frequency (making identification easy on an oscilloscope). For very large capacitors, there will be significant losses in the capacitors, and the system will still draw noticeable current at the resonant frequency.
Using the resonant circuit, currents in the coil were up to 15.4 times larger than the current supplied by the 100 watt amplifier used to drive the system (due to the energy storage). Coil currents between 14.4 and 22.8 Amps peak were measured using current shunt A. The very low value of this resistor minimized the unavoidable losses in the resonant system. Current shunt B was used to measure the current supplied by the amplifier to prevent overload.

A highly stable RC oscillator was used to provide sine wave drive to the power amplifier. The oscillator drifted less than 0.02% over 25 minutes. The period of the sine wave was measured with an eight digit counter, averaging over at least 100 cycles, yielding a frequency accurate to 1 part in $10^8$.

To fully verify the heating trends, we needed to vary $q_n$ both above and below 1. For the low range a $2.5387\pm 0.0003$ cm lead ball with a thermocouple cast into it was used. This allowed us to measure heating for $q_n$ between 0.4 and 1.4. For the range from 1.1 to 4.4, we used machined copper and aluminum balls (2.5154 cm and 2.4928 cm respectively) with thermocouples swaged into their centers through a drilled hole. To minimize error due to the thermocouple holes, we used 0.0251 cm wire, and oriented the thermocouple exit hole parallel to the axis of the coil. Knowing the masses of the spheres ($92.40 \pm 0.005$ g, 75.33 g, and 23.29 g respectively), and having measured the heating rate due to an applied magnetic field, assuming perfect insulation leads to the power input to the spheres.
Measuring the heating rate accurately required that we minimize the heat transfer and measure the temperature. Minimizing heat transfer was done with 2.54 cm of Styrofoam insulation on all sides of the sphere. The thermal time constant of the system was measured to be 1500 seconds, much longer than the experimental period of 300 seconds (shorter if the sphere temperature change reached 19.9° C). Thus, for our experiments, the sphere can be assumed to be thermally isolated, and a transient heating analysis is valid.

To achieve reasonable accuracy with heating rates down to 32 mW, we built a thermocouple reader producing 10 millivolts per °C with an adjustable zero, feeding a 3 1/2 digit meter. This was calibrated and yielded ±0.01 °C measurements, an order of magnitude more accurate than conventional instruments. This device is sufficiently sensitive that it must be shielded from air currents to prevent drift. If this is not done, it can exhibit random drift as much as ±0.15 °C in a laboratory environments. The schematic is shown in figure 14.

The repeatability of the apparatus was tested by taking multiple experimental runs using a variac to provide a stable 60 Hz wave. These are necessarily less repeatable than data taken with the power amplifier and function generator, due to the limited accuracy of setting the variac (± 0.1 amp). These tests were performed with equipment only able to measure current to ±0.1 amp, four times less accurate than the equipment used in the main experiments, yet still yielded a 95% confidence interval of 10.53±0.09 milliwatts, or ±0.86% accuracy, implying that the experimental apparatus produces repeatable results.
Figure 14 High Resolution Thermocouple Meter
THEORETICAL PREDICTIONS

The power dissipated in a sphere in a magnetic field is proportional to the square of the current induced in it and can be expressed as in the work of Lohofer (1989)

\[ P_s = \frac{1}{2\sigma_s R_s} \sum_{n=1}^{N} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} H_l(q_n) \left( |I_{n,l,m}|^2 + 2 \sum_{n' > n} \delta_{\omega_n, \omega_n'} \text{Re} \{ I_{n,l,m} \cdot I_{n',l,m}^* \} \right) \]  

(1)

The first sum, \( n \), is over the loops. The other sums, \( l \) and \( m \), are indices of spherical harmonics involved in the solution. For the case where all the loops carry currents of the same frequency (the typical implementation), this can be simplified to

\[ P_s = \frac{1}{2\sigma_s R_s} \sum_{l=0}^{\infty} \sum_{m=0}^{l} H_l(q_n)(2 - \delta_{m,0}) \left| \sum_{n=1}^{N} I_{n,l,m} \right|^2 \]

The key parameter we are interested in is \( q_n \) and its effect on the dimensionless power \( P_s / N^2 P_0 \). High frequency currents flow primarily near the surface of conductors, and the skin depth is where the current density has decayed to \( 1/e \) of its value at the surface. For higher frequency and higher conductivity, this depth decreases. The characteristic power \( P_0 \) is a measure of the maximum power that could be dissipated in a sphere of radius \( a_0 \).

\[ P_0 = I_0^2 / \sigma_s a_0 \]
\( N \) is the number of turns in the coil, and \( I_0 \) is the peak coil current. The "skin depth" function \( H_t(q_n) \) and the vector \( I_{n,l,m} \) are defined in Bayazitoglu and Sathuvalli (1994) as

\[
H_t(q_n) = -4q_n^2 \text{Im} \left[ \frac{1}{2(1+i)q_n} \frac{I_{l+1/2}}{I_{l-1/2}} \left\{ (1+i)q_n \right\} \right]
\]

\[
I_{n,l,m} = \frac{x_{i+1/2,k}^2}{\mu_0 R_s^{3/2} J_{l+1/2}(x_{i+1/2,k})} \int_0^{R_s} \int_0^{2\pi} \left[ f(\phi,u,r) \right] d\phi du dr
\]

\[
f(\phi,u,r) = r^{3/2} J_{l+1/2}(x_{i+1/2,k}) r/R_s Y_t^{m^*}(u,\phi) A_n(r,u,\phi)
\]

\[
Y_t^{m}(u,\phi) = (-1)^m \left( \frac{2l+1}{4\pi} \frac{(1-m)!}{(1+m)!} \right)^{1/2} P_l^m(u) e^{i\phi}
\]

Where \( Y_t^{m}(u,\phi) \) are spherical harmonics, and \( P_l^m(u) \) are associated Legendre polynomials of the first kind. The function \( H_t(q_n) \) represents the effect of the current distribution within the sphere on the power dissipated, and the vector \( I_{n,l,m} \) is known as the form function of the magnetic field.

For line currents, \( I_{n,l,m} \) may be expressed as in Bayazitoglu and Sathuvalli (1994, 1994, and 1992)

\[
I_{n,l,m} = I_n R_s^l F_{n,l,m}
\]

Where \( F_{n,l,m} \) is integrated over the current loop, rather than the sphere volume. It is found by integrating
\[ \mathbf{F}_{n,l,m} = \hat{\Phi}(r'_n)^{-(l+1)} Y'^m_l(u'_n, \phi'_n) d\tilde{s}' \]

The primed coordinate system is centered on the sphere. If a sphere is positioned at \((x_0, 0, z_0)\) with respect to the center of a current loop of radius \(a_0\), \(\mathbf{F}_{n,l,m}\) can be evaluated using the following variables.

\[
\begin{align*}
  r' &= \sqrt{a_0^2 + x_0^2 + z_0^2 - 2a_0x_0 \cos \phi} \\
  u' &= \cos \left( \arctan \left( z_0 / \sqrt{a_0^2 + x_0^2 - 2a_0x_0 \cos \phi} \right) \right) \\
  \phi' &= \arctan \left( a_0 \sin \phi / a_0 \cos \phi - x_0 \right) \\
  d\tilde{s}' &= \hat{\Phi} a_0 \cos \phi d\phi
\end{align*}
\]

The integration is performed for \(\phi\) from zero to two \(\pi\) with a \(\phi\) directed result.

In the case of a sphere that is small relative to the coil, only the \(l=0, m=0\) and \(l=1, m=\pm 1\) are important in the power calculations, and the following formulae from Bayazitoglu and Sathuvalli (1994) and Bayazitoglu et alia (1996) are useful

\[
\begin{align*}
  H_0(q_n) &= q \frac{\sinh 2q - \sin 2q}{\cosh 2q + \cos 2q} \\
  H_1(q_n) &= q \frac{\sinh 2q + \sin 2q}{\cosh 2q - \cos 2q} - 1
\end{align*}
\]  

(2)
\[ I_{n,0,0} = 2\sqrt{\pi} / \mu_0 A_n(r_0, \nu_0, \phi_0) \]
\[ I_{n,1,1} = \sqrt{3\pi/2} (iu_x + u_y)(BR_i / \mu_0) \]
\[ I_{n,1,-1} = -I_{n,1,1}^* \] (3)

Where \( A_n(r_0, \nu_0, \phi_0) \) is the vector potential of the applied field at the center of the sphere, and \( B \) is the magnitude of the magnetic field at the same point. For a single loop, these can be found in Smythe (1989):

\[ k^2 = 4a_0 r / \left[ (a_0 + r)^2 + z^2 \right] \]
\[ m = \left[ 1 - (1 - k^2)^{1/2} \right] / \left[ 1 + (1 - k^2)^{1/2} \right] \]
\[ A_n = \frac{\mu I_0}{\pi} \left( \frac{a_0}{mr} \right)^{1/2} \left[ K(m) - E(m) \right] = \frac{\mu I_0}{4} \left( \frac{a_0}{r} \right)^{1/2} m^{3/2} \left( 1 + \frac{3}{8} m^2 + \frac{15}{64} m^4 + \cdots \right) \]
\[ B = [B_r^2 + B_z^2]^{1/2} \]
\[ B_r = \frac{\mu I_0}{2\pi} \frac{z}{r \left( (a + r)^2 + z^2 \right)^{1/2}} \left[ -K(m) + \frac{a^2 + r^2 + z^2}{(a - r)^2 + z^2} E(m) \right] \]
\[ B_z = \frac{\mu I_0}{2\pi} \frac{1}{r \left( (a + r)^2 + z^2 \right)^{1/2}} \left[ K(m) + \frac{a^2 - r^2 - z^2}{(a - r)^2 + z^2} E(m) \right] \]

\( K \) and \( E \) are complete elliptic integrals of the first and second kind respectively. Multiple loops are treated by summing the contributions of each loop. \( H_0(q_n) \) and \( H_1(q_n) \) can be expressed in the following form in order to develop limiting cases for the power generation

\[ \lim_{q_n \to 0} H_0(q_n) = q_n^4 \]
\[ H_1(q_n) = 0.091q_n^4 \]
\[ q_n \leq 1 \]
\[ \begin{align*}
H_0(q_n) &= q_n \\
H_1(q_n) &= q_n - 1
\end{align*} \quad q_n \geq 2
\]

These imply that the power absorbed by the sphere will vary as \( q_n^4 \) for \( q_n < 1 \), and as \( q_n \) for \( q_n > 1 \).

Routines were developed to evaluate the small sphere approximation, a single loop exact solution, and a 100 loop exact solution. Using the model for one turn, it was verified that for this system, only the \( l=0 \) and \( l=1 \) modes are important (the \( l=2 \) mode makes a 0.3% contribution). Comparing their results to the highest \( q_n \) experimental value (4.4) we find that the predicted heating decreases with the accuracy of the model

\begin{align*}
\text{Small Sphere} & \quad P_s/N^2P_0 = 0.785 \\
\text{1 Loop} & \quad P_s/N^2P_0 = 0.745 \\
\text{100 Loops} & \quad P_s/N^2P_0 = 0.702 \\
\text{Experiment} & \quad P_s/N^2P_0 = 2.019
\end{align*}

There is a 12% difference between the exact theoretical solution and the approximation, implying that the approximation is reasonable. However, at this extreme, there is a large difference between any of the theoretical solutions and the measured value.

**EXPERIMENTAL RESULTS**

Using the equation (2) for \( H_l(q_n) \) and equation (3) for \( I_{n,l,m} \) in solving equation (1), we developed a fast and simple routine for predicting the heat
generation for reasonably sized spheres. Theory predicts that the function $P_s/N^2P_0$ will vary as $q_n^4$ for $q_n$ less than one, and as $q_n$ for $q_n$ greater than one. The experimental values clearly show this behavior, and agree well with the predictions up to a $q_n$ of about 2. For the low $q_n$ regime, both the theoretical and experimental results can be fitted to the following $q_n^4$ curve with correlation coefficients of 0.98745 and 0.99482 respectively.

$$P_s/n^2P_0 = 0.027898q_n^4$$

The agreement is not as good for higher $q_n$. While both the copper and aluminum spheres show the expected linear relationship between $q_n$ and $P_s/N^2P_0$, they have different slopes, and the theory shows a third slope. For $q_n$ greater than 2, the slopes are

- Copper: $0.7341q_n$
- Aluminum: $0.9706q_n$
- Theory: $0.2280q_n$

The correlation coefficients are 0.9846, 0.9755, and 0.9998 respectively.

Comparing the experimental results and theoretical predictions over two orders of magnitude of $q_n$ and 5 orders of magnitude of $P_s/N^2P_0$ in Figure 15, the agreement is very good, except for high values of $q_n$. At the low end of the range, the experimental data shows increasing noise due to the very low heating values.
Figure 15 Experimental and Theoretical Relationship Between Normalized Power \( \frac{P_s}{N^2 P_0} \) and the Ratio of Radius to Skin Depth \( q_n \) at \( x=0.48 \text{ cm} \) and \( z=0.58 \text{ cm} \)
UNCERTAINTY ANALYSIS

The uncertainty analysis is based on linear regression and the method outlined by Bragg (1974). We are interested in the uncertainty in the two dimensionless groups: $q_n$ and $P_s/N^2P_0$. The uncertainty in $q_n$ can be expressed as

$$
\partial q_n = \left( \left( \left( \pi f \mu_0 \sigma_s \right)^{\frac{1}{2}} \delta R_s \right)^2 + \left( R_s \left( \frac{\pi \mu_0 \sigma_s}{f} \right)^{\frac{1}{2}} \delta f \right)^2 + \left( R_s \left( \frac{\pi \mu_0 \sigma_s}{f} \right)^{\frac{1}{2}} \delta \sigma_s \right)^2 \right)^{\frac{1}{2}}
$$

Where $\delta f$ is $\pm 0.01$ ppm, $\delta R_s$ is $\pm 2.54 \times 10^{-6}$, and $\delta \sigma_s$ is $\pm 0.39\%$. For the range of $q_n$ considered in this work (0.4 to 4.4) this is only 0.02\%.

For the denominator of the second dimensionless group we find

$$
\delta(N^2P_0) = \left( \left( \delta I_0N^2I_0/\sigma_sa_0 \right)^2 + \left( -\delta a_0N^2I_0^2/\sigma_s^2a_0^2 \right)^2 + \left( -\delta \sigma_sN^2I_0^2/\sigma_s^2a_0^2 \right)^2 \right)^{\frac{1}{2}}
$$

While there is no uncertainty in $N$ and $a_0$ is known to $\pm 0.001$ m, $I_0$ is only known to $\pm 0.03$ amps, yielding worst case $N^2P_0$ uncertainty of $\pm 1.4\%$. $\delta P_s$ is found from

$$
\delta P_s = \left( (cm \delta \beta)^2 + (\beta c \delta m)^2 \right)^{\frac{1}{2}}
$$

$\beta$ and $\delta \beta$ are found from the experimental data using linear regression. The 95\% confidence limit for variance in the heating rate, $\delta \beta$, is $\pm 2.2\%$ and $\delta m$ is $5 \times 10^{-6}$. The 95\% confidence interval for power is $\pm 2.2\%$. Finally, the variance in dimensionless power, $P_s/N^2P_0$ is
\[
\delta(P_s/N^2P_0) = \left[ (\delta P_s/N^2P_0)^2 + \left( -\delta(N^2P_0)P_s/(N^2P_0)^2 \right)^2 \right]^{1/2}
\]

The confidence interval for this is 2.6%

CONCLUSIONS

The results of this work clearly indicate that a simplified theoretical model can be used to predict the heat generated in levitated or heated spheres with good accuracy. This has been verified for radius to skin depth ratios \(q_n\) between 0.4 and 4.4, covering the critical transition range where the electromagnetic field just penetrates to the center of the sphere. For \(q_n\) below one, the experimental data agrees well with the theoretical prediction that power dissipated will be proportional to \(q_n^4\). For \(q_n\) above one, the experimental results provide very reliable data supporting the theoretical prediction that power absorbed is proportional to \(q_n\). Experimental results show excellent quantitative agreement with theoretical predictions up to a \(q_n\) of about 2, and some difference above this.
V. SUMMARY

When applying electromagnetic levitation, four key questions need to be answered. First, what kinds of forces can be generated. Second, how much power will be dissipated in the specimen. Third, is there torque on the specimen; will it spin by itself. Finally, what shape will the specimen assume when it melts. Theoretical results exist for all four of these questions in connection with conventional conical electromagnetic levitators. Some experimental results backing up the force predictions have been published, but there is very little experimental data on heating. The question of torque has only been addressed experimentally with the observation that the levitated specimens spin rapidly (although the author has not observed this tendency personally). Some data exists for the free shape of levitated droplets in connection with micro gravity studies and measurements of surface tension.

The first portion of this work experimentally investigates the heating experienced in conical levitators and compares this with theoretical predictions. The result is that the theoretical predictions are found to be quite good, even for a simple model.

The second portion introduces a new type of levitator that promises to finally allow commercial application of levitation. Theoretical prediction for the forces, heating, and torque are developed. These predictions are compared with extensive experimental work on measuring forces and heating, and found to give excellent agreement. Experimental results indicate that the results for torque are also correct. Additional formulae sufficient to allow this type of levitator to be designed are also given.
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APPENDIX 1: PROGRAM FOR THE FORCES AND HEATING IN A LONGITUDINAL LEVITATOR

A Mathematica program to calculate the force on and the heating in a cylinder in a longitudinal levitator. This program assumes that the conductors are on a circle of radius coilr meters, with adjacent conductors carrying current in opposite directions.

\[
\begin{align*}
\text{coilr} &= 0.0299 \\
\text{a} &= 1.624 \times 2.54 / 200 \\
\text{poles} &= 8 \\
\mu_0 &= 4 \pi \times 10^{-7} \\
\sigma &= 5.8 \times 10^7 \\
\omega &= 1.834 \times 10^6 \\
\bar{i} &= 568 \\
k_1 &= a \sqrt{-i \omega \mu_0 \sigma} \\
wire &= 0.004787 / 2 \\
\sigma &= 0.5 \\
X &= N[\text{coilr} \cos(\pi/poles) - \sqrt{(a + wire)^2 - (\text{coilr} \sin(\pi/poles))^2}] \\
q &= a \sqrt{0.5 \sigma^2} \\
c &= N[\text{Table} \{\text{coilr} \cos(2\pi (m-o)/poles) + i \sin(2\pi (m-o)/poles) \}, \{m, 1, \text{poles}\}]] \\
d[m_, z_] &= N[Abs[z - c[[m]]] - wire] \\
test[z_] &= N[\text{Sum}[\text{Sign}[d[m, z]] - 1, \{m, 1, \text{poles}\}] + 0.00001] > 0.0
\end{align*}
\]
f1[m_,z_] := N[a/(c[[m]]-z)]
f2[m_,z_] := N[a/(Conjugate[c[[m]]]-Conjugate[z])]

fconst := mu0 (ihat^2)/(4 Pi a)
pconst := -(ihat^2)omega mu0/(4 Pi)

term1[n_,z_] := N[Sum[(f1[m,z]^n)((-1)^(m-1)),{m,1,poles}]]
term2[n_,z_] := N[Sum((f2[m,z]^(n+1))((-1)^(m-1)),{m,1,poles}]]
termj[n_] := N[BesselJ[n+1,k1a]/BesselJ[n-1,k1a]]

f[z_,t_] := 0.0; !test[z]
p[z_,t_] := 0.0; !test[z]

f[z_,t_] := N[fconst Sum[term1[n,z]term2[n,z]Re[termj[n]],[n,1,t]]]/test[z]
p[z_,t_] := N[pconst Sum[(Abs[term1[n,z]]^2)Im[termj[n]]/n,[n,1,t]]]/test[z]
APPENDIX 2: SPHERE HEATING PROGRAM

program power

c calculates the power dissipated in a sphere positioned in the

c magnetic field of a loop

implicit none

real qn, hlqn, ps, p0, phom, psbar, phombar, temp, t1, t2, t3
real rs, fn, sigma, mu, pi, frequency, n
real rho, z, a, i, aphi, b, at, zt, brho, bz
integer j,k,l

pi = 2.0 * asin(1.0)

write(*,*) 'assuming relative permeability is 1'
mu = pi * 4.0e-7
write(*,*) ' copper conductivity = 58.005e6'
write(*,*) ' aluminum conductivity = 35.361e6'
write(*,*) ' lead conductivity = 4.8431e6'
rs = 0.9995 * .0254 / 2.0

c write(*, *) 'input sphere radius'
c read(*, *) rs

sigma = 4.8431e6

c write(*, *) 'input sphere conductivity'
c read(*, *) sigma
write(*,*) 'sphere radius = ', rs
write(*,*) 'sphere conductivity = ', sigma

! The distances are between the coil center and the sphere center
! rho = 0.0001 * 2.54 / 100
rho = 0.19 * 2.54 / 100
write(*,*) 'input off axis distance'
read(*,*) rho
z = 0.231 * 2.54/100
write(*,*) 'input height'
read(*,*) z
a=0.071755
write(*,*) 'input coil radius'
read (*,*) a
i=10
n = 100.0
write(*,*) 'input coil current'
read (*,*) i
write(*,*) 'distance off axis=', rho, ' height=', z
write(*,*) 'coil radius = ', a
write(*,*) 'coil current = ', i, ' amps peak'
write(*,*) 'number of turns = ', n

! this is the characteristic power for non-dimensionalization
p0 = ((n*i)**2)/(sigma*a)
write(*,*) 'p0=',p0

C calculate the magnetic field and gradient at the sphere center
C call magnetic(aphi, b, mu, i, a, rho, z)

      do 100 j=1,33
      C this is sphere radius over skin depth
         qn = j / 6.0
      C calculate the frequency from qn
         fn = frequency(rs, qn, sigma, mu)
         phom = 0.0
         ps = 0.0
         aphi = 0.0
         brho = 0.0
         bz = 0.0
      do 90 k = 1,10
         at = a - 0.0103 + (k * 0.0018735)
      do 80 l = 1,10
         zt = z - 0.0111265 + (l * 0.002023)
         call magnetic(t3, t1, t2, mu, i, at, rho, zt)
         brho = brho + t1
         bz = bz + t2
         aphi = aphi + t3

     80 continue
continue
b = ((brho**2) + (bz**2))**0.5

temp = hlqn(1,qn) * 3 * pi * (b**2) * (rs**2) / (mu**2)
phom = phom + temp

ps = ps + temp + hlqn(0,qn) * 8 * pi * (aphi**2) / (mu**2)
phom = phom / (2*sigma * rs)
ps = ps / (2*sigma * rs)

psbar = ps/p0
phombar = phom/p0

c write(*,*) 'qn=',qn,' ps = ',ps, ' phom = ',phom

c write(*,*) 'qn=',qn,' ps/p0= ',psbar,' phom/p0=',phombar
write(*,*) qn,'

100 continue

stop
end

C *******************************************************

real function hlqn (l,qn)
c calculates the H(l,qn) function

integer l
real hlqn
real pi, qn, order, hlqn
complex z, ratio, gratbes

pi = 2.0 * asin(1.0)

order = l + 0.5
z = (1, 1) * qn

ratio = gratbes(order, z)

hlqn = -1.0 * Real(z / ratio)

return
derend

c *******************************************************

real function frequency(rs, qn, sigma, mu)

c calculates the frequency from qn
implicit none

real  pi, rs, qn, sigma, mu, frequency, temp

pi = 2.0 * asin(1.0)

temp = (qn*qn) / (rs*rs)
frequency = temp / (pi*mu*sigma)

return
end

c  ************************************************************

subroutine magnetic (aphi, brho, bz, mu, i, a, rho, z)

c  calculates the magnetic field (b) and the gradient

real aphi, b, mu, i, a, rho, z, ksq, m, pi, brho, bz, temp
real elk, ele, k

pi = 2.0 * asin(1.0)

c  these are parameters used to simplify the calculation
ksq = 4 * a * rho / (((a+rho)**2) + (z**2))
k = ksq **0.5
m = (1 - ((1-ksq)**0.5))/(1 + ((1-ksq)**0.5))

aphi = mu*i/pi
bz = aphi / 2
aphi = aphi * ((a/(m*rho))**0.5)
aphi = aphi * (elk(m) - ele(m))

bz = bz / (((a*rho)**2)+(z**2))**0.5
brho = bz
brho = (brho * z) / rho
temp = ((a**2) + (rho**2) + (z**2)) / (((a-rho)**2) + (z**2))
brho = brho * ((temp * ele(m)) - elk(m))
temp = ((a**2) - (rho**2) - (z**2)) / (((a-rho)**2) + (z**2))
bz = bz * ((temp * ele(m)) + elk(m))

b = ((brho**2) + (bz**2))**0.5

c      write(*,*) i,aphi,b

return
end

c      *****************************************************************

real function ele(m)
c calculates the elliptic integral E(m)
implicit none
real ele,m,temp,pi

pi = 2.0 * asin(1.0)

temp = 0.5 + ((m**2)/8) + (3*(m**4)/16) + (15*(m**6)/96)
ele = pi * temp
return
end

c ******************************************************

real function elk(m)
c calculates the elliptic integral K(m)
implicit none
real elk,m,temp,pi

pi = 2.0 * asin(1.0)

temp = 0.5 - ((m**2)/8) - (3*(m**4)/128) - (45*(m**6)/4608)
clk = pi * temp
return
end
complex function gratbes(x, z)

implicit none

complex  a, z, y1, y2, num, den, dif, gratbes, gan3, gan4
real  x, tolera
integer n, m

c  z = (1, 1)
c  x = 9.5

n = 1

34 if (n .ge. 100) then
   return
   endif

   y1 = a(x, z, n)
   y2 = a(x, z, n + 1)
if (n .eq. 1) then

    num = y2 + 1 / y1
    den = y2

    gratbes = y1 * (num / den)
    n = n + 1
    go to 34

else

    num = y2 + 1 / num
    call checksum(num, n, m)

    if (n .ne. m) then
        call skip(num, den, x, z, n, gan3, gan4)
        num = gan3
        den = gan4
        n = n + 2
        go to 38

    else

        den = y2 + 1 / den
        call checksum(den, n, m)
if (n .ne. m) then
    call skip(den, num, x, z, n, gan3, gan4)
    num = gan4
    den = gan3
    n = n + 2
    goto 38
else
    dif = num - den
    tolera = sqrt(real(dif) ** 2 + imag(dif) ** 2)
    if (tolera .lt. 1.0d-14) then
        gratbes = gratbes * (num / den)
        write(*, *) 'ratio =', gratbes
        write(*, *) 'n =', n
        return
    else
        gratbes = gratbes * (num / den)
        n = n + 1
        go to 34
    endif
endif
endif
endif

return
end

C

complex function a(x, z, n)

complex a, z, temp1, temp2
real x, t2
integer n, t1

t1 = (-1) ** (n + 1)
t2 = 2.0 * (x + n - 1)

temp1 = cmplx(t1 * t2)
temp2 = z ** (-1)

a = temp1 * temp2

return
end

c

**********

subroutine checksum(gan, n, m)

implicit none
complex gan
real mag
integer n, m

c

**********

mag = sqrt(real(gan) ** 2 + imag(gan) ** 2)

if (mag . gt. 1.0d-14) then
  m = n
else
  m = n + 2
endif

return
end
subroutine skip(gan1, gan2, x, z, n, gan3, gan4)

implicit none
complex gan1, gan2, gan3, gan4, gan41
complex z, an1, an2, zeta, beta, a
real x
integer n

beta = gan1
an1 = a(x, z, n + 1)
zeta = an1 * beta + 1
an2 = a(x, z, n + 2)

gan3 = (an2 * zeta + beta) / zeta

gan41 = an1 + (1 / gan2)
gan4 = an2 + (1 / gan41)

return
end