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RICE UNIVERSITY

TRANSIENT PERMEATE FLUX ANALYSIS, COST ESTIMATION, AND DESIGN OPTIMIZATION IN CROSSFLOW MEMBRANE FILTRATION

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE DOCTOR OF PHILOSOPHY

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ABSTRACT

Transient Permeate Flux Analysis, Cost Estimation, and Design Optimization in Crossflow Membrane Filtration

by

Sandeep Sethi

A generalized model is formulated to predict the time-dependent permeate flux by extending previous models to include the particle transport mechanisms of Brownian diffusion, shear-induced diffusion, inertial lift and concentrated flowing layers. A new model for estimating the capital costs of membrane plants is developed which incorporates individual cost correlations for different categories of manufactured equipment. The effects of particle size, design, and operating variables on permeate flux and treatment costs are investigated numerically. Optimization problems are formulated and solved to investigate (a) optimal membrane design and system operation, (b) optimal backflushing frequency, and (c) optimal selection of hybrid filtration configurations, over variable raw water quality.

The combined theory predicts an unfavorable particle size, on the order of $10^{-1}$ μm, where net back-transport is at a minimum. This implies minimum permeate fluxes in the size range of $10^{-2}$ μm - $10^{-1}$ μm, depending on the operating time. These results support experimental observations of minima in back-transport (Chellam and Wiesner, 1996) and permeate flux (Fane, 1984). Inside-out hollow fiber geometry is predicted to be favorable for feed suspensions with small particles and/or low concentrations. The constant
pressure mode of operation is predicted to yield higher specific permeate fluxes compared to the constant flux mode, particularly for particles which demonstrate mass-transfer limited behavior. Comparisons and parameter estimations made with available experimental data on polydisperse suspensions give solidosity estimates ranging from 0.70 to 0.77.

Membrane design is predicted to be optimized at values of fiber radius (narrow) and length (short) where the permeate fluxes are maximized. Particles affected by mass-transport limitations demonstrate comparatively lower optimal transmembrane pressures. For unfavorable particles, treatment costs are predicted to be minimized at intermediate recoveries and backflushing frequencies. At small capacities, the hybrid hollow fiber ultrafiltration and spiral wound nanofiltration system with higher non-membrane capital costs is predicted to be largely non-optimal compared to hollow fiber nanofiltration. Membrane costs are expected to play a significant role in determining the optimal configuration at large capacities, where the hybrid configuration is predicted to become largely optimal.
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CHAPTER 1

INTRODUCTION

Energy-saving membrane technologies have found widespread use in several fields and industries such as water purification, wastewater treatment, food and beverage industries, pharmaceutical, and various other chemical industries. Recently, membrane filtration is finding increased use in water and wastewater treatment. Improved standards with respect to product quality coupled with the reliable filtration efficiency of membrane processes have encouraged their use in the water industry. However, several challenges associated with these processes exist, such as: understanding and predicting the mass transport process and the associated permeate flux decline, estimating the capital and operating costs associated with the system, and, predicting design, operating conditions, and process configurations which may lead to better separation performance per unit cost. Over the years, partial solutions have been found to the problems facing membranes. However, understanding and mitigating the major limitation of membrane fouling, estimating process costs, and predicting better designs and operating conditions still remain a challenge, and are a focus of research today.

The aim of this work was to investigate theoretical descriptions of mass transport, estimation of treatment costs, and optimization considerations in low-pressure crossflow membrane filtration systems, specifically, ultrafiltration (UF) and microfiltration (MF). In effect, this work improves upon previous
descriptions of mass transfer and cost estimation techniques to investigate system performance and costs as a function of raw water quality (in terms of particle size and concentration) and other key design and operating variables. Further, design and optimization problems are formulated and solved in an endeavor to investigate preferred system geometry, operation and configuration as a function of raw water quality and key system parameters.

1.1. Problem Definition

System Performance: Mass transfer

Ultrafiltration and microfiltration are pressure-driven membrane processes which achieve separation by displaying different permeabilities to various components of a suspension. They can be used to separate macromolecules and materials of colloidal size and larger from suspensions. Membrane system performance is quantitatively measured in terms of the permeate flux or the rate of clean water produced per unit membrane area. The permeate flux typically declines over operating time due to the accumulation of rejected material near the membrane surface (concentration polarization) or on the membrane (cake or gel formation). The permeate flux is thus largely governed by the resistances offered to permeation by the membrane and the accumulated secondary layers. The buildup of these layers at the membrane surface is governed by various diffusive and convective mechanisms of particle transport. Estimating the permeate flux limited due to concentration polarization and cake formation involves consideration of raw water quality (in terms of particle size and concentration), operating variables (such as the driving pressure differential,
cross-flow velocity and system recovery), and the membrane geometry (diameter and length), amongst other variables.

**Cost Estimation**

The economics of membrane filtration are determined by the initial investment made towards the membrane modules along with the associated ancillary equipment and facilities, and the various operating costs (such as membrane replacement, energy and waste disposal) incurred in running the process. One of the major factors affecting treatment costs is the permeate flux which is itself governed by the hydrodynamic resistance of the particle layer accumulated on the membrane surface, as discussed above. Thus, membrane fouling translates into high capital as well as operating costs by affecting the membrane area and energy requirements. In addition to all the variables that determine permeate flux, treatment costs are also affected by module geometry, frequency of hydrodynamic cleaning, membrane plant characteristics, and other economic factors such as interest rates.

**Design and optimization**

One of the key challenges in membrane filtration is to achieve better separation performance per unit cost. The most economically efficient or optimal design and operation of a membrane filtration system is influenced by the complex interplay of all the factors and variables that determine the permeate flux and treatment cost components mentioned in the previous paragraphs. Variations in fouling with raw water quality and the economies of scale exhibited by manufactured equipment with size have major influences on
the design and optimization of membrane processes at different plant capacities and for different source waters. Optimization of membrane processes includes design variables such as membrane type and geometry, and operating variables such as the imposed pressure, crossflow velocity, system recovery, frequency of hydrodynamic cleaning, and process configuration. Compared with traditional unit separation processes, a limited historical experience coupled with the large number of variables that come into the selection and design of membrane systems makes optimization of these processes a critical, and at the same time a challenging, task.

1.2. Objectives

The objectives of this work are three fold. This study focuses on aspects of theoretical descriptions of mass transport behavior, cost estimation, and optimization of crossflow membrane filtration systems. The primary objectives of this research are to:

1. Formulate a generalized model to predict time-dependent permeate flux by extending previous mass transfer models to include the particle transport mechanisms of Brownian diffusion, shear-induced diffusion, inertial lift and concentrated flowing layers. Analyze permeate flux behavior with raw water quality (in terms of particle size and concentration), and design and operating parameters.

2. Develop a model for estimating the capital costs of membrane facilities by establishing correlations for the costs of different categories of manufactured
equipment employed in membrane systems. Estimate treatment costs for various raw waters, membrane geometries and operating conditions.

3. Formulate and solve optimization problems associated with membrane systems focusing on minimizing the total treatment costs. Investigate (a) optimal membrane design and system operation, (b) optimal frequency of backflushing, and (c) optimal selection of hybrid filtration configurations.

1.3. Significance

Permeate flux is a critical parameter in determining the performance and cost of membrane systems. Hence, it is important that mathematical models provide a good estimate of this parameter. The permeate flux is determined by the accumulation of rejected materials near the membrane surface. Quantitative estimation of permeate flux thus requires consideration of the different mass transport mechanisms involved. Previous mass transfer models have focused on a single mechanism of particle-transport to predict the permeate flux and are thus not applicable to a variety of feed suspensions or tend to approximate the mass-transport by the dominant (size-based) transport phenomenon. A related consequence is that they are not able to describe the observed minima in permeate flux at intermediate particle sizes.

In many cases, several different transport mechanisms may be important for a particular suspension. Hence, predicting and comparing permeate fluxes for feed suspensions containing materials which may vary by several orders of magnitude in effective size calls for consideration of multiple transport mechanisms in the mass transfer modeling. The applicability of previous
models can thus be improved by simultaneously considering the effects of Brownian diffusion, shear-induced diffusion, inertial lift and concentrated flowing layers on particle accumulation near the membrane. Such an approach is also able to describe the minimum in back-transport with respect to particle size reported in the literature (Chellam and Wiesner, 1996) and the consequent minimum in permeate flux with respect to particle size that is commonly observed in practice (Fane, 1984, Lahoussine-Turcaud et al., 1990).

Cost estimation is required to assess the expenses incurred in using membrane separation techniques, to compare with alternatives treatment technologies, and to design the process for better separation per unit cost. Cost estimation of unit separation processes thus aids decision making and planning efforts related to water treatment. The different categories of manufactured equipment employed in membrane systems demonstrate different economies of scale in practice. Previous approaches for modeling the capital costs of membrane processes typically present a total capital cost function which does not provide any insight on the different economies of scale associated with the different components involved. A model for estimating the capital costs of membrane plants which incorporates individual cost correlations for several different categories of manufactured equipment overcomes this limitation, and is suitable to be used in a design and optimization study.

In water and wastewater treatment, pressure driven membrane filtration has been documented to successfully achieve desired separation or removals, but due to the diversity of the chemical composition of feed waters, there is relatively little basis for anticipating under what set of design and operating
conditions a membrane process will be appropriate for a specific raw water. Numerical optimization using performance and cost models can be used to help understand the complex interplay between process variables and their impacts on process performance and economics. Knowledge of these impacts will aid in developing more efficient membrane geometries, and optimizing the operating conditions and process configurations to result in lower total treatment costs. A computational engineering approach, such as the one adopted in this work, may also be used to anticipate the most promising design and operating conditions before embarking on a pilot study. The work presented here will thus help gain understanding about membrane system performance and cost with respect to key design variables, and hence provide guidance in orienting pilot plant testing to the most promising scenarios.

1.4. Research Contributions

1. This work extends previous work in the area of permeate flux estimation by unifying several dominant particle transport mechanisms in membrane filtration, specifically, Brownian diffusion, shear-induced diffusion, inertial lift and concentrated flowing layers.

2. Theoretical comparisons between the hollow fiber and slit geometry are made and the results are interpreted to make comparisons between inside-out and outside-in hollow fiber configurations.

3. Theoretical comparisons between the constant flux and constant pressure modes of operation are made by utilizing the specific permeate flux as a measure of performance.
4. A new model for estimating the capital costs of membrane systems is presented which accounts for economies of scale of different manufactured equipment. Such a model can be used to analyze the cost behavior of individual components of a membrane system, and hence can be appropriately utilized in a design and optimization study.

5. The effects of particle size, concentration, and key design and operating parameters on the permeate flux and treatment costs of membrane filtration are analyzed.

6. Optimal fiber geometry and operating conditions are analyzed and predicted over variable raw water quality for a hollow fiber membrane system. A typical feed and bleed mode operating at steady-state and generating a continuous waste stream is considered. The optimization minimizes the treatment cost of filtration subject to an appropriate constraint set.

7. Optimal operating time (and subsequently the system recovery and backflushing frequency) are predicted over variable raw water quality for a hollow fiber membrane system operating in a transient semi-batch mode. This operation involves an intermittent waste stream that is generated only during the backflushing phase of the operation.

8. Optimal configuration for hybrid membrane processes is predicted over variable raw water quality, design capacities and feed pressures. Comparisons are made, in terms of lower treatment costs, between a single
hollow fiber nanofiltration system and a hybrid system comprised of spiral wound nanofiltration pretreated with hollow fiber ultrafiltration.

1.5. Limitations and Simplifications

1. The mass transfer models investigated and developed in this study are valid for laminar flow. This is a reasonable assumption for the two membrane geometries investigated in this work, the inside-out cylindrical hollow fiber and the flat rectangular slit, since the fluid flow in these configurations is typically laminar in mostly all practical cases.

2. The permeate flux modeling effort seeks to describe cake accumulation in a short term operation, such as between two hydrodynamic cleaning cycles. Long term fouling due to pore plugging and adsorption is neglected.

3. The model formulated is ideally applicable to monodisperse suspensions of rigid, spherical particles. For analyzing the behavior of polydisperse suspensions, an average/equivalent particle size has to be utilized.

4. Effects of rejection on the permeate flux are considered only indirectly through the effect of rejection on the modification of the bulk suspension volume fraction.

5. The model is formulated for the constant average transmembrane pressure operation mode. Simulations for the constant flux mode of operation are performed by applying the constant pressure model in an iterative fashion.
The iterations are performed at each time step till the newly calculated transmembrane pressure results in recovery of the initial permeate flux.

6. As in the shear-induced diffusion model(s) of Romero and Davis (1988, 1990), the shear stress within the concentrated flowing layer is considered constant and equal to the wall shear stress. This approximation is reasonable as long as the flowing layer thickness is small compared to the channel height/diameter.

7. Cost correlations are modeled as continuous functions. This is a simplification of the real world, where pieces of equipment come in discrete sizes and associated costs.

8. The formulation and solution of the optimization problems assumes continuous variables and cost functions.

1.6. Overview of Thesis

Chapters 2 to 4 individually describe aspects of this work which relate to describing the performance, evaluating the cost, and performing the optimization, respectively, of low-pressure membrane processes. Chapters 5 and 6 comprise the second part of the thesis and include all results, discussion, and conclusions.

Chapter 2 focuses on describing the time-dependent permeate flux in UF and MF. Transport mechanisms in crossflow membrane filtration are discussed and previous models for estimating the permeate flux in membrane filters are
reviewed. This is followed by development of an extended theory for describing the permeate flux based on multiple particle transport mechanisms. Model formulations are presented for both the flat slit and inside-out hollow fiber geometry. The numerical scheme employed to solve the governing partial differential equation is discussed.

Chapter 3 covers the economics of low-pressure membrane filtration processes. Previous capital and operating cost models are reviewed. This is followed by a description and calibration of the new capital cost model developed in this work.

Aspects of membrane system optimization and the related problems investigated are described in chapter 4. Three optimization problems are formulated which focus on (a) predicting optimal design and operating conditions, (b) optimal operating time (or recovery), and (c) optimal nanofiltration configurations, over the domain of variable raw water quality. The solution technique employed in solving the multi-dimensional optimization problem (a) is discussed.

Results and discussion pertaining to all aspects investigated in this work are presented in chapter 5. Finally, a summary of the research effort and the conclusions made from the analyses performed in this work are presented in chapter 6, along with suggestions for further research in the area.
CHAPTER 2

THEORETICAL DESCRIPTION OF PERMEATE FLUX BEHAVIOR IN ULTRAFILTRATION AND MICROFILTRATION

In this chapter background on crossflow membrane filtration and the theories used for predicting permeate flux are reviewed in sections 2.1 and 2.2. This is followed by a description of the limitations of the current models (section 2.3), which motivated the task of extending them. Subsequently, an extended model for describing the transient (and steady-state) permeate flux is presented in section 2.4. Several particle transport mechanisms are considered together in the extended formulation to obtain a model applicable over a wide range of particle sizes. The numerical scheme used to solve the governing partial differential equation is described in section 2.5.

The permeate flux model developed in this chapter is subsequently used to study and predict the effects of material properties and operating conditions on particle accumulation and the associated permeate flux decline. Results and discussion from the analysis are presented in section 5.1. Further, the model is also employed in the cost analysis and optimization studies described in chapters 3 and 4.
2.1. Background

2.1.1. Crossflow Membrane Filtration

This work investigates mass transport behavior in crossflow membrane filtration processes. Crossflow filtration is a technique used to separate the components of a suspension through the application of pressurized flow across a semi-permeable membrane. In this process the feed suspension flows tangentially along the membrane, with the permeate flow in the perpendicular direction (Figure 2.1.1). The key advantage of this configuration is that the shear provided by the tangential feed flow helps limit the buildup of rejected materials on the membrane surface.

![Diagram of crossflow filtration](image)

**Figure 2.1.1.** Schematic of crossflow filtration. Tangential flow effectively reduces the buildup of rejected materials on the membrane surface.

Based on the effective size of the membrane pores and the corresponding size of the materials they can reject, pressure driven membrane processes are classified into microfiltration (MF), ultrafiltration (UF), nanofiltration (NF) and
reverse osmosis (RO) (Figure 2.1.2). Theoretical descriptions of permeate flux discussed and developed in this work focus on UF and MF processes. As shown in Figure 2.1.2, the semi-permeable membranes employed in UF and MF roughly correspond to effective pore sizes from about 0.001 μm to 0.1 μm for UF and 0.1 μm to 10 μm for MF. Thus, UF and MF are capable of removing species in a wide size range, including macromolecules, colloids, fine particles, and cells, from suspensions.

![Diagram](image)

**Figure 2.1.2.** Classification of pressure driven membrane processes. The effective sizes of pores or contaminants removed within the range of each process are illustrated.

### 2.1.2. Concentration Polarization and Cake Buildup

The accumulation of rejected material near the membrane surface (concentration polarization) or on the membrane (cake or gel buildup) typically leads to increased hydrodynamic resistance to solvent flow through the membrane. In most practical cases, the accumulated cake provides significant
resistance to permeation, in addition to that of the membrane. Hence, concentration polarization and cake formation contribute to the decline of the permeate flux and/or increase in the pressure drop over operating time. The decrease in permeate flux is often reversible by forcing clean water or air through the membrane in the direction opposite to that of normal permeation (backflushing). This is often coupled with sending a fast pulse (fastflushing) of raw water along the membrane to sweep out the material dislodged by backflushing. These flushing operations, along with the chemical cleaning of the membrane, are collectively referred to as flux enhancement. Chemical cleaning may be required to reverse the effects of long-term permeate flux decline due to adsorption or precipitation of materials on the membrane. In this work, only short term decline in permeate flux due to cake buildup is analyzed. The phenomenon of long-term fouling of the membrane due to adsorption or pore blockage is not considered.

2.2. Literature Review: Models Based on Different Particle Transport Mechanisms

An important performance parameter in the design, operation and evaluation of membrane filtration processes is the permeation rate or the permeate flux. As described in the previous section, the permeation rate in membrane filtration declines as rejected materials accumulate on or near the membrane surface. Quantitative estimation of permeate flux requires consideration of the different mass transport mechanisms involved. Various theories of particle transport in low-pressure crossflow membrane filtration have been put forward over the previous three decades to describe the phenomenon of concentration polarization and predict the permeation rate or permeate flux.
These theories invoke different mechanisms of particle transport to describe mass transport and predict the permeate flux. The resulting models include, amongst others, the ones based on Brownian diffusion, inertial lift, shear-induced diffusion, concentrated flowing layers, and surface transport.

Hence, predictive models for permeate flux available in the literature can be divided into broad categories from the standpoint of the particle transport mechanism(s) they consider in the formulation. In the following paragraphs, the existing models based on one or more of the transport mechanisms mentioned above are reviewed.

2.2.1. Brownian Diffusion

In the case of suspensions of fine particles and solutes, deposition on the membrane can be envisioned as being limited by permeation and diffusion of these species through a thin film or concentration boundary layer (Gutman, 1987). Hence, the mathematical description of steady-state permeation under this condition is commonly known as “film theory”. In this model, steady-state permeate flux is predicted by balancing the convective transport of particles towards the membrane with the particle diffusion away from the concentration polarization layer (Brian, 1965):

\[ J \phi = D \frac{d\phi}{dy} \]  \hspace{1cm} (2.2.1)

where \( J \) = permeate flux, \( D \) = diffusivity, and \( \phi \) = particle volume fraction. In the traditional concentration-polarization (or film theory) model the back
diffusion of particles is taken to be due to Brownian diffusion, hence the diffusivity $D$ is given by the Stokes-Einstein relationship:

$$D = \frac{kT}{6\pi \mu a_p} \quad (2.2.2)$$

Integration over the boundary layer, subject to the boundary conditions $\phi = \phi_w$ at $y = 0$ and $\phi = \phi_b$ at $y = \delta$, leads to the following expression for the permeate flux:

$$J = \frac{D}{\delta} \ln\left(\frac{\phi_w}{\phi_b}\right) = K \ln\left(\frac{\phi_w}{\phi_b}\right) \quad (2.2.3)$$

where $K$ is a mass transfer coefficient, which is usually estimated using theoretical or empirical correlations. A useful expression for the mass transfer coefficient should be able to represent the effects of changing system parameters on the performance of the system (van den Berg, 1989). Since no satisfactory theory for determining the mass transfer coefficient as a function of physical properties, design and operating conditions exists, it is usually determined by introducing an empirical correlation derived from dimensional analysis (Cheryan, 1986). The most general correlation for Newtonian fluids involves the following form of the Sherwood number:

$$Sh = \frac{K d_h}{D} = A Re^\alpha Sc^\beta \left(\frac{d_h}{L}\right)^\omega \quad (2.2.4)$$

where $d_h$ = hydraulic diameter, $Re$ = Reynolds number, $Sc$ = Schmidt number and $A, \alpha, \beta$ and $\omega$ are adjustable parameters. Different correlations exist
based on whether the velocity and concentration profiles are developed or not. The development lengths required for fully developed velocity and concentration profiles are, respectively (Cheryan, 1986):

\[ L_v = 0.029 \text{Re} d_h \]  \hspace{1cm} (2.2.5)

\[ L_c = \frac{0.1 \dot{\gamma}_w d_h^3}{D} \]  \hspace{1cm} (2.2.6)

where \( \dot{\gamma}_w \) is the shear rate at the wall. In laminar flow, the following correlations are obtained (Porter, 1972):

**Leveque (or Gratz) solution for developed velocity profile**

\[ Sh = 1.86 \text{Re}^{0.33} \text{Sc}^{0.33} \left( \frac{d_h}{L} \right)^{0.33} \text{ for } L_v \leq L, L_c \geq L \]  \hspace{1cm} (2.2.7)

**Grober solution for developing velocity profile**

\[ Sh = 0.664 \text{Re}^{0.5} \text{Sc}^{0.33} \left( \frac{d_h}{L} \right)^{0.5} \text{ for } L_v \geq L, L_c \geq L \]  \hspace{1cm} (2.2.8)

For turbulent flow, both velocity and concentration profiles are established rapidly and the Sherwood number is independent of channel length. The most commonly used correlation is (Gekas and Hallstrom, 1987):

**Dittus and Boelter solution for developed flow**

\[ Sh = 0.023 \text{Re}^{0.8} \text{Sc}^{0.33} \text{ for } L_v \leq L, L_c \leq L \]  \hspace{1cm} (2.2.9)
For fully developed laminar flow, the Leveque solution for mass transfer coefficient developed for heat-transfer in non-porous tubes is usually used, resulting in the following expression for length-averaged permeate flux:

\[ J = 0.807 \left( \frac{\dot{\gamma}_D^2}{L} \right)^{1/3} \ln \left( \frac{\phi_w}{\phi_b} \right) \]  \hspace{1cm} (2.2.10)

The Leveque solution assumes that the walls are non-porous and applies only when the bulk suspension is concentrated. For porous walls additional modifications are required due to the concentration dependence of viscosity and the transverse component of the velocity. Trettin and Doshi (1980) derived similarity solutions taking the presence of a transverse velocity component into account. Their asymptotic solution can thus be applied to dilute suspensions \((\phi_b \ll \phi_w)\):

\[ J = 1.31 \left( \frac{\dot{\gamma}_D^2 \phi_w}{L \phi_b} \right)^{1/3} \]  \hspace{1cm} (2.2.11)

For the case concentrated suspensions, their result asymptotes to the Leveque solution (Eq. 2.2.10).

In general, models based on film theory are valid only above a threshold pressure and when filtration is mass transport limited. This condition occurs after formation of a cake or gel layer. Therefore, the flux is independent of transmembrane pressure and membrane resistance. Also, film theory takes
only the thickness of the gel layer into consideration and hence is permeability independent. Brownian diffusion based film theory models predict permeate flux values within 15%-30% for solutes and macromolecular solutions (Porter, 1972) but grossly under predict fluxes for colloidal suspensions. This has lead other workers (Blatt et al., 1970) to postulate on different mechanisms of particle transport which might explain the augmented back-transport and hence the "flux paradox."

Versions of models reviewed in this section (those based on film theory and others discussed subsequently) valid under the assumption of thin cake layers are compared in Figures 2.2.1 and 2.2.2. Models based on Brownian diffusion predict a decline in permeate flux with increasing particle size while those based on shear-induced diffusion and inertial lift predict monotonic increases in permeate flux with particle size. It is interesting to note that the similarity solution of Davis and Sherwood (1990) for dilute suspensions and the Zydney and Colton (1986) model for concentrated suspensions make very similar predictions of permeate flux.

2.2.2. Inertial lift

Inertial lift has been proposed as another particle transport mechanism and arises due to non-linear interactions of a particle with the surrounding flow field under conditions where the particle Reynolds number is not negligible so that the nonlinear inertial terms in the Navier-Stokes equations play a role (Belfort et al., 1994). Observations of radial displacements of neutrally buoyant particles in Poiseuille flow of dilute suspensions were first made by Serge and Silberberg (1961). A few years later, Cox and Brenner (1968) presented a
Figure 2.2.1. Predictions of permeate flux as a function of particle size. Comparison of models (reviewed in this section) which are valid for thin cake layers only. $\phi_b=0.01$, $\dot{\gamma}_0=1000$ s$^{-1}$, $L_o=100$ cm, $T=20$ °C.
Figure 2.2.2. Predictions of permeate flux as a function of particle volume fraction. Comparison of models (reviewed in this section) which are valid for thin cake layers only. $a_p=0.5\ \mu m, \ \dot{f}_0=1000\ \text{s}^{-1}, \ L_e=100\ \text{cm}, \ T=20^\circ\text{C}$
theoretical investigation of the inertial migration of a neutrally buoyant solid sphere across streamlines for laminar flow in non-porous ducts. Around this time Porter (1972) postulated that the flux paradox could perhaps be resolved by invoking the inertial lift phenomena. This was followed by other analysis which provided explicit expressions for the lift force and velocity for rectangular and tubular geometries (Ho and Leal 1974; Vasseur and Cox 1976; Ishii and Hasimoto 1980).

Following these studies in non-porous ducts, Belfort and co-workers (Green and Belfort, 1980; Altena and Belfort, 1984; Weigand, Altena and Belfort, 1985; Schonberg and Hinch 1988; Drew et al., 1991) developed the theory of inertial lift for porous channels and tubes, and proposed the lateral migration of particles due to inertial lift as a possible mechanism which supplements the back-diffusion of particles away from the membrane. The extension of the inertial lift theory from non-porous to porous ducts was pursued both for channels (Altena and Belfort, 1984) and tubes (Weigand, Altena and Belfort, 1985). This development was limited to small Reynolds numbers \((Re \ll 1)\) and the functional form of the lift velocity was given as:

\[
\nu_{lo} = \frac{bp_{o}a^{3} \nu^{2}}{16\mu_{o}} \tag{2.2.12}
\]

where the constant \(b = 1.6\) for a slit and \(b = 1.3\) for a tube. Later, Schonberg and Hinch (1988) extended the theory to all laminar flows (i.e. \(Re \gg 1\)). Under fast laminar flows \((Re \gg 1)\), Drew et al. (1991) have shown that the constant \(b=0.577\). The lift velocity is seen to be an increasing function of the cube of
particle size and square of shear-rate (Eq. 2.2.12), hence it becomes significant for large particles and high flow rates and/or narrow channels. It is noted that the expression derived by Belfort and co-workers is valid for spherical particles in dilute suspensions where particle-particle interactions are negligible.

The basic premise of the inertial lift theory is that the presence of walls induces a lift on particles, directed away from the membrane surface. Thus, particles are predicted to deposit on the membrane only if the opposing permeation velocity exceeds in magnitude to the inertial lift velocity. When the inertial lift velocity is less than the permeation velocity, particles will deposit on the membrane and form a cake layer. The cake layer will keep growing and reduce the permeation velocity, to the point where the permeation velocity is just balanced by the inertial lift velocity.

When applied to membrane filtration (where typically $Re >> 1$), the inertial lift model thus predicts the steady-state permeate flux as:

$$J = \nu_0 = \frac{0.036 \rho_o a^3 \gamma_o^2}{\mu_o}$$  \hspace{1cm} (2.2.13)

Thus the inertial lift model predicts that the permeate flux is independent of the concentration of particles in the feed suspension and the channel length. Eq. 2.2.13 is valid only for the case of thin cake layers where the increase in shear rate (and consequently the inertial lift velocity) due to channel constriction is negligible. For thick cake layers, the shear rate increases due to constriction of
the channel, and thus the inertial lift velocity increases along the axial direction as (for tubes):

$$v_l = v_{lo} \left( \frac{H_o}{H_o - \delta_c} \right)^6$$  \hspace{1cm} (2.2.14)

The permeate flux is obtained by the simultaneous solution of Eq. 2.2.14 and Darcy's expression for the permeate flux (Davis, 1992). The inertial lift velocity is significant for large particle sizes since it scales with the cube of the particle radius. Permeate flux estimates are thus not valid for sub-micron particles and are grossly under predicted. Experiments (Redkar and Davis, 1993) also indicate that inertial lift models tend to over-predict the permeate flux at high shear rates.

2.2.3. Surface Transport

In another category of models, the back-transport of particles away from the membrane is not considered at all and diffusive phenomena are assumed to play no role. Instead, the deposition of particles on the membrane due to permeation is balanced by rolling or sliding of particles along the membrane surface due to convection. Two different approaches exist: continuum and single particle.

2.2.3.1. Continuum approach

In the continuum approach it is proposed that under sufficient shear the cake layer begins to flow tangentially along the membrane surface, toward the filter
exit. Leonard and Vassilieff (1984) estimated the boundary between two distinct hypothetical regions which are assumed to comprise the bulk suspension and the boundary layer. Their analysis leads to the following coupled equations for particle layer thickness and permeate flux as a function of time and space:

\[
\frac{\gamma_{\delta}^2}{2} = v_w \frac{\phi_b}{\phi_w - \phi_b} \tag{2.2.15}
\]

\[
v_w = \frac{\delta}{t} \frac{\phi_w - \phi_b}{\phi_b} \tag{2.2.16}
\]

The model of Davis and Birdsell (1987) relaxes some of the simplifications imposed by Leonard and Vassilieff (1984), in that nonlinear velocity profiles are considered and the permeate flux is allowed to vary axially. A numerical technique is used to solve the resultant system of equations.

2.2.3.2. Single particle approach

In the single-particle models, force and torque balances are performed on a representative particle on the membrane or cake surface. These balances are used to arrive at criteria about whether the particle will adhere to the surface or be transported along the surface. Coupling with a standard filtration theory ultimately leads to an expression for the permeate flux. An expression for the axial force acting on a sphere lying on a plane wall/surface in linear shear flows was developed by O'Neil (1968):

\[
F_x = 1.7(6 \pi \mu \rho a \rho U_p) \tag{2.2.17}
\]
where \( U_p \) = axial velocity at particle mid-point. Other workers used this expression in considering the forces acting on a single particle in crossflow filtration. Formulations of force and/or torque balances on the particle were then used to lead to a criterion for deposition. Blake et al. (1992) formulated a semi-empirical frictional force balance model by equating the functionalities of the hydrodynamic and inter particle (van der Walls and electrostatic) forces acting on a particle deposited on the cake surface. The axial hydrodynamic force component was estimated using O'Neill's (1968) expression and the normal component was estimated using Stokes equation. The resulting expression for the permeate flux was:

\[
\nu_w = k_1 a_p \tau_w + k_0
\]  

(2.2.18)

where \( k_0 \) and \( k_1 \) are empirically determined constants. Hence, their model predicts that the permeation velocity monotonically increases with the wall shear stress and that the slope of this relationship is an increasing function of particle size. Lu and Ju (1989) considered the hydrodynamic forces acting on a particle and presented a criterion for selective particle deposition based on these forces and the angle of repose between the particle and the surface. Stamatakis and Tien (1993) generalized this concept and formulated a torque balance to estimate the minimum protrusion height on a cake surface required for a particle to deposit:

\[
h_{\text{min}} = \left[ 1 - \frac{1}{\sqrt{\left( \frac{F_p}{F_q} \right)^2 + 1}} \right] \frac{d_p}{2}
\]  

(2.2.19)
where $F_p$ and $F_q$ are tangential and normal forces on the particle, respectively. The tangential force is estimated from O'Neil's expression (Eq. 2.2.17) and the transverse forces include hydrodynamic drag due to permeation, lateral lift forces and buoyant forces:

$$F_q = 6 \pi \mu a_p \nu v - 6 \pi \mu a_p \left[ \frac{61 a_p^3}{576 \nu} \left( \frac{\tau_w}{\mu} \right)^2 \right] + \frac{4 \pi}{3} \left( \rho_p - \rho_o \right) g a_p^3$$

(2.2.20)

drag force lift force buoyant force

The probability of a particle to be deposited at the cake surface equals the likelihood of the local protrusion height to be greater than $h_{\text{min}}$, and is determined by assuming that the protrusion height at the cake surface is a continuous random variable and follows a uniform distribution over the range $(0, h_{\text{max}})$. By coupling the particle adhesion probability with a cake filtration model in a form of Darcy's law, a system of equations is produced which can be numerically solved for the cake growth and permeation decline with time. The theory neglects axial variation of permeate flux along the membrane.

In general, the single-particle approach under the simplifications of negligible gravitational, lift and adhesive forces, consideration of drag and contact forces leads to the following expression for the steady-state permeate flux (Belfort et al. 1994):

$$J = 2.4 a_p \dot{\gamma} \left( \frac{a_p^2 \hat{R}_c}{2} \right)^{2/5} \cot \theta$$

(2.2.21)
where \( \theta \) = angle of repose. There are several limitations to the models based on surface transport. Since these models neglect diffusive phenomena, they are limited to larger particles (> 0.1 \( \mu \)m). Furthermore, these models are not valid for thick cake layers since they assume a thin cake layer with a high specific resistance which dominates the membrane resistance and controls the flux. The continuum approach does not predict the local particle concentration in the boundary layer and, instead, assigns a fixed value to the effective viscosity. The single-particle analysis usually requires assigning a random probability density to the protrusion heights.

2.2.4. Shear-induced diffusion

Brownian diffusion of particles occurs due to random interactions of the particles with the fluid molecules. In a flowing suspension particles also interact with other particles resulting in lateral migrations of particles from their instantaneous trajectories. Two types of motion contribute towards this phenomena: a rotary motion as well as a translational motion. These random hydrodynamic particle interactions result in net particle migration from regions of high concentration to regions of low concentration, and from regions of high shear to regions of low shear (Davis, 1993). Eckstein et al. (1977) undertook the experimental determination of a diffusion coefficient characterizing the lateral migrations of rigid, spherical particles in concentrated suspensions under the conditions of a simple, linear shear flow. Their estimates postulated a self-diffusion coefficient, linearly increasing with the particle volume fraction up to \( \phi = 0.2 \):

\[
D_{sh} = 0.02 a_P^2 \gamma \phi \tag{2.2.22}
\]
For higher values of $\phi$, the behavior with respect to particle volume fraction was not clear due to experimental scatter and the following relationship was estimated for $0.2 < \phi < 0.5$:

$$D_{sh} = 0.025a_p^2 \dot{\gamma}$$  \hspace{1cm} (2.2.23)

It was thus found that the shear-induced diffusion coefficient was proportional to the shear-rate and the square of the particle size. Zydny and Colton (1986) tried to explain the flux paradox by using the phenomenon of shear-induced transport as a back-diffusion mechanism in a mass-transport model for crossflow microfiltration. They replaced the Brownian diffusion coefficient in the film theory by the shear-induced diffusion coefficient, as measured by Eckstein et al. (1977). The local mass transfer coefficient using the Leveque solution and diffusivity given by Eq. (2.2.23) can thus evaluated as:

$$k(x) = 0.052 \dot{\gamma}_w \left( \frac{a_p^4}{x} \right)^{1/3}$$  \hspace{1cm} (2.2.24)

Using Eq 2.2.3 and Eq. 2.2.24, the local values of the permeate flux and boundary layer thickness can thus be obtained as:

$$J(x) = 0.052 \dot{\gamma}_w \left( \frac{a_p^4}{x} \right)^{1/3} \ln \left( \frac{\phi_w}{\phi_B} \right)$$  \hspace{1cm} (2.2.25)

$$\delta(x) = 0.578 \left( a_p^2 x \right)^{1/3}$$  \hspace{1cm} (2.2.26)
The boundary layer thickness is predicted to be independent of the shear rate, and this interesting result can be ascribed to the linear shear rate dependence of the shear-induced diffusivity (Eq. 2.2.23). Assuming the wall shear-rate remains constant with axial position in the filter, integration of Eq. 2.2.25 over the length of the membrane yields the following expression for the length-averaged flux:

$$J(x) = 0.078 \dot{\gamma} \left( \frac{a_p^4}{L_e} \right)^{1/3} \ln \left( \frac{\phi_w}{\phi_b} \right)$$

(2.2.27)

For dilute suspensions their model would take the following form, using results of Trettin and Doshi (1980) discussed above:

$$J = 0.126 \dot{\gamma} \left( \frac{a_p^4}{L} \frac{\phi_w}{\phi_b} \right)^{V^3}$$

(2.2.28)

The resultant numerical results were found to be in good agreement with experimental data available for blood plasmapheresis, when a value of $\phi_w = 0.95$ was used for the red blood cells which are considered deformable particles.

Davis and Leighton (1987) were later puzzled by the good agreement with experimental data of Zydney and Colton's model (1986) because Leighton and Acrivos (1987a) had shown the original estimate of the shear-induced diffusion coefficient by Eckstein et al. (1977) to be inaccurate at higher concentrations
since it had violated conditions of unbounded shear field and was thus constrained by wall limitations. In their experimental determination of the shear-induced diffusion coefficient Leighton and Acrivos (1987b) were able to empirically estimate the shear-induced diffusion coefficient up to particle volume fractions of $\phi = 0.5$ by:

$$D_{sh} = a_p^2 \gamma \hat{D}_{sh}(\phi)$$  \hspace{1cm} (2.2.29)

where $\hat{D}_{sh}$ is a dimensionless function of $\phi$ and for suspensions of rigid spheres was estimated as:

$$\hat{D}_{sh} = 0.33 \phi^2 \left(1 + 0.5 e^{0.8\phi}\right)$$  \hspace{1cm} (2.2.30)

Davis and Leighton (1987) suggested two possible explanations for the surprising agreement of theory and experiment reported by Zydney and Colton (1986): first, the assumptions of a linear velocity profile across the boundary layer in the Leveque solution neglects concentration dependent viscosity and results in use of higher shear-rates; second, the shear-induced diffusion of deformable red blood cells used in the experiments might well be lower than that for rigid particles.

2.2.5. Shear-induced diffusion and Concentrated Flowing Layers

Davis and co-workers (Davis and Leighton, 1987; Romero and Davis, 1988; Romero and Davis 1990) combined the particle transport mechanisms of shear-induced diffusion and surface transport (continuum approach) to develop a new model for crossflow microfiltration. By integrating the convective-diffusion
equation for a perfectly rejecting membrane and using the shear-induced diffusivity, Davis and Leighton (1987) predicted the thickness of the concentrated layer of neutrally-buoyant spherical particles that would be flowing over a porous membrane due to the combined effects of concentration and shear gradients:

$$\delta_n = \frac{a_p^2 \tau_w}{\mu_0 \nu_w} \frac{\phi_w}{\phi_0} \int_{\phi_b}^{\phi_w} \frac{\hat{D}(\phi)}{\phi \eta(\phi)} d\phi$$  \hspace{1cm} (2.2.31)$$

where $\tau_w$ is the shear stress at the wall and $\nu_w$ is the permeation velocity. The rate by which the particles in this flowing concentrated layer are convected towards the filter exit, or the excess particle flux, was derived as:

$$Q = \frac{a_p^4 \tau_w^3}{\mu_0^3 \nu_w^2} \hat{Q}(\phi_b, \phi_w)$$  \hspace{1cm} (2.2.32)$$

where $\hat{Q}$ is the dimensionless excess particle flux given by:

$$\hat{Q}(\phi_b, \phi_w) = \frac{\phi_w}{\phi_b} \int_{\phi_b}^{\phi_w} \frac{\hat{D}(\phi')}{\phi' \eta^2(\phi')} d\phi' \left( \frac{\hat{D}(\phi)}{\phi \eta(\phi)} \right) d\phi$$  \hspace{1cm} (2.2.33)$$

A criterion for the formation of a stagnant layer of particles underlying the flowing layer is established when the wall volume fraction reaches a limiting value. Using this analysis, the authors presented the resultant nonlinear velocity and concentration profiles in the sheared particle layer. In a subsequent work (Romero and Davis, 1988), this development was extended to
include the permeate velocity as an unknown by invoking Darcy's law. This extension allowed the prediction of the axial dependence of the stagnant cake thickness and the permeate flux along the membrane. The solution involves numerical iteration due to implicit nonlinear expressions. The equation to be solved for the cake thickness profile can be written as:

\[
\frac{\mu_0 J_0^3 \phi_b (1- \delta_c / H_0)^6}{\tau_{wo} a_p^4 (1+ R_c \delta_c / R_m)^2} \int_0^x \frac{dx}{1+ R_c \delta_c / R_m} = \dot{Q}_{cr} (\phi_b, \phi_{\text{max}}) \tag{2.2.34}
\]

where the critical dimensionless excess particle flux, \( \dot{Q}_{cr} \), is given by Eq. 2.2.33 using the maximum particle volume fraction of the cake layer as the upper limit of the integrals. The steady-state permeate flux profile is then obtained by substituting the cake thickness in Darcy's law. This steady-state analysis was further extended to a transient model by Romero and Davis (1990) by considering time-dependent accumulation in the flowing and stagnant particle layers.

Davis and Sherwood (1990) also performed a similarity solution for the steady-state convective-diffusion equation, using the concept of shear-induced diffusion, in the case where a thin particle layer of fine particles is deposited on the membrane and provides the controlling resistance to filtration. For dilute suspensions (\( \phi_b < 0.1 \)), their result was:

\[
J = 0.072 \dot{\gamma}_0 \left( \frac{a_p^4 \phi_w}{L \phi_b} \right)^{1/3} \tag{2.2.35}
\]
The combined shear-induced diffusion and convective transport models of Romero and Davis (1988, 1990) appear to resolve the flux paradox to a good extent for micro-sized particles. In experimental work focused on verifying their models, Romero and Davis (1991) conducted two series of experiments, one using a rectangular glass-walled crossflow microfilter and the other using tube-bundle microfilters. Rigid spherical particles of polystyrene and acrylic were used. In contrast to other models which grossly miscalculate the permeate flux, an agreement between theoretical and observed fluxes was found within 11% for the first series and within 27% for the second series of experiments. However, the development of Romero and Davis (1988) is ideally applicable to suspensions of rigid, non-adhesive spheres. Due to this limitation, application to adhesive particles will obviously result in an over-prediction of permeate fluxes. Redkar and Davis (1993) observed the permeate flux behavior in crossflow microfiltration of yeast suspensions and compared their observations with predictions of the shear-induced diffusion model. Model predictions with the diffusivity and viscosity calculated using correlations for rigid, non-adhesive spheres gave good correlation (10-30% over-prediction) in the case of washed yeast suspensions. However, the model over predicted fluxes in the case of unwashed yeast suspensions. The authors attributed this to the fact that unwashed yeast contains extracellular proteins and other macromolecules which cause the cake to be adhesive and more resistive, thus limiting the effectiveness of shear to induce back-diffusion. Calibrating the crossflow integral given by Eq. 2.2.33 (which accounts for variations of viscosity and diffusivity in the boundary layer) with the experimental data, resulted in model predictions in good agreement with the permeate flux behavior. These results suggest that resort to calibration and data fitting may be necessary in the
application of the shear-induced diffusion model if particle adhesion is significant.

2.3. Limitations of previous models

The models described in the previous section have been analyzed for their predictive capabilities by comparison, either quantitatively or qualitatively, with published experimental data. Such comparisons have indicated that the predictive performance of a model is very much a function of the feed suspension in question (Bowen and Jenner, 1995). Previous models are essentially valid only for a particular range of particle size because they consider only a single particle transport mechanism which is believed to be dominant for a particular particle size. It should also be recognized that dominance based on particle size is not the sole governing factor in mass transport. The operating conditions also play a part in determining the importance of a transport mechanism. For example, shear-induced diffusion may become important even for small particles when flow is sufficiently fast, and/or the channel is quite narrow, and/or the suspension is very concentrated. Similarly, inertial lift might become important for smaller particles under fast flows and/or narrow channel geometry. Hence, neglecting one or more of the transport mechanisms discussed in this chapter is likely to result in an inaccurate estimation of the permeate flux.

The applicability of previous models tends to be restricted to a particular type of feed suspension due to the fact that these models invoke a single back-transport mechanism which is considered to be dominant for the feed suspension in question. However, in many cases several different transport
mechanisms may be important for a particular suspension. Hence, predicting and comparing permeate fluxes for feed suspensions containing materials which may vary by several orders of magnitude in effective size calls for consideration of multiple transport mechanisms in the mass transfer modeling. Also, since the permeate flux is a critical measure of membrane performance and plays a primary role in determining the cost of membrane filtration, it is important that mathematical models provide a very good estimate of this parameter. The applicability of previous models may be improved by simultaneously considering the effects of Brownian diffusion, shear-induced diffusion, inertial lift and concentrated flowing layers on particle accumulation near the membrane.

Another related consequence associated with previous models is that they predict a monotonic relationship of permeate flux with particle size. These models are thus unable to describe the minima in permeate flux with particle size which have been observed experimentally (Fane, 1984; Lahoussine-Turcaud et al., 1991). Until recently, this minimum in permeate flux had been described by a hypothesized minimum in back-transport as interpreted from permeate flux data. However, the first direct experimental confirmation of the existence of a minimum in back-transport based on measurements of particle residence time distributions was also recently reported (Chellam and Wiesner, 1996).

It has recently been shown in a steady-state analysis (Sethi and Wiesner, 1995) that when Brownian diffusion is considered along with shear induced diffusion, the combined theory can describe the minimum in permeate flux with
particle size observed experimentally. In what follows, several different particle transport mechanisms are combined in a transient analysis to obtain a unified theory for permeate flux.

2.4. Extended model for predicting transient permeate flux in cross-flow membrane filtration

In this section we undertake the synthesis of a unified transient model for crossflow membrane filtration that considers several particle transport mechanisms acting in concert. The approach to formulating the mass balances involved extends that employed in the transient shear-induced model of Romero and Davis (1990). We combine Brownian diffusion and inertial lift with the transport mechanisms considered in previous work. The model is formulated for a constant transmembrane pressure mode of operation in laminar flow, and later generalized to describe constant permeate flux as well (section 5.1.5).

2.4.1. Approach to Mass Transfer

Figure 2.4.1 summarizes the mass transfer approach considered in this work and mathematically described in the following paragraphs. Particles are brought to the membrane surface due to convective deposition. Back-transport of particles away from the membrane occurs due to both Brownian and shear-induced diffusion. Inertial lift is considered to effectively reduce the net convective transport of particles towards the membrane. Particles enter and are also transported in the concentrated layer that flows along the membrane due to tangential shear. As particles accumulate in this layer the local concentration
reaches a threshold where the existing tangential velocity is insufficient to maintain a completely flowing layer. At this point, formation of a compact stagnant cake is predicted underneath the concentrated flowing layer which constricts the channel and effectively increases the shear rate. This stagnant cake provides significant resistance to permeation, in addition to that of the membrane. Cake growth is described by formulating a mass balance on the control volume represented by the cake and flowing layers. The critical time and distance required for the establishment of the stagnant cake layer are also predicted. The cake growth is related to flux decline by using Darcy’s law incorporating the membrane and cake resistances.

**Figure 2.4.1.** Approach to mass transfer and schematic of flowing and cake layers. Adapted in modified form from Romero and Davis (1990).
2.4.2. Cake filtration law

The resistances produced by the membrane and the cake which accumulates on the membrane surface are considered to act in series, and the permeate flux is then described by Darcy's law:

\[ J = \frac{\Delta P}{\mu_o (R_m + R_c)} = \frac{J_m}{(1 + R_c / R_m)} \]  \hspace{1cm} (2.4.1)

where \( \Delta P \) = transmembrane pressure, \( R_m \) = membrane resistance, \( R_c \) = cake resistance and \( J_m \) = clean membrane flux = \( \Delta P / \mu_o R_m \). The resistance due to the concentrated flowing layer is assumed to be negligible compared to the stagnant cake resistance and is not included in Eq. 2.4.1. The cake resistance is given by (Davis, 1992):

\[ R_c = R_c^* \delta_c \] \hspace{1cm} (2.4.2a)

\[ R_c = R_c^* H_o \ln \left[ H_o / (H_o - \delta_c) \right] \] \hspace{1cm} inside-out cylindrical fiber \hspace{1cm} (2.4.2b)

where \( R_c^* \) = specific resistance of the cake layer, \( H_o \) = fiber radius or channel half-height, and \( \delta_c \) = cake layer thickness. Eq. 2.4.2b accounts for the change in cake area with radial growth due to curvature. The relationship between the permeate flux, \( J \), and the permeate velocity, \( v_w \), is given, from mass balance considerations, as (Davis, 1992):

\[ v_w = J \] \hspace{1cm} rectangular slit \hspace{1cm} (2.4.3a)
\[ v_w = \frac{JH_0}{(H_0 - \delta_c)} \quad \text{inside-out cylindrical fiber} \quad (2.4.3b) \]

The specific resistance of the cake layer can be estimated using the Carman-Kozeny correlation (Carman, 1938):

\[ R_c^* = \frac{45 \phi_c^2}{a_p^2 (1 - \phi_c)^3} \quad (2.4.4) \]

where \( \phi_c \) is the packing density or solidosity of the cake. If the particles are monodisperse, spherical and non-deformable \( \phi_c \) may achieve a maximum value of 0.58 (Leighton and Acrivos, 1987b). Polydisperse suspensions or suspensions of deformable particles usually produce cakes with higher solidosities.

2.4.3. Cake growth and conservation of particle flux

It has previously (Romero and Davis, 1988, 1990) been shown that the axial convection term and the transient term in the convective diffusion equation are negligible as long as the bulk suspension volume fraction is much lower than the characteristic volume fraction in the flowing layer. Under this assumption of dilute suspensions, the particle mass balance in the polarized layer is obtained by integration of a reduced form of the convective diffusion equation, which includes back-transport due to both Brownian as well as shear-induced diffusion:

\[ [D_b + D_s(\phi)] \frac{d\phi}{dy} + v_w \phi = 0 \quad (2.4.5) \]
where \( y \) is the coordinate normal to the membrane surface (Figure 2.4.1). The assumption of a perfectly rejecting membrane has been made in Eq. 2.4.5. Inertial lift, being a far field mechanism, is not considered in the mass balance within the boundary layer.

Transient accumulation of particles in the stagnant cake is described by the particle flux conservation law. When the entire flowing particle layer is considered as an elemental volume, then the net transverse convection of particles normal to the membrane surface is balanced by the axial convection tangential to the membrane surface (Figure 2.4.1). In this formulation the axial convection term is now included but the diffusion term is negligible across the boundaries of the elemental volume and is not considered. Also, the net transport of particles into the boundary layer is reduced by the effect of inertial lift on the particles:

\[
\frac{\partial}{\partial t} \left[ \int_{\delta_c}^{\delta_c+\delta} (\phi - \phi_b) dy + (\phi_c - \phi_b) \delta_c \right] + \frac{\partial}{\partial x} \int_{\delta_c}^{\delta_c+\delta} u(\phi - \phi_b) dy = (v_w - v_l) \phi_b
\]

(2.4.6)

where \( \phi_b \) = bulk suspension particle volume fraction. The first term in Eq. 2.4.6 describes particle accumulation in the flowing and cake layers, and the second term describes the "excess particle flux" (Davis and Leighton, 1987) due to the crossflow. The local velocity, \( u \), in Eq. 2.4.6 is evaluated from integration of the local velocity gradient in the polarized layer, given by (Romero and Davis, 1990):
\[ \dot{\gamma} = \left| \frac{du}{dy} \right| = \frac{\tau_w}{\mu_0 \eta(\phi)} \quad (2.4.7) \]

where \( \tau_w \) = wall shear stress and \( \eta(\phi) \) = relative viscosity. This expression assumes that the suspension flow is fully-developed and steady based on the fact that the time scale for a steady-state bulk profile is small compared to that required for the development of the concentrated layer at the membrane surface. Further, the polarized layer is assumed to be thin compared to the channel thickness, so that the \( y \) dependency of the shear-stress can be neglected. The relative viscosity is empirically estimated as (Leighton and Acivos, 1987b):

\[ \eta(\phi) = \left(1 + \frac{1.5 \phi}{1 - \phi / \phi_c} \right)^2 \quad (2.4.8) \]

and the shear stress is expressed as:

\[ \tau_w = (n+1) \bar{U} \mu_0 \eta(\phi_b) / H \quad (2.4.9) \]

where \( \bar{U} \) is the average crossflow velocity and \( H \) is the effective fiber radius or channel half-height, calculated by accounting for the constriction of the fiber or channel due to cake formation. The shear rate, and consequently the shear stress, increases with axial distance due to constriction of the membrane channel as the cake builds up, hence:
\[
\dot{\gamma} = \dot{\gamma}_0 \left( \frac{H_o}{H_o - \delta_c} \right)^n
\]  

(2.4.10)

where \( n = 2 \) for channels with rectangular cross-sections and \( n = 3 \) for channels with circular cross-sections. This relationship is established from flow continuity and neglects the small fraction of fluid lost through the membrane.

After substituting the expression for the local velocity, \( u \), in Eq. 2.4.6, and then transforming the variable of integration from \( y \) to \( \phi \) (using Eq. 2.4.5), the particle flux conservation can be expressed as:

\[
\frac{\partial}{\partial t} \left[ \frac{h}{v_w} + (\phi_c - \phi_b) \delta_c \right] + \frac{\partial}{\partial x} \left( \frac{\tau_w}{\mu_o v_w^2} l_2 \right) = (v_w - v_i) \phi_b
\]  

(2.4.11)

where \( l_1 \) and \( l_2 \) are integrals given by:

\[
l_1 = \int_{\phi_b}^{\phi_c} \left[ \frac{\tau_w a_p^2 D_s(\phi)}{\eta(\phi)} + \frac{kT}{6 \pi \mu_o a_p} \right] \left( \frac{\phi - \phi_b}{\phi} \right) d\phi
\]  

(2.4.12)

\[
l_2 = \int_{\phi_b}^{\phi_c} \int_{\phi_b}^{\phi_c} \left[ \frac{\tau_w a_p^2 D_{sh}(\phi, \phi')}{\mu_o \eta(\phi')} + \left[ \frac{kT}{3 \pi \mu_o a_p} \right] \right] d\phi' \left( \frac{\phi - \phi_b}{\phi} \right) d\phi
\]  

(2.4.13)
$l_1$ and $l_2$ can be expressed in terms of the cake thickness, using Eqs. 2.4.7 and 2.4.10, and component integrals which are a function of the particle volume fraction, $\phi$, only:

$$l_1 = \frac{\tau_w}{\mu_0(H_o - \delta_c)^n} + \frac{kT}{6\pi\mu_0a_p}l_{12}$$  \hspace{1cm} (2.4.14)

$$l_2 = \frac{\tau_w^2}{\mu_0^2(H_o - \delta_c)^{2n}}l_{21} + \frac{kT\tau_wa_p}{6\pi\mu_0^2(H_o - \delta_c)^n}l_{22} + \frac{k^2T^2}{36\pi^2\mu_0^2a_p^2}l_{24}$$  \hspace{1cm} (2.4.15)

where $l_{11}$, $l_{12}$, $l_{21}$, $l_{22}$, $l_{23}$, and $l_{24}$ are component integrals given by the following expressions:

$$l_{11} = \int_{\phi_b}^{\phi_c} \frac{\hat{D}(\phi)}{\eta(\phi)} \frac{(\phi - \phi_b)}{\phi} d\phi$$  \hspace{1cm} (2.4.16a)

$$l_{12} = \int_{\phi_b}^{\phi_c} \frac{(\phi - \phi_b)}{\phi} d\phi$$  \hspace{1cm} (2.4.16b)

$$l_{21} = \int_{\phi_b}^{\phi_w} \int_{\phi}^{\phi_w} \frac{\hat{D}_{sh}(\phi')}{\phi'\eta^2(\phi')} d\phi'(\phi - \phi_b) \frac{\hat{D}_{sh}(\phi)}{\phi\eta(\phi)}$$  \hspace{1cm} (2.4.16c)

$$l_{22} = \int_{\phi_b}^{\phi_w} \int_{\phi}^{\phi_w} \frac{\hat{D}_{sh}(\phi')}{\phi'\eta^2(\phi')} d\phi' \frac{d\phi}{\phi}$$  \hspace{1cm} (2.4.16d)

$$l_{23} = \int_{\phi_b}^{\phi_w} \int_{\phi}^{\phi_w} \frac{d\phi'}{\phi'\eta(\phi')} (\phi - \phi_b) \frac{\hat{D}_{sh}(\phi)}{\phi\eta(\phi)} d\phi$$  \hspace{1cm} (2.4.16e)
\[ l_{24} = \int_{\phi_b}^{\phi_w} \int_{\phi}^{\phi_w} \frac{d\phi'}{\phi' \eta(\phi')} (\phi - \phi_b) \frac{d\phi}{\phi} \]  

(2.4.16f)

The integrals \( l_{11} \) and \( l_{21} \) are identical to the integrals denoted as \( \tilde{\delta} \) and \( \hat{\Theta} \), respectively, in (Romero and Davis, 1990), whereas \( l_{12}, \ l_{22}, \ l_{23}, \) and \( l_{24} \) arise due to the additional consideration of Brownian diffusivity. Numerical values of these integrals (Eqs. 2.4.16a - 2.4.16f), evaluated using the routine DQDAG (IMSL, 1991), for a range of bulk suspension particle volume fractions, are presented in Figure 2.4.2. As illustrated, these integrals decrease in value with increasing concentration. For dilute solutions, the evaluation of Eq. 2.4.11 is greatly simplified due to the fact that the integrals have approximately constant values. Thus, these integrals can be approximated as: \( l_{11}=0.0282, \ l_{12}=0.58, \ l_{21}=9.84 \times 10^{-5}, \ l_{22}=4.7 \times 10^{-3}, \ l_{23}=1.53 \times 10^{-3} \) and \( l_{24}=0.207. \)

The time required to establish the cake layer growth, or the critical time, \( t_{cr} \) is given by integrating Eq. 2.4.6 from \( t = 0 \) to \( t = t_{cr} \), with \( \delta_c = 0, \ v_w = J_m \) and \( v_I = v_{lo} \) (where \( J_m \) and \( v_{lo} \) represent the permeate flux and lift velocity, respectively, in the absence of the cake layer):

\[
t_{cr} = \frac{\left( \frac{\tau_{wo} a_D^2}{\mu_0 J_m} l_{11} + \frac{k T}{6 \pi \mu_0 a_p J_m} l_{12} \right)}{(J_m - v_{lo}) \phi_b}
\]  

(2.4.17)

The distance required to establish the stagnant layer growth, or the critical length, \( x_{cr} \), is given by integrating Eq. 2.4.6 from \( x = 0 \) to \( x = x_{cr} \), with \( \delta_c = 0, \ v_w = J_m, \ v_I = v_{lo} \):
Figure 2.4.2. Numerical values of sub-integrals over a range of bulk suspension particle volume fraction.

\[
x_{cr} = \frac{\frac{r_{wo}^2}{\mu_o^3 j_m^2} \left( \frac{r_{wo}^2 a_p^4}{\mu_o^2} I_{21} + \frac{k T r_{wo} a_p}{6 \pi \mu_o^2} (I_{22} + I_{23}) + \frac{k^2 T^2}{36 \pi^2 \mu_o^2 a_p^2} I_{24} \right)}{(J_m - \nu_{lo}) \phi_b}
\]  

(2.4.18)

Using Eqs. 2.4.7 through 2.4.16, Eq. 2.4.6 may be written as:
\[
\frac{\partial}{\partial x} \left[ \frac{\tau_{wo} H_o^2}{\mu_0 \nu_w^2 (H_o - \delta_c)^n} \right]
\left[ \frac{\tau_{wo}^2 \delta_p H_o^4}{\mu_0^2 (H_o - \delta_c)^{2n}} l_{21} + \frac{kT \tau_{wo} \delta_p H_o^2}{6 \pi \mu_0^2 (H_o - \delta_c)^n} (l_{22} + l_{23}) + \frac{k^2 T^2}{36 \pi^2 \mu_0^2 \delta_p^2} l_{24} \right]
= (\nu_w - \nu_l) \phi_b
\]  
\text{(2.4.19)}

It is convenient to non-dimensionalize this governing equation by introducing the dimensionless variables defined in Table 2.4.1. Hence, Eq. 2.4.19 can be expanded for the flat and inside-out cylindrical geometries, and subsequently expressed in dimensionless form as:

**Flat slit (n=2):**

\[
\frac{\partial}{\partial t} \left[ \frac{l_{11} (1 + \beta \hat{\delta})}{P_e_s (1 - \hat{\delta})^n} \right] + \frac{l_{12} (1 + \beta \hat{\delta})}{P_e_b} + (\phi_c - \phi_b) \hat{\delta}
+ \frac{\partial}{\partial \hat{x}} \left[ \left(1 + \beta \hat{\delta}\right)^2 \left( \frac{l_{21}}{P_e_s (1 - \hat{\delta})^{3n}} + \frac{l_{22} + l_{23}}{P_e_b P_e_s (1 - \hat{\delta})^{2n}} + \frac{l_{24}}{P_e_s (1 - \hat{\delta})^n} \right) \right]
= \left[ \frac{1}{1 + \beta \hat{\delta}} - \frac{\nu_{lo}}{(1 - \hat{\delta})^{2n}} \right] \phi_b
\]  
\text{(2.4.20a)}
Table 2.4.1. Definitions of dimensionless variables

<table>
<thead>
<tr>
<th>Dimensionless variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = \frac{\delta_c}{H_o}$</td>
<td>Dimensionless cake thickness</td>
</tr>
<tr>
<td>$\beta = \frac{H_o R_c}{R_m}$</td>
<td>Dimensionless resistance</td>
</tr>
<tr>
<td>$\dot{v}_w = \frac{v_w}{J_m} = \frac{1}{1 + \beta \delta}$</td>
<td>Dimensionless permeate velocity for flat slit</td>
</tr>
<tr>
<td>$\dot{v}_w = \frac{v_w}{J_m} = \frac{1}{(1 - \delta) \left[ 1 + \beta \ln \left( \frac{1}{1 - \delta} \right) \right]}$</td>
<td>Dimensionless permeate velocity for inside-out hollow fiber</td>
</tr>
<tr>
<td>$Pe_b = \frac{6 \pi \mu_o a_p J_m H_o}{k T}$</td>
<td>Brownian diffusion based Peclet number</td>
</tr>
<tr>
<td>$Pe_s = \frac{\mu_o J_m H_o}{\tau_{wo} a_p^2}$</td>
<td>Shear-induced diffusion based Peclet number</td>
</tr>
<tr>
<td>$\dot{v}<em>{lo} = \frac{v</em>{lo}}{J_m}$</td>
<td>Dimensionless inertial lift velocity (in absence of cake layer)</td>
</tr>
<tr>
<td>$\hat{t} = \frac{J_m (t - t_{cr})}{H_o} = \frac{J_m t}{H_o} - \dot{t}_{cr}$</td>
<td>Dimensionless time</td>
</tr>
<tr>
<td>$\hat{x} = \frac{\mu_o J_m x}{H_o^2 \tau_{wo}}$</td>
<td>Dimensionless distance</td>
</tr>
</tbody>
</table>
Inside-out cylindrical hollow fiber (n=3):

\[
\frac{\partial}{\partial \hat{t}} \left[ \frac{l_{11}(1+\beta \hat{\delta})}{P_e_s(1-\hat{\delta})^n} + \frac{l_{12}(1+\beta \hat{\delta})}{P_e_b} + (\phi_c - \phi_b) \hat{\delta} \right] \\
+ \frac{\partial}{\partial \hat{x}} \left[ \left[ 1 + \beta \ln \left( \frac{1}{1-\hat{\delta}} \right) \right]^2 \left( \frac{l_{21}}{P_e_s(1-\hat{\delta})^{3n-2}} + \frac{l_{22} + l_{23}}{P_e_b P_e_s(1-\hat{\delta})^{2n-2}} + \frac{l_{24}}{P_e_b^2(1-\hat{\delta})^{n-2}} \right) \right] \\
= \left[ \frac{1}{(1-\hat{\delta}) \left[ 1 + \beta \ln \left( \frac{1}{1-\hat{\delta}} \right) \right] (1-\hat{\delta})^{2n}} \right] v_{lo} \phi_b 
\]

(2.4.20b)

The initial and boundary conditions associated with Eq. (2.4.20a) or Eq. (2.4.20b) are given by:

\[
\hat{\delta} = 0, \; \hat{\dot{t}} \leq 0 \; \text{for} \; \hat{x} \geq 0 
\]

(2.4.21a)

\[
\hat{\delta} = 0, \; \hat{\dot{x}} \leq \hat{x}_{cr} \; \text{for} \; \hat{\dot{t}} \geq 0 
\]

(2.4.21b)

where the dimensionless forms of the critical time and distance are given by:

\[
\hat{t}_{cr} = \left( \frac{l_{11} + l_{12}}{P_e_s P_e_b} \right) \frac{1}{(1-\hat{v}_{lo}) \phi_b} 
\]

(2.4.22)
\[
\hat{x}_{cr} = \frac{l_{21} + (l_{22} + l_{23}) + l_{24}}{Pe_s^2 Pe_b Pe_s Pe_b^2} (1 - \hat{v}_lo) \phi_b
\]  

(2.4.23)

When either \( x_{cr} \geq L_e \) or \( \hat{v}_lo \geq 1 \), then no cake formation is predicted and the solution is simply: \( \delta_c(x, t) = 0, \quad \nu_w(x, t) = J_m \).

Eq. 2.4.21 is a first order non-linear hyperbolic partial differential equation, which can be solved using any of the popular explicit or implicit numerical finite difference schemes. For hyperbolic problems, the explicit schemes are generally better suited than implicit schemes. Amongst the explicit schemes, the MacCormack method based on a predictor-corrector methodology is usually recommended (Hoffman and Chiang, 1993). Using the chain rule of differentiation, Eq. 2.4.21 is first recast as:

\[
g \frac{\partial \hat{\theta}}{\partial t} + h \frac{\partial \hat{\theta}}{\partial \hat{x}} = f
\]  

(2.4.24)

where the functions \( g, h \) and \( f \) are defined, for the two geometries under consideration, as:

**Flat slit (n=2):**

\[
g = \frac{l_{11}}{Pe_s} \left[ \frac{\beta}{(1 - \hat{\theta})^n} + \frac{n(1 + \beta \hat{\theta})}{(1 - \hat{\theta})^{n+1}} \right] + \frac{\beta l_{12}}{Pe_b} (\phi_c - \phi_b)
\]  

(2.4.25a)
\[ h = 2\beta (1+\beta\delta) \left[ \frac{l_{21}}{P_{es}^2 (1-\delta)^{3n}} + \frac{(l_{22} + l_{23})}{P_{eb} P_{es} (1-\delta)^{2n}} + \frac{l_{24}}{P_{eb}^2 (1-\delta)^{n}} \right] + (1+\beta\delta)^2 \left[ \frac{3nl_{21}}{P_{es}^2 (1-\delta)^{3n+1}} + \frac{2n(l_{22} + l_{23})}{P_{eb} P_{es} (1-\delta)^{2n+1}} + \frac{nl_{24}}{P_{eb}^2 (1-\delta)^{n+1}} \right] \]  

(2.4.25b)

\[ f = \phi_b \left[ \frac{1}{(1+\beta\delta)} - \frac{\dot{v}_{fo}}{(1-\delta)^{2n}} \right] \]  

(24.25c)

**Inside-out cylindrical hollow fiber (n=3):**

\[ g = \frac{l_{11}}{P_{es}} \left[ \frac{\beta}{(1-\delta)^n} + \frac{(n-1) \left[ 1 + \beta \ln \left( \frac{1}{1-\delta} \right) \right]}{(1-\delta)^n} \right] + \]  

(2.4.26a)

\[ \frac{l_{12}}{P_{eb}} \left[ \beta - \left[ 1 + \beta \ln \left( \frac{1}{1-\delta} \right) \right] \right] + (\phi_c - \phi_b) \]
\[ h = 2\beta \left[ 1 + \beta \ln \left( \frac{1}{1 - \frac{1}{\delta}} \right) \right] \left[ \frac{l_{21}}{Pe_s^2 (1 - \delta)^{3n-1}} \right. \\
+ \left. \frac{(l_{22} + l_{23})}{Pe_i Pe_s (1 - \delta)^{2n-1}} + \frac{l_{24}}{Pe_i^2 (1 - \delta)^{n-1}} \right] \\
+ \left[ 1 + \beta \ln \left( \frac{1}{1 - \frac{1}{\delta}} \right) \right]^2 \left[ \frac{(3n-2)l_{21}}{Pe_s^2 (1 - \delta)^{3n-1}} + \frac{(2n-2)(l_{22} + l_{23})}{Pe_i Pe_s (1 - \delta)^{2n-1}} + \frac{(n-2)l_{24}}{Pe_i^2 (1 - \delta)^{n-1}} \right] \]

(2.4.26b)

\[ f = \phi_b \left[ \frac{1}{(1 - \delta) \left[ 1 + \beta \ln \left( \frac{1}{1 - \frac{1}{\delta}} \right) \right]^2 (1 - \frac{1}{\delta})^{2n}} - \frac{\dot{v}_{lo}}{\left(1 - \frac{1}{\delta}\right)^2 n} \right] \]

(2.4.26c)

Using the definitions expressed in Eqs. 2.4.25 and 2.4.26, Eq. 2.4.24 is solved using the MacCormack numerical method, discussed subsequently.

### 2.5. Numerical Solution of the Governing Equation

Using the MacCormack explicit method, the first order hyperbolic PDE given by Eq. 2.4.24 can be solved for \( \delta \) at time \( t + \Delta t \) and spatial grid point \( i \) (Anderson, 1995):

\[ \delta_i^{t+\Delta t} = \delta_i^t + \left( \frac{\partial \delta}{\partial t} \right)_{avg} \Delta t \]

(2.4.27)
where \((\partial \delta / \partial t)_{avg}\) is a representative mean value of \(\partial \delta / \partial t\) between times \(t\) and \(t + \Delta t\), and is obtained using a predictor-corrector philosophy. In the predictor step, \(\partial \delta / \partial t\) is obtained at time \(t\) by using forward differences for the spatial derivative in Eq. 2.4.24

\[
\left( \frac{\partial \delta}{\partial t} \right)_i^t = -\frac{h(\delta)}{g(\delta)} \left( \frac{\delta_{i+1}^t - \delta_i^t}{\Delta x} \right) + \frac{f(\delta)}{g(\delta)} \tag{2.4.28}
\]

and then the predicted value of cake thickness, \((\delta^*)^{t+\Delta t}\), is obtained from the first two terms of a Taylor series:

\[
(\delta^*)^{t+\Delta t}_i = \delta_i^t + \left( \frac{\partial \delta}{\partial t} \right)_i^t \Delta t \tag{2.4.29}
\]

This predicted value of the cake thickness is only first order accurate since it involves only the first-order terms in the Taylor series.

In the corrector step, first a predicted value of the time derivative of the cake thickness is obtained at time \(t + \Delta t\) and then this is used along with the predicted value of the time derivative to obtain the corrected value of the cake thickness at time \(t + \Delta t\). The predicted value of the time derivative at time \(t + \Delta t\) is obtained using backward differences for the spatial derivative and substituting the predicted values of the cake thickness

\[
\left( \frac{\partial (\delta^*)}{\partial t} \right)_i^{t+\Delta t} = -\frac{h(\delta^*)}{g(\delta^*)} \left[ \frac{(\delta^*)_{i+1}^t - (\delta^*)_i^t}{\Delta x} \right] + \frac{f(\delta^*)}{g(\delta^*)} \tag{2.4.30}
\]
Finally, the corrected value of the cake thickness at time $t + \Delta t$ is obtained by using an average of the time derivatives at $t$ and $t + \Delta t$:

$$
\delta_i^{t+\Delta t} = \delta_i^t + \left( \frac{\partial \delta}{\partial t} \right)_{\text{avg}} \Delta t
$$

$$
= \delta_i^t + \frac{\Delta t}{2} \left[ \left( \frac{\partial \delta}{\partial t} \right)_i^t + \left( \frac{\partial \delta}{\partial t} \right)_i^{t+\Delta t} \right]
$$

(2.4.31)

The Courant stability condition (Anderson, 1995) is employed to calculate the appropriate time step at each iteration.
CHAPTER 3

COST ESTIMATION IN ULTRAFILTRATION AND MICROFILTRATION

Treatment costs for membrane systems depend on the initial investment made towards the membrane modules along with the associated equipment and facilities, and the various operating costs involved in running the process. Cost estimation is required to assess the expenses incurred in using the membrane separation process, to compare with alternatives treatment technologies, and to design the process for better separation per unit cost.

In this chapter, a cost model for ultrafiltration (UF) and microfiltration (MF) processes is synthesized. A new model for estimating the capital costs of membrane plants is developed which incorporates individual cost correlations for several different categories of manufactured equipment. The complete cost model is obtained by coupling the new capital cost model with equations for operating cost developed in a previous cost model (Pickering and Wiesner, 1993). General background on the economics of UF and MF is presented in section 3.1. This is followed by a literature review of the models currently available for estimating the capital (section 3.2) and operating (section 3.3) costs associated with UF and MF processes. Finally, the new capital cost model is presented in section 3.4. It is noted that in studying and comparing the various UF and MF costs reviewed in this chapter, the year of the reference should be kept in mind to account for inflation and other market factors.
3.1. Background on Economics of UF and MF Processes

Cost data on UF and MF facilities have been limited (Pickering and Wiesner, 1993). However, in recent years with increasing interest in UF and MF processes as economically competitive treatment technologies, interest in analyzing the associated costs has risen, and the body of published data is growing.

A recent study of membrane costs (Adham et al., 1996) based on cost data gathered in an international survey of UF and MF membrane facilities found that the total costs incurred by treatment plants varied from less than $0.13/m³ ($0.50/1000 gal) for plant capacities greater than 19,000 m³/d (5 MGD), to as high as $0.66/m³ ($2.50/1000 gal) for plant capacities of 38 m³/d (0.01 MGD). Experience with design of 22 full-scale UF plants for municipal drinking water supply has provided an estimated range of $200/m³/d - $600/m³/d for capital costs (Aptel, 1994). The lower value corresponds to larger plants treating relatively clean ground water while the upper value corresponds to smaller plants treating highly polluted surface water.

The main cost components related to membrane treatment include capital cost elements such as membrane modules, pumps, instrumentation and controls, piping, etc., and operating cost elements such as energy, labor, concentrate disposal, and membrane replacement. Typically, the contribution of capital costs to the total costs is greater than that of the operating costs (Eykamp, 1995). Capital costs are generally believed to fall in the range of $600/m² - $1200/m² of membrane area installed (Cheryan 1986, Kulkarni et al., 1992), with high capacity plants operating at higher permeate fluxes (or lower
solids loading) tending towards the lower end of the range. Initial capital costs and membrane replacement costs are usually the dominant cost components for an ultrafiltration plant (Kulkarni et al., 1992). These authors also observe that energy costs associated with UF are typically in the range of 10% - 15% of the total operating costs. Kulkarni and co-workers (1992) support their observations by citing relative component costs for a typical industrial ultrafiltration plant from the work of Beaton and Steadly, which are reproduced in Table 3.1.1. According to Aptel (1994) operating and maintenance costs per unit volume of water treated are primarily dependent on the raw water quality and are relatively insensitive to the plant size. A range of $0.1/m^3 - $0.2/m^3 has been quoted for operating costs including membrane replacement, power and chemicals.

**Table 3.1.1.** Percent contributions to treatment costs for a typical industrial UF plant (Kulkarni et al., 1992)

<table>
<thead>
<tr>
<th>Cost Component</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital cost</td>
<td>38</td>
</tr>
<tr>
<td>Membrane replacement</td>
<td>27</td>
</tr>
<tr>
<td>Energy/power</td>
<td>16</td>
</tr>
<tr>
<td>Labor</td>
<td>10</td>
</tr>
<tr>
<td>Cleaning solutions</td>
<td>5</td>
</tr>
<tr>
<td>Maintenance</td>
<td>4</td>
</tr>
</tbody>
</table>

Due to the initial investment and periodic replacement of membranes, capital and operating costs of crossflow filtration are a function of the composition of the membrane (polymeric, ceramic or metallic) as well as the membrane surface area. Michaels (1989) observed that membrane costs are generally only 15% - 25% of the total installed cost of the system, while the
surrounding hardware such as the pumps, piping, instrumentation and controls, makes up the major portion of capital expenditures. Typical costs reported by Michaels (1989) for membrane filters and installed systems for various types of cross-flow MF facilities are shown in Table 3.1.2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cost of membranes ($/ft^2)</th>
<th>($/m^2)</th>
<th>Total cost of installed system ($/ft^2)</th>
<th>($/m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polymers</td>
<td>6 - 90</td>
<td>65 - 969</td>
<td>160 - 650</td>
<td>1722 - 6996</td>
</tr>
<tr>
<td>Ceramics</td>
<td>80 - 300</td>
<td>861 - 3229</td>
<td>230 - 700</td>
<td>2476 - 7535</td>
</tr>
<tr>
<td>Metals</td>
<td>60 - 120</td>
<td>646 - 1292</td>
<td>220 - 900</td>
<td>2368 - 9688</td>
</tr>
</tbody>
</table>

Even though some cost data, such as those presented in the preceding paragraphs, are now available, the economics of low-pressure membrane processes for water treatment are currently not very well understood. This is due in part to the relative novelty of the processes and therefore limited history of cost data (Pickering and Wiesner, 1993), the fact that crossflow filtration is a complex operation that depends on the interplay of many variables (Mir et al., 1992), and the fact that the economics are very much application dependent (Applegate, 1984). As a result, detailed cost estimates are still scarce in the general literature, and it is difficult to generalize from available case studies to overall trends. Moreover, the existing data base of costs is limited to relatively small facilities. Extrapolation to larger facilities assumes that the existing design approaches and economies of scale will be similar for large facilities.
3.2. Current Estimates of Capital Costs and Economies of Scale

All capital cost components, e.g. membrane device cost, pump cost, instrumentation and control cost, etc., are typically observed to scale directly with the plant size, amount of filter area installed, or another design parameter, in the form of a power law:

\[
\text{Cost} = k(\text{size})^n
\]  

(3.2.1)

The manner in which the costs of an item increase with the size parameter, or the value of the exponent \(n\), determines the behavior of the associated incremental costs, defined as:

\[
IC = \frac{d(\text{cost})}{d(\text{size})} = nk(\text{size})^{n-1}
\]  

(3.2.2)

If \(n = 1\), then the incremental costs are constant (\(IC = k\)), implying that costs scale directly with the size of the equipment, and that no economies of scale are realized with size. If \(n < 1\), then increases in size produce decreasing incremental costs, and economies of scale are said to be realized with size. In practice, the exponent \(n\) has different values for different components, reflecting different economies of scale. Membrane related capital costs are thought to demonstrate insignificant economies of scale, corresponding to values of \(n\) close to unity. Other cost components associated with equipment surrounding and supporting the membrane are known to have cost functions with
corresponding values of \( n < 1 \). This suggests that the total capital costs associated with membrane facilities should exhibit economies of scale.

Based on data collected from seventy-four full scale UF and MF plants located worldwide, Adham et al. (1996) recently reported the following regression relationship between plant capital cost and capacity:

\[
C_{\text{plant}} = 1.29 \times 10^6 Q_{\text{des}}^{0.60} \quad Q_{\text{des}} = [\text{mgd}] \quad (3.2.3a)
\]

which in SI units can be expressed as:

\[
C_{\text{plant}} = 55,961.66 Q_{\text{des}}^{0.60} \quad Q_{\text{des}} = [\text{m}^3 / \text{hr}] \quad (3.2.3b)
\]

where \( Q_{\text{des}} \) is the design capacity. The plant cost was defined to include feed water pumps, backwash and recycle pumps (where applicable), air compressor (where applicable), membrane modules and racks, piping and valves, instrumentation and controls, membrane cleaning system, equipment building, electrical supply and distribution, disinfection facilities, treated water storage and facilities, and wash water recovery system.

Pickering and Wiesner (1993) correlated non-membrane capital costs to the number of modules, \( N_{\text{mod}} \), installed to meet a given design flow. The corresponding correlation for costs in 1990 dollars was given as:

\[
C_{\text{non-membrane}} = 150,037.56 (N_{\text{mod}})^{0.74} \quad (3.2.4)
\]
This correlation was based on published data and cost estimates determined from membrane manufacturers and design engineers. The non-membrane costs were broadly defined to include all equipment and facilities necessary to support the use of membranes such as pumps, monitoring equipment, valves, automation, skids, etc.

Wright and Woods (1993) used an Environmental Protection Agency cost study for unit processes (Hansen et al., 1979) to arrive at the following relationship for the physical module cost (including hollow-fiber membrane cartridges, valves, pumps, instrumentation and control, backwash cleaning equipment and housing) for an UF unit:

\[ C_{\text{physical-module}} = 40,000(Q_{\text{des}} / 70)^{0.41} \]
\[ 10 < Q_{\text{des}} < 70 \]  \hspace{1cm} (3.2.5)

\[ C_{\text{physical-module}} = 40,000(Q_{\text{des}} / 70)^{0.80} \]
\[ 70 < Q_{\text{des}} < 4000 \]  \hspace{1cm} (3.2.6)

where \( Q_{\text{des}} \) is the design flow in m\(^3\)/d and the cost is in Canadian dollars.

Fuqua et al. (1991) provided capital cost data for reverse osmosis treatment, which yields the following power law fit:

\[ C_{\text{total-capital}} = 95642.31(Q_{\text{des}})^{0.77} \]  \hspace{1cm} (3.2.7)

where the design flow, \( Q_{\text{des}} \), is in m\(^3\)/hr.
Another popular expression for estimating the non-membrane capital costs is the six-tenths power rule:

\[ C_2 = C_1 (Q_2 / Q_1)^{0.60} \]  \hspace{1cm} (3.2.8)

where the subscripts 1 and 2 correspond to known and unknown design flows and costs, respectively. Such an expression has been used by Owen et al. (1995) for estimating membrane treatment costs for water and wastewater treatment.

In analyzing the economics of antibiotics clarification as an application of cross-flow microfiltration, Mir et al. (1992) proposed the following model for capital costs:

\[ C_t = C_c + AC_m + C_h Q_r N_p \]  \hspace{1cm} (3.2.9)

where \( C_t \) = total capital cost of the system, \( C_c \) = a constant factor related to engineering and automation costs, \( A \) = total filter area in m\(^2\), \( C_m \) = filter and housing cost per unit of membrane area, \( Q_r \) = circulation rate per filter pass, m\(^3\)/h, \( N_p \) = number of filter passes in the system, and \( C_h \) = cost of circulation components per m\(^3\)/h.

In summary, capital costs for current membrane systems appear to demonstrate economies of scale characterized by exponents between 0.4 and
0.8, with the lower value corresponding to smaller plants and the upper value corresponding to larger plants.

3.3. Estimating Operating Costs

For UF and MF processes which are operated at low-pressures, the operating costs are typically dominated by the capital costs (Eykhamp, 1995). This is specially true for small plant capacities. As the plant capacity increases, the capital costs typically decrease due to economies of scale, while the operating costs, being relatively insensitive to plant size (Aptel, 1994), do not change significantly. Hence, at very high plant capacities the operating costs can be expected to become comparable or perhaps even greater than the capital costs.

Key operation and maintenance components typically include energy requirements, membrane replacement, labor, chemicals, and concentrate disposal. The typical contributions (Eykhamp, 1995) of each of these components to the total operating costs, for crossflow UF and MF processes, are listed in Table 3.3.1.

**Table 3.3.1. Typical percent contributions to operating costs in UF and MF (Eykhamp, 1995)**

<table>
<thead>
<tr>
<th>Operating Cost Item</th>
<th>% of Total Operating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane replacement</td>
<td>35 - 50</td>
</tr>
<tr>
<td>Cleaning costs</td>
<td>35 - 12</td>
</tr>
<tr>
<td>Energy</td>
<td>15 - 20</td>
</tr>
<tr>
<td>Labor</td>
<td>15 - 18</td>
</tr>
</tbody>
</table>
Operating costs can be systematically estimated for the energy utilized by the pumps, membrane replacement and chemicals. Costs associated with other components, such as concentrate disposal and labor are highly dependent on factors such as geography, scale, and application of the membrane process. Hence, developing generic mathematical expressions for these components is a difficult task. The contribution of labor and residual disposal costs to the total operating costs may range from insignificant to dominant based on location and scale (Pickering and Wiesner, 1993).

In this work, we adopt the Pickering-Wiesner (1993) model for the purposes of estimating the operating costs. In keeping with the strategy adopted in this work to express costs as a function of membrane area (section 3.4), the operating cost equations are presented here in terms of the membrane area requirement rather than as a function of the number of modules, as presented in the original work. The calculation of the membrane area is described subsequently in section 3.4.2. In the following sections, the Pickering-Wiesner model is reviewed briefly. The reader is referred to the original work as a more detailed reference and also as an additional source to a review of previous cost work.

Energy Costs

The pumping requirements for the feed water, recycling, backflushing and fastflushing are considered in calculating the costs of energy utilized by the system. These costs are a function of the fluid velocities and pressures imposed as well as the headloss produced. The ratio of the permeate flow, $Q_p$,
to the feed flow denotes the “recovery”, $R$, of the system and consequently the feed flow can be expressed as:

$$Q_f = \frac{Q_p}{R} = \frac{\bar{J}A_{mem}}{R} \quad (3.3.1)$$

where $A_{mem} =$ membrane area, and $\bar{J} =$ time-averaged permeate flux. Energy utilized by the feed pump is expressed as:

$$E_f = \frac{P_f Q_f}{\eta_f} \quad (3.3.2)$$

where $P_f =$ feed pressure, and $\eta_f =$ efficiency of the feed pump, expressed as a decimal fraction.

The headloss across the length of the membrane element and the recycle rate determines the consumption of energy by the recycle pump. For an element with a circular cross section the axial pressure drop is derived from a mechanical energy balance across the module length:

$$P_r = \frac{4L_e}{D_e} \left[ f_f - 3a_{ke} \frac{\bar{J}}{\bar{U}_m} + \frac{\bar{J}^3}{\bar{U}_m^3} \right] \left[ \frac{1}{2} \rho \bar{U}_m^2 \right] \quad (3.3.3)$$

where $D_e =$ diameter of the hollow fiber, $L_e =$ length of the hollow fiber (or module), $a_{ke} =$ kinetic energy coefficient, $f_f =$ fanning friction factor, $\rho =$ fluid density, and $\bar{U}_m =$ average crossflow velocity at midpoint of membrane given by:
\[ \bar{U}_m = \bar{U}_o - \frac{2 \bar{J} L_e}{D_e} \]  \hspace{1cm} (3.3.4)

where \( \bar{U}_o \) = average crossflow velocity at channel inlet. The values of \( a_{ke} \) and \( f_f \) depend on the Reynolds number, \( Re \), and are approximated from correlations for flow through smooth pipes:

**For laminar flow** (\( Re \leq 4000 \))

\[ f_f = 16 / Re \quad a_{ke} = 2 \]  \hspace{1cm} (3.3.5a)

**For turbulent flow** (\( Re > 4000 \))

\[ f_f = 0.0791 / Re^{0.25} \quad a_{ke} = 1 \]  \hspace{1cm} (3.3.5b)

The recycle rate, \( Q_r \), is estimated from a flow balance on the system:

\[ Q_r = Q_t - Q_f \]  \hspace{1cm} (3.3.6)

where \( Q_t \) = total flow through the modules and is calculated as:

\[ Q_t = \frac{D_e \bar{U}_o A_{mem}}{4 L_e} \]  \hspace{1cm} (3.3.7)

The energy consumed by the recycle pump is then given as:

\[ E_r = \frac{P_r Q_r}{\eta_r} \]  \hspace{1cm} (3.3.8)
where $\eta_r$ is the efficiency of the recycle pump.

The flow rates for fastflushing and backflushing are calculated as:

$$Q_{ff} = \frac{D_e \bar{U}_{ff} A_{mem}}{4 L_e} \quad (3.3.9)$$

$$Q_{bf} = J_{bf} A_{mem} \quad (3.3.10)$$

where $J_{bf}$ = backflush flux, and $\bar{U}_{ff}$ = average fastflush velocity. The energies associated with flux enhancement are then calculated as:

$$E_{bf} = \left( \frac{P_{bf} Q_{bf}}{\eta_{bf}} \right) \frac{t_{bf}}{t_{tot}} \quad (3.3.11)$$

$$E_{ff} = \left( \frac{P_f Q_{ff}}{\eta_f} \right) \frac{t_{ff}}{t_{tot}} \quad (3.3.12)$$

where $\eta_{bf}$ = efficiency of the backflush pump, $t_{ff}$ = fastflush duration, and $t_{bf}$ = backflush duration.

Finally, if $c_{KW}$ is the cost for a unit of energy then the total energy costs per unit of treated water for operating the system can be expressed as:

$$C_{energy} = \frac{c_{KW} (E_f + E_r + E_{ff} + E_{bf})}{Q_{des}} \quad (3.3.13)$$
**Chemical Costs**

The cost of treating feed water with chemicals at the head of the plant is calculated from the chemical dosage, $CH_d$, and the bulk cost of the chemical, $CH_C$:

$$C_{\text{chemical}} = \frac{Q_f}{Q_{\text{des}}} CH_d \frac{CH_C}{Q_{\text{des}}}$$  \hspace{1cm} (3.3.14)

In cases where several chemicals are needed, the chemical dose and chemical cost represent concentration-and-mass-weighted averages.

**Membrane Replacement Costs**

Assuming that membranes are replaced at fixed intervals based on their life expectancy, the cost associated with replacement of membranes is annualized and expressed as:

$$C_{mr} = \frac{c_{\text{mod}} N_{\text{mod}} (A/F)}{Q_{\text{des}}}$$  \hspace{1cm} (3.3.15)

where $(A/F)$ is the amortization factor, calculated as a function of the discount rate for membrane replacement, $i_f$, and the design life of the membrane, $ML$:

$$(A/F) = \frac{i_f (i_f + 1)^{ML}}{(i_f + 1)^{ML} - 1}$$  \hspace{1cm} (3.3.16)
Concentrate Disposal Costs

The minimum cost related to the disposal of waste or concentrate is equal to the cost of the energy and chemicals invested in the water. Disposal costs per unit volume of water treated are thus expressed as:

\[ C_{\text{disposal}} = \left( c_{kw} \frac{P_f Q_w}{\eta_f} \right) + \left( CH_d CH_c Q_w \right) \frac{Q_{des}}{} \]  

(3.3.17)

where \( Q_w \) is the waste water flow rate calculated from the following relation:

\[ Q_w = \frac{(1-R)}{R} Q_p \]  

(3.3.18)

This estimate for concentrate disposal costs considers only the resources "lost" with the concentrate and does not account for any of the costs associated with sewerage, deep well injection, or other disposal options.

3.4. Development of New Model for Capital Costs

In general, capital cost is estimated by fitting historical data and vendor quotes to a power law expression. Two broad categories considered are membrane costs and costs of all ancillary or non-membrane equipment and facilities. However, this approach to calculating capital costs is based on underlying assumptions on the design of existing membrane facilities. The relative mix of hardware and the design approach for larger facilities may differ substantially from existing practice. In this section, we develop a new model for
estimating capital costs of membrane processes. Instead of lumping together all non-membrane costs, and hence losing insight on the economies of scale of individual components, different cost correlations for major categories of capital cost equipment are developed.

Membrane system costs typically comprise the costs of pumps, pipes and valves, instrumentation and controls, tanks and frames, and the cost of the membrane modules. Data from an EPA report (Gumerman et al., 1985) and a chemical engineering handbook (Perry and Chilton, 1991) were used to establish the factors representing the economies of scale in the power law correlations for the costs of manufactured equipment. The constants in these correlations were estimated by calibrating the model using recent data on membrane system costs (Adham et. al., 1996) and typical percent contributions of the different equipment to the total cost.

3.4.1. Pump Cost Correlation

Capital costs for pumps can be expressed as a power law relation such as that expressed in Eq. 3.2.1, where the relevant design parameter is the product of pump flow and pressure:

\[ C_{pump} = k_p (\text{flow rate} \cdot \text{pressure})^{n_p} \tag{3.4.1} \]

The pump cost curve for general-purpose-single and two-stage-single-suction centrifugal pumps presented by Perry and Chilton (1991) was employed to determine the cost correlation. This correlation was expressed as:
\[ C_{\text{pump}} = I^{*} f_1^{*} f_2^{*} L^{*} 81.27 (Q^{*} P)^{0.39} \] (3.4.2)

where \( I \) = a cost index ratio used to update the cost to the recent year (base year was reported to be 1979, first quarter), \( f_1 \) = a factor to adjust for pump construction material, \( f_2 \) = a factor to adjust for suction pressure range, and \( L \) = a factor used to incorporate labor costs. The behavior of the base pump cost (excluding the index and other multipliers) with the size factor is presented in Figure 3.4.1.

![Figure 3.4.1. Behavior of pump costs with size. Curve based on Perry and Chilton (1991) for general-purpose-single and two-stage-single-suction centrifugal pumps. Base cost (C') is for first quarter of 1979 and includes pump, drive, base plate and coupling.](image)

The cost index, \( I \) in Eq. 3.4.2, can be obtained from periodicals such as "Chemical Engineering" which publish a pumps and compressors index on a regular basis. Based on a January 1979 pumps and compressors index of
269.9 and a March 1996 index of 613.5 obtained from "Chemical Engineering", a value of \( I = 2.28 \) \((613.5/269.9)\) is applied to update the pump costs to March 1996, the base year selected for this work. Labor costs of 40% are typically reported for installation of manufactured equipment (Holland et al., 1984), implying that \( L=1.4 \). The factors \( f_1 \) and \( f_2 \) are detailed in the Chemical Engineers' Handbook (Perry and Chilton, 1991) and are repeated in Table 3.4.1 for completeness. For simulations performed in this work, stainless steel (316) surfaces were assumed, implying that \( f_1 = 1.5 \). Since most UF and MF applications are conducted at pressures under 150 psi \((1034.5 \text{ kPa})\), \( f_2 = 1.0 \) was assumed throughout this work.

<table>
<thead>
<tr>
<th>Table 3.4.1. Material and suction pressure factors as reported in the Chemical Engineers' Handbook (Perry and Chilton, 1991)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
</tr>
<tr>
<td><strong>Material Factors</strong> ((f_1))</td>
</tr>
<tr>
<td>Ductile iron</td>
</tr>
<tr>
<td>304 stainless steel</td>
</tr>
<tr>
<td>316 stainless steel</td>
</tr>
<tr>
<td>Cd-4</td>
</tr>
<tr>
<td>Durimet 20</td>
</tr>
<tr>
<td>Titanium</td>
</tr>
<tr>
<td>Hastelloy C</td>
</tr>
<tr>
<td>Hastelloy B</td>
</tr>
<tr>
<td><strong>Suction Pressure Factors</strong> ((f_2))</td>
</tr>
<tr>
<td>Up to 150 psi</td>
</tr>
<tr>
<td>150 - 500 psi</td>
</tr>
<tr>
<td>500 - 1000 psi</td>
</tr>
</tbody>
</table>
3.4.2. Cost correlations for other manufactured equipment

The capital costs of a membrane plant strongly depend on the total membrane area required to meet the design flow (Mir et al., 1992). Equivalently, capital costs can be correlated to the design capacity of the plant, or to the number of membrane modules installed. Use of the design capacity in a cost correlation is somewhat limiting since such a correlation provides no information on the design and operating parameters involved, such as the design permeate flux, system recovery, and backflushing frequency. Therefore, a cost correlation formulated with the design capacity as the independent parameter cannot be used to estimate capital costs for variable hypothetical design and operating conditions. On the other hand, the number of modules is a function of design and operating variables. However, a correlation based on the number of modules as the independent parameter is too specific, since membrane modules come in specific sizes based on the manufacturer. Such a correlation will therefore only be valid for a particular module with specific characteristics. In other words, a different correlation will have to be specified and used each time one or more module characteristics (such as module and fiber diameter, length, packing density, etc.) changed. Hence, the most suitable size parameter to which the capital costs of different components can be correlated appears to be the membrane area required to meet a specified design capacity. Like the number of modules, the membrane area is also a function of the key design and operating variables. However, it is general enough to be used independent of individual module characteristics. Based on this reasoning, it was deemed in this work that correlating capital costs to membrane area should result in a cost model that is appropriate for estimating
and comparing costs of plants with different design and operating specifications, and one that can be suitably used in an optimization study.

### 3.4.2.1. Calculation of Membrane Area

The membrane area for producing a given design flow of $Q_{des}$ is calculated from the ratio of the required design capacity to the net permeate flux:

$$A_{mem} = \frac{Q_{des} t_{tot}}{J t_o - J_{bf} t_{bf}} \tag{3.4.3}$$

where $J$ is the time averaged permeate flux, $J_{bf}$ is the backflush flux, $t_o$ is the operating time between two flux enhancement cycles, $t_{tot}$ represents the total time for one complete operating and flux enhancement cycle and is calculated as the sum of $t_o$ and the backflush time, $t_{bf}$.

### 3.4.2.2. Power Law Correlations for Equipment Cost

Non-membrane equipment and facilities, other than the pumps which have already been considered in section 3.4.1, were segregated into four categories: (1) pipes and valves; (2) instruments and controls; (3) tanks and frames; and (4) miscellaneous. Based on power law relationship expressed in Eq. 3.2.1, the cost associated with each category can then be expressed as:

1. **Pipes and valves**

   $$C_{PV} = k_{PV} (A_{mem})^{n_{PV}} \tag{3.4.4}$$
2. **Instruments and controls**

\[ C_{IC} = k_{IC} (A_{mem})^{n_{IC}} \]  \hspace{1cm} (3.4.5)

3. **Tanks and frames**

\[ C_{TF} = k_{TF} (A_{mem})^{n_{TF}} \]  \hspace{1cm} (3.4.6)

4. **Miscellaneous**

\[ C_{MI} = k_{MI} (A_{mem})^{n_{MI}} \]  \hspace{1cm} (3.4.7)

Items 1-3 listed above, along with the membrane and pump cost were considered to comprise the "membrane system" costs. These "membrane costs" along with the "miscellaneous" category listed above comprised the "plant capital costs". Due to the procedure followed in the calibration of the capital cost model (section 3.4.4), the miscellaneous category was defined to include all items listed in the definition of "plant costs" in Adham et al. (1996), after excluding the "membrane system" cost items. Thus, the miscellaneous category includes process equipment building, electrical supply and distribution, disinfection facilities, treated water storage and pumping, and wash water recovery system.

Exponents representing economies of scale in Eqs 3.4.4 through 3.4.7 were established from different sources. Data from Hansen et al. (1985) on pipes and valves and instrumentation and controls were fit to log-log linear curves (Figures 3.4.2 and 3.4.3) to establish the exponents for these two categories. For tanks and frames, the exponent reported in Peters and Kimmerhaus (1991) was used. Since correlations for both plant and membrane system costs were
supplied in Adham et al. (1996), the exponent associated with the miscellaneous category was established by plotting the difference between the two costs against a range of membrane areas (Figure 3.4.4). The exponents determined from these fitting exercises for Eqs. 3.4.4 through 3.4.7 are summarized in Table 3.4.2. It is interesting to note that the exponents vary significantly for different equipment, implying different inherent economies of scale, and a changing overall economy of scale for the entire membrane system as the system design incorporates variable mixes of the components.

![Graph](image)

\[
f(x) = 4.437829E+2 \times (x^{4.220937E-1})
\]

\[
R^2 = 9.833009E-1
\]

**Figure 3.4.2.** Calculation of the exponent for the pipes and valves cost category by fitting data from Hansen et al. (1985).

The approach to determining values for the leading constants in Eqs. 3.4.4 through 3.4.7 is discussed subsequently in section 3.4.4.
Figure 3.4.3. Calculation of the exponent for the instrumentation and controls category by fitting data from Hansen et al. (1985).

Table 3.4.2. Exponents associated with Eqs. 3.4.4 through 3.4.7, representing different economies of scale associated with different equipment

<table>
<thead>
<tr>
<th>Category</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipes and valves</td>
<td>( n_{TF} = 0.42 )</td>
</tr>
<tr>
<td>Instrumentation and controls</td>
<td>( n_{IC} = 0.66 )</td>
</tr>
<tr>
<td>Tanks and frames</td>
<td>( n_{TF} = 0.53 )</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>( n_{MI} = 0.57 )</td>
</tr>
</tbody>
</table>

3.4.3. Membrane Costs

The capital costs of membranes was calculated as the product of the membrane cost per unit area and the membrane area required to meet the design capacity:

\[
C_{\text{mem}} = C_{\text{mem}}^* A_{\text{mem}}
\]  

(3.4.8)
Based on estimates provided by membrane manufacturers on the costs of MF and UF hollow-fiber membranes, a value of $C_{mem}^* = $100/m² (1996 prices) was utilized in this work. Clearly, this value is a function of the type of membrane material, geometry, and process. In addition, this value for any given membrane is likely to be strongly influenced in the near future by the rapid growth of the membrane market.

3.4.4. Calibration of Capital Cost Model to Published Data

The cost of pumps can be directly estimated using Eq. 3.4.2. However, a calibration to membrane system cost data needs to be performed to estimate the constants for the other categories of non-membrane equipment considered. Adham et al. (1996) recently published data gathered from an international survey conducted on characteristics and costs of MF and UF facilities. From their survey, based on 1993-1995 prices which the authors adjusted for inflation to represent 1996 values, they reported the following correlation for membrane system costs (including membrane modules and directly associated equipment):

$$ C_{ms} = 7.8 \times 10^5 \ Q_{des}^{0.62} $$

$$ Q_{des} = [\text{mgd}] \quad \text{(3.4.9a)} $$

which in SI units can be expressed as:

$$ C_{ms} = 33,837.28 \ Q_{des}^{0.62} $$

$$ Q_{des} = [\text{m}^3/\text{hr}] \quad \text{(3.4.9b)} $$

In addition to membrane system costs Adham et al. (1996) also reported a correlation (Eq. 3.2.3) for the plant capital costs (including membrane system
costs plus process equipment building, electrical supply and distribution, disinfection facilities, treated water storage and pumping, and wash water recovery system). The constant and exponent associated with the correlation (Eq. 3.4.7) for the miscellaneous category can be established by plotting the difference between Eqs. 3.2.3 and 3.4.9 against the membrane area required to meet a given design flow, and noting the intercept and slope (Figure 3.4.4).

![Graph](image)

**Figure 3.4.4.** Calculation of the constant and exponent for the miscellaneous cost category correlation (Eq. 3.4.7).

Calibration of the other correlations included calculating the value of the constants in Eqs. 3.4.4 through 3.4.6, such that the total costs of pumps, membranes, pipes and valves, instruments and controls, and tanks and frames (calculated using Eqs 3.4.2 through 3.4.6, and Eq. 3.4.8) equaled the membrane system costs (Eq. 3.4.9) reported by Adham et al. To accomplish this, the calibration exercise also required knowledge of the typical contribution
of each equipment category to the portion of the membrane system cost which remained after subtracting the membrane and pump costs. Based on data from Eykamp (1991), for a 130,000 gpd (20.50 m³/hr) MF surface water treatment plant, the percent contribution of the different categories for a plant this size was taken to be:

- Pipes and valves: 36%
- Instrumentation and controls: 31%
- Tanks and frames: 33%

The final forms of the calibrated correlations for membrane system components are summarized in Figure 3.4.5, which shows the behavior of different cost components with membrane area.

Calibration of cost correlations also involved estimating values of several parameters to be used in the exercise. These parameters and values are summarized in Table 3.4.3. The median design flux reported in the cost survey was 115 L/m²/hr for MF (based on 40 plants) and 100 L/m²/hr for UF (based on 20 plants). From Adham et. al's data it can be interpreted that of the 40 MF plants, about 90% had a design flux equal to 115 L/m²/hr and the remaining 10% had design fluxes between 155-170 L/m²/hr. Since the membrane system cost correlation is based primarily on MF plant data, a design flux value of 115 L/m²/hr, reported for most of the MF plants, is employed in the calibration exercise performed in this work. Similarly, a design recovery of 95% was considered to be representative of recoveries reported in the survey data.
(based on 40 MF plants and 12 UF plants), the overall range being from 90-98%.

**Figure 3.4.5.** Calibrated cost correlations for various membrane system components.

Backflushing requirements of UF and MF hollow-fiber membranes are usually in the range of once per 20-60 minutes, depending on the raw water
quality. The typical duration for a backflush cycle is about 45 seconds (Aptel, 1994). Based on these facts, an operating time of 40 minutes and a backflush duration of 45 seconds was assumed. A maximum design pressure of about 40 psi (276 kPa) is typically reported for hollow-fiber systems in manufacturer literature. This value was employed as the design pressure of the pumps in the calibration exercise.

Table 3.4.3. Parameters values assumed in the calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design flux (L/m²/hr)</td>
<td>115</td>
</tr>
<tr>
<td>Design recovery (%)</td>
<td>95</td>
</tr>
<tr>
<td>Operating time (minutes)</td>
<td>40</td>
</tr>
<tr>
<td>Backflush duration (seconds)</td>
<td>45</td>
</tr>
<tr>
<td>Design pressure (psi, kPa)</td>
<td>40, 276</td>
</tr>
</tbody>
</table>

The calibrated membrane system costs are compared to the corresponding cost correlation (Eq. 3.4.9) from Adham et al. in Figure 3.4.6. The calibrated model is seen to be in good agreement with the correlation of Adham et al. over most of the range of the plant design capacity considered. At very high design capacities, the model developed in this work based on component cost correlations predicts slightly higher costs than the lump-sum correlation of Adham et al. Since the latter is based on data available for plant capacities up to only 2400 m³/hr, and no other data for UF/MF facilities at these high capacities are available, it cannot be said whether the component cost model or the original correlation used for calibration provides better cost estimates at these capacities.
Figure 3.4.6. Calibrated model compared to correlation from Adham et al. (1996) over a wide range of plant design capacities.
CHAPTER 4

DESIGN AND OPTIMIZATION CONSIDERATIONS IN
MEMBRANE SYSTEMS

In this chapter, a brief review of previous work related to optimization of low-pressure membrane filtration processes is presented in section 4.1. The operational configuration assumed in investigating the optimization problems considered in this study is discussed in section 4.2. This is followed by a discussion of the different optimization problems that were formulated and addressed in this work: (a) optimal membrane geometry and system operation, (b) optimal frequency of backflushing, and (c) optimal selection of ultrafiltration-nanofiltration hybrid systems. The first problem involved five decision variables and hence an optimization algorithm (discussed in section 4.3.1.4) was employed to obtain the desired solution. The one dimensional nature of the remaining two problems allowed for identification of optimal solutions using numerical simulation.

4.1. Background

The fact that treatment costs are dominated by capital costs in most cases (Eykamp, 1995; Sethi 1994; Pickering and Wiesner, 1993) has a significant impact on the design of membrane systems. The behavior of permeate flux with factors including the raw water quality, membrane characteristics, and operating conditions leads to different optimal system design and operation for different source waters. Similarly, the economies of scale exhibited by membrane plant
costs at different plant capacities also influence the optimal design and operation. In effect, trade-offs between energy and investment costs play a larger role in optimizing membrane filtration design and operation at higher plant capacities.

Although much work has been published in the literature on the physical performance of low-pressure membrane processes, there is relatively little literature and experience with regard to the numerical design and optimization of these processes from an economic standpoint. This is due, in part, to the large number of variables and system parameters encountered in the design of membrane filtration systems. One of the few optimization studies includes the work of Wiley et al. (1985) who looked at the optimization of module design for brackish water desalination by reverse osmosis (RO). These authors considered a simple model for predicting permeate flux based on film theory. The objective function comprised annual costs associated with the membranes and energy requirements of pumping the feed. Capital costs of ancillary equipment and facilities did not form a part of the objective function, and no constraints were considered in formulating the optimization. The study predicted that costs are minimized for narrow and short channels, and low cross-flow velocities.

Other work has focused on optimization of sub-problems in membrane filtration. Suwandi and co-workers looked at minimization of membrane area for multistage filtration by using semi-empirical expressions based on film theory to estimate the permeate flux, both in a steady-state (Suwandi and Singh, 1988) and a transient (Suwandi, 1993) analysis. Doshi et al. (1977)
focused on maximizing permeate flux per unit shell volume for an outside-in hollow fiber module. Using estimates of clean water flux they concluded that fibers open at both ends will have greater permeation rates than fibers closed at one end.

In summary, there has been relatively little optimization or simulation work in the area of membrane system design and operation, specially from an economic standpoint. The few optimization formulations that are existent essentially pertain to desalting membranes and are based on film theory, or sometimes on a semi-empirical approach, for estimating the permeate flux. However, the models based on film theory are known to have significant limitations and are unable to describe the permeate fluxes associated with feed suspensions characterized by colloidal or larger particles (section 2.2.1 and 2.3). Hence, the results and conclusions obtained from optimizations performed on desalting (RO) processes cannot be extrapolated to particle removal with UF or MF processes. Due to these facts, the utility of previous optimization studies has been limited in giving a relatively detailed picture of preferred membrane geometry and operating conditions in UF/MF for different raw waters. In this work we perform several optimizations in membrane systems by employing the extended model for permeate flux (chapter 2) and the complete cost model (chapter 3) for estimating the total treatment costs of membrane processes.

4.2. System Configuration

The operational configuration assumed in formulating the optimization problems investigated in this work is described in this section. Membrane systems for particle removal are commonly operated in either a continuous
“feed and bleed” configuration, or in a semi-batch mode where membrane units are periodically taken out of service and hydrodynamically washed (using backflushing and fastflushing) to reverse the effects of fouling. Recirculation of the retentate stream is usually employed to maintain the desired crossflow rate through the modules. In these operational configurations (Figure 4.2.1), a portion of the concentrate stream is either continuously (feed and bleed operation) or intermittently (semi-batch operation) removed from the system as waste.

![Diagram of membrane system operational configuration](image)

**Figure 4.2.1.** Schematic of membrane system operational configuration.

When concentrated material is intermittently wasted, the waste stream is typically generated only during the hydrodynamic backflushing cycles. To optimize the frequency of the backflush cycle, or equivalently the length of the operation between backflushes, the time-dependent permeate flux behavior
must be modeled. The recovery of the system in this case is expressed as a function of the time-averaged permeate flux (Sethi and Wiesner, 1995):

\[
R = \frac{\bar{J}t_o}{Jt_o + Jbf t_{bf}}
\]  

(4.2.1)

where \( R \) = system recovery expression as a decimal fraction, \( \bar{J} \) = permeate flux averaged over the operating time \( t_o \), \( t_{bf} \) = backflush duration, and \( J_{bf} \) = backflush flux.

Continuous bleeding implies a waste stream being continuously generated. This continuous operation occurs at a quasi steady-state, where the permeate flux has stabilized with respect to short term cake accumulation. The operating time has to be specified in this operational mode to determine system performance parameters, whereas it can be calculated from the specified recovery rate (Eq. 4.2.1) for the semi-batch mode.

Various permutations of operating modes, type of membrane, etc., lead to different optimization models. Three optimization models are formulated in this work:

(a) Design of membrane system (section 4.3.1): operation under steady-state conditions

(b) Optimal operating time (section 4.3.2): transient permeate flux behavior and intermittent wastage

(c) Optimal selection of hybrid configurations (section 4.2.3) from among the following processes:
Hollow-fiber nanofiltration: continuous steady-state operation
Hollow-fiber ultrafiltration: intermittent transient operation
Spiral-wound nanofiltration: continuous steady-state operation

In addition, it is also noted that the spiral-wound nanofiltration configuration investigated is considered to be operated with no recirculation, as in done in practice. Further, due to their design, spiral wound membranes cannot be backflushed. Therefore, costs associated with the purchase and energy utilization of the recycle and backflush pumps do not form a part of the total treatment costs for the spiral-wound membrane system.

4.3. Problem formulations

Three optimization problems were formulated and solved in this work. These problems focus on predicting the values of one or more design variables that result in (a) optimal membrane design and operation, (b) optimal operating time (and consequently system recovery), and (c) optimal choice between direct and UF pretreated nanofiltration configurations. Minimization of total treatment costs per unit volume of permeate produced is the objective function, subject to the appropriate constraints.

4.3.1. Optimal Membrane Design and Operation

4.3.1.1. Decision variables

The following decision variables were considered in the first optimization model:
1. Membrane radius, $H_o$
2. Membrane length, $L_e$
3. Transmembrane pressure, $\Delta P$
4. Average crossflow velocity, $\bar{U}_o$
5. System recovery, $R$

### 4.3.1.2. Objective Function

The objective function is comprised of both capital and operating cost components expressed per unit volume of permeate produced:

$$\min \, f(X) = \text{Capital costs} + \text{Operating costs} \quad (4.3.1)$$

where, as discussed in chapter 3, the capital and operating costs include:

**Capital costs** =

- Membranes
- Pumps
- Pipes and Valves
- Instruments and Controls
- Tanks and Frames
- Miscellaneous (buildings, storage, etc.)

**Operating costs** =

- Membrane replacement
- Energy invested in pumping feed
- Energy invested in recirculation
- Energy invested in backflushing
- Energy invested in fastflushing
- Concentrate disposal
Capital costs and membrane replacement costs are amortized over the design life of the plant (assumed to be twenty years) and membranes (assumed to be five years), respectively. Capital and operating costs are in turn a function of the raw water quality which determines the membrane area needed to provide the design flow for a given membrane design and set of operating parameters. The general methodology employed in estimating the objective function is illustrated in Figure 4.3.1. Feed stream characteristics and particular values of decision variables serve as input to the permeate flux model. The output from this model was coupled with the cost model to estimate the treatment costs.

**Figure 4.3.1.** General methodology to obtain the objective function.

### 4.3.1.3. Constraints

The set of constraints included three constraints employed to adequately describe the physical system, and the bounds associated with the decision variables.

**Constrained trans-module pressure drop**

Pressure drop across the module is a function of element geometry, feed pressure, cross-flow velocity and the permeate flux. Membrane system
operation typically prescribes a maximum allowable value for this pressure drop:

\[ \Delta P_x(\overline{U}, D_e, L_e, \Delta P, \overline{J}) \leq (\Delta P_x)_{\text{max}} \]  

(4.3.3)

where \( \overline{J} \) is calculated as a function of the feed water characteristics. The explicit functional form of \( \Delta P_x \) is given in Eq. 3.3.3 and discussed in section 3.3. A typical value of \( (\Delta P_x)_{\text{max}} = 50 \) kPa was assumed in the optimizations performed in this work.

Flow continuity

A mass balance on the fluid flow for the operational configuration investigated (Figure 4.2.1) leads to the following constraint:

\[ \frac{4JL_e}{D_eU_cR} \leq 1 \]  

(4.3.4)

This constraint ensures that only physically meaningful configurations are considered in performing the optimization.

Laminar flow

Due to the applicability of the mass transport modeling efforts investigated in this study an upper limit on the Reynolds number also needs to be prescribed:

\[ \frac{\rho \overline{U}_e D_e}{\mu} \leq \text{Re}_{\text{max}} \]  

(4.3.5)
Re_{max} = 4000 was assumed based on typical values quoted in the literature (Belfort and Nagata, 1985) for the onset of turbulence during flow in porous tubes with suction.

**Bounds on decision variables**

In general, the bounds on a decision variable \( x \) will have the following form:

\[
x_{\text{min}} \leq x \leq x_{\text{max}}
\]  \hspace{1cm} (4.3.6)

Based on typical as well as physically meaningful ranges, the following bounds were defined for the decision variables:

- \( 0.125 \text{ mm} \leq H_0 \leq 1.5 \text{ mm} \) \hspace{1cm} (4.3.7a)
- \( 20 \text{ cm} \leq L_e \leq 200 \text{ cm} \) \hspace{1cm} (4.3.7b)
- \( 25 \text{ kPa} \leq \Delta P \leq 250 \text{ kPa} \) \hspace{1cm} (4.3.7c)
- \( 10 \text{ cm/s} \leq \overline{U}_o \leq 200 \text{ cm/s} \) \hspace{1cm} (4.3.7d)
- \( 55 \% \leq R \leq 99 \% \) \hspace{1cm} (4.3.7e)

These ranges reflect typical low-pressure hollow fiber configurations. Very narrow fibers are likely to plug up in surface water treatment, and also lead to very high pressure drops across the module. The lower bound on fiber radius was thus established based on practical considerations and manufacturer literature. For the same crossflow velocity, hollow fibers offer greater shear stress compared to the tubular geometry, due to a narrower flow path. The upper bound on fiber radius was constrained to a value that would result in
acceptable shear stresses. The bounds on the membrane length were approximated based on the facts that very small modules would not be able to accommodate all the inlet and outlet ports, and very long fibers would produce large pressure drops. The upper bound on transmembrane pressure was defined based on typical design pressures that hollow fibers are reported to be able to withstand. The bounds for crossflow velocity were defined based on the laminar flow operation typical of hollow fibers. Since part of the permeate produced in membrane filtration is typically utilized in backflushing the system, a lower bound of 55% was prescribed for the system recovery, to reflect a positive production.

An optimization algorithm (described subsequently) based on sequential quadratic programming (SQP) was used to solve the system defined in Eqs. 4.3.1 through 4.3.7. The algorithm further required estimates of the gradients of the objective and constraint functions. These were calculated using finite differences since analytical derivatives are not available for the problem considered. The gradient of the objective function was calculated using central differences while the gradients of the constraints were calculated using forward differences.

Figure 4.3.2 summarizes the general methodology used in solving the optimization problem. Starting with an initial guess, the optimization algorithm utilizes the performance and cost models to estimate objective and constraint function values. The process is repeated iteratively, with improved estimates of the decision vector, till a converged solution is obtained.
4.3.1.4. Solution Technique

A popular approach used for solving nonlinear optimization problems is the sequential quadratic programming method. An implementation of this method for solving constrained nonlinear problems is due to Schittkowski (1985/6) and can be used to solve the problem of optimal membrane design and operation. Schittkowski's algorithm is available as a set of FORTRAN subroutines and was integrated with the numerical codes for the performance and cost models formulated in this work, along with subroutines for calculating the constraints and finite difference estimates of the gradients. An overview of the algorithm employed is presented next.
Consider a general constrained nonlinear programming problem (Schittkowski 1985/6):

\[
\begin{align*}
\text{min.} & \quad f(x) \\
\text{subject to} & \quad g_j(x) = 0 \quad \text{for} \quad j = 1, \ldots, m_e \\
& \quad g_j(x) \geq 0 \quad \text{for} \quad j = m_e, \ldots, m \\
& \quad x_l \leq x \leq x_u \quad \text{for} \quad x \in \mathbb{R}^n
\end{align*}
\]

where \( n \) = number of decision variables and \( m \) = total number of constraints, of which \( m_e \) are equality constraints. A Lagrangian function is defined as:

\[
L(x, u) = f(x) - \sum_{j=1}^{m'} u_j g_j(x)
\]

where \( u \in \mathbb{R}^{m'} \) = vector of Lagrange multipliers, and \( m' = m + 2n \) and includes the bound constraints. A sub-problem to the system (4.3.8) is obtained by linearizing the nonlinear constraints and minimizing a quadratic approximation of the Lagrangian function:

\[
\begin{align*}
\text{min.} & \quad \frac{1}{2} s^T H_k s + \nabla f(x_k)^T s \\
\text{subject to} & \quad \nabla g_j(x_k)^T s + g_j(x_k) = 0 \quad \text{for} \quad j = 1, \ldots, m_e \\
& \quad \nabla g_j(x_k)^T s + g_j(x_k) \geq 0 \quad \text{for} \quad j = m_e, \ldots, m \\
& \quad x_l - x_k \leq s \leq x_u - x_k
\end{align*}
\]
where $H_k$ = a positive definite approximation of the Hessian matrix of the Lagrangian function, updated according to a modified BFGS formula, named after Broyden, Fletcher, Goldfarb, and Shanno (Dennis, 1983), to guarantee positive definite matrices), and $x_k$ = current iterate. The algorithm determines a new iterate using a line search:

$$x_{k+1} = x_k + \alpha_k s_k$$  \hspace{1cm} (4.3.11)

where $\alpha_k$ = a step length parameter and $s_k$ = solution of the system (4.3.10). The line search is performed such that a merit function will have a lower function value at the new point. Here the merit function is taken to be the augmented Lagrangian function. The algorithm contains additional features to overcome certain disadvantages, such as recalculation of gradients at each iteration and empty feasible regions, and is described in more detail elsewhere (Schittkowski, 1985/6).

The optimizations model was run to identify optimal membrane designs and conditions for operation as a function of raw water quality. Raw water quality in itself was parameterized by a range of particle sizes and concentrations. Results from the analysis are presented in section 5.3.1.

4.3.2. Optimal backflushing frequency for hydrodynamic cleaning

The membrane area requirement in membrane filtration, assuming an intermittent transient operation (section 4.2), is determined by the ratio of the required design flow to the net permeate flux. Thus, the membrane area is a
function of the time averaged permeate flux, the durations of the operating and backflush phases and the backflush velocity (section 3.4.2.1):  

$$A_{mem} = \frac{Q_{des}(t_o + t_{bf})}{J_{t_o} - J_{bf}t_{bf}}$$  

(3.4.3)

In a constant pressure mode of operation, permeate flux declines with time as materials accumulate on the membrane surface, increasing the hydraulic resistance to permeation. In a constant permeate flux mode of operation, pressure drop across the membrane increases over time. The rate of permeate flux decline depends on many factors including the raw water quality, membrane design and operating characteristics, leading to different transient permeate flux curves for different scenarios. It is then of interest to formulate the dynamic problem of determining the optimal operating time over which the permeate flux should be allowed to decline, before arresting the decline by subjecting the membrane to hydrodynamic cleaning by backflushing.

In this problem the operating time is a decision variable. The objective is to find the optimal operating time (and consequently the optimal backflushing frequency and system recovery) so that the total treatment costs are at a minimum. Two constraints are involved which ultimately lead to lower bounds on the operating time, positive productivity and flow continuity.

**Positive productivity**

Since a portion of the permeate produced is employed in backflushing, a constraint is needed to ensure a net positive production of permeate.
Therefore, the operating time should be such that the net water produced, or the denominator in Eq. 3.4.3, should be positive, or:

\[ \bar{J}t_o > J_{bf}t_{bf} \]  
\[ (4.3.12a) \]

Hence, positive productivity establishes a lower bound on the operating time:

\[ t_o > \frac{J_{bf}t_{bf}}{\bar{J}} \]  
\[ (4.3.12b) \]

or, equivalently:

\[ \int_0^{t_o} J(t)dt > J_{bf}t_{bf} \]  
\[ (4.3.12c) \]

since the time-averaged permeate flux, \( \bar{J} \), is defined as:

\[ \bar{J} = \frac{\int_0^{t_o} J(t)dt}{t_o} \]  
\[ (4.3.13) \]

**Flow continuity**

Since the recovery in an intermittent wastage operation is a function of the operating time (Eq. 4.2.1), the latter relationship can be substituted in Eq. 4.3.4 to give another lower bound on the operating time:
\[ t_o > \frac{4L_o (J_t o + J_b t_b)}{D_e U_o} \]  
\hspace{10cm} (4.3.14a)

or, equivalently:

\[ t_o > \frac{4L_o \left( \int_0^{t_o} J(t) dt + J_b t_{b_f} \right)}{D_e U_o} \]  
\hspace{10cm} (4.3.14b)

The greater of the two lower bounds expressed in Eqs. 4.3.12 and 4.3.14 determines the minimum allowable operating time.

Due to the inherent nature of the optimal operating time problem, the solution can be obtained using numerical simulation. Optimal operating times and recoveries were calculated over a domain of raw water quality represented by variable particle size and concentration. Results and discussion from the analysis are presented in section 5.3.2.

4.3.3. Optimal Configuration of Nanofiltration Systems

Nanofiltration (NF) membranes typically remove particulate and organic material as small as 250 Daltons in molecular weight (equivalent to about 0.001 \( \mu \text{m} \)). Hence, these membranes are commonly employed for the removal of divalent ions and organic material from raw water. NF membranes are of considerable interest in potable water treatment due to their capability of effectively removing low molecular weight dissolved organic compounds (while
offering the cost advantages of low pressure filtration). This is of special interest from the standpoint of meeting the water quality standards promulgated by the current and anticipated rules for disinfection by-products (DBPs).

For NF, two configurations commonly applicable for commercial use include the hollow-fiber and spiral-wound membrane systems. The spiral-wound (SW) membranes are cheaper than the hollow-fiber (HF) membranes due to a design that permits a higher packing density. However, a higher packing density also makes the spiral wound membranes more prone to fouling and hence they require advanced pre-treatment. This pre-treatment may be in the form of HFUF or HFMF to effectively reduce the contaminant load on the SWNF membranes. Based on its design, the HF geometry is less prone to fouling and offers the advantage of periodic hydrodynamic backflushing to (partially) recover the declining permeate flux. HFNF can therefore be used to process feed streams without advanced pre-treatment.

An optimization problem was formulated that focused on investigating the interplay between the different economies and mass transfer capabilities of the single (HFNF) and hybrid (SWNF pretreated with HFUF) nanofiltration configurations, over a domain of variable raw water quality characterized by variable particle size and concentration.

**Treatment Options**

The two treatment alternatives investigated are schematically represented in Figure 4.3.3. The analysis presented in this work focuses on predicting the optimal treatment option based on the capital and operating costs of the unit.
membrane processes. Costs for the basic pre-treatment indicated in Figure 4.3.3 are assumed to be similar for the two treatment chains, and are not considered in the total cost estimates.

**SINGLE NF**

![Diagram](image)

**HYBRID NF**

![Diagram](image)

**Figure 4.3.3.** Schematic of single system and hybrid system NF treatment alternatives

Treatment costs for the intermittently operated HFUF system were estimated by optimizing the backflushing frequency (and hence the system recovery, Eq. 4.2.1) in a manner analogous to that discussed in section 4.3.2. The permeate flux model formulated in this work is applicable to membrane channels with either rectangular or circular cross sections. Permeate flux for the spiral-wound geometry involved in this optimization problem was approximated by utilizing an equivalent hydraulic diameter calculated from the thickness of the feed spacer.
The concentration of the HFUF permeate stream, which is subsequently fed into the SWNF system was calculated using the sieve model for estimating contaminant rejection (Zeman and Wales, 1981) and a mass balance on the HFUF operational configuration. Hence, the concentration of the feed into SWNF system was calculated as:

\[ \phi_{SWNF} = \frac{\phi_{HFUF} \kappa}{1 - (1 - \kappa)R} \]  

(4.3.15)

where \( \kappa \) = sieve coefficient, \( \phi \) = feed suspension concentration, and \( R \) = recovery of the HFUF system expressed as a decimal fraction. This expression assumes that concentration polarization and removal in the cake compensate for one another and that the sieve coefficient, which corresponds to the local rejection, can be used to obtain an estimate for the apparent rejection (Sethi and Wiesner, 1995).

Results from the optimal configuration analysis are presented in section 5.3.3. Analyses are performed to investigate the effects of variable particle size, concentration, plant capacity and HFUF system feed pressure on the domains of optimal configurations.
CHAPTER 5

RESULTS AND DISCUSSION

The permeate flux model developed in chapter 2 was used to examine the effects of raw water quality (in terms of particle size and concentration), and selected design and operating parameters, on the cake growth and permeate flux behavior of typical UF/MF systems. Results from these simulations and sensitivity analyses are presented in section 5.1.

The economic performance of typical low-pressure membrane systems was investigated by coupling the permeate flux model with the cost model developed in chapter 3. Effects of particle size and concentration on parameters of economic performance, along with sensitivity analyses with respect to key design and operating parameters are presented in section 5.2.

In performing the optimizations described in chapter 4, the appropriate design and operating parameters were prescribed as unknown decision variables to be determined. Optimal values of these variables, which resulted in a minimization of the total treatment costs within the feasible domain prescribed by the appropriate constraint set, are presented and discussed in section 5.3.

Estimation of Bulk Suspension Volume Fraction

For the permeate flux behavior investigated in section 5.1, the results are presented in terms of a specified bulk suspension volume fraction. However,
results for the cost analysis (section 5.2) and optimization problems (section 5.3) are presented in terms of the feed suspension concentration (or equivalently, the feed suspension volume fraction). The bulk suspension volume fraction in the latter cases is estimated from the feed suspension volume fraction using the sieve coefficient to estimate the rejection on the membrane surface. From a mass balance on the operational configuration presented in Figure 4.2.1 the bulk suspension volume fraction, $\phi_b$, can be expressed as:

$$\phi_b = \frac{\phi_0}{1 - rR}$$  \hspace{1cm} (5.1.1)

where $\phi_0$ = feed suspension volume fraction, $r$ = apparent rejection, and $R$ = system recovery. The apparent rejection is estimated as:

$$r = 1 - \kappa$$ \hspace{1cm} (5.1.2)

A model for passage of hard spherical particles approaching cylindrical pores, with negligible diffusive transport, is used to estimate the sieve coefficient, $\kappa$ (Ferry, 1936). This model assumes uniform concentrations in the pores (equal to the concentration at the pore entrance), and no pore-particle interactions before attachment. The expression for the sieve coefficient originally proposed by Ferry (1936) was later modified (Zeman and Wales, 1981) to account for wall effects by introducing a "lag" factor: $G = \exp(-0.7472\lambda^2)$ where $\lambda$ is the particle to pore size ratio. The modified expression for the sieve coefficient is given as:
\[ \kappa = \begin{cases} 
(1-\lambda)^2 [2-(1-\lambda)^2] G & \lambda \leq 1 \\
0 & \lambda > 1
\end{cases} \quad (5.1.3) \]

Eq. 5.1.1 assumes that concentration polarization and removal in the cake compensate for one another and that the sieve coefficient, which corresponds to the local rejection, can be used to obtain an estimate for the apparent rejection (Sethi and Wiesner, 1995).

5.1. Permeate Flux Behavior

In this section the effects of variations in the model parameters determining cake growth and permeate flux are presented. Although it is convenient to solve the problem in dimensionless form, presentation of the results in this form is hindered by the fact that the dimensionless variables are closely related and it is not possible to vary one dimensionless number while keeping all others fixed. Hence, the results in this section are presented assuming typical dimensional values of the parameters. Permeate flux estimations in this section assume perfect rejection on the membrane for all particles. The simulations are performed assuming a continuous load of influent particles in the feed stream characterized by a single diameter and volume fraction, since the model ideally assumes a monodisperse suspension. The variation in particle diameter in these simulations may be interpreted as representing various monodisperse suspensions, different fractions of a single polydisperse suspension, or various polydisperse suspensions characterized by an average/equivalent particle size. For comparison with the polydisperse suspension experiments (section 5.1.6), model predictions are made by approximating the polydispersivity with the volume averaged particle size as the effective size.
Simulations were performed to study the effects of particle size and concentration on the cake thickness and permeate flux, for a typical flat slit and an inside-out hollow fiber. Particle size was varied from 0.001 - 10 μm, while other system parameters were held constant at their baseline values defined in Table 5.1.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low Value</th>
<th>Baseline Value</th>
<th>High Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmembrane pressure, ΔP (kPa)</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Bulk suspension volume fraction, ϕ_b (+)</td>
<td>0.0001</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>Fiber radius or slit half-height, H_o (mm)</td>
<td>0.25</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Average cross-flow velocity at inlet, U_o (cm/s)</td>
<td>30</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Membrane length, L_o (cm)</td>
<td>70</td>
<td>105</td>
<td>140</td>
</tr>
<tr>
<td>Membrane Resistance, R_m (1/cm)</td>
<td>-</td>
<td>1.46×10^{10}</td>
<td>-</td>
</tr>
</tbody>
</table>

5.1.1. Effects of Particle Size

5.1.1.1 Cake growth

The effect of particle size on cake growth is depicted by plotting the instantaneous length-averaged cake thickness (normalized with the channel radius or half-height) against particle diameter at various times (Figure 5.1.1). The unified model suggests an intermediate range of particle size where net transport of particles away from the membrane approaches a minimum and results in thick cake buildup. This occurs around 0.4 μm for the system simulated in this work. However, at small times (1000s curve in Figure 5.1.1)
the unfavorable size is seen to actually move towards larger particle diameters. This is due to the fact that at smaller times the relative size of the particles is significant. At larger times, when the number of particles accumulated becomes significant, it is clearly illustrated that the peak occurs at 0.4 μm. The unfavorable size window is dynamic and moves with the kinetics of cake growth, as well as the hydrodynamics of the system. Thus, the relative composition of cakes formed from polydisperse suspensions is expected to change over time.

![Diagram](image)

**Figure 5.1.1.** Particle size effects on the length-averaged cake thickness at various times for typical flat slit and inside-out hollow fiber filters.

The cake buildup at steady-state is less for an inside-out hollow fiber than for a flat slit, for all particle diameters. This is due to the fact that cake growth in the
inside-out cylindrical geometry causes greater reduction in cross-sectional area for suspension flow, leading to less permeate flux (section 5.1.1.2), and consequently less potential for cake growth. For the curve at 1000 s, however, the thicker cakes in the case of a slit for larger particles are actually due to the lower shear rate in the slit geometry. This was confirmed by performing simulations after adjusting the shear rate in the slit to be the same as in the case of the hollow fiber.

Another consequence of the different geometries is that steady-state is achieved relatively faster in the cylindrical geometry than in the slit. Therefore, the kinetics of cake growth differ in the two geometries and though the equilibrated cake thickness is higher for a slit, the instantaneous cake thickness can be lower, particularly for particles with low net diffusivities which take long times to achieve steady-state conditions.

The transient variation in the length-averaged cake thickness is illustrated in Figure 5.1.2 where the cake thickness for various representative particle sizes is plotted against time. The inset depicts the cake thickness on a logarithmic scale. Small particles (0.005 \( \mu \text{m} \), 0.01 \( \mu \text{m} \)), which form cakes with high specific resistance, demonstrate a relatively fast approach to steady state. These simulations also imply that the time to steady-state is the longest for particles favoring accumulation (0.1 \( \mu \text{m} \), 0.5 \( \mu \text{m} \)), as expected. The 1.0 \( \mu \text{m} \) particle has less cake buildup than the 0.5 \( \mu \text{m} \) particle due to shear-induced diffusion and inertial effects.
Figure 5.1.2. Cake growth for various particle sizes for typical flat slit and inside-out hollow fiber filters.

5.1.1.2 Permeate Flux

The influence of particle size on the permeate flux decline is illustrated in Figures 5.1.3 and 5.1.4, by plotting the instantaneous length-averaged permeate flux (normalized with the membrane limited flux, \( J_m \)). The "unfavorable" particle size range (Figure 5.1.3), where the largest decline in flux occurs, is determined not only by the cake thickness, but also by the cake
permeability. Since the latter decreases with decreasing particle size, the unfavorable size range for permeate flux decline shifts towards smaller particle sizes, when compared to the unfavorable particle size range for cake growth.

![Diagram showing normalized length-averaged permeate flux vs particle diameter for Hollow-fiber and Slit filters at different time points (t = 10^2 s, 10^3 s, 10^4 s) with Steady-State indicated.](image)

**Figure 5.1.3.** Particle size effects on the length-averaged permeate flux at various times for typical flat slit and inside-out hollow fiber filters.

The transient variation of the permeate flux, for a set of representative particle sizes, is illustrated in Figure 5.1.4. For large particles which significantly constrict the channel due to cake buildup, the surface area for filtration reduces in case of an inside-out cylindrical geometry. Hence the permeate flux for the inside-out hollow fiber is less than that of the slit, even though the cake for the former is thinner (at longer times and steady-state, Figure 5.1.1). For smaller
particles, which form thin cakes, curvature effects are not important, and the hollow fiber geometry (which results in a higher shear-rate than the slit for the same cross-flow velocity and channel dimensions) results in higher permeate flux. Figure 5.1.4 also illustrates that for the system simulated, the crossover from a favorable tube geometry to a favorable slit geometry, in terms of a higher equilibrated permeate flux, occurs at a particle size of about 0.06 μm.

![Graph showing permeate flux decline for various particle sizes](image)

**Figure 5.1.4.** Permeate flux decline for various particle sizes for typical flat slit and inside-out hollow fiber filters.

### 5.1.2. Effects of Concentration

Figure 5.1.5 shows the effect of particle concentration on the transient permeate flux. As expected, the permeate flux decreases with increasing bulk
suspension particle volume fraction, $\phi_b$. The time to steady-state also decreases with increasing concentration. The variation in the permeate flux with concentration is seen to be quite significant in the case of all representative particle sizes.

![Graph showing permeate flux decline for various concentrations and particle sizes for typical flat slit and inside-out hollow fiber filters.](image)

**Figure 5.1.5.** Permeate flux decline for various concentrations and particle sizes for typical flat slit and inside-out hollow fiber filters.

5.1.3. **Comparison between inside-out hollow fiber and slit geometry**

In discussing Figure 5.1.3 it was noted that a crossover from a favorable inside-out hollow fiber geometry to a favorable slit geometry, in terms of a higher equilibrated permeate flux, occurred at a particle size of about 0.06 µm. When simulations are performed over a range of bulk suspension
concentrations, the "crossover diameter" is seen to decrease with increasing concentration (Figure 5.1.6). The flat slit geometry represents a crossover in the radius of curvature from inside-out (concave) to outside-in (convex) cylindrical configuration. Thus, Figure 5.1.6 can be interpreted as illustrating the respective domains of raw water quality where these two hollow-fiber configurations are expected to be favorable. The inside-out geometry is expected to be disadvantageous in situations where there is a potential to form thick cakes, as in the case of large particles and high concentrations. This is explained by the reduction in permeation area caused by the buildup of cakes in the case of the inside-out cylindrical geometry. Thus, as demonstrated in Figure 5.1.6, the flat slit or inside-out geometry is expected to be optimal for smaller particles and/or low concentrations.

![Graph showing crossover diameter]

**Figure 5.1.6.** Domains of favorable equilibrated permeate flux for different hollow fiber configurations.
5.1.4. Effects of Operating Parameters on Permeate Flux for Inside-Out Hollow Fibers

Interactions between operating parameters and membrane fouling were investigated by performing simulations in which the operating variable being analyzed assumes three values (Table 5.1.1), while all other system parameters are held constant at their baseline values (Table 5.1.1). These simulations were conducted for feed suspensions containing one of three representative particle sizes (small particle: 0.01 μm; intermediate/unfavorable sized particle: 0.1 μm; and large particle: 1.0 μm).

5.1.4.1. Transmembrane Pressure

The effect of transmembrane pressure on the transient permeate flux is illustrated in Figure 5.1.7. The permeate flux is not presented in a normalized fashion in this plot since the membrane limited flux changes with transmembrane pressure. Colloidal particles (0.1 μm, 0.01 μm) are seen to create mass transport limited (or pressure independent) behavior since the limiting or steady-state permeate flux does changes little with pressure. However, large particles (1.0 μm) show significant variation in permeate flux with transmembrane pressure. Operating at higher pressure causes the steady-state to be achieved earlier and the permeate flux to increase over the entire operation. However, is should be noted that with increasing pressure the steady-state flux actually decreases relative to the initial flux, for all particle sizes.
Figure 5.1.7. Effect of transmembrane pressure on the length-averaged permeate flux for an inside-out hollow fiber filter.

5.1.4.2 Cross-flow Velocity

Cross-flow velocity directly influences the shear-rate in the channel. Figure 5.1.8 illustrates the effect of varying cross-flow velocity on the permeate flux. When the crossflow velocity is varied, the effect on the transient decline of the permeate flux is less significant than the effect on the steady-state flux, particularly for the unfavorable particles. The time to equilibration decreases with a higher cross-flow velocity, but the change is not very significant. The greatest variation is observed in the steady-state flux, particularly for the favorable particles.
Figure 5.1.8. Effect of cross-flow velocity on the length-averaged permeate flux for an inside-out hollow fiber filter.

5.1.4.3. Hollow-fiber Radius

Figure 5.1.9 illustrates the effect of varying the hollow-fiber radius on the permeate flux. Increasing the membrane radius decreases the shear rate and hence the equilibrated permeate flux decreases. However, in the case of 0.1 μm and 1.0 μm particles, which form thick cakes resulting in significantly reduced area available for filtration, a larger hollow fiber radius increases the filtration area. For these particles, the kinetics of cake growth coupled with the curvature effects on available filtration area determine the transient permeate flux profile. Hence, as seen in Figure 5.1.9, for these particles the transient flux can improve with increasing fiber radius. This behavior is not observed in the
case of small particles, since the cakes formed are very thin and hence the reduction in available filtration area is not significant. Again, in all cases, a narrow channel with a greater shear rate achieves equilibrium conditions earlier. Larger particles show the greatest sensitivity to variations in the fiber radius.

![Graph showing normalized length-averaged permeate flux vs. time for different fiber radii (0.01 μm, 0.1 μm, 1.0 μm) and channel heights (H₀ = 0.25 mm, 0.5 mm, 1 mm).]

**Figure 5.1.9.** Effect of fiber radius on the length-averaged permeate flux for an inside-out hollow fiber filter.

5.1.4.4. Shear Rate

Shear rate can be adjusted by varying either the cross-flow velocity or the channel radius/height. A combination and permutation of the variable cross-flow velocities and channel diameters/heights employed in Figures 5.1.8 and
5.1.9 results in five different values of the shear-rate, ranging from 1200 s\(^{-1}\) to 19200 s\(^{-1}\). Figure 5.1.10 shows the effect on the transient permeate flux as the shear-rate is varied over this range. Small particles demonstrate the same behavior in permeate flux at the same shear rate, regardless of the individual values of the cross-flow velocity and the membrane radius. However, the intermediate and large size particles show variation in the permeate flux even at the same shear rate, depending on the value of velocity and membrane radius. For these particles, at the same shear rate, a narrower channel with a lower cross-flow velocity results in a higher steady-state permeate flux, but a lower transient permeate flux. This behavior can be explained from the earlier discussion on the effects of membrane radius on the permeate flux (section 5.1.4.3).

5.1.4.5. Hollow-fiber Length

The steady-state permeate flux decreases with increasing channel length, while the effect on the transient permeate flux is seen to be minimal (Figure 5.1.11). In this case the smallest particles are seen to be the most sensitive to changes in the hollow fiber length, followed by the largest particles. The intermediate or unfavorable sized particles show almost insignificant improvement in permeate flux with shorter lengths.
Figure 5.1.10. Effect of shear-rate on the length-averaged permeate flux for an inside-out hollow fiber filter.
**Figure 5.1.11.** Effect of membrane length on the length-averaged permeate flux for an inside-out hollow fiber filter.

### 5.1.5. Comparison Between Constant Pressure and Constant Flux Operational Modes

Membrane systems are typically operated either in a constant pressure or constant permeate flux mode of operation. It is interesting to compare the permeate flux behavior of each mode as a function of particle size representing different source waters. Constant permeate flux simulations were performed by applying the constant pressure model developed in this work in an iterative fashion. The iterations were performed at each time step until the newly calculated transmembrane pressure resulted in recovery of the initial permeate flux.
Theoretical comparisons between the constant flux and constant pressure modes of operation are made in Figure 5.1.12 by utilizing the specific permeate flux as a measure of performance. For all the three representative particle sizes the constant pressure mode is predicted to yield a higher specific permeate flux.

Figure 5.1.12. Comparison between constant pressure and constant flux modes of operation for various particle sizes.

The difference is hardly discernible for the large particles. However, the intermediate and small sized particles, which were earlier shown to demonstrate mass-transfer limited (or pressure independent) behavior at high pressures (section 5.1.4.1), demonstrate significant differences in the specific
permeate fluxes. For these particles, the constant pressure mode of operation is predicted to yield a much higher specific permeate flux, particularly at large times.

5.1.6. Comparison With Experimental Data

In this section comparisons of model predictions are made with available experimental data. An ideal comparison of the model formulated in this study with would require data from controlled experiments conducted with monodisperse particle suspensions. However, data on permeate flux decline from a previous study (Chellam, 1995) were available only for polydisperse suspensions. Nevertheless, for the purposes of comparing model results with the available data and gaining insight on the rheology of cakes formed during filtration of polydisperse suspensions, comparisons were made with the data available. Comparison between theory and observation was first made by arbitrarily prescribing a solidosity of $\phi_c = 0.58$ to estimate the specific resistance of the cake layer. Cake solidosity was later prescribed as a fitting parameter, and non-linear least-square fits were performed to predict the cake solidosity which minimized the deviation between theoretical predictions and observed data.

In making the comparisons it is also noted that the data are limited to polydisperse suspensions of supra-micron particles and hence the role of Brownian diffusion as a transport mechanism can be assumed to be negligible. The fact that all particles were smaller than about 0.7 $\mu$m, combined with system parameters employed, also indicates that inertial lift will not be the dominant transport mechanism. Thus, for these suspensions the most dominant
mechanism of particle transport is thought to be shear-induced diffusion, and as such the model predictions presented in this section should closely represent the predictions of the shear-induced diffusion model (Romero and Davis, 1990).

Chellam (1995) conducted experiments in crossflow mode using polycarbonate track-etch (PTCE) membranes with a pore diameter of 0.1 μm. The experiments were conducted in a rectangular channel with one porous wall comprised of the PTCE membrane. A suspension of silica particles ($\rho_p = 2.6$ g/cm$^3$) with a relatively narrow particle size distribution was employed to approximate ideal (monodisperse) conditions. Particle size ranged from about 0.12 μm to 0.7 μm, hence all particles were larger than the membrane pore. This implies perfect rejection and minimization in pore blockage which might cause changes in membrane permeability. The three runs conducted with the silica suspensions are summarized in Table 5.1.2.

Comparison with these experiments is complicated due to the fact that the transmembrane pressure varied with time. Operation under variable transmembrane pressure and the one-walled geometry are accounted for by appropriate modifications to the theory, which are discussed in Appendix A. The time-dependent trans-membrane pressure was reported at the center of the channel and this value is used as the instantaneous pressure in the theoretical simulations. The polydispersivity was accommodated in an approximate fashion by using the volume-averaged diameter as the characteristic size. Results from these comparisons are presented next.
### Table 5.1.2. Summary of experiments conducted by Chellam (1995) with silica suspensions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expt. S1</th>
<th>Expt. S2</th>
<th>Expt. S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed suspension concentration (g/l)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial transmembrane pressure kPa (psig)</td>
<td>12.21 (1.77)</td>
<td>21.10 (3.06)</td>
<td>17.66 (2.56)</td>
</tr>
<tr>
<td>Final transmembrane pressure kPa (psig)</td>
<td>35.86 (5.20)</td>
<td>46.07 (6.68)</td>
<td>77.93 (11.30)</td>
</tr>
<tr>
<td>Time-averaged transmembrane pressure kPa (psig)</td>
<td>25.03 (3.63)</td>
<td>37.86 (5.49)</td>
<td>62.41 (9.05)</td>
</tr>
<tr>
<td>Channel half-height (mm)</td>
<td>0.381</td>
<td>0.175</td>
<td>0.381</td>
</tr>
<tr>
<td>Average cross-flow velocity at inlet (cm/s)</td>
<td>14.7</td>
<td>32.0</td>
<td>14.7</td>
</tr>
<tr>
<td>Shear rate (s(^{-1}))</td>
<td>1157</td>
<td>5486</td>
<td>1157</td>
</tr>
</tbody>
</table>

### 5.1.6.1. Results from Comparisons

An important parameter in the permeation theory is the specific resistance of the cake, which is in turn depends on the size of the particles comprising the cake and the porosity (or solidosity) of the cake. Rigid, monodisperse, spherical particles are expected to form cakes with a maximum solidosity of about 0.58 (Leighton and Acrivos, 1987b). However, polydispersivity increases the solidosity of the cake due to the percolation of fines. For cakes initially composed of particles packed at less than the maximum packing density, the solidosity can also increase with transmembrane pressure as the deposits rearrange. Hence, polydispersivity and increases in transmembrane pressure are expected to result in cakes with a solidosity higher than the value estimated for monodisperse spheres.

When a solidosity of \( \phi_c = 0.58 \) is assumed for the cake formed by the silica suspensions, the Kozeny equation (Eq. 2.4.4) predicts a specific resistance of \( 5.19 \times 10^{11} \) cm/g. As shown by the dotted lines in Figure 5.1.13, this value
results in over-prediction of the normalized specific permeate flux. However, the shape of the predicted profile for the permeate flux is seen to be in reasonable agreement with the observed data. This implies that the specific resistance employed is too low and its scale might need to be adjusted. To estimate the specific resistance of the silica feed suspensions employed, Chellam (1995) conducted experiments in the dead-end filtration mode. The specific resistance estimated form this procedure was $1.14 \times 10^{12}$ cm/g. This value is higher than the one estimated using the Kozeny equation with a solidosity of $\phi_c = 0.58$ and the volume averaged diameter. Using this alpha improves agreement for runs S1 and S2, however the over-prediction is still significant for run S3, implying that the specific resistance in cross-flow mode is higher than that observed in the dead-end filtration mode. Amongst other possible factors, higher transmembrane pressure in run S3 is expected to result in a particle size distribution in the cake which is removed or significantly different from the PSD in the feed suspension. This explains why the cake resistance estimated using dead-end filtration of feed suspensions is lower than the cake resistance expected in crossflow filtration.

5.1.6.2. Parameter Estimation of the specific cake resistance

These comparisons indicate that the cake solidosity, and hence the specific cake resistance, is probably higher than the assumed value of $\phi_c = 0.58$ due to the polydispersivity of the suspension, rearrangement of the cake, and/or the variations in the transmembrane pressure/initial permeation rate. A parameter estimation technique was employed to predict the specific cake resistance, and hence the solidosity, which reduces the sum of squares of the residuals between the normalized (with respect to the initial permeate flux) values of the
predicted and observed permeate flux. This minimization can be represented as:

\[
\text{Minimize the residual error } \sum_{t=0}^{t_{\text{end}}} \left[ \left( \frac{J(t)}{J_m} \right)_{\text{model}} - \left( \frac{J(t)}{J_m} \right)_{\text{expt}} \right]^2
\]  

(5.1.4)

Figure 5.1.13. Comparisons between experimental data and theoretical predictions.

where \( t_{\text{end}} \) is the total time over which the experiment was conducted. The optimization runs were conducted using the adaptive non-linear least-squares algorithm NL2SOL (Dennis et al., 1981). In these runs, it was assumed that the empirical expression for the relative viscosity, Eq. 2.4.8, which is ideally
applicable to solidosities up to $\phi_c = 0.58$, can be used for higher values of the solidosity. Table 5.1.3 summarizes the results of the parameter estimation runs. The solidosity is calculated from the Carman-Kozeny equation (Eq. 2.4.4) using the volume average diameter as the characteristic size and the predicted value of the specific cake resistance, $\alpha$. The dashed lines in Figure 5.1.13 represent the flux predictions made using the predicted values of the solidosity. The agreement with the observed permeate flux profile is seen to be quite good. Similar values of the predicted solidosity for the S1 and S2 runs (Table 5.1.3), which were conducted at different shear-rates, imply that shear rate is not expected to have a significant effect on the cake solidosity. However, significantly different values of the solidosity were predicted for runs S1 and S3. Experiment S3 was conducted at a much higher initial flux and average transmembrane pressure than S1. Hence, predicted cake solidosities from the parameter estimation runs imply that a higher trans-membrane pressure may cause rearrangement of the cake to result in a more compact packing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run S1</th>
<th>Run S2</th>
<th>Run S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal specific cake resistance, $\alpha$ (cm/g)</td>
<td>$1.64 \times 10^{12}$</td>
<td>$1.86 \times 10^{12}$</td>
<td>$4.02 \times 10^{12}$</td>
</tr>
<tr>
<td>Optimal cake solidosity, $\phi_c$ (-)</td>
<td>0.70</td>
<td>0.71</td>
<td>0.77</td>
</tr>
<tr>
<td>Residual error (Eq. 5.1.1)</td>
<td>0.63</td>
<td>0.24</td>
<td>0.08</td>
</tr>
</tbody>
</table>
5.2. Cost Analysis

The effect on cost of a membrane system operated with intermittent backflush and wastage was investigated by simulating the behavior of the transient permeate flux and the associated cost parameters over the domain of raw water quality at three plant capacities. Results from these simulations are presented in section 5.2.1. The continuously operated feed and bleed mode was also investigated assuming conditions of steady-state permeate flux, and results pertaining to simulations performed for this operational mode are presented in section 5.2.2. Behavior of the treatment costs for the feed and bleed operational mode over the raw water quality domain are analyzed in section 5.2.2.1. Sensitivity analyses of treatment costs with respect to key design and operating variables are presented in section 5.2.2.2.

The "baseline design configuration" assumed for a 158 m³/hr (1 MGD) low-pressure membrane plant simulated in this section is given in Table 5.2.1. In addition to the raw water quality, other parameters that vary in the sensitivity analyses performed include the plant design capacity, membrane radius, membrane length, trans-membrane pressure, crossflow velocity and system recovery.

The backflush frequency for the intermittently operated semi-batch mode is calculated as a function of the system recovery and the permeate flux (Eq. 4.2.1). Fouling rates, and hence the permeate flux, vary as a function of the raw water quality, and thus, for a given recovery and design flow, backflush frequency and membrane area will vary to meet the design parameters (Eqs.
3.4.3 and 4.2.1). For the continuously operated feed and bleed mode, where the hydrodynamic cleaning cycles are relatively infrequent (Jacangelo et al., 1994), a small backflush frequency of four times per day was assumed for the purposes of estimating the energy invested in flashflushing and backflushing.

Table 5.2.1. System parameters representing the "baseline design configuration" in the sensitivity analyses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane radius (mm)</td>
<td>0.5</td>
</tr>
<tr>
<td>Membrane length (cm)</td>
<td>105</td>
</tr>
<tr>
<td>Trans-membrane pressure (kPa)</td>
<td>100</td>
</tr>
<tr>
<td>Crossflow velocity (cm/s)</td>
<td>60</td>
</tr>
<tr>
<td>System Recovery (%)</td>
<td>90</td>
</tr>
<tr>
<td>Plant design capacity (m³/hr)</td>
<td>157.73</td>
</tr>
<tr>
<td>Plant design life (years)</td>
<td>20</td>
</tr>
<tr>
<td>Membrane MWCO (Daltons)</td>
<td>100,000</td>
</tr>
<tr>
<td>Membrane cost ($/m²)</td>
<td>100</td>
</tr>
<tr>
<td>Membrane resistance (1/cm)</td>
<td>1.46 X 10⁹</td>
</tr>
<tr>
<td>Membrane life (years)</td>
<td>5</td>
</tr>
<tr>
<td>Backflush duration (s)</td>
<td>45</td>
</tr>
<tr>
<td>Backflush flux (L/m²/hr)</td>
<td>384</td>
</tr>
<tr>
<td>Backflush pressure (kPa)</td>
<td>200</td>
</tr>
<tr>
<td>Fastflush duration</td>
<td>15</td>
</tr>
<tr>
<td>Fastflush velocity (cm/s)</td>
<td>113</td>
</tr>
<tr>
<td>Energy Cost ($/kwh)</td>
<td>0.07</td>
</tr>
<tr>
<td>Cost of capital (%)</td>
<td>10</td>
</tr>
<tr>
<td>Earning interest rate (%)</td>
<td>8</td>
</tr>
<tr>
<td>Efficiency of pumps (%)</td>
<td>70</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>20</td>
</tr>
</tbody>
</table>
5.2.1. Behavior of treatment costs with raw water quality and plant capacity for intermittent semi-batch operation

Treatment costs for the baseline design configuration are presented as a function of particle size and concentration in Figure 5.2.1 for a 158 m³/hr (1 MGD) plant. Treatment costs are significantly affected, and are essentially a mirror image of the permeate flux behavior (Figure 5.2.2) with the raw water quality. Very small and large particles which are characterized by high net back-transport and/or relatively porous cakes demonstrate high permeate fluxes, which translate into low treatment costs. Unfavorable particles in the intermediate size range where back-transport is at a minimum yield low permeate fluxes and are hence characterized by high total treatment costs.

![Figure 5.2.1](image)

**Figure 5.2.1.** Behavior of treatment costs with particle size and concentration for the intermittently operated semi-batch configuration. Plant capacity = 157.73 m³/hr (1 MGD).
Figure 5.2.2. Behavior of time-averaged permeate flux with particle size and concentration for the intermittently operated semi-batch configuration. Plant capacity = $157.73 \text{ m}^3/\text{hr}$ (1 MGD).

The behavior of treatment costs with plant capacity is illustrated in Figure 5.2.3 at a feed suspension concentration of 200 mg/L. The decrease in total costs with plant capacity is due to the economies of scale exhibited by the non-membrane capital costs. Operating costs and membrane costs per unit volume of permeate produced are assumed to remain constant with increasing capacity.
Figure 5.2.3. Behavior of treatment costs with plant capacity and particle size for the intermittently operated semi-batch configuration.

5.2.2. Treatment costs for steady-state feed and bleed operation

5.2.2.1. Behavior of treatment costs with raw water quality

As in the case of the intermittent semi-batch operation, treatment costs for the steady-state feed and bleed operation (Figure 5.2.4, Table 5.2.2) are also a mirror image of the permeate flux behavior with particle size and concentration (Figure 5.2.5). Since the system is operated and modeled at steady-state, the treatment costs are higher for the steady-state operation (Figure 5.2.4) when compared to the intermittent semi-batch operation (Figure 5.2.1). This is a direct consequence of the lower permeate fluxes at steady-state (Figure 5.2.5), compared to the time-averaged permeate fluxes for the intermittent transient operation (Figure 5.2.2).
Figure 5.2.4. Behavior of treatment costs with particle size and concentration for the continuously operated feed and bleed configuration. Plant capacity = 157.73 m$^3$/hr (1 MGD).

Table 5.2.2. Treatment costs (in $/m^3$) for various particle sizes and concentrations, for a 155.73 m$^3$/hr (1 MGD) plant.

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.072641</td>
<td>0.072641</td>
<td>0.072641</td>
<td>0.072641</td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>0.094215</td>
<td>0.108600</td>
<td>0.126530</td>
<td>0.148850</td>
<td></td>
</tr>
<tr>
<td>0.010</td>
<td>0.140440</td>
<td>0.165050</td>
<td>0.195410</td>
<td>0.233380</td>
<td></td>
</tr>
<tr>
<td>0.050</td>
<td>0.296110</td>
<td>0.352570</td>
<td>0.422200</td>
<td>0.509370</td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.378670</td>
<td>0.439420</td>
<td>0.506110</td>
<td>0.576880</td>
<td></td>
</tr>
<tr>
<td>0.500</td>
<td>0.107300</td>
<td>0.110970</td>
<td>0.114600</td>
<td>0.118160</td>
<td></td>
</tr>
<tr>
<td>1.000</td>
<td>0.079252</td>
<td>0.080448</td>
<td>0.081625</td>
<td>0.082767</td>
<td></td>
</tr>
<tr>
<td>2.000</td>
<td>0.073173</td>
<td>0.073469</td>
<td>0.073780</td>
<td>0.074092</td>
<td></td>
</tr>
<tr>
<td>5.000</td>
<td>0.072641</td>
<td>0.072641</td>
<td>0.072641</td>
<td>0.072643</td>
<td></td>
</tr>
<tr>
<td>10.000</td>
<td>0.072641</td>
<td>0.072641</td>
<td>0.072641</td>
<td>0.072641</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.2.5. Behavior of steady-state permeate flux with particle size and concentration for the continuously operated feed and bleed configuration. Plant capacity = 157.73 m³/hr (1 MGD).

5.2.2.2. Sensitivity Analyses

Sensitivity analyses were performed to study the effects of key design and operating variables on treatment costs. All simulations were based on a feed suspension concentration of 200 mg/L, and are presented here for three representative particle sizes (small particle: 0.01 μm; intermediate/unfavorable sized particle: 0.1 μm; and large particle: 1.0 μm). In each case, all parameters were kept fixed at their baseline values listed in Table 5.2.1, with the exception of the sensitivity parameter.

Membrane treatment costs are dependent on a number of design and operating variables. The interpretations and conclusions made in the following one-dimensional sensitivity analyses, performed with respect to one variable at
a time, are to an extent specific to the baseline values of the remaining variables. Nevertheless, these analyses help understand the general trends in treatment costs with variations in individual design variables. Further, such an understanding is also helpful in interpreting the results from a more complex and integrated multi-dimensional optimization analysis, such as the one performed in section 5.3.1.

5.2.2.2.1. Plant Capacity

Total treatment costs per unit volume of permeate produced are plotted for a range of plant capacities from 1.5773 m³/hr (0.01 MGD) to 15773 m³/hr (100 MGD) in Figure 5.2.6a.

![Graph showing total cost vs plant design capacity](image)

**Figure 5.2.6a.** Behavior of unit treatment costs with plant capacity for the continuously operated feed and bleed configuration.
Treatment costs for all the three particle sizes demonstrate a significant decrease with plant capacity due to the economies of scale exhibited by the non-membrane capital costs. At higher capacities the incremental cost of building the plant decreases rapidly. The costs of membranes and energy invested per unit volume of permeate produced remain constant with capacity. Costs are highest at all capacities for the intermediate sized particles which are associated with low permeate fluxes.

Total treatment costs in dollars are plotted against plant capacity in Figure 5.2.6b to illustrate the overall economies of scale associated with different particle sizes. The different exponents from the power law curve fits performed in Figure 5.2.6b result from the variable mixes of membranes and ancillary hardware associated with different particle sizes. The economies of scale are

![Graph showing treatment costs versus plant design capacity](image)

**Figure 5.2.6b.** Behavior of treatment costs with plant capacity for the continuously operated feed and bleed configuration. Overall economies of scale associated with different particle sizes are illustrated.
lowest for the unfavorable 0.1 μm particle due to the large membrane area required to produce the design flow, and the fact that membrane costs are assumed to demonstrate no economies of scale.

5.2.2.2.2. Fiber Radius

The size of the hollow-fiber radius effects the shear-rate and consequently the permeate flux (section 5.1.4.3). Since the shear-rate increases for narrow fibers, the treatment costs decrease as the fiber radius is decreased (Figure 5.2.7). For the membrane system simulated, the extra costs associated with a larger pressure drop for narrow fibers are relatively small compared to the savings in capital costs due to higher permeate fluxes. Permeate flux directly affects the membrane area required to meet a specified design capacity, which also indirectly affects the size of all associated ancillary equipment and facilities.

All particles demonstrate significant sensitivity to the fiber radius, with the unfavorable 0.1 μm sized particle showing the most dramatic effect. For the 1.0 μm particle, which results in the formation of thick cakes (section 5.1.1.1), the Reynolds number increases due to higher effective crossflow velocities in constricted channels. Simulations for the 1.0 μm particle were not performed at high fiber diameters since the constraint prescribed on the Reynolds number (section 4.3.4) was violated. The constraint on flow continuity or a feasible configuration (Eq. 4.3.3.4) effectively sets a lower limit on the fiber radius. To yield the very high permeate fluxes associated with the 1.0 μm particle the channel dimension has to be such that an adequate amount of total crossflow
can be accommodated. Thus, for the 1.0 \( \mu \text{m} \) particle a very narrow fiber radius of 0.125 mm results in an infeasible configuration and was also not simulated.

![Graph showing total cost vs. hollow fiber radius](image)

**Figure 5.2.7.** Behavior of treatment costs with hollow fiber radius for the continuously operated feed and bleed configuration.

5.2.2.2.3. Fiber Length

The behavior of treatment costs with hollow fiber length is shown in Figure 5.2.8. An increased fiber length adversely affects the permeate flux (section 5..1.4.5) and results in higher treatment costs. The 0.1 \( \mu \text{m} \) particle demonstrates the highest sensitivity to changes in fiber length while the 1.0 \( \mu \text{m} \) particle is largely independent of the fiber length.
Figure 5.2.8. Behavior of treatment costs with hollow fiber length for the continuously operated feed and bleed configuration.

5.2.2.2.4. Trans-membrane Pressure

As in the case of fiber radius and length, the 0.1 μm particle demonstrates the highest sensitivity to changes in the transmembrane pressure (Figure 5.2.9). Costs for the intermediate and large particles decrease at higher transmembrane pressures due to improvements in the permeate flux. The increased energy costs associated with pumping at a higher transmembrane pressure are more than offset by savings in the capital costs due to a reduction in the membrane area and the associated equipment. The 0.01 μm particle demonstrates a slight increase in treatment costs with increased transmembrane pressures. As discussed in section 5.1.4.1, for a 0.01 μm particle the permeate flux becomes mass transport limited. For the system configuration simulated, the steady-state permeate flux associated with the 0.01
μm particle actually demonstrated a slight decrease with an increased transmembrane pressure.

![Graph](image)

**Figure 5.2.9.** Behavior of treatment costs with trans-membrane pressure for the continuously operated feed and bleed configuration.

5.2.2.2.5. Cross-flow Velocity

Operating at higher crossflow velocities results in savings in capital costs due to improvements in the permeate flux. However, at high crossflow velocities the energy costs increase significantly. Hence, total costs are minimized at intermediate values of the crossflow velocity. This is true for the total cost curves corresponding to the 0.1 and 0.01 μm particles in Figure 5.2.10. The 1.0 μm particle demonstrates a low sensitivity to changes in the crossflow velocity. However, for this particle, the total costs increase monotonically with cross-flow velocity, indicating that incremental operating
costs are higher than savings in capital costs over the entire range of the velocities analyzed.

![Graph showing the relationship between cross-flow velocity and total cost](image)

**Figure 5.2.10.** Behavior of treatment costs with cross-flow velocity for the continuously operated feed and bleed configuration.

5.2.2.2.6. **System Recovery**

Higher recoveries are associated with increased bulk suspension concentrations which adversely affects the permeate flux of the intermediate and small sized particles, leading to higher treatment costs (Figure 5.2.11). The 1.0 μm particle demonstrates a slight decrease in treatment costs at higher recoveries due to savings associated with pumping a smaller feed flow, which are predicted to be more than the increase in capital costs with a slightly smaller permeate flux.
Figure 5.2.11. Behavior of treatment costs with system recovery for the continuously operated feed and bleed configuration.

5.3. Design and Optimization

5.3.1. Optimal Membrane Design and Operating Conditions

Results from optimizations performed on membrane design and operation for a continuously operating feed and bleed system, described in section 4.3.1, are presented in this section. Predicted optimal values of the treatment costs and permeate fluxes are presented over a range of particle sizes and concentrations depicting different source waters. This is followed by discussions on the active constraints and the individual behavior of the decision variables at the optimal solutions. The parameters characterizing the typical UF/MF system assumed in these optimizations are the same as the baseline design configuration adopted in section 5.2. (Table 5.2.1), except for the fiber radius, fiber length, trans-membrane pressure, crossflow velocity and system
recovery, which form the unknown decision variables to be determined in these optimizations.

5.3.1.1. Treatment Costs

The total treatment costs at the predicted optimal design and operation are shown in Figure 5.3.1. These costs are plotted on the same scale as that employed for the treatment costs of the typical 157.73 m³/hr (1 MGD) low-pressure membrane plant (Figure 5.2.4) simulated in section 5.2, referred to as the "base design" configuration, for easy comparison. Total treatment costs are seen to be significantly reduced at the optimal membrane geometry and operation predicted. Numbers for the optimal design costs and the base design costs are compared in Tables 5.3.1 and 5.2.2, respectively. Optimal costs as a percentage of the base design costs are presented in Table 5.3.2. In general, the optimal costs were about 26% to 64% (Table 5.3.2) of the base design costs, depending on the particle size and concentration. The lower value corresponds to particles with low permeate fluxes while the upper value corresponds to particles with high permeate fluxes. Improvements in costs are relatively lower in the case of particles with very high permeate fluxes due to the fact that the treatment costs associated with these particles are quite low even at the base design configuration, thus offering relatively little scope for further reduction in costs.
Figure 5.3.1. Behavior of optimal treatment costs over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant.

Table 5.3.1. Optimal treatment costs (in $/m³) for various particle sizes and concentrations, for a 155.73 m³/hr (1 MGD) plant

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>0.001</td>
<td>0.046741</td>
</tr>
<tr>
<td>0.005</td>
<td>0.053648</td>
</tr>
<tr>
<td>0.010</td>
<td>0.063679</td>
</tr>
<tr>
<td>0.050</td>
<td>0.112111</td>
</tr>
<tr>
<td>0.100</td>
<td>0.119077</td>
</tr>
<tr>
<td>0.500</td>
<td>0.051485</td>
</tr>
<tr>
<td>1.000</td>
<td>0.047910</td>
</tr>
<tr>
<td>2.000</td>
<td>0.047082</td>
</tr>
<tr>
<td>5.000</td>
<td>0.046770</td>
</tr>
<tr>
<td>10.000</td>
<td>0.046753</td>
</tr>
</tbody>
</table>
Table 5.3.2. Optimal treatment costs (Table 5.3.1.) as a percentage of treatment costs at the base design configuration (Table 5.2.2)

<table>
<thead>
<tr>
<th>Particle Size (μm)</th>
<th>Concentration (mg/L)</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
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</tr>
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</tr>
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<td>64.36</td>
<td>64.37</td>
<td>64.37</td>
<td>64.37</td>
</tr>
</tbody>
</table>

5.3.1.2. Permeate Flux

Significantly lower values of the optimal treatment costs are primarily due to the improvements in the permeate fluxes at the optimal design. Figures 5.3.2 and 5.3.3, and the associated tables (5.3.3 and 5.3.4) depict the behavior of the permeate fluxes with raw water quality at the optimal and base designs, respectively. At the optimal design, permeate fluxes are more than two to seven times greater than the base design fluxes (Table 5.3.5), depending on the raw water quality.
Figure 5.3.2. Behavior of optimal permeate fluxes over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant.

Table 5.3.3. Optimal permeate fluxes (in cm/s) for various particles and concentrations

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td></td>
<td>0.017032</td>
<td>0.017026</td>
<td>0.017030</td>
<td>0.017031</td>
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<tr>
<td>0.005</td>
<td></td>
<td>0.014944</td>
<td>0.013287</td>
<td>0.010981</td>
<td>0.008654</td>
</tr>
<tr>
<td>0.010</td>
<td></td>
<td>0.010234</td>
<td>0.008498</td>
<td>0.006713</td>
<td>0.005282</td>
</tr>
<tr>
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<td></td>
<td>0.003732</td>
<td>0.002949</td>
<td>0.002327</td>
<td>0.001832</td>
</tr>
<tr>
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<td></td>
<td>0.003827</td>
<td>0.003340</td>
<td>0.002959</td>
<td>0.001890</td>
</tr>
<tr>
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<td></td>
<td>0.014872</td>
<td>0.014319</td>
<td>0.014316</td>
<td>0.013935</td>
</tr>
<tr>
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<td>0.016142</td>
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<tr>
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<td>0.017028</td>
<td>0.017026</td>
<td>0.017025</td>
</tr>
</tbody>
</table>
Figure 5.3.3. Behavior of base design permeate fluxes over the raw water quality domain for a 155.73 m$^3$/hr (1 MGD) plant.

Table 5.3.4. Base design permeate fluxes (in cm/s) for various particles and concentrations

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>0.001</td>
<td>0.006813</td>
</tr>
<tr>
<td>0.005</td>
<td>0.004489</td>
</tr>
<tr>
<td>0.010</td>
<td>0.002432</td>
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<tr>
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<td>0.000836</td>
</tr>
<tr>
<td>0.100</td>
<td>0.000599</td>
</tr>
<tr>
<td>0.500</td>
<td>0.003664</td>
</tr>
<tr>
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</tr>
<tr>
<td>2.000</td>
<td>0.006732</td>
</tr>
<tr>
<td>5.000</td>
<td>0.006813</td>
</tr>
<tr>
<td>10.000</td>
<td>0.006813</td>
</tr>
</tbody>
</table>
Table 5.3.5. Optimal permeate fluxes (Table 5.3.3) as a percentage of permeate fluxes at the base design configuration (Table 5.3.4)

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>0.001</td>
<td>250</td>
</tr>
<tr>
<td>0.005</td>
<td>333</td>
</tr>
<tr>
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<td>421</td>
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<tr>
<td>0.050</td>
<td>446</td>
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<tr>
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<td>639</td>
</tr>
<tr>
<td>0.500</td>
<td>406</td>
</tr>
<tr>
<td>1.000</td>
<td>281</td>
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<tr>
<td>2.000</td>
<td>251</td>
</tr>
<tr>
<td>5.000</td>
<td>250</td>
</tr>
<tr>
<td>10.000</td>
<td>250</td>
</tr>
</tbody>
</table>

5.3.1.3. Active Constraints

In all optimizations, one or more of the constraints prescribed (section 4.3.1.3) was found to be active. This indicates that the feasible region for the optimal design and operation was always constrained. The constraints which were active at the converged solution are summarized in Table 5.3.6. The constraint on the Reynolds number (Eq. 4.3.5), prescribed to enforce laminar flow, always remained inactive. The Reynolds numbers in these optimizations tended to be low due to the fact that the optimal fiber radius tended to be small (section 5.3.1.4). An interpretation of this result is that the optimal design for hollow fiber membranes is predicted to favor the laminar flow region, under conditions typical of hollow-fiber membrane plant design.
Table 5.3.6. Active constraints at the optimal solution. Feasible configuration is determined by Eq. 4.3.4. The maximum pressure drop (Eq. 4.3.3) prescribed was 50 kPa

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>0.001</td>
<td>Configuration</td>
</tr>
<tr>
<td>0.005</td>
<td>Configuration</td>
</tr>
<tr>
<td>0.010</td>
<td>Pressure Drop</td>
</tr>
<tr>
<td>0.050</td>
<td>Pressure Drop</td>
</tr>
<tr>
<td>0.100</td>
<td>Pressure Drop</td>
</tr>
<tr>
<td>0.500</td>
<td>Configuration</td>
</tr>
<tr>
<td>1.000</td>
<td>Configuration</td>
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<tr>
<td>2.000</td>
<td>Configuration</td>
</tr>
<tr>
<td>5.000</td>
<td>Configuration</td>
</tr>
<tr>
<td>10.000</td>
<td>Configuration</td>
</tr>
</tbody>
</table>

The optimal design and operation associated with particles which demonstrate relatively high permeate fluxes (Figure 5.3.2) is predicted to be constrained by the flow continuity or feasible configuration constraint (Eq. 4.3.4), while the particles which demonstrate lower permeate fluxes are constrained by considerations of pressure drop across the membrane module. At particle sizes of 0.005 µm and 0.5 µm, where the transition from constrained pressure to constrained configuration occurs, both the constraints are seen to be active for some concentrations.
5.3.1.4. Fiber Radius

A narrow hollow-fiber offers the advantages of a higher shear rate which can significantly improve the steady-state permeate flux (section 5.1.4.3). However, costs of pumping the feed increase as the fiber radius decreases due to an increased pressure drop. Costs associated with recirculation are also expected to increase with a narrower channel if the increase in pressure drop becomes more than the decrease in recycle flow. The behavior of the optimal fiber radius as a function of water quality is presented in Figure 5.3.4 and Table 5.3.7.

Figure 5.3.4. Behavior of optimal fiber radius over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant (lower bound = 0.125 mm, upper bound = 1.5 mm).
Table 5.3.7. Optimal fiber radius (in mm) for various particles and concentrations (lower bound = 0.125 mm, upper bound = 1.5 mm)

<table>
<thead>
<tr>
<th>Particle Size (μm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>0.001</td>
<td>0.6159</td>
</tr>
<tr>
<td>0.005</td>
<td>0.1262</td>
</tr>
<tr>
<td>0.010</td>
<td>0.1250</td>
</tr>
<tr>
<td>0.050</td>
<td>0.1250</td>
</tr>
<tr>
<td>0.100</td>
<td>0.1250</td>
</tr>
<tr>
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<td>0.1250</td>
</tr>
<tr>
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<td>0.1250</td>
</tr>
<tr>
<td>2.000</td>
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<td>0.2557</td>
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<tr>
<td>10.000</td>
<td>0.3508</td>
</tr>
</tbody>
</table>

For most particle sizes and concentrations, the optimal fiber radius was predicted to be at the prescribed lower bound, 0.125 mm. The increased costs of pumping the feed and recycle flows at smaller diameters are predicted to be over-shadowed by the savings in costs due to the increased permeate flux. The optimal fiber radius is higher for particles with very high permeate fluxes (0.001 μm, 2 μm, 5 μm and 10 μm) due to the feasible configuration constraint (Eq. 4.3.3.4). This constraint sets a lower limit on the fiber radius. To yield the very high permeate fluxes associated with these particles the flow channel dimension has to be such that an adequate amount of total crossflow can be accommodated. As the permeate flux decreases with increasing concentrations, the minimum allowable radius also decreases (Table 5.3.7).
5.3.1.5. Fiber Length

Shorter fiber lengths are predicted to improve permeate fluxes (section 5.1.4.5) and hence lead to lower treatment costs. For most particle sizes and concentrations, the optimal fiber length was predicted to be at the prescribed lower bound, 20 cm (Figure 5.3.5 and Table 5.3.8). As in the case of the fiber radius, longer fiber lengths may be required for particles with very high permeate fluxes to meet the feasible configuration constraint. The hump in the optimal surface for a 0.5 µm particle size at higher concentrations is an interesting manifestation of the interplay between the fiber length and crossflow velocity (section 5.3.1.7) in a region where both pressure drop and flow continuity considerations become important (Table 5.3.6). Increasing the fiber length may decrease recirculation costs by bringing down the recycle flow. However, the permeate flux will be adversely affected by a longer fiber length. Hence, a situation, as depicted in Figures 5.3.5 and 5.3.7 might arise, where the crossflow velocity increases (Figure 5.3.7) to compensate for the adverse effects of an increased fiber length on the permeate flux. At low concentrations in this particle size range, the fiber length is low since the savings in recirculation due to a longer length, after adjusting for a decreased permeate flux, are lower than the savings in energy, due to a smaller cross flow velocity (Figure 5.3.7), after adjusting for a smaller permeate flux. Hence, to summarize, for the 0.5 µm particle (Table 5.3.8), shorter lengths and smaller crossflow velocities are optimal at low concentrations and longer lengths and higher crossflow velocities are optimal at high concentrations.
Figure 5.3.5. Behavior of optimal fiber length over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant (lower bound = 20 cm, upper bound = 200 cm).

Table 5.3.8. Optimal fiber length (in cm) for various particles and concentrations (lower bound = 20 cm, upper bound = 200 cm)

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
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<td>37.51</td>
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<td>21.58</td>
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<td>20.00</td>
<td>20.00</td>
</tr>
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<td>20.00</td>
<td>20.00</td>
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<td>20.00</td>
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<tr>
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<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
</tr>
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</tr>
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<td>20.00</td>
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<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
</tr>
</tbody>
</table>
5.3.1.6. Trans-membrane Pressure

Optimal behavior of the trans-membrane pressure is presented in Figure 5.3.6 and Table 5.3.9. Very small particles and particles larger than about 0.1 μm demonstrate an optimal pressure corresponding to the prescribed upper bound (250 kPa). For these particles the permeate flux improves considerably with pressure and increases in energy costs are more than compensated with savings in capital costs. However, for small particles and particles in the intermediate size range, the limiting trans-membrane pressure is determined by a significant increase in operating costs. Improvements in permeate flux with trans-membrane pressure are not very significant for these particles due to mass transport limitations (section 5.1.4.1).

**Figure 5.3.6.** Behavior of optimal trans-membrane pressure over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant (lower bound = 25 kPa, upper bound = 250 kPa).
Table 5.3.9. Optimal trans-membrane pressure (in kPa) for various particles and concentrations (lower bound = 25 kPa, upper bound = 250 kPa)

<table>
<thead>
<tr>
<th>Particle Size (μm)</th>
<th>Concentration (mg/L)</th>
<th>25</th>
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<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
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<td>250.00</td>
<td>250.00</td>
<td>250.00</td>
</tr>
</tbody>
</table>

5.3.1.7. Crossflow Velocity

Figure 5.3.7 and Table 5.3.10 depict the optimal behavior of the crossflow velocity. Optimal values of the velocity are determined by the constraints prescribed on the system and a significant interplay between capital and operating costs. For all particles, higher crossflow velocities increase the permeate fluxes as well as the energy costs, and hence an intermediate value exists which minimizes the total treatment costs. The optimal velocity is higher for particles with low permeate fluxes, while the very small and large particles demonstrate comparatively lower optimal velocities. This is due to the fact that improvements in permeate flux, and hence capital costs, with crossflow velocity are more critical for the unfavorable particles compared to the favorable particles. In the region where the pressure drop is not constrained (Table 5.3.6), velocity increases with concentration to maintain high permeate fluxes.
(Table 5.3.10). However, in the region where pressure drop is constrained, the velocity demonstrates a diminishing trend with concentration. Low crossflow velocities for particles characterized by high permeate fluxes suggest that the benefits of the crossflow mode of operation, as compared with the dead-end mode, are expected to be small for these particles, specially at low concentrations.

![Optimal Cross-flow Velocity Graph](image)

**Figure 5.3.7.** Behavior of optimal cross-flow velocity over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant (lower bound = 10 cm/s, upper bound = 200 cm/s).
### Table 5.3.10.
Optimal cross-flow velocity (cm/s) for various particles and concentrations (lower bound = 10 cm/s, upper bound = 200 cm/s)

<table>
<thead>
<tr>
<th>Particle Size (μm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
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<tr>
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<td>72.06</td>
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<td>66.23</td>
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<td>54.71</td>
</tr>
<tr>
<td>0.100</td>
<td>54.87</td>
</tr>
<tr>
<td>0.500</td>
<td>54.06</td>
</tr>
<tr>
<td>1.000</td>
<td>56.24</td>
</tr>
<tr>
<td>2.000</td>
<td>42.85</td>
</tr>
<tr>
<td>5.000</td>
<td>26.89</td>
</tr>
<tr>
<td>10.000</td>
<td>19.62</td>
</tr>
</tbody>
</table>

### 5.3.1.8. System Recovery

Optimal recoveries are obtained at the lower bound (55%) for particles in the low permeate flux region (Figure 5.3.8, Table 5.3.11). This occurs due to the fact that operating at a high system recovery causes an increase in the bulk suspension concentration which adversely affects the permeate flux of the intermediate and small sized particles, leading to higher capital costs. However, for large and very small particles, which demonstrate high permeate fluxes regardless of the concentration, operating at higher recoveries reduces the operating costs of pumping the feed. Hence, the optimal recovery increases as particle size becomes favorable and the permeate flux demonstrates decreasing sensitivity to changes in concentration.
Figure 5.3.8. Behavior of optimal system recovery over the raw water quality domain for a 155.73 m$^3$/hr (1 MGD) plant assuming a continuous feed and bleed operation (lower bound = 55 %, upper bound = 99 %).

The optimal recoveries for particles characterized by low permeate fluxes are at the lower bound in these optimizations due to the steady-state nature of the operation which occurs at a high final concentration. Results from optimization of the backflushing frequency (and hence the system recovery) in the case of the intermittent semi-batch mode (section 5.3.2) indicate that the optimal recovery in that mode would be relatively higher. Nevertheless, the behavior of the optimal recovery with particle size in both the modes of operation is the same (Figures 5.3.8 and 5.3.11), in that optimal recoveries for the unfavorable particles are lower than those associated with the favorable particles.
Table 5.3.11. Optimal system recovery (%) for various particles and concentrations (lower bound = 55 %, upper bound = 99 %)

<table>
<thead>
<tr>
<th>Particle Size (µm)</th>
<th>Concentration (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
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<td>55.00</td>
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<td>55.00</td>
</tr>
<tr>
<td>0.100</td>
<td>55.00</td>
</tr>
<tr>
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</tr>
<tr>
<td>1.000</td>
<td>97.44</td>
</tr>
<tr>
<td>2.000</td>
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<tr>
<td>5.000</td>
<td>99.00</td>
</tr>
<tr>
<td>10.000</td>
<td>99.00</td>
</tr>
</tbody>
</table>

5.3.2. Optimal Backflushing Frequency for Hydrodynamic Cleaning

Optimal backflushing frequencies (and hence the system recovery) in the operation of the intermittent semi-batch configuration, as described in section 4.3.2, were determined by performing simulations over a range of particle sizes, concentrations and plant capacities. The maximum value of recovery explored in these simulations was artificially restricted to a value of 99.50% to maintain reasonable computational times.

5.3.2.1. Performance and Economic Parameters as a Function of Operating Time

The kinetics of permeate flux decline and the inherent relationship between system recovery and operating time (Eq. 4.2.1) in the intermittently operated semi-batch operational mode determine an optimal operating time and recovery
depending on the raw water quality. Parameters of system performance and economics such as membrane area and the various cost components demonstrate a minimum at such an optimal operating time. Figure 5.3.9 shows the behavior of time-averaged permeate flux as a function of the operating time between two backflushing cycles, for various particle sizes. In the case of very small (0.001 μm) and very large (10 μm) particles the permeate flux is relatively high enough at all times, to permit long operating times and recoveries, and the total costs decrease monotonically with operating time (Figure 5.3.10). In other words, since the permeate flux is not a limiting criterion for these particles, the optimal operation is simply such that the amount of clean water employed for backflushing is minimized. However, for particles in the intermediate sized range, the situation is complicated due to the kinetics of the permeate flux decline. For particles that demonstrate a significant decline in permeate flux with time, very long operating times result in higher total costs (Figure 5.3.10). Hence, the optimal point in time to arrest this decline by backflushing becomes an important consideration. Very short operating times result in high costs due to the fact that most of the permeate produced is used up in backflushing and the net system production is low. As illustrated in Figure 5.3.10, this implies an intermediate operating time or backflushing frequency where the total costs are minimized. The relationship between the system recovery and backflushing frequency is illustrated in Figure 5.3.11 for various particle sizes. Results presented in Figures 5.3.9 through 5.3.11 were generated assuming a plant capacity of 157.72 m³/hr (1.0 MGD) and a feed suspension concentration of 100 mg/L. Other parameters employed in these simulations remain the same as the baseline values specified in Table 5.2.1.
Figure 5.3.9. Time-averaged flux as a function of operating time for various particle sizes (feed suspension concentration = 100 mg/L).

5.3.2.2. Optimal Operation as a Function of Raw Water Quality

The behavior of the optimal system recovery as a function of particle size and concentration is presented in Figure 5.3.12 for a plant capacity of 155.73 m³/hr (1 MGD). Optimal recoveries, as a function of raw water quality, for the three different capacities simulated (0.1 MGD, 1 MGD and 10 MGD) are given in Tables 5.3.12 through 5.3.14. At any plant capacity, high optimal recoveries are predicted for very small and large particles which yield a high permeate flux. Intermediate sized particles demonstrate lower optimal recoveries as discussed in section 5.3.2.1. For all particles, the optimal recovery increases with concentration, as the permeate flux decreases.
Figure 5.3.10. Total cost and system recovery as a function of operating time (and backflush frequency) for various representative particle sizes. Plant capacity = 157.73 m$^3$/hr (1 MGD), feed suspension concentration = 100 mg/L.
Figure 5.3.11. Relationship between system recovery and backflush frequency for various particle sizes (feed suspension concentration = 100 mg/L).

Figure 5.3.12. Behavior of optimal system recovery over the raw water quality domain for a 155.73 m³/hr (1 MGD) plant assuming an intermittent semi-batch operation.
Table 5.3.12. Optimal recoveries for various particles and concentrations for a 15.77 m³/hr (0.1 MGD) plant

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The behavior of optimal recoveries with plant capacity is illustrated in Figures 5.3.13 and 5.3.14 for various representative particle sizes and feed suspension concentrations. The investment and operating costs of pumping the feed decrease with an increased recovery. These costs increase in significance, compared to the total treatment costs, with increasing plant capacities due to economies of scale exhibited by the non-membrane equipment. Hence, as the plant capacity increases, optimal recoveries increase in order to keep the pumping costs down.
Table 5.3.13. Optimal recoveries for various particles and concentrations for a 157.7 m³/hr (1.0 MGD) plant

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Figure 5.3.13. Comparison of optimal recovery at different plant capacities. Feed suspension concentration = 25 mg/L.
Table 5.3.14. Optimal recoveries for various particles and concentrations for a 1577 m³/hr (10 MGD) plant

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Figure 5.3.14. Comparison of optimal recovery at different plant capacities. Feed suspension concentration = 200 mg/L.
5.3.3. Optimal Configuration of Nanofiltration Systems

The effects of particle size and concentration on the permeate flux and treatment costs associated with the three unit processes considered (section 4.3.3):

- Hollow fiber nanofiltration (HFNF): continuous steady-state operation
- Hollow fiber ultrafiltration (HFUF): intermittent transient operation
- Spiral wound nanofiltration (SWNF): continuous steady-state operation

were estimated assuming typical values of system parameters listed in Table 5.3.15. Most of the operating variables listed in Table 5.3.15 were chosen from previously reported pilot studies (Wiesner et al., 1994; Jacangelo et al., 1994) conducted using HFNF, SWNF and HFUF systems. The numbers in Table 5.3.15 represent baseline values which remain constant for all simulations, unless mentioned otherwise in the presentation of the results. For the HFUF system, treatment costs were estimated by optimizing the backflushing frequency (and hence the system recovery) in a manner analogous to that discussed in section 4.3.2.

5.3.3.1. Total Treatment Costs

The effect of particle size and concentration on treatment costs are depicted by plotting contours of the total cost per unit volume of permeate produced in $/m^3$ units for the single HFNF system (Figure 5.3.15) and the hybrid system comprised of SWNF pre-treated with HFUF (Figure 5.3.16). The treatment costs
Table 5.3.15. Baseline values of design and operating parameters used in simulations for optimal selection of nanofiltration configuration

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<td>Fastflush velocity (cm/s)</td>
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<td>70</td>
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<td>Temperature (°C)</td>
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<td>20</td>
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</table>

$^*$ Effective diameter for a 0.76 mm thick feed spacer
$^†$ Effective crossflow velocity
$^‡$ Not applicable to spiral-wound system
$^§$ Calculated such that treatment cost is minimized (section 4.3.2)

are highest for particles in the intermediate size range where the permeate flux is minimal due to a minimum in net back-transport away from the membrane. Costs are low and effectively constant for very large and very small particles due to the high permeate fluxes associated with these particles. Costs increase
Figure 5.3.15. Contours of total treatment costs (in $/m^3) for HFNF system.

Figure 5.3.16. Contours of total treatment costs (in $/m^3) for SWNF system pretreated with HFUF.
with concentration as the permeate flux is decreased. For the plant capacity simulated HFNF has lower costs compared to the hybrid SWNF and HFUF system for most of the raw water conditions evaluated.

5.3.3.2. Optimal configuration at baseline plant capacity and HFUF feed pressure

Domains of optimal configurations can be segregated by plotting the cost ratios of the two alternatives in a contour plot. The contour line with a value of unity represents the boundary between the two domains where each configuration is optimal. Figure 5.3.17 shows the contours of the cost ratio $z$:

$$
z = \frac{TC_{HFNF}}{TC_{HFUF} + TC_{SWNF}}$$  \hspace*{1cm} (5.3.3.1)

**Figure 5.3.17.** Contours of cost ratio of single HFNF system to hybrid HFUF and SWNF system.
where $TC = \text{total treatment cost (\$/m}^3)$. The domains where HFNF and SWNF are optimal are given by the regions where $z < 1$ and $z > 1$, respectively. For the system simulated in Figure 5.3.17, the HFNF configuration has higher membrane capital costs when compared to the combined analogous costs of HFUF and SWNF, over most of the particle sizes and concentrations, due to lower permeate fluxes compared to HFUF and SWNF. However, HFNF demonstrates lower non-membrane capital costs compared to the combined non-membrane costs of HFUF and SWNF, at all particle sizes and concentrations. Similarly, the HFNF configuration has higher membrane replacement costs but lower non-membrane operating costs when compared to the combined analogous costs of HFUF and SWNF, at most particle sizes and concentrations. At a plant capacity of 158 m$^3$/hr (1 MGD) and HFUF feed pressure of 150 kPa the non-membrane capital costs were found to comprise the dominant portion (from 53 to 74 percent, depending on the raw water quality and the membrane system) of the total treatment costs. This implies that the HFNF configuration is optimal over most of the raw water quality domain (Figure 5.3.17) predominantly due to the lower non-membrane capital costs when compared to the SWNF option. At a moderate plant capacity of 158 m$^3$/hr (1 MGD), the cost advantages of the cheaper SW membranes are predicted to be over-shadowed by the high non-membrane costs of the hybrid system. Optimal recoveries calculated for the HFUF system simulated in Figure 5.3.17 are presented in Table 5.3.20.
5.3.3.3. Optimal configuration behavior at different capacities and feed pressures

Plant capacity affects the capital costs due to the economies of scale exhibited by manufactured equipment. Increases in feed pressure effect the total treatment in a two fold manner by:

- increasing the feed pump capital and energy costs
- decreasing the treatment costs by improving the permeate flux

UF systems are typically operated at pressures ranging from about 50 kPa to 200 kPa. The effects of variable plant capacity and HFUF feed pressure on the domains of optimal configuration were investigated by performing simulations with different representative values of these parameters. The results are summarized in Figures 5.3.18 through 5.3.26 which depict the optimal domains at UF feed pressures of 100, 150 and 200 kPa and plant capacities of 15.8, 158, and 1580 m$^3$/hr (0.1, 1.0 and 10 MGD). The optimal recoveries for the HFUF system corresponding to the simulations depicted in Figures 5.3.18 through 5.3.26 are presented in Tables 5.3.16 through 5.3.24, respectively. These cost estimates indicate that operating at high UF feed pressure can lead to improvements in the cost effectiveness of the hybrid HFUF and SWNF configuration. This effect is true at all plant capacities, and becomes increasingly prominent at large capacities. Increases in the UF feed pump capital and energy costs are more than offset by savings in other components comprising the UF non-membrane capital cost, due to improvements in permeate flux with increased feed pressure.
As the capacity decreases from 158 m$^3$/hr (1 MGD) to 15.8 m$^3$/hr (0.1 MGD), the HFNF configuration becomes still more optimal compared to the SWNF configuration (Figures 5.3.18 through 5.3.20). At a HFUF feed pressure of 100 kPa, the SWNF configuration is non-optimal over the entire domain of raw water quality considered (Figure 5.3.18). However, the hybrid system begins to demonstrate regions of optimality at higher HFUF pressures. At a 15.8 m$^3$/hr (0.1 MGD) design capacity, the non-membrane costs comprised about 76 to 89 percent of the total costs (depending on the raw water quality and membrane system), making the single stage HFNF with lower non-membrane capital costs the more cost efficient process over most of the raw water quality domain.

As the capacity increases from 158 m$^3$/hr (1 MGD) to 1580 m$^3$/hr (10 MGD), economies of scale are prominently realized in the non-membrane capital costs, which now comprise only about 29 to 50 percent of the total costs (depending on the raw water quality and membrane system). Hence, at large capacities the SWNF treatment chain with lower membrane costs compared to the HFNF configuration becomes significantly cost competitive (Figures 5.3.24 through 5.3.26). The cost advantages of the SW membranes are demonstrated significantly at large plant capacities, particularly at higher HFUF feed pressures (Figures 5.3.25 and 5.3.26).
Figure 5.3.18. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 15.8 m³/hr (0.1 MGD) and HFUF feed pressure of 100 kPa.

Table 5.3.16. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 15.8 m³/hr (0.1 MGD) and HFUF feed pressure of 100 kPa.

<table>
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Figure 5.3.19. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 15.8 m$^3$/hr (0.1 MGD) and HFUF feed pressure of 150 kPa.

Table 5.3.17. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 15.8 m$^3$/hr (0.1 MGD) and HFUF feed pressure of 150 kPa

<table>
<thead>
<tr>
<th>Log Particle Diameter (µm)</th>
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Figure 5.3.20. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 15.8 m³/hr (0.1 MGD) and HFUF feed pressure of 200 kPa.

Table 5.3.18. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 15.8 m³/hr (0.1 MGD) and HFUF feed pressure of 200 kPa.

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<th>100</th>
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Figure 5.3.21. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 158 m³/hr (1 MGD) and HFUF feed pressure of 100 kPa.

Table 5.3.19. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 158 m³/hr (1 MGD) and HFUF feed pressure of 100 kPa

<table>
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<tr>
<th>Log Particle Diameter (µm)</th>
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Figure 5.3.22. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 158 m³/hr (1 MGD) and HFUF feed pressure of 150 kPa.

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<th>Log Particle Diameter (µm)</th>
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Table 5.3.20. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 158 m³/hr (1 MGD) and HFUF feed pressure of 150 kPa.
Figure 5.3.23. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 158 m$^3$/hr (1 MGD) and HFUF feed pressure of 200 kPa.

Table 5.3.21. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 158 m$^3$/hr (1 MGD) and HFUF feed pressure of 200 kPa

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Figure 5.3.24. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 1580 m³/hr (10 MGD) and HFUF feed pressure of 100 kPa.

Table 5.3.22. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 1580 m³/hr (10 MGD) and HFUF feed pressure of 100 kPa

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Figure 5.3.25. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 1580 m³/hr (10 MGD) and HFUF feed pressure of 150 kPa.

Table 5.3.23. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 1580 m³/hr (10 MGD) and HFUF feed pressure of 150 kPa

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Figure 5.3.26. Optimal domains for single (HF) and hybrid (SW) nanofiltration systems at a plant capacity of 1580 m³/hr (10 MGD) and HFUF feed pressure of 200 kPa.

Table 5.3.24. Optimal recoveries (%) for various particles and concentrations at a plant capacity of 1580 m³/hr (10 MGD) and HFUF feed pressure of 200 kPa

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CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1. Summary

A generalized model has been formulated to predict the time-dependent permeate flux by extending previous models to include the particle transport mechanisms of Brownian diffusion, shear-induced diffusion, inertial lift and concentrated flowing layers. This extension makes the theory applicable to a broad range of contaminant sizes ranging from macromolecules, colloidal and fine particles, to large particles. The extended model is used to numerically predict and study the effects of raw water quality (in terms of particle size and concentration) and design and operating parameters on the permeate flux behavior and cake growth in membrane filters.

A new model for estimating the capital costs of membrane plants has been developed which incorporates individual cost correlations for several different categories of manufactured equipment employed in membrane systems. Instead of lumping together all non-membrane costs, as in often done in the literature, and hence losing insight on the economies of scale of individual components, different cost correlations for major categories of capital cost equipment are developed. This is deemed to be a more suitable approach for estimating and comparing costs of plants with different design and operating specifications, and one that can be appropriately used in an optimization study. The capital cost model is coupled with a previously developed model for
estimating the operating costs (Pickering and Wiesner, 1993) to predict and analyze treatment costs for various raw water qualities, membrane geometries and operating conditions.

The extended model for estimating permeate flux and the cost model are coupled to compose and solve three optimization problems associated with membrane systems, focusing on minimizing the total treatment costs. Optimization problems are formulated and solved to investigate (a) optimal membrane design and system operation, (b) optimal frequency of backflushing, and (c) optimal selection of hybrid filtration configurations. The optimizations are performed over a range of particle sizes and concentrations depicting different raw water qualities. The multi-dimensional problem (a) is solved using an optimization algorithm while optimal solutions for the remaining two one-dimensional problems are obtained through numerical simulations.

6.2. Conclusions

6.2.1. Mass Transfer Description and Permeate Flux Behavior

For the typical system simulated in this work, the combined theory predicts an unfavorable particle size, on the order of $10^{-1}$ μm, where net back transport due to Brownian diffusion, shear-induced diffusion and inertial lift is at a minimum, and results in highest particle accumulation on the membrane surface. This, coupled with the decrease in cake permeability with smaller particle size, and cake growth over time, implies minimum permeate fluxes in the size range of $10^{-2}$ μm - $10^{-1}$ μm, depending on the operating time. These results support the indirect experimental observations of a minimum in
permeate flux (Fane, 1984; Lahoussine-Turcaud et al., 1990) as well as recent direct experimental confirmation of the existence of a minimum in back-transport (Chellam and Wiesner, 1996). Cakes formed from small particles are predicted to attain their steady-state thickness earliest, followed by large particles, and finally the particles in the intermediate size range.

Particle size, coupled with the effects of curved cakes in cylindrical geometries, determines whether the flat slit or inside-out hollow fiber yields higher permeate flux. For the same cross-flow velocity and diameter/height, the inside-out cylindrical geometry results in a higher shear-rate. The inside-out geometry may therefore be expected to be favorable (in terms of higher permeate flux) for feed waters with small particles which form thin resistive cakes. However, larger particles (>0.1 \( \mu \text{m} \)), which form thick cakes, may result in reduced surface area available for filtration due to curvature effects in inside-out membranes, making the slit or outside-in geometry more favorable for these particles. The outside-in configuration is predicted to be favorable over a larger particle size range as the concentration in the raw water increases.

For the typical ultrafiltration system simulated in this study, variations in transmembrane pressure produced significant changes in both the transient and equilibrated flux for the large particles, and in the transient flux for the intermediate sized particles. The equilibrated flux was not significantly affected in the case of the intermediate sized and small particles, indicating that fine particles (<0.1 \( \mu \text{m} \)) demonstrate mass transfer limited, or pressure independent, behavior. For large particles the kinetics of cake growth coupled with the curvature effects on available filtration area determine the permeate flux profile.
and the \textit{transient} flux can improve with increased fiber radius. This behavior is not observed in the case of small particles, since the cakes formed are very thin and hence the reduction in available filtration area is not significant. The time to equilibration decreases with a higher cross-flow velocity or a smaller hollow fiber radius, but the change is seen to be more significant in the latter case. Small particles demonstrate the same behavior in permeate flux at the same shear rate, regardless of the individual values of the cross-flow velocity and the membrane radius. However, the intermediate and large size particles show variation in the permeate flux even at the same shear rate, depending on the value of crossflow velocity and membrane radius. For these particles, at the same shear rate, a narrower fiber with a lower cross-flow velocity results in a higher \textit{steady-state} permeate flux, but a lower \textit{transient} permeate flux.

An ideal comparison of the model formulated in this study with would require data from controlled experiments conducted with monodisperse particle suspensions. However, data on permeate flux decline from a previous study (Chellam, 1995) were available only for polydisperse suspensions. Nevertheless, for the purposes of comparing model results with the available data and gaining insight on the rheology of cakes formed during filtration of polydisperse suspensions, comparisons were made with the data available. Application of the model to polydisperse suspensions requires that estimates of the cake solidosity be available, preferably as a function of operating parameters such as transmembrane pressure and shear-rate. In the absence of knowledge of the cake solidosity, a parameter estimation technique may be adopted to determine the optimal solidosity. Comparisons with available data on permeate flux show that assuming a cake solidosity of $\phi_c = 0.58$, as is often
approximated for suspensions of monodisperse spherical particles, might be too low for polydisperse suspensions. An optimization was performed to determine the value of the specific resistance (and hence the solidosity) of the cake layer that minimized the sum of squares of the residual errors between the observed and predicted permeate flux. Solidosities ranging from 0.70 to 0.77 were estimated, implying that the suspension formed very compact cakes which varied in structure under different operating conditions. The solidosity estimates from these permeate flux data increased with both shear-rate and transmembrane pressure. However, the effect of transmembrane pressure on the cake solidosity was predicted to be much more significant than that of the shear-rate.

6.2.2. Cost Estimation and Sensitivity

Estimates of treatment costs as a function of particle size and concentration, for common ranges of system parameters characterizing typical UF/MF facilities, indicate that costs are significantly affected by the permeate flux. In fact, total costs are essentially a mirror image of the permeate flux behavior with the raw water quality. Very small and large particles which are characterized by high net back-transport and/or relatively porous cakes demonstrate high permeate fluxes, which translate into low treatment costs. Unfavorable particles in the intermediate size range yield low permeate fluxes and are hence characterized by high total treatment costs.

Cost sensitivity analyses were performed with respect to key design and operating variables. Treatment costs demonstrate a significant decrease with plant capacity. This is attributable to the economies of scale exhibited by the
non-membrane capital costs. Hollow fiber radius and length have relatively little impact on treatment costs when suspensions of particles that allow large permeate fluxes are treated. However, when permeate flux is limited due to concentration polarization and cake growth, costs are more sensitive to membrane design. Costs for the intermediate and large particles decrease at higher transmembrane pressures due to improvements in the permeate flux. The increased energy costs associated with pumping at a higher transmembrane pressure are more than offset by savings in the capital costs due to a reduction in the membrane area and the associated equipment. Smaller particles which are affected by mass transport limitations show a permeate flux behavior that is essentially independent of transmembrane pressure, specially at high values of the latter. For these particles operating at high pressures leads to higher operating costs, while the savings in capital costs are negligible due to the relative insensitivity of permeate flux to pressure. Therefore, total cost associated with these particles may increase with transmembrane pressure.

Total costs are relatively insensitive to the cross-flow velocity and system recovery for large particles characterized with high permeate fluxes. For these particles, incremental operating costs were found to be higher than savings in capital costs over the range of the velocities analyzed. Intermediate and small sized particles show a more significant sensitivity to the crossflow velocity and system recovery.
6.2.3. Design and Optimization Considerations

Optimizations of membrane design and operation performed in this work indicate that the optimal design is largely influenced by the fact that energy costs in typical low-pressure membrane filtration systems are dominated by capital costs. Hence, the total treatment costs tend to be optimal at values of decision variables where the permeate fluxes are maximized, within the constraints prescribed by the system. This is largely true for the fiber radius and length, both of which demonstrate an optimal value at the prescribed lower bound. Small deviations from this behavior occur for particles characterized by very high permeate fluxes, mostly in order to accommodate the constraints applied.

Trans-membrane pressure is predicted to become optimal at the upper bound for particles which demonstrate significant sensitivity between the permeate flux and pressure. It should be noted that higher pressures may also make backflushing less efficient and therefore foul the membrane more. For particles in the intermediate and small size ranges, low sensitivity to the trans-membrane pressure or the pressure independent behavior of permeate fluxes results in intermediate pressures where the total treatment costs are optimal. For these particles, operating at higher trans-membrane pressures is predicted to cause an increase in energy costs which are not compensated enough by the small savings in capital costs.

Optimal values of the cross-flow velocity are largely determined by the interplay between capital and operating costs. Particles with low permeate fluxes demonstrate higher optimal velocities compared to the favorable particles.
due to the fact that improvements in permeate flux with crossflow velocity are more critical for the former particles. Relatively lower optimal recoveries are predicted to be optimal for the unfavorable particles characterized by small permeate fluxes. Operating at a high system recovery causes an increase in the bulk suspension concentration which adversely affects the permeate flux of these particles, leading to higher capital costs. However, for favorable particles, which demonstrate high permeate fluxes regardless of the concentration, operating at higher recoveries reduces the operating costs of pumping the feed, and hence optimal recoveries are predicted at or close to the upper bound. These trends in optimal recovery with particle size are observed for both the feed and bleed and the semi-batch operation.

The kinetics of permeate flux decline as a function of raw water quality and the inherent relationship between the system recovery and the operating time in the intermittently operated semi-batch operational mode determine an optimal frequency of backflushing. High optimal recoveries and low backflushing frequencies are predicted for favorable particles which demonstrate a small or insignificant decline in permeate flux over time. However, for unfavorable particles the permeate flux declines significantly with time and thus arresting this decline by backflushing becomes an important consideration. Therefore, treatment costs for the unfavorable particle sizes are predicted to be minimized at intermediate values of system recoveries determined from the kinetics of the permeate flux decline. Optimal recoveries are predicted to increase with the plant capacity. This can be explained by the fact that the investment and operating costs of pumping the feed decrease with an increased recovery. These costs increase in significance, compared to the total treatment costs, with
increasing plant capacities due to economies of scale exhibited by the non-membrane equipment. Hence, as the plant capacity increases, optimal recoveries may increase in order to keep the pumping costs down.

Optimal selection of a single hollow fiber nanofiltration (HFNF) system versus a hybrid system composed of hollow fiber ultrafiltration (HFUF) and spiral wound nanofiltration (SWNF) is theoretically explored in terms of total treatment costs. At small plant capacities, the hybrid configuration is predicted to be non-optimal compared to the HFNF configuration, over most raw water quality conditions. This is attributable to the high non-membrane capital costs of the combined HFUF and SWNF treatment chain at small design capacities. At higher plant capacities economies of scale are realized in the non-membrane capital costs, making the contribution of the membrane costs to the total treatment costs more significant and comparable to the contribution of the non-membrane costs. Therefore, the region where SWNF configuration is optimal over the raw water quality domain expands considerably. Hence, membrane costs are predicted to play a significant role in determining the optimal configuration for large plants. For these plants, the cost advantages of the cheaper SW membranes are seen to come into play. Operating the HFUF system at high feed pressures also lowers the capital costs of the SWNF treatment chain and leads to larger optimal domains with respect to particle size and concentration. This effect is more pronounced at large plant capacities. However, it should be noted that higher pressures may also make backflushing less efficient and therefore foul the membrane more.
6.3. Suggestions for Further Research

A complete theoretical description of a complex system characterized by numerous design and operating variables, feed suspension characteristics, economic parameters, and affected by mass transport, boundary layer growth, fouling, effects of hydrodynamic cleaning, amongst other factors, inevitably requires simplifying assumptions. Suggestions for additional work that may be performed to reduce some of the key simplifications inherent in the current study, and/or extend the theoretical formulations and analyses made, include the following.

1. The current formulation for description of permeate flux does not include long term fouling due to pore plugging and adsorption. Considerations of these contributions to permeate flux decline should be incorporated.

2. The permeate flux model is applicable to inside-out cylindrical configurations and flat geometries. Comparisons with the outside-in geometry are made through indirect interpretations. The current formulation can be extended to model the mass transport and permeate flux in the outside-in cylindrical configuration.

3. Although hollow fiber membranes are typically operated in laminar flow, an extension and generalization of the current work to other membrane geometries may motivate including considerations of turbulent flow.

4. The present analysis includes effects of rejection on the permeate flux only in an indirect fashion. Future work should include aspects of incomplete rejection in the mass transport equations.
5. The current description of permeate flux is ideally applicable to monodisperse suspensions. Application of the model formulated in this work to available data on permeate flux decline for polydisperse suspensions suggests that future work should try to extend the cake resistance sub-models to include more fundamental relationships between cake morphology, suspension characteristics, and operating conditions.

6. Cost correlations are presented in this work as continuous functions. However, more specific correlations may be formulated that include the discreteness of size and associated cost of manufactured equipment.

7. Optimization problems have been formulated in this work incorporating continuous decision variables. Future analysis and extensions may include discrete decision variables to provide knowledge of the optimal number of modules, banks, stages, etc., that result in a more efficient system design.
REFERENCES


APPENDIX A

MODIFICATIONS TO GOVERNING PDE TO ACCOUNT FOR VARIABLE TRANSMEMBRANE PRESSURE AND ONE-WALLED GEOMETRY

The modifications to the governing PDE for cake growth (section 2.4), to account for operation under a variable transmembrane pressure and a one-walled geometry characterizing the experimental data used for comparison purposes (section 5.1.6), are presented in this appendix. Let \( \Delta \bar{P}(t) = \Delta P(t)/\Delta P(0) \) represent the dimensionless instantaneous transmembrane pressure. Then the permeate flux, permeation velocity and the cake thickness are related by:

\[
\dot{J} = \frac{J}{J_m} = \frac{v_w}{J_m} = \frac{\Delta \bar{P}}{\left(1 + \beta \delta \right)}
\]  

(A-1)

The shear rate increases in a one-walled channel (for constant feed flow rate) as:

\[
\tau_w = \tau_{wo} \left( \frac{1}{1 - \frac{\delta}{2}} \right)^2
\]  

(A-2)

Using these relationships the functions \( g, h \) and \( f \) to be employed in Eq. 2.2.24 are given as:
One-walled flat slit with variable transmembrane pressure \((n = 2)\)

\[
g = \frac{l_{11}}{\Delta \hat{P} Pe_s} \left[ \frac{\beta}{\left(1 - \frac{\hat{\delta}}{2}\right)^n} + \frac{n(1+\beta \hat{\delta})}{\left(1 - \frac{\hat{\delta}}{2}\right)^{n+1}} \right] + \frac{\beta l_{12}}{\Delta \hat{P} Pe_b} + (\phi_c - \phi_b) \quad (A-3a)
\]

\[
h = \frac{2\beta(1+\beta \hat{\delta})}{\Delta \hat{P}^2} \left[ \frac{l_{21}}{Pe_s^2 \left(1 - \frac{\hat{\delta}}{2}\right)^{3n}} + \frac{(l_{22} + l_{23})}{Pe_b Pe_s \left(1 - \frac{\hat{\delta}}{2}\right)^{2n}} + \frac{l_{24}}{Pe_b^2 \left(1 - \frac{\hat{\delta}}{2}\right)^n} \right] + \frac{(1+\beta \hat{\delta})^2}{\Delta \hat{P}^2} \left[ \frac{3nl_{21}}{2 Pe_s^2 \left(1 - \frac{\hat{\delta}}{2}\right)^{3n+1}} + \frac{n(l_{22} + l_{23})}{Pe_b Pe_s \left(1 - \frac{\hat{\delta}}{2}\right)^{2n+1}} \right] + \frac{nl_{24}}{2 Pe_b^2 \left(1 - \frac{\hat{\delta}}{2}\right)^{n+1}} \quad (A-3b)
\]
\[
f = \phi_b \left[ \frac{\dot{\rho}}{(1 + \beta \hat{\delta})} - \frac{\dot{v}_{lo}}{(1 - \frac{\hat{\delta}}{2})^{2n}} \right]
\]

\[
- \frac{\partial}{\partial t} \left( \frac{1}{\Delta \tilde{P}} \right) \left[ \frac{l_{11} (1 + \beta \hat{\delta})}{P e_s \left(1 - \frac{\hat{\delta}}{2}\right)^n} + \frac{l_{12} (1 + \beta \hat{\delta})}{P e_b} \right]
\]

(A-3c)

The initial and boundary conditions, and the expressions for the critical time and distance are given by Eqs. 2.4.21 through 2.4.23.
APPENDIX B

NOTATION

\( a_{ke} \) kinetic energy coefficient (-)
\( a_p \) particle radius (L)
\( A_{mem} \) membrane area required to produce design flow (L^2)
\( A/F \) uniform series sinking fund factor (-)
\( C \) concentration (ML^-3)
\( c_b \) bulk suspension concentration (ML^-3)
\( c_{kw} \) cost of one kilowatt-hour of electricity ($M^{-1}L^{-2}T^2$)
\( c_o \) feed suspension concentration (ML^-3)
\( c_w \) wall concentration (ML^-3)
\( C_{chemical} \) cost of chemicals per volume of water produced ($L^{-3}$)
\( C_{disposal} \) cost of concentrate disposal per volume of water produced ($L^{-3}$)
\( C_{energy} \) cost of energy consumed per volume of water produced ($L^{-3}$)
\( C_{membrane} \) amortized capital cost of membranes per volume of water produced ($L^{-3}$)
\( C_{mr} \) cost of membrane replacement per volume of water produced ($L^{-3}$)
\( CH_c \) cost of bulk coagulant ($M^{-1}$)
\( CH_d \) coagulant dose (ML^-3)
\( d_p \) particle diameter (L)
\( D \) diffusion coefficient (L^2T^-1)
\( D_b \) Brownian diffusion coefficient (L^2T^-1)
\( D_e \) diameter of membrane element (L)
\( D_s \) shear-induced diffusion coefficient (L^2T^{-1})

\( \hat{D}_s \) dimensionless shear-induced diffusion coefficient (-)

\( DL \) design life of plant (T)

\( E_{bf} \) rate of energy consumption of backflush (ML^2T^{-3})

\( E_f \) rate of energy consumption of feed pump (ML^2T^{-3})

\( E_{ff} \) rate of energy consumption of fastflush (ML^2T^{-3})

\( E_r \) rate of energy consumption of recycle pump (ML^2T^{-3})

\( f \) dimensionless function

\( f_f \) Fanning friction factor (-)

\( g \) dimensionless function

\( G \) lag factor (-)

\( h \) dimensionless function

\( H \) effective slit half-height or hollow fiber radius \((H=H_o - \delta_o)\) (L)

\( H_o \) slit half-height or hollow fiber radius (L)

\( i_c \) cost of capital, as interest rate (%)

\( i_f \) cost of sinking fund, as interest rate (%)

\( l_1 \) integral defined by Eq. 2.4.14

\( l_{11} \) integral defined by Eq. 2.4.16a

\( l_{12} \) integral defined by Eq. 2.4.16b

\( l_2 \) integral defined by Eq. 2.4.15

\( l_{21} \) integral defined by Eq. 2.4.16c

\( l_{22} \) integral defined by Eq. 2.4.16d

\( l_{23} \) integral defined by Eq. 2.4.16e

\( l_{24} \) integral defined by Eq. 2.4.16f

\( J \) permeate flux (LT^{-1})

\( \bar{J} \) time-averaged permeate flux (LT^{-1})
\( J_{bf} \) backflush flux (LT\(^{-1}\))

\( J_m \) permeate flux limited only by membrane in absence of cake layer (LT\(^{-1}\))

\( k \) Boltzmann's constant (ML\(^2\)T\(^{-2}\)/K)

\( K \) mass transfer coefficient (LT\(^{-1}\))

\( L_e \) length of membrane element (L)

\( ML \) expected average membrane life (T)

\( n \) constant (\( n=2 \) for flat geometry, \( n=3 \) for cylindrical geometry) (-)

\( P_{bf} \) backflush pressure (ML\(^{-1}\)T\(^{-2}\))

\( Pe_b \) Peclet number based on Brownian diffusion (-)

\( Pe_s \) Peclet number based on shear-induced diffusion (-)

\( P_f \) feed pressure (ML\(^{-1}\)T\(^{-2}\))

\( P_r \) axial pressure drop across module (ML\(^{-1}\)T\(^{-2}\))

\( Q \) excess particle flux (L\(^2\)T\(^{-1}\))

\( Q_{bf} \) backflush flow rate (L\(^3\)T\(^{-1}\))

\( Q_{cr} \) critical excess particle flux (L\(^2\)T\(^{-1}\))

\( Q_{des} \) plant design capacity (L\(^3\)T\(^{-1}\))

\( Q_f \) feed flow rate (L\(^3\)T\(^{-1}\))

\( Q_{ff} \) fastflush flow rate (L\(^3\)T\(^{-1}\))

\( Q_p \) permeate flow rate (L\(^3\)T\(^{-1}\))

\( Q_r \) recycle flow rate (L\(^3\)T\(^{-1}\))

\( Q_t \) total flow rate (L\(^3\)T\(^{-1}\))

\( Q_w \) waste flow rate (L\(^3\)T\(^{-1}\))

\( R_c \) cake resistance (L\(^{-1}\))

\( R_c^* \) specific cake resistance per unit height (L\(^{-2}\))

\( R_m \) membrane resistance (L\(^{-1}\))
\( R \) \hspace{1cm} \text{recovery} \ (Q_p / Q_f) \ (-)

\( Re \) \hspace{1cm} \text{Reynolds number} \ (-)

\( Sc \) \hspace{1cm} \text{Schmidt number} \ (-)

\( Sh \) \hspace{1cm} \text{Sherwood number} \ (-)

\( t \) \hspace{1cm} \text{time} \ (T)

\( t_{bf} \) \hspace{1cm} \text{backflush duration} \ (T)

\( t_{cr} \) \hspace{1cm} \text{critical time} \ (T)

\( t_{ff} \) \hspace{1cm} \text{fastflush duration} \ (T)

\( t_o \) \hspace{1cm} \text{operating time between two flux enhancement cycles} \ (T)

\( t_{tot} \) \hspace{1cm} \text{total time for one complete operating and flux enhancement cycle} \ (T)

\( T \) \hspace{1cm} \text{absolute temperature} \ (^\circ K)

\( u \) \hspace{1cm} \text{axial velocity} \ (LT^{-1})

\( \bar{U} \) \hspace{1cm} \text{average cross-flow velocity of bulk suspension} \ (LT^{-1})

\( U_{ff} \) \hspace{1cm} \text{fastflush velocity} \ (LT^{-1})

\( \bar{U}_m \) \hspace{1cm} \text{average cross-flow velocity at mid-point of element} \ (LT^{-1})

\( \bar{U}_o \) \hspace{1cm} \text{average cross-flow velocity at channel inlet} \ (LT^{-1})

\( v \) \hspace{1cm} \text{transverse velocity} \ (LT^{-1})

\( v_l \) \hspace{1cm} \text{lift velocity} \ (LT^{-1})

\( v_{lo} \) \hspace{1cm} \text{lift velocity in absence of cake} \ (LT^{-1})

\( v_w \) \hspace{1cm} \text{permeate velocity} \ (LT^{-1})

\( x \) \hspace{1cm} \text{axial coordinate} \ (L)

\( x_{cr} \) \hspace{1cm} \text{critical length} \ (L)

\( y \) \hspace{1cm} \text{transverse coordinate} \ (L)
### Greek Symbols

- $\beta$  
  Dimensionless resistance (-)

- $\kappa$  
  Sieve coefficient (-)

- $\delta$  
  Particle layer thickness (L)

- $\delta_c$  
  Dimensionless cake thickness (-)

- $\Delta P$  
  Transmembrane pressure (ML\(^{-1}\)T\(^{-2}\))

- $\phi$  
  Particle volume fraction (-)

- $\phi_b$  
  Bulk suspension particle volume fraction (-)

- $\phi_c$  
  Maximum packing or solidosity of cake (-)

- $\phi_o$  
  Feed suspension particle volume fraction (-)

- $\phi_w$  
  Particle volume fraction at wall (-)

- $\dot{\gamma}$  
  Shear rate (T\(^{-1}\))

- $\dot{\gamma}_o$  
  Shear rate in absence of cake layer (T\(^{-1}\))

- $\dot{\gamma}_r$  
  Shear rate (T\(^{-1}\))

- $\eta$  
  Relative viscosity ((\(\mu(\phi)/\mu_o\)) (-)

- $\eta_{bf}$  
  Efficiency of backflush pump (%)  

- $\eta_f$  
  Efficiency of feed pump (%)  

- $\eta_r$  
  Efficiency of recycle pump (%)  

- $\lambda$  
  Solute to pore size ratio (-)

- $\mu$  
  Dynamic viscosity (ML\(^{-1}\)T\(^{-1}\))

- $\mu_o$  
  Dynamic viscosity of particle-free fluid (ML\(^{-1}\)T\(^{-1}\))

- $\rho$  
  Fluid density (ML\(^{-3}\))

- $\rho_p$  
  Particle density (ML\(^{-3}\))

- $\nu$  
  Kinematic viscosity (L\(^2\)T\(^{-1}\))
\( \tau_w \)  
wall shear stress \((\text{ML}^{-1}\text{T}^{-2})\)

\( \tau_{wo} \)  
wall shear stress before formation of stagnant cake \((\text{ML}^{-1}\text{T}^{-2})\)

*Superscripts*

\(^\wedge\) dimensionless value